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Distribution Natural Waves in an Infinite Viscoelastic Cylinder with Radial Cracks and Wedges

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Abstract- This paper deals with the distribution of natural waves of an infinite cylinder with radial crack and wedge. The task is put in cylindrical coordinates. Viscoelastic cylinder with radial crack is a limiting case of the wedge with an angle 360^0. With the help of the Navier equations and physical system received six differential equations. After not complicated conversion obtained spectral boundary value problem for systems of ordinary and partial differential equations complex the coefficient, which is then solved by the direct and orthogonal shooting with a combination of the method of Mueller on a complex arithmetic. A dispersion relation for a cylinder with radial crack and the wedge was got.

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6. Safarov I.I., Teshaev M.KH, Boltaev Z.I.. Mathematical modeling of wave process in a mechanical waveguide taking into account the internal friction. Germany. LAP. 2013. 243p.

Distribution Natural Waves in an Infinite Viscoelastic Cylinder with Radial Cracks and Wedges

Safarov Ismail Ibrahimovich [α], Akhmedov Maqsud Sharipovich [σ] & Boltayev Zafar Ixtiyarovich [ρ]

Annotation- This paper deals with the distribution of natural waves of an infinite cylinder with radial crack and wedge. The task is put in cylindrical coordinates. Viscoelastic cylinder with radial crack is a limiting case of the wedge with an angle 360^0. With the help of the Navier equations and physical system received six differential equations. After not complicated conversion obtained spectral boundary value problem for systems of ordinary and partial differential equations complex the coefficient, which is then solved by the direct and orthogonal shooting with a combination of the method of Mueller on a complex arithmetic. A dispersion relation for a cylinder with radial crack and the wedge was got.

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I. Introductions

One of the central tasks of dynamic elasticity theory is the study of the spread of a perturbation of the stress-strain state in deformed bodies with geometric structures that combine the concept of a mechanical waveguide [1,2,3]. The main features are the length of the waveguide in one direction, as well as restrictions and localization of the wave beam in other directions. Accounting for the damping capacity of the waveguide material plays an important role in the dynamic behavior of the structure [4,5]. It leads to a marked weakening of the natural oscillations, a significant decrease in amplitudes of forced vibrations and smoothing of the stresses in the concentration zone of the oscillations. The complexity of their solutions for many reasons, for example, rheological properties of real waveguides, not the classic geometric shapes and so on. N., Resulting in a wide variety of schematized models to describe in some approximation of real phenomena and makes it difficult to create a unified mathematical model of the mechanical system [6] . In the viscoelastic cylinder with radial crack is a limiting case of the wedge with an angle 360^0. A method and a solution algorithm for the study of wave propagation in viscoelastic cylinder with radial crack and wedge with an arbitrary angle vertex.

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II. Waves in an Infinite Cylinder with Radial Crack

The propagation of harmonic waves in an infinite elastic cylinder with radial crack is dialed. The task is put in cylindrical coordinates. The elastic cylinder with radial crack is a limiting case of the wedge with an angle 360^0. The basic equations of motion of an elastic medium, which occupies a region B are defined by three groups of relations [6]:

$$\frac{\partial \sigma_{ik}}{\partial x_k} = \rho \frac{\partial^2 u_i}{\partial t^2};$$

$$\varepsilon_{ik} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i}\right), \quad \sigma_{ik} = \tilde{\lambda}\,\theta\delta_{ik} + 2\tilde{\mu}\varepsilon_{ik}. \tag{1}$$

Here σ_{ik}- stress tensor, ε_{ik}- strain tensor, θ- volumetric deformation, $\tilde{\lambda}$ and $\tilde{\mu}$ - operator elastic moduli [7,8,9]:

$$\tilde{\lambda}\,\varphi(t) = \lambda_{01}\left[\varphi(t) - \int_0^t R_\lambda(t-\tau)\varphi(t)d\tau\right];$$

$$\tilde{\mu}\varphi(t) = \mu_{01}\left[\varphi(t) - \int_0^t R_\mu(t-\tau)\varphi(t)d\tau\right] \tag{2}$$

$\varphi(t)$– Arbitrary function of time; $R_\lambda(t-\tau)$ and $R_\mu(t-\tau)$ – core and relaxation λ_{01}, μ_{01}– Instant elastic modules. We accept the integral terms in (2) small, then the function $\varphi(t) = \psi(t)e^{-i\omega_R t}$, where $\psi(t)$- a slowly varying function of time, ω_R- real constant. Next, using the freezing procedure [9], we note the relation (3) approximate species

$$\bar{\lambda}\varphi = \lambda_{01}\left[1 - \Gamma_\lambda^C(\omega_R) - i\Gamma_\lambda^S(\omega_R)\right];$$

$$\bar{\mu}\varphi = \mu_{01}\left[1 - \Gamma_\mu^C(\omega_R) - i\Gamma_\mu^S(\omega_R)\right]\varphi, \tag{3}$$

Where

$$\Gamma_\lambda{}^C(\omega_R) = \int_0^\infty R_\lambda(\tau)\cos\omega_R\tau\, d\tau;\ \Gamma_\lambda{}^S(\omega_R) = \int_0^\infty R_\lambda(\tau)\sin\omega_R\tau\, d\tau,$$

$$\Gamma_\mu{}^C(\omega_R) = \int_0^\infty R_\mu(\tau)\cos\omega_R\tau\, d\tau, \Gamma_\mu{}^S(\omega_R) = \int_0^\infty R_\mu(\tau)\sin\omega_R\tau\, d\tau.$$

Respectively cosine and sine Fourier transform of the relaxation of the core material. As an example, the viscoelastic material take three parametric relaxation nucleus $R_\lambda(t) = R_\mu(t) = Ae^{-\beta t}/t^{1-\alpha}$. On the effect of the function $R(t-\tau)$ superimposed usual requirements integral ability, continuity (except $t=\tau$), sign certainty and monotony:

Ref

9. Koltunov M.A.. Creep and relaxation. - M.: Higher School, 1976. - 276p.

$$R \rangle 0, \quad \frac{dR(t)}{dt} \leq 0, \; 0 \langle \int_0^\infty R(t)dt \langle 1.$$

In a cylindrical coordinate system, the equation (1), (2), (3) have the form

$$\frac{\partial \sigma_{rr}}{dr} + \frac{\sigma_{rr} - \sigma_{r\varphi}}{r} + \frac{1}{r}\frac{\partial \sigma_{r\varphi}}{\partial \varphi} + \frac{\partial \sigma_{rz}}{\partial z} = \rho \frac{\partial^2 u_r}{dt^2};$$

$$\frac{1}{r}\frac{\partial \sigma_{\varphi\varphi}}{\partial \varphi} + \frac{2\sigma_{r\varphi}}{r} + \frac{\partial \sigma_{r\varphi}}{\partial r} + \frac{\partial \sigma_{z\varphi}}{\partial z} = \rho \frac{\partial^2 u_\varphi}{\partial t^2}; \qquad (4)$$

$$\frac{\partial \sigma_{zz}}{\partial z} + \frac{\partial \sigma_{rz}}{\partial r} + \frac{\sigma_{zz}}{r} + \frac{1}{r}\frac{\partial \sigma_{z\varphi}}{\partial \varphi} = \rho \frac{\partial^2 u_z}{\partial t^2};$$

$$\varepsilon_{rr} = \frac{\partial u_r}{\partial r}; \;\; \varepsilon_{zz} = \frac{\partial u_z}{\partial z}; \; \varepsilon_{\varphi\varphi} = \frac{1}{r}\frac{\partial u_r}{\partial \varphi} + \frac{u_r}{r};$$

$$\varepsilon_{r\varphi} = \frac{1}{2}\left(\frac{1}{r}\frac{\partial u_r}{\partial \varphi} + \frac{\partial u_\varphi}{\partial r} - \frac{u_\varphi}{r}\right); \varepsilon_{rz} = \frac{1}{2}\left(\frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z}\right); \qquad (5)$$

$$\varepsilon_{\varphi z} = \frac{1}{2}\left(\frac{\partial u_\varphi}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial \varphi}\right);$$

$$\sigma_{rr} = \lambda\left(\frac{\partial u_r}{\partial r} + \frac{1}{r}\frac{\partial u_\varphi}{\partial \varphi} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z}\right) + 2\mu \frac{\partial u_r}{\partial r};$$

$$\sigma_{r\varphi} = 2\mu\varepsilon_{r\varphi} = \mu\left(\frac{1}{r}\frac{\partial u_r}{\partial \varphi} + \frac{\partial u_\varphi}{\partial r} - \frac{u_\varphi}{r}\right);$$

$$\sigma_{rz} = 2\mu\varepsilon_{rz} = \mu\left(\frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z}\right);$$

$$\sigma_{\varphi\varphi} = \lambda\left(\frac{\partial u_r}{\partial r} + \frac{1}{r}\frac{\partial u_\varphi}{\partial \varphi} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z}\right) + 2\mu\left(\frac{1}{r}\frac{\partial u_\varphi}{\partial \varphi} + \frac{u_r}{r}\right);$$

$$\sigma_{\varphi z} = \mu\left(\frac{\partial u_\varphi}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial \varphi}\right),$$

$$\sigma_{zz} = \lambda\left(\frac{\partial u_r}{\partial r} + \frac{1}{r}\frac{\partial u_\varphi}{\partial r} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z}\right) + 2\mu \frac{\partial u_z}{\partial z}. \qquad (6)$$

Where $\sigma_{rr}, \sigma_{r\varphi}, \sigma_{rz}, \sigma_{\varphi\varphi}, \sigma_{\varphi z}, \sigma_{zz}$ - respectively components of the stress tensor; $\varepsilon_{rr}, \varepsilon_{r\varphi}, \varepsilon_{rz}, \varepsilon_{\varphi\varphi}, \varepsilon_{\varphi z}, \varepsilon_{zz}$ -respectively components of the strain tensor. The link between stress and strain is given in the third chapter. Equations (4), (5) and (6) after algebraic manipulations are identical to the system of six differential equations solved with respect to the first derivative of the radial coordinate [10,11]

Ref

10. Safarov I.I., Teshaev M.KH. Boltaev Z.I.. Distribution of linear waves in extended lamellar bodies. LAP, Lambert Academic Publishing (Germany). 2016. 315 p.

$$\begin{cases} \dfrac{\partial u_r}{\partial r} = \dfrac{1}{K}\sigma_{rr} - \dfrac{\lambda}{K}\left(\dfrac{1}{r}\dfrac{\partial u_\varphi}{\partial \varphi} + \dfrac{u_r}{r} + \dfrac{\partial u_z}{\partial z}\right); \\ \dfrac{\partial u_\varphi}{\partial r} = \dfrac{1}{\mu}\sigma_{r\varphi} - \dfrac{1}{r}\left(\dfrac{\partial u_r}{\partial \varphi} - u_\varphi\right); \\ \dfrac{\partial u_z}{\partial r} = \dfrac{1}{\mu}\sigma_{rz} - \dfrac{\partial u_r}{\partial z}; \\ \dfrac{\partial \sigma_{rr}}{\partial r} = \rho\dfrac{\partial^2 u_r}{\partial t^2} - \dfrac{\tilde{A}}{r} - \dfrac{1}{r}\dfrac{\partial \sigma_{r\varphi}}{\partial \varphi} - \dfrac{\partial \sigma_{rz}}{\partial z}; \\ \dfrac{\partial \sigma_{r\varphi}}{\partial r} = \rho\dfrac{\partial^2 u_\varphi}{\partial t^2} - \dfrac{1}{r}\dfrac{\partial}{\partial \varphi}\left[\sigma_{rr} - \tilde{A}\right] - \dfrac{2\sigma_{r\varphi}}{r} - \dfrac{\partial}{\partial z}\tilde{B}; \\ \dfrac{\partial \sigma_{rz}}{\partial r} = \rho\dfrac{\partial^2 u_z}{\partial t^2} - \dfrac{\partial}{\partial z}\left[\sigma_{rr} - 2\mu\left(\dfrac{\partial u_r}{\partial r} - \dfrac{\partial u_z}{\partial z}\right)\right] - \dfrac{\sigma_{rz}}{r} - \dfrac{1}{r}\dfrac{\partial}{\partial \varphi}\tilde{B}; \end{cases} \quad (7)$$

where we use the notation

$$\tilde{A} = 2\mu\left[\frac{\partial u_r}{\partial r} - \frac{1}{r}\left(\frac{\partial u_\varphi}{\partial \varphi} + u_r\right)\right];$$

$$\tilde{B} = \mu\left(\frac{\partial u_\varphi}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial \varphi}\right).$$

Boundary conditions are given in the form:

$$r = r_0 \to 0, R: \sigma_{rz} = \sigma_{rr} = \sigma_{r\varphi} = 0; \quad (8)$$

$$r = 0,\ \ 2\pi: u_\varphi = 0; \sigma_{\varphi z} = \sigma_{\varphi r} = 0. \quad (9)$$

Condition (8) $r = 0$ physically result can be interpreted as limiting the transition from the hollow cylinder with the inner free surface to the solid. Inner radius tends to zero. In the case of harmonic waves traveling along the axis *z,* solution of (7), (8), (9) admits separation of variables [12,13]

$$\begin{aligned} u_r &= w(r)\cos\frac{\varphi}{2}\cos(kz - \omega t); \\ u_\varphi &= v(r)\sin\frac{\varphi}{2}\cos(kz - \omega t); \\ u_z &= u(r)\cos\frac{\varphi}{2}\sin(kz - \omega t); \end{aligned} \quad (10,a)$$

$$\sigma_{rr} = \sigma(r)\cos\frac{\varphi}{2}\cos(kz - \omega t);$$

$$\sigma_{r\varphi} = \tau_\varphi(r)\ \sin\frac{\varphi}{2}\cos(kz - \omega t);$$

$$\sigma_{rz} = \tau_z(r)\cos\frac{\varphi}{2}\sin(kz - \omega t),$$

Ref

12. Safarov I.I, Boltaev Z. I., Akhmedov M. Sh. Properties of wave motion in a fluid-filled cylindrical shell/ LAP, Lambert Academic Publishing . 2016 -105 p.

Notes

or

$$u_r = w(r)\cos\frac{\varphi}{2}e^{i(kz-\omega t)};$$

$$u_\varphi = v(r)\sin\frac{\varphi}{2}e^{i(kz-\omega t)};$$

$$u_z = u(r)\cos\frac{\varphi}{2}e^{i(kz-\omega t)};$$

$$\sigma_{rr} = \sigma(r)\cos\frac{\varphi}{2}e^{i(kz-\omega t)};$$

$$\sigma_{r\varphi} = \tau_\varphi(r)\sin\frac{\varphi}{2}e^{i(kz-\omega t)}; \qquad (10,b)$$

$$\sigma_{rz} = \tau_z(r)\cos\frac{\varphi}{2}e^{i(kz-\omega t)}.$$

Given (10), problem (7), (8), (9) is transformed into a spectral boundary value problem for a system of ordinary differential equations with complex the coefficient:

$$\begin{cases} w' = \dfrac{\sigma}{k} - \dfrac{\lambda}{k}\left(ku + \dfrac{v}{2r} + \dfrac{w}{r}\right); \\ v' = \dfrac{\tau_\varphi}{\mu} + \dfrac{\vartheta}{r} + \dfrac{w}{2r}; \\ u' = \dfrac{\tau_z}{\mu} + kw; \\ \sigma' = -\omega^2\rho w + \dfrac{\tilde{a}}{r} - \dfrac{\tau_\varphi}{2r} - k\tau_z; \\ \tau_\varphi' = -\omega^2\rho\vartheta - \dfrac{2\tau_\varphi}{r} + (\sigma + \tilde{a})\dfrac{1}{2r} - k\tilde{b}; \\ \tau_z{}' = -\omega^2\rho u - \dfrac{\tau_z}{r} - \dfrac{\tilde{b}}{2r} + k(\sigma + 2\mu(ku - w')), \end{cases} \qquad (11)$$

$$(\ldots)' = \frac{d}{dr}.$$

Here $\tilde{a} = 2\mu\left(\dfrac{\vartheta + w}{2r} - w'\right)$; $\tilde{b} = \mu\left(-\dfrac{u}{2r} - k\vartheta\right)$.

with boundary conditions

$$\begin{aligned} r = r_0 \to 0: &\ \sigma = \tau_\varphi = \tau_z = 0; \\ r = R: &\ \sigma = \tau_\varphi = \tau_z = 0. \end{aligned} \qquad (12)$$

Thus formulated spectral boundary value problem (11), (12) describing the propagation of harmonic waves in an infinite cylinder with radial crack. Note that the choice of boundary conditions at the edges of the slit (9) led primarily to separate variables to the coordinates r and φ, which greatly simplifies the solution of the original problem. Separation of variables is also possible in the case of the following boundary conditions:

$$\begin{aligned} &\varphi = 0: \quad \sigma_{\varphi\varphi} = 0; \quad u_r = u_z = 0; \\ &\varphi = 2\pi: \sigma_{\varphi\varphi} = 0; \quad u_r = u_z = 0. \end{aligned} ; \tag{13}$$

Indeed, performing in (7), (8) the change of variables so as to satisfy the conditions (13)

$$\begin{aligned} u_r &= \tilde{w}(r)\sin\frac{\varphi}{2}\cos(kz - \omega t); \\ u_\varphi &= \tilde{\vartheta}(r)\cos\frac{\varphi}{2}\cos(kz - \omega t); \\ u_z &= \tilde{u}(r)\sin\frac{\varphi}{2}\sin(kz - \omega t); \\ \sigma_{rr} &= \tilde{\sigma}(r)\sin\frac{\varphi}{2}\cos(kz - \omega t); \\ \sigma_{r\varphi} &= \tau_\varphi(r)\cos\frac{\varphi}{2}\cos(kz - \omega t); \\ \sigma_{rz} &= \tau_z(r)\sin\frac{\varphi}{2}\sin(kz - \omega t); \end{aligned} \tag{14}$$

We obtain spectral boundary value problem with complex coefficients and roots

$$\begin{aligned} \tilde{w}' &= \frac{\tilde{\sigma}}{k} - \frac{\lambda}{k}\left(k\tilde{u} - \frac{\tilde{v}}{2r} + \frac{\tilde{w}}{r}\right); \\ \tilde{u}' &= \frac{\tilde{\tau}_\varphi}{\mu} + \frac{\tilde{v}}{r} - \frac{\tilde{w}}{2r}; \\ \tilde{u} &= \frac{\tilde{\tau}_z}{\mu} + k\tilde{w}; \\ \tilde{\sigma}' &= -\rho\omega^2\tilde{w} + \frac{2\mu}{r}\left(-\frac{\tilde{v}}{2r} + \frac{\tilde{w}}{r} - \tilde{w}'\right) + \frac{\tilde{\tau}_\varphi}{2r} - k\tilde{\tau}; \\ \tilde{\tau}_\varphi' &= -\rho\omega^2\tilde{v} - \frac{2\tilde{\tau}_\varphi}{r} - \frac{1}{2r}\left(\tilde{\sigma} + 2\mu\left(-\frac{\tilde{v}}{2r} + \frac{\tilde{w}}{r} - \tilde{w}\right) - k\left(\frac{\tilde{u}}{2r} - k\tilde{v}\right)\right); \\ \tilde{\tau}_z{}' &= -\rho\omega^2\tilde{u} - \frac{\tilde{\tau}_z}{r} + \frac{\mu}{2r}\left(\frac{u}{2r} - k\tilde{v}\right) + k\left(\tilde{\sigma} + 2\mu\left(k\tilde{u} - \tilde{w}'\right)\right), \end{aligned} \tag{15}$$

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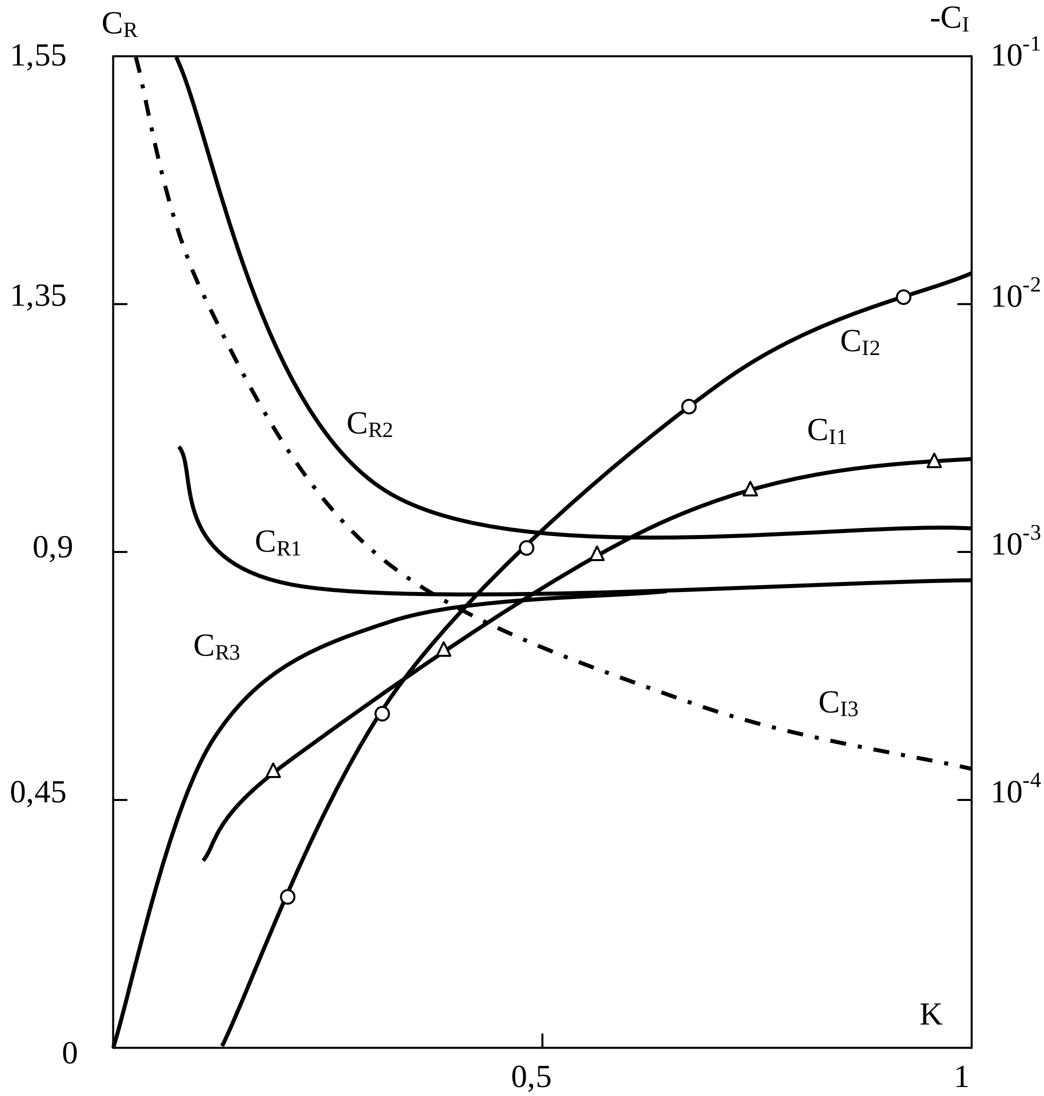

Figure 1: Changes in the real and imaginary parts of the frequency of oscillation on κ

With the boundary conditions

$$r = r_0 \to 0: \tilde{\sigma} = \tilde{\tau}_\varphi = \tilde{\tau}_z = 0;$$
$$r = R: \tilde{\sigma} = \tilde{\tau}_\varphi = \tilde{\tau}_z = 0 \qquad (16)$$

It is easy to see that the problem (15), (16) reduces to the problem (11), (12) by replacing

$$\tilde{\tau}_z = \tau_z, \;\; \tilde{\tau}_\varphi = -\tau_\varphi, \tilde{\sigma} = \sigma, \;\; \tilde{w} = w, \;\; \tilde{u}_\varphi = -u_\varphi, \;\; \tilde{u}_z = u_z.$$

The solution of (11), (12) was carried out by the orthogonal shooting Godunov [14]. Dimensionless quantities in the formulation of the problem chosen in such a way that the shear rate C_s, density ρ and the outer radius R has the single value. Fig. I shows dispersion curves of the first two modes in infinite cylinder with viscoelastic radial thickness (curves 1 and 2). For comparison, the same figure shows the dependence of the phase velocity of the wave number of the first bending mode vibrations of a solid cylinder (curve 3) without gaps. final solution of the problem has been previously found Pohgomerom Cree and with the help of special functions (5) the solution was used for testing tasks.

Ref

14. S.K. Godunov. On the numerical solution of boundary value problems for systems of linear ordinary differential equations. - Successes of Mathematical Sciences, 1061, T.16, № 3, 171-174p.

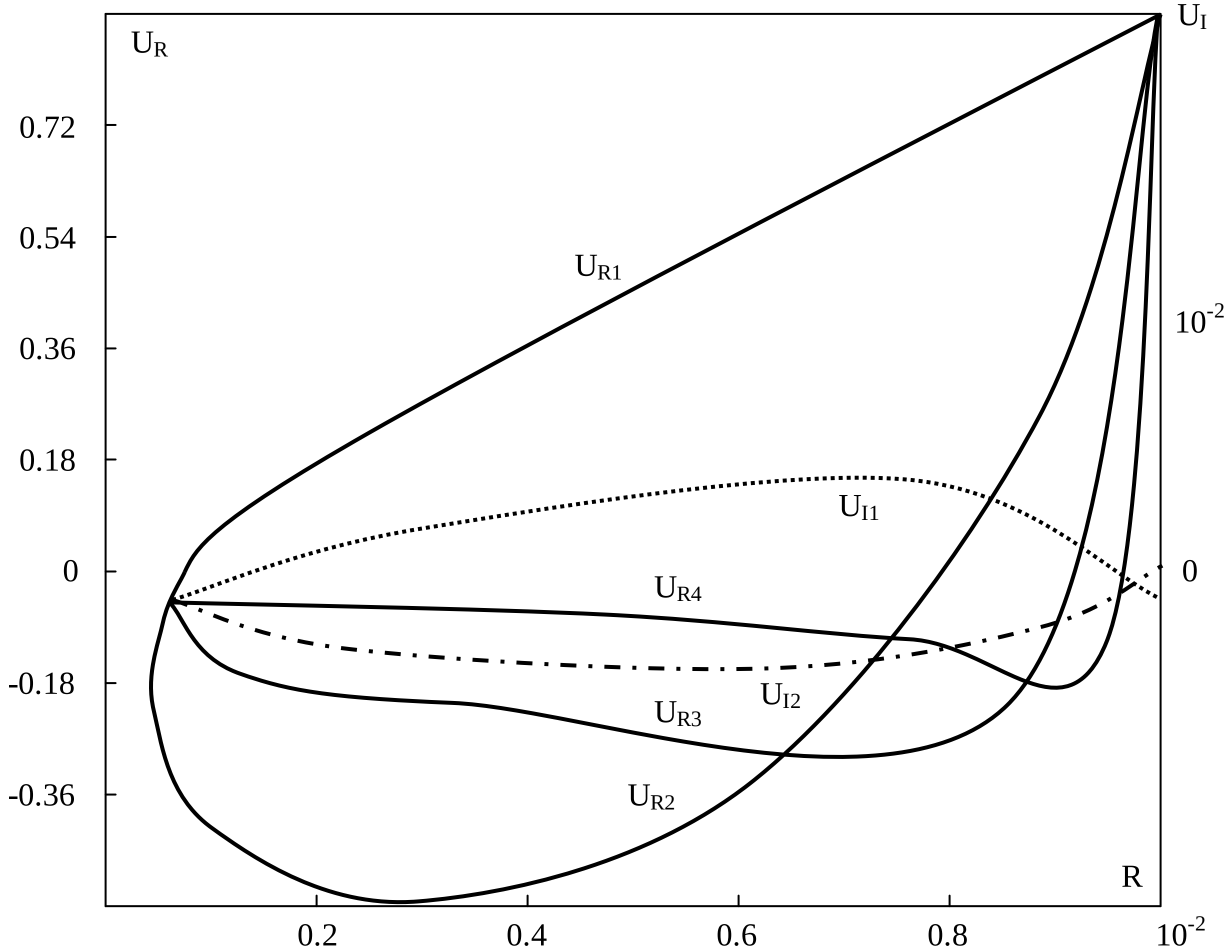

Figure 2: Changes in the real and imaginary parts of the waveform U_R and U_I on R

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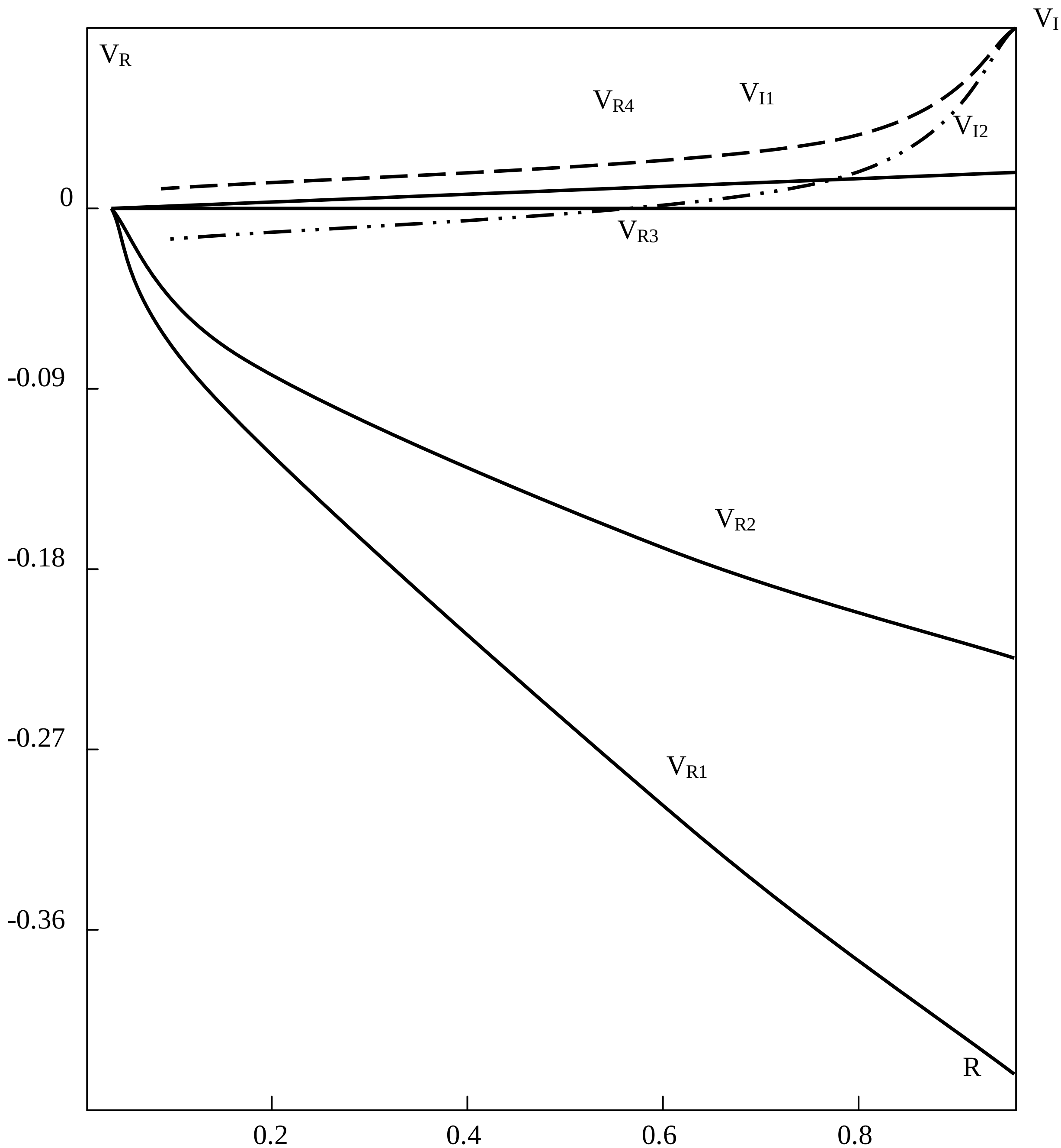

Figure 3: Changes in the real and imaginary parts of the waveform V_R and V_I on R

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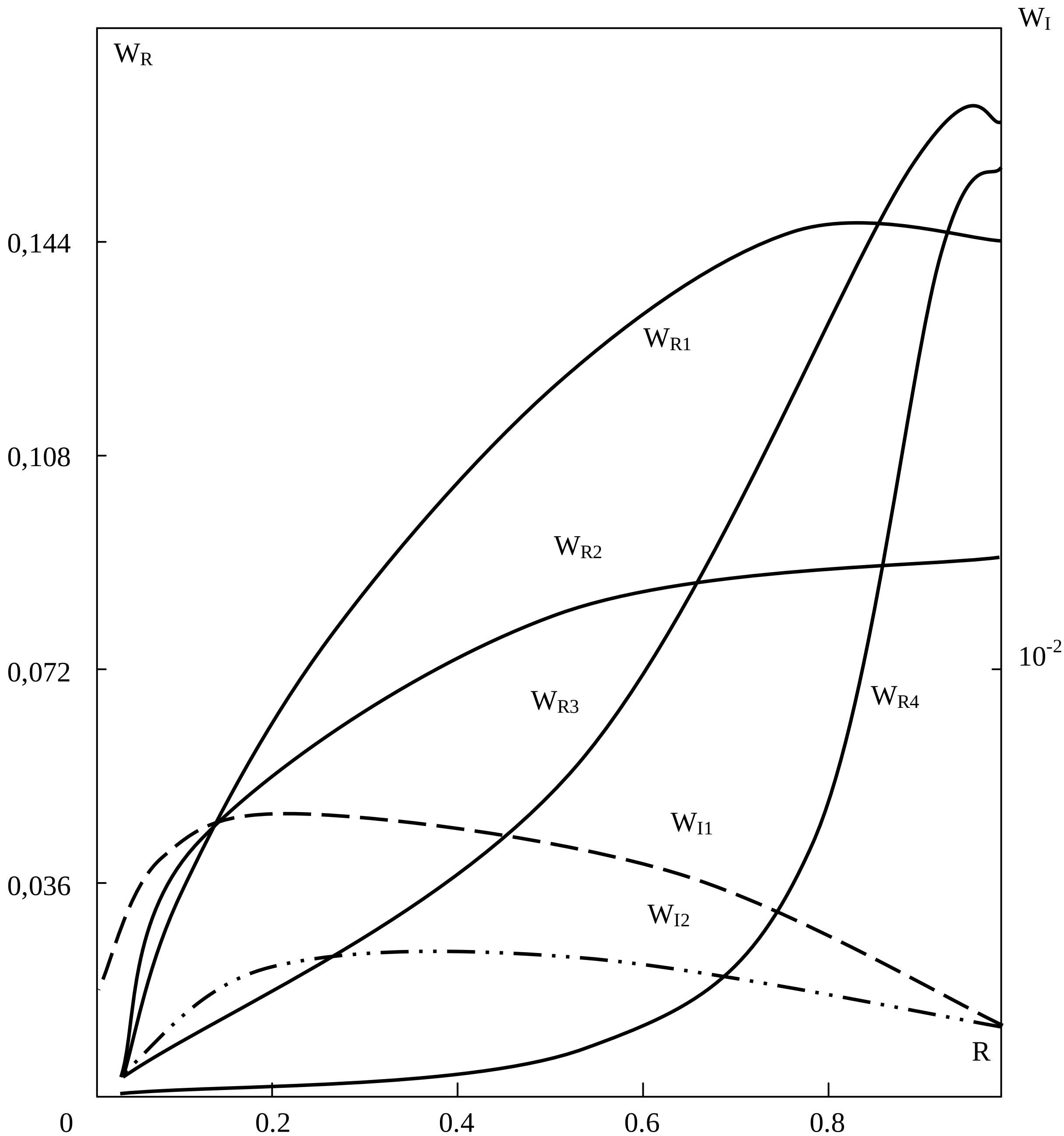

Figure 4: Changing the real and imaginary parts of the waveform on R

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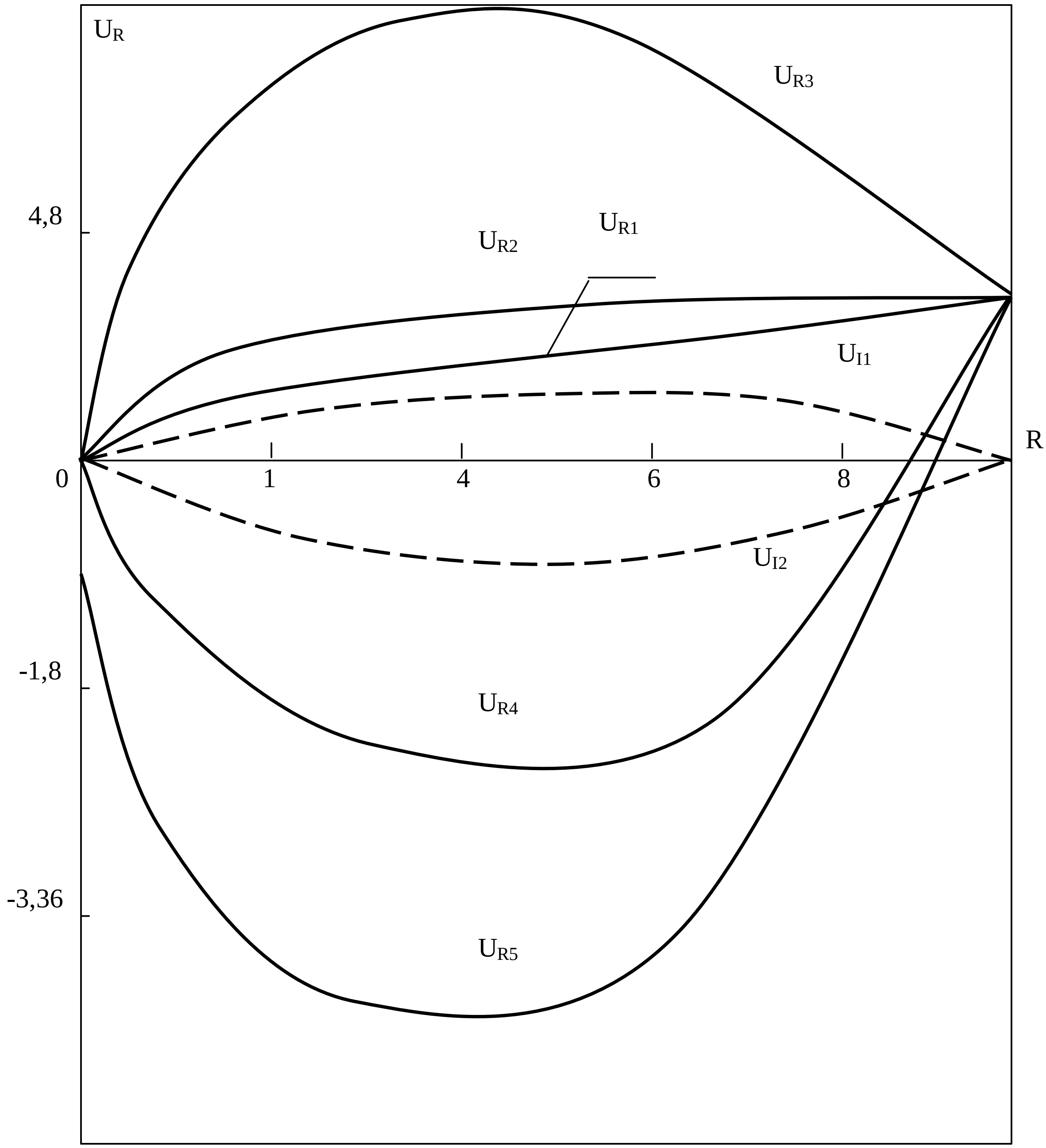

Figure 5: Changing the real and imaginary parts of the waveform on R

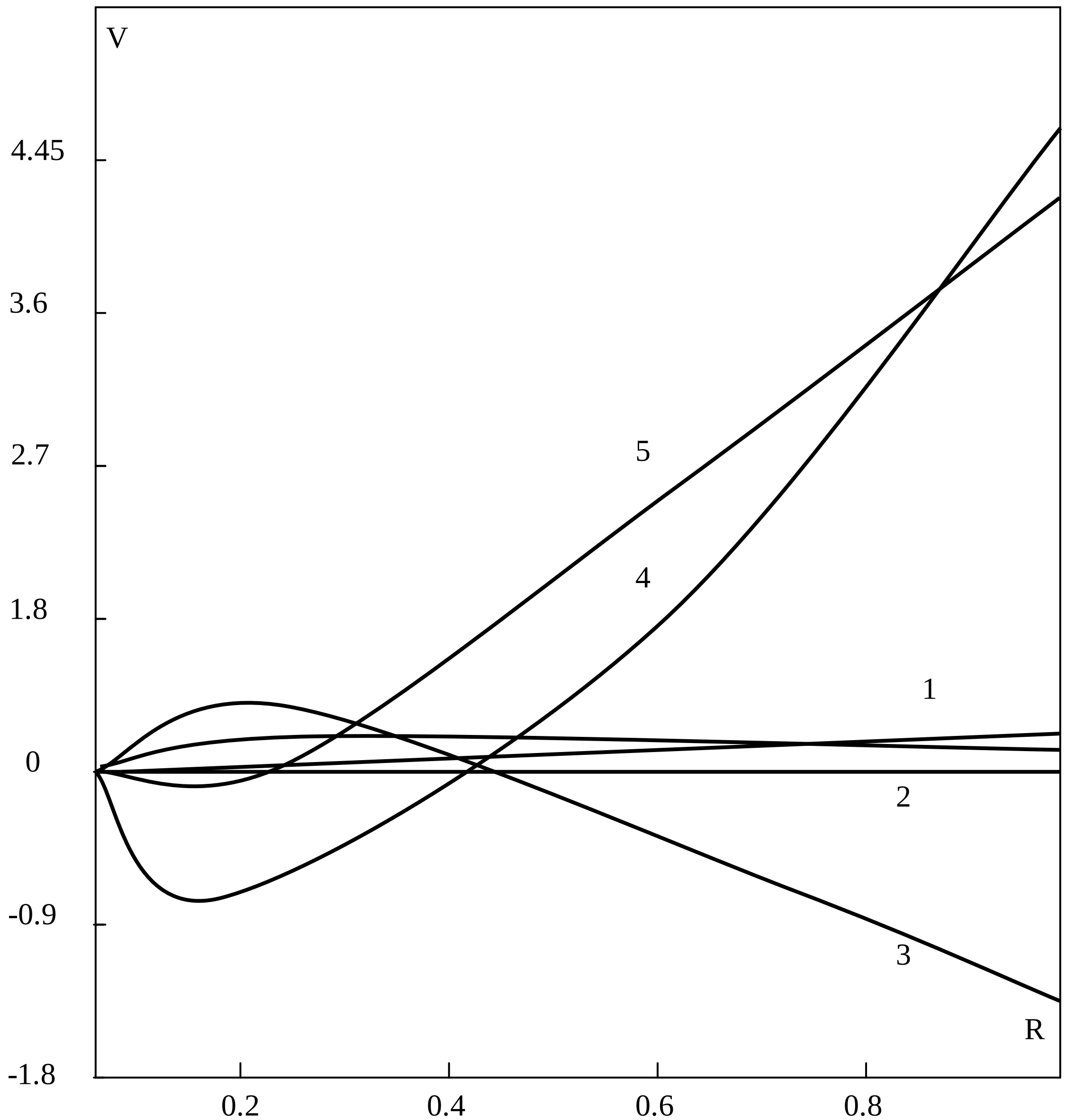

Figure 6: Change the absolute value of the waveform $V^2 = V_R^{\ 2} + V_I^{\ 2}$ on R

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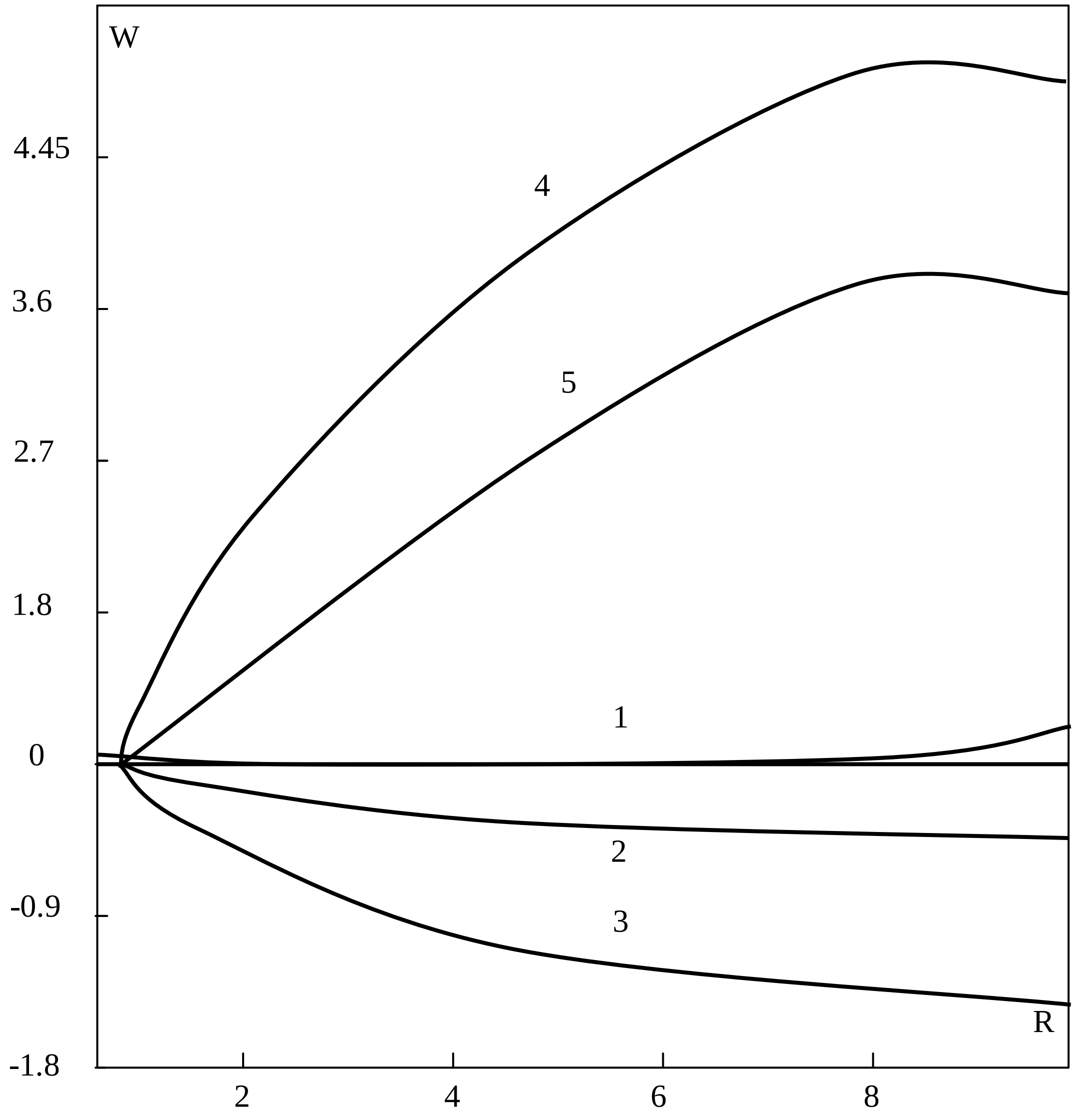

Figure 7: Change the absolute value of the waveform $W^2 = W_R^{\ 2} + W_I^{\ 2}$ on R

Note the characteristics of the curve 3: at the origin of the phase velocity is equal to zero, but not infinite, is committed to the Rayleigh wave velocity for a half. In the case of a cylinder with radial crack the first mode has a cut-off frequency and phase velocity tends to infinity. At large wave numbers limit the phase velocity of this mode also coincides with the velocity of the Rayleigh wave. At the cutoff frequency axial displacement are zero and cylinder vibrations occur in the plane strain condition. In the second mode at a cut-off frequency are observed only real and opinions of the axial displacement, circumferential and radial displacement are zero. The evolution of the form of the solution of the complex movements of the first and second modes depending on the wave number is shown in Figures 2-4 and 5-7, respectively.

The curves are numbered in order of growth κ. Note the strong dependence on the wave number of forms. With the growth of the wave number in the first mode are localized oscillations near the outer surface of the cylinder. It is characteristic that the second mode, which is on the small wave numbers, is a form of predominantly axial vibration, with growth κ gradually turning into a form of predominantly radial oscillations.

III. Waves in a Deformable Wedge with an Arbitrary Angle Vertices

In this section we consider the propagation of harmonic waves in an infinite elastic wedge with an arbitrary angle peaks. For a description of the wave process, use the above relations in the preceding paragraph (1), (2), (3). Resolving equation system coincides with the system (7) is also saved without changing the boundary conditions on the surface (8). The boundary conditions for φ for any angle of the wedge when the free lateral surfaces should be written in the form:

$$\varphi = -\frac{\varphi_0}{2}, \frac{\varphi_0}{2}: \quad \sigma_{\varphi\varphi} = \sigma_{\varphi r} = \sigma_{\varphi z} = 0, \tag{17}$$

where φ_0 - angle at the apex of the wedge. Harmonic waves propagating along the z axis, the essence of the solution of the problem (7), (8), (9), (17) periodic in z and time. Terms periodicity allows to eliminate the dependence of the main unknowns on the time axis and the z coordinate with the following change of variables:

$$\begin{aligned} u_r &= w(r,\varphi)e^{i(\kappa z-\omega t)}; \\ u_\varphi &= v(r,\varphi)e^{i(\kappa z-\omega t)}; \\ u_z &= \tilde{u}(r,\varphi)e^{i(\kappa z-\omega t)}; \\ \sigma_{rr} &= \sigma(r,\varphi)e^{i(\kappa z-\omega t)}; \\ \sigma_{rz} &= \tau_z(r,\varphi)e^{i(\kappa z-\omega t)}; \\ \sigma_{r\varphi} &= \tau_\varphi(r,\varphi)e^{i(\kappa z-\omega t)}; \end{aligned} \tag{18}$$

Under the condition (17), separation of variables r and φ, as in the previous paragraph, it is already impossible. Taking into account (18), the system of equations (7) takes the form:

$$\begin{cases} w' = \dfrac{\sigma}{k} - \dfrac{\bar{\lambda}}{k}\left(ku + \dfrac{1}{r}\left(w + \dfrac{\partial v}{\partial \varphi}\right)\right) \\ v' = \dfrac{\tau_\varphi}{\bar{\mu}} + \dfrac{1}{r}\left(v - \dfrac{\partial w}{\partial \varphi}\right) \\ u' = \dfrac{\tau_z}{\bar{\mu}} + kw \\ \sigma' = -\omega^2 \rho w + \dfrac{1}{r}\left(A - \dfrac{\partial \tau_\varphi}{\partial \varphi}\right) - k\tau_z \\ \tau'_\varphi = -\omega^2 \rho v - \dfrac{1}{r}\left(\dfrac{\partial (A+\sigma)}{\partial \varphi} + 2\tau_\varphi\right) - kB \\ \tau'_z = -\omega^2 \rho u - \dfrac{1}{r}\left(\dfrac{\partial B}{\partial \varphi} + \tau_z\right) + k(\sigma + 2\bar{\mu}(ku - w')) \end{cases} \tag{19}$$

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$$\begin{cases} w' = \dfrac{\sigma}{k} - \dfrac{\overline{\lambda}}{k}\left(ku + \dfrac{1}{r}\left(w + \dfrac{\partial v}{\partial \varphi}\right)\right) \\ v' = \dfrac{\tau_\varphi}{\overline{\mu}} + \dfrac{1}{r}\left(v - \dfrac{\partial w}{\partial \varphi}\right) \\ u' = \dfrac{\tau_z}{\overline{\mu}} + kw \\ \sigma' = -\omega^2 \rho w + \dfrac{1}{r}\left(A - \dfrac{\partial \tau_\varphi}{\partial \varphi}\right) - k\tau_z \\ \tau'_\varphi = -\omega^2 \rho v - \dfrac{1}{r}\left(\dfrac{\partial (A+\sigma)}{\partial \varphi} + 2\tau_\varphi\right) - kB \\ \tau'_z = -\omega^2 \rho u - \dfrac{1}{r}\left(\dfrac{\partial B}{\partial \varphi} + \tau_z\right) + k(\sigma + 2\overline{\mu}(ku - w')) \end{cases} \tag{19}$$

Where

$$A = 2\overline{\mu}\left(\frac{1}{2}\left(\frac{\partial v}{\partial \varphi} + w\right) - w'\right); \; B = \overline{\mu}\left(\frac{1}{r}\frac{\partial u}{\partial \varphi} - kv\right).$$

Similarly transformed boundary conditions (8)

$$r = 0, R : \sigma = \tau_\varphi = \tau_z = 0. \tag{20}$$

It is easy to see that the components of the stress tensor, $\sigma_{\varphi\varphi}, \sigma_{\varphi z}$ and σ_{zz} expressed in terms of the main unknowns on formulas:

$$\begin{aligned} \sigma_{\varphi\varphi} &= \sigma_{rr} + 2\overline{\mu}\left(\frac{1}{r}\frac{\partial u_\varphi}{\partial \varphi} + \frac{u_r}{r} - \frac{\partial u_r}{\partial r}\right); \\ \sigma_{\varphi z} &= \overline{\mu}\left(\frac{\partial u_z}{\partial \varphi} + \frac{\partial u_\varphi}{\partial z}\right); \\ \sigma_{zz} &= \sigma_{rr} + 2\overline{\mu}\left(\frac{\partial u_z}{\partial z} - \frac{\partial u_r}{\partial r}\right). \end{aligned} \tag{21}$$

Then, taking into account the first equation (21), the boundary conditions (20) take the form:

$$\begin{aligned} &\sigma_\varphi = A + \sigma_r = a\sigma_r + b\frac{1}{r}\left(\frac{\partial v}{\partial \varphi} + w\right) + cku = 0; \\ &\varphi = -\frac{\varphi_0}{2}, \frac{\varphi_0}{2} : \quad \tau_\varphi = 0, B = \overline{\mu}\left(\frac{\partial u}{r\partial \varphi} - kr\right) = 0, \end{aligned} \tag{22}$$

where

$$a = 1 + \frac{2\overline{\mu}}{k};\ b = 2\overline{\mu}\left(1 + \frac{\overline{\lambda}}{k}\right), c = 2\overline{\mu}\frac{\overline{\lambda}}{k}.$$

A boundary value problem for the alignment of the system in the frequency of derivatives (19) (20) (22) can be reduced to a boundary value problem for a system of ordinary differential equations by the method of lines that will be used in solving a software unit orthogonal shooting method. According to the method of direct rectangular domain of the function key unknown is covered by lines parallel to the axis r and evenly spaced (Figure 8).

The solution is sought only on these lines, and the directional derivative φ, is replaced by the approximate finite differences. Used a second-order approximation formulas for the first and second derivatives are of the form [14,15]:

$$y_{i,\varphi} \cong \frac{y_{i+1} - y_{i-1}}{2\Delta} \cong \frac{-3y_i + 4y_{i+1} - y_{i+2}}{2\Delta} \cong \frac{3y_i - 4y_{i-1} + y_{i-2}}{2\Delta} \tag{23}$$

$$y''_{i,\varphi} \cong \frac{y_{i+1} - 2y_i + y_{i-1}}{\Delta^2} \tag{24}$$

where i it varies from 0 to $N+1\left(\overline{i=0\ N+1}\right)$, y_i - the projection of the unknown function on the line number i; Δ – move partition to the coordinate φ

As a result, the main vector of the sample of unknown total $6N$ dimension can be written as:

$$Y = \left(\{w_i\}, \{v_i\}, \{u_i\}, \{\sigma_{ri}\}, \{\tau_{\varphi i}\}, \{\tau_{zi}\}\right)^T \quad i = \overline{1, N} \tag{25}$$

The central difference (23), (25) are used for domestic direct *(1<i<N),* the difference between the left and right (24), (25) make it possible to take into account the boundary conditions for φ. In the first case, the derivative with respect φ on the right sides of equations (19) is expressed by the formulas:

$$1 < i < N$$

$$\begin{aligned}
& w_{i,\varphi} = (w_{i+1} - w_{i-1})/2\Delta;\ u_{i,\varphi} = (u_{i+1} - u_{i-1})/2\Delta; \\
& v_{i,\varphi} = (v_{i+1} - v_{i-1})/2\Delta; \tau_{\varphi_i,\varphi} = \left(\tau_{\varphi(i+1)} - \tau_{\varphi(i-1)}\right)/2\Delta \\
& \tau_{\varphi_i,\varphi} = \left(\tau_{\varphi(i+1)} - \tau_{\varphi(i-1)}\right)/2\Delta; \\
& \sigma_{\varphi_i,\varphi} = a(\sigma_{i+1} - \sigma_{i-1})\Big/ 2\Delta + \frac{b}{r}\left[(v_{i+1} - 2v_i + v_{i-1})/\Delta^2 + w_{i,\varphi}\right] + cku_{i,\varphi}; \\
& B_i = (u_{i+1} - 2u_i + u_{i-1})/\Delta^2/k - kv_{i,\varphi}.
\end{aligned} \tag{26}$$

The boundary conditions at $\varphi = -\frac{\varphi_0}{2}$ accounted for in the equations corresponding straight $i = I$. For the main unknown outside the boundary conditions w_i, v_i, u_i Use the right difference (24):

Ref

15. B.I. Myachenkov, I.V. Grigorev. Calculation of composite shell structures on the computer: Spavochnik. -Moscow: Engineering, 1981-216 p.

$$
\begin{aligned}
w_{i,\varphi} &= \left(-3w_1 + 4w_2 - w_3\right)/2\Delta;\\
v_{i,\varphi} &= \left(-3v_1 + 4v_2 - v_3\right)/2\Delta;\\
u_{i,\varphi} &= \left(-3u_1 + 4u_2 - u_3\right)/2\Delta.
\end{aligned} \tag{27}
$$

Notes

For variable τ_φ Conditions (22) are recorded by means of the central difference

$$
\tau_{\varphi_i,\varphi} \cong \left(\tau_{\varphi_2} - \tau_{\varphi_0}\right)/2\Delta = -\tau_{\varphi_2}/2\Delta . \tag{28}
$$

The first and third of the conditions (22) is taken into account in the approximation of the derivatives of the function in the software φ σ_φ

$$
\begin{aligned}
&\sigma_{\varphi_1,\varphi} \cong \left(\sigma_{\varphi_2} - \sigma_{\varphi_0}\right)/2\Delta = \sigma_{\varphi_2}\Big/2\Delta = \left(a\sigma_{r_2} + \frac{b}{r}\left[\left(v_3 - v_1\right)/2\Delta + w_2\right] - cku_2\right)\Big/2\Delta;\\
&B_{1,\varphi} \cong \left(B_2 - B_0\right)/2\Delta = B_2/2\Delta = \left[\left(u_3 - u_1\right)/2\Delta/r - kv_2\right]/2\Delta.
\end{aligned} \tag{29}
$$

Similarly, derivatives are presented to the line with number i = N, taking into account the boundary conditions at $\varphi = \frac{\varphi_0}{2}$. The only difference is the replacement of the right finite difference Left:

$i{=}N:$

$$
\begin{aligned}
&w_{i,\varphi} = \left(3w_N - 4w_{N-1} + w_{N-2}\right)/2\Delta;\; v_{i,\varphi} = \left(3v_N - \ldots\right)/2\Delta;\\
&U_{i,\varphi} = \left(U_{i+1} - U_{i-1}\right)/2\Delta;\, u_{i,\varphi} = \left(3u_N - \ldots\right)/2\Delta;\\
&\tau_{\varphi,\varphi} = -\tau_{\varphi(N-1)}/2\Delta;\, \tau_{\varphi_i,\varphi} = \left(\tau_{\varphi(i+1)} - \tau_{\varphi(i-1)}\right)/2\Delta;\\
&\sigma_{i,\varphi} = \left(a\sigma_{N-1} + \frac{b}{r}\left[\left(v_N - v_{N-2}\right)/2\Delta + w_{N-1}\right] + cku_{N-1}\right)\Big/2\Delta = -\frac{\sigma_{N-1}}{2\Delta},\\
&B_{i,\varphi} = -\left[\left(u_N - u_{N-2}\right)/2\Delta/r - kv_{N-1}\right]\Big/2\Delta = -\frac{B_{N-1}}{2\Delta}.
\end{aligned} \tag{30}
$$

The number of lines can be reduced by half if the conditions of use of anti symmetry transverse plate vibrations at $\varphi = 0$

$$
w = u = \sigma_\varphi = 0 \tag{31}
$$

The corresponding difference ratio, taking into account the conditions (31) can be written as:

$i = N:$

$$
\begin{aligned}
&w_{i,\varphi} = -w_{N-1}/2\Delta;\; u_{i,\varphi} = -u_{N-1}/2\Delta;\\
&v_{i,\varphi} = \left(3v_N - \ldots\right)/2\Delta;\; \tau_{\varphi_i,\varphi} = \left(3\tau_{\varphi N} - 4\tau_{\varphi(N-1)} + \tau_{\varphi(N-2)}\right)/2\Delta;
\end{aligned}
$$

$$\sigma_{i,\varphi} = -\left(a\sigma_{N-1} + \frac{b}{r}\left[(v_N - v_{N-2})/2\Delta + w_{N-1}\right] + cku_{N-1}\right)\Big/ 2\Delta = -\frac{\sigma_{N-1}}{2\Delta}; \qquad (32)$$

$$B_{i,\varphi} = -(-2u_N + u_{N-1})/\Delta^2/r - kv_{i,\varphi}.$$

The resolution of a system of ordinary differential equations according to (21) has the form:

$$w_i' = \sigma_i/k - a\left(ku_i + (w_i + v_{i,\varphi})/R\right); \quad v_i' = \tau_{\varphi i} + (v_i - w_{i,\varphi})/R;$$

$$u_i' = \tau_{zi} + kw_i; \; \sigma_i' = -\omega^2 w_i + \left[2\left((w_i + v_{i,\varphi})/R - w_i'\right) - \tau_{\varphi_i,\varphi}\right]\Big/ r - k\tau_{zi};$$

$$\tau_{zi}{}' = -\omega^2 u_i - (B_{i,\varphi} + \tau_{zi})/r + k(\sigma + 2(ku_i - w_i')); \qquad (33)$$

$$\tau_{\varphi i}{}' = -\omega^2 v_i + (\sigma_{i,\varphi} + 2\tau_{\varphi i})/r - k(u_{i,\varphi}/R - kv_i).$$

In equations (33) the expression for the derivatives wiφ, $v_{i\varphi}$, $u_{i\varphi}$,$\sigma_{i,\varphi}$ $\beta_{i,\varphi}$ $\tau_{i,\varphi}$ are selected from (29) - (32) depending on the boundary conditions of the coordinate φ. free surface conditions equivalent (20) and forming together with the equations (33), the boundary value problem, is obtained in the form of

$$B_i = 0, \tau_{\varphi i} = 0, \sigma_{\varphi} = 0. \qquad (i=1,N) \qquad (34)$$

Thus, the initial spectral problem (19), (20), (22) by means of sampling coordinate φ by the method of direct reduced to the canonical problem (33), (34), for solutions which use the method of orthogonal sweep method previously used. The table shows the limit values of the phase velocity of the first edge of fashion, depending on the angle of the wedge. Found phase velocity for a material with a Poisson's ratio $\upsilon = 0{,}25$ Kirchhoff theory on plates - Love (column 3), Timoshenko - (column 4), contained within this section of the wedge method for calculating three-dimensional (column 5 - 6) and the formula $C_0 = C_{\text{ч}} \; \sin(m\varphi)$ /8/, m = 1, 2, …, $m\varphi < 90^0$ (column 6). Column 5 corresponds to the embodiment of calculation with three internal lines (N = 3) and the boundary conditions (17), column 6 corresponds to the boundary conditions:

$$\varphi = -\frac{\varphi_0}{2}: \quad \sigma_{\varphi\varphi} = \sigma_{\varphi \text{ч}} = \sigma_{\varphi z} = 0; \quad \varphi = 0: \quad u_r = u_z = \sigma_{\varphi\varphi} = 0.$$

In accordance with the numerical results and shown in Table 1, embodiments of methods for calculating the Kirchhoff - Love, a three-dimensional theory of Timoshenko and agree with each other within 7 % for a thickness of the wedge angles of the base h_2, not exceeding 0.5 (the wedge angle $\varphi_0 = 28^0$). Note that for the angle $\varphi = 90^0$ limiting phase velocity was calculated as in [16,17,18], where the value is for her 0,90I $(\nu = 0{,}25)$. Thus, in contrast to the waveguides with a rectangular cross-section in the tapered waveguide with a sufficiently small wedge angle in the analysis of the dispersion relations of the first mode is permissible to use the theory of plates Kirchhoff - Love. Established fact is explained by the phenomenon of localization waveform near the acute angle of the wedge, as described in [8]. This phenomenon should be seen as a characteristic feature of the dynamic behavior of a plate of variable thickness.

Ref

16. H.B. Tolipov, S.Yu. Gurevich, AV Gerenshteyn. Propagation of elastic waves in the wedge: Monograph Chelyabinsk. 2003 – 122 p.

Notes

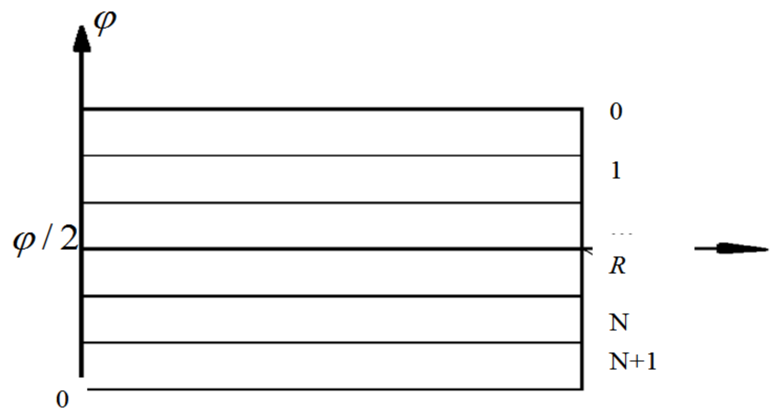

Figure 8: The settlement scheme

Table 1

R^*	φ_0	*К/Л*	*Т*	*З(1)*	*З(2)*	**По работе** [8]
0,2	11^0	0,2	0,196	-	-	0,182
0,3	17^0	0,3	0,286	0,308	0,298	0,276
0,5	28^0	0,5	0,442	0,475	0,462	0,433
0,7	38^0	0,7	0,563	0,605	0,592	0,574
1	53^0	1	0,691	0,741	0,729	0,736
2	90^0	2	0,864	0,908	-	0,92

IV. Conclusions

1. It was revealed that in an elastic cylinder with radial crack no waves having a real part of the phase velocity, localized near the axis of the cylinder.
2. The results of calculation of the maximum speed the spread of the first tapered waveguide modes in the theory of plates Kirchhoff - Love and dynamic elasticity does not differ by more than 6% to the top of the wedge angles not exceeding 28^0. *At* $28^0 < \varphi < 90^0$ characterized by certain surcharges to 20%. Thus, for small wedge angles permissible use of the simplified theory of Kirchhoff - Love and Timoshenko in the whole wavelength range. Thus, for small wedge angles permissible use of the simplified theory of Kirchhoff - Love and Timoshenko in the whole wavelength range.
3. Accounting for the viscoelastic properties of the wedge material reduces the real part of the wave propagation velocity is 10-15%, as well as to evaluate the ability of the system damping in general. The work was supported by the Foundation for Fundamental Research Ф-4-14 of the Republic of Uzbekistan.

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Notes