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Effect of Cherenkov Radiation on Superluminal Free Spin-half Particles Motion in Spacetime

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Abstract- Conservation laws, consisting of the existence of quantities which do not change in time, independent of the dynamical evolution of a system, are crucial and vital for the construction of any dynamical system theory. The basic properties such as conservation of energy, momentum, angular momentum, charge, isospin, or generalization thereof are fundamental and must be guaranteed by a physical system, if it is to give a valid description of nature. One persistent objection against the concept of superluminal entities is based on the anticipation of fast energy loss which could be incurred under Vavilov-Cherenkov radiation, with the consequent prediction that no such particles could be detected. Yet presently, no theoretical or experimental explication exists which justifies this claim. Here we show, in the limit of a kinematically permissible and non-dispersive medium, that energy conservation is feasible. Corresponding to radiation intensities from large energy-momentum transfer, when the parameter k of the generalized linear velocity of the superluminal free spin-half field is made sufficiently large, Cherenkov cone becomes flattened at 90^o with direction of motion, bringing the radiated energy to merge with the circulating energy flow in the wave field of the particle.

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I. Introduction

The blue light observed by (Cherenkov, 1934) in his experiments was originally given a theoretical explanation by (Frank and Tamm, 1937). They associated it with the radiation of a charge moving uniformly with a velocity greater than that of light in medium. Under the restriction of non-dispersive medium, they derived the radiation intensity confined to the surface of the so-called Cherenkov cone defined by

$$\mathrm{Cos}\,\theta = \frac{c}{nv}, \tag{1.1}$$

where v is particle velocity and n the medium refractive index (which is the ratio of the phase velocity of light in the medium to its velocity in free space). In the presence of dispersion, n and $\cos\theta$ vary with ω continuously. As a result, the Cherenkov radiation fills continuous sequence of Cherenkov cones, relatively to different frequencies in the frequency regions where $n > 1$. In their derivations, Tamm and Frank suggested that the charge velocity was constant, and they disregarded the recoil effects in case of which equation (1.1) involves an additive term, given by[1]

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1. (Katharina, 2014)

$$\cos\theta = \frac{1}{n\beta} + \frac{\hbar k}{2p}\left(1 - \frac{1}{n^2}\right). \tag{1.2}$$

Here, $\beta = v/c$, and p and $\hbar k$ are particle and photon momentum, respectively. Also disregarded in their calculations was the case of arbitrary emission angle produced by the use of properly engineered one dimensional metamaterials, where

$$\cos\theta = \frac{1}{n\beta} + \frac{n}{k_0}.\left(\frac{d\varphi}{n^2}\right). \tag{1.3}$$

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This last equation originates from (Ginzburg, 1940) contribution, who evaluated the photon emission angle for an arbitrary energy loss of the initial charge, found the radiation intensity in the nonrelativistic approximation and showed that corrections to the Tamm–Frank formula are negligible in the visible and ultraviolet parts of the radiation spectrum. They further assumed smallness of the photon energy with respect to the energy of the initial charge. Since then, it is usually believed that the Vavilov–Cherenkov radiation for the fixed refractive index lies on the surface of the Cherenkov cone. As an early remark, it is clear from (1.1) that the emission angle θ relative to the direction of (linear) velocity depends only on particle linear constant velocity and the refractive index, but not on the coordinate x and hence, not on the particle trajectory. In his note (Vavilov, 1934) accompanying Cherenkov paper, Vavilov suggested that the radiation observed in Cherenkov experiments was due to the electron deceleration. This was further admitted by (Afanasiev, 2004) to be at least partly right since electrons were completely stopped in Cherenkov experiments(thorough discussion on this may be found in Cherenkov's Doctor of Science dissertation; (Cherenkov, 1944)), thus exhibiting deceleration.

Cherenkov radiation is frequently used in particle identification detectors (PID), (Alaeian, 2014; Jelle, 1955; Konrad, 1998; Leroy and Rancoita, 2014), a process which separates particles such as protons, electrons, muons, pions, etc at different velocities. A threshold measurement mechanism through which the number of particles at given velocities would be determined is set on a radiated angle equation in which the particle velocity exceeds c/n and photons are generated. If the particles pass through, for example, lucite or plexiglass, for which $n \approx 1.5$, only those with $v > 0.67c$ emit Cherenkov radiation and so can be detected as an optical signal. Particles with extreme relativistic energies can be detected in gas Cherenkov detectors where the refractive index n of the gas is just greater than 1. Another application is in the detection of ultra-high energy $\gamma - rays$ when they enter the top of the atmosphere. The high energy $\gamma - ray$ initiates an electron – photon cascade and, if the electron – positron pairs acquire velocities greater than the speed of light in air, optical Cherenkov radiation is emitted which can be detected by light detectors at sea-level (Longair, 1981; 1995; 1997). At present, evaluation of radiation intensities effect of Cherenkov photons on the superluminal motion of free fermions in spacetime remains to be determined. The aim of this observation is to analyze and ascertain where the context of very large energy-momentum transfer corresponding to radiation at an angle near to 90^o leads, as far as energy conservation is concerned in superluminal motion.

The plan of our examination is as follows. In Section 2, we review the theory of Vavilov-Cherenkov radiation, using John Peacock's approach (Dunlop et al., 1990;

1996). In Section 3, the true mechanism of spin is re-exposed in order to provide a basis for correctly understanding the interaction between the fermion wave field and the right-angled radiated photons energy plane, as explained in Section 4; there, we show that, in a kinematically permissible localized region of spacetime and in the absence of dispersion, the radiated energy (conventionally believed to be lost) is a constant of motion and could merge with the circulating energy flow of the field; so, in this condition, it contributes to the field which carries the particle.

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II. Theory of Cherenkov Radiation

Geometrically, the angle of emission (Cherenkov angle) is derived by Huygens' construction (see Fig.1). The origin of the emission is best appreciated from the following two Liénard -Wiechert potentials expressions $A(r,t)$ and $\phi(r,t)$ which are given by:

$$A(r,t) = \frac{\mu_0}{4\pi r}\left[\frac{qv}{1-(\boldsymbol{v}.i_{obs})/c}\right]_{ret} ;$$

$$\phi(r,t) = \frac{1}{4\pi\varepsilon_0 r}\left[\frac{q}{1-(\boldsymbol{v}.\boldsymbol{i}_{obs})/c}\right]_{ret} , \qquad (2.1)$$

where $\boldsymbol{i}_{obs}$ is the unit vector in the direction of observation from the moving charge q with velocity v at distance r, the subscript ret stands for retarded potential, and μ_0 and ε_0 are permeability and permittivity of space, respectively. In the case of a vacuum, one of the standard results of electromagnetic theory is that a charged particle moving at constant velocity v does not radiate electromagnetic radiation. Radiation is emitted in vacuum if the particle is accelerated. In a medium with a finite permittivity ε, or refractive index n, however, the potentials in (2.1) become singular along the cone, invalidating the denominators in this equation. Explicitly, we have:

$$1 - \frac{1}{c}(\boldsymbol{v}.i_{obs}) = 0 \quad \Longrightarrow \quad \cos\theta = \frac{c}{nv}, \qquad (2.2)$$

if one takes θ to be the angle between velocity direction and the unit vector $\boldsymbol{i}_{obs}$. Thus the usual rule that only accelerated charges radiate no longer applies.

The geometric representation of this process is that, because the particle moves superluminally through the medium, a 'shock wave' is created behind the particle. The wave front of the radiation propagates at a fixed angle with respect to the velocity vector of the particle because the wave fronts only add up coherently in this direction according to Huygens' construction. The geometry of this figure shows that the angle of the wave vector with respect to the direction of motion of the particle is $\cos\theta = c/nv$.

We will now determine the main features of Cherenkov radiation, analytically. Let us consider an electron moving along the positive x- axis at a constant velocity v. This motion corresponds to a current density $\boldsymbol{J}$ where

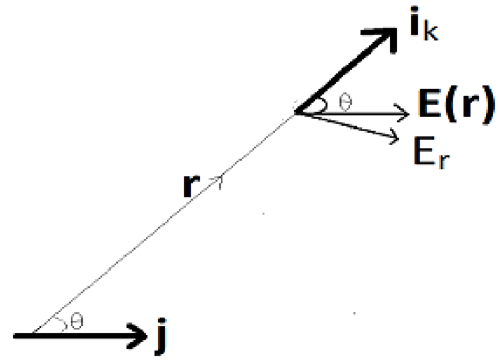

Fig. 1 : Illustrating the geometry used in the derivation of the expressions for Cherenkov angle and Cherenkov radiation

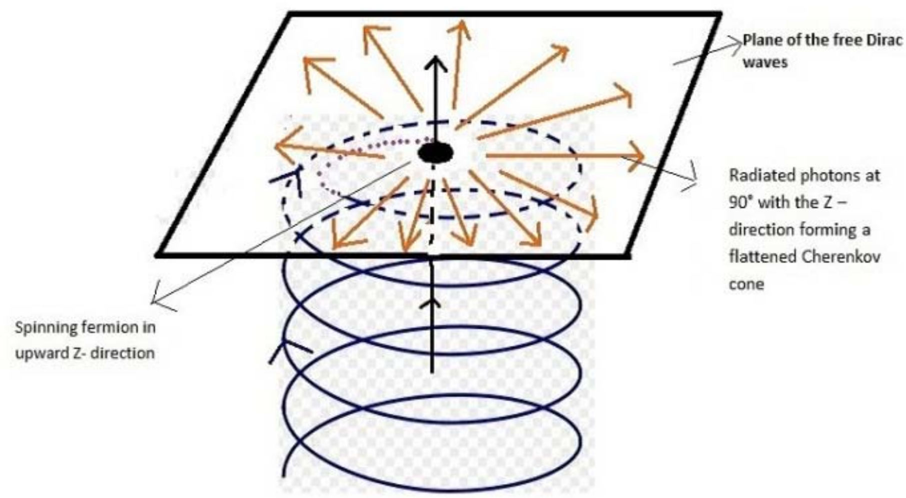

Fig. 2 : Sufficiently superluminal helical energy flow of a free fermion field for large parameter k. Photons are radiated at nearly 90^o where Cherenkov cone becomes flattened and coincides with the plane of the particle wave field . The flow is shown in the transverse direction.

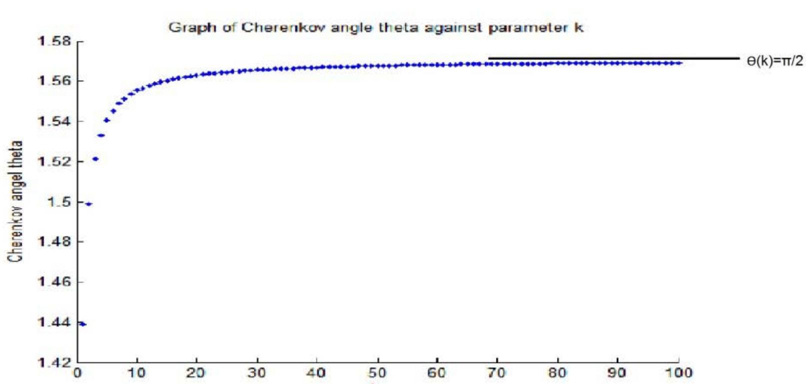

Fig. 3 : A graph illustrating the discrete evolution of Cherenkov angle θ to a limiting value, with respect to discrete parameter k

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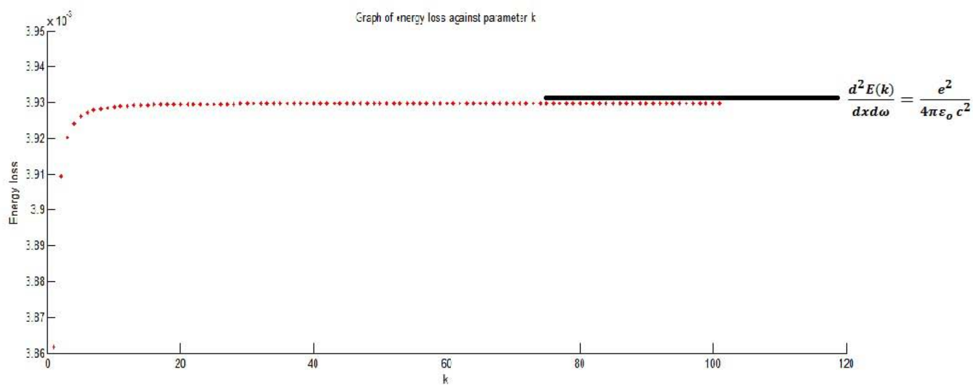

Fig. 4 : A graph illustrating the discrete evolution of the energy loss rate per unit path length and per frequency unit to a limiting value, with respect to discrete parameter k

$$\boldsymbol{J} = ev\delta(x - vt)\delta(y)\delta(z)i_x. \tag{2.3}$$

Taking the Fourier transform of this current density to find the frequency components $\boldsymbol{J}(\omega)$ corresponding to this motion, we have

$$\boldsymbol{J}(\omega) = \frac{1}{(2\pi)^{1/2}}\int \boldsymbol{J}\exp{(i\omega t)}\mathrm{dt}$$

$$= \frac{e}{(2\pi)^{1/2}}\delta(y)\delta(z)\exp{(i\omega x/v)}\boldsymbol{i}_x. \tag{2.4}$$

Equation (2.4) can be regarded as a representation of the motion of the electron by a line distribution of coherently oscillating currents. Our task is to work out the coherent emission, if any, from this distribution of oscillating currents. The quite cumbersome full treatments given in standard texts such as (Jackson, 1999) and (Clemmow and Dougherty, 1969) will not be used in this paper. Rather, we will adopt an approach developed by John Peacock in (Dunlop et al., 1990; 1996), different from the usual derivation which employs energy and momentum conservation.

First, let us review some of the standard results concerning the propagation of electromagnetic waves in a medium of permittivity ε, or refractive index $n = \varepsilon^{1/2}$. It is a standard result of classical electrodynamics that the flow of electromagnetic energy through a surface $\boldsymbol{dS}$ is given by the *Poynting vector flux*, $\boldsymbol{N}.\boldsymbol{dS} = (\boldsymbol{E}\times\boldsymbol{H}).\boldsymbol{dS}$. The electric and magnetic field strengths $\boldsymbol{E}$ and $\boldsymbol{H}$ are related to the electric flux density $\boldsymbol{D}$ and the magnetic flux density $\boldsymbol{B}$ by the constitutive relations

$$\boldsymbol{D} = \varepsilon\varepsilon_0\boldsymbol{E}, \qquad \boldsymbol{B} = \mu\mu_0\boldsymbol{H} \tag{2.5}$$

The energy density of the electromagnetic field in the medium is given by the standard formula

$$u = \int \boldsymbol{E}.d\boldsymbol{D} + \int \boldsymbol{H}.d\boldsymbol{B}. \tag{2.6}$$

If the medium has a constant real permittivity ε and permeability $\mu = 1$, the energy density in the medium is

$$u = \frac{1}{2}\varepsilon\varepsilon_0 E^2 + \frac{1}{2}\mu_0 H^2. \tag{2.7}$$

The speed of propagation of the waves is found from the dispersion relation $k^2 = \varepsilon\varepsilon_0\mu_0\,\omega^2$, that is, $c(\varepsilon) = \omega/k = (\varepsilon\varepsilon_0\mu_0)^{-1/2} = c/\varepsilon^{1/2}$. This demonstrates the well-known result that, in a linear medium, the refractive index n is $\varepsilon^{1/2}$. Another useful relation between the $\boldsymbol{E}$ and $\boldsymbol{B}$ fields in the electromagnetic wave, the ratio E/B, is $c/\varepsilon^{1/2} = c/n$. Substituting this result into the expression for the electric and magnetic field energies (2.7), it is found that these are equal. Thus, the total energy density in the wave is $u = \varepsilon\varepsilon_0 E^2$. Furthermore, the Poynting vector flux $\boldsymbol{E}\times\boldsymbol{H}$ is $\varepsilon^{1/2}\varepsilon_0 E^2 c = n\varepsilon_0 E^2 c$. This energy flow corresponds to the energy density of radiation in the wave $\varepsilon\varepsilon_0 E^2$ propagating at the velocity of light in the medium c/n. It follows that $N = n\varepsilon_0 E^2 c$, as expected.

Let us consider the expressions for the retarded values of the current which contributes to the vector potential at the point $\boldsymbol{r}$, (Fig.1). The expression for the vector potential $\boldsymbol{A}$ due to the current density $\boldsymbol{J}$ at distance r is

$$\boldsymbol{A}(\boldsymbol{r}) = \frac{\mu_0}{4\pi}\int d^3\boldsymbol{r}'\,\frac{\boldsymbol{J}(r',\ \boldsymbol{t}-|\boldsymbol{r}-r'|/c)}{|\boldsymbol{r}-r'|} = \frac{\mu_0}{4\pi}\int d^3\boldsymbol{r}'\,\frac{[\boldsymbol{J}]}{|\boldsymbol{r}-r'|}\ ; \tag{2.8}$$

the square brackets refer to retarded potentials. Taking the time derivative,

$$\boldsymbol{E}(\boldsymbol{r}) = -\frac{\partial\boldsymbol{A}}{\partial t} = -\frac{\mu_0}{4\pi}\int d^3\boldsymbol{r}'\,\frac{[\dot{\boldsymbol{J}}]}{|\boldsymbol{r}-r'|}\ . \tag{2.9}$$

In the distant far field limit, the electric field component E_r of the radiation field is perpendicular to the radial vector $\boldsymbol{r}$ and so, as indicated in Fig.1, $\boldsymbol{E}(\boldsymbol{r}) = \boldsymbol{E}(\boldsymbol{r})\times\boldsymbol{i}_k$, i.e.,

$$|\boldsymbol{E}(\boldsymbol{r})| = \frac{\mu_0\sin\theta}{4\pi}\left|\int d^3\boldsymbol{r}'\,\frac{[\dot{\boldsymbol{j}}]}{|\boldsymbol{r}-r'|}\right|. \tag{2.10}$$

It should be observed that if we substitute $\int d^3\boldsymbol{r}'[\dot{\boldsymbol{j}}] = \boldsymbol{e}\ddot{\boldsymbol{r}}$ into (2.10), we obtain

$$E_{pc} = \frac{|\ddot{\boldsymbol{p}}|\sin\theta}{4\pi\varepsilon_0 c^2 r}; \tag{2.11}$$

this is the expression for the radiation of a point charge, and $\boldsymbol{p}$ is the electric dipole moment of the charge with respect to some origin.

We now evaluate the frequency spectrum of the radiation. First of all, we work out the total radiation rate by integrating the Poynting vector flux over a sphere at a large distance $\boldsymbol{r}$,

$$\left(\frac{dE}{dt}\right)_{rad} = \int_S nc\varepsilon_0 E_r^2\,dS$$

$$= \int_\Omega \frac{nc\,\varepsilon_0\sin^2\theta}{16\pi^2}\left|\int d^3\boldsymbol{r}'\,\frac{[\dot{j}]}{|\dot{\boldsymbol{r}}-r'|}\right|^2 r^2\,d\Omega \tag{2.12}$$

Assuming the size of the emitting region is much smaller than the distance to the point of observation, i.e., $L \ll r$, we can write $|\boldsymbol{r}-\boldsymbol{r}'| = r$ and then,

$$\left(\frac{dE}{dt}\right)_{rad} = \int \frac{n\sin^2\theta}{16\pi^2\varepsilon_0 c^3}\left|\int d^3\boldsymbol{r}'[\dot{\boldsymbol{j}}]\right|^2 d\Omega. \tag{2.13}$$

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Next, we take the time integral of the radiation rate to find the total radiated energy,

$$E_{rad} = \int_{-\infty}^{\infty}\left(\frac{dE}{dt}\right)_{rad} dt$$

$$= \int_{-\infty}^{\infty}\int_{\Omega}\frac{n\sin^2\theta}{16\pi^2\varepsilon_0 c^3}\left|\int d^3\boldsymbol{r}'\left[\dot{\boldsymbol{J}}\right]\right|^2 d\Omega dt. \tag{2.14}$$

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Using Parseval's theorem to transform from an integral over time to an integral over frequency, and restricting our interest to positive frequencies only, we find:

$$E_{rad} = \int_0^{\infty}\int_{\Omega}\frac{n\sin^2\theta}{8\pi^2\varepsilon_0 c^3}\left|\int d^3\boldsymbol{r}'\left[\dot{\boldsymbol{J}}(\omega)\right]\right|^2 d\Omega d\omega. \tag{2.15}$$

Let us now evaluate the volume integral $\int d^3\boldsymbol{r}'\left[\dot{\boldsymbol{J}}(\omega)\right]$. We take $\boldsymbol{R}$ to be the vector from the origin of the coordinate system to the observer, and x to be the position vector of the current element $\boldsymbol{J}(\omega)d^3\boldsymbol{r}'$ from the origin; so that $\boldsymbol{r}' = \boldsymbol{R}-\boldsymbol{x}$. Now the waves from the current element at x propagate outwards from the emitting region at velocity c/n with phase factor $\exp[i(\omega t - \boldsymbol{k}.\boldsymbol{r}')]$ and therefore, relative to the origin at O, the phase factor of the waves, which we need to find for the retarded value of $\dot{\boldsymbol{J}}(\omega)$, is

$$\exp\left[i\left(\omega t - \boldsymbol{k}.(\boldsymbol{R}-\boldsymbol{x})\right)\right] = \exp(-i\boldsymbol{k}.\boldsymbol{R})\exp[i(\omega t + \boldsymbol{k}.\boldsymbol{x})]. \tag{2.16}$$

So, evaluating $\left[\dot{\boldsymbol{J}}(\omega)\right]$, we have

$$\left|\int d^3\boldsymbol{r}'\left[\dot{\boldsymbol{J}}(\omega)\right]\right| = \left|i\omega\int d^3\boldsymbol{r}'\left[\boldsymbol{J}(\omega)\right]\right|$$

or, by including the phase factor explicitly

$$\left|\int d^3\boldsymbol{r}'\left[\dot{\boldsymbol{J}}(\omega)\right]\right| = \left|\int d^3\boldsymbol{r}'\,\omega\boldsymbol{J}(\omega)\exp[i(\omega t + \boldsymbol{k}.\boldsymbol{x})]\right|. \tag{2.17}$$

Using (2.4) we obtain:

$$\left|\int d^3\boldsymbol{r}'\left[\dot{\boldsymbol{J}}(\omega)\right]\right| = \left|\frac{\omega e}{(2\pi)^{1/2}}\exp(i\omega t)\int\exp\left[i\left(\boldsymbol{k}.\boldsymbol{x}+\frac{\boldsymbol{\omega x}}{\boldsymbol{v}}\right)\right]dx\right|$$

$$= \left|\frac{\omega e}{(2\pi)^{1/2}}\int\exp\left[i\left(\boldsymbol{k}.\boldsymbol{x}+\frac{\boldsymbol{\omega x}}{\boldsymbol{v}}\right)\right]dx\right|. \tag{2.18}$$

This is the key integral in deciding whether or not the particle radiates. If the electron propagates in a vacuum, $\omega/k = c$ and we can write the exponent as

$$kx\left(\cos\theta + \frac{\omega}{kv}\right) = kx\left(\cos\theta + \frac{c}{v}\right) \tag{2.19}$$

In a vacuum, $c/v > 1$, and so this exponent is always greater than zero and hence the exponential integral over all x is always zero, assuming boundary limits vanish. This means that a particle moving at constant velocity in a vacuum does not radiate. However, if the medium has refractive index n, $\omega/k = c/n$, and then the exponent is zero if $\cos\theta = -c/nv$. This is the origin of the Cherenkov radiation phenomenon. The radiation is only coherent along the angle θ corresponding to the Cherenkov cone derived from Huygens' construction, i.e., given by (1.1).

We can therefore write down formally the energy spectrum using the relation (2.22) (below) of the average number of photons in a given state in the phase space, recalling that the radiation is only emitted at an angle $\cos\theta = c/nv$. But first, we need the equation which describes how the spectrum of radiation evolves towards the so called Bose-Einstein distribution (Einstein, 1905; 1915).

In the non-relativistic limit, this equation is known as the Kompaneets equation. It is written in terms of the occupation number of photons in phase space, because we need to include both spontaneous and induced processes in the calculation. Let us compare this approach with that involving the coefficients of emission and absorption of radiation. As a good reference for understanding the basic physics of spontaneous and induced processes, we present in the following Feynman's enunciation of the key rule for the emission and absorption of photons, which are spin-1 bosons (Feynman, Leighton and Sands, 1965; Feynman, 1972):

$$\textit{The probability that an atom will emit a photon in a particular final state is increased by a factor } (n+1) \textit{ if there are already } n \textit{ photons in that state.}$$

The statement is made in terms of probabilities rather than quantum mechanical amplitudes; in the latter case, the amplitude would be increased by a factor $\sqrt{n+1}$. We will use probabilities in our analysis. The number n will turn out to be the occupation number. To derive the Planck spectrum, consider an atom which can be in two states, an upper state 2 with energy $\hbar\omega$ greater than the lower state 1. N_1 is the number of atoms in the lower state and N_2 the number in the upper state. In thermodynamic equilibrium, the ratio of the numbers of atoms in these states is given by the Boltzmann relation,

$$\frac{N_2}{N_1} = \exp(-\Delta E/kt) = \exp\left(-\frac{\hbar\omega}{kT}\right), \qquad (2.20)$$

where $\nabla E = \hbar\omega$ and the corresponding statistical weights g_2 and g_1 are assumed to be the same. When a photon of energy $\hbar\omega$ is absorbed, the atom is excited from state 1 to state 2 and, when a photon of the same energy is emitted from state 2, the atom de-excites from state 2 to state 1. In thermodynamic equilibrium, the rates for the emission and absorption of photons between the two levels must be exactly balanced. These rates are proportional to the product of the probability of the events occurring and the number of atoms present in the appropriate state. Suppose $\bar{n}$ is the average number of photons in a given state in the phase space of the photons with energy $\hbar\omega$. Then, the absorption rate of these photons by the atoms in the state 1 is $N_1\bar{n}\,p_{12}$, where p_{12} is the probability that the photon will be absorbed by an atom in state 1, which is then excited to state 2. According to the rule enunciated above by Feynman, the rate of emission of photons when the atom de-excites from state 2 to state 1 is $N_2(\bar{n}+1)p_{12}$. At the quantum mechanical level, the probabilities p_{12} and p_{21} are equal. This is because the matrix element for, say, the p_{12} transition is the complex conjugate of the transition p_{21} and, since the probabilities depend upon the square of the magnitude of the matrix elements, these must be equal. This is called the *principle of jump rate symmetry*. Therefore,

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$$N_1\bar{n} = N_2(\bar{n}+1). \tag{2.21}$$

Solving for $\bar{n}$ and using (2.20), we obtain

$$\bar{n} = \frac{1}{e^{\hbar\omega/kT}-1} \tag{2.22}$$

as the required average number of photons in a given state in the phase space. Now from (2.15) and (2.22) we have

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$$\frac{dE_{rad}}{d\omega} = \int_\Omega \frac{n\omega^2 e^2 \sin^2\theta}{16\pi^2\varepsilon_0 c^3}\left|\int \exp\left[ikx\left(\cos\theta+\frac{\omega}{kv}\right)\right]dx\right|^2 d\Omega$$

$$= \frac{n\omega^2 e^2}{16\pi^2\varepsilon_0 c^3}\left(1-\frac{c^2}{n^2v^2}\right)\int_\Omega \left|\int \exp\left[ikx\left(\cos\theta+\frac{\omega}{kv}\right)\right]dx\right|^2 d\Omega\ . \tag{2.23}$$

If one sets $k(\cos\theta+\omega/kv)=\alpha$, one gets

$$\int_\Omega \left|\int \exp\left[ikx\left(\cos\theta+\frac{\omega}{kv}\right)\right]dx\right|^2 d\Omega$$

$$= \int_\theta \left|\int \exp(i\alpha x)dx\right|^2 2\pi\sin\theta\, d\theta\ . \tag{2.24}$$

We will evaluate the line integral along a finite path length from $-L$ to L, avoiding however the use of contour integration at values of θ ranging from $-\infty$ to ∞. The integral should be taken over a small finite range of angles about $\theta = \cos^{-1}(c/nv)$ for which $(\cos\theta+\omega/kv)$ or $(-\cos\theta+\omega/kv)$ is close to zero. Integration therefore can be taken over all values of θ or $\alpha = k(-\cos\theta+\omega/kv)$ knowing that most of the integral is contributed by values of θ very close to $\cos^{-1}(c/nv)$; so that $d\alpha = d(k(-\cos\theta+\omega/kv)) = k\sin\theta\, d\theta$. Thence, with respect to α, the integral (2.24) becomes

$$\int_\Omega \left|\int \exp\left[ikx\left(\cos\theta+\frac{\omega}{kv}\right)\right]dx\right|^2 d\Omega$$

$$= 8\pi\int \frac{\sin^2\alpha L}{\alpha^2}\frac{d\alpha}{k}, \tag{2.25}$$

This is an improper integral to be taken over all values of α from $-\infty$ to ∞. Combining test for convergence methods for such integrals and integration by parts, (2.25) is evaluated as

$$8\pi\int \frac{\sin^2\alpha L}{\alpha^2}\frac{d\alpha}{k} = 8\pi^2\left(\frac{L}{k}\right) = \frac{8\pi^2 c}{n\omega}L. \tag{2.26}$$

It follows that the energy radiated per unit bandwidth is

$$\frac{du}{d\omega} = \frac{\omega e^2}{2\pi\varepsilon_0 c^2}\left(1-\frac{c^2}{n^2v^2}\right)L. \tag{2.27}$$

We obtain the energy loss rate per unit path length directly by dividing by $2L$. Thus, the energy loss rate per unit path length follows as

$$\frac{du(\omega)}{dx} = \frac{\omega e^2}{4\pi\varepsilon_0 c^2}\left(1-\frac{c^2}{n^2v^2}\right). \tag{2.28}$$

Finally, the energy loss rate per unit path length and per frequency unit is obtained :

$$\frac{d^2E}{dxd\omega} = \frac{e^2}{4\pi\varepsilon_0 c^2}\left(1 - \frac{c^2}{n^2v^2}\right), \tag{2.29}$$

where it should be recalled that $v$is the particle superluminal velocity in the medium. Equations (1.1) and (2.29) are valid for arbitrary dependence $n(\omega)$.

As an important side remark, notice from (2.29) that the energy loss rate is a constant of motion with respect to constant ultra-relativistic (i.e., superluminal) velocity v and the medium refractive index n. We will now investigate what this means in the special context of free spin-1/2 particles superluminal motion in space time. However, before coming to the application in our field of interest, let us capture the true mechanism underlying the spin phenomenon.

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III. Understanding the True Mechanism of the Free Spin-1/2 Field

What is spin? This is a short but exact query which had been perfectly clarified in (Belinfante, 1939; Ohanian, 1984). As echo of these references, we say that persistent prevailing speculations would have the spin of the electron or of some other particle a mysterious internal process for which no concrete physical picture exists, and for which there is no classical analogue. Judging from arguments which surface in scientific criticisms and statements found in modem textbooks on atomic physics and quantum theory, it is surprising to observe that our understanding of spin (or the lack thereof) has not made any advance since the early years of quantum mechanics (Dirac, 1928). It is usually believed that the spin is a nonorbital, "internal," "intrinsic," or "inherent" angular momentum (these words being often incorrectly used as synonyms), and often treated as an irreducible entity that cannot be explained further. Sometimes, the speculation goes that the spin is a product of an (unspecified) internal structure of the electron, or arises in a natural way from Dirac's equation or from the analysis of the representations of the Lorentz group. The mathematical formalism of the Dirac equation and of group theory resort to the existence of the spin to achieve the conservation of angular momentum and to construct the generators of the rotation group, but when it comes to understanding the physical mechanism that produces the spin, no explication is given. This lack of a concrete picture of the spin leaves a grievous gap in our understanding of quantum mechanics, and hinder the derivation of applications therefrom. However, the solution of this problem has been at hand since (Belinfante, 1939) who, on the basis of an old calculation, was able to give the true (concrete) picture of the spin. He established that the spin could be regarded as due to a circulating flow of energy, or a momentum density, in the electron wave field. He stressed that this picture of the spin is valid not only for electrons, but also for photons, vector mesons, and gravitons, and in all cases the spin angular momentum is due to a circulating energy flow in the fields. Thus, in contradistinction to the common prejudice, the spin of the electron has a close classical analogue; It is an angular momentum of exactly the same kind as carried by the fields of a classical circularly polarized electromagnetic wave. Moreover, according to a demonstration by (Gordon, 1928), the magnetic moment of the electron is due to the circulating flow of charge in the electron wave field. Definitely, as a result, neither the spin nor the magnetic moment are internal properties of the electron and other particles: they have nothing to do with the internal structure of the electron, but only depend on the structure of its wave field.

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Further, a comparison between calculations of angular momentum in the Dirac field and the electromagnetic fields shows that the spin of the electron is entirely analogous to the angular momentum carried by a classical circularly polarized wave (Ohanian, 1984). From a theoretical point [cf.(Greiner,2000)], Maxwell (electromagnetic) equations given by

$$curl\, \boldsymbol{E} + \frac{1}{c}\frac{\partial \boldsymbol{H}}{\partial t} = 0, \quad curl\, \boldsymbol{H} - \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} = \frac{4\pi}{c}\boldsymbol{j}$$

can be represented in the form of the Dirac equations (spinor equation)

$$\frac{1}{i}\sum_{r=0}^{3} \hat{\alpha}^r \frac{\partial}{\partial x^r}\psi = \frac{4\pi}{c}\phi.$$

This relates the physics of self-interaction of the field of the particle, where it is known that a moving electron generates an electromagnetic field. All of these put together corroborate the above observations.

Having clarified the mechanism of spin, it is noteworthy that the Dirac (free) field is a plane wave, and so the axis of rotation of a free Dirac particle (which coincides with the direction of the field linear phase velocity) is perpendicular to this plane, Fig.2. Thus, a spinning free fermion is carried by a circularly polarized electromagnetic plane wave; in other words, a spinning free fermion rolls helically in an electromagnetic plane wave.

IV. Conservation of Energy by Superluminal Free Spin-1/2 Particles under Cherenkov Radiation

Theoretically (Afanasiev et al., 1999) and experimentally (Stevens et al., 2001; Wahlstr and and Merlin, 2003) it has been shown that the inclusion of the medium dispersion (a case we will not however consider in this work) leads to the appearance of additional radiation intensity maxima (or striped-like structure) in the angular distribution of the radiation.

Let us consider a free spin-1/2 particle moving in a localized, kinematically permissible region of spacetime with superluminal generalized linear velocity component of parameter k, given by(Gazoya et al., 2015; 2016):

$$V_{\text{Sup}}(k) = \left[\cos^{-1}\left(\frac{1}{4}\right)\right] \times c \approx \left(\frac{21\pi}{50} + 2\pi k\right) \times c,$$

$$k = 0, 1, 2, \ldots \tag{4.1}$$

where c is the universal value of the speed of light in a vacuum. In the absence of dispersion, the Cherenkov angle expression (1.1), as a function of the parameter k takes the form

$$\cos\theta(k) = \frac{c}{nV_{\text{Sup}}(k)} = \frac{50}{n\pi}\left(\frac{1}{100k+21}\right), \tag{4.2}$$

that is,

$$\theta(k) = \arccos\left(\frac{50}{n\pi}\left(\frac{1}{100k+21}\right)\right). \tag{4.3}$$

Clearly, as the parameter k assumes large numerical values, the argument of the inverse cosine function in (4.3) tends to zero, this brings the direction of the radiated

Cherenkov photons to an angle near to 90^o with the direction of the particle linear velocity; at this point Cherenkov cone becomes flattened. Mathematically, we write

$$\lim_{k\to\infty} \theta(k) = \frac{\pi}{2} \ . \tag{4.4}$$

This situation corresponds to Cherenkov radiation at an angle of approximately 90^o of moving free spin-1/2 particles with 'sufficiently' superluminal linear velocity in spacetime (of refractive index $n = 1.000277$ taken at Standard Temperature and Pressure (STP), for example) (see Fig.2).

On the other hand, the expression (2.29) of the radiated energy loss rate per unit path length and per frequency unit in terms of the parameter k becomes

$$\frac{d^2E(k)}{dxd\omega} = \frac{e^2}{4\pi\varepsilon_0 c^2}\left[1 - \frac{2500}{n^2\pi^2}\left(\frac{1}{100k+21}\right)^2\right]. \tag{4.5}$$

Upon taking the limit as k assumes large numerical values in (4.5) we obtain

$$\lim_{k\to\infty}\left[\frac{d^2E(k)}{dxd\omega}\right] = \frac{e^2}{4\pi\varepsilon_0 c^2} = \text{const.} \tag{4.6}$$

Thus, in the particular case of large k, the radiated energy is still a constant of motion, independent of the medium refractive index $n(\omega)$, and so independent of frequency.

V. Discussions

In light of the true mechanism of the free spin-1/2 field exposed in Section 3 above, the question arises: Is the radiated energy in (4.6) really lost, as conventionally claimed so far? For sufficiently superluminal motion induced by large parameter k, the direction of the radiated photons tends near to 90^o with that of the plane wave spin-half field. There, the Cherenkov cone becomes flattened to a plane which in turn coincides with the plane wave field of the particle (see Fig. 2). As a result, the radiated energy could be regarded as merging with the planewave of the circulating energy flow which carries the particle. It could not be considered lost; it could rather contribute to the fermion wave field, or, precisely, 're-invested' in the fermion wave field. Clearly, in case this radiated energy (being a constant of motion) could really go into waste, this could not significantly affect the superluminal nature of the propagation. In Fig.3 and Fig.4, graphs of the parameter k against angle $\theta(k)$ and energy loss rate $d^2E(k)/dxd\omega$ are plotted. A limiting value is reached in each case by these functions as the parameter k takes on big values. Thus, clearly, the result of this demonstration completely contradicts the speculative anticipation of instant collapse of such superluminal particles due to fast energy loss under Cherenkov radiation.

VI. Conclusions

Theoretically it has been shown that, in the limit of a kinematically permissible medium with absence of dispersion, the superluminal motion of free spin-half particles could be a reliable dynamical system in conformity with one of nature's basic laws of conservation of energy. The other side-argument which anticipates and insists on fast energy loss that would bring this kind of systems to instant collapse in their dynamical evolution could not hold. The radiated energy, which is a constant of motion in this case, whether lost or re-invested (as shown) in the wave field of the particle, could not

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significantly affect the superluminal nature of the propagation. Moreover, the larger the number the parameter k assumes in the quantization of the superluminal linear velocity, the less energy loss would be expected. In other words, the highly superluminal the propagation,the less the radiated energy wouldbe gone into waste.

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