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Effect of First Order Chemical Reaction for Coriolis Force and Dust Particles for Small Reynolds Number in the Atmosphere Over Territory

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Effect of First Order Chemical Reaction for Coriolis Force and Dust Particles for Small Reynolds Number in the Atmosphere Over Territory

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Abstract- In the atmospheric boundary layer there are horizontal pressure gradient and Coriolis force to the velocity distribution signify that, the horizontal component of the velocity in the boundary layer turns left (right) with increasing height in the Southern (Northern) Hemisphere, downward (upward) motion occurs on the windward (lee) side of the mountain, and downward (upward) motion also occurs on the slope to the right (left) of the geostrophic wind in the Northern Hemisphere, whereas in the Southern Hemisphere upward (downward) motion occurs on the slope to the left (right) of the geostrophic wind. Using Navier-Stokes equations for three-dimensional stationary flows, hydrostatic and continuity equations, the effects of first order reactant of Coriolis force and dust particles for small Reynolds number in the atmospheric boundary layer over territory is obtained.

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I. Introduction

In recent years, a large number of analytical and numerical models for the study of the mean and turbulent motions in the planetary boundary layer under various thermal stratifications have been constructed. The main difficulty of solving the Navier-Stokes equations exactly is the contribution of the nonlinear terms representing fluid inertia which then troubled the conventional analysis in general case. Lyberg and Tryggeson [1, 2007] discussed the analytical solution of the Navier-Stokes equations for internal flows. Chae and Choe [2, 1999] studied the regularity of solutions to the Navier-Stokes equations. Nugroho *et al.,* [3, 2009] explained on a special class of analytical solutions to the three-dimensional incompressible Navier-Stokes equations. Wang [4, 1991] studied the exact solutions of the steady Navier-Stokes equations. Thailert [5, 2005] explained a one class of regular partially invariant solutions of the Navier-Stokes equations. Shapiro [6, 1993] studied the use of an exact solution of the Navier-Stokes equations in a validation test of a three- dimensional nonhydrostatic numerical model. Rajagopal [7, 1984] explained a class of exact solutions to the Navier-Stokes equations. Nugroho [8, 2013] discussed on analytical solutions to the three-dimensional incompressible Navier-Stokes equations with general forcing functions and their relation to turbulence. Bkar Pk *et al.*, [9, 2012] also studied the decay of energy of MHD turbulence for four-point correlation. He [10, 2013] studied the decay of dusty fluid MHD turbulence for four-point correlation in a rotating system. He [11,2013] also discussed the decay of MHD turbulence prior to the ultimate phase in presence of dust particle for four-point correlation. Kao [12, 1976] discussed a model for turbulent diffusion over terrain. Velocity distribution in the atmospheric boundary layer over a flat surface was first studied by Ekman [13, 1905] who assumed a constant eddy viscosity in the planetary boundary layer, and obtained an exact solution to the Navier-Stokes equations for the balance between Coriolis, pressure-gradient and viscous forces. However, most of these investigations have emphasized on flows in the boundary layer over flat surfaces.

For flows in the planetary boundary layer over flat surfaces, the mean motion may be assumed to be homogeneous in the horizontal, therefore, the Navier-Stokes equations become linear Ekman and the motion is horizontal. However, when the homogeneity in the topographical configuration of the earth's surface is taken into account, the motion is three-dimensional and the equations of motion are no longer linear. Because of increasing concern about atmospheric pollution in many population centers, industrial and power plants, which are located in valleys and terrain and since atmospheric motion is the mechanism for the transport and dispersion of pollutants, there is a growing interest in the atmospheric motion and pollution in these regions. Kao [14, 1981] also discussed an analytical solution for three dimensional stationary flows in the atmospheric boundary layer over terrain. In this work we have obtained an analytical solution to the Navier-Stokes equation for small Reynolds number and dust particles in the atmosphere over mountain-terrain and to analyze the effects of dust particle and Coriolis force on the velocity distribution in the boundary layer due to first

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order chemical reaction and the result has been discussed graphically.

II. Mathematical Formulation

Consider stationary flows of small Reynolds number and dust particles in the planetary and surface boundary layers over terrain. The Navier-Stokes equations, hydrostatic and continuity equations may be expressed as

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=f(v-v_g)+f^*(v-v_g)+R(v-v_g)+K\frac{\partial^2 u}{\partial z^2} \tag{1}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=-f(u-u_g)-f^*(u-u_g)-R(v-v_g)+K\frac{\partial^2 v}{\partial z^2} \tag{2}$$

$$\frac{\partial p}{\partial z}=-\rho g,$$

and

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0.$$

where

p is the pressure,
g is the gravity acceleration,
ρ is the density,
f is the Coriolis parameter,
f^* is dust particles parameter,
R is first order chemical reaction,
K is the coefficient of eddy diffusivity assumed constant as a first approximation and u, v, w are the components of the velocity of x, y, z respectively.

The equation of motion in the surface boundary layer for stationary flow can be articulated as

$$\frac{\partial}{\partial z}|u+iv|=\frac{u_*}{k}\left(\frac{1}{z}+\frac{a}{L}\right) \tag{3}$$

where

$$i=(-1)^{\frac{1}{2}},\ \ u_*=\left(\frac{\tau}{\rho}\right)^{\frac{1}{2}},$$

a is a constant which has been experimentally determined,
u_* is the friction velocity,
L is Monin-Obukhov length and
k the von Karman's constant.

The boundary conditions for this model are

$$u=v=w=0,\quad at\ z=h(x,y) \tag{4}$$

$$u\to u_g, v\to v_g,\quad as\ z\to\infty \tag{5}$$

$$u+iv=A\frac{\partial}{\partial z}(u+iv), at\ z=h(x,y)+h_s \tag{6}$$

where

$h(x,y)$ is the topographical configuration of the terrain,
h_s is the thickness of the surface boundary layer,
$G=u_g+iv_g$ is the geostrophic wind velocity and
A is a constant.

Because at the solid boundary i.e., $z=h(x,y)$, all components are zero. If z tends to infinity then the velocity tends to the geostrophic velocity. As a result, the wind direction coincides with the wind stress at the lower boundary of the planetary boundary layer.

Combining equations (1) and (2), we get

$$\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}+w\frac{\partial}{\partial z}\right)(u+iv-G)=\left(K\frac{\partial^2}{\partial z^2}-if-if^*-iR\right)(u+iv-G) \tag{7}$$

It has the following solution which satisfies the boundary conditions (5):

$$(u+iv)=G+(B_r+iB_i)\exp\{-(1+i)\mu\delta\} \tag{8}$$

$$w = [u_g + \exp\{-\mu\delta\}\{B_r \cos\mu\delta\}] + B_i \sin\mu\delta \frac{\partial h}{\partial x} + [(v_g + \exp\{-\mu\delta\})$$

$$\{B_i \cos\mu\delta] - B_r \sin\mu\delta\}\frac{\partial h}{\partial y} \tag{9}$$

where B_r and B_i are constants, and $\mu = (\frac{f+R+f^*}{2K})^{\frac{1}{2}}$.

In view of the fact that wind has no direction in the surface boundary layer, its velocity in the surface boundary layer may be expressed as

$$u + iv = |u + iv|e^{i\alpha} \tag{10}$$

At the lower boundary of the planetary boundary layer, equation (8) yields

$$(u + iv)_{z=h(x,y)+h_s} = G + (B_r + iB_i) \tag{11}$$

Combining equation (10) and (11) we get

$$(B_r + iB_i) = |u + iv|_{z=h(x,y)+h_s} e^{i\alpha} - G, \tag{12}$$

Then equation (8) can be written for dust particle as

$$(u + iv) = G + \{|u + iv|_{z=h(x,y)+h_s} e^{i\alpha} - G\} \times \exp\{-(1+i)\mu\delta\} \tag{13}$$

for $z \geq h(x,y) + h_s$.

where α is the angle made between the geostrophic wind and the wind in the surface boundary layer.

Applying boundary conditions from equations (6) to equation (13) we can write

$$|u + iv|_{z=h(x,y)+h_s}\{A\mu(\cos\alpha - \sin\alpha) + \cos\alpha\} - A\mu(u_g - v_g) = 0 \tag{14}$$

$$|u + iv|_{z=h(x,y)+h_s}\{A\mu(\cos\alpha - \sin\alpha) + \sin\alpha\} - A\mu(u_g + v_g) = 0 \tag{15}$$

The solutions of equations (14) and (15) for A and $|u + iv|_{z=h(x,y)+h_s}$, we get

$$A = \frac{u_g(\cos\alpha - \sin\alpha) + v_g(\cos\alpha + \sin\alpha)}{2\mu(u_g \sin\alpha - v_g \cos\alpha)}$$

$$|u + iv|_{z=h(x,y)+h_s} = u_g(\cos\alpha - \sin\alpha) + v_g(\cos\alpha + \sin\alpha) \tag{16}$$

Using equation (16) into equation (13), we can write

$$u + iv = (u_g + iv_g) - \exp\{-(1+i)\mu\delta\}$$
$$\times \{(u_g + iv_g) - [u_g(\cos\alpha - \sin\alpha) + v_g(\cos\alpha + \sin\alpha)]e^{i\alpha}\}$$

$$u + iv = (u_g + iv_g) - \{\cos(1+i)\mu\delta - i\sin(1+i)\mu\delta\} \times \{(u_g + iv_g) -$$
$$[u_g(\cos\alpha - \sin\alpha) + v_g(\cos\alpha + \sin\alpha)](\cos\alpha + i\sin\alpha)\}$$

$$u + iv = (u_g + iv_g) - (u_g + iv_g)\cos(1+i)\mu\delta + i\,(u_g + iv_g)\sin(1+i)\mu\delta$$
$$+ \{\cos(1+i)\mu\delta - i\sin(1+i)\mu\delta\}[u_g(\cos\alpha - \sin\alpha) + v_g(\cos\alpha + \sin\alpha)](\cos\alpha + i\sin\alpha)$$

$$u + iv = (u_g + iv_g) - (u_g + iv_g)\cos(1+i)\mu\delta + i\,(u_g + iv_g)\sin(1+i)\mu\delta$$
$$+ \{\cos(1+i)\mu\delta - i\sin(1+i)\mu\delta\}[u_g\cos\alpha\,(\cos\alpha - \sin\alpha) + v_g\cos\alpha\,(\cos\alpha + \sin\alpha)$$
$$+ iu_g\sin\alpha\,(\cos\alpha - \sin\alpha) + iv_g\sin\alpha\,(\cos\alpha + \sin\alpha)]$$

$$u + iv = (u_g + iv_g) - (u_g + iv_g)\cos(1+i)\mu\delta + i\,(u_g + iv_g)\sin(1+i)\mu\delta$$
$$+ u_g\cos\alpha\,(\cos\alpha - \sin\alpha)\cos(1+i)\mu\delta + v_g\cos\alpha\,(\cos\alpha + \sin\alpha)\cos(1+i)\mu\delta$$
$$+ iu_g\sin\alpha\,(\cos\alpha - \sin\alpha)\cos(1+i)\mu\delta + iv_g\sin\alpha\,(\cos\alpha + \sin\alpha)\cos(1+i)\mu\delta$$
$$- iu_g\cos\alpha\,(\cos\alpha - \sin\alpha)\sin(1+i)\mu\delta - iv_g\cos\alpha\,(\cos\alpha + \sin\alpha)\sin(1+i)\mu\delta$$
$$+ u_g\sin\alpha\,(\cos\alpha - \sin\alpha)\sin(1+i)\mu\delta + v_g\sin\alpha\,(\cos\alpha + \sin\alpha)\sin(1+i)\mu\delta$$

$$
\begin{aligned}
u + iv = {} & (u_g + iv_g) - (u_g + iv_g)(\cos\mu\delta\cos i\mu\delta - \sin\mu\delta\sin i\mu\delta) \\
& + (iu_g - v_g)(\sin\mu\delta\cos i\mu\delta + \cos\mu\delta\sin i\mu\delta) \\
& + (u_g\cos^2\alpha - u_g\sin\alpha\cos\alpha)(\cos\mu\delta\cos i\mu\delta - \sin\mu\delta\sin i\mu\delta) \\
& + (v_g\cos^2\alpha + v_g\sin\alpha\cos\alpha)(\cos\mu\delta\cos i\mu\delta - \sin\mu\delta\sin i\mu\delta) \\
& + (iu_g\sin\alpha\cos\alpha - iu_g\sin^2\alpha)(\cos\mu\delta\cos i\mu\delta - \sin\mu\delta\sin i\mu\delta) \\
& + (iv_g\sin\alpha\cos\alpha + iv_g\sin^2\alpha)(\cos\mu\delta\cos i\mu\delta - \sin\mu\delta\sin i\mu\delta) \\
& - (iu_g\cos^2\alpha - iu_g\sin\alpha\cos\alpha)(\sin\mu\delta\cos i\mu\delta + \cos\mu\delta\sin i\mu\delta) \\
& - (iv_g\cos^2\alpha + iv_g\sin\alpha\cos\alpha)(\sin\mu\delta\cos i\mu\delta + \cos\mu\delta\sin i\mu\delta) \\
& + (u_g\sin\alpha\cos\alpha - u_g\sin^2\alpha)(\sin\mu\delta\cos i\mu\delta + \cos\mu\delta\sin i\mu\delta) \\
& + (u_g\sin\alpha\cos\alpha - u_g\sin^2\alpha)(\sin\mu\delta\cos i\mu\delta + \cos\mu\delta\sin i\mu\delta)
\end{aligned}
$$

$$
\begin{aligned}
u + iv = {} & (u_g + iv_g) - (u_g + iv_g)(\cos\mu\delta\cosh\mu\delta - i\sin\mu\delta\sinh\mu\delta) \\
& + (iu_g - v_g)(\sin\mu\delta\cosh\mu\delta + i\cos\mu\delta\sinh\mu\delta) \\
& + (u_g\cos^2\alpha - u_g\sin\alpha\cos\alpha)(\cos\mu\delta\cosh\mu\delta - i\sin\mu\delta\sinh\mu\delta) \\
& + (v_g\cos^2\alpha + v_g\sin\alpha\cos\alpha)(\cos\mu\delta\cosh\mu\delta - i\sin\mu\delta\sinh\mu\delta) \\
& + (iu_g\sin\alpha\cos\alpha - iu_g\sin^2\alpha)(\cos\mu\delta\cosh\mu\delta - i\sin\mu\delta\sinh\mu\delta) \\
& + (iv_g\sin\alpha\cos\alpha + iv_g\sin^2\alpha)(\cos\mu\delta\cosh\mu\delta - i\sin\mu\delta\sinh\mu\delta) \\
& - (iu_g\cos^2\alpha - iu_g\sin\alpha\cos\alpha)(\sin\mu\delta\cosh\mu\delta + i\cos\mu\delta\sinh\mu\delta) \\
& - (iv_g\cos^2\alpha + iv_g\sin\alpha\cos\alpha)(\sin\mu\delta\cosh\mu\delta + i\cos\mu\delta\sinh\mu\delta) \\
& + (u_g\sin\alpha\cos\alpha - u_g\sin^2\alpha)(\sin\mu\delta\cosh\mu\delta + i\cos\mu\delta\sinh\mu\delta) \\
& + (u_g\sin\alpha\cos\alpha - u_g\sin^2\alpha)(\sin\mu\delta\cosh\mu\delta + i\cos\mu\delta\sinh\mu\delta)
\end{aligned} \tag{17}
$$

Separating real and imaginary part in equation (17), we get respectively,

$$
\begin{aligned}
u = {} & u_g - u_g\cos\mu\delta\cosh\mu\delta + v_g\sin\mu\delta\sinh\mu\delta + u_g\cos\mu\delta\sinh\mu\delta \\
& - v_g\sin\mu\delta\cosh\mu\delta + (u_g\cos^2\alpha - u_g\sin\alpha\cos\alpha)\cos\mu\delta\cosh\mu\delta \\
& + (v_g\cos^2\alpha + v_g\sin\alpha\cos\alpha) + u_g\sin\alpha\cos\alpha\sin\mu\delta\sinh\mu\delta \\
& - u_g\sin^2\alpha\sin\mu\delta\sinh\mu\delta + v_g\sin\alpha\cos\alpha\sin\mu\delta\sinh\mu\delta \\
& + v_g\sin^2\alpha\sin\mu\delta\sinh\mu\delta + u_g\cos^2\alpha\cos\mu\delta\sinh\mu\delta \\
& - u_g\sin\alpha\cos\alpha\cos\mu\delta\sinh\mu\delta + v_g\cos^2\alpha\cos\mu\delta\sinh\mu\delta \\
& + v_g\sin\alpha\cos\alpha\cos\mu\delta\sinh\mu\delta \\
& + (u_g\sin\alpha\cos\alpha - u_g\sin^2\alpha)\sin\mu\delta\cosh\mu\delta \\
& + (v_g\sin\alpha\cos\alpha + v_g\sin^2\alpha)\sin\mu\delta\cosh\mu\delta
\end{aligned}
$$

$$
\begin{aligned}
u = {} & u_g - \exp\{-\mu\delta\} \times \{u_g\cos\mu\delta + v_g\sin\mu\delta \\
& - [u_g(\cos\alpha - \sin\alpha) + v_g(\cos\alpha + \sin\alpha)] \times \cos(\mu\delta - \alpha)\}
\end{aligned} \tag{18}
$$

$$
\begin{aligned}
v = {} & v_g + u_g\sin\mu\delta\sinh\mu\delta - v_g\cos\mu\delta\cosh\mu\delta + u_g\sin\mu\delta\cosh\mu\delta \\
& - v_g\cos\mu\delta\sinh\mu\delta - (u_g\cos^2\alpha - u_g\sin\alpha\cos\alpha)\sin\mu\delta\sinh\mu\delta \\
& - (v_g\cos^2\alpha + v_g\sin\alpha\cos\alpha)\sin\mu\delta\sinh\mu\delta \\
& + u_g\sin\alpha\cos\alpha\cos\mu\delta\cosh\mu\delta - u_g\sin^2\alpha\cos\mu\delta\cosh\mu\delta \\
& + v_g\sin\alpha\cos\alpha\cos\mu\delta\cosh\mu\delta + v_g\sin^2\alpha\cos\mu\delta\cosh\mu\delta \\
& - u_g\cos^2\alpha\sin\mu\delta\cosh\mu\delta + u_g\sin\alpha\cos\alpha\sin\mu\delta\cosh\mu\delta
\end{aligned}
$$

$$-v_g\cos^2\alpha\sin\mu\delta\cosh\mu\delta - v_g\sin\alpha\cos\alpha\sin\mu\delta\cosh\mu\delta$$
$$+\left(u_g\sin\alpha\cos\alpha - u_g\sin^2\alpha\right)\cos\mu\delta\sinh\mu\delta$$
$$+\left(v_g\sin\alpha\cos\alpha + v_g\sin^2\alpha\right)\cos\mu\delta\sinh\mu\delta$$
$$v = v_g - \exp\{-\mu\delta\}\times\{-u_g\sin\mu\delta + v_g\cos\mu\delta$$
$$+\left[u_g(\cos\alpha-\sin\alpha)+v_g(\cos\alpha+\sin\alpha)\right]\times\sin(\mu\delta-\alpha)\} \tag{19}$$

with

$$w = u\frac{\partial h}{\partial x}+v\frac{\partial h}{\partial y}, \quad \text{where } \delta=[z-h(x,y)-h_s]$$

In the surface boundary layer with the wind at the lower boundary of the planetary layer, let the wind distribution in the surface boundary layer be

$$|u+iv| = \frac{u_*}{k}\{\ln\left[\frac{\delta+h_s+z_0}{z_0}\right]+a[\frac{\delta+h_s}{L}]\} \tag{20}$$

At the lower boundary of the planetary boundary layer, equation (3) becomes

$$|u+iv|_{z=h(x,y)+h_s} = \frac{u_*}{k}[\ln(\frac{h_s+z_0}{z_0})+a\frac{h_s}{L}],$$

Substituting this value in equation (16) we get

$$u_* = \frac{k[u_g(\cos\alpha-\sin\alpha)+v_g(\cos\alpha+\sin\alpha)]}{[\ln\left(\frac{h_s+z_0}{z_0}\right)+a\frac{h_s}{L}]} \tag{21}$$

It is noticed that u_* depends on G, L and α.
Substitution of (21) into (20) gives the velocity profiles in the surface boundary layer,

$$|u+iv| = \frac{[u_g(\cos\alpha-\sin\alpha)+v_g(\cos\alpha+\sin\alpha)]}{[\ln\left(\frac{h_s+z_0}{z_0}\right)+a\frac{h_s}{L}]}\times\{\ln\left[\frac{\delta+h_s+z_0}{z_0}\right]+a[\frac{\delta+h_s}{L}]\} \tag{22}$$

$$w = |u+iv|(\cos\alpha\frac{\partial h}{\partial x}+\sin\alpha\frac{\partial h}{\partial y}),$$

$$\text{for } h(x,y)\le z\le h(x,y)+h_s.$$

If v_s and v_n be the velocity components along the s and n axes respectively. it is evidently that s and n axes be respectively oriented parallel and horizontally perpendicular to the geotrophic wind. Therefore, G becomes a real number, then α is the cross-isobaric angle of the wind at the surface, and equation (17) reduces into the form

$$v_s + iv_n = G + G\{(\cos\alpha-\sin\alpha)e^{i\alpha}-1\}\times\exp\{-(1+i)\mu\delta\}$$
$$= G + G\sin\alpha\{-(\sin\alpha+\cos\alpha)+i(\cos\alpha-\sin\alpha)\}\times\exp\{-(1+i)\mu\delta\} \tag{23}$$

Since

$$e^{i[\alpha+(\frac{3}{4\pi})]} = \cos\left[\alpha+\left(\frac{3}{4\pi}\right)\right]+i\sin[\alpha+(\frac{3}{4\pi})]$$

Equation (23) can be written as

$$v_s + iv_n = G[1+\sqrt{2}\sin\alpha\exp\left(-\mu\delta+i\left\{\alpha+\left(\frac{3}{4\pi}\right)-\mu\delta\right\}\right) \tag{24}$$

Therefore,

$$v_s = G\left(1+2^{\frac{1}{2}}\exp\{-\mu\delta\right\}\sin\alpha\ \cos\{\alpha+\frac{3}{4\pi}-\mu\delta\}) \tag{25}$$

$$v_n = 2^{\frac{1}{2}}G\exp\{-\mu\delta\}\sin\alpha\ \sin\{\alpha+\frac{3}{4\pi}-\mu\delta\} \tag{26}$$

$$w = G\left(1+2^{\frac{1}{2}}\exp\{-\mu\delta\right\}\sin\alpha\ \cos\{\alpha+\frac{3}{4\pi}-\mu\delta\})\ \frac{\partial h}{\partial x}$$

$$+2^{\frac{1}{2}}\, G\exp\{-\mu\delta\}\sin\alpha\ \sin\{\alpha+\frac{3}{4\pi}-\mu\delta\}\frac{\partial h}{\partial y} \tag{27}$$

for $z > h(x,y)+h_s$, equation (22) becomes

$$|v_s+iv_n| = \frac{G(\cos\alpha-\sin\alpha)}{[\ln\left(\frac{h_s+z_0}{z_0}\right)+a\frac{h_s}{L}]}\times\{\ln\left[\frac{\delta+h_s+z_0}{z_0}\right]+a[\frac{\delta+h_s}{L}]\}$$

$$w = |v_s+iv_n|(\cos\alpha\frac{\partial h}{\partial x}+\sin\alpha\frac{\partial h}{\partial y}),\ \text{for}\ h(x,y)\le z\le h(x,y)+h_s. \tag{28}$$

The cross-isobaric angle α may be estimated from equation (26) by putting $v_n = 0$ at the geostrophic wind level, $z = H$. Thus,

$$\alpha = (\frac{\Omega\sin\phi}{K})^{\frac{1}{2}}[\delta-H]-\frac{3}{4\pi} \tag{29}$$

Substitutions of (29) into equations (24), (25), (26) and (28) yield, respectively,

$$v_s = G\left(1+2^{\frac{1}{2}}\ \exp\{-\mu\delta\}\times\sin\left\{\mu\delta-\frac{3}{4\pi}\right\}\cos\mu\,(H-z)\right) \tag{30}$$

$$v_n = 2^{\frac{1}{2}}\,G\exp\{-\mu\delta\}\ \times sin\left\{\mu[\delta-H]-\frac{3}{4\pi}\right\}\sin\mu\,(H-z) \tag{31}$$

$$w = G\frac{\partial h}{\partial x}+2^{\frac{1}{2}}\,G\exp\{-\mu\delta\}\times sin\left\{\mu[\delta-H]-\frac{3}{4\pi}\right\}\times[\cos\mu\,(H-z)\frac{\partial h}{\partial x}$$

$$+\ \sin\mu\,(H-z)\frac{\partial h}{\partial y}],\ \text{for}\ z > h(x,y)+h_s \tag{32}$$

$$|v_s+iv_n| = \frac{2^{\frac{1}{2}}G\cos\{\mu[\delta-H]-\frac{1}{2\pi}}{[\ln\left(\frac{h_s+z_0}{z_0}\right)+a\frac{h_s}{L}]}\times\{\ln\left[\frac{\delta+h_s+z_0}{z_0}\right]+a[\frac{\delta+h_s}{L}]\} \tag{33}$$

for $h(x,y) < z \le h(x,y)+h_s$.

For small Reynolds number and dust particles in the planetary boundary layer of topography, the pressure gradient and Coriolis forces on stationary flows due to first order reactant, we consider a model mountain of which the height takes the form

$$h^*(x^*,y^*) = a\mu H\{\frac{1}{1+\exp\{-b[(x^{*2}+y^{*2})^{\frac{1}{2}}+c]\}}+\frac{1}{1+\exp\{b[(x^{*2}+y^{*2})^{\frac{1}{2}}-c]\}}-1\} \tag{34}$$

If $f^*$and R are absent then equation (34) becomes

$$h^*(x^*,y^*) = a\vartheta H\{\frac{1}{1+\exp\{-b[(x^{*2}+y^{*2})^{\frac{1}{2}}+c]\}}+\frac{1}{1+\exp\{b[(x^{*2}+y^{*2})^{\frac{1}{2}}-c]\}}-1\} \tag{35}$$

where $\vartheta = (\frac{f}{2K})^{\frac{1}{2}}$. This is the same equation that Kao has been obtained in [14].

III. Results and Discussion

For $f = f^* = 0.125$ and $k = 1$, I have computed the non-dimensionalized velocity components, $U^* = \frac{v_s}{G}, V^* = \frac{v_n}{G}$ and $W^* = \frac{w}{G}$ with the use of equations (30)-(33) in (34) and plotted the results on Figures (1 -5) shows the effect on the height of the mountain and surface boundary thickness on the earth due to first order chemical reactant and μ. we have computed the non-dimensionalized velocity components, $U^* = \frac{v_s}{G}, V^* = \frac{v_n}{G}$ and $W^* = \frac{w}{G}$ with the use of equations (30)-(33) in (35) and plotted the results on Figures (1-5). Where $x^* = \vartheta x, y^* = \vartheta y, h^* = \vartheta h, h_s^* = \vartheta h_s$ are the non-dimensionalized coordinate x, y, height of the mountain, and the surface boundary thickness respectively.

For $f = f^* = 0.125$ and $k = 0.50, 0.25, 0.15$ Figures 6, 7 and 8 show the effect on the height of the mountain and surface boundary thickness on the earth due to first order chemical reactant and μ. The distribution of the non-dimensional velocity components in the vertical cross section passing through the mountain top, parallel to the geostrophic wind. We have computed the non-dimensionalized velocity components, $U^* = \frac{v_s}{G}, V^* = \frac{v_n}{G}$ and $W^* = \frac{w}{G}$ with the use

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of equations (30)-(33) in (34) and plotted the results on Figures 6, 7 and 8.

It is seen that $U^* = V^* = W^* = 0$ at the surface of the mountain, and that $U^* \to 1, V^* \to 0$ as $z \to \infty$. On the windward slope of the mountain, a horizontal convergence of U^* results in an upward motion, whereas on the lee side of the mountain a downward motion occurs as a consequence of a horizontal divergence of U^*.

For $a = b = c = 1,\ H = 1,\ h^*(0,0) = 0.1,\ h_s^* = 0.1$, and $z_0^* = 0.0001$. Figures (1-5) show the distribution of the non-dimensional velocity components in the vertical cross-section perpendicular to the geostrophic wind, passing through the mountain top. It is seen that that $U^* = V^* = W^* = 0$ at the surface of the mountain and that $U^* \to 1, V^* \to 0$ as $z \to \infty$.

Here $x^* = \vartheta x,\ y^* = \vartheta y,\ h^* = \vartheta h,\ h_s^* = \vartheta h_s$ are the nondimensionalized coordinate x, y, height of the mountain, and the surface boundary thickness respectively.

Figures 6, 7 and 8 show the distribution of the non-dimensional velocity components in the vertical cross section passing through the mountain top, parallel to the geostrophic wind. We have computed the non-dimensionalized velocity components, $U^* = \frac{v_s}{G}, V^* = \frac{v_n}{G}$ and $W^* = \frac{w}{G}$ with the use of equations (30)-(33) in (34) and plotted the results on Figures 6, 7 and 8. Effects on the height of the mountain and surface boundary thickness due to some values of R and μ are shown in the figures. Figure 3 and 6 are same though the values of μ are different.

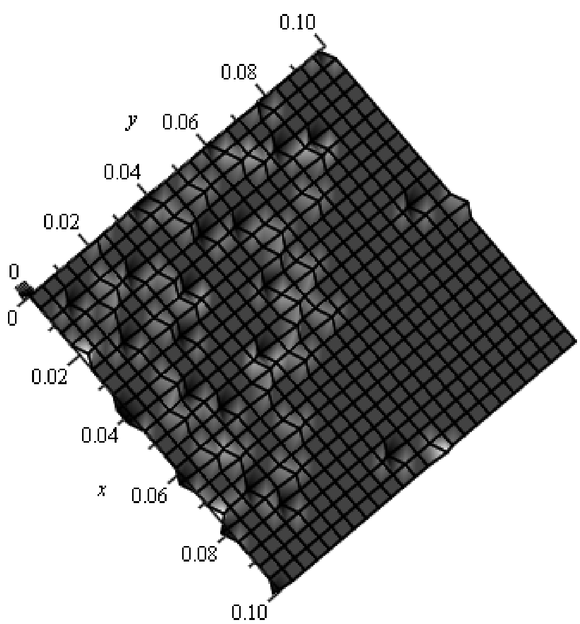

Figure 1 : Effects on the height of the mountain and surface boundary thickness due to

$R = 0 \ and \ \mu = 0.35.$

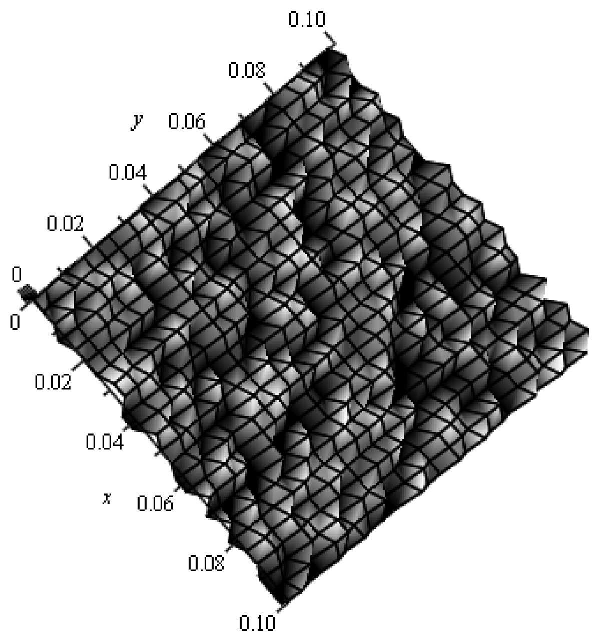

Figure 2 : Effects on the height of the mountain and surface boundary thickness due to $R = 0.125 \; and \; \mu = 0.45.$

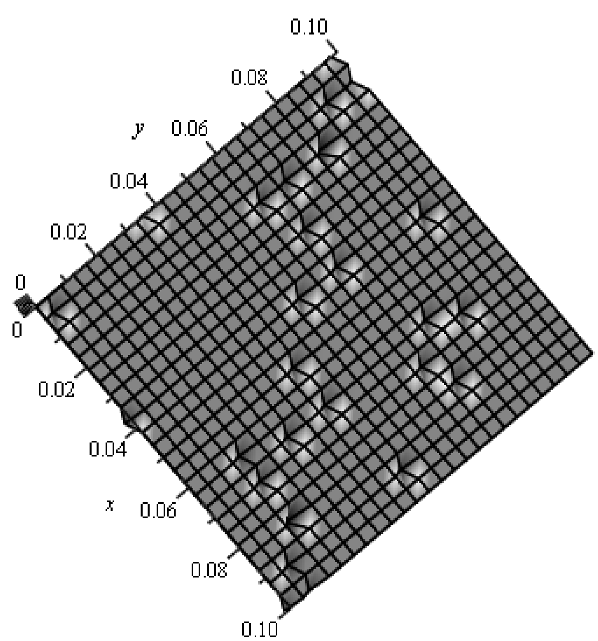

Figure 3 : First order chemical reaction due to $R = 0.250 \; and \; \mu = 0.50.$

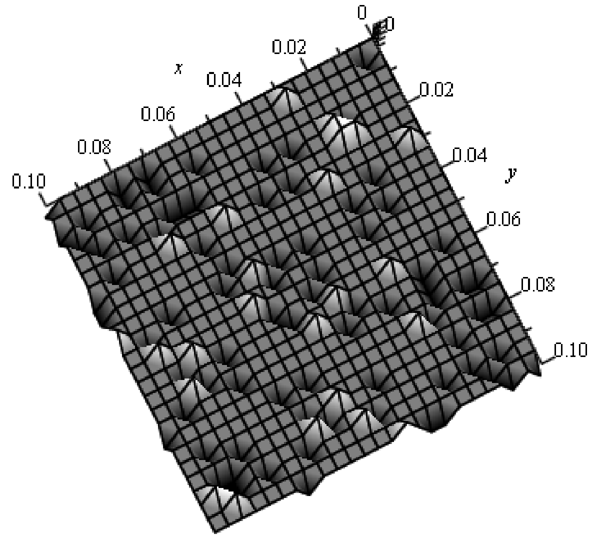

Figure 4 : Effects on the height of the mountain and surface boundary thickness due to $R = 0.50 \; and \; \mu = 0.61.$

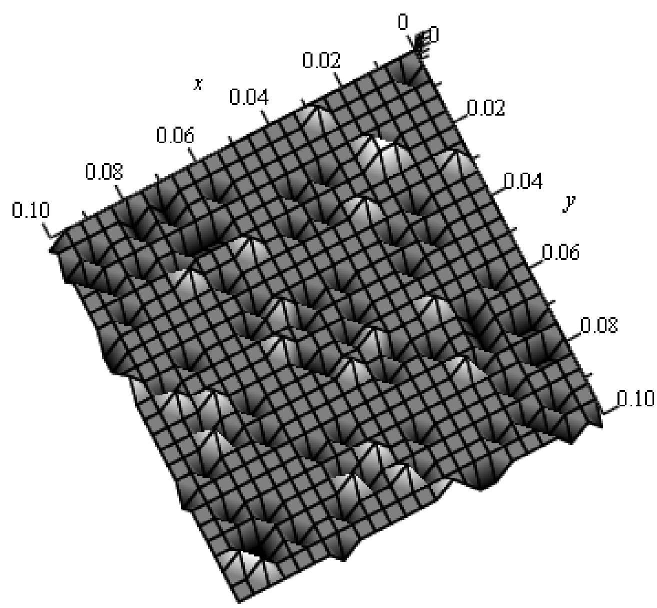

Figure 5 : First order chemical reaction due to $R = 1 \; and \; \mu = 0.80$

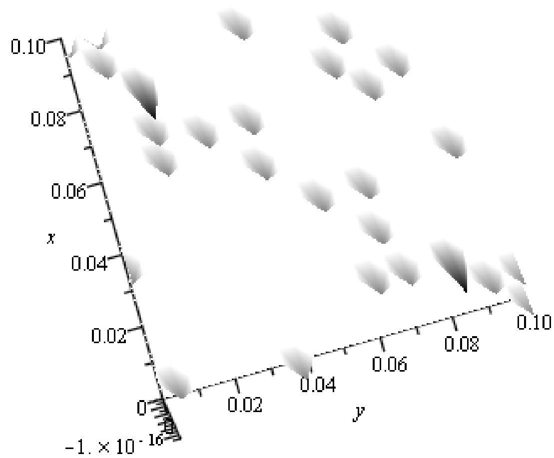

Figure 6 : Effects on the height of the mountain and surface boundary thickness due to $R = 0 \; and \; \mu = 0.50$

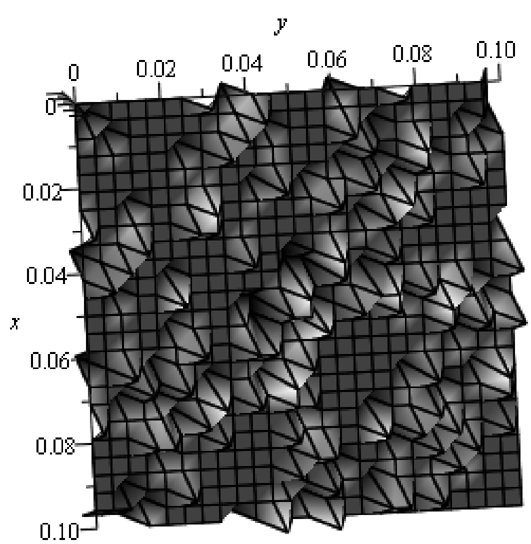

Figure 7 : First order chemical reaction due to $R = 0.50 \; and \; \mu = 1.22.$

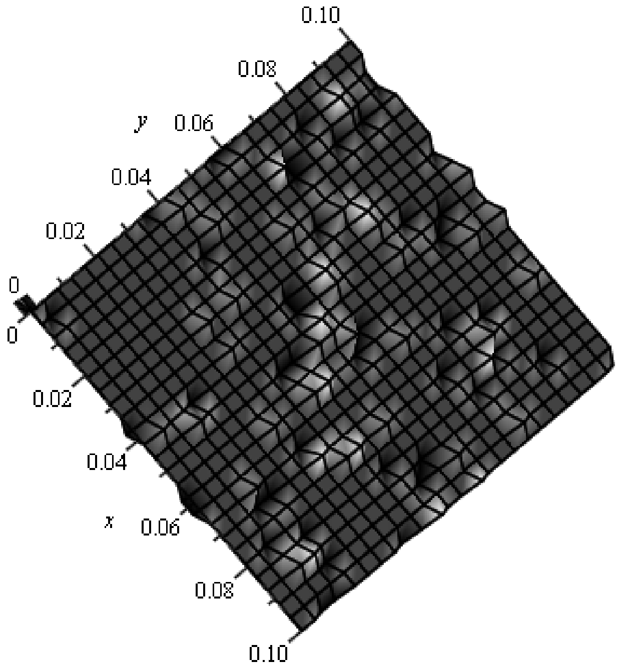

Figure 8 : First order chemical reaction due to $R = 1 \text{ and } \mu = 2.03$

IV. Conclusion

Effects of first order reactant of dust particle, Coriolis force and small Reynolds number on topography, horizontal pressure gradient forces on the velocity distribution in the atmospheric boundary layer increases due to increases of μ which indicates that:

- The cross-isobaric angle is a function of the coefficient of eddy diffusivity.
- The horizontal component of the velocity in the boundary layer turns right with increasing height in the Northern Hemisphere.
- Downward (upward) motion occurs on the windward (lee) side of the mountain.
- Downward (upward) motion also occurs on the slope to the left (right) of the geostrophic wind in the Southern Hemisphere, whereas in the Northern Hemisphere upward (downward) motion occurs on the slope to the left (right) of the geostrophic wind.
- Cross-isobaric angle α decreases with increasing height of the topography.

Therefore, there would be more chance of getting precipitation on the windward side due to absent of dust particle and chemical reaction and on the slope to the left (right) of the geostrophic wind in the Southern (Northern) Hemisphere there would be less possibility of getting precipitation due to present of more dust particles and chemical reaction. Because of increasing concern about atmospheric pollution in many population centers, industrial regions and power plants, which are located in valleys and terrain and since atmospheric motion is the mechanism for the transport and dispersion of pollutants.

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