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# Pseudo Ricci-Symmetric $(LCS)_n$-Manifolds

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22. Z.I. Szabo, *Structure theorems on Riemannian space satisfying* $R(X, Y).R = 0$, *I, The local version*, J. Differential Geom., 17, 531 (1982).

# Pseudo Ricci-Symmetric $(LCS)_n$ -Manifolds

N. S. Ravikumar [α] & K. Nagana Gouda [σ]

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## I. Introduction

The study of Riemann symmetric manifolds began with the work of Cartan [5]. According to Cartan, a Riemannian manifold is said to be locally symmetric if its curvature tensor $R$ satisfies the relation $DR = 0$, where $D$ is the covariant differentiation operator with respect to the metric tensor $g$. During the last five decades the notion of locally symmetric manifolds has been weakened by many authors in several ways to a different extent such as recurrent manifold [27], semi symmetric manifold [22], pseudo symmetric manifold [6, 14] etc.,

If the Ricci tensor $S$ of type $(0, 2)$ in a Riemannian manifold $M$ satisfies the relation $DS = 0$, then $S$ is said to be Ricci symmetric. The notion of Ricci symmetry has been studied extensively by many authors in several ways to a different extent viz., Ricci recurrent manifold [18], Ricci semi symmetric manifold [22], Ricci pseudo symmetric manifold [7, 13], weakly Ricci symmetric manifold [23].

A $(2n + 1)$-dimensional non-flat Riemannian manifold $M$ is said to be pseudo Ricci symmetric if its Ricci tensor $S$ of type $(0, 2)$ is not identically zero and satisfies the relation

$$(D_X S)(Y, Z) = 2A(X)S(Y, Z) + A(Y)S(X, Z) + A(Z)S(X, Y),$$

for any vector fields $X$, $Y$ and $Z$, where $A$ is a nowhere vanishing 1-form on $M$. The pseudo Ricci symmetric manifolds have also been studied by Arslan et. al [1], De and Mazumder [9], De et. al [11] and many others.

The present paper is organized as follows: In Section 2 we give the definitions and some preliminary results that will be needed thereafter. In Section 3 we discuss generalized pseudo-Ricci symmetric $(LCS)_n$-manifold and it is shown that the sum of $2A$, $B$ and $C$ is always nonzero. Section 4 is devoted to the study of almost pseudo Ricci-symmetric $(LCS)_n$-manifold and obtain that the sum $3A + B$ is nowhere zero. In section 5 we consider semi pseudo Ricci-symmetric $(LCS)_n$-manifold and proved that the 1-form $A$ is always non zero.

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## II. PRELIMINARIES

The notion of Lorentzian concircular structure manifolds (briefly $(LCS)_n$-manifolds) was introduced by A.A. Shaikh [20] in 2003. An $n$-dimensional Lorentzian manifold $M$ is a smooth connected paracompact Hausdorff manifold with a Lorentzian metric $g$, that is, $M$ admits a smooth symmetric tensor field $g$ of type $(0,2)$ such that for each point $p \in M$, the tensor $g_p : T_pM \times T_pM \to R$ is a non-degenerate inner product of signature $(-,+,.....,+)$, where $T_pM$ denotes the tangent vector space of $M$ at $p$ and $R$ is the real number space.

***Definition 2.1.*** *In a Lorentzian manifold $(M,g)$, a vector field $P$ defined by*

$$g(X,P) = A(X),$$

*for any vector field $X \in \chi(M)$ is said to be a concircular vector field if*

$$(\nabla_X A)(Y) = \alpha[g(X,Y) + \omega(X)A(Y)],$$

*where $\alpha$ is a non-zero scalar function, $A$ is a 1-form and $\omega$ is a closed 1-form.*

Let $M$ be a $n$-dimensional Lorentzian manifold admitting a unit timelike concircular vector field $\xi$, called the characteristic vector field of the manifold. Then we have

$$g(\xi,\xi) = -1. \tag{2.1}$$

Since $\xi$ is a unit concircular vector field, there exists a non-zero 1-form $\eta$ such that

$$g(X,\xi) = \eta(X), \tag{2.2}$$

the equation of the following form holds

$$(\nabla_X \eta)(Y) = \alpha[g(X,Y) + \eta(X)\eta(Y)], \quad (\alpha \neq 0) \tag{2.3}$$

for all vector fields $X, Y$, where $\nabla$ denotes the operator of covariant differentiation with respect to Lorentzian metric $g$ and $\alpha$ is a non-zero scalar function satisfying

$$\nabla_X \alpha = (X\alpha) = d\alpha(X) = \rho\eta(X), \tag{2.4}$$

$\rho$ being a certain scalar function given by $\rho = -(\xi\alpha)$. If we put

$$\phi X = \frac{1}{\alpha}\nabla_X \xi, \tag{2.5}$$

then from (2.3) and (2.4), we have

$$\phi X = X + \eta(X)\xi, \tag{2.6}$$

from which it follows that $\phi$ is a symmetric $(1,1)$ tensor. Thus the Lorentzian manifold $M$ together with the unit timelike concircular vector field $\xi$, its associated 1-form $\eta$ and $(1,1)$ tensor field $\phi$ is said to be a Lorentzian concircular structure manifold (briefly $(LCS)_n$-manifold). Especially, if we take $\alpha = 1$, then we can obtain the Lorentzian para-Sasakian structure of Matsumoto [16]. In a $(LCS)_n$-manifold, the following relations hold ([20], [21]):

Ref

20. A.A. Shaikh, *Lorentzian almost paracontact manifolds with a structure of concircular type*, Kyungpook Math. J., 43 (2) (2003), 305-314.

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$$\eta(\xi) = -1, \quad \phi\xi = 0, \quad \eta(\phi X) = 0, \tag{2.7}$$

$$g(\phi X, \phi Y) = g(X,Y) + \eta(X)\eta(Y), \tag{2.8}$$

$$\eta(R(X,Y)Z) = (\alpha^2 - \rho)[g(Y,Z)\eta(X) - g(X,Z)\eta(Y)], \tag{2.9}$$

$$(\nabla_X \phi)(Y) = \alpha[g(X,Y)\xi + 2\eta(X)\eta(Y)\xi + \eta(Y)X], \tag{2.10}$$

$$S(X,\xi) = (n-1)(\alpha^2 - \rho)\eta(X), \tag{2.11}$$

$$S(\phi X, \phi Y) = S(X,Y) + (n-1)(\alpha^2 - \rho)\eta(X)\eta(Y), \tag{2.12}$$

for any vector fields $X, Y, Z$, where $R, S$ denote respectively the curvature tensor and the Ricci tensor of the manifold.

## III. Generalized Pseudo-Ricci Symmetric $(LCS)n$-Manifold

Let $M$ be a $(2n+1)$-dimensional generalized pseudo-Ricci symmetric $(LCS)_n$-manifold. Then by definition, we have

$$(D_X S)(Y,Z) = 2A(X)S(Y,Z) + B(Y)S(X,Z) + C(Z)S(X,Y), \tag{3.1}$$

where $A$, $B$ and $C$ are three non-zero 1-forms.
Putting $Z = \xi$ in (3.1) and using (2.11), we get

$$(D_X S)(Y,\xi) = 2(n-1)(\alpha^2 - \rho)A(X)\eta(Y) + (n-1)(\alpha^2 - \rho)B(Y)\eta(X) + C(\xi)S(X,Y). \tag{3.2}$$

Also we have

$$(D_X S)(Y,\xi) = (n-1)(2\alpha\rho - \beta)\eta(X)\eta(Y) + (n-1)(\alpha^2 - \rho)\alpha[g(X,Y) + \eta(X)\eta(Y)] - \alpha S(X,Y). \tag{3.3}$$

By using (3.3) in (3.2), we get

$$(n-1)(2\alpha\rho - \beta)\eta(X)\eta(Y) + (n-1)(\alpha^2 - \rho)\alpha[g(X,Y) + \eta(X)\eta(Y)] - \alpha S(X,Y) = 2(n-1)(\alpha^2 - \rho)A(X)\eta(Y) + (n-1)(\alpha^2 - \rho)B(Y)\eta(X) + C(\xi)S(X,Y). \tag{3.4}$$

Again putting $X = Y = \xi$ in (3.2), yields

$$2A(\xi) + B(\xi) + C(\xi) = -\frac{2\alpha\rho - \beta}{\alpha^2 - \rho}. \tag{3.5}$$

Taking $X = \xi$ in (3.2) and by virtue of (3.5), we have

$$B(Y) = -\eta(Y)B(\xi). \tag{3.6}$$

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Similarly by taking $Y = \xi$ in (3.2) and using (3.5), we get

$$A(X) = -\eta(X)A(\xi). \tag{3.7}$$

Substituting $X = Y = \xi$ in (3.1) and by virtue of (3.5), we obtain

$$C(Z) = -C(\xi)\eta(Z). \tag{3.8}$$

In view of (3.6), (3.7) and (3.8), we have

$$2A(X) + B(X) + C(X) = \frac{2\alpha\rho - \beta}{\alpha^2 - \rho}\eta(X) \ \ for \ \ all \ \ X.$$

Hence we can state the following theorem:

*Theorem 3.1.* *In a (2n+1)-dimensional generalized pseudo Ricci-symmetric $(LCS)n$-manifold the sum of $2A$, $B$ and $C$ is always nonzero.*

## IV. Almost Pseudo Ricci-Symmetric $(LCS)n$-Manifold

In 2007, Chaki and Kawaguchi [8] introduced a type of non-flat Riemannian manifold whose Ricci tensor $S$ of type $(0, 2)$ satisfies the condition

$$(D_X S)(Y, Z) = [A(X) + B(X)]S(Y, Z) + A(Y)S(X, Z) + A(Z)S(X, Y), \tag{4.1}$$

where $A$, $B$ are non-zero 1-forms called the associated 1-forms and $D$ denotes the operator of covariant differentiation with respect to the metric $g$. Such a manifold was called an almost pseudo Ricci symmetric manifold. If in particular $A = B$ then the manifold becomes a pseudo Ricci symmetric manifold introduced by Chaki [7].

Let us consider $M$ be an almost pseudo Ricci-symmetric $N(k)$-contact metric manifold. Now putting $Z = \xi$ in (4.1), we get

$$(D_X S)(Y, \xi) = [A(X) + B(X)]S(Y, \xi) + A(Y)S(X, \xi) + A(\xi)S(X, Y). \tag{4.2}$$

By using (2.3), (2.4), (2.5) and (2.11) in (4.2), we have

$$\begin{aligned} (n-1)(2\alpha\rho - \beta)\eta(X)\eta(Y) &= (n-1)(\alpha^2 - \rho)[A(X) + B(X)]\eta()Y \\ &+ (n-1)(\alpha^2 - \rho)A(Y)\eta(X) + A(\xi)S(X, Y). \end{aligned} \tag{4.3}$$

Putting $X = \xi$ in (4.3), we get

$$\begin{aligned} -(2\alpha\rho - \beta)\eta(Y) &= (\alpha^2 - \rho)[A(\xi) + B(\xi)]\eta(Y) \\ &- (\alpha^2 - \rho)A(Y) + A(\xi)(\alpha^2 - \rho)\eta(Y). \end{aligned} \tag{4.4}$$

Again putting $Y = \xi$ in (4.4), gives

$$3A(\xi) + B(\xi) = -\frac{(2\alpha\rho - \beta)}{(\alpha^2 - \rho)}. \tag{4.5}$$

Ref

7. M.C. Chaki, *On pseudo Ricci symmetric manifolds*, Bulg. J. Phys., 15 (1988), 526-531.

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Next putting $Y = \xi$ in (4.3), we get

$$-(2\alpha\rho - \beta)\eta(X) = -(\alpha^2 - \rho)[A(X) + B(X)] + 2(\alpha^2 - \rho)A(\xi)\eta(X). \tag{4.6}$$

Now replacing $Y$ by $X$ in (4.4) and adding with (4.6), and by virtue of (4.5), we obtain

$$3A(X) + B(X) = \frac{(2\alpha\rho - \beta)}{(\alpha^2 - \rho)}\eta(X). \tag{4.7}$$

Hence we can state the following:

*Theorem 4.2. An $(2n+1)$-dimensional $(LCS)_n$-manifold is almost pseudo Ricci-symmetric, if the sum $3A + B$ is nowhere zero.*

## V. SEMI PSEUDO RICCI-SYMMETRIC $(LCS)n$-MANIFOLD

The notion of semi pseudo Ricci Symmetric Manifolds introduced by Tarafdar and Jawarneh in 1993 [24], a non flat Riemannian manifold whose Ricci tensor $S$ satisfies:

$$(D_X S)(Y, Z) = A(Y)S(X, Z) + A(Z)S(X, Y), \tag{5.1}$$

where $A$ is a non-zero 1-form.
Put $Z = \xi$ in (5.1), we get

$$(n-1)(2\alpha\rho - \beta)\eta(X)\eta(Y) + (n-1)(\alpha^2 - \rho)\alpha[g(X, Y) + \eta(X)\eta(Y)] - \alpha S(X, Y) \tag{5.2}$$

$$= 2(n-1)(\alpha^2 - \rho)A(X)\eta(Y) + (n-1)(\alpha^2 - \rho)B(Y)\eta(X) + C(\xi)S(X, Y).$$

Putting $X = \xi$ in (5.2), gives

$$(\alpha^2 - \rho)A(Y) = [(2\alpha\rho - \beta) + (\alpha^2 - \rho)A(\xi)]\eta(Y). \tag{5.3}$$

Again putting $Y = \xi$ in (5.3), we have

$$A(\xi) = -\frac{(2\alpha\rho - \beta)}{(\alpha^2 - \rho)}. \tag{5.4}$$

Using (5.4) in (5.3), gives

$$A(Y) = \frac{(2\alpha\rho - \beta)}{(\alpha^2 - \rho)}\eta(Y). \tag{5.5}$$

Hence we can state the following theorem:

*Theorem 5.3. In a semi pseudo Ricci-symmetric $(LCS)_n$-manifold, the 1-form $A$ is always non zero and is given by (5.5).*

Also from above theorem we can state the following corollary:
*Corollary 5.1. There exists a semi pseudo Ricci-symmetric $(LCS)_n$-manifold.*

Ref

24. M. Tarafdar and M.A. Jawarneh, *Semi-Pseudo Ricci Symmetric manifold*, J. Indian. Inst. of Science, 73, (1993), 591-596.

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