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Two Curious Summation Formulae in the Monograph of Salahuddin Et Al

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TWOCURIOUSSUMMATIONFORMULAEINTHEMONOGRAPHOFSALAHUDDINETAL

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Ref

2. Prudnikov, A.P., Brychkov, Yu. A. and Marichev, O.I.; Integral and Series Vol 3: More Special Functions, Nauka, Moscow, 2003.

Two Curious Summation Formulae in the Monograph of Salahuddin Et Al

Salahuddin [α] & R. K. Khola [σ]

Abstract- In this paper we have developed two summation formulae which are unique with the help of contiguous relation.

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I. Introduction and Results Required

Special functions are very useful in Mathematical Analysis, Applied Mathematics, Physical sciences and so many other branches of science and engineering. The uses of summation formulae are spectacular. So many scientists are involved to develop summation formulae in the field of Hypergeometric function. Contiguous relations are also useful with summation formulae.

Prudnikov et al[2,p.414] developed the following seven summation formulae

$$ {}_2F_1\left[\begin{matrix} a, & -a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] = \frac{\sqrt{\pi}\,\Gamma(c)}{2^c}\Big[\frac{1}{\Gamma(\frac{c+a+1}{2})\,\Gamma(\frac{c-a}{2})} + \frac{1}{\Gamma(\frac{c+a}{2})\,\Gamma(\frac{c-a+1}{2})}\Big] \tag{1} $$

$$ {}_2F_1\left[\begin{matrix} a, & 1-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] = \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-1}}\Big[\frac{1}{\Gamma(\frac{c+a}{2})\,\Gamma(\frac{c-a+1}{2})}\Big] \tag{2} $$

$$ {}_2F_1\left[\begin{matrix} a, & 2-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] = \frac{\sqrt{\pi}\,\Gamma(c)}{(a-1)\;2^{c-2}}\Big[\frac{1}{\Gamma(\frac{c+a-2}{2})\,\Gamma(\frac{c-a+1}{2})} - \frac{1}{\Gamma(\frac{c+a-1}{2})\,\Gamma(\frac{c-a}{2})}\Big] \tag{3} $$

$$ {}_2F_1\left[\begin{matrix} a, & 3-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] = \frac{\sqrt{\pi}\,\Gamma(c)}{(a-1)(a-2)\;2^{c-3}}\Big[\frac{(c-2)}{\Gamma(\frac{c+a-2}{2})\,\Gamma(\frac{c-a+1}{2})} - \frac{2}{\Gamma(\frac{c+a-3}{2})\,\Gamma(\frac{c-a}{2})}\Big] \tag{4} $$

$$ {}_2F_1\left[\begin{matrix} a, & 4-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] = \frac{\sqrt{\pi}\,\Gamma(c)}{(1-a)(2-a)(3-a)\;2^{c-4}}\Big[\frac{(a-2c+3)}{\Gamma(\frac{c+a-4}{2})\,\Gamma(\frac{c-a+1}{2})} + \frac{(a+2c-7)}{\Gamma(\frac{c+a-3}{2})\,\Gamma(\frac{c-a}{2})}\Big] \tag{5} $$

$$ {}_2F_1\left[\begin{matrix} a, & 5-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] = \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-5}\{\prod_{\gamma=1}^{4}(\gamma-a)\}}\Big[\frac{\{2(c-2)(c-4)-(a-1)(a-4)\}}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-4}{2})} + \frac{(12-4c)}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-5}{2})}\Big] \tag{6} $$

$$ {}_2F_1\left[\begin{matrix} a, & 6-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] = $$

$$ = \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-6}\{\prod_{\delta=1}^{5}(\delta-a)\}}\Big[\frac{(4c^2+2ac-a^2-a-34c+62)}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-5}{2})} - \frac{(4c^2-2ac-a^2+13a-22c+20)}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-6}{2})}\Big] \tag{7} $$

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The contiguous relation is defined as Abramowitz et al[1,p.558]

$$b\ {}_2F_1\left[\begin{array}{ll} a, & b+1\ ; \\ c & ; \end{array} z\right] = (b-c+1)\ {}_2F_1\left[\begin{array}{ll} a, & b\ ; \\ c & ; \end{array} z\right] + (c-1)\ {}_2F_1\left[\begin{array}{ll} a, & b\ ; \\ c-1 & ; \end{array} z\right] \quad (8)$$

Salahuddin et al[3,4,5] derived the following fifteen summation formulae

$${}_2F_1\left[\begin{array}{ll} a, & 7-a\ ; \\ c & ; \end{array} \frac{1}{2}\right] =$$

$$= \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-7}\{\prod_{\varsigma=1}^{6}(\varsigma-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-6}{2})}(-3a^2c+12a^2+21ac-84a+4c^3-48c^2+158c-120)+$$

$$+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-7}{2})}(2a^2-14a-8c^2+64c-108)\Big] \quad (9)$$

$${}_2F_1\left[\begin{array}{ll} a, & 8-a\ ; \\ c & ; \end{array} \frac{1}{2}\right] =$$

$$= \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-8}\{\prod_{\xi=1}^{7}(\xi-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-7}{2})}(-a^3-4a^2c+30a^2+4ac^2-4ac-107a+8c^3-124c^2+576c-762)+$$

$$+\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-8}{2})}(-a^3+4a^2c-6a^2+4ac^2-68ac+181a-8c^3+92c^2-288c+210)\Big] \quad (10)$$

$${}_2F_1\left[\begin{array}{ll} a, & 9-a\ ; \\ c & ; \end{array} \frac{1}{2}\right] =$$

$$= \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-9}\{\prod_{\varpi=1}^{8}(\varpi-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-8}{2})}(a^4-18a^3-8a^2c^2+80a^2c-85a^2+72ac^2-720ac+1494a+8c^4-$$

$$-160c^3+1056c^2-2560c+1680)+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-9}{2})}(8a^2c-40a^2-72ac+360a-16c^3+240c^2-1072c+1360)\Big] \quad (11)$$

$${}_2F_1\left[\begin{array}{ll} a, & 10-a\ ; \\ c & ; \end{array} \frac{1}{2}\right] =$$

$$= \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-10}\{\prod_{\upsilon=1}^{9}(\upsilon-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-10}{2})}(-a^4-4a^3c+42a^3+12a^2c^2-72a^2c-107a^2+8ac^3-252ac^2+$$

$$+1772ac-3054a-16c^4+312c^3-2000c^2+4704c-3024)+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-9}{2})}(a^4-4a^3c+2a^3-12a^2c^2+192a^2c-$$

$$-553a^2+8ac^3-12ac^2-868ac+3406a+16c^4-392c^3+3320c^2-11224c+12264)\Big] \quad (12)$$

$${}_2F_1\left[\begin{array}{ll} a, & 11-a\ ; \\ c & ; \end{array} \frac{1}{2}\right] =$$

$$= \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-11}\{\prod_{\varphi=1}^{10}(\varphi-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-10}{2})}(5a^4c-30a^4-110a^3c+660a^3-20a^2c^3+360a^2c^2-1305a^2c-$$

$$-810a^2+220ac^3-3960ac^2+21010ac-31020a+16c^5-480c^4+5240c^3-25200c^2+50544c-30240)+$$

$$+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-11}{2})}(-2a^4+44a^3+24a^2c^2-288a^2c+530a^2-264ac^2+3168ac-8492a-32c^4+768c^3-6352c^2+$$

$$+20928c-22320)\Big] \quad (13)$$

$${}_2F_1\left[\begin{array}{ll} a, & 12-a\ ; \\ c & ; \end{array} \frac{1}{2}\right] =$$

Ref

4. Salahuddin, Khola, R. K.; New hypergeometric summation formulae arising from the summation formulae of Prudnikov, South Asian Journal of Mathematics, 4(2014),192-196.

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Notes

$$= \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-12}\{\prod_{\chi=1}^{11}(\chi-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-12}{2})}(a^5-6a^4c+9a^4-12a^3c^2+300a^3c-1103a^3+32a^2c^3-$$

$$-408a^2c^2+46a^2c+6351a^2+16ac^4-800ac^3+10364ac^2-46852ac+62182a-32c^5+944c^4-10112c^3+47656c^2-$$

$$-93776c+55440)+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-11}{2})}(a^5+6a^4c-69a^4-12a^3c^2+12a^3c+769a^3-32a^2c^3+840a^2c^2-$$

$$-5662a^2c+8301a^2+16ac^4-32ac^3-4612ac^2+42380ac-96002a+32c^5-1136c^4+15104c^3-$$

$$-92536c^2+255392c-245640)\Big] \tag{14}$$

$${}_2F_1\left[\begin{matrix} a, & 13-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right]=$$

$$= \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-13}\{\prod_{\beta=1}^{12}(\beta-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-12}{2})}(-a^6+39a^5+18a^4c^2-252a^4c+275a^4-468a^3c^2+6552a^3c$$

$$-18135a^3-48a^2c^4+1344a^2c^3-9834a^2c^2+5964a^2c+74246a^2+624ac^4-17472ac^3+167388ac^2-631176ac$$

$$+752856a+32c^6-1344c^5+21824c^4-172032c^3+674384c^2-1187424c+665280)+$$

$$+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-13}{2})}(-12a^4c+84a^4+312a^3c-2184a^3+64a^2c^3-1344a^2c^2$$

$$+6620a^2c-2436a^2-832ac^3+17472ac^2-112424ac+216216a-64c^5+2240c^4-29312c^3+176512c^2-$$

$$-478752c+453600)\Big] \tag{15}$$

$${}_2F_1\left[\begin{matrix} a, & 14-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right]=$$

$$= \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-14}\{\prod_{\gamma=1}^{13}(\gamma-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-14}{2})}(a^6+6a^5c-87a^5-24a^4c^2+150a^4c+925a^4-32a^3c^3+1392a^3c^2$$

$$-12706a^3c+24615a^3+80a^2c^4-1728a^2c^3+5368a^2c^2+58986a^2c-242486a^2+32ac^5-2320ac^4+47328ac^3$$

$$-391568ac^2+1344076ac-1496568a-64c^6+2656c^5-42560c^4+330752c^3-1278144c^2+2222160c-1235520)+$$

$$+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-13}{2})}(-a^6+6a^5c-$$

$$-3a^5+24a^4c^2-570a^4c+2225a^4-32a^3c^3+48a^3c^2+7454a^3c-39225a^3-80a^2c^4+3072a^2c^3-35608a^2c^2+133626a^2c-$$

$$-68104a^2+32ac^5-80ac^4-19872ac^3+313808ac^2-1676564ac+2856228a+64c^6-3104c^5+59360c^4-566848c^3+$$

$$+2810304c^2-6724560c+5897520)\Big] \tag{16}$$

$${}_2F_1\left[\begin{matrix} a, & 15-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right]=$$

$$= \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-15}\{\prod_{\varepsilon=1}^{14}(\varepsilon-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-14}{2})}(-7a^6c+56a^6+315a^5c-2520a^5+56a^4c^3-1344a^4c^2+5103a^4c+$$

$$+16520a^4-1680a^3c^3+40320a^3c^2-271215a^3c+449400a^3-112a^2c^5+4480a^2c^4-54040a^2c^3+150080a^2c^2+845824a^2c-$$

$$-3383296a^2+1680ac^5-67200ac^4+999600ac^3-6787200ac^2+20482140ac-21070560a+64c^7-3584c^6+80864c^5-$$

$$-940800c^4+5987520c^3-20296192c^2+32464368c-17297280)+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-15}{2})}(2a^6-90a^5-48a^4c^2+768a^4c-$$

$$-1474a^4+1440a^3c^2-23040a^3c+77970a^3+160a^2c^4-5120a^2c^3+46640a^2c^2-90880a^2c-226192a^2-2400ac^4+$$

$$+76800ac^3-861600ac^2+3955200ac-6138120a-128c^6+6144c^5-116160c^4+1095680c^3-5363584c^2+$$

$$+12679168c-11009376)\Big] \tag{17}$$

$${}_2F_1\left[\begin{matrix} a, & 16-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right]=$$

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$$= \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-16}\{\prod_{\zeta=1}^{15}(\zeta-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-16}{2})}(-a^7+8a^6c-12a^6+24a^5c^2-792a^5c+3710a^5-80a^4c^3+1080a^4c^2+$$

$$+6280a^4c-66600a^4-80a^3c^4+5280a^3c^3-85480a^3c^2+435480a^3c-458929a^3+192a^2c^5-6240a^2c^4+45200a^2c^3+$$

$$+271560a^2c^2-3746640a^2c+8942052a^2+64ac^6-6336ac^5+186000ac^4-2408160ac^3+15005072ac^2-42553152ac+$$

$$+41722740a-128c^7+7104c^6-158720c^5+1827360c^4-11505152c^3+38596416c^2-61194240c+32432400)+$$

$$+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-15}{2})}(-a^7-8a^6c+124a^6+24a^5c^2-24a^5c-2818a^5+80a^4c^3-3000a^4c^2+26360a^4c-40760a^4-80a^3c^4+$$

$$+160a^3c^3+45080a^3c^2-534760a^3c+1499471a^3-192a^2c^5+10080a^2c^4-175760a^2c^3+1189560a^2c^2-2226480a^2c-$$

$$-2760884a^2+64ac^6-192ac^5-75120ac^4+1782560ac^3-16394608ac^2+65703616ac-93008652a+128c^7-8128c^6+$$

$$+210944c^5-2878240c^4+22080512c^3-94015552c^2+202146816c-165145680)\Big] \tag{18}$$

$${}_2F_1\left[\begin{matrix} a, & 17-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right]=$$

$$= \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-17}\{\prod_{\vartheta=1}^{16}(\vartheta-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-16}{2})}(a^8-68a^7-32a^6c^2+576a^6c-638a^6+1632a^5c^2-29376a^5c+101320a^5+$$

$$+160a^4c^4-5760a^4c^3+44640a^4c^2+129600a^4c-1341071a^4-5440a^3c^4+195840a^3c^3-2303840a^3c^2+$$

$$+9743040a^3c-9832052a^3-256a^2c^6+13824a^2c^5-246560a^2c^4+1411200a^2c^3+4297408a^2c^2-64103040a^2c+$$

$$+143207628a^2+4352ac^6-235008ac^5+4977600ac^4-52289280ac^3+282566656ac^2-727036416ac+670152240a+$$

$$+128c^8-9216c^7+275456c^6-4423680c^5+41249792c^4-224907264c^3+683065344c^2-1014128640c+518918400+$$

$$+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-17}{2})}(16a^6c-144a^6-816a^5c+7344a^5-160a^4c^3+4320a^4c^2-22480a^4c-30960a^4+5440a^3c^3-$$

$$-146880a^3c^2+1157360a^3c-2484720a^3+384a^2c^5-17280a^2c^4+247840a^2c^3-1092960a^2c^2-1901760a^2c+$$

$$+15669504a^2-6528ac^5+293760ac^4-4999360ac^3+39804480ac^2-146267456ac+194890176a-256c^7+16128c^6-$$

$$-414976c^5+5610240c^4-42628864c^3+179788032c^2-383195904c+310867200)\Big] \tag{19}$$

$${}_2F_1\left[\begin{matrix} a, & 18-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right]=$$

$$= \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-18}\{\prod_{\eta=1}^{17}(\eta-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-18}{2})}(-a^8-8a^7c+148a^7+40a^6c^2-256a^6c-3362a^6+80a^5c^3-4440a^5c^2+$$

$$+49664a^5c-103400a^5-240a^4c^4+5520a^4c^3+18760a^4c^2-849520a^4c+3240271a^4-192a^3c^5+17760a^3c^4-440560a^3c^3+$$

$$+4091160a^3c^2-12923320a^3c+3622852a^3+448a^2c^6-20352a^2c^5+253360a^2c^4+576240a^2c^3-31091248a^2c^2+$$

$$+192701168a^2c-344444908a^2+128ac^7-16576ac^6+660032ac^5-12228640ac^4+118499872ac^3-604789504ac^2+$$

$$+1488844864ac-1324543920a-256c^8+18304c^7-542976c^6+8650240c^5-79993344c^4+432549376c^3-1303568384c^2+$$

$$+1923025920c-980179200)+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-17}{2})}(a^8-8a^7c+4a^7-40a^6c^2+1264a^6c-6214a^6+80a^5c^3-120a^5c^2-$$

$$-32416a^5c+213904a^5+240a^4c^4-12720a^4c^3+186440a^4c^2-743120a^4c-456391a^4-192a^3c^5+480a^3c^4+216080a^3c^3-$$

$$-4278120a^3c^2+27569480a^3c-52277444a^3-448a^2c^6+30720a^2c^5-745840a^2c^4+7817520a^2c^3-30345632a^2c^2-$$

$$-19224224a^2c+253516684a^2+128ac^7-448ac^6-259264ac^5+8556320ac^4-118218848ac^3+813195488ac^2-$$

$$-2692403360ac+3335839536a+256c^8-20608c^7+696192c^6-12817024c^5+139638144c^4-913535872c^3+3463541888c^2-$$

$$-6848013696c+5284782720)\Big] \tag{20}$$

$${}_2F_1\left[\begin{matrix} a, & 19-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right]=$$

Notes

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$$= \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-19}\{\prod_{\lambda=1}^{18}(\lambda-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-18}{2})}(9a^8c-90a^8-684a^7c+6840a^7-120a^6c^3+3600a^6c^2-14046a^6c-99540a^6+$$

$$+6840a^5c^3-205200a^5c^2+1664856a^5c-2968560a^5+432a^4c^5-21600a^4c^4+277080a^4c^3+327600a^4c^2-20793831a^4c+$$

$$+70898310a^4-16416a^3c^5+820800a^3c^4-14644440a^3c^3+111013200a^3c^2-315518940a^3c+131909400a^3-576a^2c^7+$$

$$+40320a^2c^6-992880a^2c^5+9324000a^2c^4+4429536a^2c^3-636886080a^2c^2+3695816316a^2c-6211091160a^2+10944ac^7-$$

$$-766080ac^6+21827808ac^5-325310400ac^4+2707726176ac^3-12394025280ac^2+28254838896ac-23908836960a+256c^9-$$

$$-23040c^8+880512c^7-18627840c^6+238347264c^5-1891123200c^4+9158978048c^3-25507261440c^2+35661692160c-$$

$$-17643225600)+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-19}{2})}(-2a^8+152a^7+80a^6c^2-1600a^6c+3148a^6-4560a^5c^2+91200a^5c-371488a^5-$$

$$-480a^4c^4+19200a^4c^3-185680a^4c^2-126400a^4c+4559182a^4+18240a^3c^4-729600a^3c^3+9799440a^3c^2-50068800a^3c+$$

$$+73373288a^3+896a^2c^6-53760a^2c^5+1107680a^2c^4-8467200a^2c^3-743936a^2c^2+274718720a^2c-822056088a^2-$$

$$-17024ac^6+1021440ac^5-24338240ac^4+292569600ac^3-1853708096ac^2+5798641920ac-6885423072a-$$

$$-512c^8+40960c^7-1374464c^6+25123840c^5-271685888c^4+1764075520c^3-6639757056c^2+13042437120c-$$

$$-10013310720)\Big] \tag{21}$$

$${}_2F_1\left[\begin{matrix} a, & 20-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right]=$$

$$= \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-20}\{\prod_{\Upsilon=1}^{19}(\Upsilon-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-20}{2})}(33522128640+47215599696a+14182895460a^2+345040520a^3-$$

$$-140133105a^4+962073a^5+330750a^6-9330a^7+15a^8+a^9-67958134272c-57343402272ac-9605975576a^2c+$$

$$+295428296a^3c+58846422a^4c-2100880a^5c-32820a^6c+1640a^7c-10a^8c+48842214912c^2+$$

$$+25998562336ac^2+2187966784a^2c^2-168954152a^3c^2-6101120a^4c^2+380720a^5c^2-2240a^6c^2-40a^7c^2-17641896960c^3-$$

$$-5917427456ac^3-182014144a^2c^3+27821280a^3c^3-9440a^4c^3-19680a^5c^3+160a^6c^3+3666323456c^4+750095264ac^4-$$

$$-1895280a^2c^4-1926160a^3c^4+23280a^4c^4+240a^5c^4-465172736c^5-54369728ac^5+1155616a^2c^5+55104a^3c^5-672a^4c^5+$$

$$+36595328c^6+2174144ac^6-61824a^2c^6-448a^3c^6-1740800c^7-41984ac^7+1024a^2c^7+45824c^8+256ac^8-512c^9)+$$

$$+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-19}{2})}(-190253266560-131460917904a-15315714660a^2+1718684120a^3+100625805a^4-10839927a^5+$$

$$+135450a^6+7470a^7-195a^8+a^9+258458522112c+117489033888ac+5199265016a^2c-1259577944a^3c+961578a^4c+$$

$$+3256720a^5c-84780a^6c+40a^7c+10a^8c-139931759232c^2-40815588704ac^2+198370336a^2c^2+283436248a^3c^2-$$

$$-7330880a^4c^2-224080a^5c^2+7840a^6c^2-40a^7c^2+40472263680c^3+7213462784ac^3-274206656a^2c^3-26053920a^3c^3+$$

$$+1017440a^4c^3-480a^5c^3-160a^6c^3-6993636736c^4-700147936ac^4+42392880a^2c^4+896240a^3c^4-47280a^4c^4+240a^5c^4+$$

$$+757008896c^5+36475712ac^5-2849056a^2c^5+1344a^3c^5+672a^4c^5-51764608c^6-836416ac^6+88704a^2c^6-448a^3c^6+$$

$$+2170880c^7-1024ac^7-1024a^2c^7-50944c^8+256ac^8+512c^9)\Big] \tag{22}$$

$${}_2F_1\left[\begin{matrix} a, & 21-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right]=$$

$$= \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-21}\{\prod_{\Psi=1}^{20}(\Psi-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-20}{2})}(670442572800+946321185600a+284169369024a^2+4885689900a^3-$$

$$-3333875180a^4+41694345a^5+10037727a^6-381150a^7+1230a^8+105a^9-a^{10}-1394694005760c-1198379286720ac-$$

$$-203053089360a^2c+8433107760a^3c+1530533620a^4c-70408800a^5c-1146200a^6c+92400a^7c-1100a^8c+$$

$$+1048586614272c^2+578478838560ac^2+49539606520a^2c^2-4805882760a^3c^2-177714670a^4c^2+15397200a^5c^2-$$

$$-141500a^6c^2-4200a^7c^2+50a^8c^2-404078540800c^3-143591669760ac^3-4354528640a^2c^3+902932800a^3c^3-$$

$$-2094400a^4c^3-1108800a^5c^3+17600a^6c^3+91700259840c^4+20464187520ac^4-122473120a^2c^4-77439600a^3c^4+$$

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$$+1402800a^4c^4+25200a^5c^4-400a^6c^4-13092907520c^5-1739633280ac^5+50240960a^2c^5+3104640a^3c^5-73920a^4c^5+$$
$$+1209103616c^6+87071040ac^6-3652320a^2c^6-47040a^3c^6+1120a^4c^6-72089600c^7-2365440ac^7+112640a^2c^7+$$
$$+2677760c^8+26880ac^8-1280a^2c^8-56320c^9+512c^{10})+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-21}{2})}(362387520000+268742591040a+$$
$$+41471452880a^2-1867829040a^3-305673060a^4+14303520a^5+225720a^6-18480a^7+220a^8-494250063360c-$$
$$-247867413696ac-19713479280a^2c+1984361232a^3c+69962284a^4c-6179040a^5c+56920a^6c+1680a^7c-20a^8c+$$
$$+268936121344c^2+89644203264ac^2+2511762176a^2c^2-547968960a^3c^2+1404480a^4c^2+665280a^5c^2-10560a^6c^2-$$
$$-78226625536c^3-16719935232ac^3+112602112a^2c^3+62139840a^3c^3-1126720a^4c^3-20160a^5c^3+320a^6c^3+$$
$$+13598953984c^4+1756191360ac^4-51029440a^2c^4-3104640a^3c^4+73920a^4c^4-1480941056c^5-104786304ac^5+$$
$$+4397120a^2c^5+56448a^3c^5-1344a^4c^5+101871616c^6+3311616ac^6-157696a^2c^6-4296704c^7-43008ac^7+2048a^2c^7+$$
$$+101376c^8-1024c^9)\Big] \qquad (23)$$

II. Main Summation Formulae

$${}_2F_1\left[\begin{matrix} a, & 22-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right]=$$

$$=\frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-22}\{\prod_{\Xi=1}^{21}(\Xi-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-22}{2})}(-1279935820800-1868233671360a-628352859744a^2-$$
$$-34417212780a^3+5753119700a^4+134236095a^5-20700687a^6+312270a^7+8730a^8-225a^9+a^{10}+2668809669120c+$$
$$+2417863186656ac+489345655848a^2c-219480864a^3c-3388493178a^4c+53042458a^5c+4970700a^6c-142820a^7c+$$
$$+390a^8c+10a^9c-2014029186048c^2-1197461040576ac^2-138256171792a^2c^2+5911683120a^3c^2+583619652a^4c^2-$$
$$-22165920a^5c^2-176120a^6c^2+10800a^7c^2-60a^8c^2+779711413248c^3+306554335232ac^3+17513420736a^2c^3-$$
$$-1499242976a^3c^3-32128320a^4c^3+2199680a^5c^3-13440a^6c^3-160a^7c^3-177857647616c^4-45404661120ac^4-$$
$$-823493664a^2c^4+154420560a^3c^4-467600a^4c^4-75600a^5c^4+560a^6c^4+25531683072c^5+4062167872ac^5-$$
$$-27880608a^2c^5-7519456a^3c^5+86688a^4c^5+672a^5c^5-2370643968c^6-219093504ac^6+4625152a^2c^6+161280a^3c^6-$$
$$-1792a^4c^6+142098432c^7+6765568ac^7-178176a^2c^7-1024a^3c^7-5305344c^8-103680ac^8+2304a^2c^8+112128c^9+$$
$$+512ac^9-1024c^{10})+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-21}{2})}(7610141548800+5664039006240a+862754799384a^2-50618670580a^3-$$
$$-7467040370a^4+438809595a^5+6355587a^6-793890a^7+14040a^8-5a^9-a^{10}-10762094073600c-5490993903456ac-$$
$$-428082370072a^2c+53697863232a^3c+1803933278a^4c-214732742a^5c+2793980a^6c+100060a^7c-2370a^8c+10a^9c+$$
$$+6167102701056c^2+2126343680320ac^2+53972779984a^2c^2-16286791024a^3c^2+92193948a^4c^2+28580160a^5c^2-$$
$$-673960a^6c^2+240a^7c^2+60a^8c^2-1923629552128c^3-434298545536ac^3+5057543680a^2c^3+2145900064a^3c^3-$$
$$-52633280a^4c^3-1200640a^5c^3+38080a^6c^3-160a^7c^3+366720941312c^4+51431687104ac^4-1928215296a^2c^4-$$
$$-133374640a^3c^4+4718000a^4c^4-1680a^5c^4-560a^6c^4-45108419328c^5-3603513536ac^5+200868192a^2c^5+$$
$$+3361568a^3c^5-160608a^4c^5+672a^5c^5+3654604800c^6+142266880ac^6-10065664a^2c^6+3584a^3c^6+1792a^4c^6-$$
$$-193800192c^7-2561024ac^7+245760a^2c^7-1024a^3c^7+6471168c^8-2304ac^8-2304a^2c^8-123392c^9+512ac^9+1024c^{10})\Big]$$

$${}_2F_1\left[\begin{matrix} a, & 23-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right]= \qquad (24)$$

$$=\frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-23}\{\prod_{\Omega=1}^{22}(\Omega-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-22}{2})}(-28158588057600-41169473009280a-13846175136288a^2-$$
$$-688985043120a^3+149373094200a^4+2886127860a^5-648260844a^6+12963720a^7+382800a^8-15180a^9+132a^{10}+$$
$$+60062080780800c+55217638146528ac+11293160726232a^2c-82022959260a^3c-93645815450a^4c+1995539777a^5c+$$

Notes

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Notes

$$+178029929a^6c - 6909430a^7c + 31460a^8c + 1265a^9c - 11a^10c - 47111453908992c^2 - 28820579344128ac^2 -$$
$$-3392485386048a^2c^2 + 183098051136a^3c^2 + 17810133264a^4c^2 - 867081600a^5c^2 - 6985440a^6c^2 + 728640a^7c^2 - 7920a^8c^2 +$$
$$+19258856466432c^3 + 7922696165824ac^3 + 461791289168a^2c^3 - 49584004624a^3c^3 - 1091526436a^4c^3 + 105693280a^5c^3 -$$
$$-988680a^6c^3 - 20240a^7c^3 + 220a^8c^3 - 4723308327936c^4 - 1289002826496ac^4 - 22320173568a^2c^4 + 5914856640a^3c^4 -$$
$$-30824640a^4c^4 - 5100480a^5c^4 + 73920a^6c^4 + 745452131072c^5 + 130485126464ac^5 - 1364040832a^2c^5 - 359725520a^3c^5 +$$
$$+6190800a^4c^5 + 85008a^5c^5 - 1232a^6c^5 - 78371758080c^6 - 8302221312ac^6 + 235834368a^2c^6 + 10881024a^3c^6 -$$
$$-236544a^4c^6 + 5546010624c^7 + 322674176ac^7 - 12539648a^2c^7 - 129536a^3c^7 + 2816a^4c^7 - 260941824c^8 - 6994944ac^8 +$$
$$+304128a^2c^8 + 7822848c^9 + 64768ac^9 - 2816a^2c^9 - 135168c^{10} + 1024c^{11}) + \frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-23}{2})}(-14558535129600-$$
$$-11503844032320a - 2137013714928a^2 + 23993967080a^3 + 17223845140a^4 - 382057830a^5 - 32189094a^6 + 1259940a^7 -$$
$$-5760a^8 - 230a^9 + 2a^{10} + 20652447375360c + 11426734414848ac + 1249606186752a^2c - 70894693632a^3c -$$
$$-6453736128a^4c + 319011840a^5c + 2486400a^6c - 264960a^7c + 2880a^8c - 11881425202176c^2 - 4559948772992ac^2 -$$
$$-249358186400a^2c^2 + 27700361696a^3c^2 + 584111304a^4c^2 - 57805440a^5c^2 + 541520a^6c^2 + 11040a^7c^2 - 120a^8c^2 +$$
$$+3722781351936c^3 + 967717718016ac^3 + 15372177408a^2c^3 - 4341281280a^3c^3 + 23278080a^4c^3 + 3709440a^5c^3 -$$
$$-53760a^6c^3 - 713155826176c^4 - 120677707136ac^4 + 1319899392a^2c^4 + 327847520a^3c^4 - 5645920a^4c^4 - 77280a^5c^4 +$$
$$+1120a^6c^4 + 88159518720c^5 + 9128189952ac^5 - 260370432a^2c^5 - 11870208a^3c^5 + 258048a^4c^5 - 7178121216c^6 -$$
$$-411665408ac^6 + 16002560a^2c^6 + 164864a^3c^6 - 3584a^4c^6 + 382500864c^7 + 10174464ac^7 - 442368a^2c^7 - 12831744c^8 -$$
$$-105984ac^8 + 4608a^2c^8 + 245760c^9 - 2048c^{10})\Big] \qquad (25)$$

III. Derivation of the Main Formulae

Involving the contiguous relation (8) and the formula of Salahuddin et al(23), one can established the result(24) and on the same way result(25) can be established.

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