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Ducting in Extended Plates of Variable Thickness

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Abstract- The paper deals with the spread of its own waves on the visco elastic plate with variable thickness. Basic relations of the classical theory of plates of variable thickness obtained on the basis of the principles of virtual displacements. The spectral problem, which is not self ad joint. Built for the task biorthogonality conditions, based on the Lagrange formula. The numerical solution of the spectral tasks performed on the computer software system based on the method of orthogonal shooting S.K. Godunov in combination with the method of Muller.

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1. I. P. Bestuzheva, V.N. Durov. On the propagation of edge waves in elastic media. - Akust. Journal. 1981, 27, №4, p. 487-490.

Ducting in Extended Plates of Variable Thickness

Ismail Ibrahimovich Safarov [α], Maqsud Sharipovich Akhmedov [σ] & Zafar Ihterovich Boltaev [ρ]

Annotation- The paper deals with the spread of its own waves on the visco elastic plate with variable thickness. Basic relations of the classical theory of plates of variable thickness obtained on the basis of the principles of virtual displacements. The spectral problem, which is not self ad joint. Built for the task biorthogonality conditions, based on the Lagrange formula. The numerical solution of the spectral tasks performed on the computer software system based on the method of orthogonal shooting S.K. Godunov in combination with the method of Muller.

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I. Introduction

Known [1,2,3] that normal wave deformable layer (Lamb wave) is not orthogonal thickness, i.e. the integral of the scalar product of vectors of displacements of two different waves, considered as a function of position perpendicular to the surface layer is not zero. They are also not orthogonal conjugate wave, which is obtained from a consideration of the dual problem. This introduces additional difficulties in solving practical problems. This article is based conjugate spectral biorthogonality objectives and conditions for the problem.

a) The mathematical formulation of the problem

We consider the visco elastic waveguide as an infinite axial x_1 variable thickness (Figure 1).

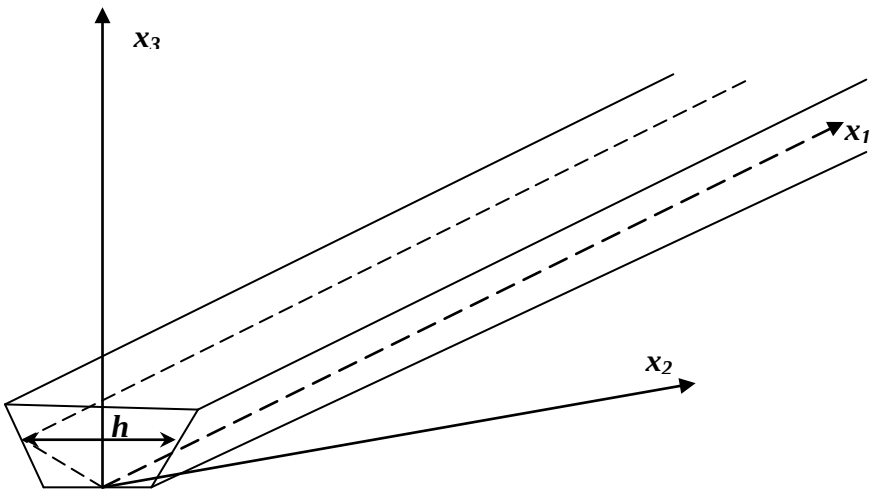

Figure 1 : Design scheme: plates of variable thickness

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Basic relations of the classical theory of plates of variable thickness can be obtained on the basis of the principles of virtual displacements. The variation equation problem visco elasticity theory in three-dimensional statement has the form

$$\iiint_{V}(\sigma_{ij}\delta\varepsilon_{ij}+\rho u_i\delta u_{i\ i})dx_3dx_2dx_1=0 \qquad (i=1,2,3;\ \ j=1,2,3) \tag{1}$$

where ρ- material density; u_i – displacement components; σ_{ij} and ε_{ij} – components of the stress tensor and strain; h- plate thickness; V – the volume occupied by the body. In accordance with the hypotheses of Kirchhoff - Love

$$\sigma_{12}=\sigma_{23}=\sigma_{33}=0, \qquad u_i=-x_3\frac{\partial w}{\partial x_i},\ w(x_3,t)=w. \tag{2}$$

Neglecting in (1) the members of which take into account the inertia of rotation normal to the middle plane, will have the following variation equation:

$$\int_S ds\int_{-\frac{h}{2}}^{\frac{h}{2}}(\sigma_{11}\delta\varepsilon_{11}+2\tau_{12}\delta\varepsilon_{12}+\tau_{22}\delta\varepsilon_{22})dx_3+\int_S ds\int_{-\frac{h}{2}}^{\frac{h}{2}}\rho\frac{\partial^2 w}{\partial t^2}\delta w dx_3=0 \tag{3}$$

Based on the geometric relationships and relations of the generalized Hooke's law, taking into account the kinematic hypotheses (2), the expressions for the components of the strain and stress tensor has the form

$$\varepsilon_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)-x_3\frac{\partial^2 w}{\partial x_i\partial x_j}, \qquad i,j=1,2;$$

$$\sigma_{11}=\frac{\tilde{E}}{1-\nu}(\varepsilon_{11}+\nu\varepsilon_{22}); \quad \sigma_{22}=\frac{\tilde{E}}{1-\nu}(\varepsilon_{22}+\nu\varepsilon_{11}); \tag{4}$$

$$\sigma_{12}=\frac{\tilde{E}}{1+\nu}\varepsilon_{12},$$

$$\tilde{E}_n\varphi(t)=E_{0n}\left[\varphi(t)-\int_0^t R_{En}(t-\tau)\varphi(t)d\tau\right], \tag{5,a}$$

where $\varphi(t)$– arbitrary function of time; ν – Poisson's ratio; $R_{En}(t-\tau)$– the core of relaxation; E_{01} – instantaneous modulus of elasticity; We accept the integral terms in (5, a) small, then the function $\varphi(t)=\psi(t)e^{-i\omega_R t}$, where $\psi(t)$- slowly varying function of time, ω_R - real constant. The [7], we replace of (5,a) approximate species

$$\overline{E}_n\varphi=E_{0j}\left[1-\Gamma_j^C(\omega_R)-i\Gamma_j^S(\omega_R)\right]\varphi, \tag{5,б}$$

where $\Gamma_n{}^C(\omega_R)=\int_0^\infty R_{En}(\tau)\cos\omega_R\tau\,d\tau$, $\Gamma_n{}^S(\omega_R)=\int_0^\infty R_{En}(\tau)\sin\omega_R\tau\,d\tau$, respectively, cosine and sine Fourier transforms relaxation kernel material. As an example, the visco elastic material take three parametric relaxation nucleus $R_{En}(t)=A_n e^{-\beta_n t}/t^{1-\alpha_n}$. Here

Ref
7. M. A. Tangles. Creep and relaksatsiya. Moscow. High school, 1976, 273 p.

A_n , α_n , β_n - parameters relaxation nucleus. On the effect of the function $R_{En}(t-\tau)$ superimposed usual requirements inerrability, continuity (except $t=\tau$), signs - certainty and monotony:

$$R_{En}\rangle 0\ ,\ \frac{dR_{En}}{dt}\le 0,\ 0\langle \int_0^{\infty} R_{En}(t)dt\langle 1.$$

Notes

Introducing the notation for points

$$M_{11}=\overline{D}\left(\frac{\partial^2 w}{\partial x_1^2}+v\frac{\partial^2 w}{\partial x_2^2}\right);M_{22}=\overline{D}\left(\frac{\partial^2 w}{\partial x_2^2}+v\frac{\partial^2 w}{\partial x_1^2}\right);$$

$$M_{12}=\overline{D}(1-v)\frac{\partial^2 w}{\partial x_1\,\partial x_2}\qquad \overline{D}=\frac{\overline{E}h^3}{12(1-\nu^2)}.$$

When $R_{En}(t-\tau)$=0, then $D=\frac{Eh^3}{12(1-\nu^2)}$. Here E is the modulus of elasticity.

Integrating (3) in the strip thickness leads to the following form

$$\int_5\left(M_{11}\frac{\partial^2\delta w}{\partial x_1^2}+2M_{12}\frac{\partial^2\delta w}{\partial x_1\partial x_2}+M_{22}\frac{\partial^2\delta w}{\partial x_2^2}\right)ds-\int_s\rho h\frac{\partial^2 w}{\partial t^2}\delta w ds=0. \tag{6}$$

Integrating twice by parts and alignment to zero, the coefficients of variation δ w inside the body and on its boundary, and we obtain the following differential equation

$$\frac{\partial^2 M_{11}}{\partial x^2}+2\frac{\partial^2 M_{12}}{\partial x_1\partial x_2}+\frac{\partial^2 M_{22}}{\partial x_2^2}+\rho h\ddot{w}=0,\ \left(\ddot{w}=\partial^2\ w/\partial t^2\right) \tag{7}$$

with natural boundary conditions:

$$\begin{cases}\dfrac{\partial w}{\partial x_2}=0\\ w=0;\ x_2=0:l_2\end{cases} \tag{8}$$

$$\begin{cases}\dfrac{\partial w}{\partial x_1}=0 & x_1=0;l_1\\ w=0\ ;\end{cases}. \tag{9}$$

The main alternative boundary conditions to them

$$\begin{cases}M_{22}=0\\ \dfrac{\partial M_{22}}{\partial x_2}+2\dfrac{\partial M_{12}}{\partial x_1}=0;\ x_2=0\ ;\quad l_2\end{cases} \tag{10}$$

$$\begin{cases} M_{11} = 0 \\ \dfrac{\partial M_{11}}{\partial x_1} + 2\dfrac{\partial M_{12}}{\partial x_2} = 0\,;\; x_1 = 0 : l_1. \end{cases} \tag{11}$$

For, we construct a spectral problem by entering the following change of variables

$$w = W; \quad \varphi = \frac{\partial W}{\partial x_2};$$

$$M = \left(\frac{\partial^2 W}{\partial x_1^2} + \frac{\partial^2 W}{\partial x_2^2} \right); \quad Q = \frac{\partial M_{22}}{\partial x_2} + \frac{\partial M_{12}}{\partial x_1} \; . \tag{12}$$

Substituting (12) into (7) we obtain the differential equation of the system relatively sparse on the first derivatives x_2 :

$$\frac{\partial Q}{\partial x_2} + \frac{\partial^2 M}{\partial x_1^{\,2}} + \overline{D}'(1-\nu)\frac{\partial^2 \varphi}{\partial x_1^{\,2}} + \rho h \frac{\partial^2 W}{\partial t^2} = 0\,;$$

$$\frac{\partial M}{\partial x_2} - Q - \overline{D}''(1-\nu)\frac{\partial^2 W}{\partial x_1^{\,2}} = 0\,;$$

$$\overline{D}\frac{\partial \varphi}{\partial x_2} - M + \overline{D}\frac{\partial^2 W}{\partial x_1^{\,2}} = 0\,; \tag{13}$$

$$\frac{\partial W}{\partial x_2} - \varphi = 0 :$$

and alternative boundary conditions $x_{2=}0$: $x_2 = l_2$:

$$\varphi = 0 \;\text{ or }\; M - \overline{D}(1-\nu)\frac{\partial^2 M}{\partial x_1^2} = 0; \tag{14}$$

$$W = 0 \quad \text{or} \quad Q + \overline{D}(1-\nu)\frac{\partial^2 \varphi}{\partial x_1^2} = 0.$$

and $x_1 = 0$, $x_1 = l_1$,

$$\varphi = 0 \;\text{ or }\; M - \overline{D}(1-\nu)\frac{\partial^2 M}{\partial x_1^2} = 0;$$

$$W = 0 \;\text{ or }\; Q + \overline{D}(1-\nu)\frac{\partial^2 \varphi}{\partial x_1^2} = 0 \tag{15}$$

Now consider the infinite along the axis x_1 band with an arbitrary thickness changes $h = h(x_2)$. We seek a solution of problem (13) - (15) in the for

$$(Q, M, \varphi, W)^T = (\overline{Q}, \overline{M}, \overline{\varphi}, \overline{W})^T e^{i(\alpha x_1 - \omega t)} \tag{16}$$

Describing the harmonic plane waves propagating along the axis x_1. Here $(\overline{Q}, \overline{M}, \overline{\varphi}, \overline{W})^T$ – complex amplitude - function; k – wave number; C ($C = C_R + iC_i$)– complex phase velocity; ω – complex frequency.

Notes

To clarify their physical meaning, consider two cases:

1) $k = k_R$; $C = C_R + iC_i$, ($\omega_R = \omega_I + i\omega_I$)then the solution of differential equations (13) has the form of a sine wave at x_1, whose amplitude decays over time;
2) $k = k_R + ik_I$; $C = C_R$, Then at each point x_1 fluctuations established, but x_1 attenuated.

In both cases, the imaginary part k_I or C_I characterized by the intensity of the dissipative processes. Substituting (16) in (17), we obtain a system of first order differential equations solved for the derivative

$$\begin{aligned} &\overline{Q}' - \alpha^2 \overline{M} - \alpha^2 \overline{D}'(1-\nu)\overline{\varphi} - \rho h \omega^2 \overline{W} = 0; \\ &\overline{M}' - \overline{Q} + \alpha^2 \overline{D}'(1-\nu)\overline{W} = 0; \\ &\overline{\varphi}' - \frac{1}{\overline{D}} \overline{M} - \alpha^2 \overline{W} = 0; \\ &\overline{W}' - \overline{\varphi} = 0 \end{aligned} \tag{17}$$

with boundary conditions at the ends of the band $x_2=0, l_2$, one of the four types

а. swivel bearing: $\overline{W} = \overline{M} = 0;$ (18)

б. sliding clamp: $\overline{Q} = \overline{\varphi} = 0$ (19)

в. anchorage: $\overline{W} = \overline{\varphi} = 0$ (20)

г. free edge: $\begin{cases} \overline{M} + \alpha^2 \overline{D}(1-\nu)\overline{W} = 0 \\ \overline{Q} - \alpha^2 (1-\nu)\overline{D}\,\overline{\varphi} = 0 \end{cases}$ (21)

Thus, the spectral formulated task (17) and (21) the parameter α^2, describes the propagation of flexural waves in planar waveguide made as a band with an arbitrary coordinate on the thickness change x_2. It is shown that the spectral parameter α^2 It takes complex values (in the case of $R_{En}(t-\tau) \neq 0$) If $R_{En}(t-\tau) = 0$, whereas the spectral parameter α^2 It takes only real values. Transform this system (17). We have

$$\overline{Q}' = \overline{M}'' + \overline{D}''(1-\nu)\alpha^2 \overline{W} + \overline{D}'(1-\nu)\alpha^2 \overline{\varphi}$$

From whence

$$M'' + \overline{D}''(1-\nu)\alpha^2 \overline{W} - \alpha^2 \overline{M} - \rho h w^2 \overline{W} = 0$$

Moreover

$$\overline{W} - \frac{1}{\overline{D}} \overline{M} - \alpha^2 \overline{W} = 0.$$

Thus, the conversion system is of the form

$$\begin{cases} \overline{M}'' - \alpha^2 \overline{M} - \left(\rho h \omega^2 - \overline{D}''(1-\nu)\alpha^2\right) \overline{W} = 0 \\ W'' - \alpha^2 \overline{W} - \frac{1}{\overline{D}} \overline{M} = 0 \end{cases} \tag{22}$$

The boundary conditions (18) - (21) in alternating $\overline{W}, \overline{M}$ it has the form:

а. swivel bearing: $\overline{W} = \overline{M} = 0;$ (23)

б. sliding clamp: $\overline{W}' = \overline{M}' - \alpha^2 \overline{D}'(1-\nu)\overline{W} = 0;$ (24)

в. anchorage: $\overline{W} = \overline{W}' = 0$ (25)

г. free edge: $\overline{M}' + \alpha^2 \overline{D}(1-\nu)\overline{W} = 0$, $\overline{M}' - \alpha^2 (1-\nu)(D\overline{W})' = 0$ (26)

at $x_2=0$ or $x_2=+l_2$

Let $\overline{M}$ and $\overline{W}$ some own functions of the system (22) - (26) may have a complex meaning. Multiply the equation system (22) to function $\hat{M}$ and $\hat{W}$, complex conjugate to $\overline{M}$ and $\overline{W}$. Identical converting the first equation, we integrate the resulting equality x_2 and composed of the following linear combination

$$\int_0^{l_2} \overline{M}''\hat{W}dx_2 - \alpha^2(1-\nu)\int_0^{l_2} (\overline{D}\overline{W})''\hat{W}dx_2 + \alpha^2(1-\nu)\int_0^{l_2} (\overline{D}\overline{W})''\hat{W}dx_2 -$$

$$-\alpha^2 \int_0^{l_2} \overline{M}\hat{W}dx_2 - \omega^2 \int_0^{l_2} \rho h \overline{W}\hat{W}dx_2 - \alpha^2(1-\nu)\int_0^{l_2} \overline{D}''\overline{W}\hat{W}dx_2 + \tag{27}$$

$$+\int_0^{l_2} \overline{W}''\hat{M}dx_2 - \alpha^2 \int_0^{l_2} \overline{W}\hat{M}dx_2 - \int_0^{l_2} \frac{\overline{M}\hat{M}}{\overline{D}} dx_2 = 0$$

Integrating (27) by parts,

$$\left[\overline{M}' - \alpha^2(1-\nu)(D\overline{W}')\right]\hat{W}\Big|_0^{l_2} - \int_0^{l_2} [\overline{M}'\hat{W}' + \hat{M}'\overline{W}']dx_2 +$$

$$+\alpha^2(1-\nu)\int_0^{l_2} \overline{D}\,\overline{W}'\hat{W}'dx_2 - \alpha^2 \int_0^{l_2} [\overline{M}\hat{W}' + \hat{M}\overline{W}]dx_2 +$$

$$+2\alpha^2(1-\nu)\int_0^{l_2} \overline{D}''\overline{W}\,\hat{W}\,dx_2 - \omega^2 \int_0^{l_2} \rho h \overline{W}\hat{W}dx_2 - \int_0^{l_2} \frac{\overline{M}\hat{M}}{\overline{D}} dx_2 + \overline{W}'\hat{M}\Big|_0^{l_2} +$$

$$+2\alpha^2(1-\nu)\int_0^{l_2} \overline{D}\overline{W}'\hat{W}dx_2 + \alpha^2(1-\nu)\int_0^{l_2} \overline{D}'\overline{W}\hat{W}'dx_2 + \alpha^2(1-\nu)\int_0^{l_2} D\overline{W}''\hat{W}dx_2 = 0$$

or

$$\left[\overline{M}' - \alpha^2(1-\nu)\left(D\overline{W}\right)'\right]\widehat{W}\Big|_0^{l_2} + \left[\widehat{M} + \alpha^2(1-\nu)\overline{D}\,\widehat{W}\right]\overline{W}'\Big|_0^{l_2}$$

$$-\int_0^{l_2}\left(\overline{M}'\widehat{W}' + \widehat{M}'\overline{W}'\right)dx_2 - \alpha^2\int_0^{l_2}\left(\overline{W}\widehat{M} + \widehat{W}\overline{M}\right)dx_2 - \tag{28}$$

$$-\int_0^{l_2}\frac{\overline{M}\widehat{M}}{\overline{D}}dx_2 - \omega^2\int_0^{l_2}\rho h\overline{W}\widehat{W}dx_2 - 2\alpha^2(1-\nu)\int_0^{e_2}\overline{D}''\widehat{W}\overline{W}dx_2 + \alpha^2(1-\nu)\int_0^{e_2}\overline{D}'\left(\widehat{W}\overline{W}\right)'dx_2 = 0.$$

It is easy to make sure that is the integral terms of (28) vanish at any combination of the boundary conditions (23) - (26). It should also be noted that all the functions under the integral valid at $R_{En}(t-\tau)=0$. The expressing α^2 (28) We find that

$$\alpha^2 = \frac{\int_0^{l_2}(\overline{M}'\widehat{W}' + \widehat{M}'\overline{W}')\,dx_2 + \int_0^{l_2}\frac{\overline{M}\widehat{M}}{\overline{D}}dx_2 + \omega^2\int_0^{l_2}\rho h\widehat{\overline{W}}\widehat{W}dx_2}{\int_0^{l_2}(\overline{M}\widehat{W} + \widehat{M}\overline{W})\,dx_2 - 2(1-\nu)\int_0^{l_2}\overline{D}''\overline{W}\,\widehat{W}dx_2 - (1-\nu)\int_0^{l_2}\overline{D}'(\overline{W}\widehat{W})'dx_2}$$ - real number.

Thus (with $R_{En}(t-\tau)=0$), It is shown that the square of the wave number for own endless strip of varying thickness is valid for any combination of boundary conditions. If $R_{En}(t-\tau)\neq 0$, then α^2 It is a complex value for any combination of boundary conditions.

II. Adjoin Spectral Problem, Orthogonality Condition

The resulting spectral problem (17) - (21) is not self-adjoin. Built for her adjoin problem using this Lagrange formula [16]

$$\int_0^l L(U)\cdot V^*dx = Z(U,V^*)\Big|_0^l - \int_0^l L^*(V^*)\cdot Udx, \tag{29}$$

where L and L* - direct and adjoin linear differential operators; U and V* - arbitrary decisions of relevant boundary value problems.
In our case

$$L = \begin{bmatrix} \frac{\partial}{\partial x_2} & -\alpha^2 & -\alpha^2\overline{D}'(1-\nu) & -\rho h\omega^2 \\ -1 & \frac{\partial}{\partial x_2} & 0 & -\alpha^2\overline{D}'(1-\nu) \\ 0 & \frac{1}{\overline{D}} & \frac{\partial}{\partial x_2} & -\alpha^2 \\ 0 & 0 & -1 & \frac{\partial}{\partial x_2} \end{bmatrix}, \tag{30}$$

on the left-hand side of equation (29) will be as follows

Ref

16. M. A. Neumark, Linear differential operators. Publishing Moscow, 1969. - 526 p.

$$\int_0^{l_2}\Big[\overline{Q}'\overline{Q}^{\bullet}-\alpha^2\overline{M}\,\overline{Q}^{\bullet}-\alpha^2\overline{D}'(1-\nu)\overline{\varphi}Q^{\bullet}-\rho h\omega^2\overline{W}\overline{Q}^{\bullet}+\overline{M}'\overline{M}^{\bullet}-\overline{Q}\,\overline{M}^{\bullet}+$$

$$+\alpha^2\,\overline{D}'(1-\nu)\overline{W}\,\overline{M}^{\bullet}+\overline{\varphi}'\overline{\varphi}^{\bullet}-\frac{1}{\overline{D}}\overline{M}\overline{\varphi}^{\bullet}-\alpha^2\overline{W}\overline{\varphi}^{\bullet}+\overline{W}'\overline{W}^{\bullet}-\overline{\varphi}\overline{W}^{\bullet}\Big]dx_2=0 \tag{31}$$

or, integrating parts

$$\Big[\overline{Q}\overline{Q}^{\bullet}+\overline{M}\,\overline{M}^{\bullet}+\overline{\varphi}\overline{\varphi}^{\bullet}+\overline{W}\overline{W}^{\bullet}\Big]\Big|_0^{l_2}-\int_0^{l_2}\Big[(\overline{Q}^{\bullet\,\prime}+\overline{M}^{\bullet})\overline{Q}+(\overline{M}^{\bullet\prime}+\alpha^2\overline{Q}^{\bullet}+$$

$$+\frac{1}{\overline{D}}\overline{\varphi}^{\bullet})\overline{M}+(\overline{\varphi}^{\bullet\,\prime}+\overline{W}^{\bullet}+\alpha^2\overline{D}'(1-\nu)\overline{Q}^{\bullet})\overline{\varphi}+(\overline{W}^{\bullet\,\prime}+\alpha^2\overline{\varphi}^{\bullet}- \tag{32}$$

$$-\alpha^2\overline{D}'(1-\nu)\overline{M}^{\bullet}+\rho h\omega^2\overline{Q}^{\bullet})\overline{W}\Big]dx_2=0$$

Thus the conjugate (30) - (32), the system has the form

$$\begin{cases}\overline{Q}^{\bullet\prime}+\overline{M}^{\bullet}=0\\ \overline{M}^{\bullet\prime}+\alpha^2\overline{Q}^{\bullet}+\dfrac{1}{D}\overline{\varphi}^{\bullet}=0\\ \overline{\varphi}^{\bullet\,\prime}+\overline{W}^{\bullet}+\alpha^2\overline{D}'(1-\nu)\overline{Q}^{\bullet}=0\\ \overline{W}^{\bullet\prime}-\alpha^2\overline{D}'(1-\nu)\overline{M}^{\bullet}+\alpha^2\overline{\varphi}^{\bullet}+\rho h\omega^2\overline{Q}^{\bullet}=0\end{cases} \tag{33}$$

Moreover, we get the conjugate boundary conditions of equality to zero is integral members $Z(U,V^*)\Big|_0^{l_2}$ expression in (32):

а. swivel bearing: $\overline{\varphi}^{\bullet}=\overline{Q}^{\bullet}=0,\qquad x_2=0,\ \ l_2$ (34)

б. sliding clamp: $\overline{W}^{\bullet}=\overline{M}^{\bullet}=0,\qquad x_2=0,\ \ l_2$ (35)

в. anchorage: $\overline{M}^{\bullet}=\overline{Q}^{\bullet}=0,\qquad x_2=0,\ \ l_2$ (36)

г. free edge: $\begin{cases}\overline{\varphi}^{\bullet}+\alpha^2\overline{D}(1-\nu)\overline{Q}^{\bullet}=0,\\ \overline{W}^{\bullet}-\alpha^2\overline{D}(1-\nu)\overline{M}^{\bullet}=0,\end{cases}\quad x_2=0,\ \ l_2$

For conditions biorthogonality solutions once again use the Lagrange formula (29) in the form

$$\int_0^{l}\Big[L(U)V^{\bullet}+L^{\bullet}(V^{\bullet})U\Big]dx=Z(U,V^{\bullet})\Big|_0^{l_2}, \tag{37}$$

that leads to the consideration of the following integral

$$\int_0^{l_2}\Big[\overline{Q}_i'\,\overline{Q}_j^{\bullet}-\alpha_i^2\overline{M}_i\overline{Q}_j^{\bullet}-\alpha_i^2\overline{D}'(1-\nu)\overline{\varphi}_i\overline{Q}_j^{\bullet}-\rho h\omega^2\overline{W}_i\overline{Q}_j^{\bullet}+\overline{M}_i'\,\overline{M}_j^{\bullet}-$$

$$-\overline{Q}_i\overline{M}_j^{\bullet}+\alpha_i^2\overline{D}'(1-\nu)\overline{W}_i\overline{M}_j^{\bullet}+\overline{\varphi}_i'\overline{\varphi}_j^{\bullet}-\frac{1}{\overline{D}}\overline{M}_i\overline{\varphi}_j^{\bullet}-\alpha^2{}_i\ \overline{W}_i\overline{\varphi}_j^{\bullet}+$$

$$+\overline{W}_i'\overline{W}_j^{\bullet}-\overline{\varphi}_i\overline{W}_j^{\bullet}+\overline{Q}_j^{\bullet'}\overline{Q}_i+\overline{M}_j^{\bullet}\overline{Q}_i+\overline{M}_j^{\bullet'}\overline{M}_i+\alpha^2{}_j\overline{M}_i\overline{Q}_j^{\bullet}+ \tag{38}$$

$$+\frac{1}{\overline{D}}\overline{M}_i\overline{\varphi}_j^{\bullet}+\overline{\varphi}_i\overline{\varphi}_j^{\bullet'}+\overline{W}_j^{\bullet}\overline{\varphi}_i+\alpha_j^2D'(1-\nu)\overline{Q}_j^{\bullet}\overline{\varphi}_i+\overline{W}_i\overline{W}_j^{\bullet}{}'+$$

$$+\alpha_j^2\overline{W}_i\overline{\varphi}_j^{\bullet}-\alpha_j^2\overline{D}'(1-\nu)\overline{W}_i\overline{M}_j^{\bullet}+\rho h\omega^2\overline{Q}_j^{\bullet}\overline{W}_i]dx_2=0,$$

where $(\overline{Q}_i\ ,\ \overline{M}_{i,}\overline{\varphi}_i,\overline{W}_i)^T$ - own form, corresponding to the Eigen value α_i original spectral problem; $(\overline{Q}_j^{\bullet},\overline{M}_j^{\bullet},\overline{\varphi}_j^{\bullet},\overline{W}_j^{\bullet})^T$ - own form, corresponding to the Eigen value α_j adjoin. Integrating (38) by parts

$$\left(\alpha_i^2-\alpha_j^2\right)\left[\int_0^{l_2}\left[-\overline{M}_i\overline{Q}_j^{\bullet}-\overline{D}'(1-\nu)\overline{Q}_j^{\bullet}\overline{\varphi}_i+\overline{D}'(1-\nu)\overline{W}_i\overline{M}_j^{\bullet}-\overline{W}_i\overline{\varphi}_j^{\bullet}\right]dx_2+\right. \tag{39}$$

$$\left.+\left[\overline{D}(1-\nu)\overline{Q}_j^{\bullet}\overline{\varphi}_i-\overline{D}(1-\nu)\overline{W}_i\overline{M}_j^{\bullet}\right]_0^{l_2}=0,\right.$$

where to i≠j we have the condition biorthogonality forms:

$$\int_0^{l_2}\left[\overline{M}_i+\overline{D}'(1-\nu)\overline{\varphi}_i)\overline{Q}_j^{\bullet}+\overline{W}_i(\overline{\varphi}_j^{\bullet}=\overline{D}'(1-\nu)\overline{M}_j^{\bullet})\right]dx_2+$$

$$+\overline{D}(1-\nu)\left[\overline{W}_i\overline{M}_j^{\bullet}-\overline{Q}_j^{\bullet}\overline{\varphi}_i\right]\Big|_0^{l_2}=\delta_{ij} \tag{40}$$

The expression $\overline{W}_i\overline{M}_j^{\bullet}-\overline{Q}_j^{\bullet}\overline{\varphi}_i$ zero, if the border is set to any of the conditions (18) - (21) in addition to the conditions of the free edge.

III. Fixed Problem for a Semi-Infinite Strip of Variable Thickness.

Consider a semi-infinite axial x_1 lane variable section, wherein at the end (x_1 =0) harmonic set time exposure of one of two types of:

$$W=f_W(x_2)e^{i\omega t},\ \ M_{11}=f_W(x_2)e^{i\omega t},\ \ \mathrm{x}_1=0 \tag{41}$$

or

$$\varphi_1=f_\varphi(\mathrm{x}_2)\mathrm{e}^{i\omega t},\qquad Q_1=f_Q(x_2)e^{i\omega t},\ \ x_1=0 \tag{42}$$

where

$$\varphi_1=\frac{\partial W}{\partial x_1},\quad Q_1=\overline{D}\left[\frac{\partial^3W}{\partial x_1^3}+2(1-\nu)\frac{\partial^3W}{\partial x_1\partial x_2^2}\right] \tag{43}$$

Transform the boundary conditions (41) so that they contain only selected our variables $W,\varphi,\ M$ and Q

$$W = f_w(x_2)e^{i\omega t},\quad \overline{D}\left(\frac{\partial^2 W}{\partial x_1^2}+\nu\frac{\partial^2 W}{\partial x_2^2}\right)=f_M(x_2)e^{i\omega t},\ x_1=0,$$

$$\frac{\partial W}{\partial x_1}=f_\varphi(x_2)e^{i\omega t},\quad \overline{D}\left(\frac{\partial^3 W}{\partial x_1^3}+2(1-\nu)\frac{\partial^3 W}{\partial x_1\partial x_2^2}\right)=f_Q(x_2)e^{i\omega t},\quad x_1=0,$$

or

$$W = f_w(x_2)e^{i\omega t},\quad \overline{D}\left(\frac{\partial^2 W}{\partial x_1^2}+\nu\frac{\partial^2 W}{\partial x_2^2}\right)-\overline{D}(1-\nu)\frac{\partial^2 W}{\partial x_2^2}=f_M(x_2)e^{i\omega t},\ x_1=0,$$

$$\frac{\partial W}{\partial x_1}=f_\varphi(x_2)e^{i\omega t},\ \frac{\partial}{\partial x_1}\left[\overline{D}\left(\frac{\partial^2 W}{\partial x_1^2}+\frac{\partial^2 W}{\partial x_2^2}\right)\right]+\overline{D}(1-\nu)\frac{\partial^2}{\partial x_2^2}\left(\frac{\partial W}{\partial x_1}\right)=f_Q(x_2)e^{i\omega t},\quad x_1=0,$$

Of finally

$$W = f_w(x_2)e^{i\omega t},\ M=\left[f_M(x_2)+\overline{D}(1-\nu)f_w''(x_2)\right]e^{i\omega t},\ x_1=0 \tag{44}$$

$$\frac{\partial W}{\partial x_1}=f_\varphi(x_2)e^{i\omega t},\quad \frac{\partial M}{\partial x_1}=\left[f_Q(x_2)-\overline{D}(1-\nu)f_\varphi''(x_2)\right]e^{i\omega t},\quad x_1=0. \tag{45}$$

Assume that the desired solution of the no stationary problem can be expanded in a series in Eigen functions of the solution of the spectral problem. In the case of constant thickness it is evident, and in general, the question remains open.
The solution of the stationary problem (17) - (21) (41) - (42) will seek a

$$\begin{pmatrix} W\\ \varphi\\ M\\ Q\end{pmatrix}=\sum_{k=1}^{N}a_k\begin{pmatrix}\overline{W}_k(x_2)\\ \overline{\varphi}_k(x_2)\\ \overline{M}_k(x_2)\\ \overline{Q}_k(x_2)\end{pmatrix}e^{-i(\alpha_k x_1-\omega t)} \tag{46}$$

where $\overline{W}_k,\ \overline{\varphi}_k,\ \overline{\theta}_k,\ \overline{M}_k$ - biorthonormal own forms of the spectral problem (17) – (21).

The representation (46) gives us the solution to the problem of non-stationary wave in the far field, i.e., where it has faded not propagating modes. The number of propagating modes used N course for each specific frequency ω, since the cutoff frequency is greater than the other ω.

Consider two cases of excitation of stationary waves in the band:

а) $f_w=0$ – antisymmetric relative x_1;

б) $f_\varphi=0$ – symmetric.

In the case of antisymmetric excitation, substituting (46) into (44) and expressing $f_M(x_2)$, obtain

$$f_M(x_2)=\sum_{k=1}^{N}\alpha_k\overline{M}_k(x_2). \tag{47}$$

Value biorthogonality (40) gives expression to determine the unknown coefficients

$$a_k = \int_0^{e_2} f_M(x_2)\overline{Q}_k^{\bullet}(x_2)dx_2 .$$

In the case of a symmetrical excitation $f_Q(x_2)$ We obtain rearranging (46) to (45) in the following form

$$f_Q(x_2) = \sum_{k=1}^{N}(-i\alpha_k a_k \overline{M}_k(x_2)) \tag{48}$$

Biorthogonality ratio (30) gives

$$a_k = \frac{i}{\alpha_k}\int_0^{l_2} f_Q(x_2)\overline{Q}_k^{\bullet}dx_2 \tag{49}$$

IV. Testing Software System and Study the Properties of Propagation of Flexural Waves in a Band of Variable Thickness

Testing program was carried out on the task of distributing the flexural waves in a plate of constant thickness. Consider the floor plate of constant thickness infinitely satisfying Kirchhoff-Love hypotheses, with supported long edges (Figure 1).
at end face $x_1=0$ specified:

$$w=f_1(x_2)e^{i\omega t}, \qquad M_{11}=f_2(x_2)e^{i\omega t} \tag{50}$$

Spread along the axis x_1 flexural wave is described by the differential control system (13)

$$\begin{cases} Q' + \dfrac{\partial^2 M}{\partial x_1^2} + \rho h w'' = 0 \left(w'' = \partial^2 w / \partial t^2\right), \\ M' - \theta = 0 , \\ \varphi' - \dfrac{1}{\overline{D}} M + \dfrac{\partial^2 w}{\partial x_1^2} = 0 \\ w' - \varphi = 0, \quad h = h_0 \end{cases} \tag{51}$$

with boundary conditions of the form (15)

$$w=0, M - \overline{D}(1-\nu)\frac{\partial^2 w}{\partial x^2{}_1} = 0, \quad x_2 = 0, \ \pi \quad . \tag{52}$$

Introducing the desired motion vector in the form of

$$\begin{pmatrix} Q \\ M \\ \varphi \\ w \end{pmatrix} = \begin{pmatrix} \overline{Q} \\ \overline{M} \\ \overline{\varphi} \\ \overline{W} \end{pmatrix} e^{-i(a_k x_1 - \omega t)} \tag{53}$$

Go to the spectral problem

$$\begin{cases} \overline{Q}' - \alpha^2 \overline{M} - \omega^2 \rho h \overline{W} = 0, \\ \overline{M}' - \overline{Q} = 0, \\ \overline{\varphi}' - \dfrac{1}{\overline{D}} \overline{M} - \alpha^2 \overline{W} = 0, \\ \overline{W}' - \overline{\varphi} = 0, \end{cases} \tag{54}$$

with the boundary conditions

$$W = 0, \ \overline{M} + \overline{D}(1-\nu)\alpha^2 \overline{W} = 0, \ x_2 = 0, \ \pi$$

or

$$\overline{W} = 0, \ \overline{M} = 0, \ x_2 = 0, \quad \pi . \tag{55}$$

Rewrite the system (54) as follows

$$\begin{cases} \overline{W}'' - \dfrac{1}{\overline{D}} \overline{M} - \alpha^2 \overline{W} = 0, \\ \overline{M}'' - \alpha^2 \overline{M} - \omega^2 \rho h \overline{W} = 0, \end{cases} \tag{56}$$

and $\overline{W} = 0, \overline{M} = 0, \ x_2 = 0, \quad \pi.$

We seek the solution of (56) in the form

$$\begin{aligned} \overline{W} &= a_w \sin n x_2, \\ M &= a_M \sin n x_2, \ (\ n = 1, 2, ...) \end{aligned} \tag{57}$$

satisfying the boundary conditions (55).

We obtain an algebraic homogeneous system

$$-n^2 a_W - \alpha^2 a_W - \frac{1}{\overline{D}} a_M = 0 \ ;$$

$$-n^2 a_M - \alpha^2 a_M - \omega^2 \rho h a_w = 0 \ . \tag{58}$$

For the existence of a nontrivial solution, which is necessary to require the vanishing of its determinant

$$\det \begin{vmatrix} n^2 + \alpha^2 & \dfrac{1}{\overline{D}} \\ \omega^2 \rho h & n^2 + \alpha^2 \end{vmatrix} = 0,$$

or

$$\alpha_{1,2}{}^2 + n^2 = \pm \omega \sqrt{\frac{\rho h}{\overline{D}_k}}, \tag{59,a}$$

where, when $R_{En}(t-\tau)=0$

$$\alpha_{1,2}{}^{2} = -n^{2} \pm \omega\sqrt{\frac{\rho h}{D}}. \tag{59,b}$$

Notes

Ownership of constant thickness strip bending vibrations are of the form

$$\overline{W}_{n}^{1,2} = \sin n x_2;$$

$$M_{n}^{1,2} = \pm\omega\sqrt{\rho h \overline{D}}\sin n x_2; \tag{60}$$

$$\varphi_{n}^{1,2} = n\cos n x_2;$$

$$Q_{n}^{1,2} = \mp\omega\sqrt{\rho h \overline{D}}\cos n x_2.$$

We construct the solution of the problem adjoin to (54)-(55)

$$\begin{cases} \overline{W}^{\bullet\prime} + \alpha^{2}\overline{\varphi}^{\bullet} + \rho h\omega^{2}\overline{Q}^{*} = 0, \\ \overline{\varphi}^{\bullet\prime} + W^{\bullet} = 0 \\ \overline{M}^{*\prime} + \frac{1}{\overline{D}}\overline{\varphi}^{\bullet} + \alpha^{2}\overline{Q}^{\bullet} = 0, \\ \overline{Q}^{\bullet\prime} + \overline{M}^{\bullet} = 0 \end{cases} \tag{61}$$

and

$$\overline{\varphi}^{*} = 0,\ \overline{Q}^{*} = 0,\ x_2 = 0,\quad \pi. \tag{62}$$

Transforming (61) - (62) we obtain the following system of first order differential equations

$$\begin{cases} \overline{\varphi}^{\bullet\prime\prime} - \alpha^{2}\overline{\varphi}^{\bullet} - \rho h\omega^{2}\overline{Q}^{\bullet} = 0 \\ \overline{Q}^{\bullet\prime\prime} - \alpha^{2}\overline{Q}^{\bullet} - \frac{1}{\overline{D}}\overline{\varphi}^{\bullet} = 0, \end{cases}$$

$$x_2 = 0, \pi:\ \overline{\varphi}^{\bullet} = 0, \overline{Q}^{\bullet} = 0. \tag{63}$$

The solution of (63) in the form

$$\overline{\varphi}^{\bullet} = a_{\varphi}^{\bullet}\sin n x_2,\quad \overline{Q}^{\bullet} = a_{Q}^{\bullet}\sin n x_2 \tag{64}$$

From whence $\alpha_{1,2}^{2}$ It has the same form (59a), own forms of vibrations are of the form:

$$\begin{aligned}
\overline{\varphi}_n^{\bullet 1,2} &= \pm\omega\sqrt{ph\overline{D}}\,\sin nx_2,\\
\overline{Q}_n^{\bullet 1,2} &= \sin n\,x_2,\\
\overline{W}_n^{\bullet 1,2} &= \pm n\omega\sqrt{ph\overline{D}}\,\cos nx_2\\
\overline{M}_n^{\bullet 1,2} &= n\cos n\,x_2.
\end{aligned} \tag{65}$$

For biorthogonality conditions direct solutions and the adjoin problem is necessary to consider the following equation

$$\begin{aligned}
&\int_0^{\pi}\Big[\overline{Q}_i'\overline{Q}_j^{\bullet} - \alpha_i{}^2\overline{M}_i\,\overline{Q}_j^{\bullet} - \omega^2\rho h\overline{W}_i\overline{Q}_j^{\bullet} + \overline{Q}_j^{\bullet\prime}\overline{Q}_i + \overline{M}_j^{\bullet}\overline{Q}_i + \overline{M}_i'\overline{M}_j^{\bullet} - \overline{Q}_i\overline{M}_j^{\bullet} +\\
&+\overline{M}_j^{\bullet\prime}\overline{M}_i + \frac{1}{\overline{D}}\overline{\varphi}_j^{\bullet}\overline{M}_i + \alpha_j^2\overline{Q}_j^{\bullet}\overline{M}_i + \overline{\varphi}_i'\overline{\varphi}_j^{\bullet} - \frac{1}{\overline{D}}\overline{M}_i\,\overline{\varphi}_j^{\bullet} - \alpha_i{}^2\overline{W}_i\overline{\varphi}_j^{\bullet} + \overline{\varphi}_j^{\bullet\prime}\overline{\varphi}_i +\\
&+\overline{W}_j^{\bullet}\overline{\varphi}_i + \overline{W}_i'\overline{W}_j^{\bullet} - \overline{\varphi}_i\overline{W}_j^{\bullet} + \overline{W}_j^{\bullet\prime}\overline{W}_i + \alpha^2\overline{\varphi}_j^{\bullet}\overline{W}_i + \rho h\omega^2\overline{Q}_j^{\bullet}\overline{W}_i\Big]dx_2 = \delta_{ij}
\end{aligned} \tag{66}$$

where $\overline{Q}_i$, $\overline{M}_i$, $\overline{\varphi}_i$, $\overline{W}_i$ – own form for the direct problem, the corresponding Eigen values α_i, and $Q_j^{\bullet}$, $\overline{M}_j^{\bullet}$, $\overline{\varphi}_j^{\bullet}$, $\overline{W}_j^{\bullet}$ - own form of the dual problem, the corresponding Eigen value α_j. Integrating by parts in (66), using the boundary conditions (55) and (62) we obtain the desired condition:

$$\int_0^{\pi}\Big[\overline{M}_i\overline{Q}_j^{\bullet} + \overline{W}_i\overline{\varphi}_j^{\bullet}\Big]dx_2 = \delta_{ij}\,. \tag{67}$$

We now verify biorthogonality received their own forms (60) and (65) using the condition biorthogonality (67)

$$\begin{aligned}
&\int_0^{\pi}\Big[\sqrt{\rho h\overline{D}}\;Sin(ix_2)\cdot Sin(jx_2) + \sqrt{\rho h\overline{D}}\,Sin(ix_2)Sin(jx_2)\Big]dx_2 =\\
&= 2\sqrt{\rho h\overline{D}}\int_0^{\pi}Sin(ix_2)Sin(jx_2)dx_2 = \pi\sqrt{\rho h\overline{D}}\;\delta_{ij}
\end{aligned}$$

The normalized adjoin eigenvector on $\pi\sqrt{\rho h\overline{D}}$, we have a system of eigenvectors satisfying the condition (67).

We now obtain the solution of the problem of the distribution of the stationary wave in the semi-infinite strip of constant thickness. Suppose that at the border $x_1=0$ set the following stationary disturbance:

$$\begin{aligned}
w &= \overline{W}e^{i\omega t} = b_W\sin(nx_2)e^{i\omega t},\\
M &= \overline{M}e^{i\omega t} = b_M\sin(nx_2)e^{i\omega t}, \qquad x_1 = 0
\end{aligned} \tag{68}$$

We seek a solution of a problem

$$w(x_1,x_2,t) = \sum_{k=1}^{\infty}a_kW_k\;, M(x_1,x_2,t) = \sum_{k=1}^{\infty}a_kM_k \tag{69}$$

where

$$W_k = \overline{W}(x_{21})e^{-i(a_k x_l - \omega t)}, \quad M_k = \overline{M}(x_2)e^{-i(a_k x_l - \omega t)},$$

a $\overline{W}_k$ u $\overline{M}_k$ - own form (60), corresponding to α_k

It is evident from the band at the end face at $x_1=0$ decision (69) must satisfy the boundary conditions (68)

$$b_W \sin(nx_2)e^{i\omega t} = \sum_{k=1}^{\infty} a_k \overline{W}_k(x_2)e^{i\omega t},$$

$$b_M \sin(nx_2)e^{i\omega t} = \sum_{k=1}^{\infty} a_k \overline{M}_k(x_2)e^{i\omega t},$$

or go to the amplitude values

$$b_w \sin(nx_2)e^{i\omega t} = \sum_{k=1}^{\infty} a_k \overline{W}_k,$$

$$b_M \sin(nx_2)e^{i\omega t} = \sum_{k=1}^{\infty} a_k \overline{M}_k, \tag{70}$$

Consider the following integral

$$\int_0^{\pi} \left[\overline{M}\overline{Q}_j^{\bullet} + \overline{W}\overline{\varphi}_j^{\bullet}\right]dx_2 = \int_0^{\pi} \left[\sum_{k=1}^{\infty} a_k \overline{M}_k \overline{Q}_j^{\bullet} + \sum_{k=1}^{\infty} a_k \overline{W}_k \overline{\varphi}_j^{\bullet}\right]dx_2 =$$

$$= \sum_{k=1}^{\infty} a_k \int_0^{\pi} \left[\overline{M}_k \overline{Q}_j^{\bullet} + \overline{W}_k \overline{\varphi}_j^{\bullet}\right]dx_2 = a_j. \tag{71}$$

On the other hand on the edge $x_1 = 0$ the same integral as follows

$$\int_0^{\pi} \left[b_M \sin(nx_2)\overline{Q}_j^{\bullet} + b_W \sin(nx_2)\overline{\varphi}_j^{\bullet}\right]dx_2 \tag{72}$$

Substituting in (72) from the normalized own form (65) we obtain

$$\int_0^{\pi} \left[b_M \sin(n x_2)\frac{1}{\pi\sqrt{\rho h \overline{D}}}\sin(jx_2) \pm b_w \sin(n x_2)\frac{\omega}{\pi}\sin(jx_2)\right]dx_2 =$$

$$= \left[\frac{b_M}{\pi\sqrt{\rho h \overline{D}}} \pm \frac{b_w \omega}{\pi}\right] \cdot \int_0^{\pi} \sin(n x_2) \cdot \sin(jx_2)\right]dx_2 \tag{73}$$

From a comparison of the formulas (71) and (73) it is clear that under such boundary conditions is excited only "n" - Single private form:

$$a_j^{\pm} = \delta_{nj}\left[\frac{b_M}{2\sqrt{\rho h \overline{D}}} \pm \frac{b_w \omega}{2}\right] \tag{74}$$

Thus, the solution of the no stationary problem for a half-strip of constant thickness has the form

$$\overline{W} = \left(a_n^+ \overline{W}_n^+ + a_n^- \overline{W}_n^-\right) e^{-i(\alpha x_1 - \omega t)}$$
$$\overline{M} = \left(a_n^- \overline{M}_n^+ + a_n^- \overline{M}_n^-\right) e^{-i(\alpha x_1 - \omega t)} \quad (75)$$

where

$\overline{W}_n^{\pm} = \pm \sin n x_2$, $\overline{M}_n^{\pm} = \pm\omega\sqrt{\rho h \overline{D}} \sin n x_2$, a $a_n^{\pm}$ - determined from the ratio (74).

Now suppose that the steady influence on the border of semi-infinite strip $x_1=0$ it has the form

$$w = f_w(x_2) e^{i\omega t}, \quad M = f_M(x_2) e^{i\omega t} \quad (76)$$

Let us expand the function f_w and f_M Fourier series of sinus in the interval $[0,\pi]$

$$f_w(x_2) = \sum_{k=1}^{\infty} B_w^k \sin(k x_2), \quad f_M(x_2) = \sum_{k=1}^{\infty} b_M^k \sin(k x_2) \quad (77)$$

Using the results of the previous problem, we find that the solution can be represented as a Fourier series:

$$W = \sum_{k=1}^{\infty} \left(a_k^+ W_k^+ + a_k^- W_k^-\right) e^{-i(\alpha x_1 - \omega t)}$$
$$M = \sum_{k=1}^{\infty} \left(a_k^+ M_k^+ + a_k^- M_k^-\right) e^{-i(\alpha x_1 - \omega t)} \quad (78)$$

where $a_k^{\pm} = \dfrac{b_M^k}{2\sqrt{\rho h \overline{D}}} \pm \dfrac{b_w^k \omega}{2}$, $a\overline{W}_k^{\pm}$, $\overline{M}_k^{\pm}$ determined from the ratio (60).

V. Numerical Results and Analysis

The numerical solution of spectral problems carried out by computer software system based on the method of orthogonal shooting S.K. Godunov [4] combined with the method of Muller. The results obtained in testing with the same software package analytically up to 4-5 mark frequency range from 0.01 to 100. Hereinafter, the entire analysis is conducted in dimensionless variables, in which the density of the material ρ, half the width of the waveguide l_2 and E modulus taken to be unity, and the parameters of relaxation kernel $A = 0{,}048$; $\beta = 0{,}05$; $\alpha = 0{,}1$.

Ref

4. SK Godunov, On the numerical solution of boundary value problems for systems of linear ordinary differential equations. - Russian Mathematical Surveys, 1061, T.16, vol.3, 171-174 p.

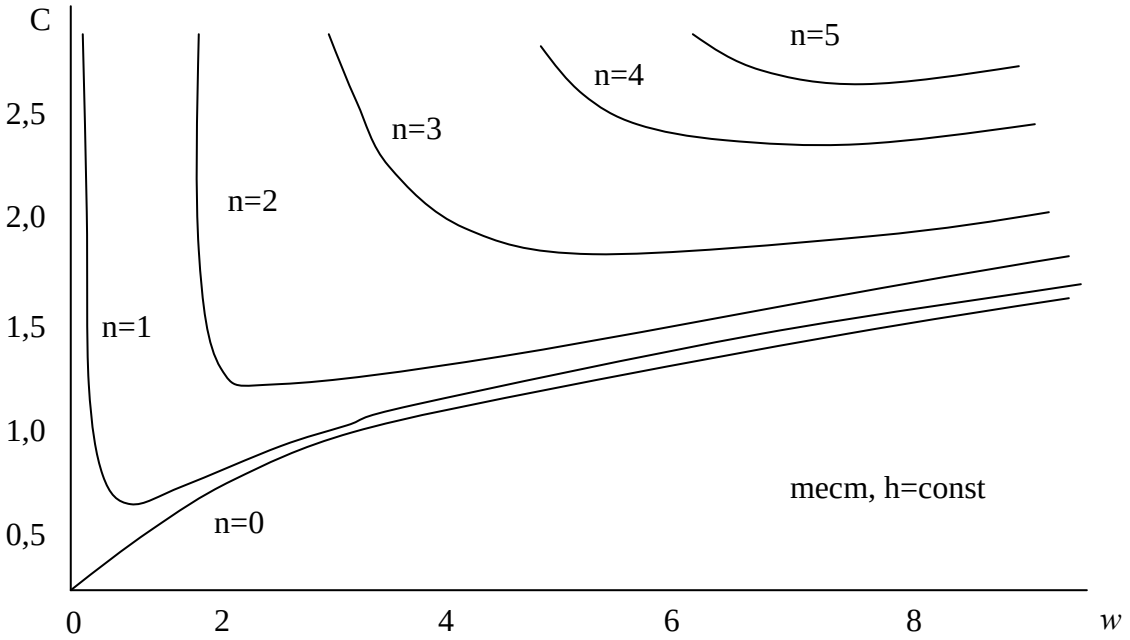

Figure 2 : The dependence of the phase velocity on frequency

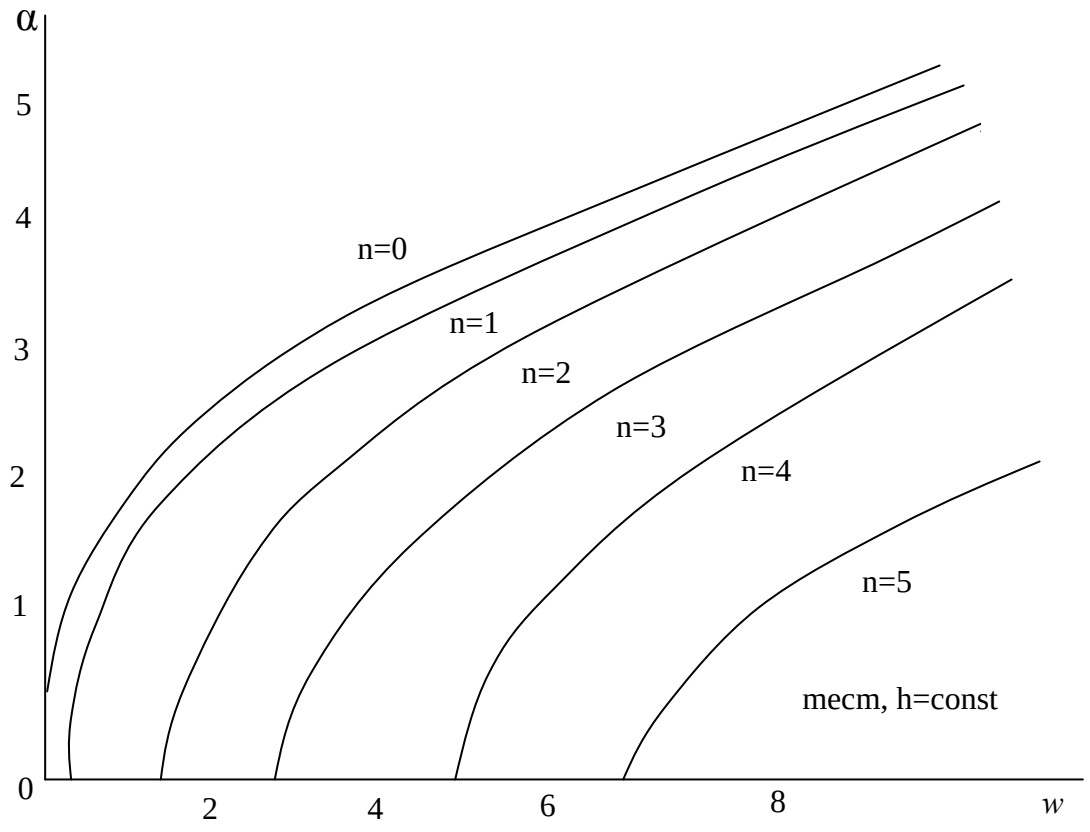

Figure 3 : The dependence of the frequency of the wave

The calculation results are obtained when A = 0. Figure 2 shows the spectral curves of the lower modes of oscillation of constant thickness plate, the corresponding n=0, 1, 2, 3, 4, 5 for Poisson's ratio υ=0,25. Analysis of the data shows that the range of applicability of the theory of Kirchhoff-Love to a plate of constant thickness is

limited by the low frequency range. For example, for the first mode (h = 0), the range of application of the theory 0≤ω≤3 because of the unlimited growth of the phase propagation velocity with increasing frequency, for high frequencies $C_f C_s \sim\sqrt{\omega}$.

At high frequencies, where the wavelength is comparable or less than the fashion of strip thickness, there is, as is well known, localized in the faces of the Rayleigh wave band at a speed slower speed C_s, however, as is obvious, this formulation of the problem, in principle, does not allow to obtain this result. However, it should be noted that in the application of the theory of Kirchhoff - Love platinum constant thickness is obtained the correct conclusion about the growth of the number of propagating modes with increasing frequency that is well seen from the spectral curves of Figure 2 and Figure 3, which shows the dependence of the wave number α the frequency for the same modes of waves.

Figure 4 shows the obtained numerical form for the above modes of oscillations coincided with the same accuracy (4-5 decimal places) in the division of bandwidth by 90 equal segments.

Figure 5 illustrates the solution of the stationary problem: the amplitude of the excited oscillation modes linearly depends on the frequency ω.

We proceed to the propagation of flexural waves in a symmetric band Kirchhoff - Love of variable thickness. Let us first consider a waveguide with a linear thickness change, presented in Figure 6 and 7 which are free edges. Figure 8 shows the dispersion curves for the first mode, depending on the verge of tilt angle $\varphi/2$. Curve I corresponds to a strip of constant thickness $h_0=h_1$. Curve 2 corresponds to a waveguide with an angle of inclination of faces $\varphi/2 =\pi/4$ or $tg\ \varphi/2=1$ and curve 3 corresponds to a waveguide $tg\ \varphi/2=0{,}2$. The figure shows that, unlike the bands in the case of constant cross-section of the waveguide with a small tapered angle at the base of the wedge α (Curve 3) there exists a finite limit of the phase velocity of the fashion spread, and

$$\lim_{\omega\to\infty}\tilde{C}_f = 2C_s\ tg\frac{\varphi}{2}$$

where C_s -. The speed of shear waves, which coincides with the results of other studies [5,6,14,15] Thus, it is shown that -Lava Kirchhoff theory provides a wave propagating in the waveguide is tapered with a sufficiently small angle at the base of the wedge-speed, lower shear wave velocity and different from the Rayleigh wave velocity. Moreover, these waves from a frequency distributed without dispersion. This wave is called "wave Troyanovskiy - Safarov" [10, 12,13].

Figure 9 shows the waveform of the same frequency for $\omega=10$, from which it follows that the strip of constant thickness behaves like a rod while at the wedge-shaped strip there is a significant localization of waves in the area of acute viburnum, and the more, the smaller the angle φ. The above fact explains the Kirchhoff theory -Lava applicability for studying wave propagation in waveguides is tapered, as the frequency increases with decreasing length of one side of the wave modes, with different wave localizes with the sharp edge of the wedge so that the ratio of the wavelength and the effective thickness of the material is in the field of applicability of the theory. This statement is true, the smaller the angle at the base of the wedge.

It should also be noted that the numerical analysis of the dispersion equation (33) does not allow to show the presence of strictly limit the speed of wave propagation

Ref

5. V. T. Grinchenko, VV Myaleshka, harmonic oscillations and waves in elastic bodies. - Kiev: 1981, 284p.

modes, since the computer cannot handle infinitely large quantities. We can only speak about the numerical stability result in a large frequency range, which is confirmed by research. For example, when $tg\varphi/2=0,2$ value of the phase velocity of a measured without shear wave velocity at $\omega=3$ and $\omega=40$ It differs fifth sign that corresponds to the accuracy of calculations, resulting in test problem.

Notes

In the example $h_0 = 0,0001$, it certainly gives an increase of the phase velocity when the frequency increases further, since such a strong localization of the wave to the thin edge of the wedge, starts to affect the characteristic dimension - the thickness of the truncated wedge, and Kirchhoff hypothesis -Lava stops working. To solve the problem of acute wedge numerically is not possible, since the dispersion equation contains a term D^{-1}, and the thickness tends to zero flexural rigidity D behaves as a cube and the thickness goes to zero. This significantly increases the "rigidity" (ie the ratio between the small and large coefficient) system, increases dramatically the computing time and decreases the accuracy of the results. However, it is clear that you can trust the results obtained where the agreed parameters h_0 and α. We note also that the numerical experiment showed no significant dependence of the phase velocity of the first mode of the Poisson's ratio ν, and the fact that a family of dispersion curves with different apex angles of the wedge have a similarity property: the ratio of the phase velocity to the limit does not depend on the angle of the wedge φ. On the modes, starting from the second, the speed limit dependence on Poisson's ratio becomes noticeable - about 8.5% for the second mode when changing $0 \le \nu \le 0,5$. Generally, the limit speed increases with the stronger and the more the mode number.

Figure 10 shows the dispersion curves for the first modal wedge $tg\varphi/2=0,2$. The figure shows that the speed of the first mode (curve I) is equal to zero for $\omega=0$ and since the frequency $\omega=1$, virtually unchanged. The speed of the second mode (curve 2) is nonzero and finite for $\omega=0$ and stabilized at $\omega=3$. The rest of the modes (curve number matches the number of fashion) have a cut-off frequency, which can be easily determined from Figure 11 (the dependence of the wave number α of the frequency), and decreasing, stabilized (seen 3 and 4 modes) at top speed.

Figure 12 shows the evolution of the first waveform with the frequency ω for frequencies $\omega=0,5;\ 1;\ 5$ и 20. Pronounced localized form with increasing frequency. Figure 13-16 shows the own forms respectively for 2-4 modes of vibration for different frequencies: $\omega = 1, 2, 3$ and 4 (the number of grid points corresponds to the number form). And here there are localized forms in the area of thin wedge edge. Figure 16 gives an idea of the degree of localization of the forms at the frequency ω = 1, obviously, the lower the number of forms, the stronger it is localized at the edge of the wedge.

Figures 17-19 shows the spectral curves of the first three events in the case of the nonlinear dependence of the thickness of the strip from the coordinates x_2.

$$h(x_2) = h_0 + hx_2{}^p, \qquad 0\langle x_2 \le 1,$$

where the parameter ρ It was assumed to be 1.5; 2; 2.5; 3 (curves 1, 2, 3, 4, respectively, curve "0" corresponds to p = 1 - linear relationship).

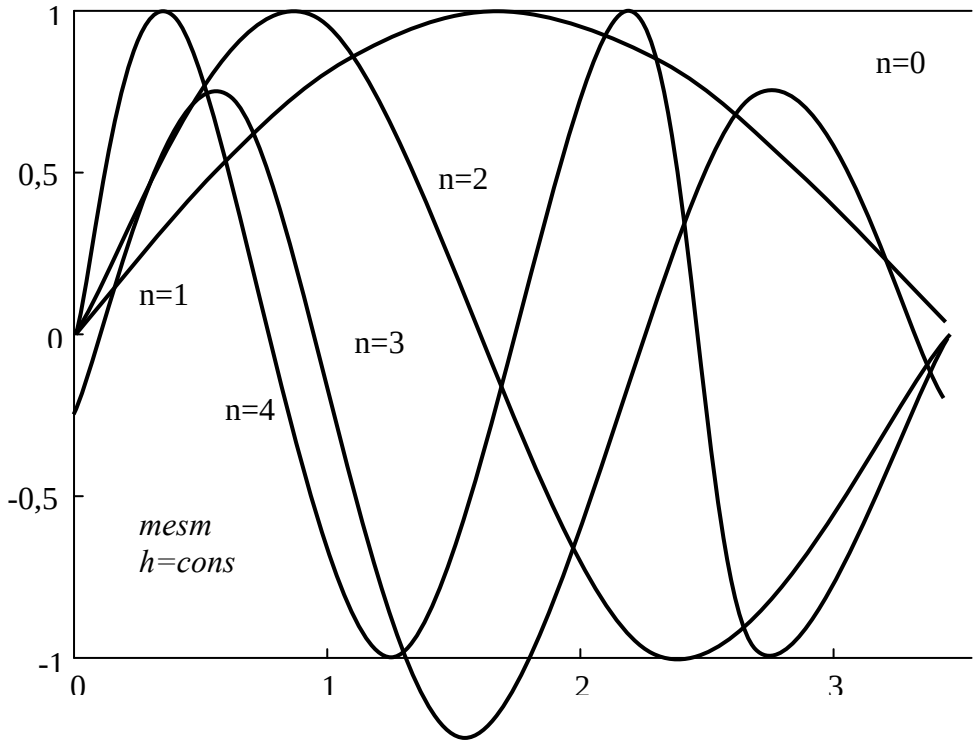

Figure 4 : Form for the higher oscillation modes

From the equation of "0" with the remaining curve shows that they are located on the horizontal high-frequency asymptote, monotonically to zero. The midrange is observed a characteristic peak which is shifted to lower frequencies with an increase in "p". In accordance with the charts of waveforms at ris.20-22 quicker and localization of motion near the edge of the waveguide.

Thus, it can be concluded that the phase velocity of the wave in the localized waveguide edge is defined as the frequency increases the rate of change of thickness in the vicinity of the sharp edge.

Figures 23-28 illustrate the solution of the stationary problem for a wedge-shaped waveguide with a linear change in the thickness of the coordinates x_2 depending on the location of the excitation zone, from which it is clear that the main contribution to the resulting solution brings a sharp edge excited waveguides. Analysis of figures 23-25 shows that, if aroused sharp edge of the wedge is raised mostly first oscillation mode, and ratio α_1 increases with increasing frequency.

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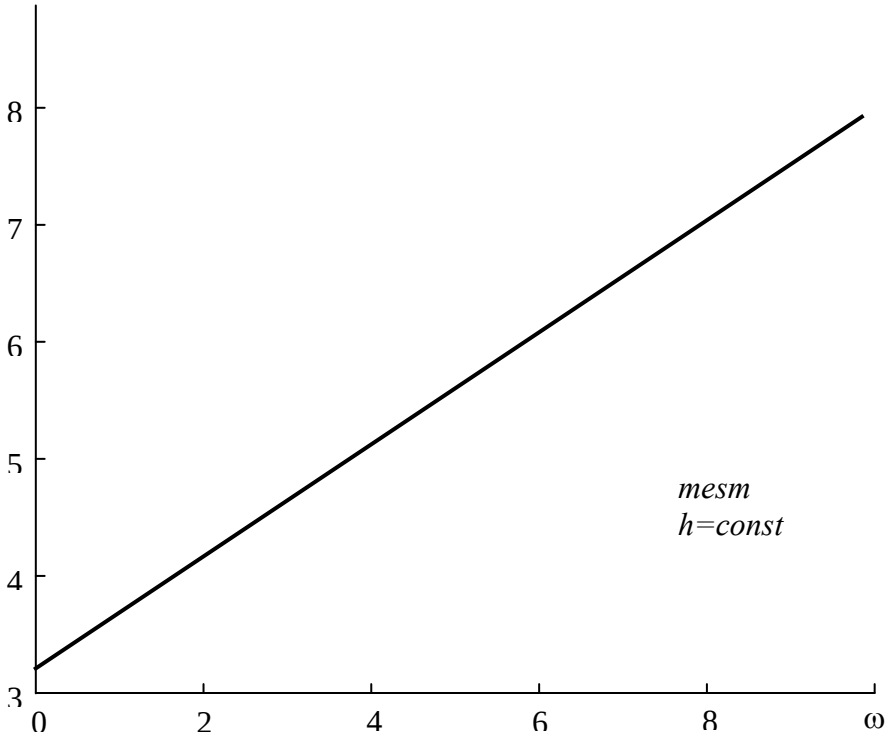

Figure 5 : The amplitude of the excited mode depending on the frequency

The amplitude of the remaining modes is not more than 5% from the first (ω = 10). Upon excitation of the central waveguide portion (fig.26 and fig.27) the amplitude of oscillation is 20-50 times lower than when excited sharp edge and decreases with increasing frequency. On Figure 28 shows the factors driving modes when the excitation zone does not capture any region, the center of the waveguide. The oscillation amplitude is also here oscillations 20-50 times less than in the first case. 26-28 of the drawings can be made and another conclusion that in this case the entire frequency range can be divided into zones, in which one of the modes propagates mainly. For example, in the case of Figure 25:

$0 \le \omega \le 2$ *I* fashion; $2 \le \omega \le 5$ *II* fashion; $5 \le \omega \le 10$ *III* fashion, t. i.

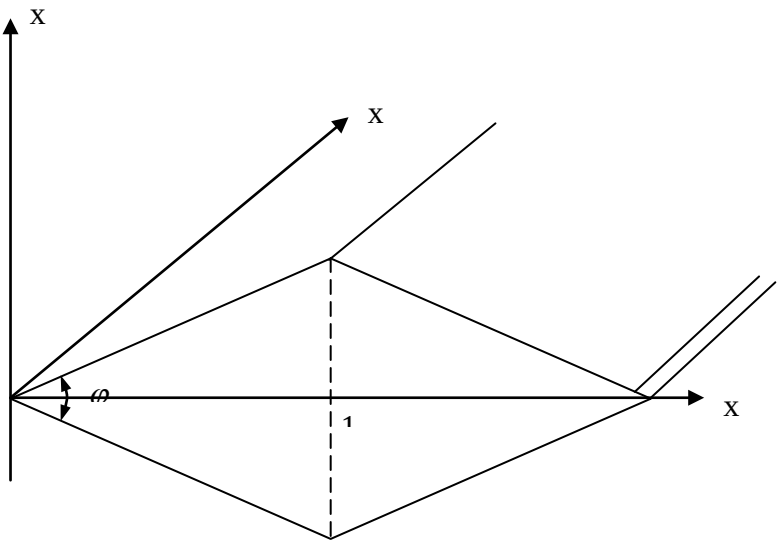

Figure 6 : The settlement scheme

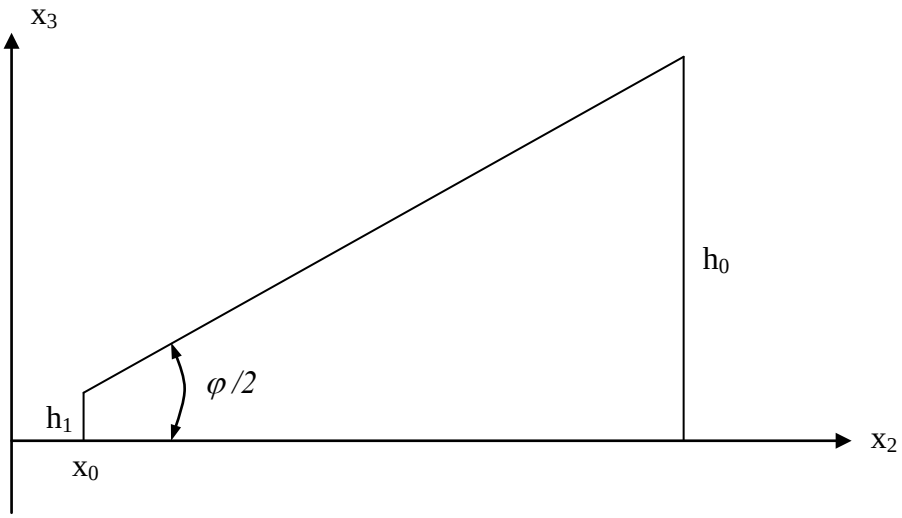

Figure 7 : The settlement scheme

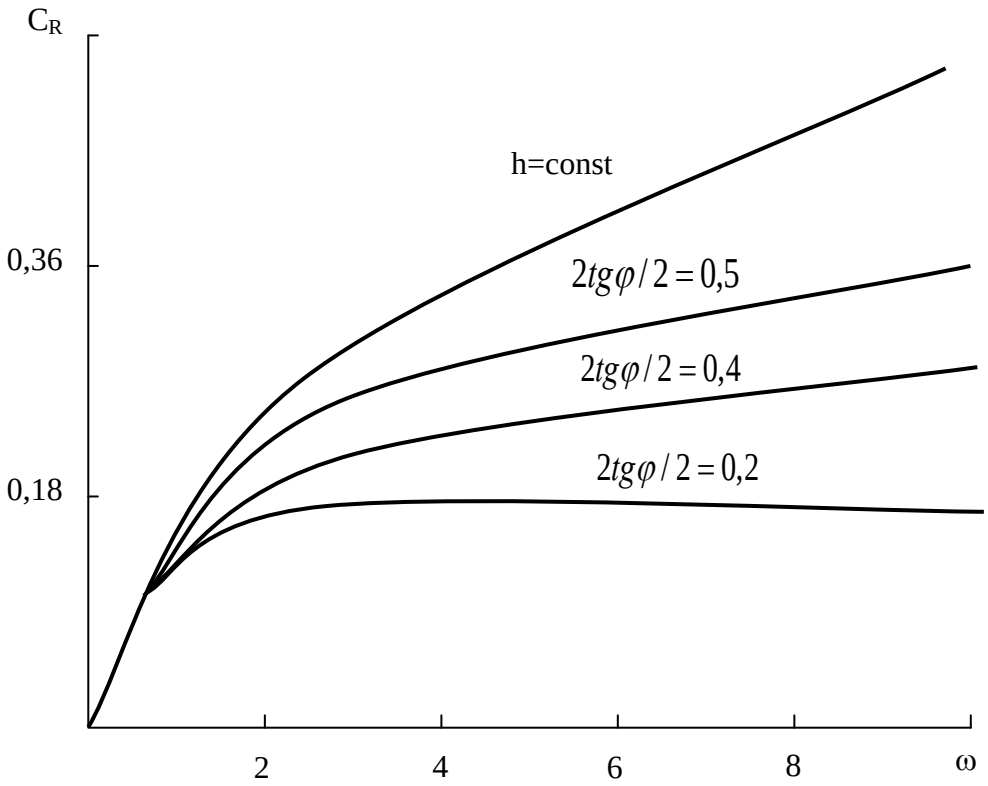

Figure 8 : The dependence of the real and imaginary parts of the phase velocity on frequency

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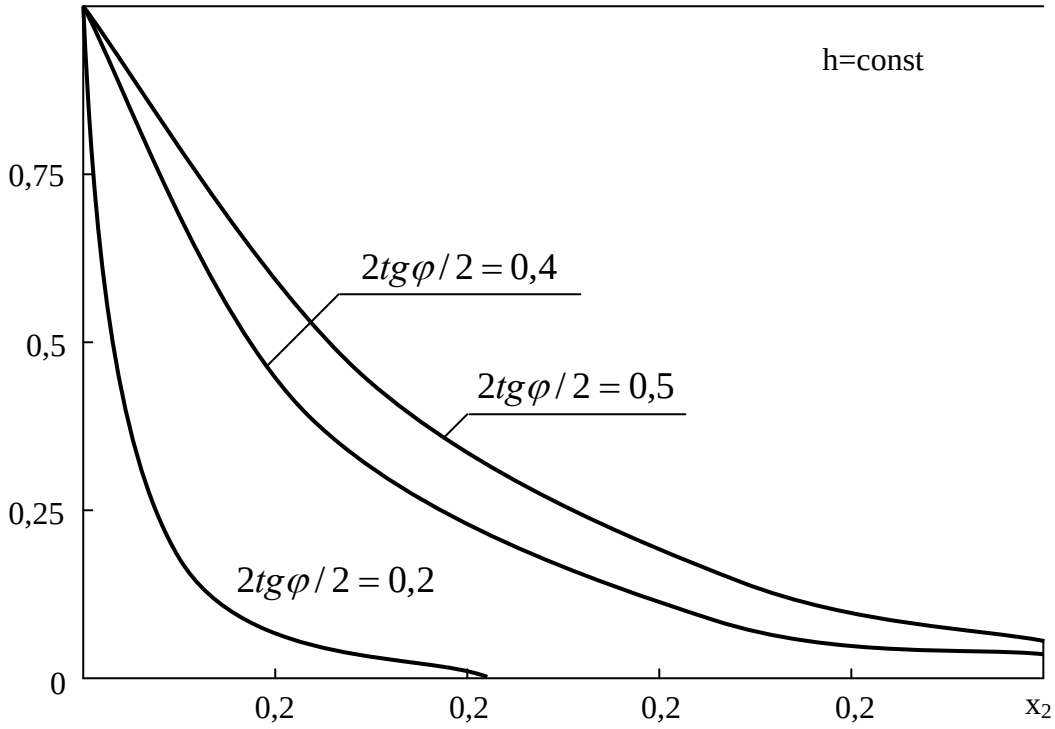

Figure 9 : The forms of the coordinate fluctuations X_2

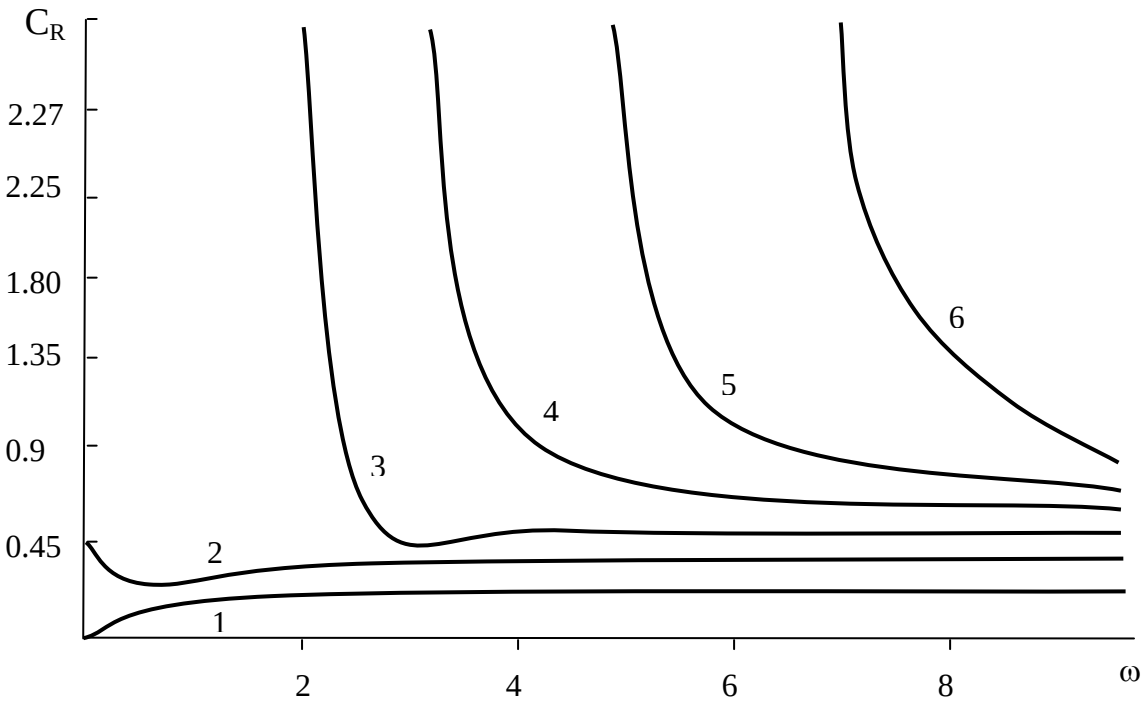

Figure 10 : Dependence of the real and imaginary parts of the phase velocity on frequency

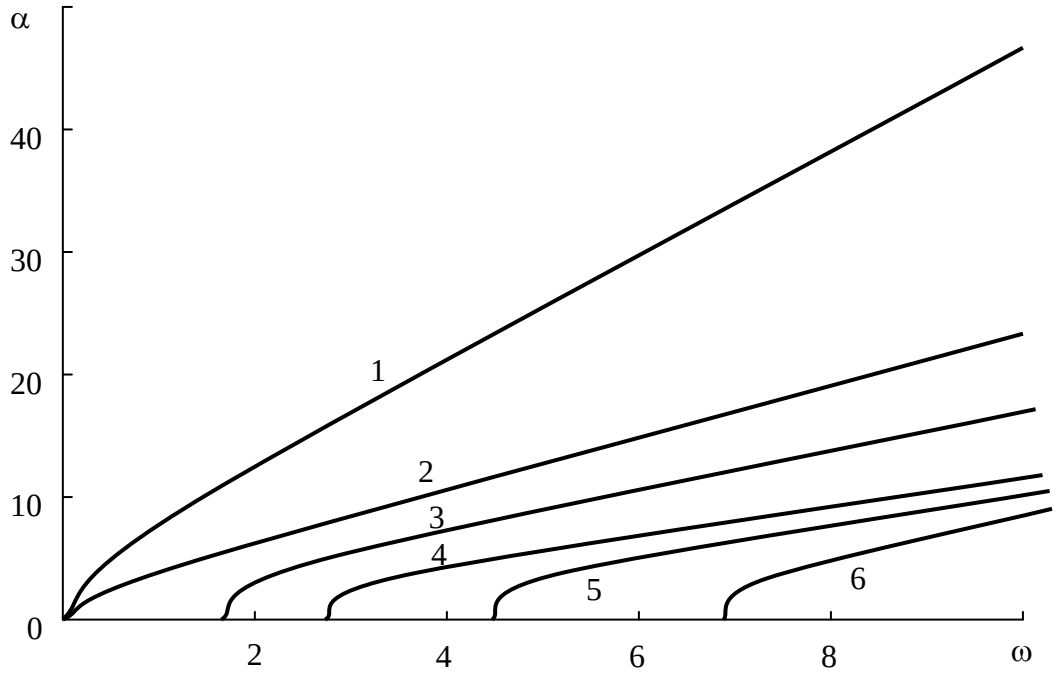

Figure 11 : Dependence of the frequency of the wave

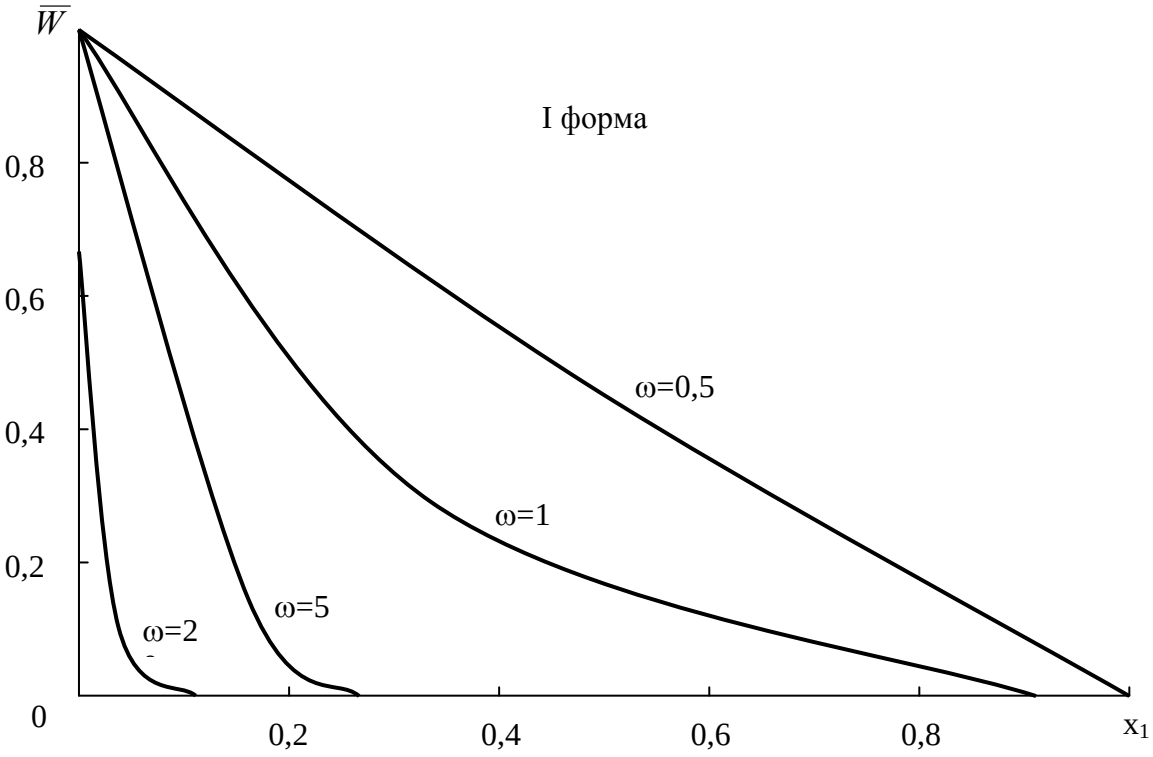

Figure 12 : Changing the shape of the coordinate fluctuations X_1

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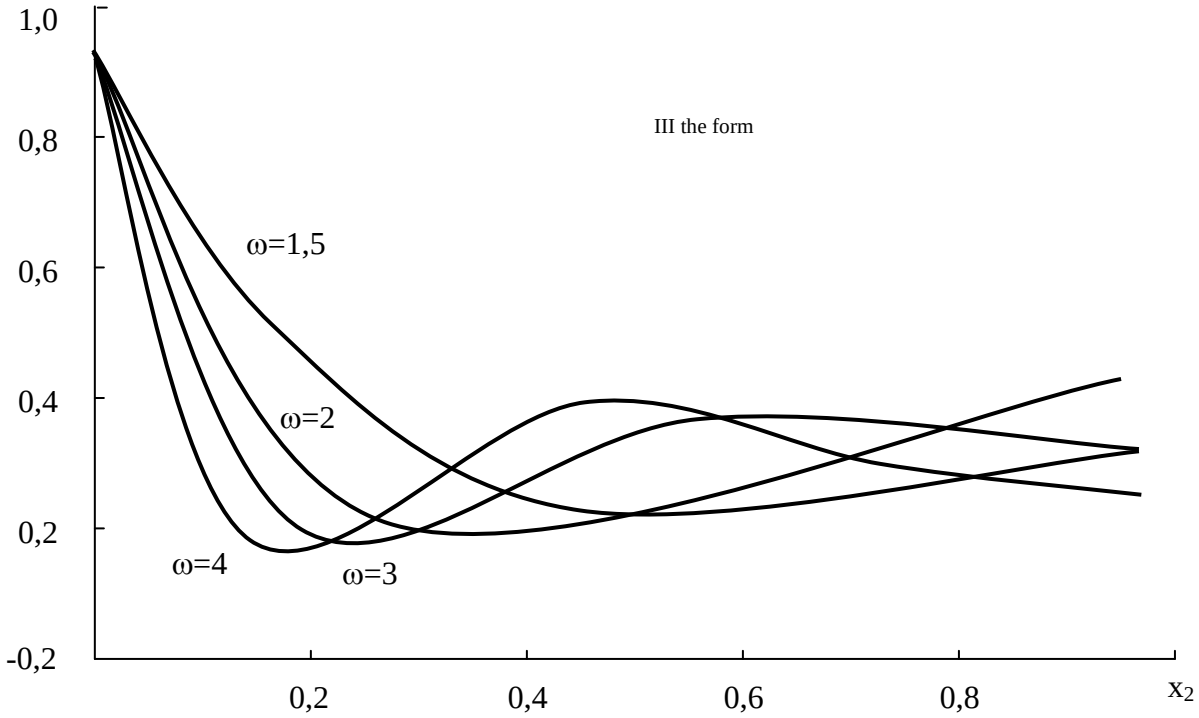

Figure 13 : Changing the shape of the coordinate fluctuations X_2

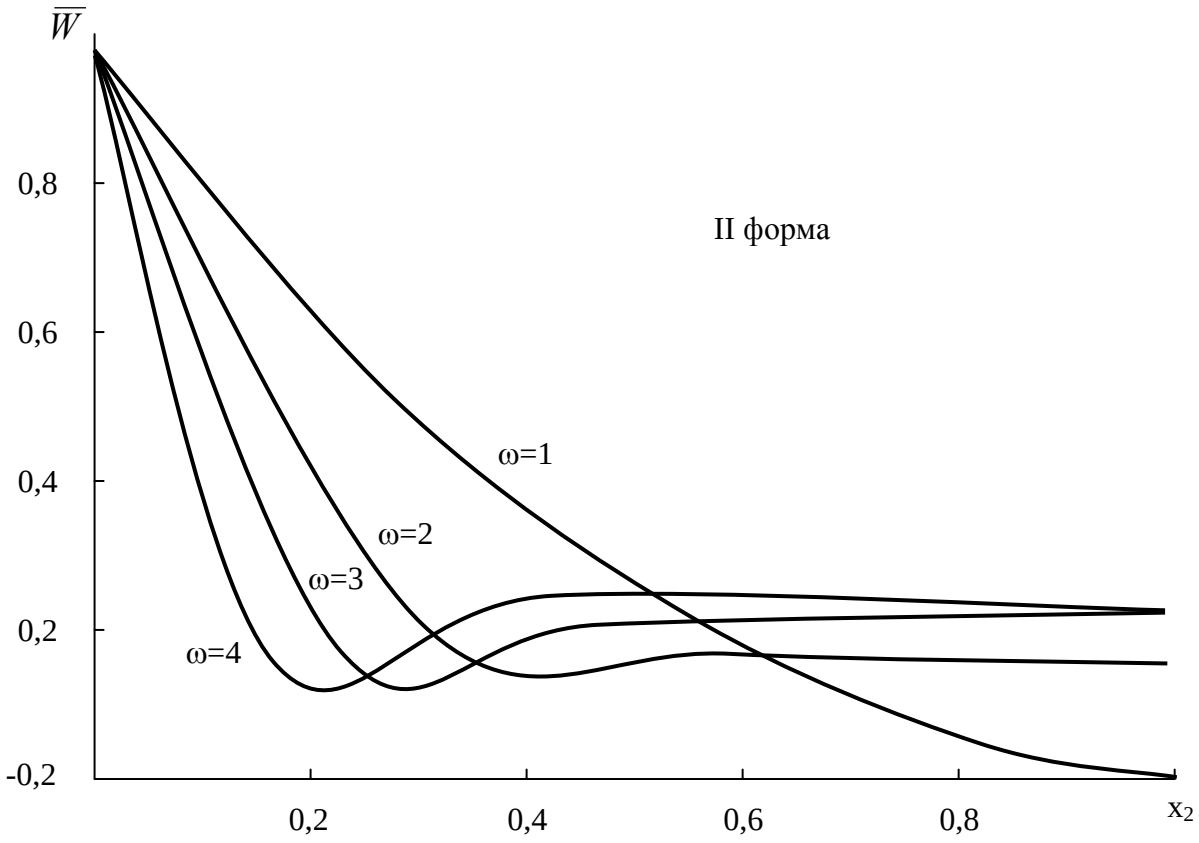

Figure 14 : Changing the shape of the coordinate fluctuations

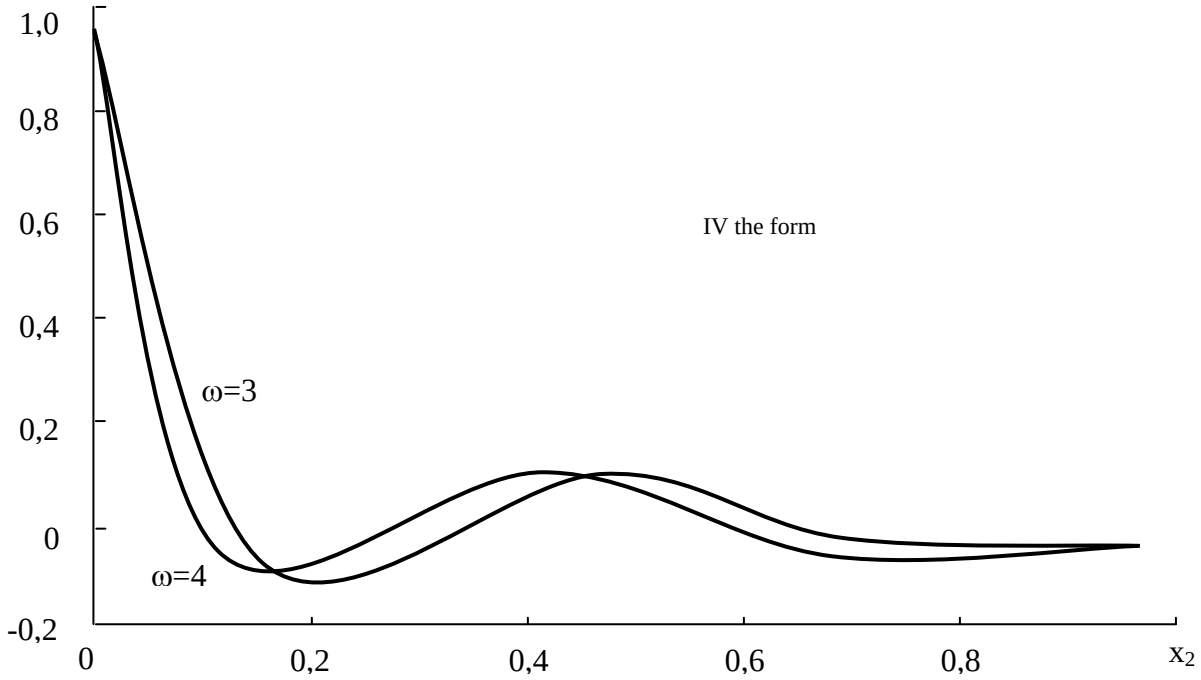

Figure 15 : Changing the shape of the coordinate fluctuations

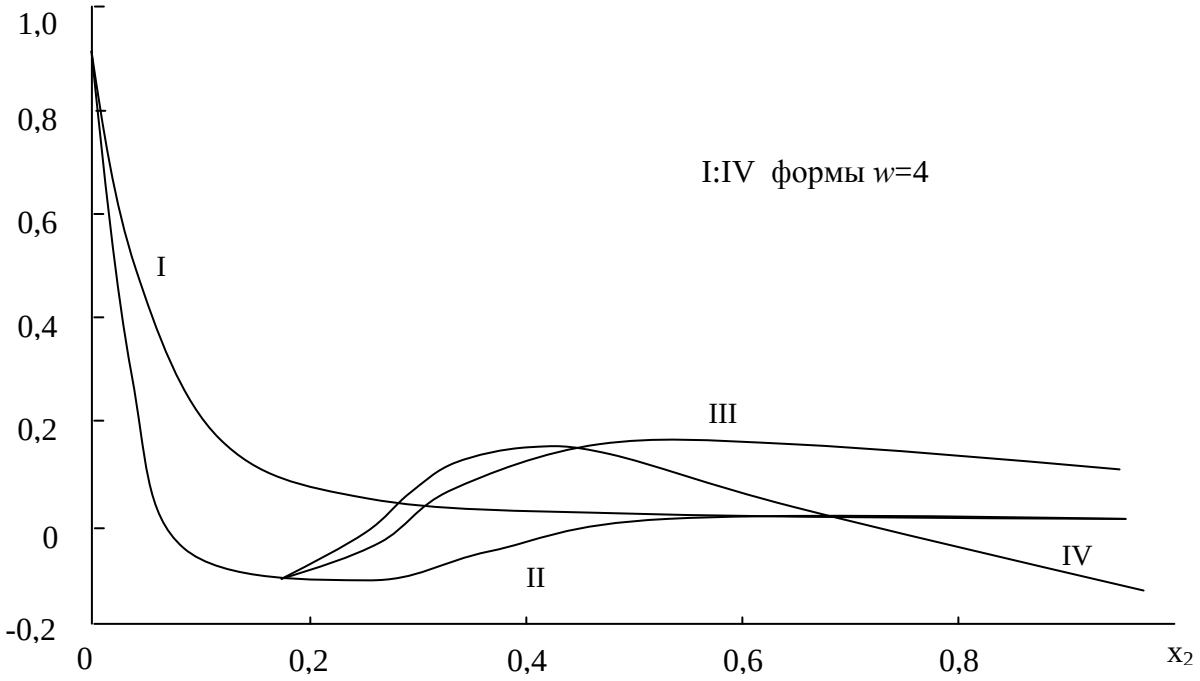

Figure 16 : Changing the shape of the coordinate fluctuations

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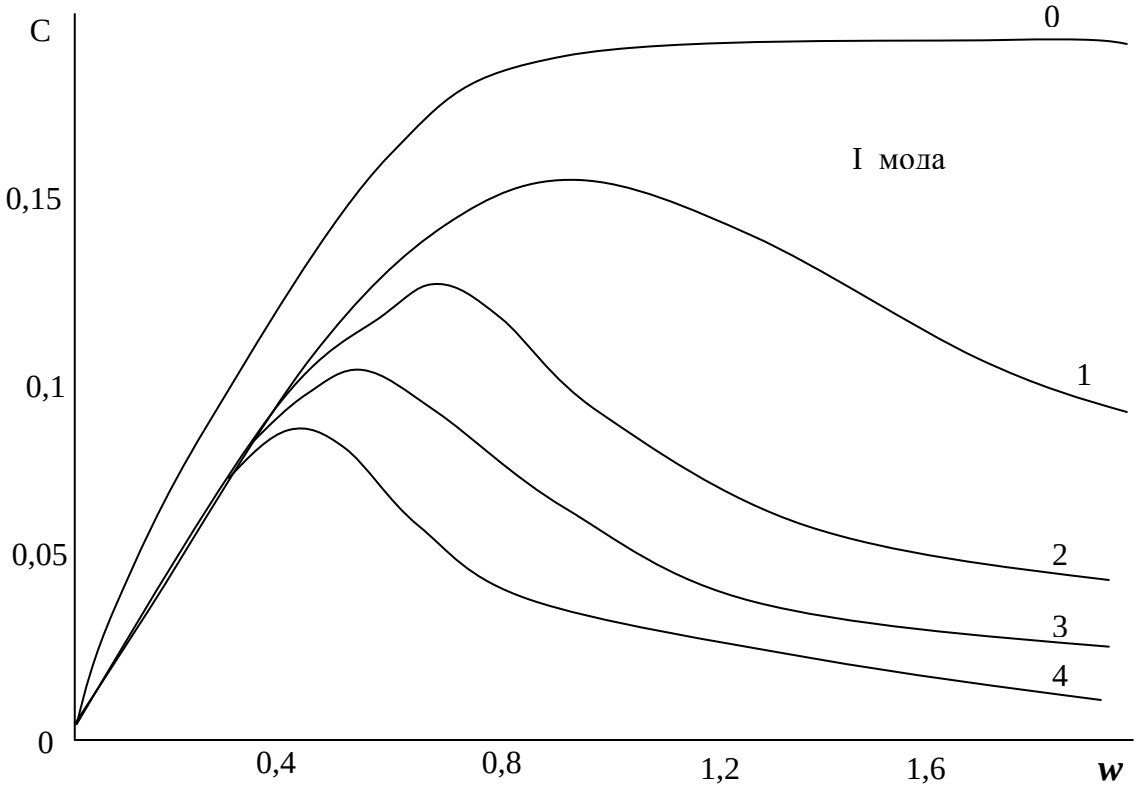

Figure 17 : Changing the phase velocity as a function of frequency

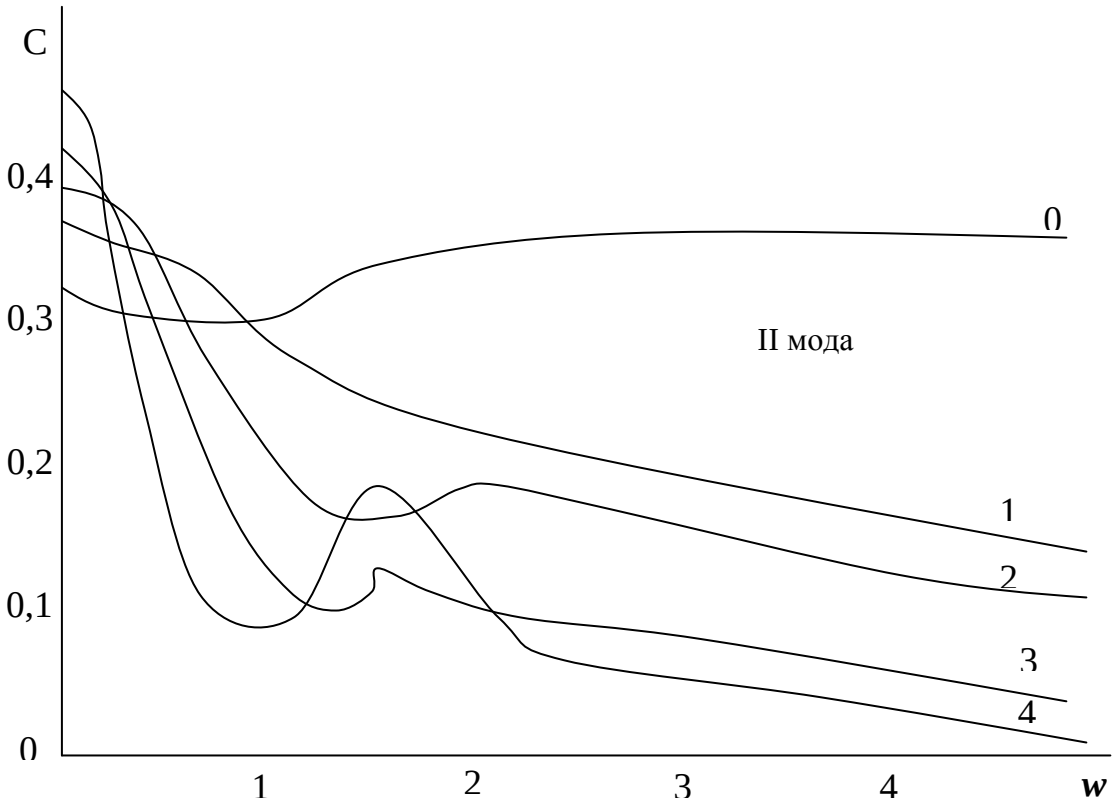

Figure 18 : Changing the phase velocity as a function of frequency

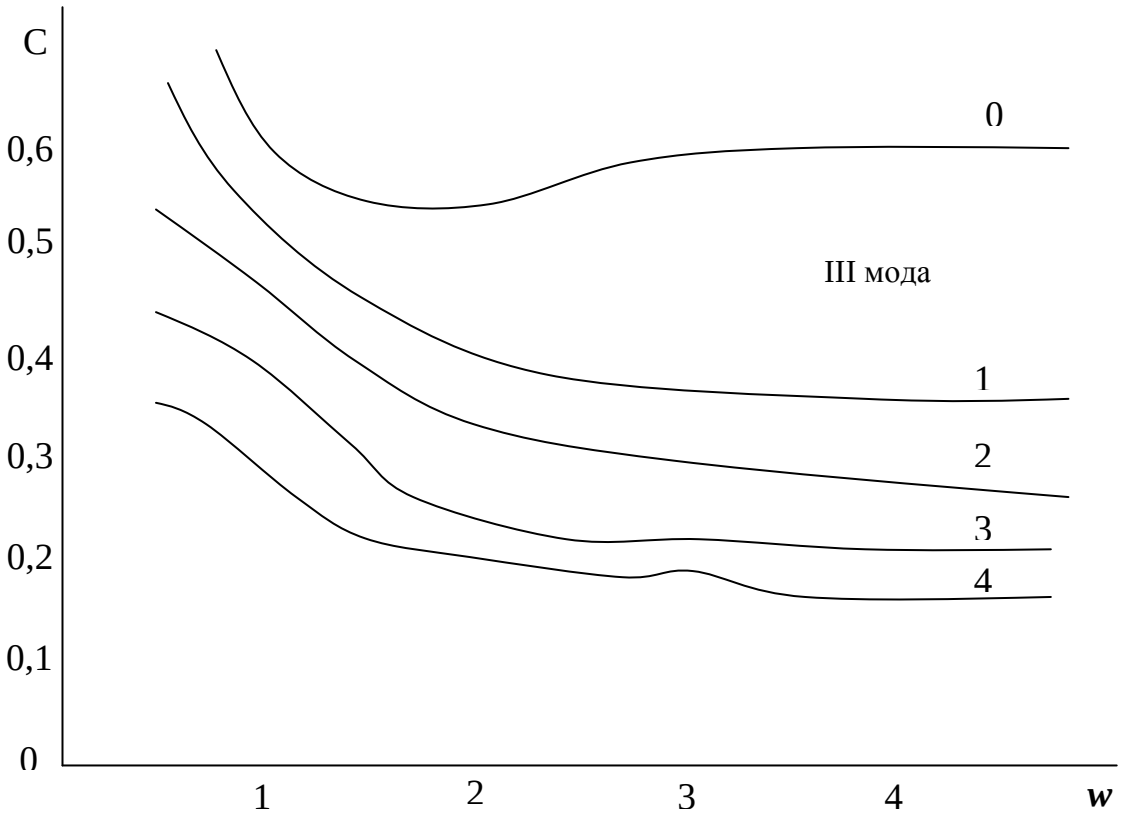

Figure 19 : Changing the phase velocity as a function of frequency

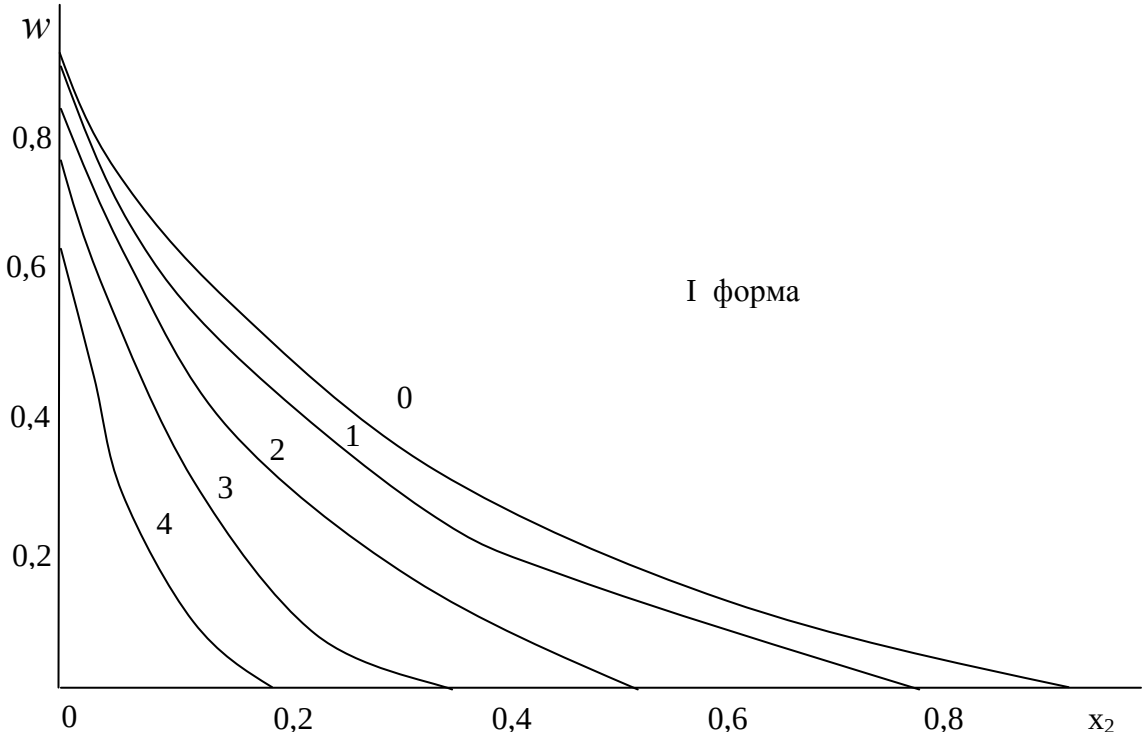

Figure 20 : Changing the shape of the coordinate fluctuations

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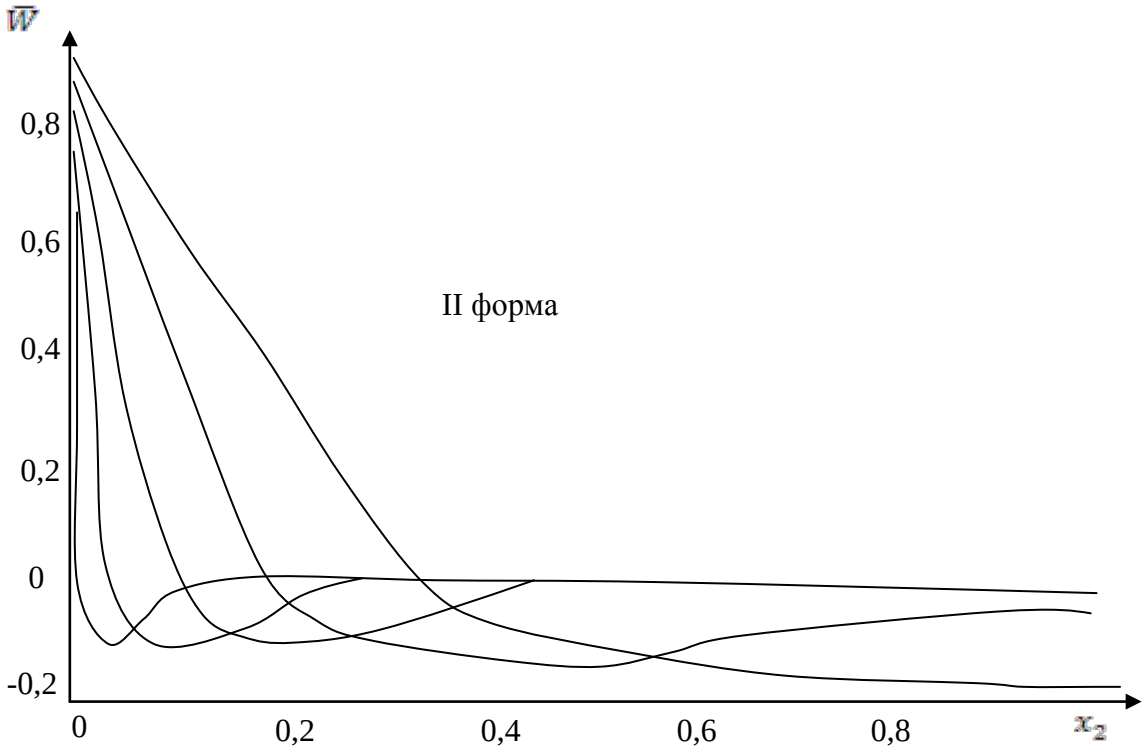

Figure 21 : Changing the shape of the coordinate fluctuations

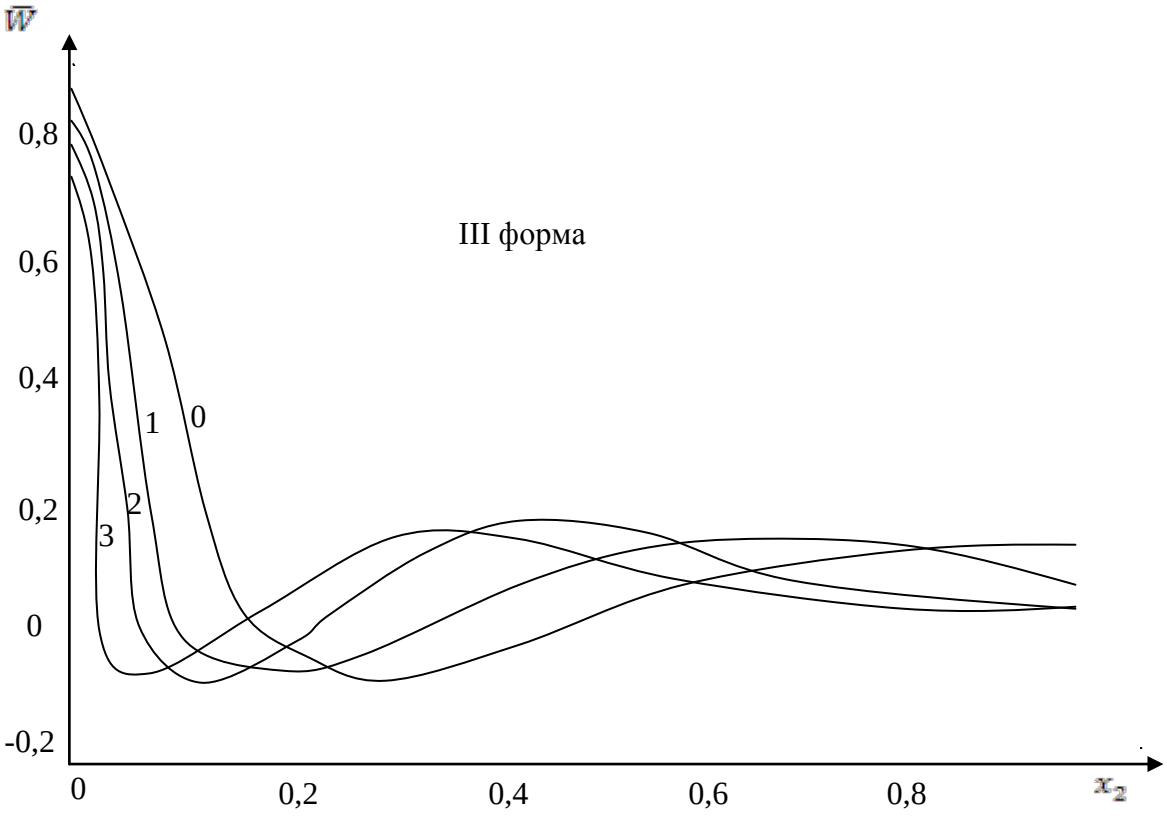

Figure 22 : Changing the shape of the coordinate fluctuations

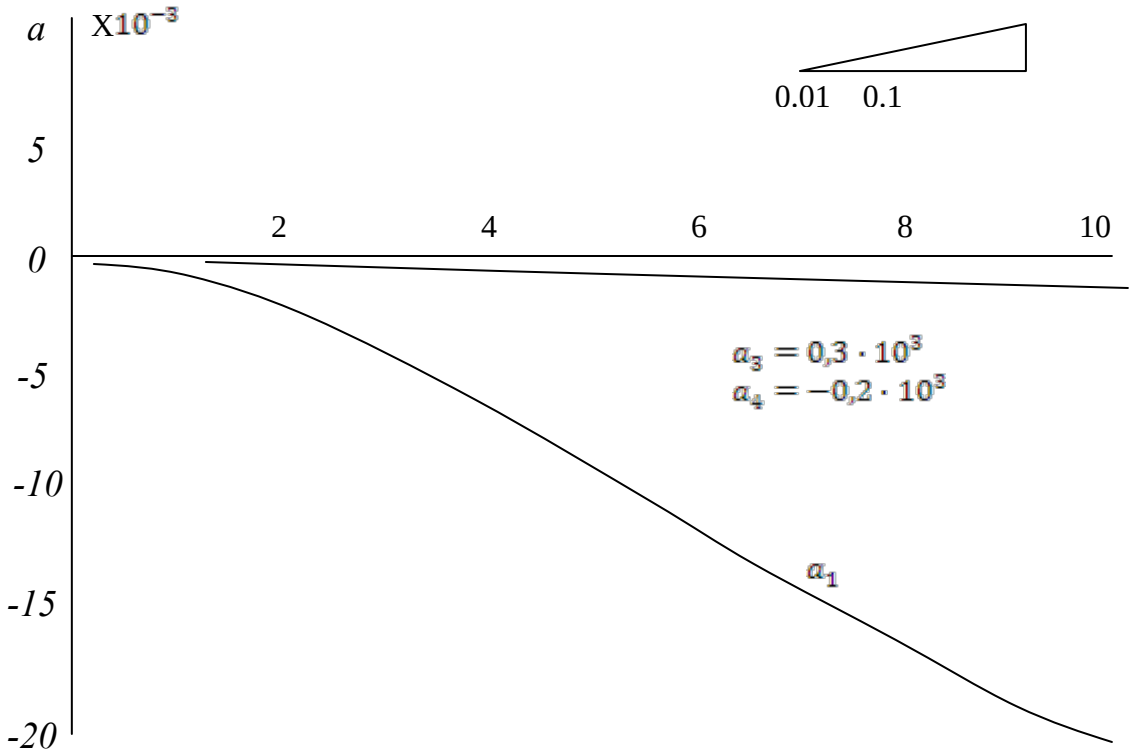

Figure 23 : The change factor *a* depending on the frequency

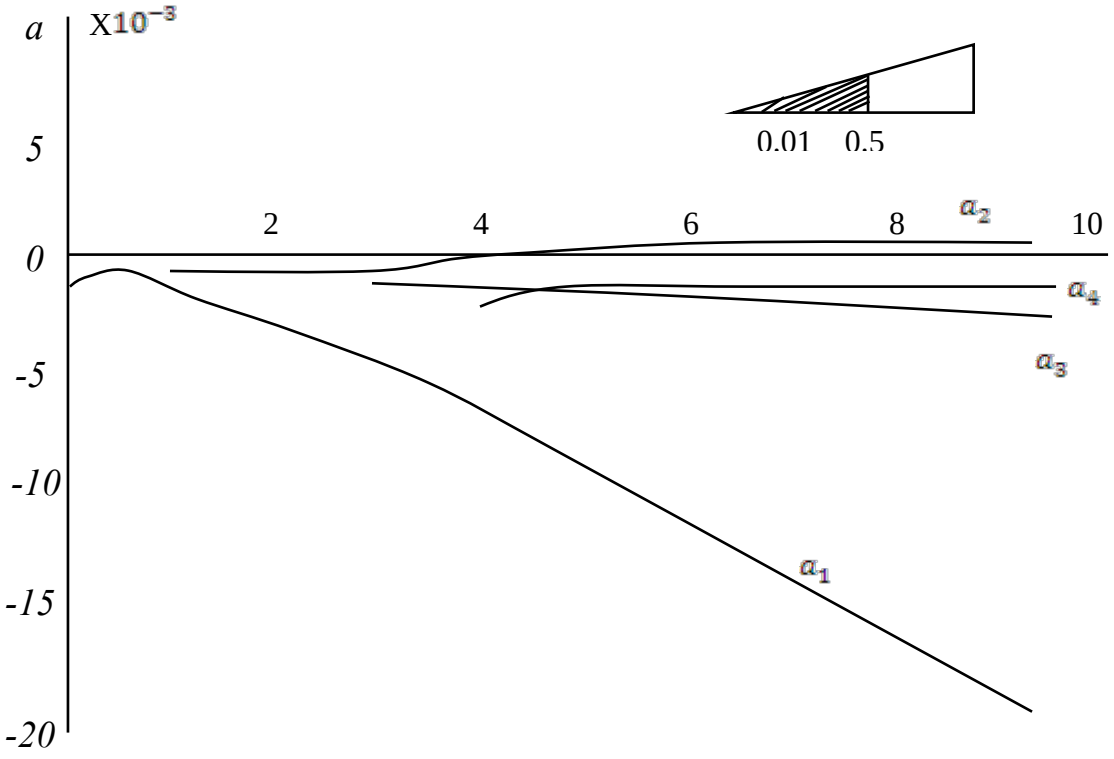

Figure 24 : The change factor *a* depending on the frequency

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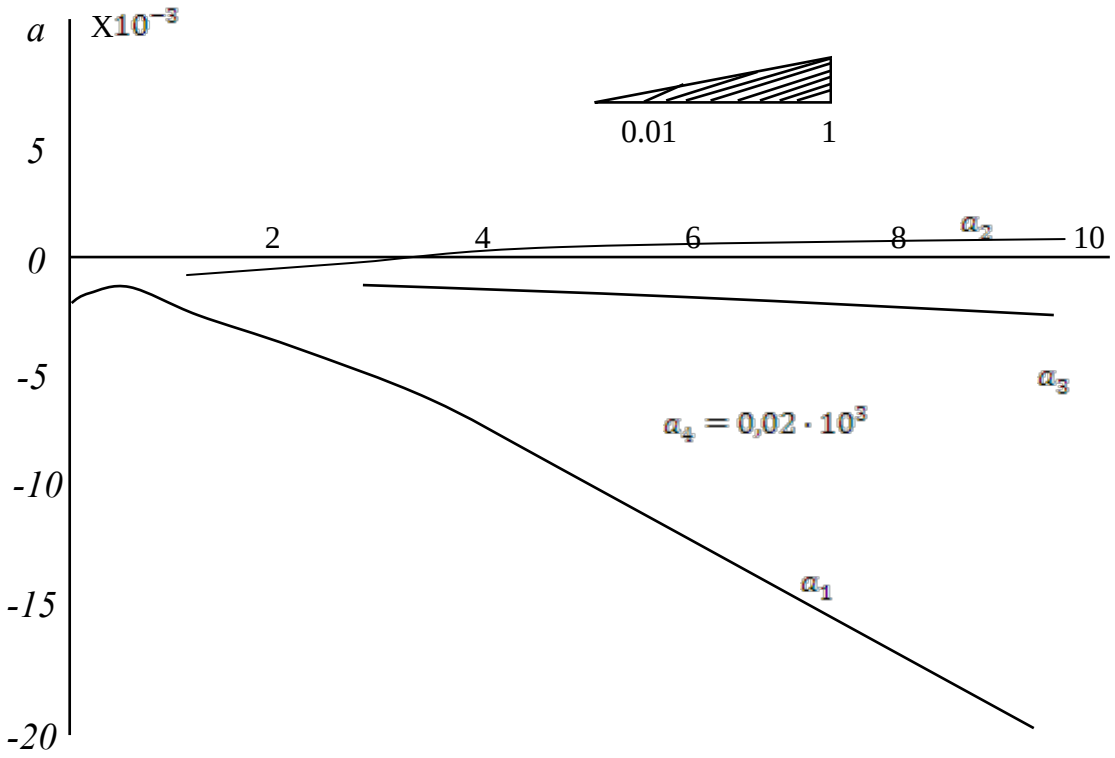

Figure 25 : The change factor a depending on the frequency

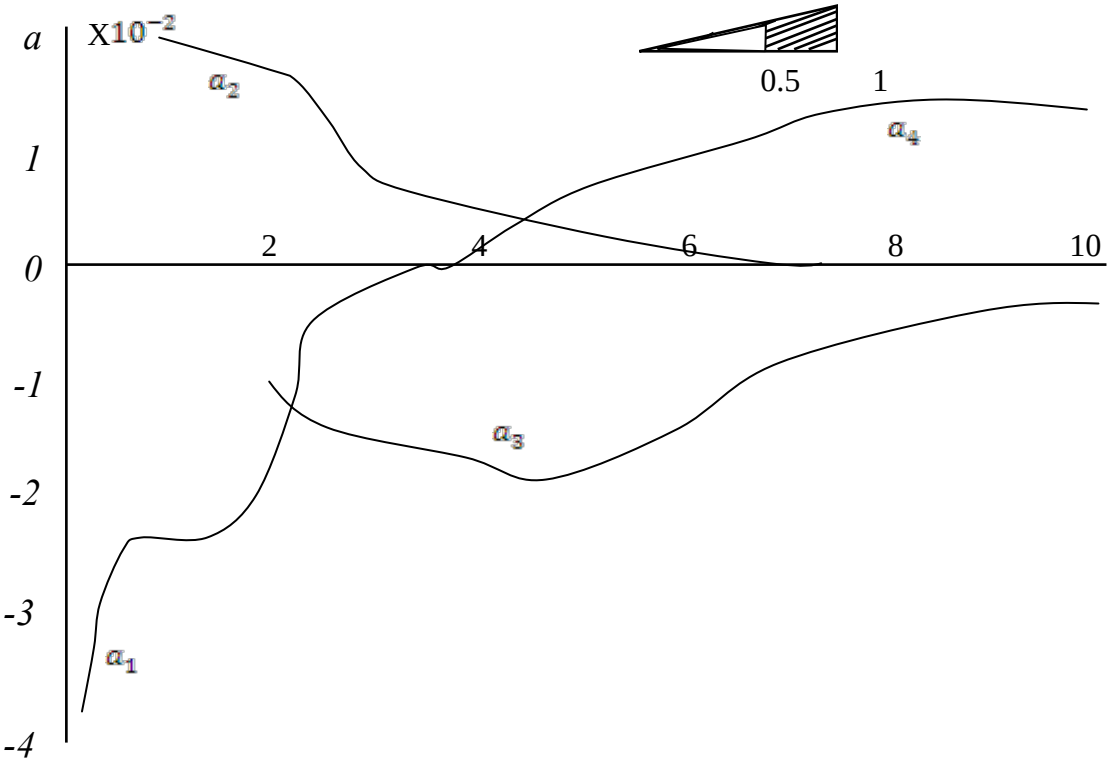

Figure 26 : The change factor a depending on the frequency

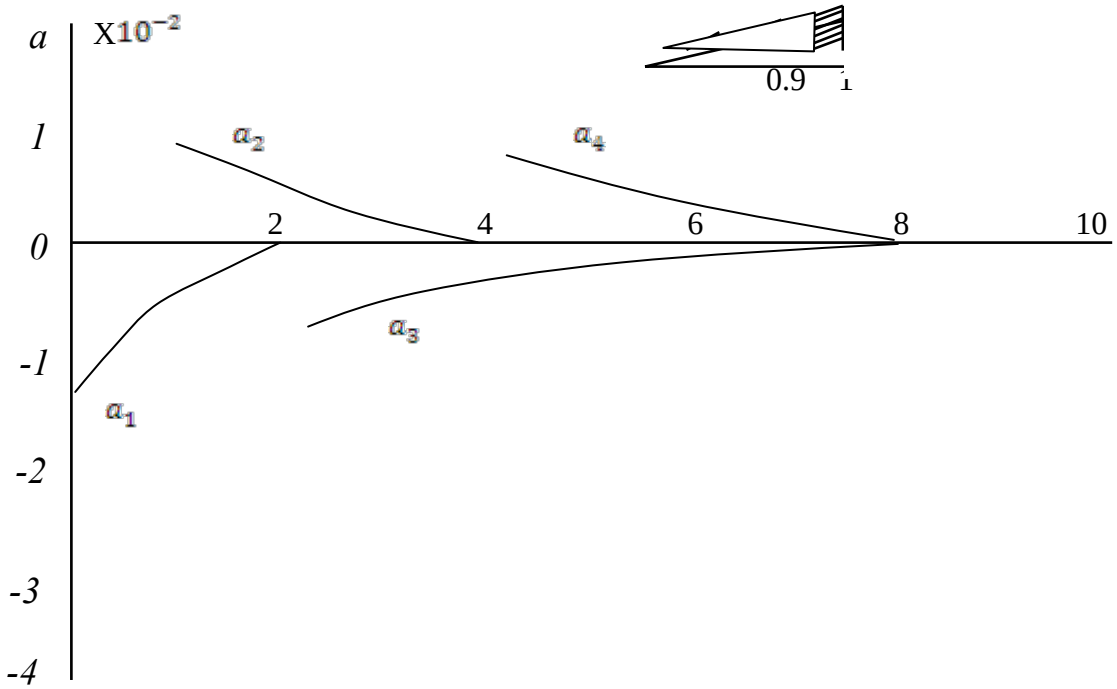

Figure 27 : The change factor *a* depending on the frequency

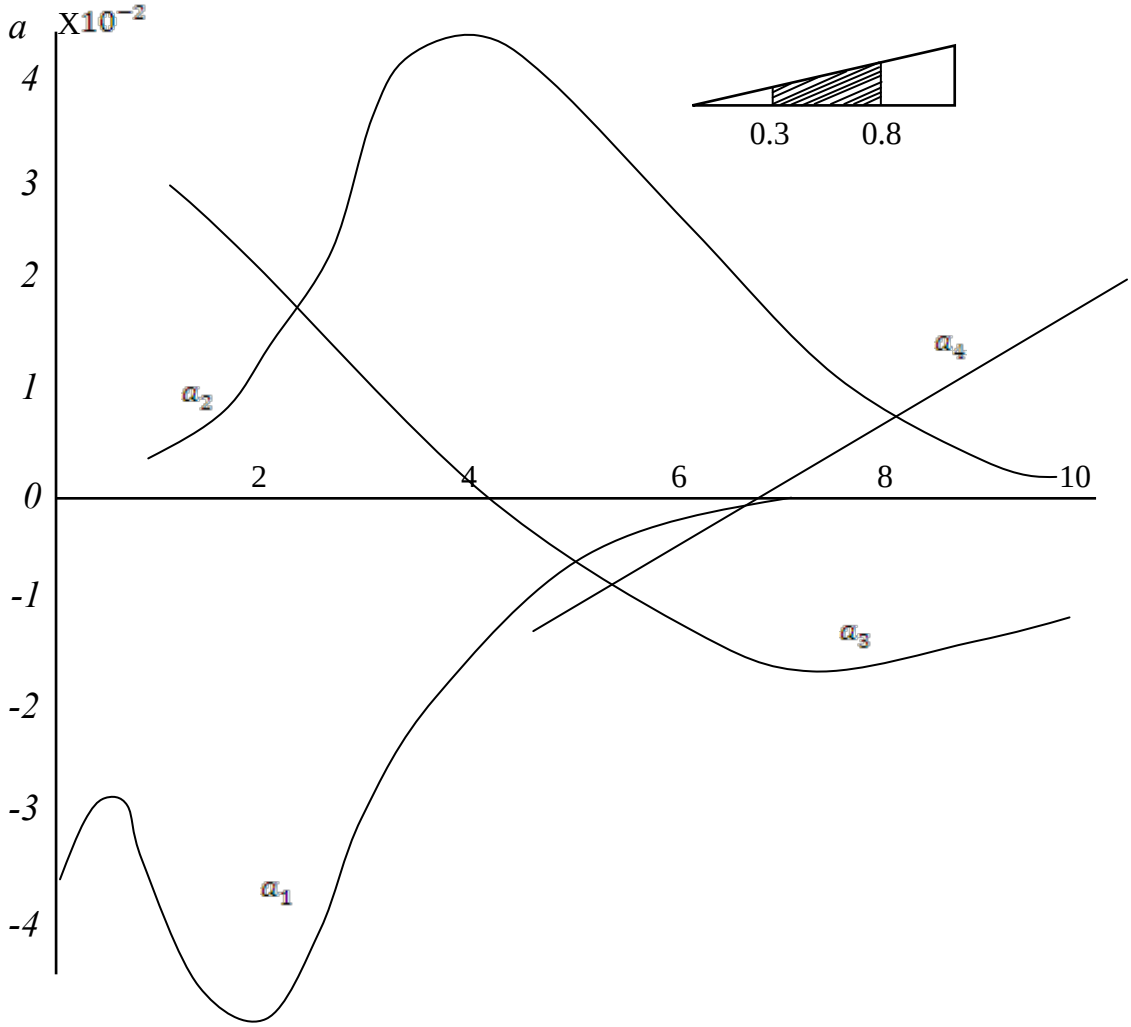

Figure 28 : The change factor *a* depending on the frequency

Notes

On the basis of these results the following conclusions:

- On the basis of the variation equations of elasticity theory, the mathematical formulation of the problem of wave propagation in the extended plates of variable thickness. A system of differential equations with the appropriate boundary conditions.
- Showing that the square of the wave number for own endless bands of variable thickness in any combination of the action of the boundary conditions.
- Obtained spectral problem is not self-adjoin. Built conjugate problem for her. Coupling system consists of ordinary differential equations with the appropriate boundary conditions. With the help of the Lagrange formula obtained conditions biorthogonality forms. The problem is solved numerically by the method of orthogonal shooting S.K. Godunov in conjunction with the method of Muller.
- Analysis of the data shows that the region with the imaginary theory of Kirchhoff-Love to the plate of constant thickness is limited by the low frequency range. At high frequencies, when wavelength comparable to fashion or less than the thickness of the plate theory Kirchhoff -Love does not yield reliable results.
- For the phase velocity of propagation modes in the band of variable thickness, there is final repartition unlike the constant cross-section strip.

Notes

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