Global Journal of Science Frontier Research: A
Physics and Space Science
Volume 16 Issue 6 Version 1.0 Year 2016
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Fluids through Inclined Granular Media

By A .T. Ngiangia & Sozo T Harry
University of Port Harcourt

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GJSFR-A Classification: *FOR Code: 020303*

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Abstract- A theoretical analysis of inclined fluids through granular media was carried out. Modified Darcy's equation and its approximate solution showed that for the isothermal case, increase in the argument and permeability of the granules result in a corresponding increase in the pressure exerted on the photosphere while increase in porosity and viscosity brings about a decrease in the pressure. For the adiabatic fluid case, the pressure is not affected as a result of increase in the angle of the granules. The same physical results occur in both the isothermal fluid case and that of the adiabatic fluid case as a result of increase in viscosity, porosity and permeability of the granules in the pressure exerted on the photosphere.

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I. Introduction

A look into the atmosphere at the surface of the sun appears completely opaque at a point called the photosphere. It is regarded as an imaginary surface from which the solar light appears to be emitted. Photosphere is the densest level of the solar atmosphere with a high temperature of not less than $5510\,^0C$ [1]. It is a star's outer shell from which light is radiated and a pressure of about the fraction 0.001 that of earth's atmosphere at sea level. The photosphere as a negative hydrogen ions, block, absorb and emit light, all of which prevent light from passing directly through a cloud of hydrogen ions. As a result of convection currents, pressure is exerted continuously on the photosphere and it breaks into tiny bright points called granules. The granules are in constant turmoil and change as a result of pressure exerted on the photosphere of the sun caused by convection currents of plasma within the sun's convection zone. The flow within the granules can reach supersonic speed and generates waves on the sun surface, at this level, the Navier-Stokes equation would be an appropriate equation to model the physical condition. Ulysses spacecraft and the Solar and Heliospheric Observatory (SOHO) of the United States and European Space Agency which was launched in 1990 and 1995 respectively, has jointly contributed to the understanding of solar wind in regions above the sun poles. Fluid flow through granular medium and its attendant effect forms the core for the study of astrophysics and its sub disciplines. There has been studies and many ongoing on the porosity, permeability, viscosity and its attendant effect on flow of fluid and this has necessitated the use of Darcy's equation and its modification in the study of these myriads of problems. [2] and [3] considered anthropogenic and non anthropogenic factors on the depletion of the ozone layer. They deduced that reaction of fluids mainly gases led to its depletion. [4], critically analyzed the seepage of polar fluids through porous media and deduced that, permeability and porosity are causes of the change in the pressure of the fluid. [5], examined porosity and permeability using modeling and strongly describe the existence of correlation between grain size and hydraulic conductivity.[6-11], specifically mentioned porous medium as the plank of their study and their recommendation greatly enrich the study of fluids through porous media. [12], considered the effect of fluid salinity on permeability of oil reserviour and opined that, increase concentration of fluid salinity, enhanced the recovery of core contents as a result of increased permeability. Measurement of flow material in the photosphere can be tackled using the principle of Doppler Effect. These measurements reveal additional features such as super granules, large scale flows and pattern of waves and oscillations. However, our aim is to theoretically examine the pressure of the fluid through an inclined granules and when it is not as a result of the effect of viscosity, porosity and permeability.

Author α: Department of Physics, University of Port Harcourt, Choba, Port Harcourt, Nigeria. e-mail: kellydap08@gmail.com
Author σ: Ignatius Ajuru University of Education, Rumuolumeni, Port Harcourt.

II. Formalism

For flow of fluid through porous medium, the smoothed continuity equation and the Darcy's equation respectively are

$$\xi\frac{\partial\rho}{\partial t}=-\left(\nabla.\rho v_0\right) \tag{1}$$

$$v_0=-\frac{\kappa}{\mu}\left(\nabla p-\rho g Cos\varphi\right) \tag{2}$$

where $\xi,\kappa,\rho,\mu,v_0,p,t,g,\varphi$ are respectively the porosity, permeability, density of fluid, fluid viscosity, superficial velocity, pressure of fluid, time, acceleration due to gravity and arguement. Combination of equations (1) and (2) result in

$$\left(\frac{\xi\mu}{\kappa}\right)\frac{\partial\rho}{\partial t}=\left(\nabla.\rho\left(\nabla p-\rho g Cos\varphi\right)\right) \tag{3}$$

with the boundary conditions

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$$\rho(0)=0 \text{ and } \rho(-1)=1 \tag{4}$$

We write the equation of state for this study following the argument of [11] as

$$\rho=\rho_0 p^m e^{\beta p} \tag{5}$$

where ρ_0 is the fluid density at unit pressure, m and β are integers.

III. Method of Solution

For Isothermal expansion of fluids, m = 1, $\beta=0$ and equations (3) and (5) reduced to

$$\frac{2\xi\mu\rho_0}{\kappa}\frac{\partial\rho}{\partial t}=\nabla^2\rho^2-\nabla\rho^2 g\rho_0 Cos\varphi \tag{6}$$

We approximate ρ^2 by discarding powers of ρ greater than unity using Taylor's series expansion about 0 and rewriting equation (6) in one dimension, we get

$$\frac{\xi\mu\rho_0}{\kappa}\frac{\partial\rho}{\partial t}=\frac{\partial^2\rho}{\partial x^2}-\frac{\partial\rho}{\partial x}\rho g\rho_0 Cos\varphi \tag{7}$$

To seek for solution of equation (7), we assume a solution of the form

$$\rho(x,t)=\theta(x)e^{-\lambda t} \tag{8}$$

where λ is a constant and the boundary conditions also transformed into

$$\theta(0)=0 \text{ and } \theta(-1)=e^{\lambda t} \tag{9}$$

Substitution of equation (8) into equation (7) and simplify we obtain

$$\theta''(x)-\varpi\theta'(x)+\lambda f\theta(x)=0 \tag{10}$$

$f=\dfrac{\xi\mu\rho_0}{\kappa}$ and $\varpi=g\rho_0 Cos\varphi$

The solution of equation (10) and the imposition of the boundary conditions of equation (9) as well as applying the transformation of equation (8), we get

$$\rho(x,t)=\frac{1}{1-\exp\left(\frac{\varpi-\sqrt{\varpi^2-4f\lambda}}{2}\right)}\exp\left(\frac{\varpi+\sqrt{\varpi^2-4f\lambda}}{2}\right)x+\frac{1}{\exp\left(\frac{\varpi-\sqrt{\varpi^2-4f\lambda}}{2}\right)-1}\exp\left(\frac{\varpi-\sqrt{\varpi^2-4f\lambda}}{2}\right)x \tag{11}$$

Using the equation of state as described in equation (5) and considering the case of isothermal expansion of fluids, we write equation (11) as

$$p(x)=\frac{1}{\rho_0}\frac{1}{1-\exp\left(\frac{\varpi-\sqrt{\varpi^2-4f\lambda}}{2}\right)}\exp\left(\frac{\varpi+\sqrt{\varpi^2-4f\lambda}}{2}\right)x+\frac{1}{\rho_0}\frac{1}{\exp\left(\frac{\varpi-\sqrt{\varpi^2-4f\lambda}}{2}\right)-1}\exp\left(\frac{\varpi-\sqrt{\varpi^2-4f\lambda}}{2}\right)x \tag{12}$$

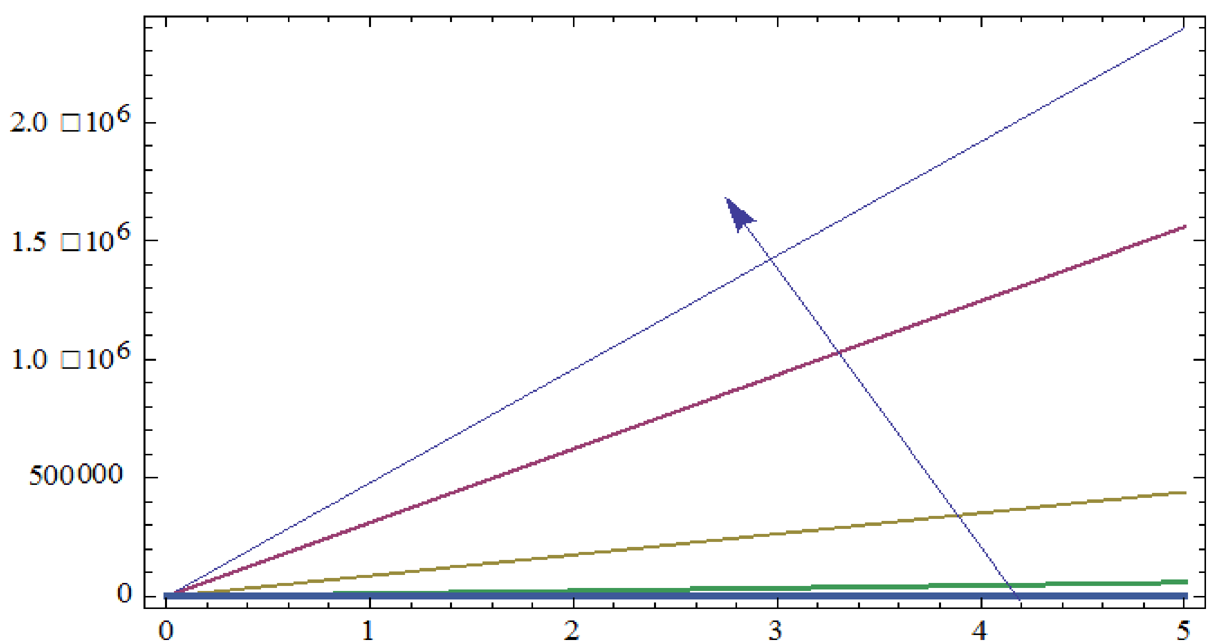

Figure 1: Pressure profile p against boundary layer x for varying angle (φ)

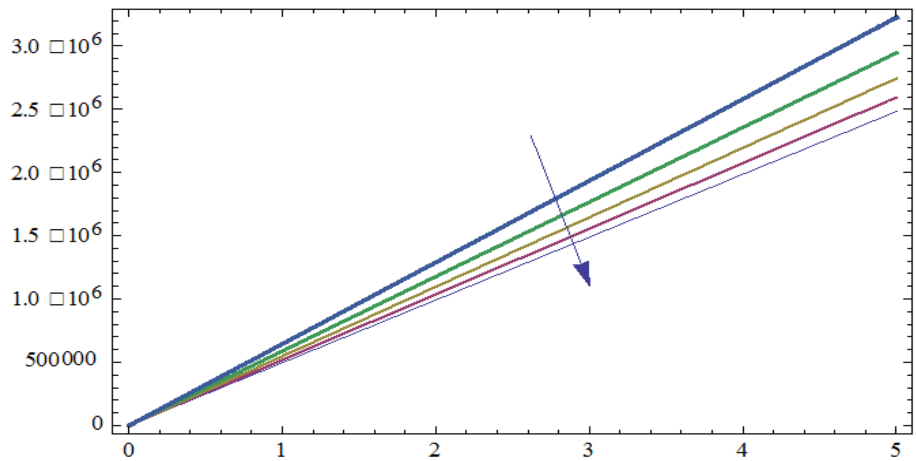

Figure 2: Pressure profile p against boundary layer x for varying porosity term. ξ

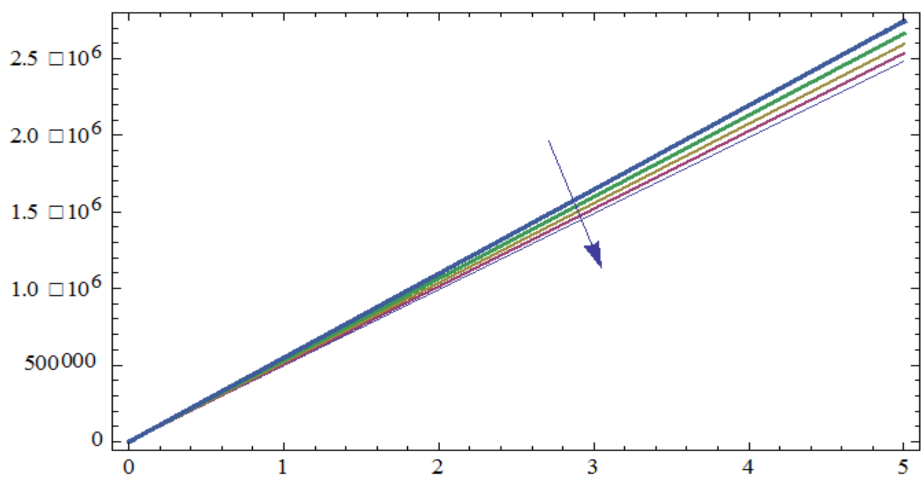

Figure 3: Pressure profile p against boundary layer x for varying viscous term (μ)

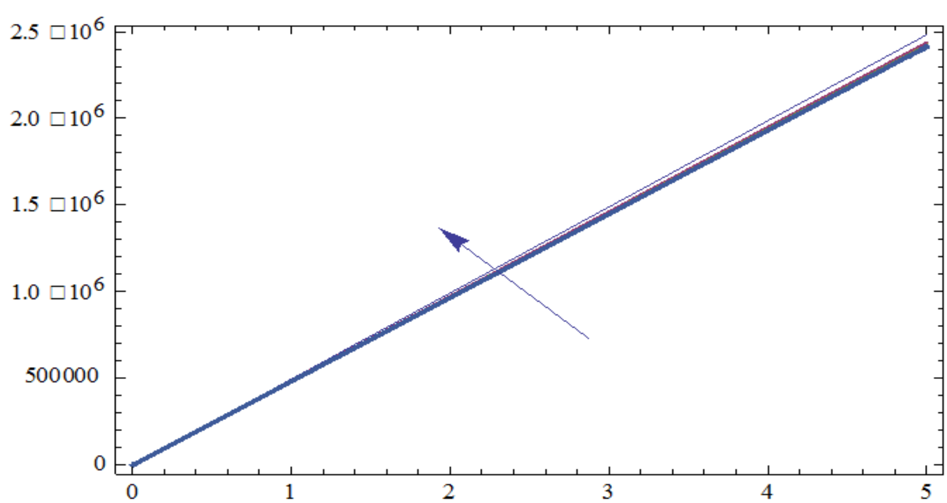

Figure 4: Pressure profile p against boundary layer x for varying permeability term (κ)

For adiabatic expansion of fluids, $\beta = 0$, $m = \frac{C_v}{C_p}$ (C_v is specific heat capacity at constant volume and C_p at constant pressure) and combination of equations (3) and (5) reduced to

$$\frac{(m+1)\xi\mu\rho_0^{\frac{1}{m}}}{\kappa}\frac{\partial\rho}{\partial t}=\nabla^2\rho^{\left(\frac{1+m}{m}\right)}-\nabla\rho^{\left(\frac{m+1}{m}\right)}g\rho_0^{\left(\frac{1}{m}\right)}Cos\varphi \quad (13)$$

Following the same procedure adopted for the isothermal case, we write equation (13) as

$$\frac{(m+1)\xi\mu\rho_0^{\frac{1}{m}}}{\kappa}\frac{\partial\rho}{\partial t}=\frac{(1+m)}{m}\frac{\partial^2\rho}{\partial x^2}-\frac{m+1}{m}\frac{\partial\rho}{\partial x}g\rho_0^{\frac{1}{m}}Cos\varphi \quad (14)$$

Applying the solution technique of equation (8), transform equation (14) into

$$\theta''(x)-\frac{\beta_3}{\beta_2}\theta'(x)+\frac{\beta_1}{\beta_2}\lambda\theta(x)=0 \quad (15)$$

where $\beta_1=\frac{(m+1)\xi\mu\rho_0^{\frac{1}{m}}}{\kappa}$, $\beta_2=\frac{m+1}{m}$, $\beta_3=\frac{m+1}{m}g\rho_0^{\frac{1}{m}}Cos\varphi$

The solution of equation (15) and the imposition of the boundary conditions of equation (9) as well as applying the transformation of equation (8), we get

$$\rho(x,t)=\frac{1}{\exp\left(\frac{\frac{\beta_3}{\beta_2}+\sqrt{\left(\frac{\beta_3}{\beta_2}\right)^2-4\frac{\beta_1}{\beta_2}\lambda}}{2}\right)-1}\exp\left(\frac{\frac{\beta_3}{\beta_2}+\sqrt{\left(\frac{\beta_3}{\beta_2}\right)-4\frac{\beta_1}{\beta_2}\lambda}}{2}\right)x+$$
$$\frac{1}{1-\exp\left(\frac{\frac{\beta_3}{\beta_2}+\sqrt{\left(\frac{\beta_3}{\beta_2}\right)-4\frac{\beta_1}{\beta_2}\lambda}}{2}\right)}\exp\left(\frac{\frac{\beta_3}{\beta_2}-\sqrt{\left(\frac{\beta_3}{\beta_2}\right)-4\frac{\beta_1}{\beta_2}\lambda}}{2}\right)x \quad (16)$$

Using the equation of state as described in equation (5) and considering the case of adiabatic expansion of fluids, we write equation (16) as

$$\log p=\frac{1}{m}\log\left[\frac{1}{\rho_0}\rho(x,t)\right] \quad (17)$$

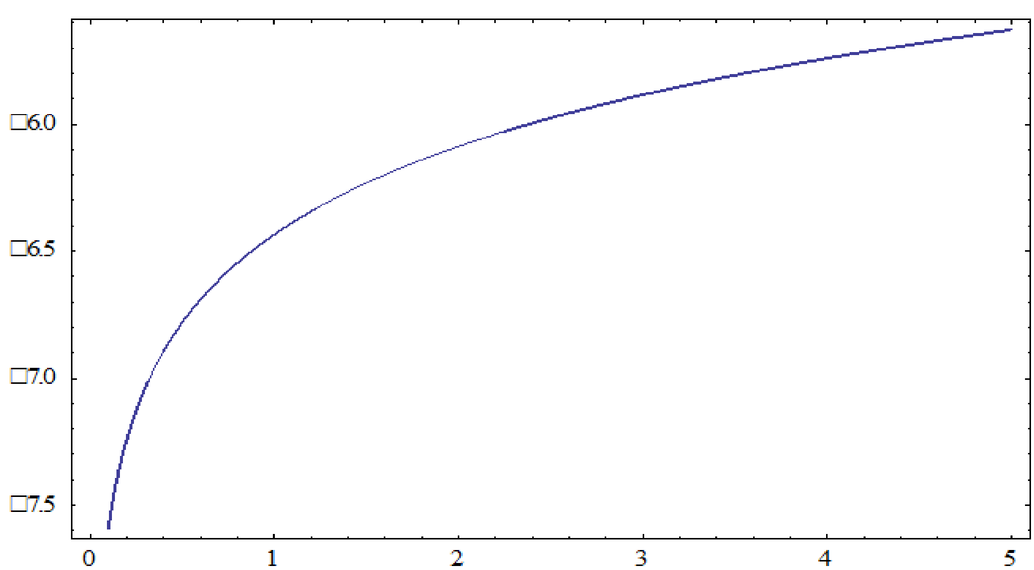

Figure 5: Pressure profile p against boundary layer x for varying angle (φ)

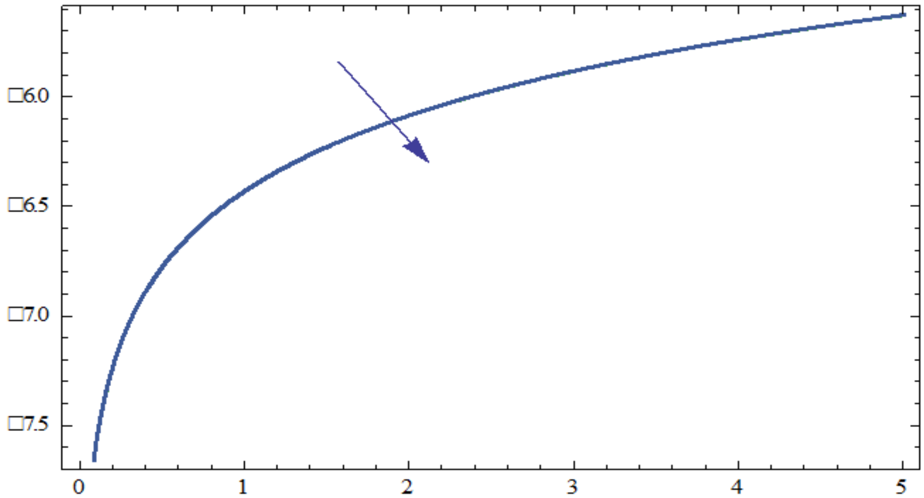

Figure 6: Pressure profile p against boundary layer x for varying porosity term. ξ

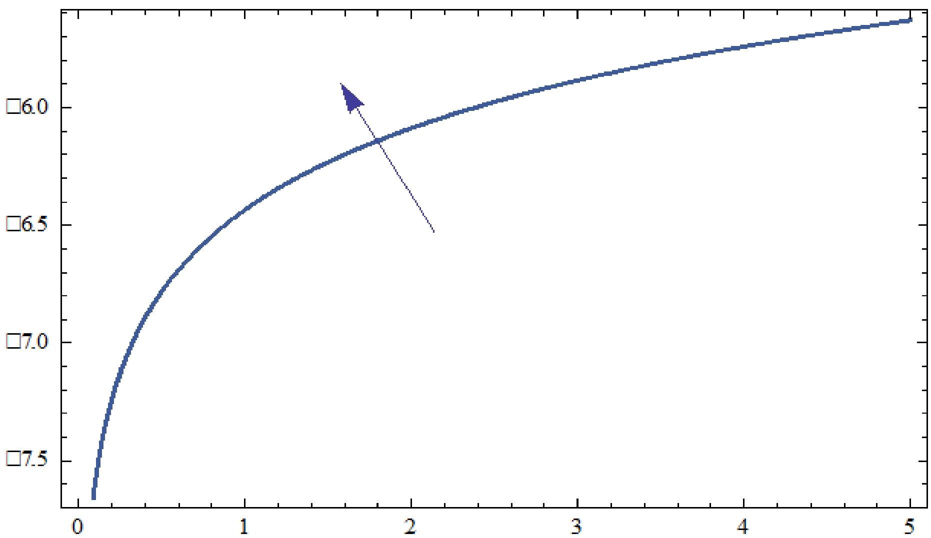

Figure 7: Pressure profile p against boundary layer x for varying permeability term (κ)

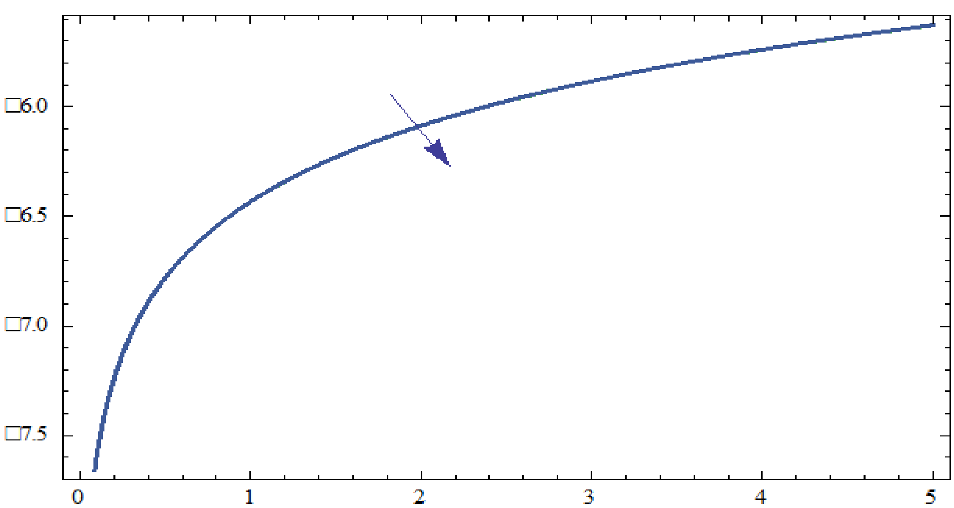

Figure 8: Pressure profile p against boundary layer x for varying viscous term (μ)

IV. Results and Discussion

For numerical validation and physical insight of the problem, an approximate value of the ratio of specific heat capacity at constant volume to that at constant pressure $\left(\frac{C_v}{C_p}=2\right)$, constant viscosity of fluid at $20^0 C$ (air $\mu = 1.0 N.sm^{-2}$) and constant $(\lambda = 0.0035)$ is chosen. The values of other parameters made use of are

$$\xi = 0.6, 1.2, 1.8, 2.4, 3.0$$

$$\kappa = 0.5, 1.0, 1.5, 2.0, 2.5$$

$$\rho_0 = 1.29 kgm^{-3}$$

$$\mu = 1.0, 1.5, 2.0, 2.5, 3.0$$

$$\varphi = 0, \frac{\pi}{12}, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}$$

Analysis of figure 1 shows that increase in the tilting of the granules result in an increase in the pressure exerted on the photosphere in the isothermal fluid case but from figure 5 in the adiabatic fluid case, increase in the angle of the granules does not affect the pressure on the granules. Figure 2 depicts the behaviour of increase in porosity on the pressure of the fluid, it is shown that increase in porosity leads to a corresponding decrease on the pressure of the fluid thereby reducing the granules. The same behaviour is observed in the adiabatic fluid case as illustrated in figure 6. Figure 3 shows that increase in viscosity results in decreasing the pressure of the photosphere therby reducing the pressure of the permeability of the fluid. The same physical situation occurs in the adiabatic fluid case as it is observed in figure 8. Increase in permeability, result in an increase in the pressure of the fluid as shown in figure 4, for the isothermal case and same situation is observed in the adiabatic fluid case as depicted in figure 7. The physical observation of porosity and permeability are inagreement with the work of [4].

V. Conclusion

For gases, it is customary to neglect the gravity (g) term, since it is small comp compared with the pressure terms as in the case of non gravitating gas but our study is for gravitating gas as in astrophysics and hence the gravity term is not neglected. In addition studying the physics of tilting the granules is new which we have analyzed.

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