Global Journal of Science Frontier Research: F
Mathematics and Decision Sciences
Volume 16 Issue 5 Version 1.0 Year 2016
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

The Distribution of Cube Root Transformation of the Error Component of the Multiplicative Time Series Model

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Keywords: power transformations, probability density function, error component, mean, variance, multiplicative time series.

GJSFR-F Classification : FOR Code : 65H04

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The Distribution of Cube Root Transformation of the Error Component of the Multiplicative Time Series Model

Dike A. O. [α], Otuonye E. L. [σ] & Chikezie D. C. [ρ]

Abstract- In this paper, the probability density function (pdf), of the cube root transformation was derived from the $\mathbf{n^{th}}$ power transformation of the error component of the multiplicative time series model. The mean and variance of the cube root transformation were equally established. From the simulated results it was found that the cube root transformed error component was normal with unit mean and variance approximately $\frac{1}{9}$ times that of the original error before transformation. Furthermore, the Kolmogorov-Smirnov test for normality was used ascertain the effect of cube root transformation as regard to normalization, from the results of the test at p-value of 0.05, we accepted normality for $\boldsymbol{\sigma}$ values of 0.001 to 0.22. Hence, a successful transformation is achieved when $\mathbf{0 \le \boldsymbol{\sigma} \le 0.22}$ depending on the decimal places desired.

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I. Introduction

The Gausian distribution (commonly called the normal distribution) is the best well known and most frequently used in probability distribution theory. It is widely used in natural and social sciences to represent real- valued random variables whose distributions are not known.

The normal distribution derived its usefulness from the central limit theorem. The central limit theorem states that the averages of random variables independently drawn from independent distributions converges in distribution to the normal, that is, becomes normally distributed when the number of random variables is sufficiently large. Physical quantities that are expected to be the sum of many independent processes (such as measurement errors) often have distributions that are nearly normal.

The probability density function (pdf) of the normal distribution is given in Uche(2003)as

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}\ e^{-\frac{1}{2}\left(\frac{x-1}{\sigma}\right)^2}, x \ge 0, \sigma^2 > 0 \qquad (1)$$

The error component $\mathrm{e_t}$ of the multiplicative time series model has a pdf $N(1, \sigma^2)$ where $\mathrm{e_t} > 0$, Iwueze(2007) established the distribution of the left-truncated normal distribution and is given by

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$$f(x) = \frac{e^{-\frac{1}{2}\left(\frac{x-1}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}\left[1-\Phi\left(\frac{-1}{\sigma}\right)\right]}, x \geq 0, \sigma^2 > 0 \quad (2)$$

With mean $E(X)$ and variance $Var(X)$ given by

$$E(X) = 1 + \frac{\sigma e^{-\frac{1}{2}\sigma^2}}{\sigma\sqrt{2\pi}\left[1-\Phi\left(\frac{-1}{\sigma}\right)\right]}, x \geq 0, \sigma^2 > 0 \quad (3)$$

and

$$Var(X) = \frac{\sigma^2}{2\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)}\left(\left[1 + P_r\left(\chi^2_{(1)} \leq \frac{1}{\sigma^2}\right)\right]\right) - \frac{\sigma e^{-\frac{1}{2}\sigma^2}}{\sqrt{2\pi}\left[1-\Phi\left(\frac{-1}{\sigma}\right)\right]} - \left[\frac{\sigma e^{-\frac{1}{2}\sigma^2}}{\sqrt{2\pi}\left[1-\Phi\left(\frac{-1}{\sigma}\right)\right]}\right]^2 \quad (4)$$

He examined some implications of truncating the $N(1,\sigma^2)$ to the left.

The truncated normal distribution have gained much acceptance in various fields of human endeavours, these include inventory management, regression analysis, operation management, time series analysis and so on. Johnson and Thomopoules (2004) considered the use of the left truncated distribution for improving achieved service levels. They presented the table of the cumulative distribution function of the left truncated normal distribution and derived the characteristic parameters of the distribution, and also presented the table of the partial expectation of the left truncated normal distribution A time series is a collection of ordered observation made sequentially in time. Examples abound in Sciences, Engineering, Economics etc and methods of analysing time series constitute a vital area in the field of Statistics.

According to Spiegel and Stephens (1999) the general time series model is always considered as a mixture of four major components, namely the Trend T_t, Seasonal movements S_t, cyclical movements C_t and irregular or Random Movements e_t. The general multiplicative time series model is given as

$$X_t = T_t S_t C_t e_t \quad (5)$$

In short term series the trend and cyclical components are merged to give the trend-cycle component; hence equation (5) can be rewritten as

$$X_t = M_t S_t e_t \quad (6)$$

where M_t is the trend cycle component and e_t is independent identically distributed (iid) normal errors with mean 1 and variance $\sigma^2 > 0$ $[e_t \sim N(1,\sigma^2)]$

Cox (2007) observed that the cube (X^3) transformation is a fairly strong transformation with a substantial effect on distribution shape. It is also used for reducing right skewness, and has the advantage that it can be applied to zero and non negative values. A similar property is possessed by any other root whose power is the inverse of an odd positive integer example 1/3,1/5, 1/7, etc. The cubic transformation is stronger than the square (X^2) transformation, though weaker than the logarithm transformation.

a) Data Transformation

According to Cox (2007) transformation is the replacement of a variable by a function of that variable, for example, replacing a variable x by the square root of x or the logarithm of x. In a stronger sense, a transformation is a replacement that changes

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the shape of a distribution or relationship. Reasons for transformation include stabilizing variance, normalizing, reducing the effect of outliers, making a measurement scale more meaningful, and to linearize a relationship. For more references see Bartlett (1947) Box and Cox (1964), etc.

Many time series analysis assume normality and it is well known that variance stabilization implies normality of the series. The most popular and common transformation are the logarithm transformation and the power transformations (square, square root, inverse, inverse square, and inverse square root). It is important to note that, if we apply the cube root transformation on model (6), we still obtain a multiplicative time series model given by

$$Y_t = M_t^3 \, S_t^3 \, e_t^3 = M_t^* \quad S_t^* \quad e_t^* \tag{7}$$

Where $M_t^3 = M_t^*, S_t^3 = S_t^*, \ e_t^3 = e_t^*$

Several studies abound in statistical literature on effects of power transformations on the error component of a multiplicative time series model whose error component is classified under the characteristics given in (3). The sole aim of such studies is to establish the conditions for successful transformation. A successful transformation is achieved when the derivable statistical properties of a data set remain unchanged after transformation, there basic properties or assumptions of interest for the studies are (i) unit mean and (ii) constant variance. Also Nwosu et al (2010) studied the effects of inverse and square root transformation respectively on the error component of the same model and discovered that the inverse transform $Y = \frac{1}{e_t}$ can be assumed to be normally distributed with mean, one and the same variance provided, $\sigma < 0.07$. Similarly Otuonye et al (2011) discovered that the square root transform; $\sqrt{e_t}$ can be assumed to be normally distributed with unit mean and variance $4\sigma^2$, for $\sigma_1 \leq 0.3$ where σ_1^2 is the variance of the original error component before transformation.

Ibeh et al (2013) studied the inverse square transformation of error the component of the multiplicative time series model, the results of the research showed that the basic assumptions of the error term of the multiplicative model which is normally distributed with mean 1 and finite variance can only be maintained if the standard deviation of the untransformed error term is less than or equal to 0.07 (σ ¡ 0.07). the study also revealed that the variance of the transformed of the error term is 4 times the variance of the untransformed for $\sigma \leq 0.07$.

Ajibade et al (2015) studied the distribution of the inverse square root transformed error component of the multiplication time series model and found out that the means are the same and variance $Var(e_t^*) \approx \frac{1}{4} Var(e_t)$ for $\sigma \leq 0$.

Dike et al (2016) generalized the power transformation by establishing the n^{th} power transformation; they showed that any power transformation can be derived from the formular.

In this paper the cube root transformation was carried out, the probability density function, the mean and variance of the distribution were established using the n^{th} power transformation given by Dike *et al*(2016) by substituting $n = \frac{1}{3}$

According to Osborne (2002) caution should be exercised in the transformation so that the basic structure of the original series is not altered.

To this end comparison would be made between the transformed and the untransformed series for their mean and variances respectively to know the condition under which the transformation would be successful.

Otuonye *et al* (2011) showed that the variance of the untransformed is 4 times the variance of the transformed series for $o<\sigma\leq 0.3$

In this cube root transformation we would demonstrate whether the transformation is normal with mean 1 and same variance. This would be done by simulating the original series for specified values σ and carry out the transformation.

The normality test of the cube root transformed series would be done using the kolmogorov- Smirnov normality test for varying values of σ.

To validate our finding simulation and practical example would be use to drive the research home.

b) The Probability Density Function (pdf) of the Cube Root Transformation

The pdf of the general equation of the n^{th} power transformation given by Dike *et al* (2016) is

$$f(y)=\frac{\frac{1}{|n|}.y^{\frac{1}{n}-1}\,e^{-\frac{1}{2}\left(\frac{y^{\frac{1}{n}}-1}{\sigma}\right)^{2}}}{\sigma\sqrt{2\pi}\left[1-\Phi\left(\frac{-1}{\sigma}\right)\right]} \tag{8}$$

Substituting $n=\frac{1}{3}$ in_the_General_equation given in (8) yields the pdf of the cube root transformation given as

\

$$f(y)=\frac{\frac{1}{3}.y^{\frac{1}{3}-1}\,e^{-\frac{1}{2}\left(\frac{\frac{1}{3}-1}{\sigma}\right)^{2}}}{\sigma\sqrt{2\pi}\left[1-\Phi\left(\frac{-1}{\sigma}\right)\right]} \tag{9}$$

From equation (9) we derive the moments (mean and variance) of the cube root transformation

c) The Mean for cube root transformation

Given

$$E(Y)=1+\frac{n\sigma}{\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}e^{-\frac{1}{2\sigma^{2}}}+\frac{n(n-1)\sigma^{2}}{2!\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(-\frac{e^{-\frac{1}{2\sigma^{2}}}}{\sigma}+\frac{\sqrt{2\pi}}{2}\left[1+P_{r}\left(\chi^{2}_{(1)}\leq\frac{1}{\sigma^{2}}\right)\right]\right)+\frac{2n(n-1)(n-2)\sigma^{3}}{3!\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(1+\frac{1}{2\sigma^{2}}\right)e^{-\frac{1}{2\sigma^{2}}} \tag{10}$$

For $n=\frac{1}{3}$

$$E(Y)=1+\frac{\left(\frac{1}{3}\right)\sigma}{\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}e^{-\frac{1}{2\sigma^{2}}}+\frac{\left(\frac{1}{3}\right)\left(\frac{1}{3}-1\right)\sigma^{2}}{2!\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(-\frac{e^{-\frac{1}{2\sigma^{2}}}}{\sigma}+\frac{\sqrt{2\pi}}{2}\left[1+P_{r}\left(\chi^{2}_{(1)}\leq\frac{1}{\sigma^{2}}\right)\right]\right)+\frac{2\left(\frac{1}{3}\right)\left(\frac{1}{3}-1\right)\left(\frac{1}{3}-2\right)\sigma^{3}}{3!\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(1+\frac{1}{2\sigma^{2}}\right)e^{-\frac{1}{2\sigma^{2}}} \tag{11}$$

$$=1+\frac{\sigma}{3\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}e^{-\frac{1}{2\sigma^{2}}}+\frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\sigma^{2}}{2\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(-\frac{e^{-\frac{1}{2\sigma^{2}}}}{\sigma}+\frac{\sqrt{2\pi}}{2}\left[1+P_{r}\left(\chi^{2}_{(1)}\leq\frac{1}{\sigma^{2}}\right)\right]\right)+\frac{2\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)\sigma^{3}}{6\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(1+\frac{1}{2\sigma^{2}}\right)e^{-\frac{1}{2\sigma^{2}}} \tag{12}$$

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$$= 1 + \frac{\sigma}{3\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}} - \frac{\sigma^2}{9\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(-\frac{e^{-\frac{1}{2\sigma^2}}}{\sigma} + \frac{\sqrt{2\pi}}{2}\left[1 + P_r\left(\chi^2_{(1)} \le \frac{1}{\sigma^2}\right)\right]\right) + \frac{10\sigma^3}{81\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}$$

$$\left(1 + \frac{1}{2\sigma^2}\right) e^{-\frac{1}{2\sigma^2}} \quad (13)$$

$$= 1 + \frac{\sigma}{3\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}} + \frac{\sigma}{9\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}} - \frac{\sigma^2}{18\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(1 + P_r\left(\chi^2_{(1)} \le \frac{1}{\sigma^2}\right)\right) + \frac{10\sigma^3}{81\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}} +$$

$$\frac{5\sigma}{81\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}} \quad (14)$$

$$= 1 + \frac{(27\sigma + 9\sigma + 5\sigma)}{81\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}} - \frac{\sigma^2}{18\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(1 + P_r\left(\chi^2_{(1)} \le \frac{1}{\sigma^2}\right)\right) + \frac{10\sigma^3}{81\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}} \quad (15)$$

$$= 1 + \frac{41\sigma}{16\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}} - \frac{\sigma^2}{18\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(1 + P_r\left(\chi^2_{(1)} \le \frac{1}{\sigma^2}\right)\right) + \frac{10\sigma^3}{81\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}} \quad (16)$$

$$\therefore E(Y) = 1 + \frac{41\sigma}{16\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}} - \frac{\sigma^2}{18\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(1 + P_r\left(\chi^2_{(1)} \le \frac{1}{\sigma^2}\right)\right) + \frac{10\sigma^3}{81\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}} \quad (17)$$

d) Variance for cube root transformation

For $E(Y^2)$

Given

$$E(Y^2) = 1 + \frac{2n\sigma}{\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}} + \frac{2n(2n-1)\sigma^2}{2!\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(-\frac{e^{-\frac{1}{2\sigma^2}}}{\sigma} + \frac{\sqrt{2\pi}}{2}\left[1 + P_r\left(\chi^2_{(1)} \le \frac{1}{\sigma^2}\right)\right]\right) +$$

$$\frac{2(2n)(2n-1)(2n-2)\sigma^3}{3!\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(1 + \frac{1}{2\sigma^2}\right) e^{-\frac{1}{2\sigma^2}} \quad (18)$$

For $n = \frac{1}{3}$

$$E(Y^2) = 1 + \frac{2\left(\frac{1}{3}\right)\sigma}{\sqrt{2\pi}\left[1-\Phi\left(\frac{-1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}} + \frac{2\left(\frac{1}{3}\right)\left[2\left(\frac{1}{3}\right)-1\right]\sigma^2}{2!\sqrt{2\pi}\left[1-\Phi\left(\frac{-1}{\sigma}\right)\right]}\left(-\frac{e^{-\frac{1}{2\sigma^2}}}{\sigma} + \frac{\sqrt{2\pi}}{2}\left[1 + P_r\left(\chi^2_{(1)} \le \frac{1}{\sigma^2}\right)\right]\right) + \frac{2\left[2\left(\frac{1}{3}\right)\right]\left[2\left(\frac{1}{3}\right)-1\right]\left[2\left(\frac{1}{3}\right)-2\right]\sigma^3}{3!\sqrt{2\pi}\left[1-\Phi\left(\frac{-1}{\sigma}\right)\right]}$$

$$\left(1 + \frac{1}{2\sigma^2}\right) e^{-\frac{1}{2\sigma^2}} \quad (19)$$

$$= 1 + \frac{2\sigma}{3\sqrt{2\pi}\left[1-\Phi\left(\frac{-1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}} + \frac{\left(\frac{2}{3}\right)\left(\frac{-1}{3}\right)\sigma^2}{2\sqrt{2\pi}\left[\left[1-\Phi\left(\frac{-1}{\sigma}\right)\right]\right]}\left(-\frac{e^{-\frac{1}{2\sigma^2}}}{\sigma} + \frac{\sqrt{2\pi}}{2}\left[1 + P_r\left(\chi^2_{(1)} \le \frac{1}{\sigma^2}\right)\right]\right) + \frac{2\left(\frac{2}{3}\right)\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)\sigma^3}{6\sqrt{2\pi}\left[1-\Phi\left(\frac{-1}{\sigma}\right)\right]}$$

$$\left(1 + \frac{1}{2\sigma^2}\right) e^{-\frac{1}{2\sigma^2}} \quad (20)$$

$$= 1 + \frac{2\sigma}{3\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}} - \frac{\sigma^2}{9\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(-\frac{e^{-\frac{1}{2\sigma^2}}}{\sigma} + \frac{\sqrt{2\pi}}{2}\left[1 + P_r\left(\chi^2_{(1)} \le \frac{1}{\sigma^2}\right)\right]\right) + \frac{8\sigma^3}{81\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}$$

$$\left(1 + \frac{1}{2\sigma^2}\right) e^{-\frac{1}{2\sigma^2}} \quad (21)$$

$$= 1 + \frac{2\sigma}{3\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}} + \frac{\sigma}{9\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}} - \frac{\sigma^2}{18\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(1 + P_r\left(\chi^2_{(1)} \le \frac{1}{\sigma^2}\right)\right) + \frac{8\sigma^3}{81\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}} +$$

$$\frac{4\sigma}{81\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}} \quad (22)$$

$$= 1 + \frac{(54+9+4)\sigma}{81\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}} - \frac{\sigma^2}{18\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(1 + P_r\left(\chi^2_{(1)} \leq \frac{1}{\sigma^2}\right)\right) + \frac{8\sigma^3}{81\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}} \quad (23)$$

$$\therefore E(Y^2) = 1 + \frac{67\sigma}{81\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}} - \frac{\sigma^2}{18\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(1 + P_r\left(\chi^2_{(1)} \leq \frac{1}{\sigma^2}\right)\right) + \frac{8\sigma^3}{81\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}} \quad (24)$$

Without any lost in generality, the subsequent terms in $E(Y^2)$ *and* $E(Y)$ with the factor $e^{-\frac{1}{2\sigma^2}}$ will decay fast to zero for values of $\sigma \geq 0$

$$E(Y^2) = 1 - \frac{\sigma^2}{18\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(1 + P_r\left(\chi^2_{(1)} \leq \frac{1}{\sigma^2}\right)\right)$$

And

$$\therefore E(Y) = 1 - \frac{\sigma^2}{18\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(1 + P_r\left(\chi^2_{(1)} \leq \frac{1}{\sigma^2}\right)\right)$$

$$\Rightarrow Var(Y) = E(Y^2) - [E(y)]^2$$

$$= \left[1 - \frac{\sigma^2}{18\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(1 + P_r\left(\chi^2_{(1)} \leq \frac{1}{\sigma^2}\right)\right)\right] - \left[1 - \frac{\sigma^2}{18\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(1 + P_r\left(\chi^2_{(1)} \leq \frac{1}{\sigma^2}\right)\right)\right]^2$$

$$\therefore Var(Y) = \left[\frac{\sigma^2}{18\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(1 + P_r\left(\chi^2_{(1)} \leq \frac{1}{\sigma^2}\right)\right)\right] - \left[\frac{\sigma^2}{18\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(1 + P_r\left(\chi^2_{(1)} \leq \frac{1}{\sigma^2}\right)\right)\right]^2 \quad (25)$$

II. Numerical Illustration

The graph forms of the probability density function of the cube root transformation for specified values of σ are given on figures 3.9 to 3.12. for want of space only graph of σ = 0.22 to 0.25 are shown here.

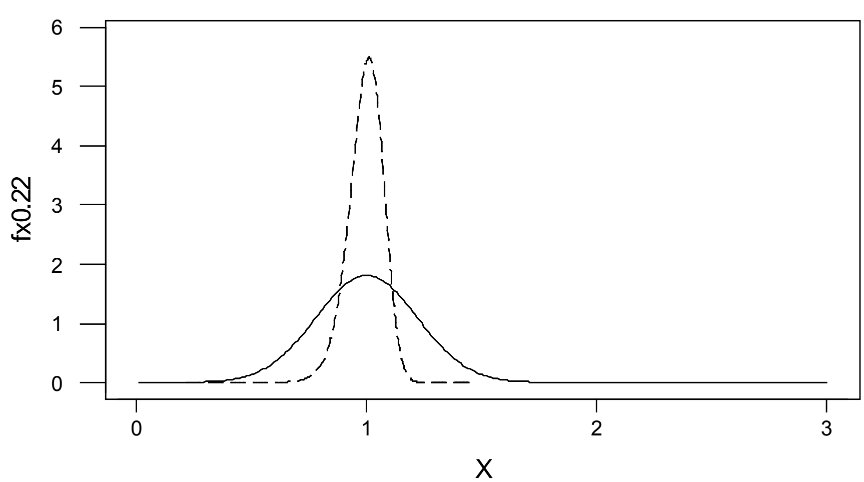

Figure 3.9 : Graph of $f(x)$ and $f(y)$ for $\sigma = 0.22$

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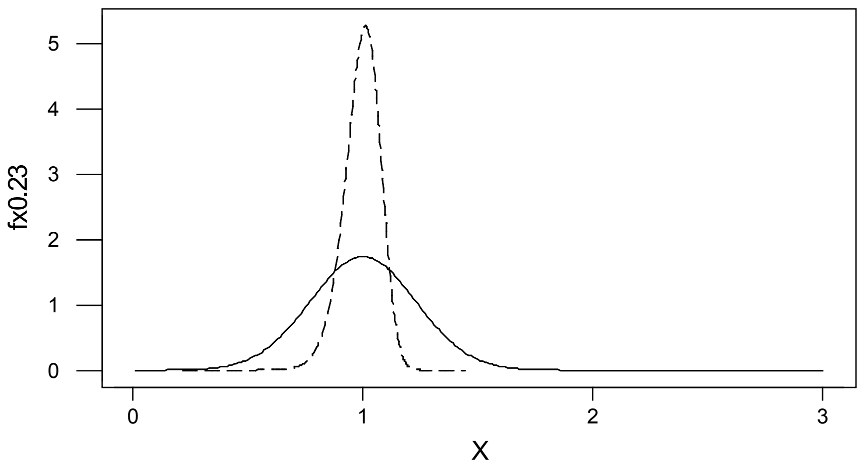

Figure 3.10 : Graph of $f(x)$ and $f(y)$ for $\sigma = 0.23$

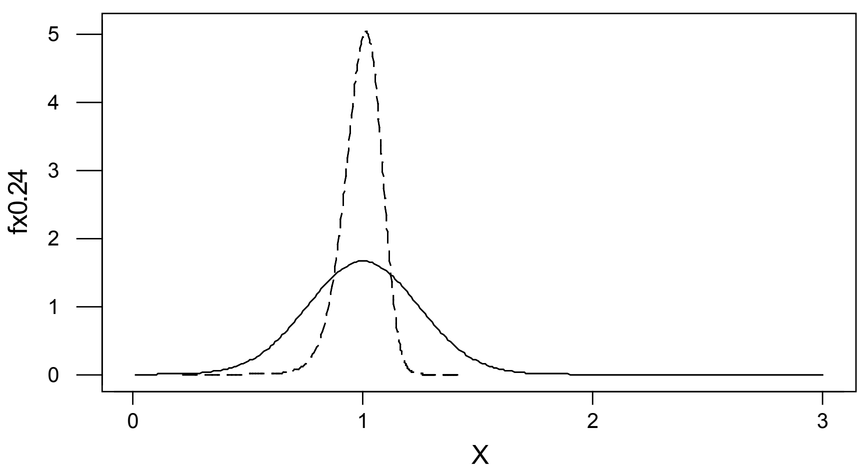

Figure 3.11 : Graph of $f(x)$ and $f(y)$ for $\sigma = 0.24$

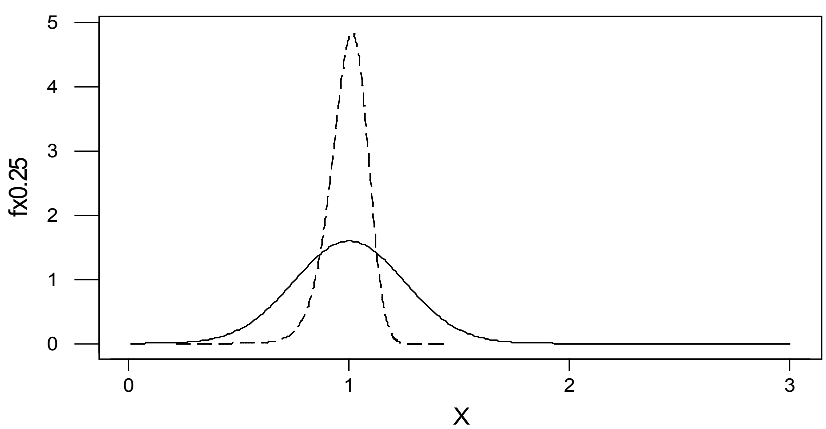

Figure 3.12 : Graph of $f(x)$ and $f(y)$ for $\sigma = 0.25$

The graphs of $f(x)$ and $f(y)$ look bell-shaped and there is none of the graphs where the two graphs coincide.

The numerical comparison of simulation between the cube root transformed distribution and the left-truncated $(1, \sigma^2)$ distribution for their means and variances for $0 < \sigma \leq 0.6$. the data is presented in table 1

Table 1 : Simulated values of E(X), E(Y), Var(X) and Var(Y)

S/no	sigma	E(X)	E(Y)	VAR(X)	VAR(Y)	var(x)/var(y)
1	0.001	1.00000	1.00000	0.000001	0.0000001	9.0000
2	0.002	1.00000	1.00000	0.000004	0.0000004	9.0000
3	0.003	1.00000	1.00000	0.000009	0.0000010	9.0000
4	0.004	1.00000	1.00000	0.000016	0.0000018	9.0000
5	0.005	1.00000	1.00000	0.000025	0.0000028	9.0000
6	0.006	1.00000	1.00000	0.000036	0.0000040	9.0000
7	0.007	1.00000	0.99999	0.000049	0.0000054	9.0000
8	0.008	1.00000	0.99999	0.000064	0.0000071	9.0001
9	0.009	1.00000	0.99999	0.000081	0.0000090	9.0001
10	0.010	1.00000	0.99999	0.000100	0.0000111	9.0001
11	0.011	1.00000	0.99999	0.000121	0.0000134	9.0001
12	0.012	1.00000	0.99998	0.000144	0.0000160	9.0001
13	0.013	1.00000	0.99998	0.000169	0.0000188	9.0002
14	0.014	1.00000	0.99998	0.000196	0.0000218	9.0002
15	0.015	1.00000	0.99997	0.000225	0.0000250	9.0002
16	0.016	1.00000	0.99997	0.000256	0.0000284	9.0003
17	0.017	1.00000	0.99997	0.000289	0.0000321	9.0003
18	0.018	1.00000	0.99996	0.000324	0.0000360	9.0003
19	0.019	1.00000	0.99996	0.000361	0.0000401	9.0004
20	0.020	1.00000	0.99996	0.000400	0.0000444	9.0004
21	0.021	1.00000	0.99995	0.000441	0.0000490	9.0004
22	0.022	1.00000	0.99995	0.000484	0.0000538	9.0005
23	0.023	1.00000	0.99994	0.000529	0.0000588	9.0005
24	0.024	1.00000	0.99994	0.000576	0.0000640	9.0006
25	0.025	1.00000	0.99993	0.000625	0.0000694	9.0006
26	0.026	1.00000	0.99992	0.000676	0.0000751	9.0007
27	0.027	1.00000	0.99992	0.000729	0.0000810	9.0007
28	0.028	1.00000	0.99991	0.000784	0.0000871	9.0008
29	0.029	1.00000	0.99991	0.000841	0.0000934	9.0008
30	0.030	1.00000	0.99990	0.000900	0.0001000	9.0009

Result of simulation of σ = 0.001 to 0.600, but for want of space only 0.001 to 0.030 are shown in table 1 above.

Depending on the level of accuracy needed, we have the following conditions for the means to be equal to 1.0 and variance of the left truncated $\boldsymbol{N(1,\sigma^2)}$ to be equal to 9 times the variance of the cube root transformed left truncated $\boldsymbol{N(1,\sigma^2)}$

Table 2 : Conditions for the means and variances to be equal

S/No	Decimal Places	E(x) = E(y)	Var(x) = 9*Var(y)
1	4	$0 \le \sigma \le 0.022$	$0 \le \sigma \le 0.007$
2	3	$0 \le \sigma \le 0.067$	$0 \le \sigma \le 0.021$
3	2	$0 \le \sigma \le 0.212$	$0 \le \sigma \le 0.070$
4	1	$0 \le \sigma \le 0.567$	$0 \le \sigma \le 0.221$

By adopting 1 decimal place, the interval where the left truncated $N(1,\sigma^2)$distribution and its cube root transformed counterpart have means equal to 1.0 and variance of the former equal to 9 times the variance of the later, that is given by $0 \le \sigma \le 0.221$.

a) Comparison of the means and standard deviations of the Simulated Series

In this section the summary o the simulated means, standard deviations, variances, ratios of standard deviations and variances of the untransformed and

Notes

transformed distributions would be presented on table 1 to observe if there are departures from the earlier established results. The means, standard deviations, variances, ratios of standard deviations and variances of the of the simulated values for the untransformed and transformed distributions are presented in Table 2.

Also in this Section, we would test for the normality of the cube root transformed values using Kolmogorov-Smirnov Goodness-of-Fit test (One sample case).

Notes

b) The Test Statistic

The difference between the theoretical cumulative distribution function F(x) and the sample cumulative distribution $F^0(x)$ is measured by the statistic D, and it is the greatest vertical distance between F(x) and $F^0(x)$. For a two- sided test of the hypothesis, the null hypothesis and alternative is given by

$$H_o: F^0(x) = F(x) \ \forall x$$

$$H_1: F^0(x) \neq F(x) \text{ for at least one x}$$

The test statistic is $D = \text{Sup}\left|F^*(x) - F(x)\right|$

The null hypothesis is rejected at $\alpha = 0.05$ level of significance if the computed value of D exceeds the value read from a statistical table, and the sample size is n = 300. For ease of computations, Minitab software was used and the results summarized in table 2

Table 3 : Summary of Kolmogorov-Smirnov Test of Normality for the transformed series (For specified values of σ)

Σ	D^+	D^-	D	Approx p-value	α	Decision
0.001	0.043	0.033	0.043	0.072	0.05	Accept normality
0.005	0.031	0.037	0.037	0.15	,,	Accept normality
0.01	0.30	0.035	0.035	0.15	,,	Accept normality
0.05	0.023	0.033	0.033	0.15	,,	Accept normality
0.10	0.020	0.031	0.031	0.15	,,	Accept normality
0.15	0.023	0.033	0.033	0.15	,,	Accept normality
0.20	0.017	0.031	0.031	0.15	,,	Accept normality
0.21	0.018	0.031	0.031	0.15	,,	Accept normality
0.22	0.250	0.037	0.037	0.15	,,	Accept normality
0.23	0.028	0.054	0.054	<0.01	,,	Reject normality
0.24	0.022	0.060	0.060	<0.01	,,	Reject normality
0.25	0.027	0.049	0.049	<0.028	,,	Reject normality

From the summary of test of normality of the transformed series, we observe that the cube root transformed series is normal $0 < \sigma \leq 0.22$

III. Summary and Recommendations

The findings of the research work are summarised, conclusions were also drawn and suggestion for further research and recommendation using the n^{th} root transformation and to establish the cube root transformation.

a) Summary

The finding of the research are summarised as follows

(a) The probability density function of the cube root transformation derived from n^{th} root transformed error component of multiplicative time series model Dike et al (2016) and is given by

$$f(y) = \frac{\frac{1}{|3|} \cdot y^{\frac{1}{|3|}-1} e^{-\frac{1}{2}\left(\frac{y^{\frac{1}{|3|}}-1}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}\left[1 - \Phi\left(\frac{-1}{\sigma}\right)\right]}$$

The work went further by establishing the moments (mean and variance) of the cube root transformation .

(b) The graph forms of the probability density function were shown to be bell-shaped and symmetric for some values of σ.

(c) The mean and variance of the cube root transformation are in terms of the cumulative density function and the chi-square distribution with 1 degree of freedom (df). The mean given by

$$E(Y) = 1 + \frac{41\sigma}{16\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}} - \frac{\sigma^2}{18\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(1 + P_r\left(\chi^2_{(1)} \le \frac{1}{\sigma^2}\right)\right) +$$

$$\frac{10\sigma^3}{81\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}}$$

also the variance is given by

(d) $Var(Y)=\left[\frac{\sigma^2}{18\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(1 + P_r\left(\chi^2_{(1)} \le \frac{1}{\sigma^2}\right)\right)\right]-\left[\frac{\sigma^2}{18\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(1 + P_r\left(\chi^2_{(1)} \le \frac{1}{\sigma^2}\right)\right)\right]^2$ Using simulated values it was found that the condition under which the mean and variance are equal as shown in table 4

Table 4 : Conditions for Successful transformation

S/No	Decimal Places	E(x) = E(y)	Var(x) = 9*Var(y)
1	4	$0 \le \sigma \le 0.022$	$0 \le \sigma \le 0.007$
2	3	$0 \le \sigma \le 0.067$	$0 \le \sigma \le 0.021$
3	2	$0 \le \sigma \le 0.212$	$0 \le \sigma \le 0.070$
4	1	$0 \le \sigma \le 0.567$	$0 \le \sigma \le 0.221$

(e) From the result in table 4 it was shown that the means of the error component of the original and the cube root transformed series is 1 and the variance of the original series is 9 times that of the transformed series for $0 < \sigma \le 0.22$ depending on the decimal places desired.

(f) The test of normality using the Kolomogorov-Simirnov test shows in table 4.16 showed that the cube root transformed series is normal for $0 < \sigma \le 0.22$

IV. Conclusion

In situation where the cube root is to be applied the following steps should be adopted for it to be successful and serve the need for which it was adopted. This would be achieved by ensuring that

Notes

(i) The model used to analyse the error component is multiplicative
(ii) It fits the transformation to be adopted
(iii) The untransformed and the transformed meet the conditions for normality. These measures will guarantee that data satisfy the assumption inherent in the statistical inference to be applied and ensure improved interpretation as expressed by Osborne(2002); who expressed that caution should be exercised on the choice of transformation to be used so that the fundamental structure of the series is not altered

Notes

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