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Solitary Wave Solutions for the Generalized Zakharov-Kuznetsov- Benjamin-Bona-Mahony Nonlinear Evolution Equation

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Abstract- In this paper, we employ the exp $(-\varphi(\xi))$-expansion method to find the exact traveling wave solutions involving parameters of nonlinear evolution equations. When these parameters are taken to be special values, the solitary wave solutions are derived from the exact traveling wave solutions. It is shown that the proposed method provides a more powerful mathematical tool for constructing exact traveling wave solutions for many other nonlinear evolution equations.

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I. Introduction

In recent years, due to the wide applications of soliton theory in mathematics, physics, chemistry, biology, communications, astrophysics and geophysics, etc., the search for explicit exact solutions, in particular, solitary wave solutions of nonlinear evolution equations (NEEs) has played an important role in the soliton theory. Various effective methods have been developed,. Such methods are tanh - sech method [1]-[3], extended tanh - method [4]-[6], sine - cosine method [7]-[9], homogeneous balance method [10, 11], F-expansion method [12]-[14], exp-function method [15, 16], trigonometric function series method [17], $(\frac{G'}{G})$-expansion method [18]-[21], Jacobi elliptic function method [22]-[25], the exp (-φ (ξ))-expansion method [26]-[28] and so on. The objective of this article is to apply the exp (-φ (ξ))-expansion method for finding the exact traveling wave solution of the generalized Zakharov-kuznetsov- Benjamin-Bona-Mahony nonlinear evolution equation system which play an important role in mathematical physics.

The rest of this paper is organized as follows: In Section 2, we give the description of the exp (-φ (ξ)) expansion method. In Section 3, we use this method to find the exact solutions of the nonlinear evolution equations pointed out above. In Section 5, we give the physical interpretations of the solutions. In Section 5, conclusions are given.

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II. Description of Method

Let us we have the following nonlinear evolution equation

$$P(u, u_t, u_x, u_{tt}, u_{xx}, \ldots\ldots) = 0, \tag{2.1}$$

since, P is a polynomial in $u(x,t)$ and its partial derivatives. In the following, we give the main steps of this method.

Step 1. We use the traveling wave solution in the form

$$u(x,t) = u(\xi), \qquad \xi = x - ct, \tag{2.2}$$

where c is a positive constant, to reduce Eq. (2.1) to the following ODE:

$$p(u, u', u'', u''', \ldots\ldots) = 0, \tag{2.3}$$

where P is a polynomial in u (ξ) and its total derivatives.

Step 2. Suppose that the solution of ODE (2.3) can be expressed by a polynomial in $exp(-\varphi(\xi))$ as follow

$$u(\xi) = a_m(\exp(-\varphi(\xi))^m + \cdots, a_m \neq 0, \tag{2.4}$$

where $\varphi(\xi)$ satisfies the ODE in the form

$$\varphi'(\xi) = \exp(-\varphi(\xi)) + \mu \exp(\varphi(\xi)) + \lambda. \tag{2.5}$$

The solutions of ODE (2.5) are:

When $\lambda^2 - 4\mu > 0, \mu \neq 0,$

$$\varphi(\xi) = ln\left(\frac{-\sqrt{\lambda^2-4\mu}\, tanh(\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi+c_1))-\lambda}{2\mu}\right), \tag{2.6}$$

and

$$\varphi(\xi) = ln\left(\frac{-\sqrt{\lambda^2-4\mu}\, coth(\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi+c_1))-\lambda}{2\mu}\right). \tag{2.7}$$

When $\lambda^2 - 4\mu > 0, \mu = 0,$

$$\varphi(\xi) = -ln\left(\frac{\lambda}{exp(\lambda(\xi+C_1))-1}\right). \tag{2.8}$$

When $\lambda^2 - 4\mu = 0, \mu \neq 0, \lambda \neq 0,$

$$\varphi(\xi) = ln\left(-\frac{2(\lambda(\xi+C_1)+2)}{\lambda^2(\xi+C_1)}\right). \tag{2.9}$$

When $\lambda^2 - 4\mu = 0, \mu \neq 0, \lambda = 0,$

$$\varphi(\xi) = \ln(\xi + C_1). \tag{2.10}$$

When $\lambda^2 - 4\mu < 0$

$$\varphi(\xi) = ln\left(\frac{\sqrt{4\mu-\lambda^2}tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}(\xi+c_1)\right)-\lambda}{2\mu}\right), \tag{2.11}$$

and

$$\varphi(\xi) = ln\left(\frac{\sqrt{4\mu-\lambda^2}cot\left(\frac{\sqrt{4\mu-\lambda^2}}{2}(\xi+c_1)\right)-\lambda}{2\mu}\right). \tag{2.12}$$

Where $a_m, \ldots, \lambda, \mu$ are constants to be determined later.

Step 3 Substitute Eq. (2.4) along Eq. (2.5) into Eq. (2.3) and collecting all the terms of the same power $exp(-m\varphi(\xi))$ (m = 0,1,2,3 ...) and equating them to zero, we obtain a system of algebraic equations, which can be solved by Maple or Mathematica to get the values of it.

Step 4. Substituting these values and the solutions of Eq. (2.5) into Eq. (2.3) we obtain the exact solutions of Eq. (2.1).

It is to be noted here that the construction of the $exp(-\varphi(\xi))$–expansion method is similar to the construction of the $(\frac{G\prime}{G})$ -expansion. For better understanding of the duality of both methods we cite [29]-[31].

III. Application

Here, we will apply the exp (-φ (ξ))-expansion method described in Sec.2 to find the exact traveling wave solutions and the solitary wave solutions of the generalized Zakharov-kuznetsov- Benjamin-Bona-Mahony nonlinear evolution equation [32, 33]. We consider the generalized Zakharov-kuznetsov-Benjamin-Bona-Mahony nonlinear evolution equation.

$$u_t + u_x + \alpha(u^n)_x + \beta\left(u_{xt} + u_{yy}\right)_x = 0, \quad n > 1. \tag{3.1}$$

Where (α, β) are real constants. By using the wave transformation $u(\xi)=u(x,y,t)$, since $\xi=x+y-ct$, we get:

$$-cu' + u' + \alpha(u^n)' + \beta(-cu'' + u'')' = 0. \tag{3.2}$$

By integration Eq. (3.2) and neglect the constant of integration we obtain:

$$(1-c)u' + \alpha(u^n) + \beta(1-c)u'' = 0. \tag{3.2}$$

Balancing between the highest order derivatives and nonlinear terms appearing in Eq. (3.3) $\left(u^n and\, u'' \Rightarrow m = \frac{2}{n-1}\right)$. So that we use transformation $\left[u = v^{\frac{2}{n-1}}\right]$ and substituting this transformation into Eq. (3.3) we get:

$$\begin{gathered}(1-c)(n-1)^2v^2 + \alpha(n-1)^2v^4 + \beta(1-c)(6-2n)v'^2 \\ +3\beta(1-c)(n-1)vv'' = 0.\end{gathered} \tag{3.2}$$

Balancing between the highest order derivatives and nonlinear terms appearing in Eq. (3.4) $(v^4 and\, vv'' \Rightarrow m = 1)$. So that, by using Eq. (2.4) we get the formal solution of Eq. (3.5)

$$v(\xi) = a_0 + a_1 \exp(-\phi(\xi)). \tag{3.5}$$

Substituting Eq. (3.5) and its derivative into Eq. (3.4) and collecting all term with the same power of $\exp(-4\phi(\xi)), \exp(-3\phi(\xi)), \exp(-2\phi(\xi)), \exp(-\phi(\xi)), \exp(0\phi(\xi))$ we get:

$$\begin{cases} \alpha(n-1)^2(a_1^4)+\beta(1-c)(6-2n)(a_1^2)+3\beta(1-c)(n-1)(2a_1^2)=0, \\ \alpha(n-1)^2(4a_0a_1^3)+\beta(1-c)(6-2n)(2a_1^2\lambda) \\ +3\beta(1-c)(n-1)(2a_0a_1+3a_1^2\lambda)=0, \\ (1-c)(n-1)^2(a_1^2)+\alpha(n-1)^2(6a_0^2a_1^2)+\beta(1-c)(6-2n)\begin{pmatrix}2a_1^2\mu \\ +a_1^2\lambda^2\end{pmatrix} \\ +3\beta(1-c)(n-1)(3a_0a_1\lambda+2a_1^2\mu+a_1^2\lambda^2)=0, \\ (1-c)(n-1)^2(2a_0a_1)+\alpha(n-1)^2(4a_0^3a_1)+\beta(1-c)(6-2n)(2a_1^2\mu\lambda) \\ +3\beta(1-c)(n-1)(2a_0a_1\mu+a_0a_1\lambda^2+a_1^2\mu\lambda)=0, \\ (1-c)(n-1)^2(a_0^2)+\alpha(n-1)^2(a_0^4)+\beta(1-c)(6-2n)(a_1^2\mu) \\ +3\beta(1-c)(n-1)(a_0a_1\lambda\mu)=0. \end{cases} \tag{3.6}$$

Solving above system by using maple 16, we get:

$$n=3,\alpha=\frac{-4(c-1)}{a_1^2(-\lambda^2+4\mu)},\beta=\frac{-4}{3(-\lambda^2+4\mu)},a_0=\frac{a_1\lambda}{2},a_1=a_1.$$

Thus the solution is

$$v(\xi)=\frac{a_1\lambda}{2}+a_1\exp\big(-\phi(\xi)\big). \tag{3.7}$$

Now, we discuss the following cases:

When $\lambda^2 — 4\mu > 0, \mu \neq 0,$

$$v(\xi)=\frac{a_1\lambda}{2}+a_1\left(\frac{2\mu}{-\sqrt{\lambda^2-4\mu}\,tanh(\frac{\sqrt{\lambda^2-4\mu}}{2}\,(\xi+c_1))-\lambda}\right), \tag{3.8}$$

and

$$v(\xi)=\frac{a_1\lambda}{2}+a_1\left(\frac{2\mu}{-\sqrt{\lambda^2-4\mu}\,coth(\frac{\sqrt{\lambda^2-4\mu}}{2}\,(\xi+c_1))-\lambda}\right) \tag{3.9}$$

When $\lambda^2 — 4\mu > 0, \mu = 0,$

$$v(\xi)=\frac{a_1\lambda}{2}+a_1\left(\frac{\lambda}{exp\big(\lambda(\xi+C_1)\big)-1}\right). \tag{3.10}$$

When $\lambda^2 — 4\mu = 0, \mu \neq 0, \lambda \neq 0,$

$$v(\xi)=\frac{a_1\lambda}{2}-a_1\left(\frac{\lambda^2(\xi+C_1)}{2(\lambda(\xi+C_1)+2}\right). \tag{3.11}$$

When $\lambda^2 — 4\mu = 0, \mu \neq 0, \lambda = 0,$

$$v(\xi)=\frac{a_1\lambda}{2}+a_1\left(\frac{1}{(\xi+C_1)}\right). \tag{3.12}$$

When $\lambda^2 — 4\mu < 0$

$$v(\xi)=\frac{a_1\lambda}{2}+a_1\left(\frac{2\mu}{\sqrt{4\mu-\lambda^2}tan\,(\frac{\sqrt{4\mu-\lambda^2}}{2}\,(\xi+c_1))-\lambda}\right), \tag{3.13}$$

and

$$v(\xi)=\frac{a_1\lambda}{2}+a_1\left(\frac{2\mu}{\sqrt{4\mu-\lambda^2}cot\,(\frac{\sqrt{4\mu-\lambda^2}}{2}\,(\xi+c_1))-\lambda}\right). \tag{3.13}$$

- *Note That:*

All the obtained results have been checked with Maple 16 by putting them back into the original equation and found correct.

IV. Conclusion

The exp (-φ (ξ))-expansion method has been applied in this paper to find the exact traveling wave solutions and then the solitary wave solutions of the generalized Zakharov-kuznetsov- Benjamin-Bona-Mahony nonlinear evolution equation. Let us compare between our results obtained in the present article with the well-known results obtained by other authors using different methods as follows: Our results of Nonlinear dynamics of the generalized Hirota-Satsuma couple KdV system are new and different from those obtained in [32,33], and figures show the solitary traveling wave solution of the generalized Zakharov-kuznetsov-Benjamin-Bona-Mahony nonlinear evolution equation. We can conclude that the exp (-φ (ξ))-expansion method is a very powerful and efficient technique in finding exact solutions for wide classes of nonlinear problems and can be applied to many other nonlinear evolution equations in mathematical physics. Another possible merit is that the reliability of the method and the reduction in the size of computational domain give this method a wider applicability.

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(Corresponding author: Mostafa M. A. Khater) I would like to dedicate this article to my mother and the soul of my father, he was there for the beginning of this degree, and did not make it to the end. His love, support, and constant care will never be forgotten. He is very much missed.

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