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Visualizing Finite Field of Order p^2 through Matrices

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Visualizing Finite Field of Order p^2 through Matrices

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Abstract- In this article we consider finite field of order $p^2(p \neq 2)$. We provide matrix representation of finite field of order at most $p^2 = 121$. However this article provides a technique to yield matrix field of order p^2 for every prime $p > 2$.

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I. Introduction

There is a finite field of order p^n for every positive prime p and positive integer n. If F is a finite field of order p and $F[x]$ is the polynomial domain defined over F then one can construct a finite field $K = \frac{F[x]}{[f(x)]}$ of order p^n, where $f(x) \in F[x]$ is an irreducible polynomial of degree n and $[f(x)]$ is the ideal generated by $f(x)$. This field K has a subfield isomorphic with the field F. One may refer [1, 2] for further details. This is the usual method of construction of finite fields.

In this article we construct finite matrix fields of order p^2 without adopting the usual method of construction of finite fields. We give finite matrix field of order at most 121. However following the approach given in this article one can construct finite matrix field of order p^2 for each $p \neq 2$.

II. Finite Matrix Field of Order p^2

In [3] one can find different matrix representations of a finite field of order p. Let p be a prime number. Then $Z_p = \{0,1,2,3,4,5...p-1\}$ is a field under addition and multiplication modulo p. One matrix representation of Z_p as given in [3] is $F_p = \left\{ \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} : a \in Z_p \right\}$. Every field of characteristic p contains a copy of a field of order

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p. All the fields given below are finite matrix field of order p^2. One can easily see that all these fields contain F_p.

We use very simple technique to obtain matrix fields of order p^2. It is seen that $F = \left\{ \begin{pmatrix} a & b \\ -b & a \end{pmatrix} : a, b \in R \right\}$ is isomorphic to the field C of complex numbers. Here R stands for the field of real numbers. We notice that if we replace R by Z_p then we obtain a finite matrix field of order p^2. It may be noted that this approach does not work for $p = 2$.

a) Finite Matrix Field Of Order 9

$$M_9 = \left\{ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 2 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 2 & 1 \\ 2 & 2 \end{pmatrix}, \begin{pmatrix} 2 & 2 \\ 1 & 2 \end{pmatrix} \right\}$$

This is a finite field of order 9 under addition and multiplication of matrices modulo 3. It is a field of characteristic 3 and it contains $F_3 = \left\{ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \right\}$.

b) Finite Matrix Field Of Order 25

$$M_{25} = \left\{ \begin{array}{l} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}, \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}, \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 4 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix}, \begin{pmatrix} 2 & 4 \\ 1 & 2 \end{pmatrix}, \begin{pmatrix} 3 & 4 \\ 1 & 3 \end{pmatrix}, \begin{pmatrix} 3 & 1 \\ 4 & 3 \end{pmatrix}, \\ \begin{pmatrix} 4 & 1 \\ 4 & 4 \end{pmatrix}, \begin{pmatrix} 4 & 4 \\ 1 & 4 \end{pmatrix}, \begin{pmatrix} 0 & 4 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 4 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 3 \\ 2 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 3 \\ 2 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 2 \\ 3 & 0 \end{pmatrix}, \begin{pmatrix} 2 & 2 \\ 3 & 2 \end{pmatrix}, \begin{pmatrix} 2 & 3 \\ 2 & 2 \end{pmatrix}, \begin{pmatrix} 3 & 3 \\ 2 & 3 \end{pmatrix}, \\ \begin{pmatrix} 3 & 2 \\ 3 & 3 \end{pmatrix}, \begin{pmatrix} 4 & 2 \\ 3 & 4 \end{pmatrix}, \begin{pmatrix} 4 & 3 \\ 2 & 4 \end{pmatrix} \end{array} \right\}$$

This is a finite field of order 25 under addition and multiplication of matrices modulo 5. One can see that it is a field of characteristic 5 and it contains

$$F_5 = \left\{ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}, \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}, \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} \right\}.$$

c) Finite Matrix Field Of Order 49

$$M_{49} = \left\{ \begin{array}{l} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}, \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}, \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}, \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}, \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}, \begin{pmatrix} 0 & 6 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 6 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 2 \\ 5 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 5 \\ 2 & 0 \end{pmatrix}, \\ \begin{pmatrix} 0 & 4 \\ 3 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 3 \\ 4 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 6 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 6 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 2 \\ 5 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 5 \\ 2 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 4 \\ 3 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 3 \\ 4 & 1 \end{pmatrix}, \begin{pmatrix} 2 & 3 \\ 4 & 2 \end{pmatrix}, \begin{pmatrix} 2 & 4 \\ 3 & 2 \end{pmatrix}, \begin{pmatrix} 2 & 5 \\ 2 & 2 \end{pmatrix}, \\ \begin{pmatrix} 2 & 2 \\ 5 & 2 \end{pmatrix}, \begin{pmatrix} 2 & 6 \\ 1 & 2 \end{pmatrix}, \begin{pmatrix} 2 & 1 \\ 6 & 2 \end{pmatrix}, \begin{pmatrix} 3 & 4 \\ 3 & 3 \end{pmatrix}, \begin{pmatrix} 3 & 3 \\ 4 & 3 \end{pmatrix}, \begin{pmatrix} 3 & 5 \\ 2 & 3 \end{pmatrix}, \begin{pmatrix} 3 & 2 \\ 5 & 3 \end{pmatrix}, \begin{pmatrix} 3 & 1 \\ 6 & 3 \end{pmatrix}, \begin{pmatrix} 3 & 6 \\ 1 & 3 \end{pmatrix}, \begin{pmatrix} 4 & 4 \\ 3 & 4 \end{pmatrix}, \begin{pmatrix} 4 & 3 \\ 4 & 4 \end{pmatrix}, \\ \begin{pmatrix} 4 & 5 \\ 2 & 4 \end{pmatrix}, \begin{pmatrix} 4 & 2 \\ 5 & 4 \end{pmatrix}, \begin{pmatrix} 4 & 1 \\ 6 & 4 \end{pmatrix}, \begin{pmatrix} 4 & 6 \\ 1 & 4 \end{pmatrix}, \begin{pmatrix} 5 & 4 \\ 3 & 5 \end{pmatrix}, \begin{pmatrix} 5 & 3 \\ 4 & 5 \end{pmatrix}, \begin{pmatrix} 5 & 5 \\ 2 & 5 \end{pmatrix}, \begin{pmatrix} 5 & 2 \\ 5 & 5 \end{pmatrix}, \begin{pmatrix} 5 & 6 \\ 1 & 5 \end{pmatrix}, \begin{pmatrix} 5 & 1 \\ 6 & 5 \end{pmatrix}, \begin{pmatrix} 6 & 4 \\ 3 & 6 \end{pmatrix}, \\ \begin{pmatrix} 6 & 3 \\ 4 & 6 \end{pmatrix}, \begin{pmatrix} 6 & 5 \\ 2 & 6 \end{pmatrix}, \begin{pmatrix} 6 & 2 \\ 5 & 6 \end{pmatrix}, \begin{pmatrix} 6 & 1 \\ 6 & 6 \end{pmatrix}, \begin{pmatrix} 6 & 6 \\ 1 & 6 \end{pmatrix} \end{array} \right\}$$

This is a finite field of order 49 under addition and multiplication of matrices modulo 7. One can see that it is a field of characteristic 7 and it contains

$$F_7=\left\{\begin{pmatrix}0&0\\0&0\end{pmatrix},\begin{pmatrix}1&0\\0&1\end{pmatrix},\begin{pmatrix}2&0\\0&2\end{pmatrix},\begin{pmatrix}3&0\\0&3\end{pmatrix},\begin{pmatrix}4&0\\0&4\end{pmatrix},\begin{pmatrix}5&0\\0&5\end{pmatrix},\begin{pmatrix}6&0\\0&6\end{pmatrix}\right\}.$$

Notes

d) Finite Matrix Field Of Order 121

$$M_{121}=\left\{\begin{array}{l}
\begin{pmatrix}0&0\\0&0\end{pmatrix},\begin{pmatrix}1&0\\0&1\end{pmatrix},\begin{pmatrix}2&0\\0&2\end{pmatrix},\begin{pmatrix}3&0\\0&3\end{pmatrix},\begin{pmatrix}4&0\\0&4\end{pmatrix},\begin{pmatrix}5&0\\0&5\end{pmatrix},\begin{pmatrix}6&0\\0&6\end{pmatrix},\begin{pmatrix}7&0\\0&7\end{pmatrix},\begin{pmatrix}8&0\\0&8\end{pmatrix},\begin{pmatrix}9&0\\0&9\end{pmatrix},\begin{pmatrix}10&0\\0&10\end{pmatrix},\\
\begin{pmatrix}0&10\\1&0\end{pmatrix},\begin{pmatrix}0&1\\10&0\end{pmatrix},\begin{pmatrix}0&9\\2&0\end{pmatrix},\begin{pmatrix}0&2\\9&0\end{pmatrix},\begin{pmatrix}0&8\\3&0\end{pmatrix},\begin{pmatrix}0&3\\8&0\end{pmatrix},\begin{pmatrix}0&4\\7&0\end{pmatrix},\begin{pmatrix}0&7\\4&0\end{pmatrix},\begin{pmatrix}0&6\\5&0\end{pmatrix},\begin{pmatrix}0&5\\6&0\end{pmatrix},\begin{pmatrix}1&10\\1&1\end{pmatrix},\\
\begin{pmatrix}1&1\\10&1\end{pmatrix},\begin{pmatrix}1&2\\9&1\end{pmatrix},\begin{pmatrix}1&9\\2&1\end{pmatrix},\begin{pmatrix}1&8\\3&1\end{pmatrix},\begin{pmatrix}1&3\\8&1\end{pmatrix},\begin{pmatrix}1&4\\7&1\end{pmatrix},\begin{pmatrix}1&7\\4&1\end{pmatrix},\begin{pmatrix}1&6\\5&1\end{pmatrix},\begin{pmatrix}1&5\\6&1\end{pmatrix},\begin{pmatrix}2&10\\1&2\end{pmatrix},\begin{pmatrix}2&1\\10&2\end{pmatrix},\\
\begin{pmatrix}2&9\\2&2\end{pmatrix}\begin{pmatrix}2&2\\9&2\end{pmatrix},\begin{pmatrix}2&8\\3&2\end{pmatrix},\begin{pmatrix}2&3\\8&2\end{pmatrix},\begin{pmatrix}2&7\\4&2\end{pmatrix},\begin{pmatrix}2&4\\7&2\end{pmatrix},\begin{pmatrix}2&6\\5&2\end{pmatrix},\begin{pmatrix}2&5\\6&2\end{pmatrix},\begin{pmatrix}3&10\\1&3\end{pmatrix},\begin{pmatrix}3&1\\10&3\end{pmatrix},\begin{pmatrix}3&9\\2&3\end{pmatrix},\\
\begin{pmatrix}3&2\\9&3\end{pmatrix},\begin{pmatrix}3&8\\3&3\end{pmatrix},\begin{pmatrix}3&3\\8&3\end{pmatrix},\begin{pmatrix}3&7\\4&3\end{pmatrix},\begin{pmatrix}3&4\\7&3\end{pmatrix},\begin{pmatrix}3&6\\5&3\end{pmatrix},\begin{pmatrix}3&5\\6&3\end{pmatrix},\begin{pmatrix}4&10\\1&4\end{pmatrix},\begin{pmatrix}4&1\\10&4\end{pmatrix},\begin{pmatrix}4&2\\9&4\end{pmatrix},\begin{pmatrix}4&9\\2&4\end{pmatrix},\\
\begin{pmatrix}4&8\\3&4\end{pmatrix},\begin{pmatrix}4&3\\8&4\end{pmatrix},\begin{pmatrix}4&7\\4&4\end{pmatrix},\begin{pmatrix}4&4\\7&4\end{pmatrix},\begin{pmatrix}4&6\\5&4\end{pmatrix},\begin{pmatrix}4&5\\6&4\end{pmatrix},\begin{pmatrix}5&10\\1&5\end{pmatrix},\begin{pmatrix}5&1\\10&5\end{pmatrix},\begin{pmatrix}5&9\\2&5\end{pmatrix},\begin{pmatrix}5&2\\9&5\end{pmatrix},\begin{pmatrix}5&3\\8&5\end{pmatrix},\\
\begin{pmatrix}5&8\\3&5\end{pmatrix},\begin{pmatrix}5&7\\4&5\end{pmatrix},\begin{pmatrix}5&4\\7&5\end{pmatrix},\begin{pmatrix}5&6\\5&5\end{pmatrix},\begin{pmatrix}5&5\\6&5\end{pmatrix},\begin{pmatrix}6&10\\1&6\end{pmatrix},\begin{pmatrix}6&1\\10&6\end{pmatrix},\begin{pmatrix}6&9\\2&6\end{pmatrix},\begin{pmatrix}6&2\\9&6\end{pmatrix},\begin{pmatrix}6&8\\3&6\end{pmatrix},\begin{pmatrix}6&3\\8&6\end{pmatrix},\\
\begin{pmatrix}6&7\\4&6\end{pmatrix},\begin{pmatrix}6&4\\7&6\end{pmatrix},\begin{pmatrix}6&6\\5&6\end{pmatrix},\begin{pmatrix}6&5\\6&6\end{pmatrix},\begin{pmatrix}7&10\\1&7\end{pmatrix},\begin{pmatrix}7&1\\10&7\end{pmatrix},\begin{pmatrix}7&9\\2&7\end{pmatrix},\begin{pmatrix}7&2\\9&7\end{pmatrix},\begin{pmatrix}7&8\\3&7\end{pmatrix},\begin{pmatrix}7&3\\8&7\end{pmatrix},\begin{pmatrix}7&7\\4&7\end{pmatrix},\\
\begin{pmatrix}7&4\\7&7\end{pmatrix},\begin{pmatrix}7&6\\5&7\end{pmatrix},\begin{pmatrix}7&5\\6&7\end{pmatrix},\begin{pmatrix}8&10\\1&8\end{pmatrix},\begin{pmatrix}8&1\\10&8\end{pmatrix},\begin{pmatrix}8&9\\2&8\end{pmatrix},\begin{pmatrix}8&2\\9&8\end{pmatrix},\begin{pmatrix}8&8\\3&8\end{pmatrix},\begin{pmatrix}8&3\\8&8\end{pmatrix},\begin{pmatrix}8&7\\4&8\end{pmatrix},\begin{pmatrix}8&4\\7&8\end{pmatrix},\\
\begin{pmatrix}8&6\\5&8\end{pmatrix},\begin{pmatrix}8&5\\6&8\end{pmatrix},\begin{pmatrix}9&10\\1&9\end{pmatrix},\begin{pmatrix}9&1\\10&9\end{pmatrix},\begin{pmatrix}9&9\\2&9\end{pmatrix},\begin{pmatrix}9&2\\9&9\end{pmatrix},\begin{pmatrix}9&8\\3&9\end{pmatrix},\begin{pmatrix}9&3\\8&9\end{pmatrix},\begin{pmatrix}9&7\\4&9\end{pmatrix},\begin{pmatrix}9&4\\7&9\end{pmatrix},\begin{pmatrix}9&6\\5&9\end{pmatrix},\\
\begin{pmatrix}9&5\\6&9\end{pmatrix},\begin{pmatrix}10&10\\1&10\end{pmatrix},\begin{pmatrix}10&1\\10&10\end{pmatrix},\begin{pmatrix}10&9\\2&10\end{pmatrix},\begin{pmatrix}10&2\\9&10\end{pmatrix},\begin{pmatrix}10&8\\3&10\end{pmatrix},\begin{pmatrix}10&3\\8&10\end{pmatrix},\begin{pmatrix}10&4\\7&10\end{pmatrix},\begin{pmatrix}10&7\\4&10\end{pmatrix},\\
\begin{pmatrix}10&6\\5&10\end{pmatrix},\begin{pmatrix}10&5\\6&10\end{pmatrix}
\end{array}\right\}$$

This is a finite field of order 121 under addition and multiplication of matrices modulo 11. It has characteristic 11 and it contains

$$F_{11}=\left\{\begin{pmatrix}0&0\\0&0\end{pmatrix},\begin{pmatrix}1&0\\0&1\end{pmatrix},\begin{pmatrix}2&0\\0&2\end{pmatrix},\begin{pmatrix}3&0\\0&3\end{pmatrix},\begin{pmatrix}4&0\\0&4\end{pmatrix},\begin{pmatrix}5&0\\0&5\end{pmatrix},\begin{pmatrix}6&0\\0&6\end{pmatrix},\begin{pmatrix}7&0\\0&7\end{pmatrix},\begin{pmatrix}8&0\\0&8\end{pmatrix},\begin{pmatrix}9&0\\0&9\end{pmatrix},\begin{pmatrix}10&0\\0&10\end{pmatrix}\right\}$$

Following same technique one can find finite matrix field of order p^2 for $p>11$.

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Notes