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## Physics and Space Science



MHD Turbulent Flow

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Production from Well Logs

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Discovering Thoughts, Inventing Future

VOLUME 16 ISSUE 1 VERSION 1.0



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PHYSICS & SPACE SCIENCE

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# First Order Reactant in the Statistical Theory of Three- Point Distribution Functions in Dusty Fluid MHD Turbulent Flow for Velocity, Magnetic Temperature and Concentration in Presence of Coriolis Force

By M. A. K. Azad, M. Mamun Miah, Mst. Mumtahirah, Abdul Malek  
& M. Masidur Rahman

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**Abstract-** In this paper, an attempt is made to study the three-point distribution function for simultaneous velocity, magnetic, temperature and concentration fields in dusty fluid MHD turbulence in presence of coriolis force under going a first order reaction. The various properties of constructed distribution functions have been sufficiently discussed. In this study, the transport equation for three-point distribution function under going a first order reaction has been obtained. The resulting equation is compared with the first equation of BBGKY hierarchy of equations and the closure difficulty is to be removed as in the case of ordinary turbulence.

**Keywords:** *dust particles, coriolis force, magnetic temperature, concentration, three-point distribution functions, MHD turbulent flow, first order reactant.*

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# First Order Reactant in the Statistical Theory of Three- Point Distribution Functions in Dusty Fluid MHD Turbulent Flow for Velocity, Magnetic Temperature and Concentration in Presence of Coriolis Force

M. A. K. Azad <sup>α</sup>, M. Mamun Miah <sup>σ</sup>, Mst. Mumtahirah <sup>ρ</sup>, Abdul Malek <sup>ω</sup> & M. Masidur Rahman <sup>¥</sup>

**Abstract-** In this paper, an attempt is made to study the three-point distribution function for simultaneous velocity, magnetic, temperature and concentration fields in dusty fluid MHD turbulence in presence of coriolis force under going a first order reaction. The various properties of constructed distribution functions have been sufficiently discussed. In this study, the transport equation for three-point distribution function under going a first order reaction has been obtained. The resulting equation is compared with the first equation of BBGKY hierarchy of equations and the closure difficulty is to be removed as in the case of ordinary turbulence.

**Keywords:** dust particles, coriolis force, magnetic temperature, concentration, three-point distribution functions, MHD turbulent flow, first order reactant.

## 1. INTRODUCTION

At present, two major and distinct areas of investigations in non-equilibrium statistical mechanics are the kinetic theory of gases and statistical theory of fluid mechanics. In molecular kinetic theory in physics, a particle's distribution function is a function of several variables. Particle distribution functions are used in plasma physics to describe wave-particle interactions and velocity-space instabilities. Distribution functions are also used in fluid mechanics, statistical mechanics and nuclear physics. Various analytical theories in the statistical theory of turbulence have been discussed in the past by Hopf (1952) Kraichanan (1959), Edward (1964) and Herring (1965).

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Lundgren (1967) derived a hierarchy of coupled equations for multi-point turbulence velocity distribution functions, which resemble with BBGKY hierarchy of equations of Ta-You (1966) in the kinetic theory of gasses.

Kishore (1978) studied the Distributions functions in the statistical theory of MHD turbulence of an incompressible fluid. Pope (1979) studied the statistical theory of turbulence flames. Pope (1981) derived the transport equation for the joint probability density function of velocity and scalars in turbulent flow. Kollman and Janicka (1982) derived the transport equation for the probability density function of a scalar in turbulent shear flow and considered a closure model based on gradient – flux model. Kishore and Singh (1984) derived the transport equation for the bivariate joint distribution function of velocity and temperature in turbulent flow. Also Kishore and Singh (1985) have been derived the transport equation for the joint distribution function of velocity, temperature and concentration in convective turbulent flow. On a rotating earth the Coriolis force acts to change the direction of a moving body to the right in the Northern Hemisphere and to the left in the Southern Hemisphere. This deflection is not only instrumental in the large-scale atmospheric circulation, the development of storms, and the sea-breeze circulation Also a first-order reaction is defined a reaction that proceeds at a rate that depends linearly only on one reactant concentration. Later, the following some researchers included coriolis force and first order reaction rate in their works.

Dixit and Upadhyay (1989) considered the distribution functions in the statistical theory of MHD turbulence of an incompressible fluid in the presence of the coriolis force. Sarker and Kishore (1991) discussed the distribution functions in the statistical theory of convective MHD turbulence of an incompressible fluid. Also Sarker and Kishore (1999) studied the distribution functions in the statistical theory of convective MHD turbulence of mixture of a miscible incompressible fluid. Sarker and Islam (2002) studied the Distribution

functions in the statistical theory of convective MHD turbulence of an incompressible fluid in a rotating system. In the continuation of the above researcher, Azad and Sarker(2003) considered the decay of MHD turbulence before the final period for the case of multi-point and multi-time in presence of dust particle. Azad and Sarker (2004a) discussed statistical theory of certain distribution functions in MHD turbulence in a rotating system in presence of dust particles. Sarker and Azad (2004b) studied the decay of MHD turbulence before the final period for the case of multi-point and multi-time in a rotating system. Sarker and Azad(2006), Islam and Sarker (2007) studied distribution functions in the statistical theory of MHD turbulence for velocity and concentration undergoing a first order reaction. Azad and Sarker(2008) deliberated the decay of temperature fluctuations in homogeneous turbulence before the final period for the case of multi- point and multi- time in a rotating system and dust particles. Azad and Sarker(2009a) had measured the decay of temperature fluctuations in MHD turbulence before the final period in a rotating system. Azad, M. A. Aziz and Sarker (2009b, 2009c) studied the first order reactant in Magneto-hydrodynamic turbulence before the final Period of decay with dust particles and rotating System. Aziz et al (2009d, 2010c) discussed the first order reactant in Magneto- hydrodynamic turbulence before the final period of decay for the case of multi-point and multi-time taking rotating system and dust particles. Aziz et al (2010a, 2010b) studied the statistical theory of certain Distribution Functions in MHD turbulent flow undergoing a first order reaction in presence of dust particles and rotating system separately. Azad et al (2011) studied the statistical theory of certain distribution Functions in MHD turbulent flow for velocity and concentration undergoing a first order reaction in a rotating system. Azad et al (2012) derived the transport equatoin for the joint distribution function of velocity, temperature and concentration in convective tubulent flow in presence of dust particles. Molla et al (2012) studied the decay of temperature fluctuations in homogeneous turbulenc before the final period in a rotating system. Bkar Pk. et al (2012) studed the First-order reactant in homogeneou dusty fluid turbulence prior to the ultimate phase of decay for four-point correlation in a rotating system. Azad and Mumtahirah(2013) studied the decay of temperature fluctuations in dusty fluid homogeneous turbulence prior to final period. Bkar Pk. et al (2013a,2013b) discussed the first-order reactant in homogeneous turbulence prior to the ultimate phase of decay for four-point correlation with dust particle and rotating system. Bkar Pk.et al (2013,2013c, 2013d)

studied the decay of MHD turbulence before the final period for four- point correlation in a rotating system and dust particles. Very recent Azad et al (2014a) derived the transport equations of three point distribution functions in MHD turbulent flow for velocity, magnetic temperature and concentration, Azad and Nazmul (2014b) considered the transport equations of three point distribution functions in MHD turbulent flow for velocity, magnetic temperature and concentration in a rotating system, Nazmul and Azad (2014) studied the transport equations of three-point distribution functions in MHD turbulent flow for velocity, magnetic temperature and concentration in presence of dust particles. Azad and Mumtahirah (2014) further has been studied the transport equatoin for the joint distribution functions in convective tubulent flow in presence of dust particles undergoing a first order reaction.Very recently, Bkar Pk.et al (2015) considering the effects of first-order reactant on MHD turbulence at four-point correlation. Azad et al (2015) derived a transport equation for the joint distribution functions of certain variables in convective dusty fluid turbulent flow in a rotating system under going a first order reaction. Bkar Pk et al (2015), Azad et al (2015a, 2015b, 2015c, and 2015d) have done their research on MHD turbulent flow considering 1<sup>st</sup> order chemical reaction for three- point distribution function. Also Bkar Pk. (2015a) studied 4-point correlations of dusty fluid MHD turbulent flow in a 1<sup>st</sup> order reaction. Bkar Pk (2015b) extended the above problem consioldering Coriolis force.

In present paper, the main purpose is to study the statistical theory of three-point distribution function for simultaneous velocity, magnetic, temperature, concentration fields in MHD turbulence in a rotating system in presence of dust particles Under going a first order reaction. Through out the study, the transport equations for evolution of distribution functions have been derived and various properties of the distribution function have been discussed. The obtained three-point transport equation is compared with the first equation of BBGKY hierarchy of equations and the closure difficulty is to be removed as in the case of ordinary turbulence.

## II. MATERIAL AND METHODS

### Basic Equations

The equations of motion and continuity for viscous incompressible dusty fluid MHD turbulent flow in a rotating system with constant reaction rate, the diffusion equations for the temperature and concentration are given by

$$\frac{\partial u_{\alpha}}{\partial t} + \frac{\partial}{\partial x_{\beta}}(u_{\alpha}u_{\beta} - h_{\alpha}h_{\beta}) = -\frac{\partial w}{\partial x_{\alpha}} + \nu \nabla^2 u_{\alpha} - 2 \epsilon_{m\alpha\beta} \Omega_m u_{\alpha} + f(u_{\alpha} - v_{\alpha}) \quad (1)$$

$$\frac{\partial h_{\alpha}}{\partial t} + \frac{\partial}{\partial x_{\beta}} (h_{\alpha} u_{\beta} - u_{\alpha} h_{\beta}) = \lambda \nabla^2 h_{\alpha}, \quad (2)$$

$$\frac{\partial \theta}{\partial t} + u_{\beta} \frac{\partial \theta}{\partial x_{\beta}} = \gamma \nabla^2 \theta, \quad (3)$$

$$\frac{\partial c}{\partial t} + u_{\beta} \frac{\partial c}{\partial x_{\beta}} = D \nabla^2 c - Rc \quad (4)$$

$$\text{with } \frac{\partial u_{\alpha}}{\partial x_{\alpha}} = \frac{\partial v_{\alpha}}{\partial x_{\alpha}} = \frac{\partial h_{\alpha}}{\partial x_{\alpha}} = 0, \quad (5)$$

where

$u_{\alpha}(x, t)$ ,  $\alpha$  – component of turbulent velocity;  
 $h_{\alpha}(x, t)$ ;  $\alpha$  – component of magnetic field;  $\theta(x, t)$ ,  
 temperature fluctuation;  $c$ , concentration of  
 contaminants;  $v_{\alpha}$ , dust particle velocity;  $\epsilon_{m\alpha\beta}$ ,  
 alternating tensor;  $f = \frac{KN}{\rho}$ , dimension of frequency;  $N$ ,  
 constant number of density of the dust particle;  
 $w(\hat{x}, t) = \frac{P}{\rho} + \frac{1}{2} |\vec{h}|^2 + \frac{1}{2} |\hat{\Omega} \times \hat{x}|^2$ , total pressure;  
 $P(\hat{x}, t)$ , hydrodynamic pressure;  $\rho$ , fluid density;  $\Omega$ ,  
 angular velocity of a uniform rotation;  $v$ , Kinematic

viscosity;  $\lambda = (4\pi\mu\sigma)^{-1}$ , magnetic diffusivity;  $\gamma = \frac{k_T}{\rho c_p}$ ,

thermal diffusivity;  $c_p$ , specific heat at constant pressure;  
 $k_T$ , thermal conductivity;  $\sigma$ , electrical conductivity;  $\mu$ ,  
 magnetic permeability;  $D$ , diffusive co-efficient for  
 contaminants;  $R$ , constant reaction rate.

The repeated suffices are assumed over the  
 values 1, 2 and 3 and unrepeated suffices may take any  
 of these values. In the whole process  $u$ ,  $h$  and  $x$  are the  
 vector quantities.

The total pressure  $w$  which, occurs in equation  
 (1) may be eliminated with the help of the equation  
 obtained by taking the divergence of equation (1)

$$\nabla^2 w = - \frac{\partial^2}{\partial x_{\alpha} \partial x_{\beta}} (u_{\alpha} u_{\beta} - h_{\alpha} h_{\beta}) = - \left[ \frac{\partial u_{\alpha}}{\partial x_{\beta}} \frac{\partial u_{\beta}}{\partial x_{\alpha}} - \frac{\partial h_{\alpha}}{\partial x_{\beta}} \frac{\partial h_{\beta}}{\partial x_{\alpha}} \right] \quad (6)$$

In a conducting infinite fluid only the particular solution of the Equation (6) is related, so that

$$w = \frac{1}{4\pi} \int \left[ \frac{\partial u'_{\alpha}}{\partial x'_{\beta}} \frac{\partial u'_{\beta}}{\partial x'_{\alpha}} - \frac{\partial h'_{\alpha}}{\partial x'_{\beta}} \frac{\partial h'_{\beta}}{\partial x'_{\alpha}} \right] \frac{\partial \bar{x}'}{|\bar{x}' - \bar{x}|} \quad (7)$$

Hence equation (1) – (4) becomes

$$\begin{aligned} \frac{\partial u_{\alpha}}{\partial t} + \frac{\partial}{\partial x_{\beta}} (u_{\alpha} u_{\beta} - h_{\alpha} h_{\beta}) = & - \frac{1}{4\pi} \frac{\partial}{\partial x_{\alpha}} \int \left[ \frac{\partial u'_{\alpha}}{\partial x'_{\beta}} \frac{\partial u'_{\beta}}{\partial x'_{\alpha}} - \frac{\partial h'_{\alpha}}{\partial x'_{\beta}} \frac{\partial h'_{\beta}}{\partial x'_{\alpha}} \right] \frac{d\bar{x}'}{|\bar{x}' - \bar{x}|} + \nu \nabla^2 u_{\alpha} \\ & - 2 \epsilon_{m\alpha\beta} \Omega_m u_{\alpha} + f(u_{\alpha} - v_{\alpha}), \end{aligned} \quad (8)$$

$$\frac{\partial h_{\alpha}}{\partial t} + \frac{\partial}{\partial x_{\beta}} (h_{\alpha} u_{\beta} - u_{\alpha} h_{\beta}) = \lambda \nabla^2 h_{\alpha}, \quad (9)$$

$$\frac{\partial \theta}{\partial t} + u_{\beta} \frac{\partial \theta}{\partial x_{\beta}} = \gamma \nabla^2 \theta, \quad (10)$$

$$\frac{\partial c}{\partial t} + u_{\beta} \frac{\partial c}{\partial x_{\beta}} = D \nabla^2 c - Rc, \quad (11)$$

### Formulation of the Problem

It has considered that the turbulence and the concentration fields are homogeneous, the chemical reaction and the local mass transfer have no effect on the velocity field and the reaction rate and the diffusivity are constant. It is also considered a large ensemble of identical fluids in which each member is an infinite incompressible reacting and heat conducting fluid in turbulent state. The fluid velocity  $u$ , Alfven velocity  $h$ , temperature  $\theta$  and concentration  $c$  are randomly distributed functions of position and time and satisfy their field. Different members of ensemble are subjected to different initial conditions and the aim is to find out a way by which we can determine the ensemble averages at the initial time.

Certain microscopic properties of conducting fluids such as total energy, total pressure, stress tensor which are nothing but ensemble averages at a particular time can be determined with the help of the bivariate distribution functions (defined as the averaged distribution functions with the help of Dirac delta-functions). The present aim is to construct a joint distribution function for its evolution of three-point distribution functions in dusty fluid MHD turbulent flow in

a rotating system under going first order reaction, study its properties and derive a transport equation for the joint distribution function of velocity, temperature and concentration in dusty fluid MHD turbulent flow in a rotating system under going a first order reaction.

### Distribution Function in MHD Turbulence and Their Properties

In MHD turbulence, it is considered that the fluid velocity  $u$ , Alfven velocity  $h$ , temperature  $\theta$  and concentration  $c$  at each point of the flow field. Corresponding to each point of the flow field, there are four measurable characteristics represent by the four variables by  $v$ ,  $g$ ,  $\phi$  and  $\psi$  and denote the pairs of these

variables at the points  $\bar{x}^{(1)}, \bar{x}^{(2)}, \dots, \bar{x}^{(n)}$  as  $(\bar{v}^{(1)}, \bar{g}^{(1)}, \bar{\phi}^{(1)}, \bar{\psi}^{(1)}), (\bar{v}^{(2)}, \bar{g}^{(2)}, \bar{\phi}^{(2)}, \bar{\psi}^{(2)}), \dots, (\bar{v}^{(n)}, \bar{g}^{(n)}, \bar{\phi}^{(n)}, \bar{\psi}^{(n)})$  at a fixed instant of time.

It is possible that the same pair may be occurred more than once; therefore, it simplifies the problem by an assumption that the distribution is discrete (in the sense that no pairs occur more than once). Symbolically we can express the bivariate distribution as

$$\{ (\bar{v}^{(1)}, \bar{g}^{(1)}, \bar{\phi}^{(1)}, \bar{\psi}^{(1)}), (\bar{v}^{(2)}, \bar{g}^{(2)}, \bar{\phi}^{(2)}, \bar{\psi}^{(2)}), \dots, (\bar{v}^{(n)}, \bar{g}^{(n)}, \bar{\phi}^{(n)}, \bar{\psi}^{(n)}) \}$$

Instead of considering discrete points in the flow field, if it is considered the continuous distribution of the variables  $\bar{v}, \bar{g}, \bar{\phi}$  and  $\bar{\psi}$  over the entire flow field, statistically behavior of the fluid may be described by the distribution function  $F(\bar{v}, \bar{g}, \bar{\phi}, \bar{\psi})$  which is normalized so that

$$\int F(\bar{v}, \bar{g}, \bar{\phi}, \bar{\psi}) d\bar{v} d\bar{g} d\bar{\phi} d\bar{\psi} = 1$$

where the integration ranges over all the possible values of  $v, g, \phi$  and  $\psi$ . We shall make use of the same normalization condition for the discrete distributions also.

The distribution functions of the above quantities can be defined in terms of Dirac delta function.

The one-point distribution function  $F_1^{(1)}(\bar{v}^{(1)}, \bar{g}^{(1)}, \bar{\phi}^{(1)}, \bar{\psi}^{(1)})$ , defined so that  $F_1^{(1)}(\bar{v}^{(1)}, \bar{g}^{(1)}, \bar{\phi}^{(1)}, \bar{\psi}^{(1)}) d\bar{v}^{(1)} d\bar{g}^{(1)} d\bar{\phi}^{(1)} d\bar{\psi}^{(1)}$  is the probability that the fluid velocity, Alfven velocity, temperature and concentration at a time  $t$  are in the element  $d\bar{v}^{(1)}$  about  $\bar{v}^{(1)}$ ,  $d\bar{g}^{(1)}$  about  $\bar{g}^{(1)}$ ,  $d\bar{\phi}^{(1)}$  about  $\bar{\phi}^{(1)}$  and  $d\bar{\psi}^{(1)}$  about  $\bar{\psi}^{(1)}$  respectively and is given by

$$F_1^{(1)}(\bar{v}^{(1)}, \bar{g}^{(1)}, \bar{\phi}^{(1)}, \bar{\psi}^{(1)}) = \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \rangle \quad (12)$$

where  $\delta$  is the Dirac delta-function defined as

$$\int \delta(\bar{u} - \bar{v}) d\bar{v} = \begin{cases} 1 & \text{at the point } \bar{u} = \bar{v} \\ 0 & \text{elsewhere} \end{cases}$$

Two-point distribution function is given by

$$F_2^{(1,2)} = \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \rangle \quad (13)$$

and three point distribution function is given by

$$F_3^{(1,2,3)} = \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \\ \times \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \rangle \quad (14)$$

Similarly, we can define an infinite numbers of multi-point distribution functions  $F_4^{(1,2,3,4)}$ ,  $F_5^{(1,2,3,4,5)}$  and so on. The following properties of the constructed distribution functions can be deduced from the above definitions:

$$\begin{aligned}\int F_1^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} &= 1, \\ \int F_2^{(1,2)} dv^{(2)} dg^{(2)} d\phi^{(2)} d\psi^{(2)} &= F_1^{(1)}, \\ \int F_3^{(1,2,3)} dv^{(3)} dg^{(3)} d\phi^{(3)} d\psi^{(3)} &= F_2^{(1,2)}\end{aligned}$$

And so on. Also the integration with respect to any one of the variables, reduces the number of Delta-functions from the distribution function by one as

$$\begin{aligned}\int F_1^{(1)} dv^{(1)} &= \langle \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \rangle, \\ \int F_1^{(1)} dg^{(1)} &= \langle \delta(u^{(1)} - v^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \rangle, \\ \int F_1^{(1)} d\phi^{(1)} &= \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \rangle,\end{aligned}$$

and

$$\begin{aligned}\int F_2^{(1,2)} dv^{(2)} &= \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(h^{(2)} - g^{(2)}) \\ &\quad \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \rangle\end{aligned}$$

#### b) Separation Properties

If two points are far apart from each other in the flow field, the pairs of variables at these points are statistically independent of each other i.e.,

$$\lim_{|\vec{x}^{(2)} - \vec{x}^{(1)}| \rightarrow \infty} F_2^{(1,2)} = F_1^{(1)} F_1^{(2)}$$

and similarly,

$$\lim_{|\vec{x}^{(3)} - \vec{x}^{(2)}| \rightarrow \infty} F_3^{(1,2,3)} = F_2^{(1,2)} F_1^{(3)} \text{ etc.}$$

#### c) Co-incidence Properties

When two points coincide in the flow field, the components at these points should be obviously the same that is  $F_2^{(1,2)}$  must be zero.

Thus  $\bar{v}^{(2)} = \bar{v}^{(1)}$ ,  $g^{(2)} = g^{(1)}$ ,  $\phi^{(2)} = \phi^{(1)}$  and  $\psi^{(2)} = \psi^{(1)}$ , but  $F_2^{(1,2)}$  must also have the property.

$$\int F_2^{(1,2)} dv^{(2)} dg^{(2)} d\phi^{(2)} d\psi^{(2)} = F_1^{(1)}$$

and hence it follows that

$$\lim_{|\vec{x}^{(2)} - \vec{x}^{(1)}| \rightarrow \infty} \int F_2^{(1,2)} = F_1^{(1)} \delta(v^{(2)} - v^{(1)}) \delta(g^{(2)} - g^{(1)}) \delta(\phi^{(2)} - \phi^{(1)}) \delta(\psi^{(2)} - \psi^{(1)})$$

Similarly,

$$\lim_{|\vec{x}^{(3)} - \vec{x}^{(2)}| \rightarrow \infty} \int F_3^{(1,2,3)} = F_2^{(1,2)} \delta(v^{(3)} - v^{(1)}) \delta(g^{(3)} - g^{(1)}) \delta(\phi^{(3)} - \phi^{(1)}) \delta(\psi^{(3)} - \psi^{(1)}) \text{ etc.}$$

d) *Symmetric Conditions*

$$F_n^{(1,2,r,-----s,-----n)} = F_n^{(1,2,-----s,---r,---n)}.$$

e) *Incompressibility Conditions*

$$(i) \int \frac{\partial F_n^{(1,2,---n)}}{\partial x_\alpha^{(r)}} v_\alpha^{(r)} d\bar{v}^{(r)} d\bar{h}^{(r)} = 0$$

$$(ii) \int \frac{\partial F_n^{(1,2,---n)}}{\partial x_\alpha^{(r)}} h_\alpha^{(r)} d\bar{v}^{(r)} d\bar{h}^{(r)} = 0$$

*Continuity Equation in Terms of Distribution Functions*

The continuity equations can be easily expressed in terms of distribution functions. An infinite number of continuity equations can be derived for the convective MHD turbulent flow and are obtained directly by using  $\text{div } \mathbf{u} = 0$

Taking ensemble average of equation (5), we get

$$\begin{aligned} 0 &= \left\langle \frac{\partial u_\alpha^{(1)}}{\partial x_\alpha^{(1)}} \right\rangle = \left\langle \frac{\partial}{\partial x_\alpha^{(1)}} u_\alpha^{(1)} \int F_1^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} \right\rangle \\ &= \frac{\partial}{\partial x_\alpha^{(1)}} \left\langle u_\alpha^{(1)} \int F_1^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} \right\rangle \\ &= \frac{\partial}{\partial x_\alpha^{(1)}} \int \left\langle u_\alpha^{(1)} \right\rangle F_1^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} \\ &= \frac{\partial}{\partial x_\alpha^{(1)}} \int v_\alpha^{(1)} F_1^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} \\ &= \int \frac{\partial F_1^{(1)}}{\partial x_\alpha^{(1)}} v_\alpha^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} \end{aligned} \quad (15)$$

and similarly,

$$0 = \int \frac{\partial F_1^{(1)}}{\partial x_\alpha^{(1)}} g_\alpha^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} \quad (16)$$

Equation (15) and (16) are the first order continuity equations in which only one point distribution function is involved. For second-order continuity equations, if we multiply the continuity equation by

$$\delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)})$$

and if we take the ensemble average, we obtain

$$\begin{aligned} 0 &= \left\langle \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \frac{\partial u_\alpha^{(1)}}{\partial x_\alpha^{(1)}} \right\rangle \\ &= \frac{\partial}{\partial x_\alpha^{(1)}} \left\langle \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) u_\alpha^{(1)} \right\rangle \\ &= \frac{\partial}{\partial x_\alpha^{(1)}} \left[ \int \left\langle u_\alpha^{(1)} \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \right\rangle \right] \end{aligned}$$

$$\times \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} ]$$

$$= \frac{\partial}{\partial x_{\alpha}^{(1)}} \int v_{\alpha}^{(1)} F_2^{(1,2)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} \quad (17)$$

and similarly,

$$o = \frac{\partial}{\partial x_{\alpha}^{(1)}} \int g_{\alpha}^{(1)} F_2^{(1,2)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} \quad (18)$$

The Nth – order continuity equations are

$$o = \frac{\partial}{\partial x_{\alpha}^{(1)}} \int v_{\alpha}^{(1)} F_N^{(1,2,\dots,N)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} \quad (19)$$

and

$$o = \frac{\partial}{\partial x_{\alpha}^{(1)}} \int g_{\alpha}^{(1)} F_N^{(1,2,\dots,N)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} \quad (20)$$

The continuity equations are symmetric in their arguments i.e.

$$\frac{\partial}{\partial x_{\alpha}^{(r)}} \left( v_{\alpha}^{(r)} F_N^{(1,2,\dots,r,N)} dv^{(r)} dg^{(r)} d\phi^{(r)} d\psi^{(r)} \right) = \frac{\partial}{\partial x_{\alpha}^{(s)}} \int v_{\alpha}^{(s)} F_N^{(1,2,\dots,r,s,\dots,N)} dv^{(s)} dg^{(s)} d\phi^{(s)} d\psi^{(s)} \quad (21)$$

Since the divergence property is an important property and it is easily verified by the use of the property of distribution function as

$$\frac{\partial}{\partial x_{\alpha}^{(1)}} \int v_{\alpha}^{(1)} F_1^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} \frac{\partial}{\partial x_{\alpha}^{(1)}} \langle u_{\alpha}^{(1)} \rangle = \left\langle \frac{\partial u_{\alpha}^{(1)}}{\partial x_{\alpha}^{(1)}} \right\rangle = o \quad (22)$$

and all the properties of the distribution function obtained in section (4) can also be verified.

*Equations for evolution of one –point distribution function  $F_1^{(1)}$*

It shall make use of equations (8) - (11) to convert these into a set of equations for the variation of the distribution function with time. This, in fact, is done by making use of the definitions of the constructed distribution functions, differentiating them partially with respect to time, making some suitable operations on the right-hand side of the equation so obtained and lastly replacing the time derivative of  $u, h, \theta$  and  $c$  from the equations (8) - (11).

Differentiating equation (12) with respect to time and using equation (8) - (11), we get,

$$\begin{aligned} \frac{\partial F_1^{(1)}}{\partial t} &= \frac{\partial}{\partial t} \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \rangle \\ &= \langle \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \frac{\partial}{\partial t} \delta(u^{(1)} - v^{(1)}) \rangle \\ &\quad + \langle \delta(u^{(1)} - v^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \frac{\partial}{\partial t} \delta(h^{(1)} - g^{(1)}) \rangle \\ &\quad + \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \frac{\partial}{\partial t} \delta(\theta^{(1)} - \phi^{(1)}) \rangle \\ &\quad + \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \frac{\partial}{\partial t} \delta(c^{(1)} - \psi^{(1)}) \rangle \\ &= \langle -\delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \frac{\partial u^{(1)}}{\partial t} \frac{\partial}{\partial v^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle \end{aligned}$$

$$\begin{aligned}
 & + \left\langle -\delta(u^{(1)} - v^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \frac{\partial h^{(1)}}{\partial t} \frac{\partial}{\partial g^{(1)}} \delta(h^{(1)} - g^{(1)}) \right\rangle \\
 & + \left\langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \frac{\partial \theta^{(1)}}{\partial t} \frac{\partial}{\partial \phi^{(1)}} \delta(\theta^{(1)} - \phi^{(1)}) \right\rangle \\
 & + \left\langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \frac{\partial c^{(1)}}{\partial t} \frac{\partial}{\partial \psi^{(1)}} \delta(c^{(1)} - \psi^{(1)}) \right\rangle \quad (23)
 \end{aligned}$$

Using equations (8) to (11) in the equation (23), we get

$$\begin{aligned}
 \frac{\partial F_1^{(1)}}{\partial t} &= \left\langle -\delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \left\{ -\frac{\partial}{\partial x_\beta^{(1)}} (u_\alpha^{(1)} u_\beta^{(1)} - h_\alpha^{(1)} h_\beta^{(1)}) \right. \right. \\
 & - \frac{1}{4\pi} \frac{\partial}{\partial x_\alpha^{(1)}} \int \left[ \frac{\partial u_\alpha^{(1)}}{\partial x_\beta^{(1)}} \frac{\partial u_\beta^{(1)}}{\partial x_\alpha^{(1)}} - \frac{\partial h_\alpha^{(1)}}{\partial x_\beta^{(1)}} \frac{\partial h_\beta^{(1)}}{\partial x_\alpha^{(1)}} \right] \frac{d\bar{x}'}{|\bar{x}' - \bar{x}|} + \nu \nabla^2 u_\alpha^{(1)} - 2 \epsilon_{m\alpha\beta} \Omega_m u_\alpha^{(1)} \\
 & + f(u_\alpha^{(1)} - v_\alpha^{(1)}) \left. \right\} \times \frac{\partial}{\partial v_\alpha^{(1)}} \delta(u^{(1)} - v^{(1)}) \\
 & + \left\langle -\delta(u^{(1)} - v^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \left\{ -\frac{\partial}{\partial x_\beta^{(1)}} (h_\alpha^{(1)} u_\beta^{(1)} - u_\alpha^{(1)} h_\beta^{(1)}) + \lambda \nabla^2 h_\alpha^{(1)} \right\} \right. \\
 & \times \frac{\partial}{\partial g_\alpha^{(1)}} \delta(h^{(1)} - g^{(1)}) \left. \right\rangle + \left\langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \left\{ -u_\beta^{(1)} \frac{\partial \theta^{(1)}}{\partial x_\beta^{(1)}} + \gamma \nabla^2 \theta^{(1)} \right\} \right. \\
 & \times \frac{\partial}{\partial \phi^{(1)}} \delta(\theta^{(1)} - \phi^{(1)}) \left. \right\rangle + \left\langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \left\{ -u_\beta^{(1)} \frac{\partial c^{(1)}}{\partial x_\beta^{(1)}} + D \nabla^2 c - Rc^1 \right\} \right. \\
 & \times \frac{\partial}{\partial \psi^{(1)}} \delta(c^{(1)} - \psi^{(1)}) \left. \right\rangle \\
 & = \left\langle \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \frac{\partial u_\alpha^{(1)} u_\beta^{(1)}}{\partial x_\beta^{(1)}} \frac{\partial}{\partial v_\alpha^{(1)}} \delta(u^{(1)} - v^{(1)}) \right\rangle \\
 & + \left\langle -\delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \frac{\partial h_\alpha^{(1)} h_\beta^{(1)}}{\partial x_\beta^{(1)}} \frac{\partial}{\partial v_\alpha^{(1)}} \delta(u^{(1)} - v^{(1)}) \right\rangle \\
 & + \left\langle \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \frac{1}{4\pi} \frac{\partial}{\partial x_\alpha^{(1)}} \int \left[ \frac{\partial u_\alpha^{(1)}}{\partial x_\beta^{(1)}} \frac{\partial u_\beta^{(1)}}{\partial x_\alpha^{(1)}} - \frac{\partial h_\alpha^{(1)}}{\partial x_\beta^{(1)}} \frac{\partial h_\beta^{(1)}}{\partial x_\alpha^{(1)}} \right] \right. \\
 & \times \frac{d\bar{x}'}{|\bar{x}' - \bar{x}|} \frac{\partial}{\partial v_\alpha^{(1)}} \delta(u^{(1)} - v^{(1)}) \left. \right\rangle \\
 & + \left\langle -\delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \times \nu \nabla^2 u_\alpha^{(1)} \frac{\partial}{\partial v_\alpha^{(1)}} \delta(u^{(1)} - v^{(1)}) \right\rangle \\
 & + \left\langle \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \times 2 \epsilon_{m\alpha\beta} \Omega_m u_\alpha^{(1)} \frac{\partial}{\partial v_\alpha^{(1)}} \delta(u^{(1)} - v^{(1)}) \right\rangle
 \end{aligned}$$

$$\begin{aligned}
 & + \langle -\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)}) \times f(u_\alpha^{(1)} - v_\alpha^{(1)}) \frac{\partial}{\partial v_\alpha^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle \\
 & + \langle \delta(u^{(1)} - v^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)}) \times \frac{\partial h_\alpha^{(1)} u_\beta^{(1)}}{\partial x_\beta^{(1)}} \frac{\partial}{\partial g_\alpha^{(1)}} \delta(h^{(1)} - g^{(1)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)}) \times \frac{\partial u_\alpha^{(1)} h_\beta^{(1)}}{\partial x_\beta^{(1)}} \frac{\partial}{\partial g_\alpha^{(1)}} \delta(h^{(1)} - g^{(1)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)}) \times \lambda \nabla^2 h_\alpha^{(1)} \frac{\partial}{\partial g_\alpha^{(1)}} \delta(h^{(1)} - g^{(1)}) \rangle \\
 & + \langle \delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(c^{(1)} - \psi^{(1)}) \times u_\beta^{(1)} \frac{\partial \theta^{(1)}}{\partial x_\beta^{(1)}} \frac{\partial}{\partial \phi^{(1)}} \delta(\theta^{(1)} - \phi^{(1)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(c^{(1)} - \psi^{(1)}) \times \gamma \nabla^2 \theta^{(1)} \frac{\partial}{\partial \phi^{(1)}} \delta(\theta^{(1)} - \phi^{(1)}) \rangle \\
 & + \langle \delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)}) \times u_\beta^{(1)} \frac{\partial c^{(1)}}{\partial x_\beta^{(1)}} \frac{\partial}{\partial \psi^{(1)}} \delta(c^{(1)} - \psi^{(1)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)}) \times D \nabla^2 c^{(1)} \frac{\partial}{\partial \psi^{(1)}} \delta(c^{(1)} - \psi^{(1)}) \rangle \\
 & + \langle \delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)}) \times Rc^{(1)} \frac{\partial}{\partial \psi^{(1)}} \delta(c^{(1)} - \psi^{(1)}) \rangle \quad (24)
 \end{aligned}$$

Various terms in the above equation can be simplified as that they may be expressed in terms of one point and two point distribution functions.

The 1<sup>st</sup> term in the above equation is simplified as follows:

$$\begin{aligned}
 & \langle \delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)}) \frac{\partial u_\alpha^{(1)} u_\beta^{(1)}}{\partial x_\beta^{(1)}} \frac{\partial}{\partial v_\alpha^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle \\
 & = \langle u_\beta^{(1)} \delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)}) \frac{\partial u_\alpha^{(1)}}{\partial x_\beta^{(1)}} \frac{\partial}{\partial v_\alpha^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle \\
 & = \langle -u_\beta^{(1)} \delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)}) \frac{\partial u_\alpha^{(1)}}{\partial x_\beta^{(1)}} \frac{\partial}{\partial x_\beta^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle \\
 & = \langle -u_\beta^{(1)} \delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)}) \frac{\partial}{\partial x_\beta^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle ; (\text{since } \frac{\partial u_\alpha^{(1)}}{\partial v_\alpha^{(1)}} = 1) \\
 & = \langle -\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)}) u_\beta^{(1)} \frac{\partial}{\partial x_\beta^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle \quad (25)
 \end{aligned}$$

Similarly, 7<sup>th</sup>, 10<sup>th</sup> and 12<sup>th</sup> terms of right hand-side of equation (24) can be simplified as follows;  
7<sup>th</sup> term

$$\langle \delta(u^{(1)} - v^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)}) \frac{\partial h_\alpha^{(1)} u_\beta^{(1)}}{\partial x_\beta^{(1)}} \frac{\partial}{\partial g_\alpha^{(1)}} \delta(h^{(1)} - g^{(1)}) \rangle \quad (26)$$

10<sup>th</sup> term,

$$\begin{aligned} & \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \mu_{\beta}^{(1)} \frac{\partial \theta^{(1)}}{\partial x_{\beta}^{(1)}} \frac{\partial}{\partial \phi^{(1)}} \delta(\theta^{(1)} - \phi^{(1)}) \rangle \\ &= \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \mu_{\beta}^{(1)} \frac{\partial}{\partial x_{\beta}^{(1)}} \delta(\theta^{(1)} - \phi^{(1)}) \rangle \end{aligned} \quad (27)$$

and 12<sup>th</sup> term

$$\begin{aligned} & \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \mu_{\beta}^{(1)} \frac{\partial c^{(1)}}{\partial x_{\beta}^{(1)}} \frac{\partial}{\partial \psi^{(1)}} \delta(c^{(1)} - \psi^{(1)}) \rangle \\ &= \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \mu_{\beta}^{(1)} \frac{\partial}{\partial x_{\beta}^{(1)}} \delta(c^{(1)} - \psi^{(1)}) \rangle \end{aligned} \quad (28)$$

Adding these equations from (25) to (28), we get

$$\begin{aligned} & \langle -\delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \mu_{\beta}^{(1)} \frac{\partial}{\partial x_{\beta}^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle \\ &+ \langle -\delta(u^{(1)} - v^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \mu_{\beta}^{(1)} \frac{\partial}{\partial x_{\beta}^{(1)}} \delta(h^{(1)} - g^{(1)}) \rangle \\ &+ \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \mu_{\beta}^{(1)} \frac{\partial}{\partial x_{\beta}^{(1)}} \delta(\theta^{(1)} - \phi^{(1)}) \rangle \\ &+ \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \mu_{\beta}^{(1)} \frac{\partial}{\partial x_{\beta}^{(1)}} \delta(c^{(1)} - \psi^{(1)}) \rangle \\ &= -\frac{\partial}{\partial x_{\beta}^{(1)}} \langle u_{\beta}^{(1)} \rangle \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \rangle \\ &= -\frac{\partial}{\partial x_{\beta}^{(1)}} v_{\beta}^{(1)} F_1^{(1)} \quad [\text{Applying the properties of distribution function}] \\ &= -v_{\beta}^{(1)} \frac{\partial F_1^{(1)}}{\partial x_{\beta}^{(1)}} \end{aligned} \quad (29)$$

Similarly second and eighth terms on the right hand-side of the equation (24) can be simplified as

$$\begin{aligned} & \langle -\delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \frac{\partial h_{\alpha}^{(1)} h_{\beta}^{(1)}}{\partial x_{\beta}^{(1)}} \frac{\partial}{\partial v_{\alpha}^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle \\ &= -g_{\beta}^{(1)} \frac{\partial g_{\alpha}^{(1)}}{\partial v_{\alpha}^{(1)}} \frac{\partial}{\partial x_{\beta}^{(1)}} F_1^{(1)} \end{aligned} \quad (30)$$

and

$$\begin{aligned} & \langle -\delta(u^{(1)} - v^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \frac{\partial u_{\alpha}^{(1)} h_{\beta}^{(1)}}{\partial x_{\beta}^{(1)}} \frac{\partial}{\partial g_{\alpha}^{(1)}} \delta(h^{(1)} - g^{(1)}) \rangle \\ &= -g_{\beta}^{(1)} \frac{\partial v_{\alpha}^{(1)}}{\partial g_{\alpha}^{(1)}} \frac{\partial}{\partial x_{\beta}^{(1)}} F_1^{(1)} \end{aligned} \quad (31)$$

4<sup>th</sup> term can be reduced as

$$\begin{aligned}
 & \left\langle -\nu \nabla^2 u_{\alpha}^{(1)} \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \frac{\partial}{\partial v_{\alpha}^{(1)}} \delta(u^{(1)} - v^{(1)}) \right\rangle \\
 &= -\nu \frac{\partial}{\partial v_{\alpha}^{(1)}} \left\langle \nabla^2 u_{\alpha}^{(1)} \left[ \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \right] \right\rangle \\
 &= -\nu \frac{\partial}{\partial v_{\alpha}^{(1)}} \frac{\partial^2}{\partial x_{\beta}^{(1)} \partial x_{\beta}^{(1)}} \left\langle u_{\alpha}^{(1)} \left[ \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \right] \right\rangle \\
 &= -\nu \frac{\partial}{\partial v_{\alpha}^{(1)}} \lim_{\bar{x}^{(2)} \rightarrow \bar{x}^{(1)}} \frac{\partial^2}{\partial x_{\beta}^{(2)} \partial x_{\beta}^{(2)}} \left\langle u_{\alpha}^{(2)} \left[ \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \right] \right\rangle \\
 &= -\nu \frac{\partial}{\partial v_{\alpha}^{(1)}} \lim_{\bar{x}^{(2)} \rightarrow \bar{x}^{(1)}} \frac{\partial^2}{\partial x_{\beta}^{(2)} \partial x_{\beta}^{(2)}} \left\langle \int u_{\alpha}^{(2)} \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \right. \\
 &\quad \times \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) dv^{(2)} dg^{(2)} d\phi^{(2)} d\psi^{(2)} \rangle \\
 &= -\nu \frac{\partial}{\partial v_{\alpha}^{(1)}} \lim_{\bar{x}^{(2)} \rightarrow \bar{x}^{(1)}} \frac{\partial^2}{\partial x_{\beta}^{(2)} \partial x_{\beta}^{(2)}} \int v_{\alpha}^{(2)} F_2^{(1,2)} dv^{(2)} dg^{(2)} d\phi^{(2)} d\psi^{(2)} \quad (32)
 \end{aligned}$$

9<sup>th</sup>, 11<sup>th</sup> and 13<sup>th</sup> terms of the right hand side of equation (24)

9<sup>th</sup> term,

$$\begin{aligned}
 & \left\langle -\delta(u^{(1)} - v^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \lambda \nabla^2 h_{\alpha}^{(1)} \frac{\partial}{\partial g_{\alpha}^{(1)}} \delta(h^{(1)} - g^{(1)}) \right\rangle \\
 &= \left\langle -\lambda \nabla^2 h_{\alpha}^{(1)} \frac{\partial}{\partial g_{\alpha}^{(1)}} \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \right\rangle \\
 &= -\lambda \frac{\partial}{\partial g_{\alpha}^{(1)}} \lim_{\bar{x}^{(2)} \rightarrow \bar{x}^{(1)}} \frac{\partial^2}{\partial x_{\beta}^{(2)} \partial x_{\beta}^{(2)}} \int g_{\alpha}^{(2)} F_2^{(1,2)} dv^{(2)} dg^{(2)} d\phi^{(2)} d\psi^{(2)} \quad (33)
 \end{aligned}$$

11<sup>th</sup> term,

$$\begin{aligned}
 & \left\langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \eta \nabla^2 \theta^{(1)} \frac{\partial}{\partial \phi^{(1)}} \delta(\theta^{(1)} - \phi^{(1)}) \right\rangle \\
 &= \left\langle -\eta \nabla^2 \theta^{(1)} \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \frac{\partial}{\partial \phi^{(1)}} \delta(\theta^{(1)} - \phi^{(1)}) \right\rangle \quad (34)
 \end{aligned}$$

13<sup>th</sup> term,

$$\begin{aligned}
 & \left\langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) D \nabla^2 c^{(1)} \frac{\partial}{\partial \psi^{(1)}} \delta(c^{(1)} - \psi^{(1)}) \right\rangle \\
 &= \left\langle -D \nabla^2 c^{(1)} \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \frac{\partial}{\partial \psi^{(1)}} \delta(c^{(1)} - \psi^{(1)}) \right\rangle
 \end{aligned}$$

$$= -D \frac{\partial}{\partial \psi^{(1)}} \lim_{\bar{x}^{(2)} \rightarrow \bar{x}^{(1)}} \frac{\partial^2}{\partial x_{\beta}^{(2)} \partial x_{\beta}^{(2)}} \int \psi^{(2)} F_2^{(1,2)} dv^{(2)} dg^{(2)} d\phi^{(2)} d\psi^{(2)} \quad (35)$$

We reduce the 3<sup>rd</sup> term of right hand side of equation (24), we get

$$\begin{aligned} & \langle \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \frac{1}{4\pi} \frac{\partial}{\partial x_{\alpha}^{(1)}} \int \left[ \frac{\partial u_{\alpha}^{(1)}}{\partial x_{\beta}^{(1)}} \frac{\partial u_{\beta}^{(1)}}{\partial x_{\alpha}^{(1)}} - \frac{\partial h_{\alpha}^{(1)}}{\partial x_{\beta}^{(1)}} \frac{\partial h_{\beta}^{(1)}}{\partial x_{\alpha}^{(1)}} \right] \frac{d\bar{x}'}{|\bar{x}' - \bar{x}|} \frac{\partial}{\partial v_{\alpha}^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle \\ &= \frac{\partial}{\partial v_{\alpha}^{(1)}} \left[ \frac{1}{4\pi} \int \frac{\partial}{\partial x_{\alpha}^{(1)}} \left( \frac{1}{|\bar{x}^{(2)} - \bar{x}^{(1)}|} \right) \left( \frac{\partial v_{\alpha}^{(2)}}{\partial x_{\beta}^{(2)}} \frac{\partial v_{\beta}^{(2)}}{\partial x_{\alpha}^{(2)}} - \frac{\partial g_{\alpha}^{(2)}}{\partial x_{\beta}^{(2)}} \frac{\partial g_{\beta}^{(2)}}{\partial x_{\alpha}^{(2)}} \right) F_2^{(1,2)} dx^{(2)} dv^{(2)} dg^{(2)} d\phi^{(2)} d\psi^{(2)} \right] \end{aligned} \quad (36)$$

5<sup>th</sup> and 6<sup>th</sup> terms of right hand side of equation (24)

$$\begin{aligned} & \langle \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \times 2 \in_{m\alpha\beta} \Omega_m u_{\alpha}^{(1)} \frac{\partial}{\partial v_{\alpha}^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle \\ &= \langle 2 \in_{m\alpha\beta} \Omega_m u_{\alpha}^{(1)} \frac{\partial}{\partial v_{\alpha}^{(1)}} [ \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) ] \rangle \\ &= 2 \in_{m\alpha\beta} \Omega_m \frac{\partial}{\partial v_{\alpha}^{(1)}} \langle u_{\alpha}^{(1)} \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \rangle \\ &= 2 \in_{m\alpha\beta} \Omega_m \frac{\partial u_{\alpha}^{(1)}}{\partial v_{\alpha}^{(1)}} \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \rangle \\ &= 2 \in_{m\alpha\beta} \Omega_m F_1^{(1)} \end{aligned} \quad (37)$$

$$\begin{aligned} & \text{and } \langle -\delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) f(u_{\alpha}^{(1)} - v_{\alpha}^{(1)}) \frac{\partial}{\partial v_{\alpha}^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle \\ &= -\langle f(u_{\alpha}^{(1)} - v_{\alpha}^{(1)}) \frac{\partial}{\partial v_{\alpha}^{(1)}} [ \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) ] \rangle \\ &= -f(u_{\alpha}^{(1)} - v_{\alpha}^{(1)}) \frac{\partial}{\partial v_{\alpha}^{(1)}} \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \rangle \\ &= -f(u_{\alpha}^{(1)} - v_{\alpha}^{(1)}) \frac{\partial}{\partial v_{\alpha}^{(1)}} F_1^{(1)} \end{aligned} \quad (38)$$

and the last term,

$$\langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \times Rc^{(1)} \frac{\partial}{\partial \psi^{(1)}} \delta(c^{(1)} - \psi^{(1)}) \rangle = Rc^{(1)} \frac{\partial}{\partial \psi^{(1)}} F_1^{(1)} \quad (39)$$

Substituting the results (25) to (39) in equation (24) we get the transport equation for one point distribution function in MHD turbulent flow in a rotating system in presence of dust particles under going a first order reaction as

$$\begin{aligned} & \frac{\partial F_1^{(1)}}{\partial t} + v_{\beta}^{(1)} \frac{\partial F_1^{(1)}}{\partial x_{\beta}^{(1)}} + g_{\beta}^{(1)} \left( \frac{\partial g_{\alpha}^{(1)}}{\partial v_{\alpha}^{(1)}} + \frac{\partial v_{\alpha}^{(1)}}{\partial g_{\alpha}^{(1)}} \right) \frac{\partial F_1^{(1)}}{\partial x_{\beta}^{(1)}} - \frac{\partial}{\partial v_{\alpha}^{(1)}} \left[ \frac{1}{4\pi} \int \frac{\partial}{\partial x_{\alpha}^{(1)}} \left( \frac{1}{|\bar{x}^{(2)} - \bar{x}^{(1)}|} \right) \right. \\ & \quad \times \left( \frac{\partial v_{\alpha}^{(2)}}{\partial x_{\beta}^{(2)}} \frac{\partial v_{\beta}^{(2)}}{\partial x_{\alpha}^{(2)}} - \frac{\partial g_{\alpha}^{(2)}}{\partial x_{\beta}^{(2)}} \frac{\partial g_{\beta}^{(2)}}{\partial x_{\alpha}^{(2)}} \right) F_2^{(1,2)} dx^{(2)} dv^{(2)} dg^{(2)} d\phi^{(2)} d\psi^{(2)} \end{aligned}$$

$$\begin{aligned}
 & + \nu \frac{\partial}{\partial v_{\alpha}^{(1)}} \lim_{\bar{x}^{(2)} \rightarrow \bar{x}^{(1)}} \frac{\partial^2}{\partial x_{\beta}^{(2)} \partial x_{\beta}^{(2)}} \int v_{\alpha}^{(2)} F_2^{(1,2)} dv^{(2)} dg^{(2)} d\phi^{(2)} d\psi^{(2)} \\
 & + \lambda \frac{\partial}{\partial g_{\alpha}^{(1)}} \lim_{\bar{x}^{(2)} \rightarrow \bar{x}^{(1)}} \frac{\partial^2}{\partial x_{\beta}^{(2)} \partial x_{\beta}^{(2)}} \int g_{\alpha}^{(2)} F_2^{(1,2)} dv^{(2)} dg^{(2)} d\phi^{(2)} d\psi^{(2)} \\
 & + \gamma \frac{\partial}{\partial \phi^{(1)}} \lim_{\bar{x}^{(2)} \rightarrow \bar{x}^{(1)}} \frac{\partial^2}{\partial x_{\beta}^{(2)} \partial x_{\beta}^{(2)}} \int \phi^{(2)} F_2^{(1,2)} dv^{(2)} dg^{(2)} d\phi^{(2)} d\psi^{(2)} \\
 & + D \frac{\partial}{\partial \psi^{(1)}} \lim_{\bar{x}^{(2)} \rightarrow \bar{x}^{(1)}} \frac{\partial^2}{\partial x_{\beta}^{(2)} \partial x_{\beta}^{(2)}} \int \psi^{(2)} F_2^{(1,2)} dv^{(2)} dg^{(2)} d\phi^{(2)} d\psi^{(2)} \\
 & + 2 \epsilon_{m\alpha\beta} \Omega_m F_1^{(1)} + f(u_{\alpha}^{(1)} - v_{\alpha}^{(1)}) \frac{\partial}{\partial v_{\alpha}^{(1)}} F_1^{(1)} - R \psi^{(1)} \frac{\partial}{\partial \psi^{(1)}} F_1^{(1)} = 0
 \end{aligned} \tag{39}$$

Equations for two-point distribution function  $F_2^{(1,2)}$

Differentiating equation (13) with respect to time, we get,

$$\begin{aligned}
 \frac{\partial F_2^{(1,2)}}{\partial t} &= \frac{\partial}{\partial t} \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \\
 &\quad \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \rangle \\
 &= \langle \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 &\quad \delta(c^{(2)} - \psi^{(2)}) \frac{\partial}{\partial t} \delta(u^{(1)} - v^{(1)}) \rangle + \langle \delta(u^{(1)} - v^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \\
 &\quad \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \frac{\partial}{\partial t} \delta(h^{(1)} - g^{(1)}) \rangle \\
 &\quad + \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 &\quad \delta(c^{(2)} - \psi^{(2)}) \frac{\partial}{\partial t} \delta(\theta^{(1)} - \phi^{(1)}) \rangle + \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \\
 &\quad \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \frac{\partial}{\partial t} \delta(c^{(1)} - \psi^{(1)}) \rangle \\
 &\quad + \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 &\quad \delta(c^{(2)} - \psi^{(2)}) \frac{\partial}{\partial t} \delta(u^{(2)} - v^{(2)}) \rangle + \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \\
 &\quad \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \frac{\partial}{\partial t} \delta(h^{(2)} - g^{(2)}) \rangle \\
 &\quad + \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \\
 &\quad \delta(c^{(2)} - \psi^{(2)}) \frac{\partial}{\partial t} \delta(\theta^{(2)} - \phi^{(2)}) \rangle + \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \\
 &\quad \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \frac{\partial}{\partial t} \delta(c^{(2)} - \psi^{(2)}) \rangle \\
 &= \langle -\delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 &\quad \delta(c^{(2)} - \psi^{(2)}) \frac{\partial u^{(1)}}{\partial t} \frac{\partial}{\partial v^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle
 \end{aligned}$$

$$\begin{aligned}
 & + \langle -\delta(u^{(1)} - v^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \frac{\partial h^{(1)}}{\partial t} \frac{\partial}{\partial g^{(1)}} \delta(h^{(1)} - g^{(1)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \frac{\partial \theta^{(1)}}{\partial t} \frac{\partial}{\partial \phi^{(1)}} \delta(\theta^{(1)} - \phi^{(1)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \frac{\partial c^{(1)}}{\partial t} \frac{\partial}{\partial \psi^{(1)}} \delta(c^{(1)} - \psi^{(1)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \frac{\partial u^{(2)}}{\partial t} \frac{\partial}{\partial v^{(2)}} \delta(u^{(2)} - v^{(2)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \frac{\partial h^{(2)}}{\partial t} \frac{\partial}{\partial g^{(2)}} \delta(h^{(2)} - g^{(2)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \frac{\partial \theta^{(2)}}{\partial t} \frac{\partial}{\partial \phi^{(2)}} \delta(\theta^{(2)} - \phi^{(2)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \\
 & \delta(\theta^{(2)} - \phi^{(2)}) \frac{\partial c^{(2)}}{\partial t} \frac{\partial}{\partial \psi^{(2)}} \delta(c^{(2)} - \psi^{(2)}) \rangle
 \end{aligned}$$

Using equations (8) to (11) we get,

$$\begin{aligned}
 \frac{\partial F_2^{(1,2)}}{\partial t} & = \langle -\delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \left\{ -\frac{\partial}{\partial x_\beta^{(1)}} (u_\alpha^{(1)} u_\beta^{(1)} - h_\alpha^{(1)} h_\beta^{(1)}) - \frac{1}{4\pi} \frac{\partial}{\partial x_\alpha^{(1)}} \int \left[ \frac{\partial u_\alpha^{(1)}}{\partial x_\beta^{(1)}} \frac{\partial u_\beta^{(1)}}{\partial x_\alpha^{(1)}} - \frac{\partial h_\alpha^{(1)}}{\partial x_\beta^{(1)}} \frac{\partial h_\beta^{(1)}}{\partial x_\alpha^{(1)}} \right] \right. \\
 & \times \frac{d\bar{x}''}{|\bar{x}'' - \bar{x}|} + \nu \nabla^2 u_\alpha^{(1)} - 2 \epsilon_{m\alpha\beta} \Omega_m u_\alpha^{(1)} + f(u_\alpha^{(1)} - v_\alpha^{(1)}) \left. \right\} \times \frac{\partial}{\partial v_\alpha^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \left\{ -\frac{\partial}{\partial x_\beta^{(1)}} (h_\alpha^{(1)} u_\beta^{(1)} - u_\alpha^{(1)} h_\beta^{(1)}) + \lambda \nabla^2 h_\alpha^{(1)} \right\} \times \frac{\partial}{\partial g_\alpha^{(1)}} \delta(h^{(1)} - g^{(1)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\
 & \left\{ -u_\beta^{(1)} \frac{\partial \theta^{(1)}}{\partial x_\beta^{(1)}} + \gamma \nabla^2 \theta^{(1)} \right\} \times \frac{\partial}{\partial \phi^{(1)}} \delta(\theta^{(1)} - \phi^{(1)}) \rangle + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(u^{(2)} - v^{(2)}) \\
 & \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \left\{ -u_\beta^{(1)} \frac{\partial c^{(1)}}{\partial x_\beta^{(1)}} + D \nabla^2 c^{(1)} - R c^{(1)} \right\} \times \frac{\partial}{\partial \psi^{(1)}} \delta(c^{(1)} - \psi^{(1)}) \rangle
 \end{aligned}$$

$$\begin{aligned}
 & + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\
 & \{ -\frac{\partial}{\partial x_{\beta}^{(2)}} (u_{\alpha}^{(2)} u_{\beta}^{(2)} - h_{\alpha}^{(2)} h_{\beta}^{(2)}) - \frac{1}{4\pi} \frac{\partial}{\partial x_{\alpha}^{(2)}} \int [ \frac{\partial u_{\alpha}^{(2)}}{\partial x_{\beta}^{(2)}} \frac{\partial u_{\beta}^{(2)}}{\partial x_{\alpha}^{(2)}} - \frac{\partial h_{\alpha}^{(2)}}{\partial x_{\beta}^{(2)}} \frac{\partial h_{\beta}^{(2)}}{\partial x_{\alpha}^{(2)}} ] \frac{d\bar{x}''}{|\bar{x}'' - \bar{x}|} \\
 & + \nabla^2 u_{\alpha}^{(2)} - 2 \epsilon_{m\alpha\beta} \Omega_m u_{\alpha}^{(2)} + f(u_{\alpha}^{(2)} - v_{\alpha}^{(2)}) \} \times \frac{\partial}{\partial v_{\alpha}^{(2)}} \delta(u^{(2)} - v^{(2)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \{ -\frac{\partial}{\partial x_{\beta}^{(2)}} (h_{\alpha}^{(2)} u_{\beta}^{(2)} - u_{\alpha}^{(2)} h_{\beta}^{(2)}) + \lambda \nabla^2 h_{\alpha}^{(2)} \} \times \frac{\partial}{\partial g_{\alpha}^{(2)}} \delta(h^{(2)} - g^{(2)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \{ -u_{\beta}^{(2)} \frac{\partial \theta^{(2)}}{\partial x_{\beta}^{(2)}} + \nabla^2 \theta^{(2)} \} \times \frac{\partial}{\partial \phi^{(2)}} \delta(\theta^{(2)} - \phi^{(2)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \\
 & \delta(\theta^{(2)} - \phi^{(2)}) \{ -u_{\beta}^{(2)} \frac{\partial c^{(2)}}{\partial x_{\beta}^{(2)}} + D \nabla^2 c^{(2)} - R c^{(2)} \} \times \frac{\partial}{\partial \psi^{(2)}} \delta(c^{(2)} - \psi^{(2)}) \rangle \\
 & = \langle \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \times \frac{\partial u_{\alpha}^{(1)} u_{\beta}^{(1)}}{\partial x_{\beta}^{(1)}} \frac{\partial}{\partial v_{\alpha}^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle \\
 & + \langle -\delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \times \frac{\partial h_{\alpha}^{(1)} h_{\beta}^{(1)}}{\partial x_{\beta}^{(1)}} \frac{\partial}{\partial v_{\alpha}^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle \\
 & + \langle \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \times \frac{1}{4\pi} \frac{\partial}{\partial x_{\alpha}^{(1)}} \int [ \frac{\partial u_{\alpha}^{(1)}}{\partial x_{\beta}^{(1)}} \frac{\partial u_{\beta}^{(1)}}{\partial x_{\alpha}^{(1)}} - \frac{\partial h_{\alpha}^{(1)}}{\partial x_{\beta}^{(1)}} \frac{\partial h_{\beta}^{(1)}}{\partial x_{\alpha}^{(1)}} ] \frac{d\bar{x}''}{|\bar{x}'' - \bar{x}|} \frac{\partial}{\partial v_{\alpha}^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle \\
 & + \langle -\delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \\
 & \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \times \nabla^2 u_{\alpha}^{(1)} \frac{\partial}{\partial v_{\alpha}^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle \\
 & + \langle \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \times 2 \epsilon_{m\alpha\beta} \Omega_m u_{\alpha}^{(1)} \frac{\partial}{\partial v_{\alpha}^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle
 \end{aligned}$$

$$\begin{aligned}
 & + \langle -\delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \times f(u_\alpha^{(1)} - v_\alpha^{(1)}) \frac{\partial}{\partial v_\alpha^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle \\
 & + \langle \delta(u^{(1)} - v^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \times \frac{\partial h_\alpha^{(1)} u_\beta^{(1)}}{\partial x_\beta^{(1)}} \frac{\partial}{\partial g_\alpha^{(1)}} \delta(h^{(1)} - g^{(1)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \times \frac{\partial u_\alpha^{(1)} h_\beta^{(1)}}{\partial x_\beta^{(1)}} \frac{\partial}{\partial g_\alpha^{(1)}} \delta(h^{(1)} - g^{(1)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \times \lambda \nabla^2 h_\alpha^{(1)} \frac{\partial}{\partial g_\alpha^{(1)}} \delta(h^{(1)} - g^{(1)}) \rangle \\
 & + \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \times u_\beta^{(1)} \frac{\partial \theta^{(1)}}{\partial x_\beta^{(1)}} \frac{\partial}{\partial \phi^{(1)}} \delta(\theta^{(1)} - \phi^{(1)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \times \gamma \nabla^2 \theta^{(1)} \frac{\partial}{\partial \phi^{(1)}} \delta(\theta^{(1)} - \phi^{(1)}) \rangle \\
 & + \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \times u_\beta^{(1)} \frac{\partial c^{(1)}}{\partial x_\beta^{(1)}} \frac{\partial}{\partial \psi^{(1)}} \delta(c^{(1)} - \psi^{(1)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \times D \nabla^2 c^{(1)} \frac{\partial}{\partial \psi^{(1)}} \delta(c^{(1)} - \psi^{(1)}) \rangle \\
 & + \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \times R c^{(1)} \frac{\partial}{\partial \psi^{(1)}} \delta(c^{(1)} - \psi^{(1)}) \rangle \\
 & + \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \times \frac{\partial u_\alpha^{(2)} u_\beta^{(2)}}{\partial x_\beta^{(2)}} \frac{\partial}{\partial v_\alpha^{(2)}} \delta(u^{(2)} - v^{(2)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \times \frac{\partial h_\alpha^{(2)} h_\beta^{(2)}}{\partial x_\beta^{(2)}} \frac{\partial}{\partial v_\alpha^{(2)}} \delta(u^{(2)} - v^{(2)}) \rangle \\
 & + \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \times \frac{1}{4\pi} \frac{\partial}{\partial x_\alpha^{(2)}} \int \left[ \frac{\partial u_\alpha^{(2)}}{\partial x_\beta^{(2)}} \frac{\partial u_\beta^{(2)}}{\partial x_\alpha^{(2)}} - \frac{\partial h_\alpha^{(2)}}{\partial x_\beta^{(2)}} \frac{\partial h_\beta^{(2)}}{\partial x_\alpha^{(2)}} \right] \frac{d\bar{x}''}{|\bar{x}'' - \bar{x}'|} \frac{\partial}{\partial v_\alpha^{(2)}} \delta(u^{(2)} - v^{(2)}) \rangle
 \end{aligned}$$

$$\begin{aligned}
 & + \langle -\delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(h^{(2)} - g^{(2)})\delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \times \nabla^2 u_\alpha^{(2)} \frac{\partial}{\partial v_\alpha^{(2)}} \delta(u^{(2)} - v^{(2)}) \rangle \\
 & + \langle \delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(h^{(2)} - g^{(2)})\delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \times 2 \epsilon_{m\alpha\beta} \Omega_m u_\alpha^{(2)} \frac{\partial}{\partial v_\alpha^{(2)}} \delta(u^{(2)} - v^{(2)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(h^{(2)} - g^{(2)})\delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \times f(u_\alpha^{(2)} - v_\alpha^{(2)}) \frac{\partial}{\partial v_\alpha^{(2)}} \delta(u^{(2)} - v^{(2)}) \rangle \\
 & + \langle \delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \times \frac{\partial h_\alpha^{(2)} u_\beta^{(2)}}{\partial x_\beta^{(2)}} \frac{\partial}{\partial g_\alpha^{(2)}} \delta(h^{(2)} - g^{(2)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \times \frac{\partial u_\alpha^{(2)} h_\beta^{(2)}}{\partial x_\beta^{(2)}} \frac{\partial}{\partial g_\alpha^{(2)}} \delta(h^{(2)} - g^{(2)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \times \lambda \nabla^2 h_\alpha^{(2)} \frac{\partial}{\partial g_\alpha^{(2)}} \delta(h^{(2)} - g^{(2)}) \rangle \\
 & + \langle \delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(h^{(2)} - g^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \times u_\beta^{(2)} \frac{\partial \theta^{(2)}}{\partial x_\beta^{(2)}} \frac{\partial}{\partial \phi^{(2)}} \delta(\theta^{(2)} - \phi^{(2)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(h^{(2)} - g^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \times \nabla^2 \theta^{(2)} \frac{\partial}{\partial \phi^{(2)}} \delta(\theta^{(2)} - \phi^{(2)}) \rangle \\
 & + \langle \delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(h^{(2)} - g^{(2)}) \\
 & \delta(\theta^{(2)} - \phi^{(2)}) \times u_\beta^{(2)} \frac{\partial c^{(2)}}{\partial x_\beta^{(2)}} \frac{\partial}{\partial \psi^{(2)}} \delta(c^{(2)} - \psi^{(2)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(h^{(2)} - g^{(2)}) \\
 & \delta(\theta^{(2)} - \phi^{(2)}) \times D \nabla^2 c^{(2)} \frac{\partial}{\partial \psi^{(2)}} \delta(c^{(2)} - \psi^{(2)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(h^{(2)} - g^{(2)}) \\
 & \delta(\theta^{(2)} - \phi^{(2)}) \times R c^{(2)} \frac{\partial}{\partial \psi^{(2)}} \delta(c^{(2)} - \psi^{(2)}) \rangle
 \end{aligned} \tag{41}$$

Various terms in the above equation can be simplified as that they may be expressed in terms of one point, two- point and three-point distribution functions.

The 1<sup>st</sup> term in the above equation is simplified as follows:

$$\begin{aligned}
 & \langle \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \times \frac{\partial u_{\alpha}^{(1)} u_{\beta}^{(1)}}{\partial x_{\beta}^{(1)}} \frac{\partial}{\partial v_{\alpha}^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle \\
 & = \langle u_{\beta}^{(1)} \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \times \frac{\partial u_{\alpha}^{(1)}}{\partial x_{\beta}^{(1)}} \frac{\partial}{\partial v_{\alpha}^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle \\
 & = \langle -u_{\beta}^{(1)} \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \times \frac{\partial u_{\alpha}^{(1)}}{\partial v_{\alpha}^{(1)}} \frac{\partial}{\partial x_{\beta}^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle; (\text{since } \frac{\partial u_{\alpha}^{(1)}}{\partial v_{\alpha}^{(1)}} = 1) \\
 & = \langle -\delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \times u_{\beta}^{(1)} \frac{\partial}{\partial x_{\beta}^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle
 \end{aligned} \tag{42}$$

Similarly, 7<sup>th</sup>, 10<sup>th</sup> and 12<sup>th</sup> terms of right hand-side of equation (41) can be simplified as follows;  
7<sup>th</sup> term,

$$\begin{aligned}
 & \langle \delta(u^{(1)} - v^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \times \frac{\partial h_{\alpha}^{(1)} u_{\beta}^{(1)}}{\partial x_{\beta}^{(1)}} \frac{\partial}{\partial g_{\alpha}^{(1)}} \delta(h^{(1)} - g^{(1)}) \rangle \\
 & = \langle -\delta(u^{(1)} - v^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \times u_{\beta}^{(1)} \frac{\partial}{\partial x_{\beta}^{(1)}} \delta(h^{(1)} - g^{(1)}) \rangle
 \end{aligned} \tag{43}$$

10<sup>th</sup> term,

$$\begin{aligned}
 & \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \times u_{\beta}^{(1)} \frac{\partial \theta^{(1)}}{\partial x_{\beta}^{(1)}} \frac{\partial}{\partial \phi^{(1)}} \delta(\theta^{(1)} - \phi^{(1)}) \rangle \\
 & = \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \times u_{\beta}^{(1)} \frac{\partial}{\partial x_{\beta}^{(1)}} \delta(\theta^{(1)} - \phi^{(1)}) \rangle
 \end{aligned} \tag{44}$$

and 12<sup>th</sup> term

$$\begin{aligned}
 & \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \times u_{\beta}^{(1)} \frac{\partial c^{(1)}}{\partial x_{\beta}^{(1)}} \frac{\partial}{\partial \psi^{(1)}} \delta(c^{(1)} - \psi^{(1)}) \rangle
 \end{aligned}$$

$$= \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \times u_{\beta}^{(1)} \frac{\partial}{\partial x_{\beta}^{(1)}} \delta(c^{(1)} - \psi^{(1)}) \rangle. \quad (45)$$

Adding these equations from (42) to (45), we get

$$\begin{aligned} & \langle -\delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\ & \delta(c^{(2)} - \psi^{(2)}) \times u_{\beta}^{(1)} \frac{\partial}{\partial x_{\beta}^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle \\ & + \langle -\delta(u^{(1)} - v^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\ & \delta(c^{(2)} - \psi^{(2)}) \times u_{\beta}^{(1)} \frac{\partial}{\partial x_{\beta}^{(1)}} \delta(h^{(1)} - g^{(1)}) \rangle \\ & + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\ & \delta(c^{(2)} - \psi^{(2)}) \times u_{\beta}^{(1)} \frac{\partial}{\partial x_{\beta}^{(1)}} \delta(\theta^{(1)} - \phi^{(1)}) \rangle \\ & + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\ & \delta(c^{(2)} - \psi^{(2)}) \times u_{\beta}^{(1)} \frac{\partial}{\partial x_{\beta}^{(1)}} \delta(c^{(1)} - \psi^{(1)}) \rangle \\ & = -\frac{\partial}{\partial x_{\beta}^{(1)}} \langle u_{\beta}^{(1)} \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \\ & \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \rangle \rangle \\ & = -\frac{\partial}{\partial x_{\beta}^{(1)}} v_{\beta}^{(1)} F_2^{(1,2)} \text{ [Applying the properties of distribution functions]} \\ & = -v_{\beta}^{(1)} \frac{\partial F_2^{(1,2)}}{\partial x_{\beta}^{(1)}} \end{aligned} \quad (46)$$

Similarly, 15<sup>th</sup>, 21<sup>st</sup>, 24<sup>th</sup> and 26<sup>th</sup> terms of right hand-side of equation (41) can be simplified as follows;  
15<sup>th</sup> term,

$$\begin{aligned} & \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\ & \delta(c^{(2)} - \psi^{(2)}) \times \frac{\partial u_{\alpha}^{(2)} u_{\beta}^{(2)}}{\partial x_{\beta}^{(2)}} \frac{\partial}{\partial v_{\alpha}^{(2)}} \delta(u^{(2)} - v^{(2)}) \rangle \\ & = \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\ & \delta(c^{(2)} - \psi^{(2)}) \times u_{\beta}^{(2)} \frac{\partial}{\partial x_{\beta}^{(2)}} \delta(u^{(2)} - v^{(2)}) \rangle \end{aligned} \quad (47)$$

21<sup>st</sup> term,

$$\begin{aligned} & \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\ & \delta(c^{(2)} - \psi^{(2)}) \times \frac{\partial h_{\alpha}^{(2)} u_{\beta}^{(2)}}{\partial x_{\beta}^{(2)}} \frac{\partial}{\partial g_{\alpha}^{(2)}} \delta(h^{(2)} - g^{(2)}) \rangle \end{aligned}$$

$$= \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \times u_{\beta}^{(2)} \frac{\partial}{\partial x_{\beta}^{(2)}} \delta(h^{(2)} - g^{(2)}) \rangle \quad (48)$$

24<sup>th</sup> term,

$$\begin{aligned} & \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \\ & \delta(c^{(2)} - \psi^{(2)}) \times u_{\beta}^{(2)} \frac{\partial \theta^{(2)}}{\partial x_{\beta}^{(2)}} \frac{\partial}{\partial \phi^{(2)}} \delta(\theta^{(2)} - \phi^{(2)}) \rangle \quad (49) \\ & = \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \\ & \delta(c^{(2)} - \psi^{(2)}) \times u_{\beta}^{(2)} \frac{\partial}{\partial x_{\beta}^{(2)}} \delta(\theta^{(2)} - \phi^{(2)}) \rangle \end{aligned}$$

and 26<sup>th</sup> term,

$$\begin{aligned} & \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \\ & \delta(\theta^{(2)} - \phi^{(2)}) \times u_{\beta}^{(2)} \frac{\partial c^{(2)}}{\partial x_{\beta}^{(2)}} \frac{\partial}{\partial \psi^{(2)}} \delta(c^{(2)} - \psi^{(2)}) \rangle \\ & = \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \\ & \delta(\theta^{(2)} - \phi^{(2)}) \times u_{\beta}^{(2)} \frac{\partial}{\partial x_{\beta}^{(2)}} \delta(c^{(2)} - \psi^{(2)}) \rangle \quad (50) \end{aligned}$$

Adding these equations from (46) to (49), we get

$$\begin{aligned} & -\frac{\partial}{\partial x_{\beta}^{(2)}} \langle u_{\beta}^{(2)} \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \\ & \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \rangle \rangle \\ & = -v_{\beta}^{(2)} \frac{\partial F_2^{(1,2)}}{\partial x_{\beta}^{(2)}} \quad (51) \end{aligned}$$

Similarly, the terms 2<sup>nd</sup>, 8<sup>th</sup>, 16<sup>th</sup> and 22<sup>nd</sup> of right hand-side of equation (41) can be simplified as follows;  
2<sup>nd</sup> term,

$$\begin{aligned} & \langle -\delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\ & \delta(c^{(2)} - \psi^{(2)}) \times \frac{\partial h_{\alpha}^{(1)} h_{\beta}^{(1)}}{\partial x_{\beta}^{(1)}} \frac{\partial}{\partial v_{\alpha}^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle \\ & = -g_{\beta}^{(1)} \frac{\partial g_{\alpha}^{(1)}}{\partial v_{\alpha}^{(1)}} \frac{\partial F_2^{(1,2)}}{\partial x_{\beta}^{(1)}} \quad (52) \end{aligned}$$

8<sup>th</sup> term,

$$\begin{aligned} & \langle -\delta(u^{(1)} - v^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\ & \delta(c^{(2)} - \psi^{(2)}) \times \frac{\partial u_{\alpha}^{(1)} h_{\beta}^{(1)}}{\partial x_{\beta}^{(1)}} \frac{\partial}{\partial g_{\alpha}^{(1)}} \delta(h^{(1)} - g^{(1)}) \rangle \end{aligned}$$

$$= -g_{\beta}^{(1)} \frac{\partial v_{\alpha}^{(1)}}{\partial g_{\alpha}^{(1)}} \quad (53)$$

16<sup>th</sup> term,

$$\begin{aligned} & \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\ & \delta(c^{(2)} - \psi^{(2)}) \times \frac{\partial h_{\alpha}^{(2)} h_{\beta}^{(2)}}{\partial x_{\beta}^{(2)}} \frac{\partial}{\partial v_{\alpha}^{(2)}} \delta(u^{(2)} - v^{(2)}) \rangle \\ & = -g_{\beta}^{(2)} \frac{\partial g_{\alpha}^{(2)}}{\partial v_{\alpha}^{(2)}} \frac{\partial F_2^{(1,2)}}{\partial x_{\beta}^{(2)}} \end{aligned} \quad (54)$$

and 22<sup>nd</sup> term,

$$\begin{aligned} & \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\ & \delta(c^{(2)} - \psi^{(2)}) \times \frac{\partial u_{\alpha}^{(2)} h_{\beta}^{(2)}}{\partial x_{\beta}^{(2)}} \frac{\partial}{\partial g_{\alpha}^{(2)}} \delta(h^{(2)} - g^{(2)}) \rangle \\ & = -g_{\beta}^{(2)} \frac{\partial v_{\alpha}^{(2)}}{\partial g_{\alpha}^{(2)}} \frac{\partial F_2^{(1,2)}}{\partial x_{\beta}^{(2)}}. \end{aligned}$$

Fourth term can be reduced as

$$\begin{aligned} & \langle -\delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\ & \delta(c^{(2)} - \psi^{(2)}) \times \nabla^2 u_{\alpha}^{(1)} \frac{\partial}{\partial v_{\alpha}^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle \\ & = -v \frac{\partial}{\partial v_{\alpha}^{(1)}} \langle \nabla^2 u_{\alpha}^{(1)} [ \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \\ & \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) ] \rangle \\ & = -v \frac{\partial}{\partial v_{\alpha}^{(1)}} \frac{\partial^2}{\partial x_{\beta}^{(1)} \partial x_{\beta}^{(1)}} \langle u_{\alpha}^{(1)} [ \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \\ & \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) ] \rangle \\ & = -v \frac{\partial}{\partial v_{\alpha}^{(1)}} \lim_{\bar{x}^{(3)} \rightarrow \bar{x}^{(1)}} \frac{\partial^2}{\partial x_{\beta}^{(3)} \partial x_{\beta}^{(3)}} \langle u_{\alpha}^{(3)} [ \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \\ & \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) ] \rangle \\ & = -v \frac{\partial}{\partial v_{\alpha}^{(1)}} \lim_{\bar{x}^{(3)} \rightarrow \bar{x}^{(1)}} \frac{\partial^2}{\partial x_{\beta}^{(3)} \partial x_{\beta}^{(3)}} \langle \int u_{\alpha}^{(3)} \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \end{aligned}$$

$$\begin{aligned} & \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(1)} - v^{(1)}) \\ & \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) dv^{(3)} dg^{(3)} d\phi^{(3)} d\psi^{(3)} \rangle \\ & = -v \frac{\partial}{\partial v_{\alpha}^{(1)}} \lim_{\bar{x}(3) \rightarrow \bar{x}(1)} \frac{\partial^2}{\partial x_{\beta}^{(3)} \partial x_{\beta}^{(3)}} \int v_{\alpha}^{(3)} F_3^{(1,2,3)} dv^{(3)} dg^{(3)} d\phi^{(3)} d\psi^{(3)} \end{aligned} \quad (56)$$

Similarly, 9<sup>th</sup>, 11<sup>th</sup>, 13<sup>th</sup>, 18<sup>th</sup>, 23<sup>rd</sup>, 25<sup>th</sup> and 27<sup>th</sup> terms of right hand-side of equation (41) can be simplified as follows;  
9<sup>th</sup> term,

$$\begin{aligned} & \langle -\delta(u^{(1)} - v^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\ & \delta(c^{(2)} - \psi^{(2)}) \times \lambda \nabla^2 h_{\alpha}^{(1)} \frac{\partial}{\partial g_{\alpha}^{(1)}} \delta(h^{(1)} - g^{(1)}) \rangle \\ & = -\lambda \frac{\partial}{\partial g_{\alpha}^{(1)}} \lim_{\bar{x}(3) \rightarrow \bar{x}(1)} \frac{\partial^2}{\partial x_{\beta}^{(3)} \partial x_{\beta}^{(3)}} \int g_{\alpha}^{(3)} F_3^{(1,2,3)} dv^{(3)} dg^{(3)} d\phi^{(3)} d\psi^{(3)} \end{aligned} \quad (57)$$

11<sup>th</sup> term,

$$\begin{aligned} & \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\ & \delta(c^{(2)} - \psi^{(2)}) \times \gamma \nabla^2 \theta^{(1)} \frac{\partial}{\partial \phi^{(1)}} \delta(\theta^{(1)} - \phi^{(1)}) \rangle \\ & = -\gamma \frac{\partial}{\partial \phi^{(1)}} \lim_{\bar{x}(3) \rightarrow \bar{x}(1)} \frac{\partial^2}{\partial x_{\beta}^{(3)} \partial x_{\beta}^{(3)}} \int \phi^{(3)} F_3^{(1,2,3)} dv^{(3)} dg^{(3)} d\phi^{(3)} d\psi^{(3)} \end{aligned} \quad (58)$$

13<sup>th</sup> term,

$$\begin{aligned} & \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\ & \delta(c^{(2)} - \psi^{(2)}) \times D \nabla^2 c^{(1)} \frac{\partial}{\partial \psi^{(1)}} \delta(c^{(1)} - \psi^{(1)}) \rangle \\ & = -D \frac{\partial}{\partial \psi^{(1)}} \lim_{\bar{x}(3) \rightarrow \bar{x}(1)} \frac{\partial^2}{\partial x_{\beta}^{(3)} \partial x_{\beta}^{(3)}} \int \psi^{(3)} F_3^{(1,2,3)} dv^{(3)} dg^{(3)} d\phi^{(3)} d\psi^{(3)} \end{aligned} \quad (59)$$

18<sup>th</sup> term,

$$\begin{aligned} & \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\ & \delta(c^{(2)} - \psi^{(2)}) \times v \nabla^2 u_{\alpha}^{(2)} \frac{\partial}{\partial v_{\alpha}^{(2)}} \delta(u^{(2)} - v^{(2)}) \rangle \\ & = -v \frac{\partial}{\partial v_{\alpha}^{(2)}} \lim_{\bar{x}(3) \rightarrow \bar{x}(2)} \frac{\partial^2}{\partial x_{\beta}^{(3)} \partial x_{\beta}^{(3)}} \int v_{\alpha}^{(3)} F_3^{(1,2,3)} dv^{(3)} dg^{(3)} d\phi^{(3)} d\psi^{(3)} \end{aligned} \quad (60)$$

23<sup>rd</sup> term,

$$\begin{aligned} & \langle -\delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(\theta^{(2)} - \phi^{(2)}) \\ & \delta(c^{(2)} - \psi^{(2)}) \times \lambda \nabla^2 h_\alpha^{(2)} \frac{\partial}{\partial g_\alpha^{(2)}} \delta(h^{(2)} - g^{(2)}) \rangle \\ & = -\lambda \frac{\partial}{\partial g_\alpha^{(2)}} \lim_{\bar{x}^{(3)} \rightarrow \bar{x}^{(2)}} \frac{\partial^2}{\partial x_\beta^{(3)} \partial x_\beta^{(3)}} \int g_\alpha^{(3)} F_3^{(1,2,3)} dv^{(3)} dg^{(3)} d\phi^{(3)} d\psi^{(3)} \end{aligned} \quad (61)$$

25<sup>th</sup> term,

$$\begin{aligned} & \langle -\delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(h^{(2)} - g^{(2)}) \\ & \delta(c^{(2)} - \psi^{(2)}) \times \gamma \nabla^2 \theta^{(2)} \frac{\partial}{\partial \phi^{(2)}} \delta(\theta^{(2)} - \phi^{(2)}) \rangle \\ & = -\gamma \frac{\partial}{\partial \phi^{(2)}} \lim_{\bar{x}^{(3)} \rightarrow \bar{x}^{(2)}} \frac{\partial^2}{\partial x_\beta^{(3)} \partial x_\beta^{(3)}} \int \phi^{(3)} F_3^{(1,2,3)} dv^{(3)} dg^{(3)} d\phi^{(3)} d\psi^{(3)} \end{aligned} \quad (62)$$

27<sup>th</sup> term,

$$\begin{aligned} & \langle -\delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(h^{(2)} - g^{(2)}) \\ & \delta(\theta^{(2)} - \phi^{(2)}) \times D \nabla^2 c^{(2)} \frac{\partial}{\partial \psi^{(2)}} \delta(c^{(2)} - \psi^{(2)}) \rangle \\ & = -D \frac{\partial}{\partial \psi^{(2)}} \lim_{\bar{x}^{(3)} \rightarrow \bar{x}^{(2)}} \frac{\partial^2}{\partial x_\beta^{(3)} \partial x_\beta^{(3)}} \int \psi^{(3)} F_3^{(1,2,3)} dv^{(3)} dg^{(3)} d\phi^{(3)} d\psi^{(3)} \end{aligned} \quad (63)$$

We reduce the third term of right - hand side of equation (41),

$$\begin{aligned} & \langle \delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(h^{(2)} - g^{(2)})\delta(\theta^{(2)} - \phi^{(2)}) \\ & \delta(c^{(2)} - \psi^{(2)}) \times \frac{1}{4\pi} \frac{\partial}{\partial x_\alpha^{(1)}} \int \left[ \frac{\partial u_\alpha^{(1)}}{\partial x_\beta^{(1)}} \frac{\partial u_\beta^{(1)}}{\partial x_\alpha^{(1)}} - \frac{\partial h_\alpha^{(1)}}{\partial x_\beta^{(1)}} \frac{\partial h_\beta^{(1)}}{\partial x_\alpha^{(1)}} \right] \frac{d\bar{x}''}{|\bar{x}'' - \bar{x}|} \frac{\partial}{\partial v_\alpha^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle \\ & = \frac{\partial}{\partial v_\alpha^{(1)}} \left[ \frac{1}{4\pi} \int \frac{\partial}{\partial x_\alpha^{(1)}} \left( \frac{1}{|\bar{x}^{(3)} - \bar{x}^{(1)}|} \right) \left( \frac{\partial v_\alpha^{(3)}}{\partial x_\beta^{(3)}} \frac{\partial v_\beta^{(3)}}{\partial x_\alpha^{(3)}} - \frac{\partial g_\alpha^{(3)}}{\partial x_\beta^{(3)}} \frac{\partial g_\beta^{(3)}}{\partial x_\alpha^{(3)}} \right) F_3^{(1,2,3)} \times d\bar{x}^{(3)} dv^{(3)} dg^{(3)} d\phi^{(3)} d\psi^{(3)} \right] \end{aligned} \quad (64)$$

Similarly, term 17<sup>th</sup>,

$$\begin{aligned} & \langle \delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(h^{(2)} - g^{(2)})\delta(\theta^{(2)} - \phi^{(2)}) \\ & \delta(c^{(2)} - \psi^{(2)}) \times \frac{1}{4\pi} \frac{\partial}{\partial x_\alpha^{(2)}} \int \left[ \frac{\partial u_\alpha^{(2)}}{\partial x_\beta^{(2)}} \frac{\partial u_\beta^{(2)}}{\partial x_\alpha^{(2)}} - \frac{\partial h_\alpha^{(2)}}{\partial x_\beta^{(2)}} \frac{\partial h_\beta^{(2)}}{\partial x_\alpha^{(2)}} \right] \frac{d\bar{x}''}{|\bar{x}'' - \bar{x}|} \frac{\partial}{\partial v_\alpha^{(2)}} \delta(u^{(2)} - v^{(2)}) \rangle \\ & = \frac{\partial}{\partial v_\alpha^{(2)}} \left[ \frac{1}{4\pi} \int \frac{\partial}{\partial x_\alpha^{(2)}} \left( \frac{1}{|\bar{x}^{(3)} - \bar{x}^{(2)}|} \right) \left( \frac{\partial v_\alpha^{(3)}}{\partial x_\beta^{(3)}} \frac{\partial v_\beta^{(3)}}{\partial x_\alpha^{(3)}} - \frac{\partial g_\alpha^{(3)}}{\partial x_\beta^{(3)}} \frac{\partial g_\beta^{(3)}}{\partial x_\alpha^{(3)}} \right) F_3^{(1,2,3)} \times d\bar{x}^{(3)} dv^{(3)} dg^{(3)} d\phi^{(3)} d\psi^{(3)} \right] \end{aligned} \quad (65)$$

5<sup>th</sup> and 6<sup>th</sup> terms of right hand side of equation (41), we get

5<sup>th</sup> term,

$$\begin{aligned}
 & \langle \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \times 2 \in_{m\alpha\beta} \Omega_m u_\alpha^{(1)} \frac{\partial}{\partial v_\alpha^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle \\
 & = \langle 2 \in_{m\alpha\beta} \Omega_m u_\alpha^{(1)} \frac{\partial}{\partial v_\alpha^{(1)}} [ \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \\
 & \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) ] \rangle \\
 & = 2 \in_{m\alpha\beta} \Omega_m \frac{\partial}{\partial v_\alpha^{(1)}} \langle u_\alpha^{(1)} \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \\
 & \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \rangle \\
 & = 2 \in_{m\alpha\beta} \Omega_m \frac{\partial u_\alpha^{(1)}}{\partial v_\alpha^{(1)}} \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \\
 & \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \rangle \\
 & = 2 \in_{m\alpha\beta} \Omega_m F_2^{(1,2)}
 \end{aligned} \tag{66}$$

and 6<sup>th</sup> term,

$$\begin{aligned}
 & \langle -\delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \times f(u_\alpha^{(1)} - v_\alpha^{(1)}) \frac{\partial}{\partial v_\alpha^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle \\
 & = -\langle f(u_\alpha^{(1)} - v_\alpha^{(1)}) \frac{\partial}{\partial v_\alpha^{(1)}} [ \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \\
 & \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) ] \rangle \\
 & = -f(u_\alpha^{(1)} - v_\alpha^{(1)}) \frac{\partial}{\partial v_\alpha^{(1)}} \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \\
 & \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \rangle \\
 & = -f(u_\alpha^{(1)} - v_\alpha^{(1)}) \frac{\partial}{\partial v_\alpha^{(1)}} F_2^{(1,2)}
 \end{aligned} \tag{67}$$

Similarly, 19<sup>th</sup> and 20<sup>th</sup> terms of right hand side of equation (41),  
19<sup>th</sup> term,

$$\begin{aligned}
 & \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \times 2 \in_{m\alpha\beta} \Omega_m u_\alpha^{(2)} \frac{\partial}{\partial v_\alpha^{(2)}} \delta(u^{(2)} - v^{(2)}) \rangle \\
 & = 2 \in_{m\alpha\beta} \Omega_m F_2^{(1,2)}
 \end{aligned} \tag{68}$$

and 20<sup>th</sup> term,

$$\begin{aligned} & \langle -\delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(h^{(2)} - g^{(2)})\delta(\theta^{(2)} - \phi^{(2)}) \\ & \delta(c^{(2)} - \psi^{(2)}) \times f(u_\alpha^{(2)} - v_\alpha^{(2)}) \frac{\partial}{\partial v_\alpha^{(2)}} \delta(u^{(2)} - v^{(2)}) \rangle \\ & = -f(u_\alpha^{(2)} - v_\alpha^{(2)}) \frac{\partial}{\partial v_\alpha^{(2)}} F_2^{(1,2)} \end{aligned} \quad (69)$$

14<sup>th</sup> and 28<sup>th</sup> terms of equation (41)

$$\begin{aligned} & \langle \delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(h^{(2)} - g^{(2)})\delta(\theta^{(2)} - \phi^{(2)}) \\ & \delta(c^{(2)} - \psi^{(2)}) \times Rc^{(1)} \frac{\partial}{\partial \psi^{(1)}} \delta(c^{(1)} - \psi^{(1)}) \rangle \\ & = R\psi^{(1)} \frac{\partial}{\partial \psi^{(1)}} F_2^{(1,2)} \end{aligned} \quad (70)$$

And 28<sup>th</sup> term

$$\begin{aligned} & \langle -\delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(h^{(2)} - g^{(2)}) \\ & \delta(\theta^{(2)} - \phi^{(2)}) \times Rc^{(2)} \frac{\partial}{\partial \psi^{(2)}} \delta(c^{(2)} - \psi^{(2)}) \rangle \\ & = R\psi^{(2)} \frac{\partial}{\partial \psi^{(2)}} F_2^{(1,2)} \end{aligned} \quad (71)$$

Substituting the results (42) – (71) in equation (41) we get the transport equation for two point distribution function  $F_2^{(1,2)}(v, g, \phi, \psi)$  in MHD turbulent flow in a rotating system in presence of dust particles as

$$\begin{aligned} & \frac{\partial F_2^{(1,2)}}{\partial t} + \left( v_\beta^{(1)} \frac{\partial}{\partial x_\beta^{(1)}} + v_\beta^{(2)} \frac{\partial}{\partial x_\beta^{(2)}} \right) F_2^{(1,2)} + g_\beta^{(1)} \left( \frac{\partial g_\alpha^{(1)}}{\partial v_\alpha^{(1)}} + \frac{\partial v_\alpha^{(1)}}{\partial g_\alpha^{(1)}} \right) \frac{\partial}{\partial x_\beta^{(1)}} F_2^{(1,2)} \\ & + g_\beta^{(2)} \left( \frac{\partial g_\alpha^{(2)}}{\partial v_\alpha^{(2)}} + \frac{\partial v_\alpha^{(2)}}{\partial g_\alpha^{(2)}} \right) \frac{\partial}{\partial x_\beta^{(2)}} F_2^{(1,2)} - \frac{\partial}{\partial v_\alpha^{(1)}} \left[ \frac{1}{4\pi} \int \frac{\partial}{\partial x_\alpha^{(1)}} \left( \frac{1}{|\bar{x}^{(3)} - \bar{x}^{(1)}|} \right) \right. \\ & \times \left( \frac{\partial v_\alpha^{(3)}}{\partial x_\beta^{(3)}} \frac{\partial v_\beta^{(3)}}{\partial x_\alpha^{(3)}} - \frac{\partial g_\alpha^{(3)}}{\partial x_\beta^{(3)}} \frac{\partial g_\beta^{(3)}}{\partial x_\alpha^{(3)}} \right) F_3^{(1,2,3)} dx^{(3)} dv^{(3)} dg^{(3)} d\phi^{(3)} d\psi^{(3)} \left. \right] \\ & - \frac{\partial}{\partial v_\alpha^{(2)}} \left[ \frac{1}{4\pi} \int \frac{\partial}{\partial x_\alpha^{(2)}} \left( \frac{1}{|\bar{x}^{(3)} - \bar{x}^{(2)}|} \right) \left( \frac{\partial v_\alpha^{(3)}}{\partial x_\beta^{(3)}} \frac{\partial v_\beta^{(3)}}{\partial x_\alpha^{(3)}} - \frac{\partial g_\alpha^{(3)}}{\partial x_\beta^{(3)}} \frac{\partial g_\beta^{(3)}}{\partial x_\alpha^{(3)}} \right) \right. \\ & \times F_3^{(1,2,3)} dx^{(3)} dv^{(3)} dg^{(3)} d\phi^{(3)} d\psi^{(3)} \left. \right] \\ & + v \left( \frac{\partial}{\partial v_\alpha^{(1)}} \lim_{\bar{x}^{(3)} \rightarrow \bar{x}^{(1)}} + \frac{\partial}{\partial v_\alpha^{(2)}} \lim_{\bar{x}^{(3)} \rightarrow \bar{x}^{(2)}} \right) \frac{\partial^2}{\partial x_\beta^{(3)} \partial x_\beta^{(3)}} \int v_\alpha^{(3)} F_3^{(1,2,3)} dv^{(3)} dg^{(3)} d\phi^{(3)} d\psi^{(3)} \\ & + \lambda \left( \frac{\partial}{\partial g_\alpha^{(1)}} \lim_{\bar{x}^{(3)} \rightarrow \bar{x}^{(1)}} + \frac{\partial}{\partial g_\alpha^{(2)}} \lim_{\bar{x}^{(3)} \rightarrow \bar{x}^{(2)}} \right) \frac{\partial^2}{\partial x_\beta^{(3)} \partial x_\beta^{(3)}} \int g_\alpha^{(3)} F_3^{(1,2,3)} dv^{(3)} dg^{(3)} d\phi^{(3)} d\psi^{(3)} \end{aligned}$$

$$\begin{aligned}
 & + \gamma \left( \frac{\partial}{\partial \phi^{(1)}} \lim_{\bar{x}^{(3)} \rightarrow \bar{x}^{(1)}} + \frac{\partial}{\partial \phi^{(2)}} \lim_{\bar{x}^{(3)} \rightarrow \bar{x}^{(2)}} \right) \frac{\partial^2}{\partial x_\beta^{(3)} \partial x_\beta^{(3)}} \int \phi^{(3)} F_3^{(1,2,3)} dv^{(3)} dg^{(3)} d\phi^{(3)} d\psi^{(3)} \\
 & + D \left( \frac{\partial}{\partial \psi^{(1)}} \lim_{\bar{x}^{(3)} \rightarrow \bar{x}^{(1)}} + \frac{\partial}{\partial \psi^{(2)}} \lim_{\bar{x}^{(3)} \rightarrow \bar{x}^{(2)}} \right) \frac{\partial^2}{\partial x_\beta^{(3)} \partial x_\beta^{(3)}} \int \psi^{(3)} F_3^{(1,2,3)} dv^{(3)} dg^{(3)} d\phi^{(3)} d\psi^{(3)} \\
 & + 4 \epsilon_{m\alpha\beta} \Omega_m F_2^{(1,2)} + \left[ f(u_a^{(1)} - v_a^{(1)}) \frac{\partial}{\partial v_a^{(1)}} + f(u_a^{(2)} - v_a^{(2)}) \frac{\partial}{\partial v_a^{(2)}} \right] F_2^{(1,2)} \\
 & - R \psi^{(1)} \frac{\partial}{\partial \psi^{(1)}} F_2^{(1,2)} - R \psi^{(2)} \frac{\partial}{\partial \psi^{(2)}} F_2^{(1,2)} = 0
 \end{aligned} \tag{72}$$

Equations for three-point distribution function  $F_3^{(1,2,3)}$ :

Differentiating equation (14) with respect to time, we get

$$\begin{aligned}
 \frac{\partial F_3^{(1,2,3)}}{\partial t} &= \frac{\partial}{\partial t} \left\langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \right. \\
 & \quad \left. \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \right\rangle \\
 &= \left\langle \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \right. \\
 & \quad \left. \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \frac{\partial}{\partial t} \delta(u^{(1)} - v^{(1)}) \right\rangle \\
 &+ \left\langle \delta(u^{(1)} - v^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \right. \\
 & \quad \left. \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \frac{\partial}{\partial t} \delta(h^{(1)} - g^{(1)}) \right\rangle \\
 &+ \left\langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \right. \\
 & \quad \left. \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \frac{\partial}{\partial t} \delta(\theta^{(1)} - \phi^{(1)}) \right\rangle \\
 &+ \left\langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \right. \\
 & \quad \left. \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \frac{\partial}{\partial t} \delta(c^{(1)} - \psi^{(1)}) \right\rangle \\
 &+ \left\langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \right. \\
 & \quad \left. \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \frac{\partial}{\partial t} \delta(u^{(2)} - v^{(2)}) \right\rangle \\
 &+ \left\langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \right. \\
 & \quad \left. \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \frac{\partial}{\partial t} \delta(h^{(2)} - g^{(2)}) \right\rangle \\
 &+ \left\langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \right. \\
 & \quad \left. \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \frac{\partial}{\partial t} \delta(u^{(2)} - v^{(2)}) \right\rangle
 \end{aligned}$$

$$\begin{aligned}
 & \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \frac{\partial}{\partial t} \delta(\theta^{(2)} - \phi^{(2)}) \rangle \\
 & + \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \\
 & \delta(\theta^{(2)} - \phi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \frac{\partial}{\partial t} \delta(c^{(2)} - \psi^{(2)}) \rangle \\
 & + \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \\
 & \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \frac{\partial}{\partial t} \delta(u^{(3)} - v^{(3)}) \rangle \\
 & + \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \\
 & \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \frac{\partial}{\partial t} \delta(h^{(3)} - g^{(3)}) \rangle \\
 & + \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \\
 & \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \frac{\partial}{\partial t} \delta(\theta^{(3)} - \phi^{(3)}) \rangle \\
 & + \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \\
 & \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \frac{\partial}{\partial t} \delta(c^{(3)} - \psi^{(3)}) \rangle \\
 & = \langle -\delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \frac{\partial u^{(1)}}{\partial t} \frac{\partial}{\partial v^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\
 & \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \frac{\partial h^{(1)}}{\partial t} \frac{\partial}{\partial g^{(1)}} \delta(h^{(1)} - g^{(1)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\
 & \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \frac{\partial \theta^{(1)}}{\partial t} \frac{\partial}{\partial \phi^{(1)}} \delta(\theta^{(1)} - \phi^{(1)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\
 & \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \frac{\partial c^{(1)}}{\partial t} \frac{\partial}{\partial \psi^{(1)}} \delta(c^{(1)} - \psi^{(1)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\
 & \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \frac{\partial u^{(2)}}{\partial t} \frac{\partial}{\partial v^{(2)}} \delta(u^{(2)} - v^{(2)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\
 & \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \frac{\partial h^{(2)}}{\partial t} \frac{\partial}{\partial g^{(2)}} \delta(h^{(2)} - g^{(2)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(c^{(2)} - \psi^{(2)})
 \end{aligned}$$

$$\begin{aligned}
 & \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \frac{\partial \theta^{(2)}}{\partial t} \frac{\partial}{\partial \phi^{(2)}} \delta(\theta^{(2)} - \phi^{(2)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \frac{\partial c^{(2)}}{\partial t} \frac{\partial}{\partial \psi^{(2)}} \delta(c^{(2)} - \psi^{(2)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \frac{\partial u^{(3)}}{\partial t} \frac{\partial}{\partial v^{(3)}} \delta(u^{(3)} - v^{(3)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \frac{\partial h^{(3)}}{\partial t} \frac{\partial}{\partial g^{(3)}} \delta(h^{(3)} - g^{(3)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \frac{\partial \theta^{(3)}}{\partial t} \frac{\partial}{\partial \phi^{(3)}} \delta(\theta^{(3)} - \phi^{(3)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \frac{\partial c^{(3)}}{\partial t} \frac{\partial}{\partial \psi^{(3)}} \delta(c^{(3)} - \psi^{(3)}) \rangle
 \end{aligned}$$

Using equations (8) to (11), we get from the above equation

$$\begin{aligned}
 & \frac{\partial F_3^{(1,2,3)}}{\partial t} = \langle -\delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \rangle \\
 & \{ -\frac{\partial}{\partial x_\beta^{(1)}} (u_\alpha^{(1)} u_\beta^{(1)} - h_\alpha^{(1)} h_\beta^{(1)}) - \frac{1}{4\pi} \frac{\partial}{\partial x_\alpha^{(1)}} \int [ \frac{\partial u_\alpha^{(1)}}{\partial x_\beta^{(1)}} \frac{\partial u_\beta^{(1)}}{\partial x_\alpha^{(1)}} - \frac{\partial h_\alpha^{(1)}}{\partial x_\beta^{(1)}} \frac{\partial h_\beta^{(1)}}{\partial x_\alpha^{(1)}} ] \frac{d\bar{x}''}{|\bar{x}'' - \bar{x}''|} \\
 & + \nu \nabla^2 u_\alpha^{(1)} - 2 \epsilon_{m\alpha\beta} \Omega_m u_\alpha^{(1)} + f(u_\alpha^{(1)} - v_\alpha^{(1)}) \} \times \frac{\partial}{\partial v_\alpha^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \rangle \\
 & \{ -\frac{\partial}{\partial x_\beta^{(1)}} (h_\alpha^{(1)} u_\beta^{(1)} - u_\alpha^{(1)} h_\beta^{(1)}) + \lambda \nabla^2 h_\alpha^{(1)} \} \times \frac{\partial}{\partial g_\alpha^{(1)}} \delta(h^{(1)} - g^{(1)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \rangle \\
 & \times \{ -u_\beta^{(1)} \frac{\partial \theta^{(1)}}{\partial x_\beta^{(1)}} + \gamma \nabla^2 \theta^{(1)} \} \times \frac{\partial}{\partial \phi^{(1)}} \delta(\theta^{(1)} - \phi^{(1)}) \rangle
 \end{aligned}$$

$$\begin{aligned}
 & + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \\
 & \left\{ -u_{\beta}^{(1)} \frac{\partial c^{(1)}}{\partial x_{\beta}^{(1)}} + D \nabla^2 c^{(1)} - R c^{(1)} \right\} \times \frac{\partial}{\partial \psi^{(1)}} \delta(c^{(1)} - \psi^{(1)}) \rangle \\
 & \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \\
 & \left\{ -\frac{\partial}{\partial x_{\beta}^{(2)}} (u_{\alpha}^{(2)} u_{\beta}^{(2)} - h_{\alpha}^{(2)} h_{\beta}^{(2)}) - \frac{1}{4\pi} \frac{\partial}{\partial x_{\alpha}^{(2)}} \int \left[ \frac{\partial u_{\alpha}^{(2)}}{\partial x_{\beta}^{(2)}} \frac{\partial u_{\beta}^{(2)}}{\partial x_{\alpha}^{(2)}} - \frac{\partial h_{\alpha}^{(2)}}{\partial x_{\beta}^{(2)}} \frac{\partial h_{\beta}^{(2)}}{\partial x_{\alpha}^{(2)}} \right] \frac{d\bar{x}'''}{|\bar{x}''' - \bar{x}|} \right. \\
 & \quad \left. + \nu \nabla^2 u_{\alpha}^{(2)} - 2 \epsilon_{m\alpha\beta} \Omega_m u_{\alpha}^{(2)} + f(u_{\alpha}^{(2)} - v_{\alpha}^{(2)}) \right\} \times \frac{\partial}{\partial v_{\alpha}^{(2)}} \delta(u^{(2)} - v^{(2)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \\
 & + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \\
 & \delta(\theta^{(2)} - \phi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \\
 & \left\{ -u_{\beta}^{(2)} \frac{\partial c^{(2)}}{\partial x_{\beta}^{(2)}} + D \nabla^2 c^{(2)} - R c^{(2)} \right\} \times \frac{\partial}{\partial \psi^{(2)}} \delta(c^{(2)} - \psi^{(2)}) \rangle \\
 & \left\{ -\frac{\partial}{\partial x_{\beta}^{(2)}} (h_{\alpha}^{(2)} u_{\beta}^{(2)} - u_{\alpha}^{(2)} h_{\beta}^{(2)}) + \lambda \nabla^2 h_{\alpha}^{(2)} \right\} \times \frac{\partial}{\partial g_{\alpha}^{(2)}} \delta(h^{(2)} - g^{(2)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \\
 & \times \left\{ -u_{\beta}^{(2)} \frac{\partial \theta^{(2)}}{\partial x_{\beta}^{(2)}} + \gamma \nabla^2 \theta^{(2)} \right\} \times \frac{\partial}{\partial \phi^{(2)}} \delta(\theta^{(2)} - \phi^{(2)}) \rangle \\
 & \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \\
 & \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \\
 & \left\{ -\frac{\partial}{\partial x_{\beta}^{(3)}} (u_{\alpha}^{(3)} u_{\beta}^{(3)} - h_{\alpha}^{(3)} h_{\beta}^{(3)}) - \frac{1}{4\pi} \frac{\partial}{\partial x_{\alpha}^{(3)}} \int \left[ \frac{\partial u_{\alpha}^{(3)}}{\partial x_{\beta}^{(3)}} \frac{\partial u_{\beta}^{(3)}}{\partial x_{\alpha}^{(3)}} - \frac{\partial h_{\alpha}^{(3)}}{\partial x_{\beta}^{(3)}} \frac{\partial h_{\beta}^{(3)}}{\partial x_{\alpha}^{(3)}} \right] \frac{d\bar{x}'''}{|\bar{x}''' - \bar{x}|} \right. \\
 & \quad \left. + \nu \nabla^2 u_{\alpha}^{(3)} - 2 \epsilon_{m\alpha\beta} \Omega_m u_{\alpha}^{(3)} + f(u_{\alpha}^{(3)} - v_{\alpha}^{(3)}) \right\} \times \frac{\partial}{\partial v_{\alpha}^{(3)}} \delta(u^{(3)} - v^{(3)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \\
 & \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \\
 & \left\{ -\frac{\partial}{\partial x_{\beta}^{(3)}} (h_{\alpha}^{(3)} u_{\beta}^{(3)} - u_{\alpha}^{(3)} h_{\beta}^{(3)}) + \lambda \nabla^2 h_{\alpha}^{(3)} \right\} \times \frac{\partial}{\partial g_{\alpha}^{(3)}} \delta(h^{(3)} - g^{(3)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \\
 & \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)})
 \end{aligned}$$

$$\begin{aligned}
 & \times \left\{ -u_{\beta}^{(3)} \frac{\partial \theta^{(3)}}{\partial x_{\beta}^{(3)}} + \gamma \nabla^2 \theta^{(3)} \right\} \times \frac{\partial}{\partial \phi^{(3)}} \delta(\theta^{(3)} - \phi^{(3)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \\
 & \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \rangle \\
 & \left\{ -u_{\beta}^{(3)} \frac{\partial c^{(3)}}{\partial x_{\beta}^{(3)}} + D \nabla^2 c^{(3)} - R c^{(3)} \right\} \frac{\partial}{\partial \psi^{(3)}} \delta(c^{(3)} - \psi^{(3)}) \rangle \\
 & = \langle \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\
 & \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times \frac{\partial u_{\alpha}^{(1)} u_{\beta}^{(1)}}{\partial x_{\beta}^{(1)}} \frac{\partial}{\partial v_{\alpha}^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle \\
 & + \langle -\delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\
 & \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times \frac{\partial h_{\alpha}^{(1)} h_{\beta}^{(1)}}{\partial x_{\beta}^{(1)}} \frac{\partial}{\partial v_{\alpha}^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle \\
 & + \langle \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\
 & \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times \frac{1}{4\pi} \frac{\partial}{\partial x_{\alpha}^{(1)}} \int \left[ \frac{\partial u_{\alpha}^{(1)}}{\partial x_{\beta}^{(1)}} \frac{\partial u_{\beta}^{(1)}}{\partial x_{\alpha}^{(1)}} - \frac{\partial h_{\alpha}^{(1)}}{\partial x_{\beta}^{(1)}} \frac{\partial h_{\beta}^{(1)}}{\partial x_{\alpha}^{(1)}} \right] \\
 & \times \frac{d\bar{x}'''}{|\bar{x}''' - \bar{x}|} \frac{\partial}{\partial v_{\alpha}^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle + \langle -\delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \\
 & \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times \gamma \nabla^2 u_{\alpha}^{(1)} \frac{\partial}{\partial v_{\alpha}^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle \\
 & + \langle \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\
 & \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times 2 \epsilon_{m\alpha\beta} \Omega_m u_{\alpha}^{(1)} \frac{\partial}{\partial v_{\alpha}^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle \\
 & + \langle -\delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\
 & \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times f(u_{\alpha}^{(1)} - v_{\alpha}^{(1)}) \frac{\partial}{\partial v_{\alpha}^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle \\
 & + \langle \delta(u^{(1)} - v^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\
 & \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times \frac{\partial h_{\alpha}^{(1)} u_{\beta}^{(1)}}{\partial x_{\beta}^{(1)}} \frac{\partial}{\partial g_{\alpha}^{(1)}} \delta(h^{(1)} - g^{(1)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\
 & \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times \frac{\partial u_{\alpha}^{(1)} h_{\beta}^{(1)}}{\partial x_{\beta}^{(1)}} \frac{\partial}{\partial g_{\alpha}^{(1)}} \delta(h^{(1)} - g^{(1)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\
 & \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times \lambda \nabla^2 h_{\alpha}^{(1)} \frac{\partial}{\partial g_{\alpha}^{(1)}} \delta(h^{(1)} - g^{(1)}) \rangle \\
 & + \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\
 & \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times u_{\beta}^{(1)} \frac{\partial \theta^{(1)}}{\partial x_{\beta}^{(1)}} \frac{\partial}{\partial \phi^{(1)}} \delta(\theta^{(1)} - \phi^{(1)}) \rangle
 \end{aligned}$$

$$\begin{aligned}
 & + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\
 & \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times \gamma \nabla^2 \theta^{(1)} \frac{\partial}{\partial \phi^{(1)}} \delta(\theta^{(1)} - \phi^{(1)}) \rangle \\
 & + \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\
 & \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times u_{\beta}^{(1)} \frac{\partial c^{(1)}}{\partial x_{\beta}^{(1)}} \frac{\partial}{\partial \psi^{(1)}} \delta(c^{(1)} - \psi^{(1)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\
 & \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times D \nabla^2 c^{(1)} \frac{\partial}{\partial \psi^{(1)}} \delta(c^{(1)} - \psi^{(1)}) \rangle \\
 & + \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\
 & \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times Rc^{(1)} \frac{\partial}{\partial \psi^{(1)}} \delta(c^{(1)} - \psi^{(1)}) \rangle \\
 & + \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\
 & \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times \frac{\partial u_{\alpha}^{(2)} u_{\beta}^{(2)}}{\partial x_{\beta}^{(2)}} \frac{\partial}{\partial v_{\alpha}^{(2)}} \delta(u^{(2)} - v^{(2)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\
 & \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times \frac{\partial h_{\alpha}^{(2)} h_{\beta}^{(2)}}{\partial x_{\beta}^{(2)}} \frac{\partial}{\partial v_{\alpha}^{(2)}} \delta(u^{(2)} - v^{(2)}) \rangle \\
 & + \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\
 & \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times \frac{1}{4\pi} \frac{\partial}{\partial x_{\alpha}^{(2)}} \int \left[ \frac{\partial u_{\alpha}^{(2)}}{\partial x_{\beta}^{(2)}} \frac{\partial u_{\beta}^{(2)}}{\partial x_{\alpha}^{(2)}} - \frac{\partial h_{\alpha}^{(2)}}{\partial x_{\beta}^{(2)}} \frac{\partial h_{\beta}^{(2)}}{\partial x_{\alpha}^{(2)}} \right] \\
 & \times \frac{d\bar{x}'''}{|\bar{x}''' - \bar{x}'|} \frac{\partial}{\partial v_{\alpha}^{(2)}} \delta(u^{(2)} - v^{(2)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\
 & \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times \nu \nabla^2 u_{\alpha}^{(2)} \frac{\partial}{\partial v_{\alpha}^{(2)}} \delta(u^{(2)} - v^{(2)}) \rangle \\
 & + \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\
 & \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times 2 \epsilon_{m\alpha\beta} \Omega_m u_{\alpha}^{(2)} \frac{\partial}{\partial v_{\alpha}^{(2)}} \delta(u^{(2)} - v^{(2)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\
 & \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times f(u_{\alpha}^{(2)} - v_{\alpha}^{(2)}) \frac{\partial}{\partial v_{\alpha}^{(2)}} \delta(u^{(2)} - v^{(2)}) \rangle \\
 & + \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\
 & \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times \frac{\partial h_{\alpha}^{(2)} u_{\beta}^{(2)}}{\partial x_{\beta}^{(2)}} \frac{\partial}{\partial g_{\alpha}^{(2)}} \delta(h^{(2)} - g^{(2)}) \rangle
 \end{aligned}$$

$$\begin{aligned}
 & + \langle -\delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(\theta^{(2)} - \phi^{(2)})\delta(c^{(2)} - \psi^{(2)}) \\
 & \delta(u^{(3)} - v^{(3)})\delta(h^{(3)} - g^{(3)})\delta(\theta^{(3)} - \phi^{(3)})\delta(c^{(3)} - \psi^{(3)}) \times \frac{\partial u_{\alpha}^{(2)} h_{\beta}^{(2)}}{\partial x_{\beta}^{(2)}} \frac{\partial}{\partial g_{\alpha}^{(2)}} \delta(h^{(2)} - g^{(2)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(\theta^{(2)} - \phi^{(2)})\delta(c^{(2)} - \psi^{(2)}) \\
 & \delta(u^{(3)} - v^{(3)})\delta(h^{(3)} - g^{(3)})\delta(\theta^{(3)} - \phi^{(3)})\delta(c^{(3)} - \psi^{(3)}) \times \lambda \nabla^2 h_{\alpha}^{(2)} \frac{\partial}{\partial g_{\alpha}^{(2)}} \delta(h^{(2)} - g^{(2)}) \rangle \\
 & + \langle \delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(h^{(2)} - g^{(2)})\delta(c^{(2)} - \psi^{(2)}) \\
 & \delta(u^{(3)} - v^{(3)})\delta(h^{(3)} - g^{(3)})\delta(\theta^{(3)} - \phi^{(3)})\delta(c^{(3)} - \psi^{(3)}) \times u_{\beta}^{(2)} \frac{\partial \theta^{(2)}}{\partial x_{\beta}^{(2)}} \frac{\partial}{\partial \phi^{(2)}} \delta(\theta^{(2)} - \phi^{(2)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(h^{(2)} - g^{(2)})\delta(c^{(2)} - \psi^{(2)}) \\
 & \delta(u^{(3)} - v^{(3)})\delta(h^{(3)} - g^{(3)})\delta(\theta^{(3)} - \phi^{(3)})\delta(c^{(3)} - \psi^{(3)}) \times \gamma \nabla^2 \theta^{(2)} \frac{\partial}{\partial \phi^{(2)}} \delta(\theta^{(2)} - \phi^{(2)}) \rangle \\
 & + \langle \delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(h^{(2)} - g^{(2)})\delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(u^{(3)} - v^{(3)})\delta(h^{(3)} - g^{(3)})\delta(\theta^{(3)} - \phi^{(3)})\delta(c^{(3)} - \psi^{(3)}) \times u_{\beta}^{(2)} \frac{\partial c^{(2)}}{\partial x_{\beta}^{(2)}} \frac{\partial}{\partial \psi^{(2)}} \delta(c^{(2)} - \psi^{(2)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(h^{(2)} - g^{(2)})\delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(u^{(3)} - v^{(3)})\delta(h^{(3)} - g^{(3)})\delta(\theta^{(3)} - \phi^{(3)})\delta(c^{(3)} - \psi^{(3)}) \times D \nabla^2 c^{(2)} \frac{\partial}{\partial \psi^{(2)}} \delta(c^{(2)} - \psi^{(2)}) \rangle \\
 & + \delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(h^{(2)} - g^{(2)})\delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(u^{(3)} - v^{(3)})\delta(h^{(3)} - g^{(3)})\delta(\theta^{(3)} - \phi^{(3)})\delta(c^{(3)} - \psi^{(3)}) \times Rc^{(2)} \frac{\partial}{\partial \psi^{(2)}} \delta(c^{(2)} - \psi^{(2)}) \rangle \\
 & + \langle \delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(h^{(2)} - g^{(2)})\delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)})\delta(h^{(3)} - g^{(3)})\delta(\theta^{(3)} - \phi^{(3)})\delta(c^{(3)} - \psi^{(3)}) \times \frac{\partial u_{\alpha}^{(3)} u_{\beta}^{(3)}}{\partial x_{\beta}^{(3)}} \frac{\partial}{\partial v_{\alpha}^{(3)}} \delta(u^{(3)} - v^{(3)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(h^{(2)} - g^{(2)})\delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)})\delta(h^{(3)} - g^{(3)})\delta(\theta^{(3)} - \phi^{(3)})\delta(c^{(3)} - \psi^{(3)}) \times \frac{\partial h_{\alpha}^{(3)} h_{\beta}^{(3)}}{\partial x_{\beta}^{(3)}} \frac{\partial}{\partial v_{\alpha}^{(3)}} \delta(u^{(3)} - v^{(3)}) \rangle \\
 & + \langle \delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(h^{(2)} - g^{(2)})\delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)})\delta(h^{(3)} - g^{(3)})\delta(\theta^{(3)} - \phi^{(3)})\delta(c^{(3)} - \psi^{(3)}) \times \frac{1}{4\pi} \frac{\partial}{\partial x_{\alpha}^{(3)}} \int \left[ \frac{\partial u_{\alpha}^{(3)}}{\partial x_{\beta}^{(3)}} \frac{\partial u_{\beta}^{(3)}}{\partial x_{\alpha}^{(3)}} - \frac{\partial h_{\alpha}^{(3)}}{\partial x_{\beta}^{(3)}} \frac{\partial h_{\beta}^{(3)}}{\partial x_{\alpha}^{(3)}} \right]
 \end{aligned}$$

$$\begin{aligned}
 & \times \frac{d\bar{x}'''}{|\bar{x}''' - \bar{x}''|} \frac{\partial}{\partial v_{\alpha}^{(3)}} \delta(u^{(3)} - v^{(3)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times v \nabla^2 u_{\alpha}^{(3)} \frac{\partial}{\partial v_{\alpha}^{(3)}} \delta(u^{(3)} - v^{(3)}) \rangle \\
 & + \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times 2 \epsilon_{m\alpha\beta} \Omega_m u_{\alpha}^{(3)} \frac{\partial}{\partial v_{\alpha}^{(3)}} \delta(u^{(3)} - v^{(3)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times f(u_{\alpha}^{(3)} - v_{\alpha}^{(3)}) \frac{\partial}{\partial v_{\alpha}^{(3)}} \delta(u^{(3)} - v^{(3)}) \rangle \\
 & + \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times \frac{\partial h_{\alpha}^{(3)} u_{\beta}^{(3)}}{\partial x_{\beta}^{(3)}} \frac{\partial}{\partial g_{\alpha}^{(3)}} \delta(h^{(3)} - g^{(3)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times \frac{\partial u_{\alpha}^{(3)} h_{\beta}^{(3)}}{\partial x_{\beta}^{(3)}} \frac{\partial}{\partial g_{\alpha}^{(3)}} \delta(h^{(3)} - g^{(3)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times \lambda \nabla^2 h_{\alpha}^{(3)} \frac{\partial}{\partial g_{\alpha}^{(3)}} \delta(h^{(3)} - g^{(3)}) \rangle \\
 & + \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times u_{\beta}^{(3)} \frac{\partial \theta^{(3)}}{\partial x_{\beta}^{(3)}} \frac{\partial}{\partial \phi^{(3)}} \delta(\theta^{(3)} - \phi^{(3)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times \nabla^2 \theta^{(3)} \frac{\partial}{\partial \phi^{(3)}} \delta(\theta^{(3)} - \phi^{(3)}) \rangle \\
 & + \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \times u_{\beta}^{(3)} \frac{\partial c^{(3)}}{\partial x_{\beta}^{(3)}} \frac{\partial}{\partial \psi^{(3)}} \delta(c^{(3)} - \psi^{(3)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \times D \nabla^2 c^{(3)} \frac{\partial}{\partial \psi^{(3)}} \delta(c^{(3)} - \psi^{(3)}) \rangle
 \end{aligned}$$

$$+ \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\ \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \times Rc^{(3)} \frac{\partial}{\partial \psi^{(3)}} \delta(c^{(3)} - \psi^{(3)}) \rangle \quad (73)$$

Various terms in the above equation can be simplified as that they may be expressed in terms of one-, two-, three- and four - point distribution functions.

The 1<sup>st</sup> term in the above equation is simplified as follows

$$\langle \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\ \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times \frac{\partial u_{\alpha}^{(1)} u_{\beta}^{(1)}}{\partial x_{\beta}^{(1)} \partial v_{\alpha}^{(1)}} \frac{\partial}{\partial v_{\alpha}^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle \\ = \langle u_{\beta}^{(1)} \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\ \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times \frac{\partial u_{\alpha}^{(1)}}{\partial x_{\beta}^{(1)}} \frac{\partial}{\partial v_{\alpha}^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle \\ = \langle -u_{\beta}^{(1)} \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\ \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times \frac{\partial u_{\alpha}^{(1)}}{\partial v_{\alpha}^{(1)}} \frac{\partial}{\partial x_{\beta}^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle; (\text{since } \frac{\partial u_{\alpha}^{(1)}}{\partial v_{\alpha}^{(1)}} = 1) \\ = \langle -\delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\ \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times u_{\beta}^{(1)} \frac{\partial}{\partial x_{\beta}^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle \quad (74)$$

Similarly, 7<sup>th</sup>, 10<sup>th</sup> and 12<sup>th</sup> terms of right hand-side of equation (73) can be simplified as follows;  
7<sup>th</sup> term,

$$\langle \delta(u^{(1)} - v^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\ \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times \frac{\partial h_{\alpha}^{(1)} u_{\beta}^{(1)}}{\partial x_{\beta}^{(1)} \partial g_{\alpha}^{(1)}} \frac{\partial}{\partial g_{\alpha}^{(1)}} \delta(h^{(1)} - g^{(1)}) \rangle \quad (75)$$

10<sup>th</sup> term,

$$\langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\ \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times u_{\beta}^{(1)} \frac{\partial \theta^{(1)}}{\partial x_{\beta}^{(1)}} \frac{\partial}{\partial \phi^{(1)}} \delta(\theta^{(1)} - \phi^{(1)}) \rangle \\ = \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\ \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times u_{\beta}^{(1)} \frac{\partial}{\partial x_{\beta}^{(1)}} \delta(\theta^{(1)} - \phi^{(1)}) \rangle \quad (76)$$

and 12<sup>th</sup> term

$$\langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\ \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times u_{\beta}^{(1)} \frac{\partial c^{(1)}}{\partial x_{\beta}^{(1)}} \frac{\partial}{\partial \psi^{(1)}} \delta(c^{(1)} - \psi^{(1)}) \rangle \\ = \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\ \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times u_{\beta}^{(1)} \frac{\partial}{\partial x_{\beta}^{(1)}} \delta(c^{(1)} - \psi^{(1)}) \rangle \quad (77)$$

Adding these equations from (74) to (77), we get

$$\begin{aligned}
 & \langle -\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(h^{(2)} - g^{(2)})\delta(\theta^{(2)} - \phi^{(2)})\delta(c^{(2)} - \psi^{(2)}) \\
 & \delta(u^{(3)} - v^{(3)})\delta(h^{(3)} - g^{(3)})\delta(\theta^{(3)} - \phi^{(3)})\delta(c^{(3)} - \psi^{(3)}) \times u_{\beta}^{(1)} \frac{\partial}{\partial x_{\beta}^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(h^{(2)} - g^{(2)})\delta(\theta^{(2)} - \phi^{(2)})\delta(c^{(2)} - \psi^{(2)}) \\
 & \delta(u^{(3)} - v^{(3)})\delta(h^{(3)} - g^{(3)})\delta(\theta^{(3)} - \phi^{(3)})\delta(c^{(3)} - \psi^{(3)}) \times u_{\beta}^{(1)} \frac{\partial}{\partial x_{\beta}^{(1)}} \delta(h^{(1)} - g^{(1)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(h^{(2)} - g^{(2)})\delta(\theta^{(2)} - \phi^{(2)})\delta(c^{(2)} - \psi^{(2)}) \\
 & \delta(u^{(3)} - v^{(3)})\delta(h^{(3)} - g^{(3)})\delta(\theta^{(3)} - \phi^{(3)})\delta(c^{(3)} - \psi^{(3)}) \times u_{\beta}^{(1)} \frac{\partial}{\partial x_{\beta}^{(1)}} \delta(\theta^{(1)} - \phi^{(1)}) \rangle \\
 & + \langle -\delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(h^{(2)} - g^{(2)})\delta(\theta^{(2)} - \phi^{(2)})\delta(c^{(2)} - \psi^{(2)}) \\
 & \delta(u^{(3)} - v^{(3)})\delta(h^{(3)} - g^{(3)})\delta(\theta^{(3)} - \phi^{(3)})\delta(c^{(3)} - \psi^{(3)}) \times u_{\beta}^{(1)} \frac{\partial}{\partial x_{\beta}^{(1)}} \delta(c^{(1)} - \psi^{(1)}) \rangle \\
 & = -\frac{\partial}{\partial x_{\beta}^{(1)}} \langle u_{\beta}^{(1)} \langle -\delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(h^{(2)} - g^{(2)}) \\
 & \delta(\theta^{(2)} - \phi^{(2)})\delta(c^{(2)} - \psi^{(2)})\delta(u^{(3)} - v^{(3)})\delta(h^{(3)} - g^{(3)})\delta(\theta^{(3)} - \phi^{(3)})\delta(c^{(3)} - \psi^{(3)}) \rangle \\
 & = -\frac{\partial}{\partial x_{\beta}^{(1)}} v_{\beta}^{(1)} F_3^{(1,2,3)} \quad [\text{Applying the properties of distribution functions}] \tag{78}
 \end{aligned}$$

Similarly, 15<sup>th</sup>, 21<sup>st</sup>, 24<sup>th</sup> and 26<sup>th</sup> terms of right hand-side of equation (73) can be simplified as follows;  
15<sup>th</sup> term,

$$\begin{aligned}
 & \langle \delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(h^{(2)} - g^{(2)})\delta(\theta^{(2)} - \phi^{(2)})\delta(c^{(2)} - \psi^{(2)}) \\
 & \delta(u^{(3)} - v^{(3)})\delta(h^{(3)} - g^{(3)})\delta(\theta^{(3)} - \phi^{(3)})\delta(c^{(3)} - \psi^{(3)}) \times \frac{\partial u_{\alpha}^{(2)} u_{\beta}^{(2)}}{\partial x_{\beta}^{(2)}} \frac{\partial}{\partial v_{\alpha}^{(2)}} \delta(u^{(2)} - v^{(2)}) \rangle \\
 & = \langle -\delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(h^{(2)} - g^{(2)})\delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)})\delta(u^{(3)} - v^{(3)})\delta(h^{(3)} - g^{(3)})\delta(\theta^{(3)} - \phi^{(3)})\delta(c^{(3)} - \psi^{(3)}) \times u_{\beta}^{(2)} \frac{\partial}{\partial x_{\beta}^{(2)}} \delta(u^{(2)} - v^{(2)}) \rangle \tag{79}
 \end{aligned}$$

21<sup>st</sup> term,

$$\begin{aligned}
 & \langle \delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(\theta^{(2)} - \phi^{(2)})\delta(c^{(2)} - \psi^{(2)}) \\
 & \delta(u^{(3)} - v^{(3)})\delta(h^{(3)} - g^{(3)})\delta(\theta^{(3)} - \phi^{(3)})\delta(c^{(3)} - \psi^{(3)}) \times \frac{\partial h_{\alpha}^{(2)} u_{\beta}^{(2)}}{\partial x_{\beta}^{(2)}} \frac{\partial}{\partial g_{\alpha}^{(2)}} \delta(h^{(2)} - g^{(2)}) \rangle \tag{80}
 \end{aligned}$$

24<sup>th</sup> term,

$$\begin{aligned}
 & \langle \delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(h^{(2)} - g^{(2)})\delta(c^{(2)} - \psi^{(2)}) \\
 & \delta(u^{(3)} - v^{(3)})\delta(h^{(3)} - g^{(3)})\delta(\theta^{(3)} - \phi^{(3)})\delta(c^{(3)} - \psi^{(3)}) \times u_{\beta}^{(2)} \frac{\partial \theta^{(2)}}{\partial x_{\beta}^{(2)}} \frac{\partial}{\partial \phi^{(2)}} \delta(\theta^{(2)} - \phi^{(2)}) \rangle
 \end{aligned}$$

$$= \langle -\delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(h^{(2)} - g^{(2)})\delta(c^{(2)} - \psi^{(2)}) \\ \delta(u^{(3)} - v^{(3)})\delta(h^{(3)} - g^{(3)})\delta(\theta^{(3)} - \phi^{(3)})\delta(c^{(3)} - \psi^{(3)}) \times u_{\beta}^{(2)} \frac{\partial}{\partial x_{\beta}^{(2)}} \delta(\theta^{(2)} - \phi^{(2)}) \rangle \quad (81)$$

And 26<sup>th</sup> term,

$$\langle \delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(h^{(2)} - g^{(2)})\delta(\theta^{(2)} - \phi^{(2)}) \\ \delta(u^{(3)} - v^{(3)})\delta(h^{(3)} - g^{(3)})\delta(\theta^{(3)} - \phi^{(3)})\delta(c^{(3)} - \psi^{(3)}) \times u_{\beta}^{(2)} \frac{\partial c^{(2)}}{\partial x_{\beta}^{(2)}} \frac{\partial}{\partial \psi^{(2)}} \delta(c^{(2)} - \psi^{(2)}) \rangle \\ = \langle -\delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(h^{(2)} - g^{(2)})\delta(\theta^{(2)} - \phi^{(2)}) \\ \delta(u^{(3)} - v^{(3)})\delta(h^{(3)} - g^{(3)})\delta(\theta^{(3)} - \phi^{(3)})\delta(c^{(3)} - \psi^{(3)}) \times u_{\beta}^{(2)} \frac{\partial}{\partial x_{\beta}^{(2)}} \delta(c^{(2)} - \psi^{(2)}) \rangle \quad (82)$$

Adding equations (79) to (82), we get

$$-\frac{\partial}{\partial x_{\beta}^{(2)}} \langle u_{\beta}^{(2)} \langle \delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(h^{(2)} - g^{(2)}) \\ \delta(\theta^{(2)} - \phi^{(2)})\delta(c^{(2)} - \psi^{(2)})\delta(u^{(3)} - v^{(3)})\delta(h^{(3)} - g^{(3)})\delta(\theta^{(3)} - \phi^{(3)})\delta(c^{(3)} - \psi^{(3)}) \rangle \\ = -v_{\beta}^{(2)} \frac{\partial F_3^{(1,2,3)}}{\partial x_{\beta}^{(2)}} \quad (83)$$

Similarly, 29<sup>th</sup>, 35<sup>th</sup>, 38<sup>th</sup> and 40<sup>th</sup> terms of right hand-side of equation (73) can be simplified as follows;  
29<sup>th</sup> term,

$$\langle \delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(h^{(2)} - g^{(2)})\delta(\theta^{(2)} - \phi^{(2)}) \\ \delta(c^{(2)} - \psi^{(2)})\delta(h^{(3)} - g^{(3)})\delta(\theta^{(3)} - \phi^{(3)})\delta(c^{(3)} - \psi^{(3)}) \times \frac{\partial u_{\alpha}^{(3)} u_{\beta}^{(3)}}{\partial x_{\beta}^{(3)}} \frac{\partial}{\partial v_{\alpha}^{(2)}} \delta(u^{(3)} - v^{(3)}) \rangle \\ = \langle -\delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(3)} - v^{(3)})\delta(h^{(2)} - g^{(2)}) \\ \delta(\theta^{(2)} - \phi^{(2)})\delta(c^{(2)} - \psi^{(2)})\delta(h^{(3)} - g^{(3)})\delta(\theta^{(3)} - \phi^{(3)})\delta(c^{(3)} - \psi^{(3)}) \times u_{\beta}^{(3)} \frac{\partial}{\partial x_{\beta}^{(3)}} \delta(u^{(3)} - v^{(3)}) \rangle \quad (84)$$

35<sup>th</sup> term,

$$\langle \delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(h^{(2)} - g^{(2)})\delta(\theta^{(2)} - \phi^{(2)}) \\ \delta(c^{(2)} - \psi^{(2)})\delta(u^{(3)} - v^{(3)})\delta(\theta^{(3)} - \phi^{(3)})\delta(c^{(3)} - \psi^{(3)}) \times \frac{\partial h_{\alpha}^{(3)} u_{\beta}^{(3)}}{\partial x_{\beta}^{(3)}} \frac{\partial}{\partial g_{\alpha}^{(3)}} \delta(h^{(3)} - g^{(3)}) \rangle \\ = \langle -\delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(h^{(2)} - g^{(2)})\delta(\theta^{(2)} - \phi^{(2)}) \\ \delta(c^{(2)} - \psi^{(2)})\delta(u^{(3)} - v^{(3)})\delta(\theta^{(3)} - \phi^{(3)})\delta(c^{(3)} - \psi^{(3)}) \times u_{\beta}^{(3)} \frac{\partial}{\partial x_{\beta}^{(3)}} \delta(h^{(3)} - g^{(3)}) \rangle \quad (85)$$

38<sup>th</sup> term,

$$\begin{aligned}
 & \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times u_{\beta}^{(3)} \frac{\partial \theta^{(3)}}{\partial x_{\beta}^{(3)}} \frac{\partial}{\partial \phi^{(3)}} \delta(\theta^{(3)} - \phi^{(3)}) \rangle \\
 & = \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times u_{\beta}^{(3)} \frac{\partial}{\partial x_{\beta}^{(3)}} \delta(\theta^{(3)} - \phi^{(3)}) \rangle
 \end{aligned} \tag{86}$$

and 40<sup>th</sup> term,

$$\begin{aligned}
 & \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \times u_{\beta}^{(3)} \frac{\partial c^{(3)}}{\partial x_{\beta}^{(3)}} \frac{\partial}{\partial \psi^{(3)}} \delta(c^{(3)} - \psi^{(3)}) \rangle \\
 & = \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
 & \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \times u_{\beta}^{(3)} \frac{\partial}{\partial x_{\beta}^{(3)}} \delta(c^{(3)} - \psi^{(3)}) \rangle
 \end{aligned} \tag{87}$$

Adding equations (84) to (87), we get

$$\begin{aligned}
 & -\frac{\partial}{\partial x_{\beta}^{(3)}} \langle u_{\beta}^{(3)} \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \\
 & \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \rangle \\
 & = -v_{\beta}^{(3)} \frac{\partial F_3^{(1,2,3)}}{\partial x_{\beta}^{(3)}}
 \end{aligned} \tag{88}$$

Similarly, 2<sup>nd</sup>, 8<sup>th</sup>, 16<sup>th</sup>, 22<sup>nd</sup>, 30<sup>th</sup> and 36<sup>th</sup> terms of right hand-side of equation (73) can be simplified as follows;  
2<sup>nd</sup> term,

$$\begin{aligned}
 & \langle -\delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\
 & \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times \frac{\partial h_{\alpha}^{(1)} h_{\beta}^{(1)}}{\partial x_{\beta}^{(1)}} \frac{\partial}{\partial v_{\alpha}^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle \\
 & = -g_{\beta}^{(1)} \frac{\partial g_{\alpha}^{(1)}}{\partial v_{\alpha}^{(1)}} \frac{\partial F_3^{(1,2,3)}}{\partial x_{\beta}^{(1)}}
 \end{aligned} \tag{89}$$

8<sup>th</sup> term,

$$\begin{aligned}
 & \langle -\delta(u^{(1)} - v^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\
 & \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times \frac{\partial u_{\alpha}^{(1)} h_{\beta}^{(1)}}{\partial x_{\beta}^{(1)}} \frac{\partial}{\partial g_{\alpha}^{(1)}} \delta(h^{(1)} - g^{(1)}) \rangle \\
 & = -g_{\beta}^{(1)} \frac{\partial v_{\alpha}^{(1)}}{\partial g_{\alpha}^{(1)}} \frac{\partial F_3^{(1,2,3)}}{\partial x_{\beta}^{(1)}}
 \end{aligned} \tag{90}$$

16<sup>th</sup> term,

$$\begin{aligned} & \langle -\delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(h^{(2)} - g^{(2)})\delta(\theta^{(2)} - \phi^{(2)})\delta(c^{(2)} - \psi^{(2)}) \\ & \delta(u^{(3)} - v^{(3)})\delta(h^{(3)} - g^{(3)})\delta(\theta^{(3)} - \phi^{(3)})\delta(c^{(3)} - \psi^{(3)}) \times \frac{\partial h_a^{(2)} h_\beta^{(2)}}{\partial x_\beta^{(2)}} \frac{\partial}{\partial v_a^{(2)}} \delta(u^{(2)} - v^{(2)}) \rangle \\ & = -g_\beta^{(2)} \frac{\partial g_\alpha^{(2)}}{\partial v_\alpha^{(2)}} \frac{\partial F_3^{(1,2,3)}}{\partial x_\beta^{(2)}} \end{aligned} \quad (91)$$

22<sup>nd</sup> term,

$$\begin{aligned} & \langle -\delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(\theta^{(2)} - \phi^{(2)})\delta(c^{(2)} - \psi^{(2)}) \\ & \delta(u^{(3)} - v^{(3)})\delta(h^{(3)} - g^{(3)})\delta(\theta^{(3)} - \phi^{(3)})\delta(c^{(3)} - \psi^{(3)}) \times \frac{\partial u_\alpha^{(2)} h_\beta^{(2)}}{\partial x_\beta^{(2)}} \frac{\partial}{\partial g_\alpha^{(2)}} \delta(h^{(2)} - g^{(2)}) \rangle \\ & = -g_\beta^{(2)} \frac{\partial v_\alpha^{(2)}}{\partial g_\alpha^{(2)}} \frac{\partial F_3^{(1,2,3)}}{\partial x_\beta^{(2)}} \end{aligned} \quad (92)$$

30<sup>th</sup> term,

$$\begin{aligned} & \langle -\delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(h^{(2)} - g^{(2)})\delta(\theta^{(2)} - \phi^{(2)}) \\ & \delta(c^{(2)} - \psi^{(2)})\delta(h^{(3)} - g^{(3)})\delta(\theta^{(3)} - \phi^{(3)})\delta(c^{(3)} - \psi^{(3)}) \times \frac{\partial h_a^{(3)} h_\beta^{(3)}}{\partial x_\beta^{(3)}} \frac{\partial}{\partial v_\alpha^{(3)}} \delta(u^{(3)} - v^{(3)}) \rangle \\ & = -g_\beta^{(3)} \frac{\partial g_\alpha^{(3)}}{\partial v_\alpha^{(3)}} \frac{\partial F_3^{(1,2,3)}}{\partial x_\beta^{(3)}} \end{aligned} \quad (93)$$

and 36<sup>th</sup> term,

$$\begin{aligned} & \langle -\delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(h^{(2)} - g^{(2)})\delta(\theta^{(2)} - \phi^{(2)}) \\ & \delta(c^{(2)} - \psi^{(2)})\delta(u^{(3)} - v^{(3)})\delta(\theta^{(3)} - \phi^{(3)})\delta(c^{(3)} - \psi^{(3)}) \times \frac{\partial u_\alpha^{(3)} h_\beta^{(3)}}{\partial x_\beta^{(3)}} \frac{\partial}{\partial g_\alpha^{(3)}} \delta(h^{(3)} - g^{(3)}) \rangle \\ & = -g_\beta^{(3)} \frac{\partial v_\alpha^{(3)}}{\partial g_\alpha^{(3)}} \frac{\partial F_3^{(1,2,3)}}{\partial x_\beta^{(3)}} \end{aligned} \quad (94)$$

4<sup>th</sup> term can be reduced as

$$\begin{aligned} & \langle -\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(h^{(2)} - g^{(2)})\delta(\theta^{(2)} - \phi^{(2)})\delta(c^{(2)} - \psi^{(2)}) \\ & \delta(u^{(3)} - v^{(3)})\delta(h^{(3)} - g^{(3)})\delta(\theta^{(3)} - \phi^{(3)})\delta(c^{(3)} - \psi^{(3)}) \times v \nabla^2 u_\alpha^{(1)} \frac{\partial}{\partial v_\alpha^{(1)}} \delta(u^{(1)} - v^{(1)}) \\ & = -v \frac{\partial}{\partial v_\alpha^{(1)}} \langle \nabla^2 u_\alpha^{(1)} [ \delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(h^{(2)} - g^{(2)}) \\ & \delta(\theta^{(2)} - \phi^{(2)})\delta(c^{(2)} - \psi^{(2)})\delta(u^{(3)} - v^{(3)})\delta(h^{(3)} - g^{(3)})\delta(\theta^{(3)} - \phi^{(3)})\delta(c^{(3)} - \psi^{(3)}) ] \rangle \end{aligned}$$

$$\begin{aligned}
 &= -v \frac{\partial}{\partial v_{\alpha}^{(1)}} \frac{\partial^2}{\partial x_{\beta}^{(1)} \partial x_{\beta}^{(1)}} \left\langle u_{\alpha}^{(1)} \left[ \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \right. \right. \\
 &\quad \left. \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \right] \rangle \\
 &= -v \frac{\partial}{\partial v_{\alpha}^{(1)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(1)}} \frac{\partial^2}{\partial x_{\beta}^{(4)} \partial x_{\beta}^{(4)}} \left\langle u_{\alpha}^{(4)} \left[ \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \right. \right. \\
 &\quad \left. \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \right] \rangle \\
 &= -v \frac{\partial}{\partial v_{\alpha}^{(1)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(1)}} \frac{\partial^2}{\partial x_{\beta}^{(4)} \partial x_{\beta}^{(4)}} \left\langle \int u_{\alpha}^{(4)} \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \phi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \right. \\
 &\quad \left. \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \right. \\
 &\quad \left. \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)} \right\rangle \\
 &= -v \frac{\partial}{\partial v_{\alpha}^{(1)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(1)}} \frac{\partial^2}{\partial x_{\beta}^{(4)} \partial x_{\beta}^{(4)}} \int v_{\alpha}^{(4)} F_4^{(1,2,3,4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)} \quad (95)
 \end{aligned}$$

Similarly, 9<sup>th</sup>, 11<sup>th</sup>, 13<sup>th</sup>, 18<sup>th</sup>, 23<sup>rd</sup>, 25<sup>th</sup>, 27<sup>th</sup>, 32<sup>nd</sup>, 37<sup>th</sup>, 39<sup>th</sup> and 41<sup>st</sup> terms of right hand-side of equation (73) can be simplified as follows;

9<sup>th</sup> term,

$$\begin{aligned}
 &\langle -\delta(u^{(1)} - v^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\
 &\quad \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times \lambda \nabla^2 h_{\alpha}^{(1)} \frac{\partial}{\partial g_{\alpha}^{(1)}} \delta(h^{(1)} - g^{(1)}) \rangle \\
 &= -\lambda \frac{\partial}{\partial g_{\alpha}^{(1)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(1)}} \frac{\partial^2}{\partial x_{\beta}^{(4)} \partial x_{\beta}^{(4)}} \int g_{\alpha}^{(4)} F_4^{(1,2,3,4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)} \quad (96)
 \end{aligned}$$

11<sup>th</sup> term,

$$\begin{aligned}
 &\langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\
 &\quad \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times \gamma \nabla^2 \theta^{(1)} \frac{\partial}{\partial \phi^{(1)}} \delta(\theta^{(1)} - \phi^{(1)}) \rangle \\
 &= -\gamma \frac{\partial}{\partial \phi^{(1)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(1)}} \frac{\partial^2}{\partial x_{\beta}^{(4)} \partial x_{\beta}^{(4)}} \int \phi^{(4)} F_4^{(1,2,3,4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)} \quad (97)
 \end{aligned}$$

13<sup>th</sup> term,

$$\begin{aligned}
 &+ \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\
 &\quad \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times D \nabla^2 c^{(1)} \frac{\partial}{\partial \psi^{(1)}} \delta(c^{(1)} - \psi^{(1)}) \rangle
 \end{aligned}$$

$$= -D \frac{\partial}{\partial \psi^{(1)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(1)}} \frac{\partial^2}{\partial x_{\beta}^{(4)} \partial x_{\beta}^{(3)}} \int \psi^{(4)} F_4^{(1,2,3,4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)} \quad (98)$$

18<sup>th</sup> term,

$$\begin{aligned} & \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\ & \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times \nabla^2 u_{\alpha}^{(2)} \frac{\partial}{\partial v_{\alpha}^{(2)}} \delta(u^{(2)} - v^{(2)}) \rangle \\ & = -\nu \frac{\partial}{\partial v_{\alpha}^{(2)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(2)}} \frac{\partial^2}{\partial x_{\beta}^{(4)} \partial x_{\beta}^{(4)}} \int v_{\alpha}^{(4)} F_4^{(1,2,3,4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)} \end{aligned} \quad (99)$$

23<sup>rd</sup> term,

$$\begin{aligned} & \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\ & \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times \lambda \nabla^2 h_{\alpha}^{(2)} \frac{\partial}{\partial g_{\alpha}^{(2)}} \delta(h^{(2)} - g^{(2)}) \rangle \\ & = -\lambda \frac{\partial}{\partial g_{\alpha}^{(2)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(2)}} \frac{\partial^2}{\partial x_{\beta}^{(4)} \partial x_{\beta}^{(4)}} \int g_{\alpha}^{(4)} F_4^{(1,2,3,4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)} \end{aligned} \quad (100)$$

25<sup>th</sup> term,

$$\begin{aligned} & \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\ & \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times \nabla^2 \theta^{(2)} \frac{\partial}{\partial \phi^{(2)}} \delta(\theta^{(2)} - \phi^{(2)}) \rangle \\ & = -\gamma \frac{\partial}{\partial \phi^{(2)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(2)}} \frac{\partial^2}{\partial x_{\beta}^{(4)} \partial x_{\beta}^{(4)}} \int \phi^{(4)} F_4^{(1,2,3,4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)} \end{aligned} \quad (101)$$

27<sup>th</sup> term,

$$\begin{aligned} & \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\ & \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times D \nabla^2 c^{(2)} \frac{\partial}{\partial \psi^{(2)}} \delta(c^{(2)} - \psi^{(2)}) \rangle \\ & = -D \frac{\partial}{\partial \psi^{(2)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(2)}} \frac{\partial^2}{\partial x_{\beta}^{(4)} \partial x_{\beta}^{(4)}} \int \psi^{(4)} F_4^{(1,2,3,4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)} \end{aligned} \quad (102)$$

32<sup>nd</sup> term,

$$\begin{aligned} & \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\ & \delta(c^{(2)} - \psi^{(2)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times \nabla^2 u_{\alpha}^{(3)} \frac{\partial}{\partial v_{\alpha}^{(3)}} \delta(u^{(3)} - v^{(3)}) \rangle \end{aligned}$$

$$= -\nu \frac{\partial}{\partial v_{\alpha}^{(3)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(3)}} \frac{\partial^2}{\partial x_{\beta}^{(4)} \partial x_{\beta}^{(4)}} \int v_{\alpha}^{(4)} F_4^{(1,2,3,4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)} \quad (103)$$

37<sup>th</sup> term,

$$\begin{aligned} & \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\ & \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times \lambda \nabla^2 h_{\alpha}^{(3)} \frac{\partial}{\partial g_{\alpha}^{(3)}} \delta(h^{(3)} - g^{(3)}) \rangle \\ & = -\lambda \frac{\partial}{\partial g_{\alpha}^{(3)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(3)}} \frac{\partial^2}{\partial x_{\beta}^{(4)} \partial x_{\beta}^{(4)}} \int g_{\alpha}^{(4)} F_4^{(1,2,3,4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)} \end{aligned} \quad (104)$$

39<sup>th</sup> term,

$$\begin{aligned} & \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\ & \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times \gamma \nabla^2 \theta^{(3)} \frac{\partial}{\partial \phi^{(3)}} \delta(\theta^{(3)} - \phi^{(3)}) \rangle \\ & = -\gamma \frac{\partial}{\partial \phi^{(3)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(3)}} \frac{\partial^2}{\partial x_{\beta}^{(4)} \partial x_{\beta}^{(4)}} \int \phi^{(4)} F_4^{(1,2,3,4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)} \end{aligned} \quad (105)$$

41<sup>st</sup> term,

$$\begin{aligned} & \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\ & \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times D \nabla^2 c^{(3)} \frac{\partial}{\partial \psi^{(3)}} \delta(c^{(3)} - \psi^{(3)}) \rangle \\ & = -D \frac{\partial}{\partial \psi^{(3)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(3)}} \frac{\partial^2}{\partial x_{\beta}^{(4)} \partial x_{\beta}^{(4)}} \int \psi^{(4)} F_4^{(1,2,3,4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)} \end{aligned} \quad (106)$$

We reduce the third term of right hand side of equation (73),

$$\begin{aligned} & \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \\ & \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \\ & \times \frac{1}{4\pi} \frac{\partial}{\partial x_{\alpha}^{(1)}} \int \left[ \frac{\partial u_{\alpha}^{(1)}}{\partial x_{\beta}^{(1)}} \frac{\partial u_{\beta}^{(1)}}{\partial x_{\alpha}^{(1)}} - \frac{\partial h_{\alpha}^{(1)}}{\partial x_{\beta}^{(1)}} \frac{\partial h_{\beta}^{(1)}}{\partial x_{\alpha}^{(1)}} \right] \times \frac{d\bar{x}'''}{|\bar{x}''' - \bar{x}|} \frac{\partial}{\partial v_{\alpha}^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle \\ & = \frac{\partial}{\partial v_{\alpha}^{(1)}} \left[ \frac{1}{4\pi} \int \frac{\partial}{\partial x_{\alpha}^{(1)}} \left( \frac{1}{|\bar{x}^{(4)} - \bar{x}^{(1)}|} \right) \left( \frac{\partial v_{\alpha}^{(4)}}{\partial x_{\beta}^{(4)}} \frac{\partial v_{\beta}^{(4)}}{\partial x_{\alpha}^{(4)}} - \frac{\partial g_{\alpha}^{(4)}}{\partial x_{\beta}^{(4)}} \frac{\partial g_{\beta}^{(4)}}{\partial x_{\alpha}^{(4)}} \right) F_4^{(1,2,3,4)} dx^{(4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)} \right] \end{aligned} \quad (107)$$

17<sup>th</sup> term,

$$\begin{aligned} & \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\ & \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \rangle \end{aligned}$$

$$\begin{aligned} & \times \frac{1}{4\pi} \frac{\partial}{\partial x_{\alpha}^{(2)}} \int \left[ \frac{\partial u_{\alpha}^{(2)}}{\partial x_{\beta}^{(2)}} \frac{\partial u_{\beta}^{(2)}}{\partial x_{\alpha}^{(2)}} - \frac{\partial h_{\alpha}^{(2)}}{\partial x_{\beta}^{(2)}} \frac{\partial h_{\beta}^{(2)}}{\partial x_{\alpha}^{(2)}} \right] \times \frac{d\bar{x}'''}{|\bar{x}''' - \bar{x}''|} \frac{\partial}{\partial v_{\alpha}^{(2)}} \delta(u^{(2)} - v^{(2)}) \rangle \\ & = \frac{\partial}{\partial v_{\alpha}^{(2)}} \left[ \frac{1}{4\pi} \int \frac{\partial}{\partial x_{\alpha}^{(2)}} \left( \frac{1}{|\bar{x}^{(4)} - \bar{x}^{(2)}|} \right) \left( \frac{\partial v_{\alpha}^{(4)}}{\partial x_{\beta}^{(4)}} \frac{\partial v_{\beta}^{(4)}}{\partial x_{\alpha}^{(4)}} - \frac{\partial g_{\alpha}^{(4)}}{\partial x_{\beta}^{(4)}} \frac{\partial g_{\beta}^{(4)}}{\partial x_{\alpha}^{(4)}} \right) F_4^{(1,2,3,4)} dx^{(4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)} \right] \quad (108) \end{aligned}$$

Similarly, 31<sup>st</sup> term,

$$\begin{aligned} & \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \\ & \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \rangle \\ & \times \frac{1}{4\pi} \frac{\partial}{\partial x_{\alpha}^{(3)}} \int \left[ \frac{\partial u_{\alpha}^{(3)}}{\partial x_{\beta}^{(3)}} \frac{\partial u_{\beta}^{(3)}}{\partial x_{\alpha}^{(3)}} - \frac{\partial h_{\alpha}^{(3)}}{\partial x_{\beta}^{(3)}} \frac{\partial h_{\beta}^{(3)}}{\partial x_{\alpha}^{(3)}} \right] \frac{d\bar{x}'''}{|\bar{x}''' - \bar{x}''|} \frac{\partial}{\partial v_{\alpha}^{(3)}} \delta(u^{(3)} - v^{(3)}) \rangle \\ & = \frac{\partial}{\partial v_{\alpha}^{(3)}} \left[ \frac{1}{4\pi} \int \frac{\partial}{\partial x_{\alpha}^{(3)}} \left( \frac{1}{|\bar{x}^{(4)} - \bar{x}^{(3)}|} \right) \left( \frac{\partial v_{\alpha}^{(4)}}{\partial x_{\beta}^{(4)}} \frac{\partial v_{\beta}^{(4)}}{\partial x_{\alpha}^{(4)}} - \frac{\partial g_{\alpha}^{(4)}}{\partial x_{\beta}^{(4)}} \frac{\partial g_{\beta}^{(4)}}{\partial x_{\alpha}^{(4)}} \right) F_4^{(1,2,3,4)} dx^{(4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)} \right] \quad (109) \end{aligned}$$

5<sup>th</sup> and 6<sup>th</sup> terms of right hand side of equation (73),  
5<sup>th</sup> term,

$$\begin{aligned} & \langle \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\ & \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times 2 \epsilon_{m\alpha\beta} \Omega_m u_{\alpha}^{(1)} \frac{\partial}{\partial v_{\alpha}^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle \\ & = \langle 2 \epsilon_{m\alpha\beta} \Omega_m u_{\alpha}^{(1)} \frac{\partial}{\partial v_{\alpha}^{(1)}} [ \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \\ & \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) ] \rangle \\ & = 2 \epsilon_{m\alpha\beta} \Omega_m \frac{\partial}{\partial v_{\alpha}^{(1)}} \langle u_{\alpha}^{(1)} \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \\ & \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \rangle \\ & = 2 \epsilon_{m\alpha\beta} \Omega_m \frac{\partial u_{\alpha}^{(1)}}{\partial v_{\alpha}^{(1)}} \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \\ & \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \rangle \\ & = 2 \epsilon_{m\alpha\beta} \Omega_m F_3^{(1,2,3)} \quad (110) \end{aligned}$$

and 6<sup>th</sup> term,

$$\begin{aligned} & \langle -\delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\ & \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times f(u_{\alpha}^{(1)} - v_{\alpha}^{(1)}) \frac{\partial}{\partial v_{\alpha}^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle \end{aligned}$$

$$\begin{aligned}
 &= -\left\langle f(u_\alpha^{(1)} - v_\alpha^{(1)}) \frac{\partial}{\partial v_\alpha^{(1)}} \left[ \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \right. \right. \\
 &\quad \left. \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \right] \rangle \\
 &= -f(u_\alpha^{(1)} - v_\alpha^{(1)}) \frac{\partial}{\partial v_\alpha^{(1)}} \left\langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \right. \\
 &\quad \left. \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \right\rangle \\
 &= -f(u_\alpha^{(1)} - v_\alpha^{(1)}) \frac{\partial}{\partial v_\alpha^{(1)}} F_3^{(1,2,3)}
 \end{aligned} \tag{111}$$

Similarly, 19<sup>th</sup>, 20<sup>th</sup>, 33<sup>rd</sup> and 34<sup>th</sup> terms of right hand side of equation (73),  
19<sup>th</sup> term,

$$\begin{aligned}
 &\left\langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \right. \\
 &\quad \left. \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \right\rangle \times 2 \in_{m\alpha\beta} \Omega_m u_\alpha^{(2)} \frac{\partial}{\partial v_\alpha^{(2)}} \delta(u^{(2)} - v^{(2)}) \rangle \\
 &= 2 \in_{m\alpha\beta} \Omega_m F_3^{(1,2,3)}
 \end{aligned} \tag{112}$$

20<sup>th</sup> term,

$$\begin{aligned}
 &\left\langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \right. \\
 &\quad \left. \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \right\rangle \times f(u_\alpha^{(2)} - v_\alpha^{(2)}) \frac{\partial}{\partial v_\alpha^{(2)}} \delta(u^{(2)} - v^{(2)}) \rangle \\
 &= -f(u_\alpha^{(2)} - v_\alpha^{(2)}) \frac{\partial}{\partial v_\alpha^{(2)}} F_3^{(1,2,3)}
 \end{aligned} \tag{113}$$

33<sup>rd</sup> term,

$$\begin{aligned}
 &\left\langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \right. \\
 &\quad \left. \delta(c^{(2)} - \psi^{(2)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \right\rangle \times 2 \in_{m\alpha\beta} \Omega_m u_\alpha^{(3)} \frac{\partial}{\partial v_\alpha^{(3)}} \delta(u^{(3)} - v^{(3)}) \rangle \\
 &= 2 \in_{m\alpha\beta} \Omega_m F_3^{(1,2,3)}
 \end{aligned} \tag{114}$$

34<sup>th</sup> term,

$$\begin{aligned}
 &\left\langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \right. \\
 &\quad \left. \delta(c^{(2)} - \psi^{(2)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \right\rangle \times f(u_\alpha^{(3)} - v_\alpha^{(3)}) \frac{\partial}{\partial v_\alpha^{(3)}} \delta(u^{(3)} - v^{(3)}) \rangle \\
 &= -f(u_\alpha^{(3)} - v_\alpha^{(3)}) \frac{\partial}{\partial v_\alpha^{(3)}} F_3^{(1,2,3)}
 \end{aligned} \tag{115}$$

14<sup>th</sup> term of Equation (73)

$$\begin{aligned}
 &\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\
 &\quad \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times R c^{(1)} \frac{\partial}{\partial \psi^{(1)}} \delta(c^{(1)} - \psi^{(1)}) \rangle \\
 &= R \psi^{(1)} \frac{\partial}{\partial \psi^{(1)}} F_3^{(1,2,3)}
 \end{aligned} \tag{116}$$

28<sup>th</sup> term of Equation (73)

$$\begin{aligned} & \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\ & \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times Rc^{(2)} \frac{\partial}{\partial \psi^{(2)}} \delta(c^{(2)} - \psi^{(2)}) \rangle \\ & = R\psi^{(2)} \frac{\partial}{\partial \psi^{(2)}} F_3^{(1,2,3)} \end{aligned} \quad (117)$$

42<sup>nd</sup> term of Equation (73)

$$\begin{aligned} & \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\ & \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \times Rc^{(3)} \frac{\partial}{\partial \psi^{(3)}} \delta(c^{(3)} - \psi^{(3)}) \rangle \\ & = R\psi^{(3)} \frac{\partial}{\partial \psi^{(3)}} F_3^{(1,2,3)} \end{aligned} \quad (118)$$

### III. RESULTS

Substituting the results (74) – (118) in equation (73) we get the transport equation for three- point distribution function  $F_3^{(1,2,3)}(v, g, \phi, \psi)$  in MHD turbulent flow in a rotating system in presence of dust particles under going a first order reaction as

$$\begin{aligned} & \frac{\partial F_3^{(1,2,3)}}{\partial t} + \left( v_\beta^{(1)} \frac{\partial}{\partial x_\beta^{(1)}} + v_\beta^{(2)} \frac{\partial}{\partial x_\beta^{(2)}} v_\beta^{(3)} \frac{\partial}{\partial x_\beta^{(3)}} \right) F_3^{(1,2,3)} + \left[ g_\beta^{(1)} \left( \frac{\partial g_\alpha^{(1)}}{\partial v_\alpha^{(1)}} + \frac{\partial v_\alpha^{(1)}}{\partial g_\alpha^{(1)}} \right) \frac{\partial}{\partial x_\beta^{(1)}} \right. \\ & + g_\beta^{(2)} \left( \frac{\partial g_\alpha^{(2)}}{\partial v_\alpha^{(2)}} + \frac{\partial v_\alpha^{(2)}}{\partial g_\alpha^{(2)}} \right) \frac{\partial}{\partial x_\beta^{(2)}} + g_\beta^{(3)} \left( \frac{\partial g_\alpha^{(3)}}{\partial v_\alpha^{(3)}} + \frac{\partial v_\alpha^{(3)}}{\partial g_\alpha^{(3)}} \right) \frac{\partial}{\partial x_\beta^{(3)}} \left. \right] F_3^{(1,2,3)} \\ & + v \left( \frac{\partial}{\partial v_\alpha^{(1)}} \lim_{\bar{x}(4) \rightarrow \bar{x}(1)} + \frac{\partial}{\partial v_\alpha^{(2)}} \lim_{\bar{x}(4) \rightarrow \bar{x}(2)} + \frac{\partial}{\partial v_\alpha^{(3)}} \lim_{\bar{x}(4) \rightarrow \bar{x}(3)} \right) \\ & \times \frac{\partial^2}{\partial x_\beta^{(4)} \partial x_\beta^{(4)}} \int v_\alpha^{(4)} F_4^{(1,2,3,4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)} \\ & + \lambda \left( \frac{\partial}{\partial g_\alpha^{(1)}} \lim_{\bar{x}(4) \rightarrow \bar{x}(1)} + \frac{\partial}{\partial g_\alpha^{(2)}} \lim_{\bar{x}(4) \rightarrow \bar{x}(2)} + \frac{\partial}{\partial g_\alpha^{(3)}} \lim_{\bar{x}(4) \rightarrow \bar{x}(3)} \right) \\ & \times \frac{\partial^2}{\partial x_\beta^{(4)} \partial x_\beta^{(4)}} \int g_\alpha^{(4)} F_4^{(1,2,3,4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)} \\ & + \gamma \left( \frac{\partial}{\partial \phi^{(1)}} \lim_{\bar{x}(4) \rightarrow \bar{x}(1)} + \frac{\partial}{\partial \phi^{(2)}} \lim_{\bar{x}(4) \rightarrow \bar{x}(2)} + \frac{\partial}{\partial \phi^{(3)}} \lim_{\bar{x}(4) \rightarrow \bar{x}(3)} \right) \\ & \times \frac{\partial^2}{\partial x_\beta^{(4)} \partial x_\beta^{(4)}} \int \phi^{(4)} F_4^{(1,2,3,4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)} \end{aligned}$$

$$\begin{aligned}
 & + D \left( \frac{\partial}{\partial \psi^{(1)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(1)}} + \frac{\partial}{\partial \psi^{(2)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(2)}} + \frac{\partial}{\partial \psi^{(3)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(3)}} \right) \\
 & \times \frac{\partial^2}{\partial x_\beta^{(4)} \partial x_\beta^{(4)}} \int \psi^{(4)} F_4^{(1,2,3,4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)} \\
 & - \left[ \frac{\partial}{\partial v_\alpha^{(1)}} \left\{ \frac{1}{4\pi} \int \frac{\partial}{\partial x_\alpha^{(1)}} \left( \frac{1}{|\bar{x}^{(4)} - \bar{x}^{(1)}|} \right) \right\} + \frac{\partial}{\partial v_\alpha^{(2)}} \left\{ \frac{1}{4\pi} \int \frac{\partial}{\partial x_\alpha^{(2)}} \left( \frac{1}{|\bar{x}^{(4)} - \bar{x}^{(2)}|} \right) \right\} \right. \\
 & + \frac{\partial}{\partial v_\alpha^{(3)}} \left\{ \frac{1}{4\pi} \int \frac{\partial}{\partial x_\alpha^{(3)}} \left( \frac{1}{|\bar{x}^{(4)} - \bar{x}^{(3)}|} \right) \right\} \times \left( \frac{\partial v_\alpha^{(4)}}{\partial x_\beta^{(4)}} \frac{\partial v_\beta^{(4)}}{\partial x_\alpha^{(4)}} - \frac{\partial g_\alpha^{(4)}}{\partial x_\beta^{(4)}} \frac{\partial g_\beta^{(4)}}{\partial x_\alpha^{(4)}} \right) F_4^{(1,2,3,4)} \\
 & \left. \times dx^{(4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)} \right] + 6 \epsilon_{m\alpha\beta} \Omega_m F_3^{(1,2,3)} \\
 & + \left[ f \left( u_\alpha^{(1)} - v_\alpha^{(1)} \right) \frac{\partial}{\partial v_\alpha^{(1)}} + f \left( u_\alpha^{(2)} - v_\alpha^{(2)} \right) \frac{\partial}{\partial v_\alpha^{(2)}} + f \left( u_\alpha^{(3)} - v_\alpha^{(3)} \right) \frac{\partial}{\partial v_\alpha^{(3)}} \right] F_3^{(1,2,3)} \\
 & - R(\psi^{(1)} \frac{\partial}{\partial \psi^{(1)}} + \psi^{(2)} \frac{\partial}{\partial \psi^{(2)}} + \psi^{(3)} \frac{\partial}{\partial \psi^{(3)}}) F_3^{(1,2,3)} = 0 \tag{119}
 \end{aligned}$$

Continuing this way, we can derive the equations for evolution of  $F_4^{(1,2,3,4)}$ ,  $F_5^{(1,2,3,4,5)}$  and so on. Logically it is possible to have an equation for every  $F_n$  ( $n$  is an integer) but the system of equations so obtained is not closed. Certain approximations will be required thus obtained.

#### IV. DISCUSSIONS

If  $R=0$ , i.e the reaction rate is absent, the transport equation for three- point distribution function in MHD turbulent flow (119) becomes

$$\begin{aligned}
 & \frac{\partial F_3^{(1,2,3)}}{\partial t} + \left( v_\beta^{(1)} \frac{\partial}{\partial x_\beta^{(1)}} + v_\beta^{(2)} \frac{\partial}{\partial x_\beta^{(2)}} v_\beta^{(3)} \frac{\partial}{\partial x_\beta^{(3)}} \right) F_3^{(1,2,3)} + \left[ g_\beta^{(1)} \left( \frac{\partial g_\alpha^{(1)}}{\partial v_\alpha^{(1)}} + \frac{\partial v_\alpha^{(1)}}{\partial g_\alpha^{(1)}} \right) \frac{\partial}{\partial x_\beta^{(1)}} \right. \\
 & + g_\beta^{(2)} \left( \frac{\partial g_\alpha^{(2)}}{\partial v_\alpha^{(2)}} + \frac{\partial v_\alpha^{(2)}}{\partial g_\alpha^{(2)}} \right) \frac{\partial}{\partial x_\beta^{(2)}} + g_\beta^{(3)} \left( \frac{\partial g_\alpha^{(3)}}{\partial v_\alpha^{(3)}} + \frac{\partial v_\alpha^{(3)}}{\partial g_\alpha^{(3)}} \right) \frac{\partial}{\partial x_\beta^{(3)}} \left. \right] F_3^{(1,2,3)} \\
 & + v \left( \frac{\partial}{\partial v_\alpha^{(1)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(1)}} + \frac{\partial}{\partial v_\alpha^{(2)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(2)}} + \frac{\partial}{\partial v_\alpha^{(3)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(3)}} \right) \\
 & \times \frac{\partial^2}{\partial x_\beta^{(4)} \partial x_\beta^{(4)}} \int v_\alpha^{(4)} F_4^{(1,2,3,4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)} \\
 & + \lambda \left( \frac{\partial}{\partial g_\alpha^{(1)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(1)}} + \frac{\partial}{\partial g_\alpha^{(2)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(2)}} + \frac{\partial}{\partial g_\alpha^{(3)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(3)}} \right) \\
 & \times \frac{\partial^2}{\partial x_\beta^{(4)} \partial x_\beta^{(4)}} \int g_\alpha^{(4)} F_4^{(1,2,3,4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)}
 \end{aligned}$$

$$\begin{aligned}
 & + \gamma \left( \frac{\partial}{\partial \phi^{(1)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(1)}} + \frac{\partial}{\partial \phi^{(2)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(2)}} + \frac{\partial}{\partial \phi^{(3)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(3)}} \right) \\
 & \times \frac{\partial^2}{\partial x_{\beta}^{(4)} \partial x_{\beta}^{(4)}} \int \phi^{(4)} F_4^{(1,2,3,4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)} \\
 & + D \left( \frac{\partial}{\partial \psi^{(1)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(1)}} + \frac{\partial}{\partial \psi^{(2)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(2)}} + \frac{\partial}{\partial \psi^{(3)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(3)}} \right) \\
 & \times \frac{\partial^2}{\partial x_{\beta}^{(4)} \partial x_{\beta}^{(4)}} \int \psi^{(4)} F_4^{(1,2,3,4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)} \\
 & - \left[ \frac{\partial}{\partial v_{\alpha}^{(1)}} \left\{ \frac{1}{4\pi} \int \frac{\partial}{\partial x_{\alpha}^{(1)}} \left( \frac{1}{|\bar{x}^{(4)} - \bar{x}^{(1)}|} \right) \right\} + \frac{\partial}{\partial v_{\alpha}^{(2)}} \left\{ \frac{1}{4\pi} \int \frac{\partial}{\partial x_{\alpha}^{(2)}} \left( \frac{1}{|\bar{x}^{(4)} - \bar{x}^{(2)}|} \right) \right\} \right. \\
 & + \frac{\partial}{\partial v_{\alpha}^{(3)}} \left\{ \frac{1}{4\pi} \int \frac{\partial}{\partial x_{\alpha}^{(3)}} \left( \frac{1}{|\bar{x}^{(4)} - \bar{x}^{(3)}|} \right) \right\} \times \left( \frac{\partial v_{\alpha}^{(4)}}{\partial x_{\beta}^{(4)}} \frac{\partial v_{\beta}^{(4)}}{\partial x_{\alpha}^{(4)}} - \frac{\partial g_{\alpha}^{(4)}}{\partial x_{\beta}^{(4)}} \frac{\partial g_{\beta}^{(4)}}{\partial x_{\alpha}^{(4)}} \right) F_4^{(1,2,3,4)} \\
 & \left. \times dx^{(4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)} \right] + 6 \epsilon_{m\alpha\beta} \Omega_m F_3^{(1,2,3)} \\
 & + \left[ f \left( u_{\alpha}^{(1)} - v_{\alpha}^{(1)} \right) \frac{\partial}{\partial v_{\alpha}^{(1)}} + f \left( u_{\alpha}^{(2)} - v_{\alpha}^{(2)} \right) \frac{\partial}{\partial v_{\alpha}^{(2)}} + f \left( u_{\alpha}^{(3)} - v_{\alpha}^{(3)} \right) \frac{\partial}{\partial v_{\alpha}^{(3)}} \right] F_3^{(1,2,3)} = 0 \quad (120)
 \end{aligned}$$

which was obtained earlier by Azad et al (2015d)

If the fluid is clean then  $f=0$ , the transport equation for three-point distribution function in MHD turbulent flow (119) becomes

$$\begin{aligned}
 & \frac{\partial F_3^{(1,2,3)}}{\partial t} + \left( v_{\beta}^{(1)} \frac{\partial}{\partial x_{\beta}^{(1)}} + v_{\beta}^{(2)} \frac{\partial}{\partial x_{\beta}^{(2)}} v_{\beta}^{(3)} \frac{\partial}{\partial x_{\beta}^{(3)}} \right) F_3^{(1,2,3)} + \left[ g_{\beta}^{(1)} \left( \frac{\partial g_{\alpha}^{(1)}}{\partial v_{\alpha}^{(1)}} + \frac{\partial v_{\alpha}^{(1)}}{\partial g_{\alpha}^{(1)}} \right) \frac{\partial}{\partial x_{\beta}^{(1)}} \right. \\
 & + g_{\beta}^{(2)} \left( \frac{\partial g_{\alpha}^{(2)}}{\partial v_{\alpha}^{(2)}} + \frac{\partial v_{\alpha}^{(2)}}{\partial g_{\alpha}^{(2)}} \right) \frac{\partial}{\partial x_{\beta}^{(2)}} + g_{\beta}^{(3)} \left( \frac{\partial g_{\alpha}^{(3)}}{\partial v_{\alpha}^{(3)}} + \frac{\partial v_{\alpha}^{(3)}}{\partial g_{\alpha}^{(3)}} \right) \frac{\partial}{\partial x_{\beta}^{(3)}} \left. \right] F_3^{(1,2,3)} \\
 & + \nu \left( \frac{\partial}{\partial v_{\alpha}^{(1)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(1)}} + \frac{\partial}{\partial v_{\alpha}^{(2)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(2)}} + \frac{\partial}{\partial v_{\alpha}^{(3)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(3)}} \right) \\
 & \times \frac{\partial^2}{\partial x_{\beta}^{(4)} \partial x_{\beta}^{(4)}} \int v_{\alpha}^{(4)} F_4^{(1,2,3,4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)} \\
 & + \lambda \left( \frac{\partial}{\partial g_{\alpha}^{(1)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(1)}} + \frac{\partial}{\partial g_{\alpha}^{(2)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(2)}} + \frac{\partial}{\partial g_{\alpha}^{(3)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(3)}} \right) \\
 & \times \frac{\partial^2}{\partial x_{\beta}^{(4)} \partial x_{\beta}^{(4)}} \int g_{\alpha}^{(4)} F_4^{(1,2,3,4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)}
 \end{aligned}$$

$$\begin{aligned}
 & + \gamma \left( \frac{\partial}{\partial \phi^{(1)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(1)}} + \frac{\partial}{\partial \phi^{(2)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(2)}} + \frac{\partial}{\partial \phi^{(3)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(3)}} \right) \\
 & \times \frac{\partial^2}{\partial x_\beta^{(4)} \partial x_\beta^{(4)}} \int \phi^{(4)} F_4^{(1,2,3,4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)} \\
 & + D \left( \frac{\partial}{\partial \psi^{(1)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(1)}} + \frac{\partial}{\partial \psi^{(2)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(2)}} + \frac{\partial}{\partial \psi^{(3)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(3)}} \right) \\
 & \times \frac{\partial^2}{\partial x_\beta^{(4)} \partial x_\beta^{(4)}} \int \psi^{(4)} F_4^{(1,2,3,4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)} \\
 & - \left[ \frac{\partial}{\partial v_\alpha^{(1)}} \left\{ \frac{1}{4\pi} \int \frac{\partial}{\partial x_\alpha^{(1)}} \left( \frac{1}{|\bar{x}^{(4)} - \bar{x}^{(1)}|} \right) \right\} + \frac{\partial}{\partial v_\alpha^{(2)}} \left\{ \frac{1}{4\pi} \int \frac{\partial}{\partial x_\alpha^{(2)}} \left( \frac{1}{|\bar{x}^{(4)} - \bar{x}^{(2)}|} \right) \right\} \right. \\
 & + \frac{\partial}{\partial v_\alpha^{(3)}} \left\{ \frac{1}{4\pi} \int \frac{\partial}{\partial x_\alpha^{(3)}} \left( \frac{1}{|\bar{x}^{(4)} - \bar{x}^{(3)}|} \right) \right\} \times \left( \frac{\partial v_\alpha^{(4)}}{\partial x_\beta^{(4)}} \frac{\partial v_\beta^{(4)}}{\partial x_\alpha^{(4)}} - \frac{\partial g_\alpha^{(4)}}{\partial x_\beta^{(4)}} \frac{\partial g_\beta^{(4)}}{\partial x_\alpha^{(4)}} \right) F_4^{(1,2,3,4)} \\
 & \left. \times dx^{(4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)} \right] + 6 \epsilon_{m\alpha\beta} \Omega_m F_3^{(1,2,3)} = 0 \quad (121)
 \end{aligned}$$

This was obtained earlier by Azad et al (2014b)

In the absence of coriolis force,  $\Omega_m = 0$ , the transport equation for three- point distribution function in MHD turbulent flow (116) becomes

$$\begin{aligned}
 & \frac{\partial F_3^{(1,2,3)}}{\partial t} + \left( v_\beta^{(1)} \frac{\partial}{\partial x_\beta^{(1)}} + v_\beta^{(2)} \frac{\partial}{\partial x_\beta^{(2)}} v_\beta^{(3)} \frac{\partial}{\partial x_\beta^{(3)}} \right) F_3^{(1,2,3)} + \left[ g_\beta^{(1)} \left( \frac{\partial g_\alpha^{(1)}}{\partial v_\alpha^{(1)}} + \frac{\partial v_\alpha^{(1)}}{\partial g_\alpha^{(1)}} \right) \frac{\partial}{\partial x_\beta^{(1)}} \right. \\
 & + g_\beta^{(2)} \left( \frac{\partial g_\alpha^{(2)}}{\partial v_\alpha^{(2)}} + \frac{\partial v_\alpha^{(2)}}{\partial g_\alpha^{(2)}} \right) \frac{\partial}{\partial x_\beta^{(2)}} + g_\beta^{(3)} \left( \frac{\partial g_\alpha^{(3)}}{\partial v_\alpha^{(3)}} + \frac{\partial v_\alpha^{(3)}}{\partial g_\alpha^{(3)}} \right) \frac{\partial}{\partial x_\beta^{(3)}} \left. \right] F_3^{(1,2,3)} \\
 & + \nu \left( \frac{\partial}{\partial v_\alpha^{(1)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(1)}} + \frac{\partial}{\partial v_\alpha^{(2)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(2)}} + \frac{\partial}{\partial v_\alpha^{(3)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(3)}} \right) \\
 & \times \frac{\partial^2}{\partial x_\beta^{(4)} \partial x_\beta^{(4)}} \int v_\alpha^{(4)} F_4^{(1,2,3,4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)} \\
 & + \lambda \left( \frac{\partial}{\partial g_\alpha^{(1)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(1)}} + \frac{\partial}{\partial g_\alpha^{(2)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(2)}} + \frac{\partial}{\partial g_\alpha^{(3)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(3)}} \right) \\
 & \times \frac{\partial^2}{\partial x_\beta^{(4)} \partial x_\beta^{(4)}} \int g_\alpha^{(4)} F_4^{(1,2,3,4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)}
 \end{aligned}$$

$$\begin{aligned}
 & + \gamma \left( \frac{\partial}{\partial \phi^{(1)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(1)}} + \frac{\partial}{\partial \phi^{(2)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(2)}} + \frac{\partial}{\partial \phi^{(3)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(3)}} \right) \\
 & \times \frac{\partial^2}{\partial x_{\beta}^{(4)} \partial x_{\beta}^{(4)}} \int \phi^{(4)} F_4^{(1,2,3,4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)} \\
 & + D \left( \frac{\partial}{\partial \psi^{(1)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(1)}} + \frac{\partial}{\partial \psi^{(2)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(2)}} + \frac{\partial}{\partial \psi^{(3)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(3)}} \right) \\
 & \times \frac{\partial^2}{\partial x_{\beta}^{(4)} \partial x_{\beta}^{(4)}} \int \psi^{(4)} F_4^{(1,2,3,4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)} \\
 & - \left[ \frac{\partial}{\partial v_{\alpha}^{(1)}} \left\{ \frac{1}{4\pi} \int \frac{\partial}{\partial x_{\alpha}^{(1)}} \left( \frac{1}{|\bar{x}^{(4)} - \bar{x}^{(1)}|} \right) \right\} + \frac{\partial}{\partial v_{\alpha}^{(2)}} \left\{ \frac{1}{4\pi} \int \frac{\partial}{\partial x_{\alpha}^{(2)}} \left( \frac{1}{|\bar{x}^{(4)} - \bar{x}^{(2)}|} \right) \right\} \right. \\
 & + \frac{\partial}{\partial v_{\alpha}^{(3)}} \left\{ \frac{1}{4\pi} \int \frac{\partial}{\partial x_{\alpha}^{(3)}} \left( \frac{1}{|\bar{x}^{(4)} - \bar{x}^{(3)}|} \right) \right\} \times \left( \frac{\partial v_{\alpha}^{(4)}}{\partial x_{\beta}^{(4)}} \frac{\partial v_{\beta}^{(4)}}{\partial x_{\alpha}^{(4)}} - \frac{\partial g_{\alpha}^{(4)}}{\partial x_{\beta}^{(4)}} \frac{\partial g_{\beta}^{(4)}}{\partial x_{\alpha}^{(4)}} \right) F_4^{(1,2,3,4)} \\
 & \left. \times dx^{(4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)} \right] \\
 & + \left[ f \left( u_{\alpha}^{(1)} - v_{\alpha}^{(1)} \right) \frac{\partial}{\partial v_{\alpha}^{(1)}} + f \left( u_{\alpha}^{(2)} - v_{\alpha}^{(2)} \right) \frac{\partial}{\partial v_{\alpha}^{(2)}} + f \left( u_{\alpha}^{(3)} - v_{\alpha}^{(3)} \right) \frac{\partial}{\partial v_{\alpha}^{(3)}} \right] F_3^{(1,2,3)} = 0
 \end{aligned} \tag{122}$$

This was obtained earlier by N. M. Islam et al (2014)

If the fluid is clean and the system is nonrotating then  $f=0$ ,  $\Omega_m=0$  and the reaction rate is absent,  $R=0$ , the transport equation for three- point distribution function in MHD turbulent flow (119) becomes

$$\begin{aligned}
 & \frac{\partial F_3^{(1,2,3)}}{\partial t} + \left( v_{\beta}^{(1)} \frac{\partial}{\partial x_{\beta}^{(1)}} + v_{\beta}^{(2)} \frac{\partial}{\partial x_{\beta}^{(2)}} v_{\beta}^{(3)} \frac{\partial}{\partial x_{\beta}^{(3)}} \right) F_3^{(1,2,3)} + \left[ g_{\beta}^{(1)} \left( \frac{\partial g_{\alpha}^{(1)}}{\partial v_{\alpha}^{(1)}} + \frac{\partial v_{\alpha}^{(1)}}{\partial g_{\alpha}^{(1)}} \right) \frac{\partial}{\partial x_{\beta}^{(1)}} \right. \\
 & + g_{\beta}^{(2)} \left( \frac{\partial g_{\alpha}^{(2)}}{\partial v_{\alpha}^{(2)}} + \frac{\partial v_{\alpha}^{(2)}}{\partial g_{\alpha}^{(2)}} \right) \frac{\partial}{\partial x_{\beta}^{(2)}} + g_{\beta}^{(3)} \left( \frac{\partial g_{\alpha}^{(3)}}{\partial v_{\alpha}^{(3)}} + \frac{\partial v_{\alpha}^{(3)}}{\partial g_{\alpha}^{(3)}} \right) \frac{\partial}{\partial x_{\beta}^{(3)}} \left. \right] F_3^{(1,2,3)} \\
 & + \nu \left( \frac{\partial}{\partial v_{\alpha}^{(1)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(1)}} + \frac{\partial}{\partial v_{\alpha}^{(2)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(2)}} + \frac{\partial}{\partial v_{\alpha}^{(3)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(3)}} \right) \\
 & \times \frac{\partial^2}{\partial x_{\beta}^{(4)} \partial x_{\beta}^{(4)}} \int v_{\alpha}^{(4)} F_4^{(1,2,3,4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)} \\
 & + \lambda \left( \frac{\partial}{\partial g_{\alpha}^{(1)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(1)}} + \frac{\partial}{\partial g_{\alpha}^{(2)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(2)}} + \frac{\partial}{\partial g_{\alpha}^{(3)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(3)}} \right) \\
 & \times \frac{\partial^2}{\partial x_{\beta}^{(4)} \partial x_{\beta}^{(4)}} \int g_{\alpha}^{(4)} F_4^{(1,2,3,4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)}
 \end{aligned}$$

$$\begin{aligned}
 & + \gamma \left( \frac{\partial}{\partial \phi^{(1)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(1)}} + \frac{\partial}{\partial \phi^{(2)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(2)}} + \frac{\partial}{\partial \phi^{(3)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(3)}} \right) \\
 & \times \frac{\partial^2}{\partial x_\beta^{(4)} \partial x_\beta^{(4)}} \int \phi^{(4)} F_4^{(1,2,3,4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)} \\
 & + D \left( \frac{\partial}{\partial \psi^{(1)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(1)}} + \frac{\partial}{\partial \psi^{(2)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(2)}} + \frac{\partial}{\partial \psi^{(3)}} \lim_{\bar{x}^{(4)} \rightarrow \bar{x}^{(3)}} \right) \\
 & \times \frac{\partial^2}{\partial x_\beta^{(4)} \partial x_\beta^{(4)}} \int \psi^{(4)} F_4^{(1,2,3,4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)} \\
 & - \left[ \frac{\partial}{\partial v_\alpha^{(1)}} \left\{ \frac{1}{4\pi} \int \frac{\partial}{\partial x_\alpha^{(1)}} \left( \frac{1}{|\bar{x}^{(4)} - \bar{x}^{(1)}|} \right) \right\} + \frac{\partial}{\partial v_\alpha^{(2)}} \left\{ \frac{1}{4\pi} \int \frac{\partial}{\partial x_\alpha^{(2)}} \left( \frac{1}{|\bar{x}^{(4)} - \bar{x}^{(2)}|} \right) \right\} \right. \\
 & + \frac{\partial}{\partial v_\alpha^{(3)}} \left\{ \frac{1}{4\pi} \int \frac{\partial}{\partial x_\alpha^{(3)}} \left( \frac{1}{|\bar{x}^{(4)} - \bar{x}^{(3)}|} \right) \right\} \times \left( \frac{\partial v_\alpha^{(4)}}{\partial x_\beta^{(4)}} \frac{\partial v_\beta^{(4)}}{\partial x_\alpha^{(4)}} - \frac{\partial g_\alpha^{(4)}}{\partial x_\beta^{(4)}} \frac{\partial g_\beta^{(4)}}{\partial x_\alpha^{(4)}} \right) F_4^{(1,2,3,4)} \\
 & \left. \times dx^{(4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)} \right] = 0
 \end{aligned} \tag{123}$$

It was obtained earlier by Azad et al (2014a)

If we drop the viscous, magnetic and thermal diffusive and concentration terms from the three point evolution equation (119), we have

$$\begin{aligned}
 & \frac{\partial F_3^{(1,2,3)}}{\partial t} + \left( v_\beta^{(1)} \frac{\partial}{\partial x_\beta^{(1)}} + v_\beta^{(2)} \frac{\partial}{\partial x_\beta^{(2)}} v_\beta^{(3)} \frac{\partial}{\partial x_\beta^{(3)}} \right) F_3^{(1,2,3)} + \left[ g_\beta^{(1)} \left( \frac{\partial g_\alpha^{(1)}}{\partial v_\alpha^{(1)}} + \frac{\partial v_\alpha^{(1)}}{\partial g_\alpha^{(1)}} \right) \frac{\partial}{\partial x_\beta^{(1)}} \right. \\
 & + g_\beta^{(2)} \left( \frac{\partial g_\alpha^{(2)}}{\partial v_\alpha^{(2)}} + \frac{\partial v_\alpha^{(2)}}{\partial g_\alpha^{(2)}} \right) \frac{\partial}{\partial x_\beta^{(2)}} + g_\beta^{(3)} \left( \frac{\partial g_\alpha^{(3)}}{\partial v_\alpha^{(3)}} + \frac{\partial v_\alpha^{(3)}}{\partial g_\alpha^{(3)}} \right) \frac{\partial}{\partial x_\beta^{(3)}} \left. \right] F_3^{(1,2,3)} \\
 & - \left[ \frac{\partial}{\partial v_\alpha^{(1)}} \left\{ \frac{1}{4\pi} \int \frac{\partial}{\partial x_\alpha^{(1)}} \left( \frac{1}{|\bar{x}^{(4)} - \bar{x}^{(1)}|} \right) \right\} + \frac{\partial}{\partial v_\alpha^{(2)}} \left\{ \frac{1}{4\pi} \int \frac{\partial}{\partial x_\alpha^{(2)}} \left( \frac{1}{|\bar{x}^{(4)} - \bar{x}^{(2)}|} \right) \right\} \right. \\
 & + \frac{\partial}{\partial v_\alpha^{(3)}} \left\{ \frac{1}{4\pi} \int \frac{\partial}{\partial x_\alpha^{(3)}} \left( \frac{1}{|\bar{x}^{(4)} - \bar{x}^{(3)}|} \right) \right\} \times \left( \frac{\partial v_\alpha^{(4)}}{\partial x_\beta^{(4)}} \frac{\partial v_\beta^{(4)}}{\partial x_\alpha^{(4)}} - \frac{\partial g_\alpha^{(4)}}{\partial x_\beta^{(4)}} \frac{\partial g_\beta^{(4)}}{\partial x_\alpha^{(4)}} \right) F_4^{(1,2,3,4)} \\
 & \left. \times dx^{(4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)} \right] = 0
 \end{aligned} \tag{124}$$

The existence of the term

$$\left( \frac{\partial g_\alpha^{(1)}}{\partial v_\alpha^{(1)}} + \frac{\partial v_\alpha^{(1)}}{\partial g_\alpha^{(1)}} \right), \left( \frac{\partial g_\alpha^{(2)}}{\partial v_\alpha^{(2)}} + \frac{\partial v_\alpha^{(2)}}{\partial g_\alpha^{(2)}} \right) \text{ and } \left( \frac{\partial g_\alpha^{(3)}}{\partial v_\alpha^{(3)}} + \frac{\partial v_\alpha^{(3)}}{\partial g_\alpha^{(3)}} \right)$$

can be explained on the basis that two characteristics of the flow field are related to each other and describe the interaction between the two modes (velocity and magnetic) at point  $x^{(1)}$ ,  $x^{(2)}$  and  $x^{(3)}$ .

We can exhibit an analogy of this equation with the 1<sup>st</sup> equation in BBGKY hierarchy in the kinetic theory of gases. The first equation of BBGKY hierarchy is given Lundgren (1969) as

$$\frac{\partial F_1^{(1)}}{\partial t} + \frac{1}{m} v_\beta^{(1)} \frac{\partial}{\partial x_\beta^{(1)}} F_1^{(1)} = n \iint \frac{\partial \psi_{1,2}}{\partial x_\alpha^{(1)}} \frac{\partial F_2^{(1,2)}}{\partial v_\alpha^{(1)}} d\mathbf{x}^{(2)} d\mathbf{v}^{(2)} \quad (125)$$

where  $\psi_{1,2} = \psi \left| v_\alpha^{(2)} - v_\alpha^{(1)} \right|$  is the inter molecular potential.

Some approximations are required, if we want to close the system of equations for the distribution functions. In the case of collection of ionized particles, i.e. in plasma turbulence, it can be provided closure form easily by decomposing  $F_2^{(1,2)}$  as  $F_1^{(1)} F_1^{(2)}$ . But it will be possible if there is no interaction or correlation between two particles. If we decompose  $F_2^{(1,2)}$  as

$$F_2^{(1,2)} = (1 + \epsilon) F_1^{(1)} F_1^{(2)}$$

and

$$F_3^{(1,2,3)} = (1 + \epsilon)^2 F_1^{(1)} F_1^{(2)} F_1^{(3)}$$

Also

$$F_4^{(1,2,3,4)} = (1 + \epsilon)^3 F_1^{(1)} F_1^{(2)} F_1^{(3)} F_1^{(4)}$$

where  $\epsilon$  is the correlation coefficient between the particles. If there is no correlation between the particles,  $\epsilon$  will be zero and distribution function can be decomposed in usual way. Here we are considering such type of approximation only to provide closed form of the equation.

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## Diamagnetism Flame. Faraday Mystery

By Yuri Pivovarenko

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**Abstract-** Earth's surface acts the electromagnetic force, which distributes charges. Under action this force the positive charges are moved up and negative charges – down. Especially, this force distributes the electric charges generated in flame. For this reason, in the flames generate electric currents. So the flames interact with the artificial magnetic fields like the typical electric currents. Proposed calculation show that the true cause of diamagnetism flame is its movement in the geomagnetic field.

**Keywords:** *flame, magnetic field, flame in the magnetic field, and faraday mystery.*

**GJSFR-A Classification :** FOR Code: 040401



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# Diamagnetism Flame. Faraday Mystery

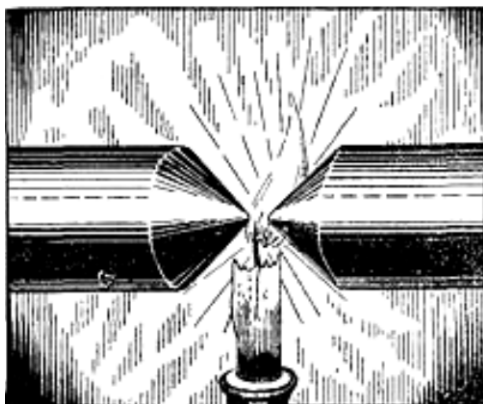
Yuri Pivovarenko

**Abstract-** Earth's surface acts the electromagnetic force, which distributes charges. Under action this force the positive charges are moved up and negative charges – down. Especially, this force distributes the electric charges generated in flame. For this reason, in the flames generate electric currents. So the flames interact with the artificial magnetic fields like the typical electric currents. Proposed calculation show that the true cause of diamagnetism flame is its movement in the geomagnetic field.

**Keywords:** flame, magnetic field, flame in the magnetic field, and faraday mystery.

## I. INTRODUCTION

It is known that flame is pushed out of magnetic field (Fig. 1).



*Fig. 1 :* The flame is pushed out of magnetic field

What the nature of diamagnetism flame? – Faraday was the first who asked this question (Faraday, 1847).

Many researchers have tried to answer this question. However, the nature of this phenomenon remains unclear (Ueno at al., 1987; Baker at al., 2001; Khaldi at al., 2010).

Faraday answering the question, we were based on such facts:

1. The Earth has daily rotation. Since studied objects located on the earth's surface, they rotate together with it.
2. The Earth has a magnetic field, namely, the geomagnetic field.
3. Magnetic fields push out the wires with electrical currents (Kuchling, 1980).

Based only on these facts, we came to the following conclusions:

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During the daily rotation of Earth all objects on its surface cross the horizontal lines of force of the geomagnetic field (Fig. 2). For this reason, all such objects, including flames, operates electromagnetic nature force (in fact, it is the Lorentz force,  $F_L = q \cdot [v \cdot B]$ ).

Under action this force, the generated in flame positively charged particles move up and negatively charged particles – down (Pivovarenko, 2015).



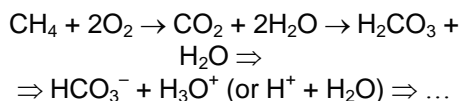
*Fig. 2 :* As the Earth has daily rotation all objects on its surface cross the horizontal lines of force of the geomagnetic field. The appeared electromagnetic force distributes the charges located near the earth's surface. Wherein the positively charged particles move up and negatively charged – down

Need to say separately, this force causes the evaporation of positively charged water vapor (Krasnogorskaja, 1984; Pivovarenko, 2015), providing the positive charge of the upper layers of Earth's atmosphere and negative charge of the earth's surface (Feinman at al., 1961; Pivovarenko, 2015).

This force causes the flow of charged particles of the flame, which interact with the artificial magnetic fields of the wire with an electric current. Here we propose our calculations.

## II. RESULTS AND DISCUSSION

The positively and negatively charged particles can be formed in many hydrocarbon flames. Because the composition of hydrocarbons is very variable, we offer our calculations made for methane flame:



(Here we don't take into account the further dissociation of carbonic acid:  $\text{HCO}_3^- \Rightarrow \text{H}^+ + \text{CO}_3^{2-}$ ).

It follows from this equation, of 1M methane (16 g) can theoretically be formed 1M of  $\text{HCO}_3^-$  with a total charge  $-96484,56 \text{ C}$  and 1M of  $\text{H}_3\text{O}^+$  with a total charge  $+96484,56 \text{ C}$ ; 96484,56 C is number of Faraday, F (Kuchling, 1980).

Based on our previous calculations (Pivovarenko, 2015), the Lorenz force,  $F_L$ , which acts on 1M of these charges, is:

$$|F_L| = \pm F \cdot |v_e| \cdot \mu_0 |H| =$$

$$\begin{aligned}&= \pm 9,648456 \cdot 10^4 \text{ A} \cdot \text{s} \cdot 463 \text{ m} \cdot \text{s}^{-1} \cdot 1,257 \cdot 10^{-6} \text{ V} \cdot \text{s} \cdot \text{A}^{-1} \cdot \text{m}^{-1} \cdot 27,06 \text{ A} \cdot \text{m}^{-1} = \sim 1519,8 \text{ kg} \cdot \text{m} \cdot \text{s}^{-2} \\ &= \sim 1,52 \cdot 10^3 \text{ N},\end{aligned}$$

where:  $\pm F$  ( $= \pm 9,648456 \cdot 10^4 \text{ A} \cdot \text{s}$ ) – the number of Faraday,

$|v_e|$  ( $= 463 \text{ m} \cdot \text{s}^{-1}$ ) – linear speed of earth's surface at equator,

$\mu_0$  ( $= 1,257 \cdot 10^{-6} \text{ V} \cdot \text{s} \cdot \text{A}^{-1} \cdot \text{m}^{-1}$ ) – magnetic constant,

$|H|$  ( $= 27,06 \text{ A} \cdot \text{m}^{-1}$ ) – intensity of geomagnetic field at equator.

This force distributes the charged particles formed in methane flame. For this reason, the negatively charged  $\text{HCO}_3^-$ -particles move downward acceleration, which is equal to:

$$\begin{aligned}|a| &= |F_L| / M(\text{HCO}_3^-) = 1,52 \cdot 10^3 \text{ kg} \cdot \text{m} \cdot \text{s}^{-2} / 61 \cdot 10^{-3} \text{ kg} = 24918 \text{ m} \cdot \text{s}^{-2},\end{aligned}$$

where:  $|F_L|$  ( $= 1,52 \cdot 10^3 \text{ N}$ ) – Lorenz force acting on the equator line,

$M(\text{HCO}_3^-)$  ( $= 61 \cdot 10^{-3} \text{ Kg}$ ) – mass 1 mole of  $\text{HCO}_3^-$ .

However, the positively charged  $\text{H}_3\text{O}^+$ -particles move upward acceleration, which is equals to:

$$\begin{aligned}|a| &= |F_L| / M(\text{H}_3\text{O}^+) = \\ &= 1,52 \cdot 10^3 \text{ kg} \cdot \text{m} \cdot \text{s}^{-2} / 19 \cdot 10^{-3} \text{ kg} = \\ &= 80000 \text{ m} \cdot \text{s}^{-2},\end{aligned}$$

where:  $|F_L|$  ( $= 1,52 \cdot 10^3 \text{ N}$ ) – Lorenz force acting on the equator line,

$M(\text{H}_3\text{O}^+)$  ( $= 19 \cdot 10^{-3} \text{ Kg}$ ) – mass 1 mole of  $\text{H}_3\text{O}^+$ .

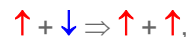
If the anions  $\text{HCO}_3^-$  are under vacuum, then for 1 second to overcome a distance greater than 12 km in down direction:

$$S = |a| t^2 / 2 = 24918 \text{ m} \cdot \text{s}^{-2} \cdot 1 \text{ s}^2 / 2 = 12459 \text{ m}.$$

If the  $\text{H}_3\text{O}^+$ -cations are under vacuum, then for 1 second to overcome a distance 40 km in up direction:

$$S = |a| t^2 / 2 = 80000 \text{ m} \cdot \text{s}^{-2} \cdot 1 \text{ s}^2 / 2 = 40000 \text{ m}.$$

Thus, the opposite charges form oppositely directed streams. At the same time, these form an electric current flows in one direction, the upwardly directed:



where:  $\uparrow$  – the current positive charges in up direction;

$\downarrow$  – the current negative charges in down direction.

Thus, assuming that 1M of methane (22,4 liters) will burn for 5 minutes (300 seconds), we theoretically obtain an electric current I directed upwards:

$$I = 2 \cdot 96484,56 \text{ C} / 300 \text{ s} = \sim 643,2 \text{ A}.$$

It is incredibly high current. But, as you know, carbonic acid is a weak: its first dissociation constant,  $K_1$  ( $20^\circ \text{C}$ )  $= [\text{H}^+][\text{HCO}_3^-] / [\text{H}_2\text{CO}_3] = 2 \cdot 10^{-4} \text{ M}$  (Nekrasov, 1974). Whereas this constant can be assumed that 1M of methane will be formed  $\sim 1,415 \cdot 10^{-2} \text{ M}$  of  $\text{H}^+$  (or  $\text{H}_3\text{O}^+$ ) and  $\sim 1,415 \cdot 10^{-2} \text{ M}$  of  $\text{HCO}_3^-$ .

Thus, given the constraint  $K_1$  ( $20^\circ \text{C}$ ), it may be formed of  $\sim 2 \cdot 1,415 \cdot 10^{-2} \text{ M} = \sim 2,83 \cdot 10^{-2} \text{ M}$  of charged particles. Therefore it can be concluded that the methane flame could be an electric current:

$$I = \sim 2,83 \cdot 10^{-2} \text{ M} \cdot 96484,56 \text{ C} / 300 \text{ s} = \sim 9,1 \text{ A}.$$

In any case, it is also a significant electric current. No surprisingly, the current is so strong interacts with an external magnetic field. (At the same time, we have ignored a significant increase in the dissociation of carbonic acid, which occurs with increasing temperature (Akolzin, 1988)

It should be noted that the distribution of charges in a hydrocarbon flames was probably used many years ago. So, the charge distribution in the flame of kerosene,  $\text{C}_9\text{H}_{20} - \text{C}_{16}\text{H}_{34}$  (Pacak, 1986), was used early to produce electric current sufficient for radio receivers (Fig. 3).

We know that the action of such a generator to explain the distribution of the electrons in the heated thermocouple. In our opinion, this explanation is incomplete because it does not take into account the distribution of charges in the flame of kerosene lamp.

In view of the proposed calculations, not also surprising that the flame has a spherical shape in space (Fig. 4): there is no movement of the flame relative to the geomagnetic field. In view of our assumptions, diamagnetism flame is not observed in the space. We think that it will be confirmed.



Fig. 3 : Charge distribution in the flame of a kerosene lamp can be used to obtain the electric current required to function the radio receiver.



Fig. 4 : It is the flame is in the space

It must be said that our ideas can also be used to explain other phenomena. In particular it is known, that pH of the water decreases when it is heated. So, it is known that increasing temperature of the water from 25 to 100 °C, followed by a decrease of the pH from 7.0 to 6.0 (Nekrasov, 1974). This could mean an increase in  $H^+$ -ion molar concentration 10 times ( $10^{-6}/10^{-7} = 10$ ). If the water is heated over an open fire, it can be hypothesized that such  $H^+$ -ions have the flame origin. Thus, increase acidity of boiling water caused its positive electrization  $H^+$ -ions generated in the flame.

### III. CONCLUSION

Objects on the earth's surface move together with it and cross the horizontal lines of force of the geomagnetic field. For this reason, the object of the earth's surface is always acting the force of electromagnetic nature. Under action this force the positive charges move up and negative charges – down. Thus, the flame generated electric currents are directed upwards. Therefore, the external magnetic fields act on the flames as on the wires with electric

currents. As a result, the flames pushed out magnetic fields.

Hence, the true cause of diamagnetism flame is its movement in the geomagnetic field.

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## Radiogenic Heat Production from Well Logs in Part of Niger Delta Sedimentary Basin, Nigeria

By Emujakporue, Godwin Omokenu

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**Abstract-** The radiogenic heat produced from radioactive elements has been investigated in some parts of Niger Delta sedimentary basin. The heat production was computed from gamma ray logs for three oil producing wells. The major lithology observed in the gamma ray log is alternation of sand and shale. The computed value ranges between  $0.35$  to  $2.0 \mu\text{Wm}^{-3}$  for well 1,  $0.34$  to  $1.78 \mu\text{Wm}^{-3}$  for well 2 and  $0.24$  to  $2.0 \mu\text{Wm}^{-3}$  for well 3. The average radiogenic heat production ranges between  $0.3$  to  $1.93 \mu\text{Wm}^{-3}$ . It was observed that the heat production within the sand lithology ranges from  $0.24$  to  $0.7 \mu\text{Wm}^{-3}$  while the computed value for the shale lithology ranges from  $0.8$  to  $2.0 \mu\text{Wm}^{-3}$ . The high radiogenic heat production in shale was as a result of high concentration of radioactive elements. The sandstone zone was deficient of radioactive elements and this resulted in the low value. The plot of depth against radiogenic heat production was scattered and it also showed an increase with depth. The increase was due to high shale lithology at greater depth of the Agbada Formation and the shale in the Akata Formation. A linear relationship was established for the radiogenic heat production and the product of gamma ray and density logs for the study Area. The relationship can be used if gamma ray spectral logs are not available.

**Keywords:** radioactive elements, heat production, well logs, niger delta, radiogenic heat.

**GJSFR-A Classification :** FOR Code: 029999



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## I. INTRODUCTION

Geothermal data have been used widely by geoscientists to provide information on temperature variations within the earth and thermal history of a sedimentary basin. Thermal history of a sedimentary basin is very important in the analysis of the organic compound maturity. The maturity of organic compound can be described as a gradual process which involves the release of hydrocarbons from buried organic materials (Beards more and Cull, 2001; Deming, 1994). Of the heat observed flowing out through the surface of the Earth about 40% originates within the outer crust (Pollack and Chapman, 1977). The heat produced by radioactive isotopic decay forms the dominant part, but there are also contributions from the friction of in traplate strain and plate motions, and heats from exothermic, metamorphic, diagenetic and other processes (Hamza and Beck, 1972).

Most heat flow analysis of sedimentary basins do not take into account the contribution from radioactive elements. Heat generated by the decay of

naturally radioactive elements in the earth's crust contributes significantly to terrestrial heat flow. Therefore the amount of heat flow in a basin should be constrained with the radiogenic heat production in order to provide a better understanding of the total heat budget (Biicker and Rybach, 1996). Heat is generated in rocks through the radioactive decay of unstable isotopes that release energy in the form of alpha, beta and gamma particles, neutrinos and antineutrinos through the decay of radioactive elements such as uranium ( $^{238}\text{U}$ ), thorium ( $^{232}\text{Th}$ ) and potassium ( $^{40}\text{K}$ ). Rocks are transparent to neutrinos and antineutrinos such that most of the energy from these particles is lost into space. However, the surrounding rocks absorb the kinetic energy associated with the other particles which is later converted to radiogenic heat (Rybach, 1986). The energy released by the decay of the uranium isotope is far greater than that released by thorium, which in turn is greater than that of potassium. However, the relative contributions of each isotope to the total heat generation are of the same order of magnitude due to their relative abundance in crustal rocks. Radiogenic heat production can be estimated from the natural gamma spectrometer (NGS) and/or gamma ray logs. The natural gamma spectrometer records the total count and frequency distribution of gamma rays and uses the data to estimate the absolute abundance by mass of each gamma-producing element in the rocks. The results are generally output as three logs - URAN (ppm uranium), THOR (ppm thorium) and POTA (percentage potassium). The gamma-ray energy spectrum emitted from a rock is the sum of the individual characteristic spectra of the radiogenic components. The objectives of this research are to determine the radiogenic heat generation in some hydrocarbon exploratory oil wells and to establish a relationship between radiogenic heat production and the product of gamma ray and density logs in part of Niger delta sedimentary basin.

## II. SUMMARY OF THE GEOLOGY OF THE NIGER DELTA

The study area is located within the Niger Delta sedimentary basin. Figure 1 is a map of the Niger Delta showing study area. The Niger Delta is the youngest sedimentary basin within the Benue Trough system. Its development began after the Eocene tectonic phase (Ekweozor and Daukoru, 1994; Doust and Omatsola, 1990). Up to 12km of deltaic and shallow marine

sediments have been accumulated in the basin. The Niger and Benue Rivers is the main supplier of sediments.

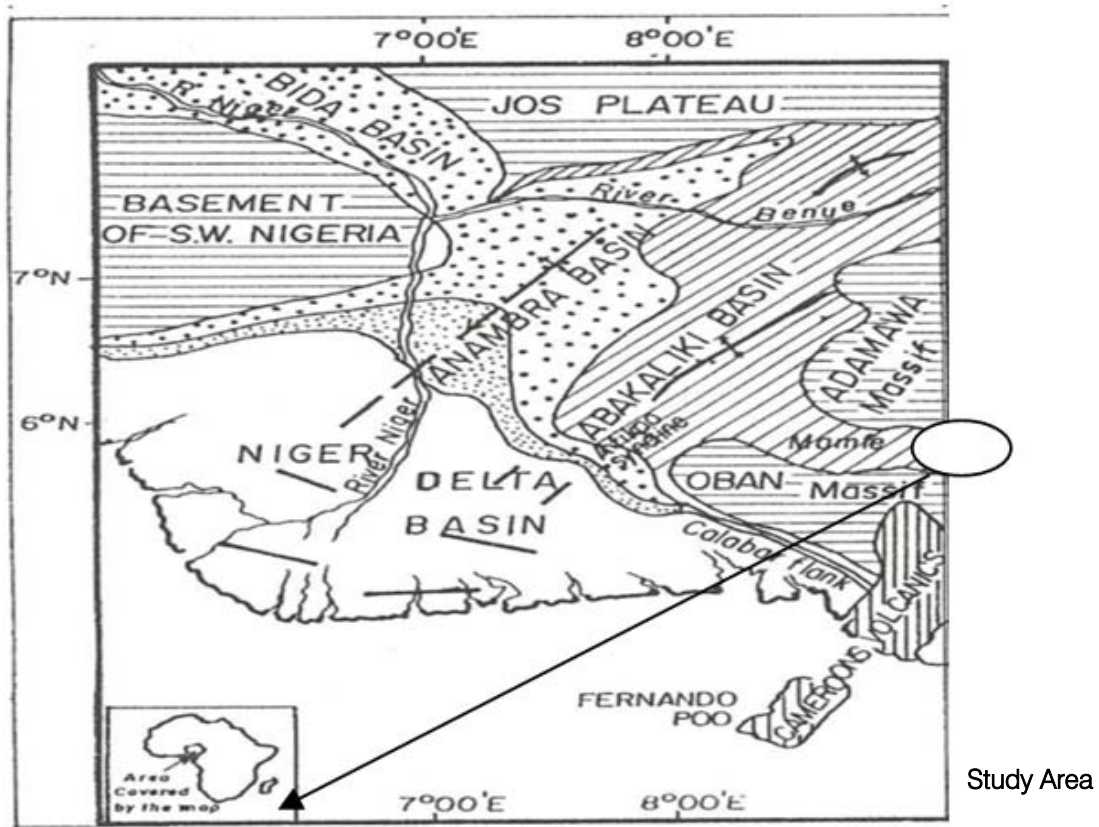
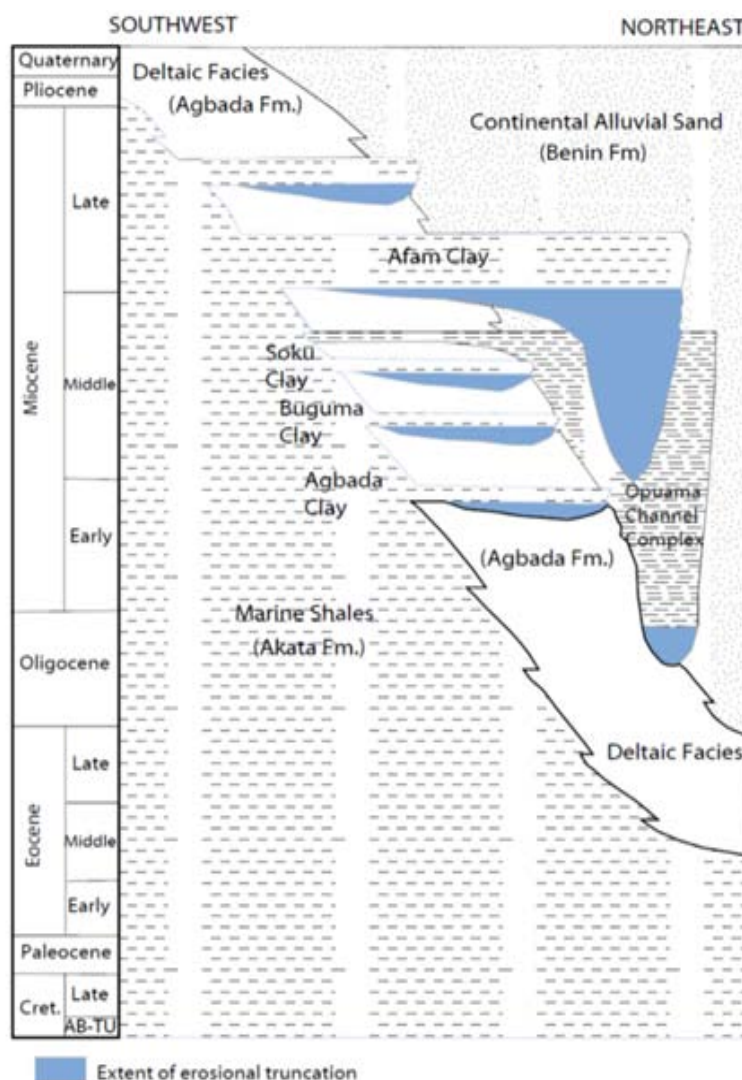


Figure 1 : Map Of Nigeria Showing Niger Delta And Study Area

The tertiary section of Niger Delta is divided into three Formations (Fig. 2.) representing prograding depositional facies that are distinguished based on sand – shale ratios (Short and Stauble, 1965). The Formations are Benin, Agbada and Akata Formations (Ekweozor and Daukoru, 1994; Kulke, 1995). The Benin Formation is the youngest of the Delta sequence. It outcrops in Benin, Onitsha and Owerri provinces in Niger Delta area. It consists mainly of sand and gravels with thickness ranging from 0 - 2100m. The sands and sandstones in this Formation are coarse – fine and commonly granular in texture and partly unconsolidated. Mostly found in the Benin formation are feldspars, hemalites, lignite streak and limonite coatings. Very little oil has been found in the Benin formation, and the formation is generally water bearing. It is the major source of portable water in the Delta area (Doust and Omatsola, 1990).



*Fig.2 :* Stratigraphic setting of Niger Delta showing the three formations of Niger Delta (modified after Doust and Omatsola, 1990)

The Agbada Formation is the major host of Niger Delta hydrocarbon and it consists of alternation of sandstones and shale. The deposition of Agbada Formation began in Eocene and continues into Pleistocene. In the lower portion, shale and sandstone beds were deposited in equal proportions. However, the upper portion is mostly with only minor shale interbed. Its thickness ranges from 300m – 4500m (Short and Stauble, 1965).

The Akata Formation forms the base of the transgressive lithologic unit of the Delta complex. It is of marine origin and it is made up of thick shale sequence (potential source rock), turbidite sand (potential reservoir in deep water) and minor amount of clay and silt (Avbovbo, 1978). Beginning in the Paleocene and through the recent, the Akata Formation was formed when terrestrial organic matter and clays were transported to deep water areas characterised by low

energy conditions and oxygen deficiency. The approximate range of thickness is from 0 – 6000m and the formation crops out subsea in the outer Delta area but rarely seen onshore. The formation outlies the entire Delta, and is typically overpressured.

### III. MATERIALS AND METHODS

The quantity of heat generated by radioactive elements is a function of the quantities present, their rate of decay and the energies of emission. According to Beardsome and Cull (2001), the radiogenic heat generation A is related to the natural gamma ray spectrometer (NGS) as

$$A = 10^{-3} \cdot \rho \cdot (96.7C_u + 26.3C_{Th} + 35.0C_K) \quad (1)$$

Where,

A = Radiogenic heat production ( $\mu\text{Wm}^{-3}$ )

$C_u$  = Concentration of Uranium in parts per million

$C_{Th}$  = Concentration of thorium in parts per million

$C_K$  = Concentration of potassium in percentage

$\rho$  = Density in  $\text{gcm}^{-3}$

The applications of equation 1 for the computation of heat production in sedimentary basin by the decay of radioactive elements depend on the availability of spectral gamma ray data. Most time these data are not available and therefore an alternative formula has to be applied. Rybach (1986) obtained a relationship between radiogenic heat production by radioactive substances and gamma ray intensity of the samples. The relationship was further modified by Biicker and Rybach (1996) as

$$A = 0.0158(GR - 0.8) \quad (2) \quad \text{Where}$$

Where

$$Y = \text{Potassium} + (0.751\text{Thorium} + 2.76\text{Uranium}) / (\text{Potassium} + 0.13\text{Thorium} + 0.36\text{Uranium}) \quad (5)$$

= Radiogenic factor

Equation 4 can also be written as

$$A = K\rho(GR) \quad (6)$$

Where

$$K = 0.035 * \left(\frac{Y}{X}\right) = A / (GR * \rho) \quad (7)$$

The radioactive heat generation in the sedimentary succession was determined from total gamma-ray logs available from three wells using equation 1. Attempt was also made to determine the value of K from available density and gamma ray logs for the study Area. The logs were obtained from Agip Oil Producing Company, Nigeria.

#### IV. RESULT AND DISCUSSION

To obtained accurate value and overall understanding of the heat flow in a sedimentary basin and the temperature gradient, the amount of heat generated by the sediments themselves is an important component, which needs to be calculated and quantified. Clastic sediments can provide reasonable amount of the total heat flow where sediment basins are deep. The depth of the gamma ray logs used for this work ranges between 1000 to 3000 metres. The depth of the density log varies between the depth 1900 – 2700 metres for wells 1 and 3 respectively while the depth is 1200 -3000 metres for well 2. The major lithology observed in the gamma ray logs are sand, shale and sandy shale. The gamma ray value ranges between 20 - 160 API while the density log reading varies between 1.7 – 2.5 $\text{g/cm}^3$ .

The heat values were computed for the total length of the wells where there is gamma ray readings. The plots of the computed radiogenic heat versus depth for the three wells are shown in Figures 3-5. The values

$GR$  = gamma ray value (API)

The gamma ray reading is related to the natural gamma ray spectral values (Beardsmore and Cull, 2001) by the equation

$$GR = X(\text{Potassium} + (0.13\text{Thorium}) + (0.36\text{Uranium})) \quad (3)$$

Where X is proportionality constant (though not really constant) and it depends on the distance into the rock that the well log tool is able to detect. The distance is also a function of the rock density, energy of the emissions and the tool type (Serra, 1984). The value of A can be related to the gamma ray (GR) value by combining equation 1 and 3 as;

$$A = 0.035\rho(GR)\left(\frac{Y}{X}\right) \quad (4)$$

ranges from 0.35 to 2.0  $\mu\text{Wm}^{-3}$  for well 1, 0.34 to 1.78 $\mu\text{Wm}^{-3}$  for well 2 and 0.24 to 2.0 $\mu\text{Wm}^{-3}$  for well 3. The average radiogenic heat production ranges between 0.3 to 1.93 $\mu\text{Wm}^{-3}$ . Due to the scattered nature of the values, it was not possible to fit a trend to the plots. This is an indication of the complex variation of the radiogenic heat production with depth. The low heat generation with value ranging from 0.24 to 0.7  $\mu\text{Wm}^{-3}$  is due to sand lithology. The maximum heat generation occurred in the Shale lithology with values ranging from 0.8 to 2.0  $\mu\text{Wm}^{-3}$ . The high radiogenic heat production in shale is expected because of its high concentration of radioactive elements. The value in sand lithology also corresponds to the low radioactive element associated with it. Similarly the low and high radiometric heat production observed in the wells can be attributed to the alternation of sand and shale lithology in the Agbada Formation.

From equation 5, the value of k was computed from the radiogenic heat and the available gamma ray and density logs in the corresponding well. The plot of k versus well depth is shown in Figures 6-8 for the three wells. The value of K ranges from 0.006 to 0.008 with a mean value of 0.007 for the three wells. The graphs show that the value of K is constant with depth. The implication of this constant value is that a plot of the radiogenic heat production versus the product of gamma ray and density will approximate a straight line with a mean gradient of 0.007. Therefore the general

equation for radiogenic heat production from gamma ray and density logs (equation 6) for the Niger Delta is given as

$$A = K (\rho x GR) = 0.007(\rho x GR) \quad (8)$$

The gradient of A versus RHOB x GR is dependent only on the relative proportions of the radioactive elements in the sediment. The radiogenic heat production is high at greater

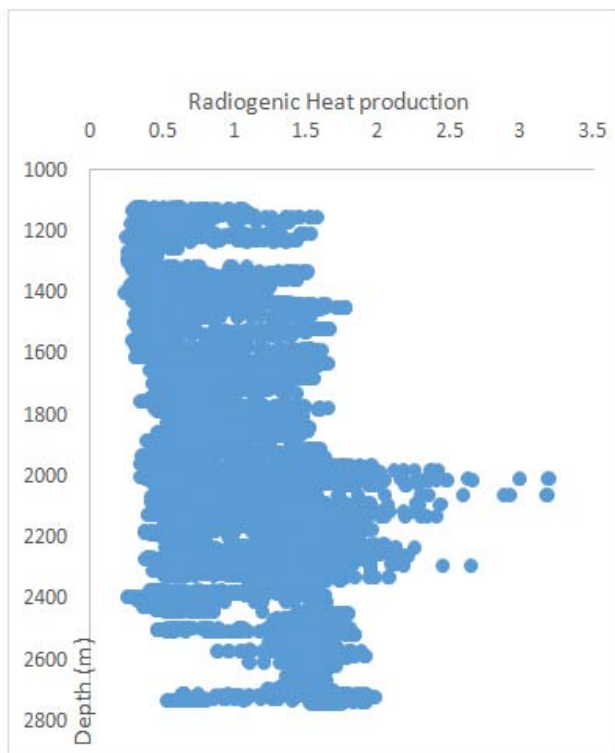


Fig. 3 : Plot of depth versus radiogenic heat production for well 1

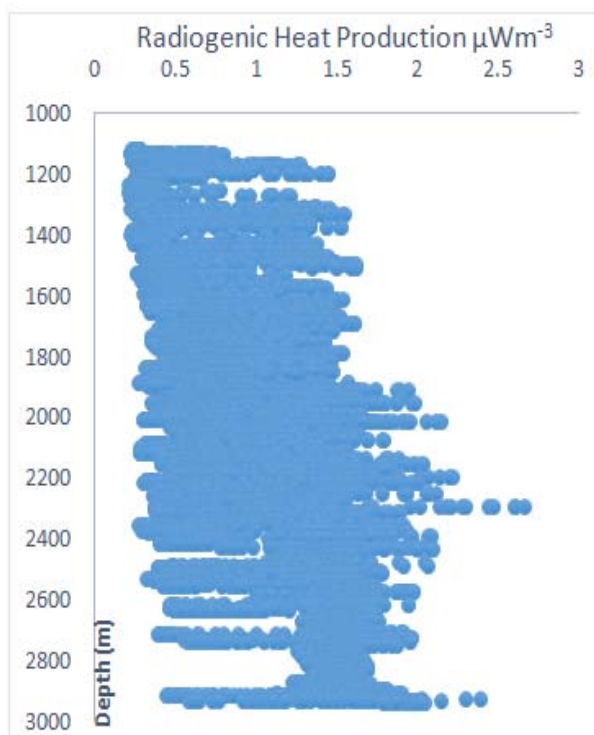


Fig. 4 : Plot of depth versus radiogenic heat production for well 2

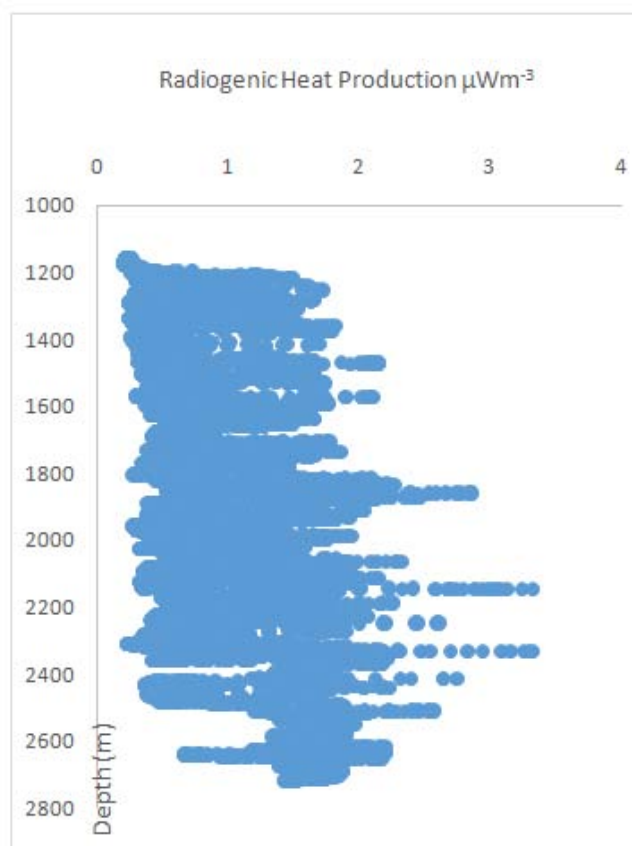


Fig. 5 : Plot of depth versus radiogenic heat production for well 3

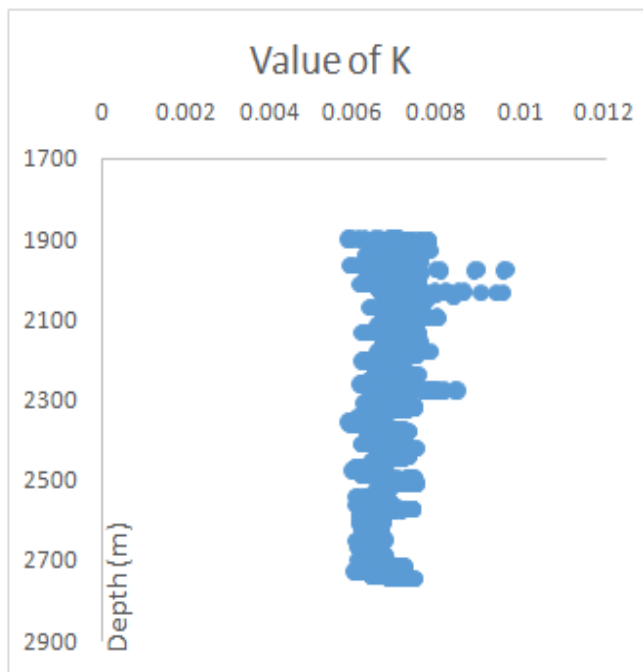


Fig. 6 : Plot of depth versus K values for well 1

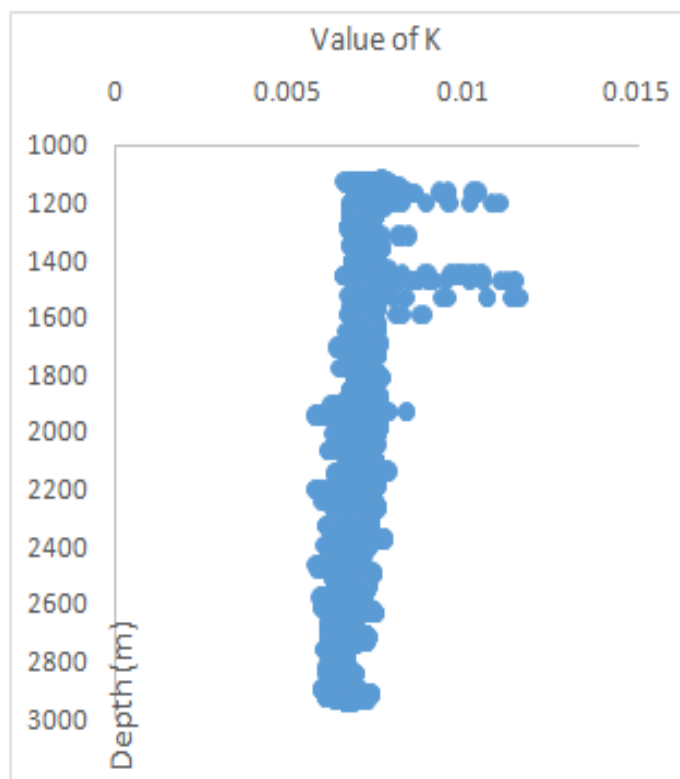


Fig. 7 : Plot of depth versus K values for well 2

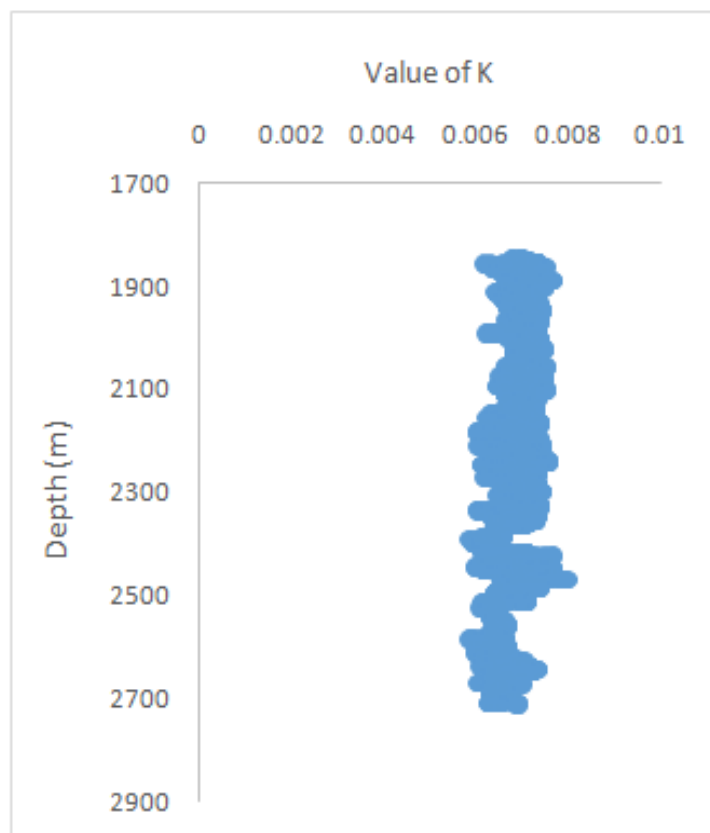


Fig. 8 : Plot of depth versus K values for well 3

Depth in the graph. This can be attributed to high shale lithology toward the base of the Agbada and top of the Akata Formations in the Niger Delta.

Comparison of the computed radiogenic heat flow for the study area (part of the Niger Delta) shows that the value is consistent with that obtained for other basins. For example, the radiogenic heat production classification by Ehinola et al., (2005) is equivalent to  $1.688 \mu\text{Wm}^{-3}$  for low generation sediments and greater than  $3.37 \mu\text{Wm}^{-3}$  for high generation sediment. Keen and Lewis (1982) observed that the heat generation from sediments of continental margin of Eastern North American ranges between  $0.3 \mu\text{Wm}^{-3}$  for limestone to  $1.4\text{--}1.8 \mu\text{Wm}^{-3}$  for shale. According to Zhang (1993), the heat generated for Cenozoic and Mesozoic lacustrine sediments in Chinese basins ranges from  $1.02\text{--}3.28 \mu\text{Wm}^{-3}$ . Furthermore, the radiogenic heat production estimated from the concentration of the radioactive elements obtained from log data ranges between  $0.17$  and  $1.90 \mu\text{Wm}^{-3}$  with an average of  $0.90$  for Chad Basin, Nigeria (Ali and Orazulike, 2010). Similarly, Chapman and Polack (1975) obtained radiogenic heat production ranging from  $0.96$  to  $1.8 \mu\text{Wm}^{-3}$  for Precambrian site on the exposed West African Craton.

#### IV. CONCLUSION

Heat generated as a result of the decay of naturally radioactive elements in the earth's crust contributes significantly to terrestrial heat flow. Heat production estimated from gamma ray logs in part of Niger Delta ranges between  $0.24\mu\text{Wm}^{-3}$  and  $2.0\mu\text{Wm}^{-3}$ . The average radiogenic heat production ranges between  $0.3$  to  $1.93\mu\text{Wm}^{-3}$ . The computed radiogenic heat production in this study area correlated very well with the results from other basin. The relationship established for the radiogenic heat production and the product of gamma ray and density logs can be used if gamma ray spectral logs are not available. Further research in radiogenic heat production should be based on natural gamma ray spectral.

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## Dielectric and Radiative Properties of Surface Water over the Persian Gulf at L-Band

By Ali Rezaei - Latifi

*Hormozgan University*

**Abstract-** For microwave remote sensing applications over the ocean using radars and radiometers, a precise knowledge of dielectric and radiative Properties of surface water is required. In present work, spatial and temporal variability of the complex permittivity and refractive index, as well as reflectivity of Persian Gulf water for normal incidence, at L-Band (1.4 GHz), which has not been investigated before, is studied by using an empirical model. The calculations results indicate a relatively significant spatial and seasonal variability in the dielectric and radiative properties of the Gulf due to large variations in temperature and salinity of surface water. The mean real refractive index  $n$  over the Gulf surface water varies from a minimum value of 7.295 in December to a maximum value of 7.875 in June with annual mean of 7.581. The extinction coefficient  $k$  reaches maximum value of 2.036 in June and minimum value of 1/303 in January and its annual mean over the Persian Gulf is 1.674.

**Keywords:** *permittivity, refractive index, reflectivity, persian gulf, L-band.*

**GJSFR-A Classification :** FOR Code: 029999p



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# Dielectric and Radiative Properties of surface Water over the Persian Gulf at L-Band

Ali Rezaei - Latifi

**Abstract-** For microwave remote sensing applications over the ocean using radars and radiometers, a precise knowledge of dielectric and radiative Properties of surface water is required. In present work, spatial and temporal variability of the complex permittivity and refractive index, as well as reflectivity of Persian Gulf water for normal incidence, at L-Band (1.4 GHz), which has not been investigated before, is studied by using an empirical model. The calculations results indicate a relatively significant spatial and seasonal variability in the dielectric and radiative properties of the Gulf due to large variations in temperature and salinity of surface water. The mean real refractive index  $n$  over the Gulf surface water varies from a minimum value of 7.295 in December to a maximum value of 7.875 in June with annual mean of 7.581. The extinction coefficient  $k$  reaches maximum value of 2.036 in June and minimum value of 1/303 in January and its annual mean over the Persian Gulf is 1.674.

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## 1. INTRODUCTION

Knowledge of dielectric properties of water is essential for calculating the radiative transfer coefficient of microwave radiation that is emitted by the ocean surface. Basis for the dielectric properties of a medium is its complex relative permittivity. Permittivity determines such intrinsic characteristics of the medium as skin (and penetration) depth, impedance, refractive index, etc. These quantities control the attenuation due to absorption and scattering and the transmission and reflection of the electromagnetic (EM) radiation in the medium. These radiative processes affect, in turn, the energy lost in and then emitted by the medium (Magdalena and Peter 2012). The real part of permittivity is known as the dielectric constant  $\epsilon'$  and is a measure of the ability of a material to be polarized and store energy. The imaginary part  $\epsilon''$  is a measure of the ability of the material to dissipate stored energy into heat.

Permittivity models for the permittivity of aqueous saline solutions and seawater given by Klien and Swift (1997), Stogryn (1971), Ellison et al. (1998), Meissner & Wenz (2004) and Blanch & Aguiasca (2004) were studied. These models provide the relative permittivity and conductivity of seawater at particular

microwave frequency as a function of the salinity and temperature. These models are based upon experimental data, performed by respective authors as well as the laboratory data others. The experimental data were fitted to empirical and theoretical expressions for calculation of relative permittivity of seawater for any frequency, salinity and temperature value desired. The Klien and swift (K&S) model is widely used for seawater dielectric coefficients. Comparison of measurement results with this model shows that real part is well in agreement. However, the experimental loss factor is slightly higher compared with the theoretical model (Joshi et al. 2012, Lang et al. 2003). Comparison between Blanch & Aguiasca(B&A) model with the other existing ones, on the other hand, showed that B&A model is close to the K&S model for the real part of permittivity and a bit high absolute values for imaginary part than the K&S model. Therefore the B&A model appears to be more accurate than well-known model of K&S.

The Persian Gulf is an important military, economic and political region owing to its oil and gas resources and is one of the busiest waterways in the world. Countries bordering the Persian Gulf are the United Arab Emirates(UAE), Saudi Arabia, Qatar, Bahrain, Kuwait and Iraq on one side and Iran on the other side (Fig. 1). The average depth of the Gulf is 36 m. Extensive shallow regions, <20m deep, are found along the coast of United Arab Emirates, around Bahrain, and at the head of the Gulf. Deeper portions, >40m deep, are found along the Iranian coast continuing into the Strait of Hormuz, which has a width of ~56 km and connects the Persian Gulf via the Gulf of Oman with the northern Indian Ocean (Kampf and Sadrasab 2006). The Persian Gulf is one of the most saline water masses in world ocean and due to its shallow nature and sub-tropical climate both the salinity and temperature experiences dramatic spatial and temporal variations (Hassanzadeh et al 2011, Hosseinibalam et al. 2011).

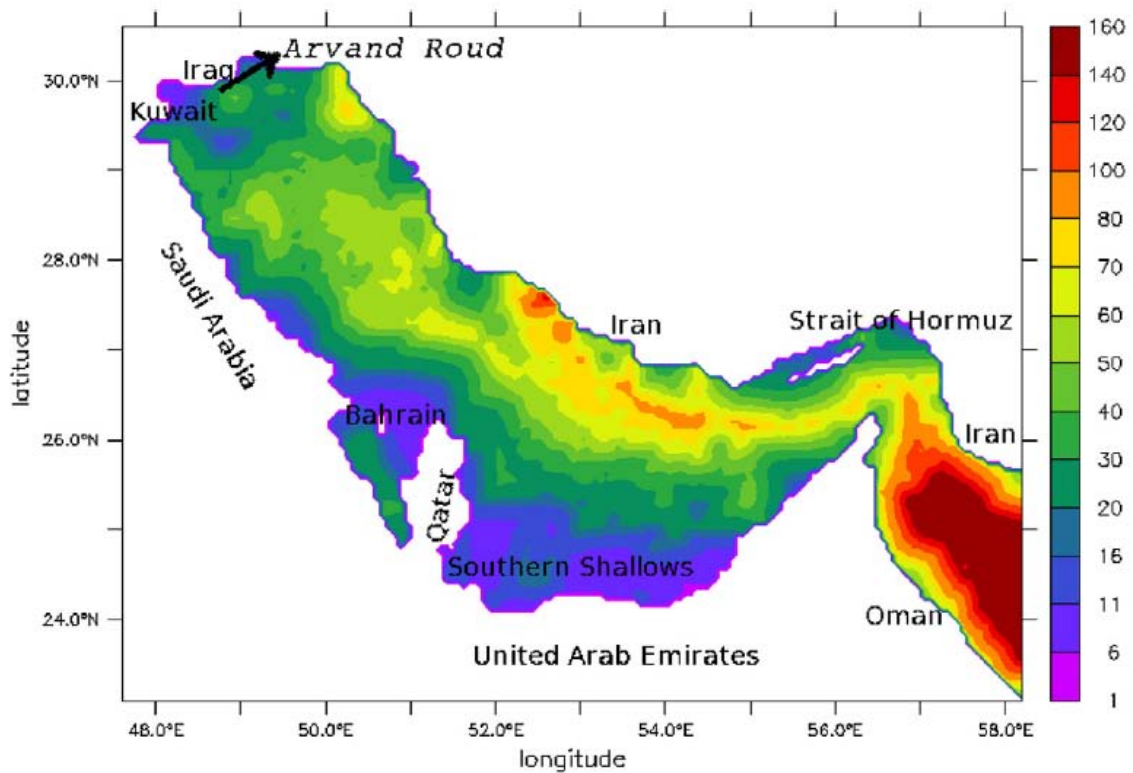


Fig. 1: Bathymetry and map of the Persian Gulf

In this work, spatial and seasonal variability of the complex permittivity and refractive index of Persian Gulf water in the long wavelength end of the microwave spectrum (1.4 GHz, L-band), is investigated by using empirical model of B&A. The salinity and temperature data of Persian Gulf water are required in order to calculate the relative permittivity and refractive index. Unfortunately, field data of salinity and temperature in the Persian Gulf are scarce and sparse. Thus, we use numerical data obtained from our previous work for estimation of permittivity. These data are in a relatively good agreement with limited direct measurements in the Persian Gulf (Hassanzadeh et al. 2011, Hosseinibalam et al. 2011, Hassanzadeh et al. 2012).

The outline of this paper is as follows. In section 2 the permittivity model reported by Blanch and Aguasca and the formulas required to estimate the refractive index is presented. The results of calculation of parts of real and imaginary of permittivity and refractive index over the Persian Gulf are analyzed in section 3. The last section provides the conclusion.

## II. MATERIALS AND METHODS

As mentioned earlier, In order to calculate the complex permittivity are required the salinity and temperature data. We previously simulated these data by using a 3-dimensional numerical model. The model is based on hydrostatic versions of the Navier-Stokes equations that embrace conservation equations for momentum, volume, heat and salt. See our previous

works for details on how to set up and run the numerical model Gulf (Hassanzadeh et al. 2011, Hosseinibalam et al. 2011, Hassanzadeh et al. 2012). The Blanch and Aguasca (2004) model, which is used to compute the complex permittivity together with formulas of refractive index are described as follows.

### a) Blanch and Aguasca model

The dielectric properties of the liquids described by the Debye model (1926).

$$\hat{\epsilon} = \epsilon' - j\epsilon'' = \epsilon_{\infty} + \frac{\epsilon_s - \epsilon_{\infty}}{1 + j\omega\tau} - j \frac{\sigma}{\omega\epsilon_0} \quad (1-2)$$

Where  $\epsilon_{\infty}$  is the permittivity at optical frequencies,  $\epsilon_s$  is the static permittivity,  $\tau$  is the relaxation time and  $\sigma$  the ionic conductivity. The value of  $\epsilon_{\infty}$  is assumed to be constant, independent of salinity and temperature (its exact value is not critical in L-narrowband,  $0.5\text{GHz} \leq \nu \leq 2.5\text{GHz}$ ) and equal to 4.9. The  $\epsilon_s$  and  $\tau$  are both modeled as product of the values for distilled water and a term that depends on temperature (T) and salinity (S):

$$\begin{aligned} \epsilon_s(T, S) &= \epsilon_s(T, 0).a(T, S) \\ \tau(T, S) &= \tau(T, 0).b(T, S) \end{aligned} \quad (2-2)$$

The conductivity follows an exponential law.

$$\sigma(T, S) = \sigma(25, S).e^{-\phi} \quad (3-2)$$

According to the above equations, the seawater permittivity is a function of frequency, temperature and salinity. Blanch and Aguasca (2004) performed a lot of measurements in wide range of values those

parameters in order to obtain the coefficients of the permittivity model. Final results in  $0.5\text{GHz} \leq \nu \leq 2.5\text{GHz}$  are:

$$\varepsilon_s(T, 0) = 87.38 - 3.436 \times 10^{-1} T - 1.912 \times 10^{-3} T^2 + 3.812 \times 10^{-5} T^3 \quad (4-2)$$

$$a(T, S) = 1 + 1.1552 \times 10^{-5} TS - 3.9073 \times 10^{-3} S + 3.0596 \times 10^{-5} S^2 \quad (5-2)$$

$$\tau(T, 0) = 17.385 - 5.78 \times 10^{-1} T + 1.084 \times 10^{-2} T^2 - 9.098 \times 10^{-5} T^3 \quad ps \quad (6-2)$$

$$b(T, S) = 1 + 2.9832 \times 10^{-4} TS - 2.3871 \times 10^{-3} S + 5.625 \times 10^{-5} S^2 \quad (7-2)$$

$$\sigma(25, S) = 1.90 \times 10^{-1} S - 2.35 \times 10^{-3} S^2 + 3.46 \times 10^{-5} S^3 \quad (8-2)$$

$$\phi = \Delta[1.9479 \times 10^{-2} + 1.6532 \times 10^{-4} \Delta - S(-1.0024 \times 10^{-6} + 6.9946 \times 10^{-7} \Delta)] \quad (9-2)$$

Where

$$\Delta = 25 - T \quad (10-2)$$

It should be noted that according to equation (1-2) the real and imaginary parts of the relative permittivity are as follows:

$$\varepsilon' = \varepsilon_\infty(T, S) + \frac{\varepsilon_s(T, S) - \varepsilon_\infty(T, S)}{1 + 4\pi^2 \nu^2 \tau^2(T, S)} \quad (11-2)$$

$$\varepsilon''(\nu, T, S) = \frac{(\varepsilon_s(T, S) - \varepsilon_\infty(T, S))2\pi\nu\tau(T, S)}{1 + 4\pi^2 \nu^2 \tau^2(T, S)} + \frac{\sigma(T, S)}{2\pi\nu\varepsilon_0} \quad (12-2)$$

#### b) Complex refractive index

The refractive index of a material is the factor that relates the phase velocity of electromagnetic radiation in that material, relative to its velocity in another medium or material. The absolute refractive index is relative to the velocity of light in the vacuum. The refractive index is a complex value,  $\hat{n} = n - jk$ . Here the real part  $n$  is the refractive index indicating the phase velocity as above, while the imaginary part  $k$  is called the extinction coefficient, which indicates the amount of absorption loss when the electromagnetic wave propagates through the material ( $j = \sqrt{-1}$ ).

The complex refractive index  $\hat{n} = n - jk$  and complex relative permittivity  $\hat{\varepsilon} = \varepsilon' - j\varepsilon''$  are related following relation (John et al. 2008):

$$\hat{\varepsilon} = \hat{n}^2 \quad (13-2)$$

By equating the real and imaginary parts we get:

$$\varepsilon' = n^2 - k^2 \quad (14-2)$$

$$\varepsilon'' = 2nk \quad (15-2)$$

These equations can be solved for  $n$  and  $k$ :

$$n = \left(\frac{\varepsilon'}{2}\right)^{1/2} \left\{ \left[ 1 + \left(\frac{\varepsilon''}{\varepsilon'}\right)^2 \right]^{1/2} + 1 \right\}^{1/2} \quad (16-2)$$

$$k = \left(\frac{\varepsilon'}{2}\right)^{1/2} \left\{ \left[ 1 + \left(\frac{\varepsilon''}{\varepsilon'}\right)^2 \right]^{1/2} - 1 \right\}^{1/2} \quad (17-2)$$

In addition, for normal incidence, the reflectivity is given by

$$r = \frac{(n-1)^2 + k^2}{(n+1)^2 + k^2} \quad (18-2)$$

For reflection at angles of incidence other than normal incidence, the reflection coefficient depends on the polarization of the electric field relative to surface.

### III. RESULTS

#### a) Variability of the relative permittivity

The calculations results indicate a significant spatial and temporal variability in the dielectric constant  $\varepsilon'$  and dielectric loss  $\varepsilon''$  of the Persian Gulf at L-band (1.4 GHz) due to large seasonal variations in temperature and salinity of surface water. The mean dielectric constant attains a minimum and maximum value during the summer and winter respectively, and varies from 68.18 in June to 71.81 in late January (Fig2 and table 1).

The dielectric loss  $\epsilon''$  varies from 73.85 in January to 93.44 in June (table 1). Dielectric loss temporal variability of Persian Gulf water is stronger than dielectric constant and it gets maximum values in June-August and minimum values in months of December-January (Fig. 3). The comparison between figures 2 and 3 shows that an inverse relationship exists between real and imaginary parts of permittivity so that when one of two increases, the other variable decreases, and vice versa. In addition, analyses of obtained results indicate in L-

band the dielectric constant of the Persian Gulf water usually decreases with increase in salinity, whereas the dielectric loss of water increases with increase in salinity. Conversely, the dielectric constant goes higher to lower value and dielectric loss lower to higher with increase in the values of temperature. These results are in agreement with previous theoretical and experimental researches (Blanch and Aguasca 2004, Joshi et al. 2012, Gadani et al. 2012).

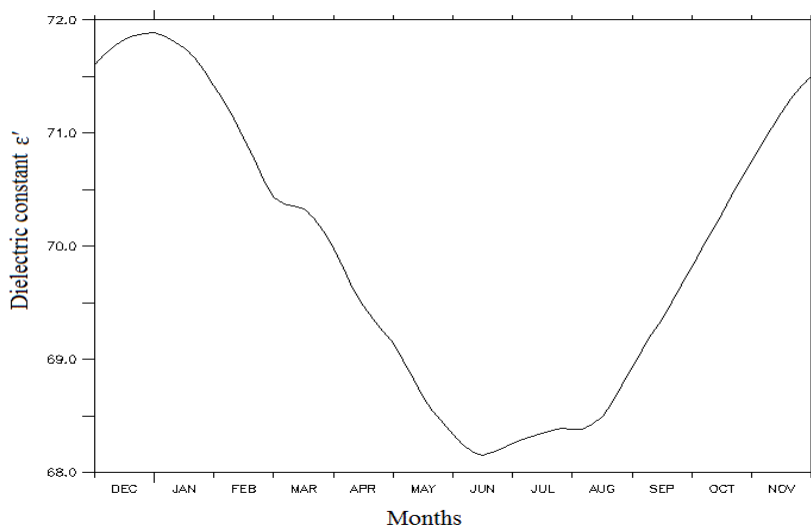


Fig. 2: Time series of domain-averaged dielectric constant  $\epsilon'$  at L-band

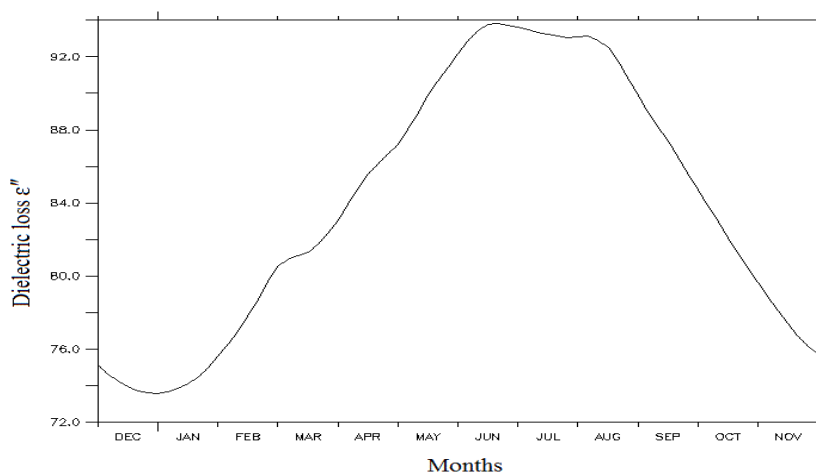


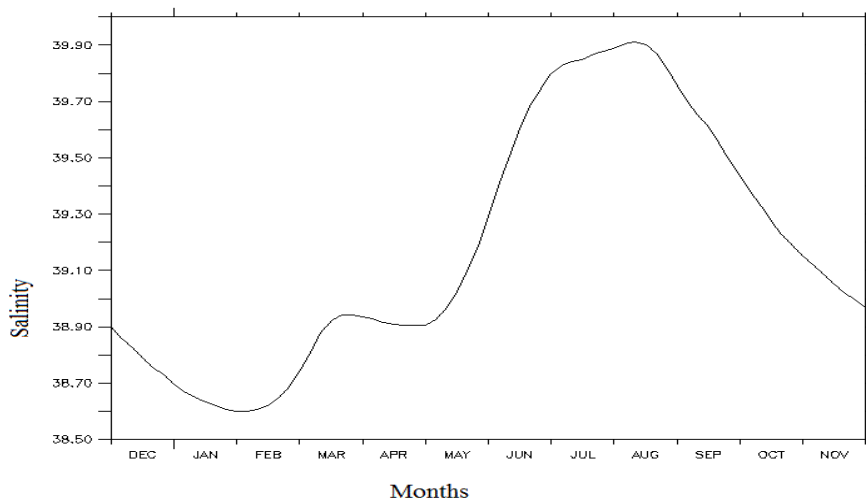
Fig. 3: Time series of domain-averaged dielectric loss  $\epsilon''$  at L-band

**Table 1:** Monthly surface mean of Dielectric constant  $\epsilon'$ , dielectric loss  $\epsilon''$ , real refractive index  $n$ , extinction coefficient  $k$ , Temperature  $T$  and salinity  $S$  of the Persian Gulf

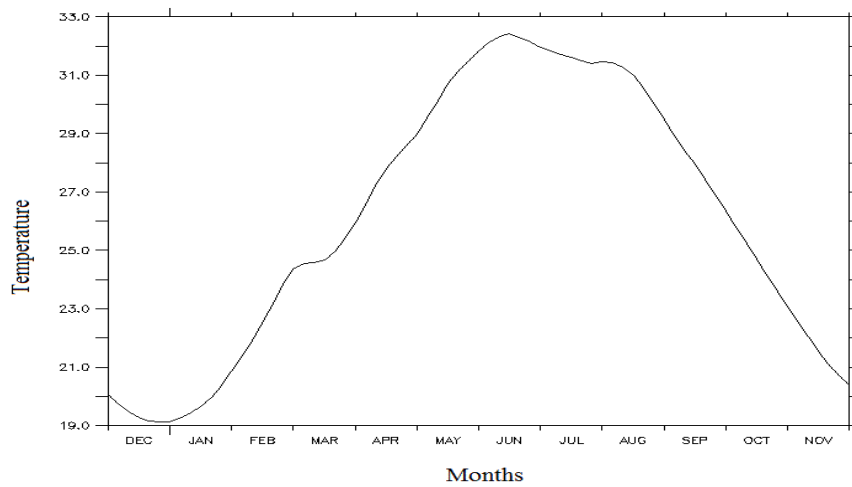
Month	$\epsilon'$	$\epsilon''$	$n$	$k$	$T$	$S$
December	71.77	74.25	7.306	1.315	19.47	38.83
January	71.78	73.85	7.295	1.303	19.43	38.65
February	71.73	76.98	7.377	1.413	21.91	38.61
March	70.36	81.11	7.493	1.562	24.59	38.82
April	69.62	84.79	7.601	1.667	27.22	38.91
May	68.82	88.97	7.729	1.863	30.14	38.97
June	68.18	93.44	7.875	2.036	32.34	39.51
July	68.32	93.35	7.873	2.028	31.70	39.84
August	68.43	92.91	7.859	2.010	31.28	39.91
September	69.22	88.16	7.707	1.824	28.42	39.65
October	70.13	82.96	7.549	1.627	24.69	39.33
November	71.04	78.10	7.410	1.450	22.03	39.09

The salinity and temperature of Persian Gulf water significantly change due to factors such as inflow seasonal variations of low- salinity surface water of Indian Ocean (<37) into the Gulf through the Hormuz Strait, the seasonal changes in the weather and the turbulence mixing processes (figs. 4,5 and table 1). Gulf-averaged temperature follows the seasonal cycle of incident solar radiation and it attains its minimum and maximum values in months of December- January and June- August, respectively (Fig. 5). Gulf-averaged salinity, on the other hand, attains minimum values during January-March and maximum values June-August each year (Fig. 4). In general, Surface water of Persian Gulf is saltier in autumn and early winter than spring. In addition, the field and numerical studies indicate that salinity over the northeast areas in winter is saltier than spring and early summer (Hasanzadeh et al. 2011, Swift and Bower 2003). A Main reason for this unusual phenomenon is inflow increase of low- salinity

surface water of Indian Ocean (<37) into the Gulf through the Hormuz Strait over the months of spring and summer due to increase in evaporation rate, and a significant reduction of inflow in autumn. The other reason is the action of lateral and vertical mixing eddies generated by the wind forcing and thermohaline fluxes in winter and autumn. during spring and summer, the density is strongly stratified and established a vertically baroclinic stability in the Gulf, mainly as are sult of the strong surface heating. In winter and autumn, together with a weakening density contrast, under the influence the atmospheric cooling, the turbulence mixing processes in the Gulf occur and rapidly deepen the surface mixed layer. Since lower layers are saltier than surface water, the deepening of the mixed layer leads to an increase in the salinity of the surface. Effects of precipitation and river run-off on salinity changes are negligible on a Gulf-wide scale.



**Fig. 4:** Time series of domain-averaged salinity over the Persian Gulf



*Fig. 5:* Time series of domain-averaged temperature over the Persian Gulf

Fig. 6 indicates spatial distribution of dielectric constant in the middle of each season. In winter, spatial distribution of dielectric constant is weaker than the other seasons due to of lateral and vertical mixing eddies generated by the wind and thermohaline forcing. In spring and summer, increasing the weather temperature the static stability of water increases as a result, the lateral stratification is reinforced and dielectric constant gradually decreases from Iran coasts toward southern shallow regions. In autumn, this regular stratification relatively decreases under the action of mesoscale eddies that start to form in this season. with the exception of small area in the northern end, both Minimum and maximum dielectric constant with values of 68 and 72.2, usually are found over southern shallow regions along the UAE and Bahrain coasts in summer and winter, respectively. This is mainly due to shallow nature and large temperature difference of  $16^{\circ}\text{C}$  between summer and winter in these regions.

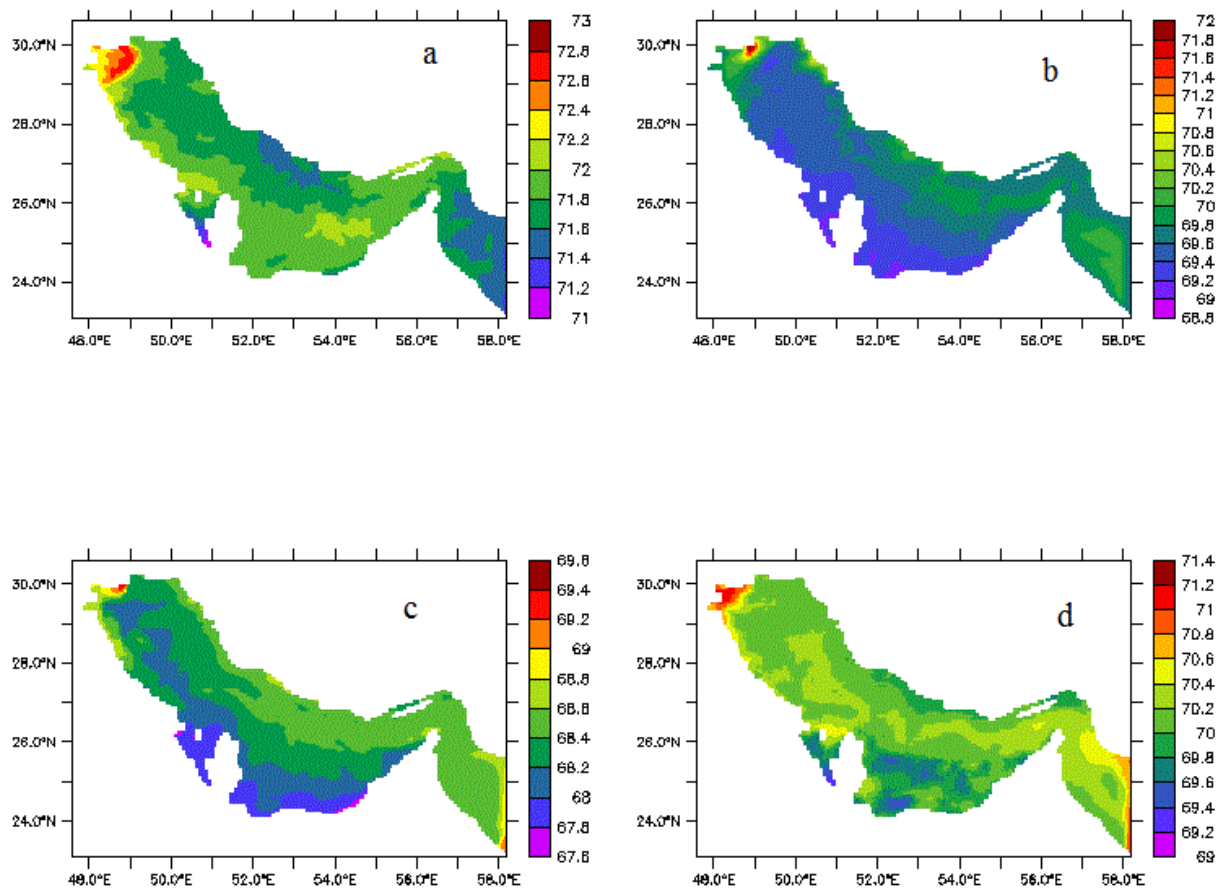


Fig. 6: The surface dielectric constant at L-band on (a) January 15, (b) April 15, (c) July 15 and (d) October 15

Spatial variability of dielectric loss at L-band is shown in fig. 7. Contrary to dielectric constant, the dielectric loss decreases from southern shallow parts to northeast deep regions. Spatial distribution of imaginary part of permittivity is to some extent uniform and varies about 72-74 over most parts of the Gulf in winter. With the exception of small area at the northwest end and south, almost a two-layer structure of dielectric loss is formed at most areas of the Gulf in Spring, with value of 80-85 within northeast regions and 85-90 over the other places of the Gulf. spatial variability in summer is stronger than the other seasons so that loss difference between southern and northeast areas reach about 20 on July 15. Main reasons for this relatively strong stratification are significant increase of salinity and temperature over the shallow regions and increase of low-salinity in flow from Hormuz Strait toward Iran coasts (Hasanzadeh et al. 2011, Swift and Bower 2003). In autumn, under the effects of lateral stirring of mesoscale eddies and convective deepening of the surface mixed layer the lateral contrast of dielectric loss is weakened and its value in most areas of gulf is about 82-86.

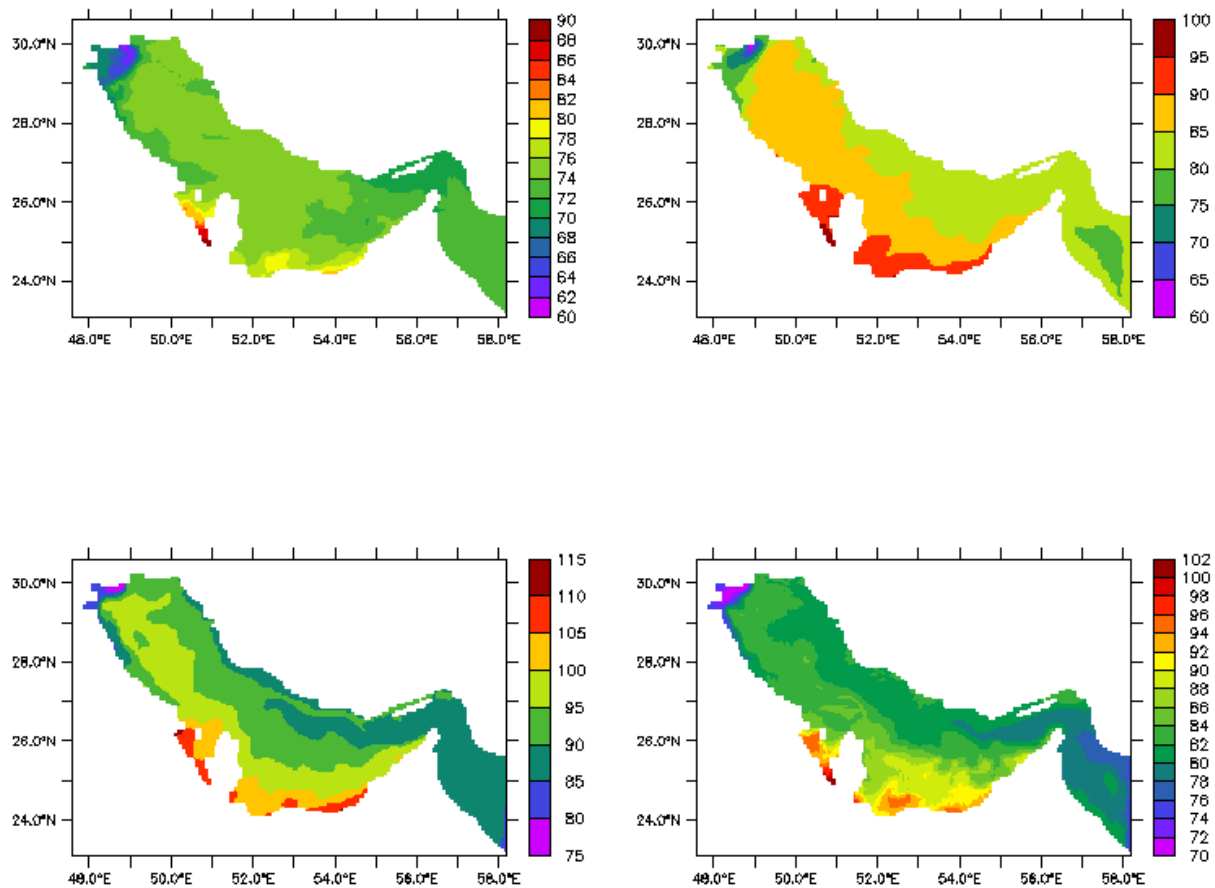


Fig. 7: The surface dielectric loss at L-band on (a) January 15, (b) April 15, (c) July 15 and (d) October 15

b) *Variability of the refractive index*

The domain-averaged of real refractive index  $n$  over the Gulf surface water varies from minimum value of 7.295 in December to maximum value of 7.875 in June (Fig.8 and table 1). The annual mean of basin-averaged is 7.581 at L-band. The highest rate of temporal variation of  $n$  is related to the spring months and its variability in summer months is relatively weak. Temporal variability of basin-averaged of imaginary refractive index  $k$  to a large extent qualitatively is similar to its counterpart  $n$  and as the real refractive index  $n$ , its variation rate is faster in spring (Fig. 9). This rapid changes in complex permittivity and refractive index in spring mainly related to a temperature increase of more than 6°C from March to June. The imaginary refractive index reach maximum value of 2.036 in June and minimum value of 1/303 in January and its annual mean over the Persian Gulf is 1.674.

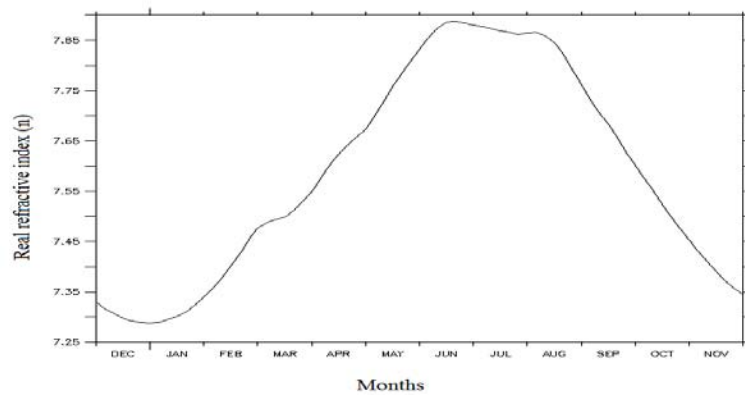


Fig. 8: Time series of domain-averaged of the real part of complex refractive index  $n$  at the Persian Gulf

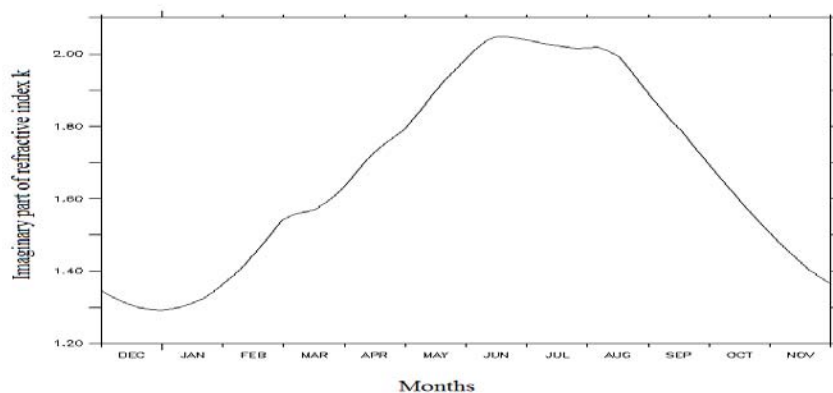
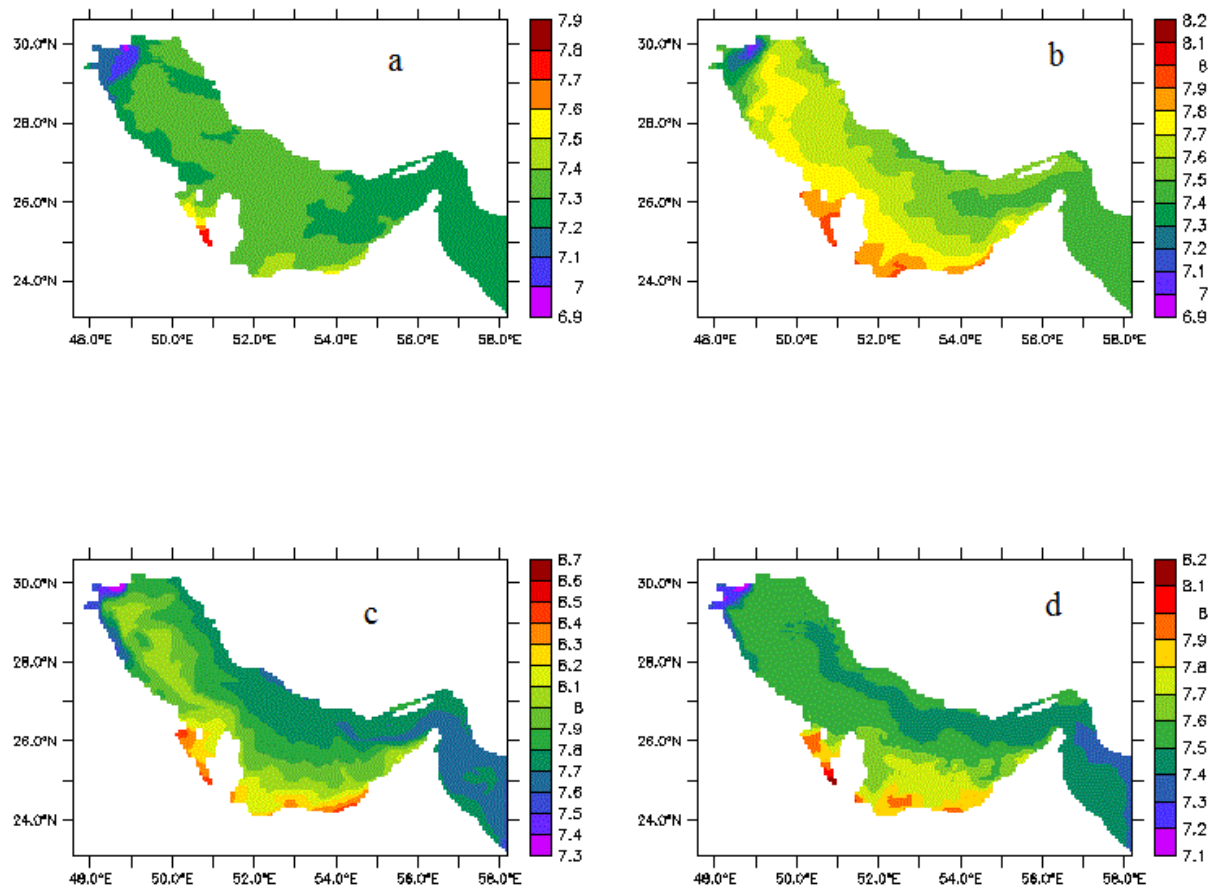


Fig. 9: Time series of domain-averaged of the imaginary part of complex refractive index  $k$  over the Persian Gulf

Figures 10 and 11 show spatial variability of parts of real and imaginary refractive index over the Gulf. In mid-winter the complex refractive index of surface water is relatively uniform and its real and imaginary parts have values of 7.3 and 1.3 respectively, at most areas of the Gulf. In mid-spring, both real and imaginary part of the refractive index from northern deep regions toward southern shallow regions increases by 0.5. The spatial distribution of real and imaginary of refractive index in mid-summer is almost similar to mid-spring but they both increase about 0.3-0.5. In autumn, over narrow region along the Iran coast real refractive index is about 7.4 and in other places with the exception southern shallow near at the UAE is about 7.6. In addition, at Iranian half of the Gulf the extinction coefficient  $k$  is about 1.4 and at other regions varies from 1.7 to 2.1 in this season. Totally, temporal variability the complex refractive index over shallow regions of Gulf is more than deep regions.



*Fig. 10:* Spatial distribution real refractive index of surface water of the Persian Gulf n at L-band on (a) January 15, (b) April 15, (c) July 15 and (d) October 15

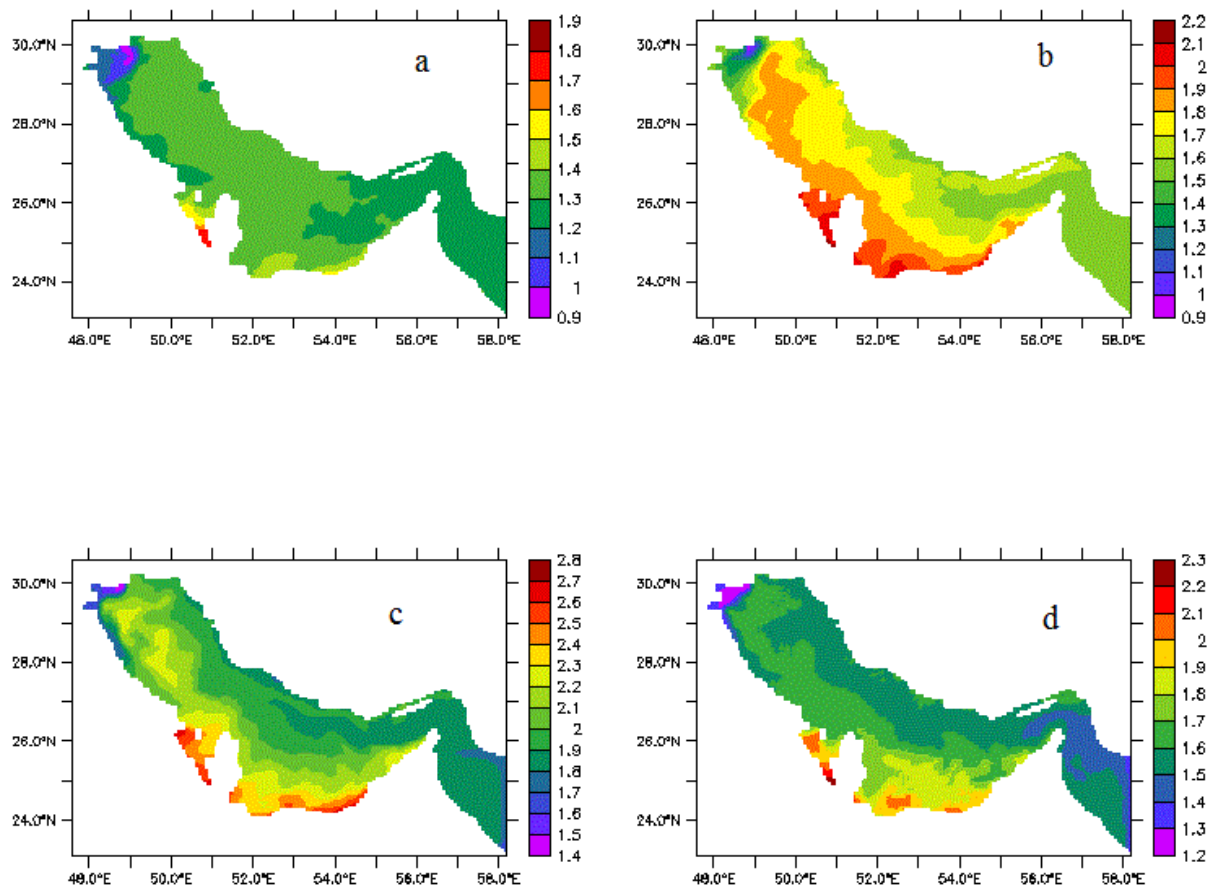


Fig. 11: Spatial distribution the extinction coefficient of surface water of the Persian Gulf  $k$  at L-band on (a) January 15, (b) April 15, (c) July 15 and (d) October 15

Now, with these values of  $n$  and  $k$ , we can calculate the reflectivity of Persian Gulf water for normal incidence at L-band by using equation (18-2). As can be seen from Fig.12, the normal reflectivity in mid-spring varies from 0.823 at the regions close to UAE coast and Bahrain–Qatar shelf to 0.830 in the northwestern end close to Arvand Roud. Furthermore, the reflectivity of water within the Iranian half of Gulf is slightly more than the Southern half close to Arabian countries coasts. Figure 13 shows temporal variability of reflectivity mean over the Gulf. The reflection coefficient mean reaches a maximum value of 0.828 on January and a minimum value of 0.824 on June–August months. In addition, its variability during summer months is very weak. Relatively high value of reflectivity is as a result of high real refractive and low imaginary refractive of the surface water over the Gulf.

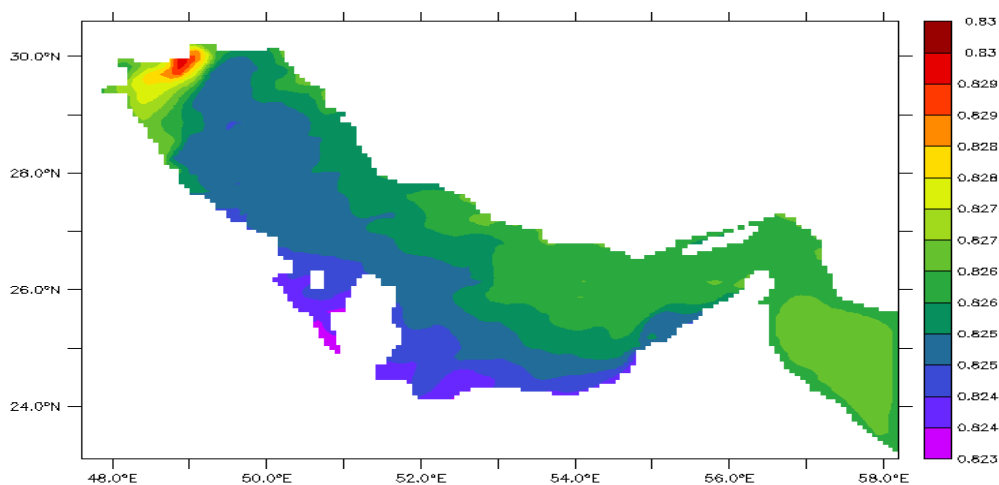


Fig. 12: Spatial variability of reflectivity of Persian Gulf water for normal incidence at L-band on April 15

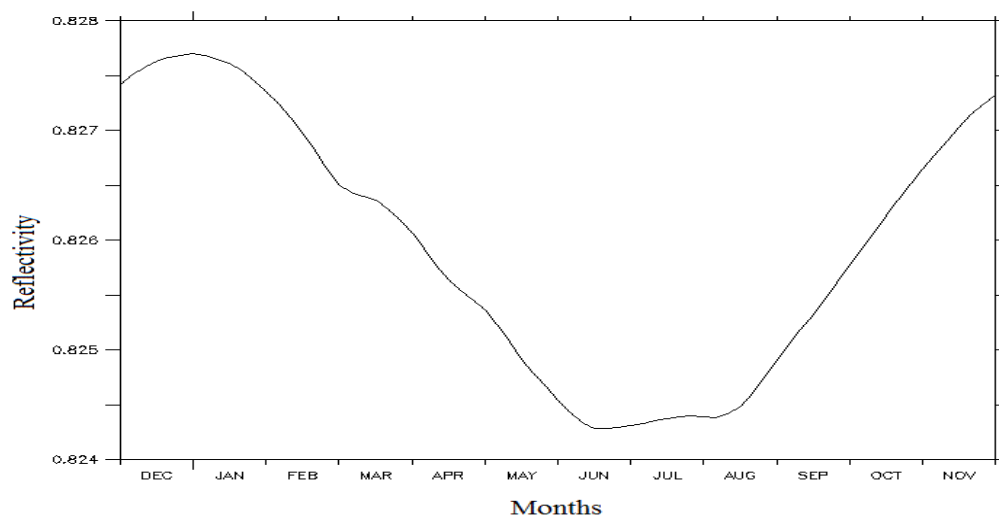


Fig. 13: Time series of domain-averaged reflectivity over the Persian Gulf

#### IV. CONCLUSION

Knowledge of the microwave dielectric Properties of ocean surface is crucial for the measurement of ocean environmental parameters from both spaceborne and airborne microwave radiometers and to understand the properties of radio wave propagation in seawater. Here, at first the complex permittivity was calculated by using Blanch and Aguasca model. All data needed for the model were provided by using a hydrodynamic 3-D numerical model described in our previous papers (Hassanzadeh et al. 2011, Hosseinibalam et al. 2011, Hassanzadeh et al. 2012). Then, permittivity results were used to calculate imaginary and real parts of the refractive index. Finally, these values were employed to estimate reflectivity for normal incidence.

The calculation results indicate that dielectric loss temporal variability of Persian Gulf water is stronger than dielectric constant and it gets maximum values in June-August and minimum values in months of

December-January. The mean dielectric constant  $\epsilon'$  attains a minimum and maximum value during the summer and winter respectively, and varies from 68.18 in June to 71.81 in late January. The dielectric loss  $\epsilon''$  varies from 73.85 in January to 93.44 in June. Contrary to dielectric constant, the dielectric loss decreases from southern shallow parts to northeast deep regions. In winter and autumn due to weak static stability conditions and under the effects of lateral stirring of mesoscale, dielectric properties of Persian Gulf water is more uniform than spring and summer. Totally, temporal variability the complex refractive index and permittivity over shallow regions of Gulf is more than deep regions due to large variations of salinity and temperature. The real refractive index mean over the Gulf surface water varies from minimum value of 7.295 in December to maximum value of 7.875 in June and its annual mean is 7.581 at L-band. The imaginary refractive index mean reach maximum value of 2.036 in June and minimum value of 1/303 in January with

annual mean value of 1.674. The normal reflectivity in mid-spring varies from 0.823 at regions close to UAE coast and Bahrain–Qatar shelf to 0.830 in the northwestern end close to Arvand Roud.

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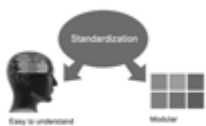
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- (a) Title should be relevant and commensurate with the theme of the paper.
- (b) A brief Summary, "Abstract" (less than 150 words) containing the major results and conclusions.
- (c) Up to ten keywords, that precisely identifies the paper's subject, purpose, and focus.
- (d) An Introduction, giving necessary background excluding subheadings; objectives must be clearly declared.
- (e) Resources and techniques with sufficient complete experimental details (wherever possible by reference) to permit repetition; sources of information must be given and numerical methods must be specified by reference, unless non-standard.
- (f) Results should be presented concisely, by well-designed tables and/or figures; the same data may not be used in both; suitable statistical data should be given. All data must be obtained with attention to numerical detail in the planning stage. As reproduced design has been recognized to be important to experiments for a considerable time, the Editor has decided that any paper that appears not to have adequate numerical treatments of the data will be returned un-refereed;
- (g) Discussion should cover the implications and consequences, not just recapitulating the results; conclusions should be summarizing.
- (h) Brief Acknowledgements.
- (i) References in the proper form.

Authors should very cautiously consider the preparation of papers to ensure that they communicate efficiently. Papers are much more likely to be accepted, if they are cautiously designed and laid out, contain few or no errors, are summarizing, and be conventional to the approach and instructions. They will in addition, be published with much less delays than those that require much technical and editorial correction.



The Editorial Board reserves the right to make literary corrections and to make suggestions to improve brevity.

It is vital, that authors take care in submitting a manuscript that is written in simple language and adheres to published guidelines.

## Format

*Language: The language of publication is UK English. Authors, for whom English is a second language, must have their manuscript efficiently edited by an English-speaking person before submission to make sure that, the English is of high excellence. It is preferable, that manuscripts should be professionally edited.*

Standard Usage, Abbreviations, and Units: Spelling and hyphenation should be conventional to The Concise Oxford English Dictionary. Statistics and measurements should at all times be given in figures, e.g. 16 min, except for when the number begins a sentence. When the number does not refer to a unit of measurement it should be spelt in full unless, it is 160 or greater.

Abbreviations supposed to be used carefully. The abbreviated name or expression is supposed to be cited in full at first usage, followed by the conventional abbreviation in parentheses.

Metric SI units are supposed to generally be used excluding where they conflict with current practice or are confusing. For illustration, 1.4 l rather than  $1.4 \times 10^{-3} \text{ m}^3$ , or 4 mm somewhat than  $4 \times 10^{-3} \text{ m}$ . Chemical formula and solutions must identify the form used, e.g. anhydrous or hydrated, and the concentration must be in clearly defined units. Common species names should be followed by underlines at the first mention. For following use the generic name should be constricted to a single letter, if it is clear.

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*Abstract, used in Original Papers and Reviews:*

### Optimizing Abstract for Search Engines

Many researchers searching for information online will use search engines such as Google, Yahoo or similar. By optimizing your paper for search engines, you will amplify the chance of someone finding it. This in turn will make it more likely to be viewed and/or cited in a further work. Global Journals Inc. (US) have compiled these guidelines to facilitate you to maximize the web-friendliness of the most public part of your paper.

### Key Words

A major linchpin in research work for the writing research paper is the keyword search, which one will employ to find both library and Internet resources.

One must be persistent and creative in using keywords. An effective keyword search requires a strategy and planning a list of possible keywords and phrases to try.

Search engines for most searches, use Boolean searching, which is somewhat different from Internet searches. The Boolean search uses "operators," words (and, or, not, and near) that enable you to expand or narrow your affords. Tips for research paper while preparing research paper are very helpful guideline of research paper.

Choice of key words is first tool of tips to write research paper. Research paper writing is an art. A few tips for deciding as strategically as possible about keyword search:



- One should start brainstorming lists of possible keywords before even begin searching. Think about the most important concepts related to research work. Ask, "What words would a source have to include to be truly valuable in research paper?" Then consider synonyms for the important words.
- It may take the discovery of only one relevant paper to let steer in the right keyword direction because in most databases, the keywords under which a research paper is abstracted are listed with the paper.
- One should avoid outdated words.

Keywords are the key that opens a door to research work sources. Keyword searching is an art in which researcher's skills are bound to improve with experience and time.

Numerical Methods: Numerical methods used should be clear and, where appropriate, supported by references.

*Acknowledgements: Please make these as concise as possible.*

## References

References follow the Harvard scheme of referencing. References in the text should cite the authors' names followed by the time of their publication, unless there are three or more authors when simply the first author's name is quoted followed by et al. unpublished work has to only be cited where necessary, and only in the text. Copies of references in press in other journals have to be supplied with submitted typescripts. It is necessary that all citations and references be carefully checked before submission, as mistakes or omissions will cause delays.

References to information on the World Wide Web can be given, but only if the information is available without charge to readers on an official site. Wikipedia and Similar websites are not allowed where anyone can change the information. Authors will be asked to make available electronic copies of the cited information for inclusion on the Global Journals Inc. (US) homepage at the judgment of the Editorial Board.

The Editorial Board and Global Journals Inc. (US) recommend that, citation of online-published papers and other material should be done via a DOI (digital object identifier). If an author cites anything, which does not have a DOI, they run the risk of the cited material not being noticeable.

The Editorial Board and Global Journals Inc. (US) recommend the use of a tool such as Reference Manager for reference management and formatting.

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*Tables: Tables should be few in number, cautiously designed, uncrowned, and include only essential data. Each must have an Arabic number, e.g. Table 4, a self-explanatory caption and be on a separate sheet. Vertical lines should not be used.*

*Figures: Figures are supposed to be submitted as separate files. Always take in a citation in the text for each figure using Arabic numbers, e.g. Fig. 4. Artwork must be submitted online in electronic form by e-mailing them.*

## Preparation of Electronic Figures for Publication

Even though low quality images are sufficient for review purposes, print publication requires high quality images to prevent the final product being blurred or fuzzy. Submit (or e-mail) EPS (line art) or TIFF (halftone/photographs) files only. MS PowerPoint and Word Graphics are unsuitable for printed pictures. Do not use pixel-oriented software. Scans (TIFF only) should have a resolution of at least 350 dpi (halftone) or 700 to 1100 dpi (line drawings) in relation to the imitation size. Please give the data for figures in black and white or submit a Color Work Agreement Form. EPS files must be saved with fonts embedded (and with a TIFF preview, if possible).

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*Figure Legends: Self-explanatory legends of all figures should be incorporated separately under the heading 'Legends to Figures'. In the full-text online edition of the journal, figure legends may possibly be truncated in abbreviated links to the full screen version. Therefore, the first 100 characters of any legend should notify the reader, about the key aspects of the figure.*

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#### TECHNIQUES FOR WRITING A GOOD QUALITY RESEARCH PAPER:

**1. Choosing the topic:** In most cases, the topic is searched by the interest of author but it can be also suggested by the guides. You can have several topics and then you can judge that in which topic or subject you are finding yourself most comfortable. This can be done by asking several questions to yourself, like Will I be able to carry our search in this area? Will I find all necessary recourses to accomplish the search? Will I be able to find all information in this field area? If the answer of these types of questions will be "Yes" then you can choose that topic. In most of the cases, you may have to conduct the surveys and have to visit several places because this field is related to Computer Science and Information Technology. Also, you may have to do a lot of work to find all rise and falls regarding the various data of that subject. Sometimes, detailed information plays a vital role, instead of short information.

**2. Evaluators are human:** First thing to remember that evaluators are also human being. They are not only meant for rejecting a paper. They are here to evaluate your paper. So, present your Best.

**3. Think Like Evaluators:** If you are in a confusion or getting demotivated that your paper will be accepted by evaluators or not, then think and try to evaluate your paper like an Evaluator. Try to understand that what an evaluator wants in your research paper and automatically you will have your answer.

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**6. Use of computer is recommended:** As you are doing research in the field of Computer Science, then this point is quite obvious.

**7. Use right software:** Always use good quality software packages. If you are not capable to judge good software then you can lose quality of your paper unknowingly. There are various software programs available to help you, which you can get through Internet.

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**10. Bookmarks are useful:** When you read any book or magazine, you generally use bookmarks, right! It is a good habit, which helps to not to lose your continuity. You should always use bookmarks while searching on Internet also, which will make your search easier.

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**12. Make all efforts:** Make all efforts to mention what you are going to write in your paper. That means always have a good start. Try to mention everything in introduction, that what is the need of a particular research paper. Polish your work by good skill of writing and always give an evaluator, what he wants.

**13. Have backups:** When you are going to do any important thing like making research paper, you should always have backup copies of it either in your computer or in paper. This will help you to not to lose any of your important.

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**16. Use proper verb tense:** Use proper verb tenses in your paper. Use past tense, to present those events that happened. Use present tense to indicate events that are going on. Use future tense to indicate future happening events. Use of improper and wrong tenses will confuse the evaluator. Avoid the sentences that are incomplete.

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**18. Pick a good study spot:** To do your research studies always try to pick a spot, which is quiet. Every spot is not for studies. Spot that suits you choose it and proceed further.

**19. Know what you know:** Always try to know, what you know by making objectives. Else, you will be confused and cannot achieve your target.

**20. Use good quality grammar:** Always use a good quality grammar and use words that will throw positive impact on evaluator. Use of good quality grammar does not mean to use tough words, that for each word the evaluator has to go through dictionary. Do not start sentence with a conjunction. Do not fragment sentences. Eliminate one-word sentences. Ignore passive voice. Do not ever use a big word when a diminutive one would suffice. Verbs have to be in agreement with their subjects. Prepositions are not expressions to finish sentences with. It is incorrect to ever divide an infinitive. Avoid clichés like the disease. Also, always shun irritating alliteration. Use language that is simple and straight forward. put together a neat summary.

**21. Arrangement of information:** Each section of the main body should start with an opening sentence and there should be a changeover at the end of the section. Give only valid and powerful arguments to your topic. You may also maintain your arguments with records.

**22. Never start in last minute:** Always start at right time and give enough time to research work. Leaving everything to the last minute will degrade your paper and spoil your work.

**23. Multitasking in research is not good:** Doing several things at the same time proves bad habit in case of research activity. Research is an area, where everything has a particular time slot. Divide your research work in parts and do particular part in particular time slot.

**24. Never copy others' work:** Never copy others' work and give it your name because if evaluator has seen it anywhere you will be in trouble.

**25. Take proper rest and food:** No matter how many hours you spend for your research activity, if you are not taking care of your health then all your efforts will be in vain. For a quality research, study is must, and this can be done by taking proper rest and food.

**26. Go for seminars:** Attend seminars if the topic is relevant to your research area. Utilize all your resources.



**27. Refresh your mind after intervals:** Try to give rest to your mind by listening to soft music or by sleeping in intervals. This will also improve your memory.

**28. Make colleagues:** Always try to make colleagues. No matter how sharper or intelligent you are, if you make colleagues you can have several ideas, which will be helpful for your research.

**29. Think technically:** Always think technically. If anything happens, then search its reasons, its benefits, and demerits.

**30. Think and then print:** When you will go to print your paper, notice that tables are not be split, headings are not detached from their descriptions, and page sequence is maintained.

**31. Adding unnecessary information:** Do not add unnecessary information, like, I have used MS Excel to draw graph. Do not add irrelevant and inappropriate material. These all will create superfluous. Foreign terminology and phrases are not apropos. One should NEVER take a broad view. Analogy in script is like feathers on a snake. Not at all use a large word when a very small one would be sufficient. Use words properly, regardless of how others use them. Remove quotations. Puns are for kids, not grunt readers. Amplification is a billion times of inferior quality than sarcasm.

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**33. Report concluded results:** Use concluded results. From raw data, filter the results and then conclude your studies based on measurements and observations taken. Significant figures and appropriate number of decimal places should be used. Parenthetical remarks are prohibitive. Proofread carefully at final stage. In the end give outline to your arguments. Spot out perspectives of further study of this subject. Justify your conclusion by at the bottom of them with sufficient justifications and examples.

**34. After conclusion:** Once you have concluded your research, the next most important step is to present your findings. Presentation is extremely important as it is the definite medium through which your research is going to be in print to the rest of the crowd. Care should be taken to categorize your thoughts well and present them in a logical and neat manner. A good quality research paper format is essential because it serves to highlight your research paper and bring to light all necessary aspects in your research.

## INFORMAL GUIDELINES OF RESEARCH PAPER WRITING

### Key points to remember:

- Submit all work in its final form.
- Write your paper in the form, which is presented in the guidelines using the template.
- Please note the criterion for grading the final paper by peer-reviewers.

### Final Points:

A purpose of organizing a research paper is to let people to interpret your effort selectively. The journal requires the following sections, submitted in the order listed, each section to start on a new page.

The introduction will be compiled from reference matter and will reflect the design processes or outline of basis that direct you to make study. As you will carry out the process of study, the method and process section will be constructed as like that. The result segment will show related statistics in nearly sequential order and will direct the reviewers next to the similar intellectual paths throughout the data that you took to carry out your study. The discussion section will provide understanding of the data and projections as to the implication of the results. The use of good quality references all through the paper will give the effort trustworthiness by representing an alertness of prior workings.



Writing a research paper is not an easy job no matter how trouble-free the actual research or concept. Practice, excellent preparation, and controlled record keeping are the only means to make straightforward the progression.

### **General style:**

Specific editorial column necessities for compliance of a manuscript will always take over from directions in these general guidelines.

To make a paper clear

- Adhere to recommended page limits

Mistakes to evade

- Insertion a title at the foot of a page with the subsequent text on the next page
- Separating a table/chart or figure - impound each figure/table to a single page
- Submitting a manuscript with pages out of sequence

In every sections of your document

- Use standard writing style including articles ("a", "the," etc.)
- Keep on paying attention on the research topic of the paper
- Use paragraphs to split each significant point (excluding for the abstract)
- Align the primary line of each section
- Present your points in sound order
- Use present tense to report well accepted
- Use past tense to describe specific results
- Shun familiar wording, don't address the reviewer directly, and don't use slang, slang language, or superlatives
- Shun use of extra pictures - include only those figures essential to presenting results

### **Title Page:**

Choose a revealing title. It should be short. It should not have non-standard acronyms or abbreviations. It should not exceed two printed lines. It should include the name(s) and address (es) of all authors.



### Abstract:

The summary should be two hundred words or less. It should briefly and clearly explain the key findings reported in the manuscript-- must have precise statistics. It should not have abnormal acronyms or abbreviations. It should be logical in itself. Shun citing references at this point.

An abstract is a brief distinct paragraph summary of finished work or work in development. In a minute or less a reviewer can be taught the foundation behind the study, common approach to the problem, relevant results, and significant conclusions or new questions.

Write your summary when your paper is completed because how can you write the summary of anything which is not yet written? Wealth of terminology is very essential in abstract. Yet, use comprehensive sentences and do not let go readability for briefness. You can maintain it succinct by phrasing sentences so that they provide more than lone rationale. The author can at this moment go straight to shortening the outcome. Sum up the study, with the subsequent elements in any summary. Try to maintain the initial two items to no more than one ruling each.

- Reason of the study - theory, overall issue, purpose
- Fundamental goal
- To the point depiction of the research
- Consequences, including definite statistics - if the consequences are quantitative in nature, account quantitative data; results of any numerical analysis should be reported
- Significant conclusions or questions that track from the research(es)

### Approach:

- Single section, and succinct
- As a outline of job done, it is always written in past tense
- A conceptual should situate on its own, and not submit to any other part of the paper such as a form or table
- Center on shortening results - bound background information to a verdict or two, if completely necessary
- What you account in an conceptual must be regular with what you reported in the manuscript
- Exact spelling, clearness of sentences and phrases, and appropriate reporting of quantities (proper units, important statistics) are just as significant in an abstract as they are anywhere else

### Introduction:

The **Introduction** should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable to comprehend and calculate the purpose of your study without having to submit to other works. The basis for the study should be offered. Give most important references but shun difficult to make a comprehensive appraisal of the topic. In the introduction, describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will have no attention in your result. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here. Following approach can create a valuable beginning:

- Explain the value (significance) of the study
- Shield the model - why did you employ this particular system or method? What is its compensation? You strength remark on its appropriateness from a abstract point of vision as well as point out sensible reasons for using it.
- Present a justification. Status your particular theory (es) or aim(s), and describe the logic that led you to choose them.
- Very for a short time explain the tentative propose and how it skilled the declared objectives.

### Approach:

- Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done.
- Sort out your thoughts; manufacture one key point with every section. If you make the four points listed above, you will need a least of four paragraphs.



- Present surroundings information only as desirable in order hold up a situation. The reviewer does not desire to read the whole thing you know about a topic.
- Shape the theory/purpose specifically - do not take a broad view.
- As always, give awareness to spelling, simplicity and correctness of sentences and phrases.

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This part is supposed to be the easiest to carve if you have good skills. A sound written Procedures segment allows a capable scientist to replacement your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt for the least amount of information that would permit another capable scientist to spare your outcome but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section. When a technique is used that has been well described in another object, mention the specific item describing a way but draw the basic principle while stating the situation. The purpose is to text all particular resources and broad procedures, so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step by step report of the whole thing you did, nor is a methods section a set of orders.

#### **Materials:**

- Explain materials individually only if the study is so complex that it saves liberty this way.
- Embrace particular materials, and any tools or provisions that are not frequently found in laboratories.
- Do not take in frequently found.
- If use of a definite type of tools.
- Materials may be reported in a part section or else they may be recognized along with your measures.

#### **Methods:**

- Report the method (not particulars of each process that engaged the same methodology)
- Describe the method entirely
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures
- Simplify - details how procedures were completed not how they were exclusively performed on a particular day.
- If well known procedures were used, account the procedure by name, possibly with reference, and that's all.

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- It is embarrassed or not possible to use vigorous voice when documenting methods with no using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result when script up the methods most authors use third person passive voice.
- Use standard style in this and in every other part of the paper - avoid familiar lists, and use full sentences.

#### **What to keep away from**

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings - save it for the argument.
- Leave out information that is immaterial to a third party.

#### **Results:**

The principle of a results segment is to present and demonstrate your conclusion. Create this part a entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Carry on to be to the point, by means of statistics and tables, if suitable, to present consequences most efficiently. You must obviously differentiate material that would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matter should not be submitted at all except requested by the instructor.



## Content

- Sum up your conclusion in text and demonstrate them, if suitable, with figures and tables.
- In manuscript, explain each of your consequences, point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation an exacting study.
- Explain results of control experiments and comprise remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or in manuscript form.

### What to stay away from

- Do not discuss or infer your outcome, report surroundings information, or try to explain anything.
- Not at all, take in raw data or intermediate calculations in a research manuscript.
- Do not present the similar data more than once.
- Manuscript should complement any figures or tables, not duplicate the identical information.
- Never confuse figures with tables - there is a difference.

### Approach

- As forever, use past tense when you submit to your results, and put the whole thing in a reasonable order.
- Put figures and tables, appropriately numbered, in order at the end of the report
- If you desire, you may place your figures and tables properly within the text of your results part.

### Figures and tables

- If you put figures and tables at the end of the details, make certain that they are visibly distinguished from any attach appendix materials, such as raw facts
- Despite of position, each figure must be numbered one after the other and complete with subtitle
- In spite of position, each table must be titled, numbered one after the other and complete with heading
- All figure and table must be adequately complete that it could situate on its own, divide from text

### Discussion:

The Discussion is expected the trickiest segment to write and describe. A lot of papers submitted for journal are discarded based on problems with the Discussion. There is no head of state for how long a argument should be. Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implication of the study. The purpose here is to offer an understanding of your results and hold up for all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of result should be visibly described. Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved with prospect, and let it drop at that.

- Make a decision if each premise is supported, discarded, or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."
- Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work
- You may propose future guidelines, such as how the experiment might be personalized to accomplish a new idea.
- Give details all of your remarks as much as possible, focus on mechanisms.
- Make a decision if the tentative design sufficiently addressed the theory, and whether or not it was correctly restricted.
- Try to present substitute explanations if sensible alternatives be present.
- One research will not counter an overall question, so maintain the large picture in mind, where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

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<i>References</i>	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring



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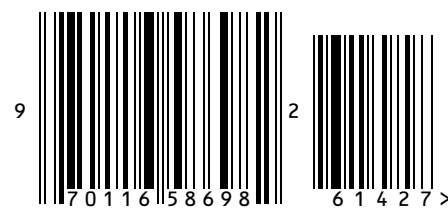


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