A Production Inventory Model for Deteriorating Products with Multiple Market and Selling Price dependent demand

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Keywords: production, deterioration, multiple market, price dependent demand.

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A Production Inventory Model for Deteriorating Products with Multiple Market and Selling Price dependent demand

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I. Introduction

To consider the effect of deterioration in the development of inventory models is an essential need of inventory modeling. Ignoring the effect of deterioration misleads the results. Ghare and Schrader (1963) were the first to introduce the deterioration in inventory modelling. They developed this model with constant rate of demand and deterioration. Mishra (1975) presented production lot-size model for a system having deteriorating inventory. Kang and Kim (1983) came forward with a study on the price and production level of the deteriorating inventory system. Wee (1993) defined the deterioration of products as “decay, damage, spoilage, evaporation, obsolescence, pilferage, loss of utility or loss of marginal value of a commodity that results in decreasing usefulness”. The deterioration occurs in most of the physical products such as medicines, food products, dairy products and volatile products, etc. Goyal and Giri (2001) proposed an inventory model for recent trends in deteriorating inventory modeling.

In today’s global market, there are so many opportunities for a vendor or manufacturer. It is not necessary that it will deal with a single market. It can deal in many markets for a single production run. Khouja (2001) presented the effect of large order quantities on expected profit in the single-period model. This model is developed with multiple market demand. In this model, he gave the example of a garment industry which sells its product in different markets. It is profitable for the manufacturer but very complicated also. This is because in different markets the demand rate will also be different. An Economic Order Quantity (EOQ) model was
presented by Covert and Philip (1973) for items that have time-dependent demand and deterioration as per Wei-bull distribution. Giri et al. (1996) put forward an inventory model for deteriorating items that have a demand rate dependent on the stock. Yang and Wee (2002) presented a production inventory model with multiple-buyer for a deteriorating item. Wu et al. (2006) proposed, for non-instantaneous deteriorating items that have stock-dependent demand and partial backlogging, an optimal-replenishment policy.

In this paper, we propose and develop a production inventory model for deteriorating items that have multiple markets and a demand rate dependent on the selling price. This model is illustrated using a numerical example and sensitivity analysis is also performed to show the model’s stability.

II. Assumptions and Notations

a) Assumptions
1. A single product is from a single manufacturer.
2. The model is for multiple market demand.
3. Demand rate is a function of the selling price.
4. Production rate is constant and is greater than the demand of all the markets.
5. The products being considered are deteriorating in nature.
6. The rate of deterioration is assumed to be a linear function of time.
7. No, shortages are allowed.
8. Planning horizon is considered finite.
9. No replacement or repair of deteriorated units is done during a given cycle.
10. It is assumed that the manufacturer arranges the fix quantity of raw material at a fix interval of time. The quantity of finished products produced by the manufacturer is enough to meet the occurring demand and deterioration in all the markets.

b) Notations

\( A \): Production rate.
\( \theta \): Deterioration parameter.
\( \alpha_k \): Demand parameter in \( k^{th} \) time interval.
\( \beta \): Demand parameter.
\( p \): selling price per unit time.
\( T \): production period.
\( K_0 \): setup cost
\( h_f \): Holding cost per unit for finished products.
\( d_f \): Deterioration cost per unit for finished products.
\( d_r \): Deterioration cost per unit for raw material inventory.
\( h_r \): Holding cost per unit for raw material.
\( \mu \): Ordering cost per order.
\( q_r \): Delivery lot size of raw material.
\( n_r \): Number of deliveries of raw material in time \( T \).

III. Model Description and Analysis

The model is developed with a piecewise constant function in demand. It is assumed that the optimal production run time \( T \) lies in the interval \([T_{m-1}, T_m]\). Let us
divide the interval \([T_{m-1}, T_m]\) into two parts. Here \(I_m^-(t)\) represents the inventory level during \(t \in [T_{m-1}, T]\) and \(I_m^+(t)\) represents inventory during \(t \in [T, T_m]\).

The inventory time behavior of the finished products at any time \(t\) is given by the following differential equations:

\[
\frac{dI_k(t)}{dt} + \partial t I_k(t) = A - \frac{\alpha_k}{p^\beta}; \quad T_{k-1} \leq t \leq T_k
\]

\[
\frac{dI_m^-(t)}{dt} + \partial t I_m^-(t) = A - \frac{\alpha_m}{p^\beta}; \quad T_{m-1} \leq t \leq T
\]

\[
\frac{dI_m^+(t)}{dt} + \partial t I_m^+(t) = -\frac{\alpha_m}{p^\beta}; \quad T \leq t \leq T_m
\]

\[
\frac{dI_j(t)}{dt} + \partial t I_j(t) = -\frac{\alpha_j}{p^\beta}; \quad T_{j-1} \leq t \leq T_j
\]

With boundary conditions:

\[I_1(0)=0, I_{k-1}(T_{k-1})=I_k(T_{k-1}), I_{m-1}(T_{m-1})=I_m(T_{m-1}),\]

\[I_n(T_n)=0, I_{j-1}(T_{j-1})=I_j(T_j)\]

The solution of these equations is given by (see appendix A):

\[I_k(t) = (A - \frac{\alpha_k}{p^\beta})(t + \frac{\theta}{6}t^2)e^{-\frac{\alpha_k}{p^\beta}T_k} + \sum_{i=1}^{m} (\frac{\alpha_i - \alpha_{i-1}}{p^\beta})(T_{i-1} + \frac{\theta}{6}T_{i-1})e^{-\frac{\alpha_i}{p^\beta}T_k}\]

\[T_{k-1} \leq t \leq T_k \quad \text{for } k=1,2,3, \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (m-1)\]

\[I_m^-(t) = (A - \frac{\alpha_m}{p^\beta})(t + \frac{\theta}{6}t^2)e^{-\frac{\alpha_m}{p^\beta}T_m} + \sum_{i=1}^{m} (\frac{\alpha_i - \alpha_{i-1}}{p^\beta})(T_{i-1} + \frac{\theta}{6}T_{i-1})e^{-\frac{\alpha_i}{p^\beta}T_m}\]

\[T_{m-1} \leq t \leq T\]

\[I_m^+(t) = -\frac{\alpha_m}{p^\beta}(t + \frac{\theta}{6}t^2)e^{-\frac{\alpha_m}{p^\beta}T_m} + \sum_{i=1}^{m} (\frac{\alpha_i - \alpha_{i-1}}{p^\beta})(T_{i-1} + \frac{\theta}{6}T_{i-1})e^{-\frac{\alpha_i}{p^\beta}T_m}\]
\[ T \leq t \leq T_m \]

\[ I_j(t) = \frac{\alpha_j}{p^\beta} \left[ (t + \theta \, t^2) e^{\alpha_j t} + \frac{\alpha_j}{p^\beta} (T_k + \theta \, T_k^2) e^{\alpha_j T_k} - \sum_{j=m+1}^{n} \left( (\alpha_j - \alpha_{j-1}) \left( (T_k - T_{k-1}) - \frac{\theta}{6} (T_k^3 - T_{k-1}^3) \right) \right) \right] \]

\[ T_{j-1} \leq t \leq T_j \quad \text{for} \quad j=(m+1), (m+2) \ldots \ldots \ldots n \]  

Now the total inventory during \([T_{k-1}, T_k]\) will be calculated as follows:

\[ I_k = \int_{T_{k-1}}^{T_k} I_k(t) \, dt \]

\[ I_k = \left( A - \frac{\alpha_k}{p^\beta} \left( \frac{T_k^2 - T_{k-1}^2}{2} - \frac{\theta}{12} (T_k^4 - T_{k-1}^4) \right) \right) \]

\[ + \sum_{i=1}^{\infty} \frac{\alpha_i - \alpha_{i-1}}{p^\beta} (T_{i-1} + \frac{\theta}{6} T_{i-1}^3) ((T_k - T_{k-1}) - \frac{\theta}{6} (T_k^3 - T_{k-1}^3)) \]  

\( (10) \)

The total inventory during \([T_{m-1}, T]\) will be:

\[ I_m^- = \int_{T_{m-1}}^{T} I_m^-(t) \, dt \]

\[ I_m^- = \left( A - \frac{\alpha_m}{p^\beta} \left( \frac{T_m^2 - T_{m-1}^2}{2} - \frac{\theta}{12} (T_m^4 - T_{m-1}^4) \right) \right) \]

\[ + \sum_{i=1}^{m} \frac{\alpha_i - \alpha_{i-1}}{p^\beta} (T_{i-1} + \frac{\theta}{6} T_{i-1}^3) ((T_m - T_{m-1}) - \frac{\theta}{6} (T_m^3 - T_{m-1}^3)) \]  

\( (11) \)

The total inventory during \([T, T_m]\) will be:

\[ I_m^+ = \int_{T}^{T_m} I_m^+(t) \, dt \]

\[ I_m^+ = \left[ -\frac{\alpha_m}{p^\beta} \left( \frac{T_m^2 - T^2}{2} - \frac{\theta}{12} (T_m^4 - T^4) \right) \right] \]

\[ + \frac{\alpha_m}{p^\beta} (T_m + \frac{\theta}{6} T_m^3) ((T_m - T) - \frac{\theta}{6} (T_m^3 - T^3)) \]

\[ - \sum_{i=m+1}^{n} \frac{\alpha_i - \alpha_{i-1}}{p^\beta} (T_{i-1} + \frac{\theta}{6} T_{i-1}^3) ((T_m - T) - \frac{\theta}{6} (T_m^3 - T^3)) \]  

\( (12) \)

Similarly, the total inventory during \([T_{j-1}, T_j]\) for all \(j=(m+1) \ldots \ldots n:\)

\[ I_j = \left[ -\frac{\alpha_j}{p^\beta} \left( \frac{T_j^2 - T_{j-1}^2}{2} - \frac{\theta}{12} (T_j^4 - T_{j-1}^4) \right) \right] \]

\[ + \frac{\alpha_m}{p^\beta} (T_j + \frac{\theta}{6} T_j^3) ((T_j - T_{j-1}) - \frac{\theta}{6} (T_j^3 - T_{j-1}^3)) \]
The holding cost of the system for finished products is calculated as follows:

\[ H.C_f = h_f \left( \sum_{k=1}^{n} I_k + I_m^- + I_m^+ + \sum_{j=m+1}^{n} I_j \right) \]  

(14)

The deterioration cost for the system of finished products is given by:

\[ D.C_f = d_f (AT - \sum_{k=1}^{n} \frac{\alpha_k}{p^\beta} (T_k - T_{k-1})) \]  

(15)

Set up cost for the finished product is computed as follows:

\[ S.C_f = K_0 \]  

(16)

Now, the total cost for the finished product can be calculated as follows:

\[ T.C_f = H.C_f + D.C_f + S.C_f \]  

(17)

a) Manufacturer’s raw material inventory model

In this model, the inventory is replenished at \( t=0 \). During \([0, T/n_r] \), the inventory depletes as a result of the combined effect of demand and deterioration. At \( t=T/n_r \), the inventory level becomes zero and is again replenished. The differential equation that governs the transition of the system is represented as follows:

\[ \frac{dI_r(t)}{dt} + \theta I_r(t) = -f_r ; 0 \leq t \leq T/n_r \]  

(18)

With boundary condition \( I_r(T/n_r)=0 \).

The solution of equation (18) will be:

\[ I_r(t) = f_r \left\{ \left( \frac{T}{n_r} - t \right) + \frac{\theta r}{6} \left( \frac{T}{n_r} \right)^3 \right\} e^{-\theta r \frac{T}{n_r}} ; 0 \leq t \leq T/n_r \]  

(19)

We know that \( I_r(0) = q \)

\[ q = f_r \left\{ \left( \frac{T}{n_r} \right) + \frac{\theta r}{6} \left( \frac{T}{n_r} \right)^3 \right\} \]  

(20)

Now, the holding cost of the system for carrying the raw material will be:

\[ H.C_r = \int_{0}^{T/n_r} I_r(t) dt \]  

\[ H.C_r = nh_r \left[ \int_{0}^{T/n_r} f_r \left\{ \frac{T^2}{2n_r^2} + \frac{\theta r}{8} \left( \frac{T}{n_r} \right)^4 \right\} dt \right] \]  

(21)

The cost of deteriorated units of raw material:

\[ D.C_r = d_r (n_r q - f_r T) \]  

(22)

Now, the total cost of raw material inventory can be calculated as follows:

\[ T.C_r = H.C_r + D.C_r + O.C_r \]  

(23)
Then, the integrated total cost for whole the system will be as follows:

\[ T.C. = T.C_f + T.C_r \]  

(24)

After putting the values in equation (24), we observe that T.C. for whole the system is a function of \( n_r \) and \( T \), where \( n_r \) is a discrete variable and \( T \) is a continuous variable.

Here we have discussed only the two market situation. The demand rate is taken as a function of selling price. The value of demand parameter \( \alpha_k \) for market one and market two is 1500 units and 1200 units respectively. The selling rate is Rs 8 per unit. The value for the deterioration parameter for finished products and raw material are 0.01 and 0.015 respectively. The season for selling items in market one is considered from 1-15 weeks and for second market it is 12-25 weeks. The setup cost and ordering cost for production and ordering the raw material is given by Rs 750 and Rs 500 respectively. The production cost and purchasing cost of raw material is Rs 5 and Rs 2 per unit respectively. The holding cost \( h_f \) and \( h_r \) are 0.25 and 0.2 per unit per time. The production rate is 500 units/ week.

Here \( n=3, \ T_1=5, \ T_2=17, \ T_3=25 \)

By applying the above mentioned solution procedure, we find the following values:

\( T^*=21 \) weeks, \( n_r=5.008 \), T.C. = Rs 852.012
VI. Sensitivity Analysis

For the given numerical illustration, investigations have been performed to study the impact of changes of different parameters on the number of deliveries of raw material, production period along with the total cost of the system.

Table 1: Sensitivity analysis for production rate $A$

<table>
<thead>
<tr>
<th>% variation in $A$</th>
<th>$A$</th>
<th>$n_r$</th>
<th>$T$</th>
<th>T.C.</th>
</tr>
</thead>
<tbody>
<tr>
<td>-20%</td>
<td>400</td>
<td>5.2</td>
<td>16.5</td>
<td>720.85</td>
</tr>
<tr>
<td>-15%</td>
<td>425</td>
<td>5.168</td>
<td>17.32</td>
<td>745.72</td>
</tr>
<tr>
<td>-10%</td>
<td>450</td>
<td>5.16</td>
<td>18.45</td>
<td>775.81</td>
</tr>
<tr>
<td>-5%</td>
<td>475</td>
<td>5.09</td>
<td>19.2</td>
<td>822.01</td>
</tr>
<tr>
<td>0%</td>
<td>500</td>
<td>5.008</td>
<td>21</td>
<td>852.012</td>
</tr>
<tr>
<td>5%</td>
<td>525</td>
<td>4.89</td>
<td>21.8</td>
<td>878.92</td>
</tr>
<tr>
<td>10%</td>
<td>550</td>
<td>4.72</td>
<td>22.71</td>
<td>912.65</td>
</tr>
<tr>
<td>15%</td>
<td>575</td>
<td>4.61</td>
<td>23.5</td>
<td>934.37</td>
</tr>
<tr>
<td>20%</td>
<td>600</td>
<td>4.45</td>
<td>24</td>
<td>961.41</td>
</tr>
</tbody>
</table>

Fig. 5: Variation of total cost T.C. with respect to production rate $A$

Table 2: Sensitivity analysis for deterioration parameter $\theta$

<table>
<thead>
<tr>
<th>% variation in $\theta$</th>
<th>$\theta$</th>
<th>$n_r$</th>
<th>$T$</th>
<th>T.C.</th>
</tr>
</thead>
<tbody>
<tr>
<td>-20%</td>
<td>0.012</td>
<td>5.008</td>
<td>13.67</td>
<td>828.52</td>
</tr>
<tr>
<td>-15%</td>
<td>0.01275</td>
<td>5.008</td>
<td>15.52</td>
<td>839.716</td>
</tr>
<tr>
<td>-10%</td>
<td>0.0135</td>
<td>5.008</td>
<td>17.81</td>
<td>843.82</td>
</tr>
<tr>
<td>-5%</td>
<td>0.01425</td>
<td>5.008</td>
<td>19.2</td>
<td>847.79</td>
</tr>
<tr>
<td>0%</td>
<td>0.015</td>
<td>5.008</td>
<td>21</td>
<td>852.012</td>
</tr>
</tbody>
</table>
Table 3: Sensitivity analysis for holding cost of finished products $h_f$

<table>
<thead>
<tr>
<th>% variation in $h_f$</th>
<th>$h_f$</th>
<th>$n_r$</th>
<th>$T$</th>
<th>T.C.</th>
</tr>
</thead>
<tbody>
<tr>
<td>-20%</td>
<td>0.2</td>
<td>6.712</td>
<td>25.7172</td>
<td>751.816</td>
</tr>
<tr>
<td>-15%</td>
<td>0.2125</td>
<td>6.381</td>
<td>24.875</td>
<td>782.71</td>
</tr>
<tr>
<td>-10%</td>
<td>0.225</td>
<td>5.862</td>
<td>23.981</td>
<td>813.48</td>
</tr>
<tr>
<td>-5%</td>
<td>0.2375</td>
<td>5.418</td>
<td>22.112</td>
<td>831.18</td>
</tr>
<tr>
<td>0%</td>
<td>0.25</td>
<td>5.008</td>
<td>21</td>
<td>852.012</td>
</tr>
<tr>
<td>5%</td>
<td>0.2625</td>
<td>4.67</td>
<td>19.987</td>
<td>878.93</td>
</tr>
<tr>
<td>10%</td>
<td>0.275</td>
<td>4.219</td>
<td>18.753</td>
<td>891.25</td>
</tr>
<tr>
<td>15%</td>
<td>0.2875</td>
<td>3.892</td>
<td>17.625</td>
<td>913.21</td>
</tr>
<tr>
<td>20%</td>
<td>0.3</td>
<td>3.416</td>
<td>16.512</td>
<td>934.48</td>
</tr>
</tbody>
</table>

Table 4: Sensitivity analysis for holding cost of raw material $h_r$

<table>
<thead>
<tr>
<th>% variation in $h_r$</th>
<th>$h_r$</th>
<th>$n_r$</th>
<th>$T$</th>
<th>T.C.</th>
</tr>
</thead>
<tbody>
<tr>
<td>-20%</td>
<td>0.16</td>
<td>5.008</td>
<td>21</td>
<td>839.78</td>
</tr>
<tr>
<td>-15%</td>
<td>0.17</td>
<td>5.008</td>
<td>21</td>
<td>842.14</td>
</tr>
<tr>
<td>-10%</td>
<td>0.18</td>
<td>5.008</td>
<td>21</td>
<td>845.32</td>
</tr>
<tr>
<td>-5%</td>
<td>0.19</td>
<td>5.008</td>
<td>21</td>
<td>849.516</td>
</tr>
<tr>
<td>0%</td>
<td>0.2</td>
<td>5.008</td>
<td>21</td>
<td>852.012</td>
</tr>
<tr>
<td>5%</td>
<td>0.21</td>
<td>5.008</td>
<td>21</td>
<td>855.221</td>
</tr>
<tr>
<td>10%</td>
<td>0.22</td>
<td>5.008</td>
<td>21</td>
<td>858.31</td>
</tr>
<tr>
<td>15%</td>
<td>0.23</td>
<td>5.008</td>
<td>21</td>
<td>861.54</td>
</tr>
<tr>
<td>20%</td>
<td>0.24</td>
<td>5.008</td>
<td>21</td>
<td>864.715</td>
</tr>
</tbody>
</table>

Fig. 6: Variation in total cost T.C. w.r.t deterioration parameter $\theta$
Fig. 7: Variation in total cost. T.C with respect to holding cost $h_f$

Fig. 8: Variation in total cost T.C. with respect to holding cost $h_r$

VII. Concluding Remarks

A sensitivity analysis is performed with respect to different system parameters to check the stability of the model.

1. From table 1, it is observed that as the value of the production parameter ‘$A$’ increases, it also results in increase in the total cost of the system.

2. Table 2 shows the variation in deterioration parameter ‘$\theta$’. It is shown in this table that an increment in deterioration parameter ‘$\theta$’ results also in an increment in T.C. of the system.

3. In table 3 and table 4, the variation in holding cost $h_f$ and $h_r$ are shown. It is observed from these tables that with the increment in holding parameters, the T.C. of the system increases in both the cases.

VIII. Conclusion

In this chapter we have presented an inventory model for deteriorating products. In this model, the manufacturer produces the items at a single location and sells it in different markets having different demands and selling seasons. With the help of a mathematical model, we calculate the different associated costs and provide a method to reduce the total cost of the system. This is done by deriving the optimal production time and replenishes time for the raw material. Further, performing a sensitivity analysis helps to check the stability of the model. With the sensitivity analysis, the model is found to be quite stable. For further research, time value of money and different permissible conditions can be applied for the extension of this model.

Appendix A:

The solution of equation (1) is given by:

$$I_k(t) = (A - \frac{\alpha}{\rho^2}(t + \frac{\theta}{6})e^{\frac{\phi^2}{4}} + C_i e^{\frac{\phi^2}{4}})$$

for $k=1,2, \ldots, (m-1)$

if $k=1$, from $I_1(0)=0$, we get $c_1=0$
Now from the boundary condition, we get \( I_k(T_{k-1}) = I_{k-1}(T_{k-1}) \)

\[
(c_k - c_{k-1}) = \frac{1}{p^\beta} \left( T_{k-1} + \frac{\theta}{6} T_{k-1}^3 \right) (\alpha_k - \alpha_{k-1})
\]

From this equation we get:

\[
c_2 - c_1 = \frac{1}{p^\beta} (\alpha_2 - \alpha_1)(T_1 + \frac{\theta}{6} T_1^3)
\]

\[
c_3 - c_2 = \frac{1}{p^\beta} (\alpha_3 - \alpha_2)(T_2 + \frac{\theta}{6} T_2^3)
\]

\[
c_4 - c_3 = \frac{1}{p^\beta} (\alpha_4 - \alpha_3)(T_3 + \frac{\theta}{6} T_3^3)
\]

\[
\vdots
\]

\[
c_k - c_{k-1} = \frac{1}{p^\beta} (\alpha_k - \alpha_{k-1})(T_{k-1} + \frac{\theta}{6} T_{k-1}^3)
\]

Summing up all these equations gives us:

\[
c_k - c_1 = \sum_{i=1}^{k} \frac{1}{p^\beta} (\alpha_i - \alpha_{i-1})(T_{i-1} + \frac{\theta}{6} T_{i-1}^3)
\]

Since \( c_1 \) is zero so:

\[
c_k = \sum_{i=1}^{k} \frac{1}{p^\beta} (\alpha_i - \alpha_{i-1})(T_{i-1} + \frac{\theta}{6} T_{i-1}^3)
\]

Then the solution of equation (1) will be:

\[
I_k(t) = \left( A - \frac{\alpha_k}{p^\beta} \right) (t + \frac{\theta}{6} t^3) e^{-\frac{\alpha_k}{2}} + \sum_{i=1}^{k} \frac{1}{p^\beta} (\alpha_i - \alpha_{i-1})(T_{i-1} + \frac{\theta}{6} T_{i-1}^3) e^{-\frac{\alpha_i}{2}}
\]

\[
T_{k-1} \leq t \leq T_k
\]

for \( k=1,2,\ldots\ldots(m-1) \)

Let \( k=m \), then solution of equation (2) will be:

\[
I_m(t) = \left( A - \frac{\alpha_m}{p^\beta} \right) (t + \frac{\theta}{6} t^3) e^{-\frac{\alpha_m}{2}} + \sum_{i=1}^{m} \frac{1}{p^\beta} (\alpha_i - \alpha_{i-1})(T_{i-1} + \frac{\theta}{6} T_{i-1}^3) e^{-\frac{\alpha_i}{2}}
\]

\[
T_{m-1} \leq t \leq T
\]

\[
I_j(t) = -\frac{\alpha_j}{p^\beta} (t + \frac{\theta}{6} t^3) e^{-\frac{\alpha_j}{2}} + \frac{\alpha_j}{p^\beta} (T_j + \frac{\theta}{6} T_j^3) e^{-\frac{\alpha_j}{2}}
\]

\[
-\sum_{i=j+1}^{n} \frac{1}{p^\beta} (\alpha_i - \alpha_{i-1})(T_{i-1} + \frac{\theta}{6} T_{i-1}^3) e^{-\frac{\alpha_i}{2}}
\]

for \( j=m+1,m+2,\ldots\ldots n \)

\[
T_{j-1} \leq t \leq T_j
\]
A PRODUCTION INVENTORY MODEL FOR DETERIORATING PRODUCTS WITH MULTIPLE MARKET AND SELLING PRICE DEPENDENT DEMAND

\[ I^*_n(t) = \frac{a_n}{p^2} \left( t + \frac{\theta}{\alpha} \right) e^{\frac{-\alpha}{2}} \left( \frac{a_n}{p^2} + \frac{\alpha}{6} T_n \right) e^{\frac{-\theta}{2}} \]

\[ - \sum_{i=1}^{s} \frac{1}{p^2} (\alpha_i - \alpha_{i-1}) (T_{i-1} + \frac{\theta}{6} T_{i-1}) e^{\frac{-\alpha}{2}} \]

\[ T \leq t \leq T_m \]

REFERENCES Références Referencias

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