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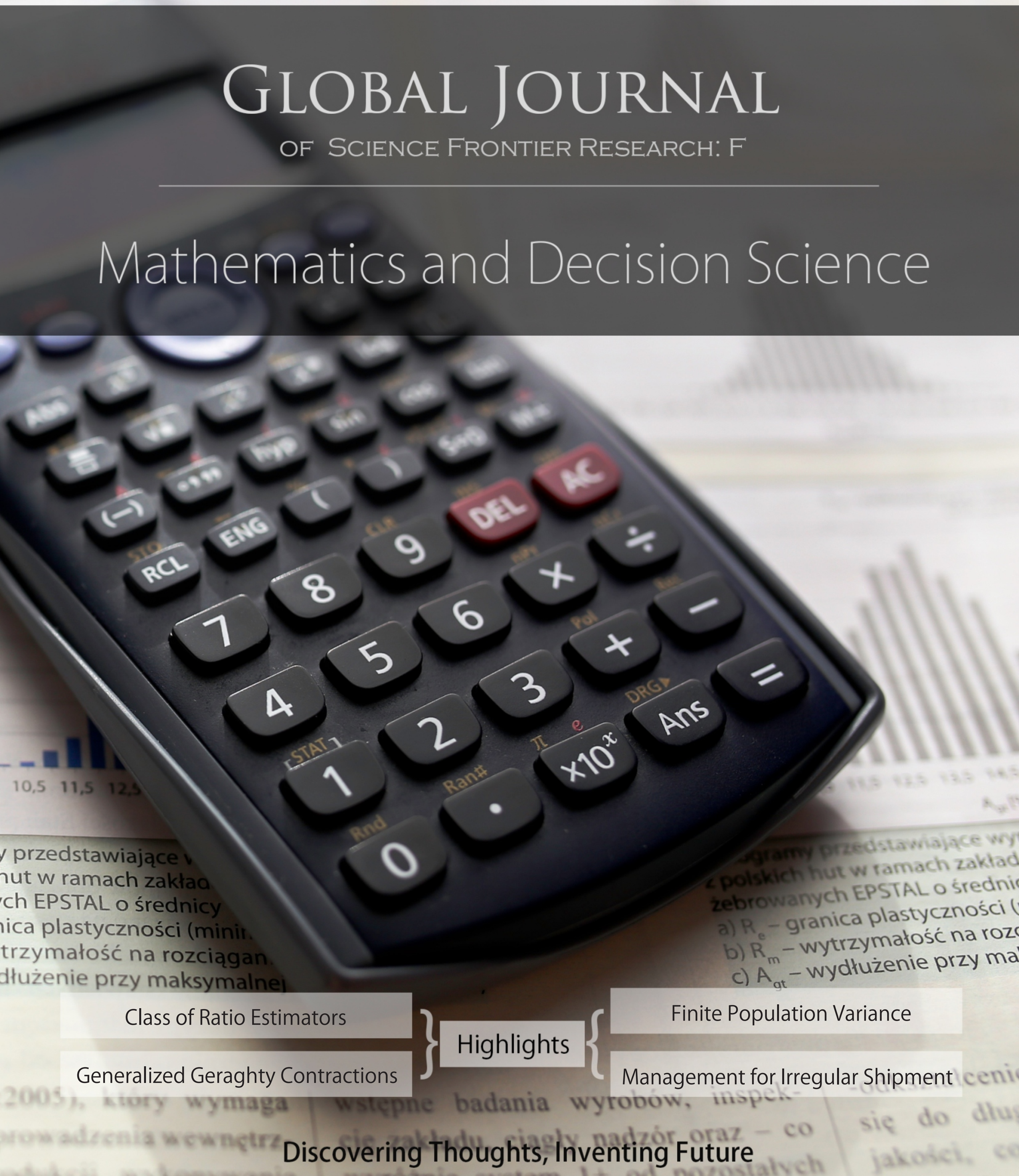

Highlights

- Class of Ratio Estimators
- Generalized Geraghty Contractions
- Finite Population Variance
- Management for Irregular Shipment

Discovering Thoughts, Inventing Future

VOLUME 16 ISSUE 2 VERSION 1.0

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Accuracy Comparison using Different Modeling Techniques under Limited Speech Data of Speaker Recognition Systems

By Satyanand Singh & Ajeet Singh
JNTU Hyderabad, India

Abstract- Pointing towards programmed machine learning by human, a technique for speaker recognition with speaker identity in light of man machine interface is an interest of science. Motivated by the same, we propose a philosophy to recognize speakers. Inside of our investigation, obtaining speech signal, analysis of spectrogram, neutralization, extraction of speaker specific features for recognition, mapping of speech using Novel Vector Quantization (NFVQ) is presented. NFVQ is particularly suitable for colossal arrangement of information and yield discourse mapping. Furthermore Speaker Recognition by utilizing NFVQ Model additionally will be exhibited in this paper. During feature extraction, traditional triangular shaped bins have been replaced by Gaussian shaped filter (GF) and Tukey filter (TF) to calculate Mel Frequency Cepstral Coefficients (MFCC). This work performs an experimental evaluation of three simple modelling techniques namely, Fuzzy c-means, FVQ2 and NFVQ. Among these NFVQ shows significant improved performance compared to Fuzzy c-means and FVQ2. For about 10 s of training and testing speech data of speakers the efficiency for NFVQ, FVQ2 and Fuzzy c-means are 98.8%, 73.33, and 8, respectively, for a set of 630 speakers taken from the TIMIT database. We additionally got 5% outright EER change for both-sex trials on the 10 s-10 s condition contrasted with the FVQ2 approach.

Index-terms: gaussian filter, triangular filter, tukey filter, subbands, MFCC, vector quantization, novel fuzzy vector quantization.

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5. Zhu Jian-wei, Sun Shui-fa, Liu Xiao-li, Lei Bang-Jun, "Pitch in Speaker Recognition." Hybrid Intelligent Systems, 2009, pp. 33-36.

Accuracy Comparison using Different Modeling Techniques under Limited Speech Data of Speaker Recognition Systems

Satyanand Singh [α] & Ajeet Singh [σ]

Abstract- Pointing towards programmed machine learning by human, a technique for speaker recognition with speaker identity in light of man machine interface is an interest of science. Motivated by the same, we propose a philosophy to recognize speakers. Inside of our investigation, obtaining speech signal, analysis of spectrogram, neutralization, extraction of speaker specific features for recognition, mapping of speech using Novel Vector Quantization (NFVQ) is presented. NFVQ is particularly suitable for colossal arrangement of information and yield discourse mapping. Furthermore Speaker Recognition by utilizing NFVQ Model additionally will be exhibited in this paper. During feature extraction, traditional triangular shaped bins have been replaced by Gaussian shaped filter (GF) and Tukey filter (TF) to calculate Mel Frequency Cepstral Coefficients (MFCC). This work performs an experimental evaluation of three simple modelling techniques namely, Fuzzy c-means, FVQ2 and NFVQ. Among these NFVQ shows significant improved performance compared to Fuzzy c-means and FVQ2. For about 10 s of training and testing speech data of speakers the efficiency for NFVQ, FVQ2 and Fuzzy c-means are 98.8%, 73.33, and 8, respectively, for a set of 630 speakers taken from the TIMIT database. We additionally got 5% outright EER change for both-sex trials on the 10 s-10 s condition contrasted with the FVQ2 approach.

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I. Introduction

A speaker recognition system mainly consists of two main modules, speaker specific feature extractor as a front end followed by a speaker modelling technique for generalized representation of extracted features [1, 2]. Since long time MFCC is considered as a reliable front end for a speaker recognition application because it has coefficients that represents audio, based on perception [3, 4]. In MFCC the frequency bands are positioned logarithmically which approximated the human auditory systems response more closely than the linear spaced frequency bands of FFT or DCT. The main speaker specific information is pitch [5], residual phase [6], prosody [7], dialectical features [8] etc. These features are related with vocal chord vibration and it is very difficult to extract speaker specific information [9]. The MFCC modeled by Fuzzy c-Means, FVQ2 and NFVQ [10] technique.

In this paper a NFVQ is proposed for speaker recognition modelling. All the above vector quantization routines perform the codebook outline by utilizing crisp choice making systems [11], the feeling that every preparation vector is allocated to one

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and only group. Inevitably, these routines overlook the likelihood that a particular preparing vector might likewise have a place with another group. Fuzzy set theory created by Zadeh has been seen as a distinct option for more conventional contemplations keeping in mind the end goal to manage perplexing, poorly characterized and less scientifically justifiable frameworks [12].

The fundamental issue in Fuzzy logic is that a particular item can be allocated to more than one bunch with specific degrees of support [13]. The use of Fuzzy systems in speaker recognition gives two fundamental advantages. Firstly, Fuzzy set hypothesis can show the vulnerability included in the information set of the preparation vectors [14]. Furthermore, it offers a computational system, which is algorithmically furnished with a strong and all around organized scientific foundation [15]. The Fuzzy logic strategies, which can be productively utilized as a part of speaker recognition, are principally in view of Fuzzy bunching investigation [16]. The most illustrative Fuzzy bunching calculation is the understood Fuzzy c-means system, which was produced by Bezdek in [17]. The Fuzzy c-means system regards every group as a Fuzzy set and along these lines, the codebook configuration is a delicate choice making procedure [18]. Since Fuzzy grouping can demonstrate the vulnerability included in the segment of the preparation vector space, it can be utilized to take out or if nothing else fundamentally diminish the reliance of the codebook outline on the introduction [19].

In this paper we propose a NFVQ algorithm for speaker recognition. In the first step, we introduce a simple modification of the fuzzy c-means objective function and reformulate this objective function. In the next step, we extract analytical learning conditions for the codebook design by minimizing the reformulated function. In many real situations, fuzzy clustering is more natural than hard clustering, as objects on the boundaries between several classes are not forced to fully belong to one of the classes, but rather are assigned membership degrees between 0 and 1 indicating their partial memberships. So the present study was undertaken with the objective of to find out the speaker recognition efficiency improving components with the help of a novel algorithms.

II. Fuzzy Clustering for Vector Quantization

In SR Vector quantization is fretful with the demonstration of a set of unlabeled data vectors $X = \{x_1, x_2, x_3 \dots, x_n\} \in \mathbb{R}^p$ by a set $V = \{v_1, v_2, v_3 \dots, v_c\} \in \mathbb{R}^p$ with $c \ll n$. Here x_k is called training vector and the set $\mathbf{X}$ is referred as training set, while each v_i is called codebook vector and the set $\boldsymbol{V}$ is referred as codebook. The key issue in vector quantization is the codebook outline. The codebook can be composed by utilizing hard or crisp choice making systems. In both cases, the nature of the last codebook is generally assessed by the accompanying normal distortion measure,

$$D = \frac{1}{n}\sum_{k=1}^{n} \min_{1 \le i \le c}\{\|x_k - v_i\|^2\} \tag{1}$$

We depict two Fuzzy grouping based vector quantization calculations in next segment, which are understood Fuzzy c-means and the FVQ2 created by Karayiannis and Pai in [20].

a) The Fuzzy c-Means Algorithm

The Fuzzy c-means is the most generally utilized calculation to deliver constrained fuzzy c-partitions in speaker recognition [21]. $\mathrm{u_{i\,k}} = \{\mathrm{u_i(x_k)}, 1 \le \mathrm{i} \le \mathrm{c}, 1 \le$

Ref

16. Li-juan Ma, Ming hu Ha., "Support Vector Machines Based on Sectional set Fuzzy K-Means Clustering," Fuzzy Info. And Engineering, vol. 54, Jan 2009, pp. 420-425.

k ≤ n } represents membership degree of the kth training vector to ith cluster. There are constrained in cluster if the next three conditions are satisfied,

$$0 \le u_{i\,k} \le 1\,, \quad \forall\, i, k$$

$$0 < \sum_{k=1}^{n} u_{i\,k} < n, \quad \forall\, i$$

$$\textstyle\sum_{i=1}^{c} u_{i\,k} = 1, \qquad \forall k \tag{2}$$

At whatever point the last situation is not fulfilled the Fuzzy c-means is said to be unconstrained. The usage of the Fuzzy c-means depends on the minimization, under the fairness imperative given in eq. (2),

$$J_m = \sum_{k=1}^{n}\sum_{i=1}^{c} (u_{i\,k})^m \|x_k - v_i\|^2 \tag{3}$$

Where $m \in (1, \infty)$ is a component to adjust the membership degree weighting effect. The cluster centers (codebook vectors) and the membership degrees that take care of the above compelled improvement issue are separately given by the accompanying mathematical statements [13],

$$v_i = \frac{\sum_{k=1}^{n} (u_{i\,k})^m \ x_k}{\sum_{k=1}^{n} (u_{i\,k})^m}\ , \qquad 1 \le i \le c \tag{4}$$

And

$$u_{i,k} = \frac{1}{\sum_{j=1}^{c} \left(\frac{\|x_k - v_i\|}{\|x_k - v_j\|} \right)^{\frac{2}{m-1}}}\ , 1 \le i \le c\, , 1 \le k \le n \tag{5}$$

Mathematical statements in eq. (4) and (5) speak to an iterative enhancement methodology, where m is the Fuzziness controlling parameter. If m takes extensive values then the participation degrees of every preparation vector tend to approach 1/c.

In what follows, results due to the case studies are presented to minimize the objective function to enhance the percentage of speaker recognition accuracy. For example, let us consider a case of 64 clusters and 35 iteration. Fuzzy c-means clustering and its minimum objective function J_m= 0.1878. Fig. 1 shows the plot of objective function by fuzzy c-means clustering.

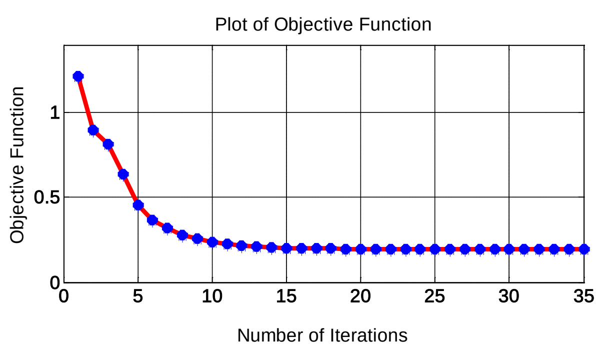

Fig. 1 : Plot of objective function J_m of Fuzzy c-means clustering

Ref

13. Jayanna, H.S., Prasanna, S.R. Mahadeva., "Fuzzy Vector Quantization for Speaker Recognition under Limited Data Conditions," IEEE Region 10 Conference, TENCON 2008, Hyderabad India, Nov 2008, pp.1-4.

Fig. 2 shows the plot of distortion of the spea kers using fuzzy c-means clustering.

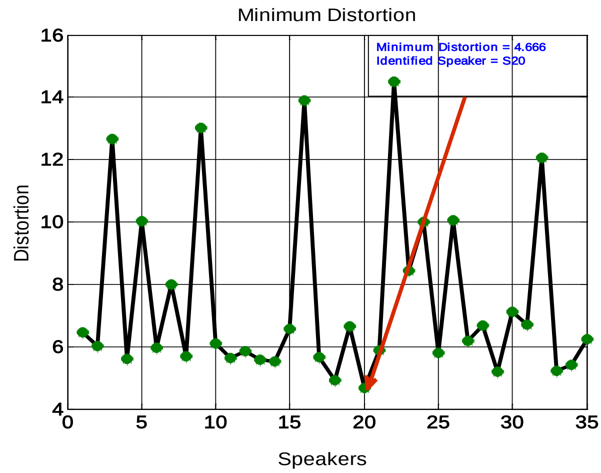

Fig. 2 : Plot of distortion measurement by fuzzy c-means clustering

Distortion of the voice data of the speakers during testing phase using fuzzy c-means clustering. D = 6.4711, 6.0261, 12.6531, 5.6056, 10.0395, 5.968, 8.0071, 5.6889, 13.0149, 6.0971, 5.6337, 5.8435, 5.5733, 5.5329, 6.5637, 13.8927, 5.6554, 4.9273, 6.6624, **4.6669**, 5.8925, 14.5138, 8.4431, 9.9943, 5.7945, 10.0552, 6.195, 6.6755, 5.192, 7.1233, 6.7116, 12.066, 5.209, 5.4083, 6.2515.

b) *Fuzzy Vector Quantization2*

Vector quantization is completed by relating every preparation vector to a solitary codebook vector. In this manner, the utilization of the Fuzzy c-means to vector quantization ought to be founded on relegating every preparation vector to the codebook vector. In any case, such a fresh understanding of the Fuzzy c-means amid the codebook configuration may affect the nature of the last codebook, since this methodology shrouds the presence of anomalies and replaces them by their nearest codebook vectors.

The arrangement of the codebook vectors that fit in with the hyper circle focused at the kth preparing vector is meant as T_k. At that point, the move from Fuzzy to crisp mode is proficient by steadily contracting the covering hyper circles amid the grouping procedure. In addition, it was found that the move speed straightforwardly influences the nature of the subsequent codebook [20]. As the configuration procedure continues, the set T_k is upgraded by taking after method:

In the ν-th iteration the set $T_k^{(v)}$ contains $\aleph T_k^{(v)}$ codebook vectors. The average distance is defined as,

$$\check{d}_k^{(v)} = \frac{1}{\aleph(T_k^{(v)})} \sum_{v_i \in T_K^{(v)}} \|x_k - v_i\|^2 \tag{6}$$

The T_k is updated in the $(v+1)th$ iteration as follows,

$$T_k^{(v+1)} = \left\{ v_i \in T_k^{(v)} : \|x_k - v_i\|^2 \le \check{d}_k^{(v)} \right\} \tag{7}$$

The above upgrading guideline requires that the enrollment degrees of x_k to the codebook vectors, which during the $(i+1)^{th}$ iteration are removed from the set $T_k^{(v)}$, are set equal to zero. In this manner, at first, every preparation vector is relegated to

Ref

21. James C. Bezdek, "Pattern Recognition with Fuzzy Objective Function Algorithms," Plenum Press, New York, 1987.

the majority of the codebook vectors. These sureties the interest of all the codebook vectors in the codebook outline process. As this improvement continues, the cardinality $\aleph(T_k^{(v)})$ of the set T_k diminishes, until T_k will incorporate stand out component. For this situation the k-th preparing vector is exchanged from Fuzzy to crisp mode. For assessment reasons, from the methodologies created in [13], we utilize the FVQ2 calculation, in light of the fact that just this calculation is specifically identified with the compelled minimization of the target capacity given in eq. (3). The usage of the FVQ2 requires that the codebook vectors are redesigned by utilizing eq. (4)· Additionally, in the v-th emphasis, if the preparation vector x_k is in fuzzy mode its participation degrees are figured as,

$$u_{i\,k} = \frac{1}{\sum_{v_j \in T_k^{(v)}} \left(\frac{\|x_k - v_i\|}{\|x_k - v_j\|} \right)^{\frac{2}{m-1}}} \text{, with } v_i \in T_k^{(v)} \tag{8}$$

while in the event that x_k is in fresh mode then the participation degrees are given by the following closest neighbor condition,

$$u_{i\,k} = \begin{cases} 1, \text{ if } \|x_k - v_i\|^2 = \min_{1 \le i \le c}\{\|x_k - v_i\|^2\} \\ 0, \text{ otherwise} \end{cases} \tag{9}$$

Alluding to the last comparison, the utilization of eq. (4) is still legitimate, subsequent to for this situation it holds that $(u_{i\,k})^m = u_{i\,k}$ notwithstanding the estimation of m. To this end, the FVQ2 calculation comprises on iteratively utilizing the eqs (4), (8) and (9) to figure the codebook vectors and the participation degrees, in blend with the beforehand dissected methodology for the move from fuzzy to crisp decisions. FVQ2 clustering and its minimum objective function $\check{d}_k^{(v)}$=0.1887. Fig. 3 shows the plot of objective function by FVQ2.

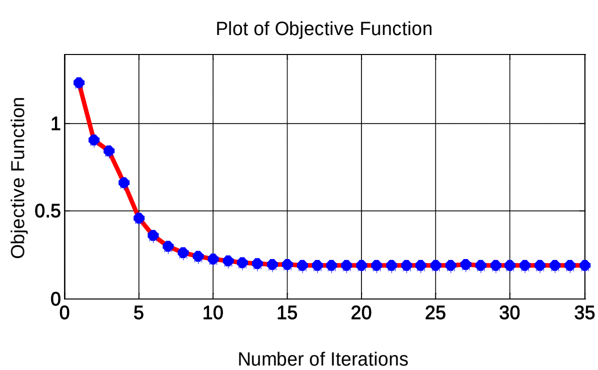

Fig. 3 : Plot of objective function of Fuzzy Vector Quantization2 clustering

Ref
13. Jayanna, H.S., Prasanna, S.R. Mahadeva., "Fuzzy Vector Quantization for Speaker Recognition under Limited Data Conditions," IEEE Region 10 Conference, TENCON 2008, Hyderabad India, Nov 2008, pp.1-4.

Fig. 4 shows the plot of distortion of the speakers using FVQ2 clustering.

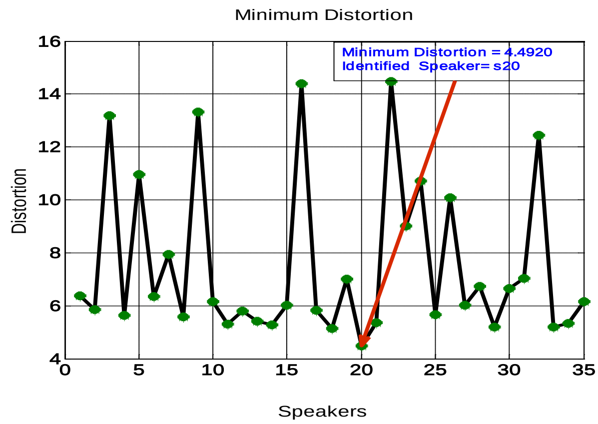

Fig. 4 : Plot of distortion measurement of Fuzzy Vector Quantization2

Distortion of the voice data of the speakers during testing phase using FVQ 2 clustering. D =6.3721, 5.8452, 13.1923, 5.6251, 10.963, 6.3408, 7.938, 5.582, 13.319, 6.163, 5.296, 5.785, 5.412, 5.264, 6.011, 14.384, 5.815, 5.131, 7.005, **4.492**, 5.362, 14.482, 9.007, 10.723, 5.671, 10.07, 6.0151, 6.7238, 5.2056, 6.6534, 7.0366, 12.4455, 5.1927, 5.3277, 6.1519.

III. The Novel Fuzzy Vector Quantization Algorithm

In this section we present a detailed analysis of the NFVQ algorithm. The algorithm is based on the following novel objective function of the fuzzy *c*-means method,

$$J = \sum_{k=1}^{n}\sum_{i=1}^{c} f(u_{i\,k})\|x_k - v_i\|^2 \tag{10}$$

With

$$f(u_{i\,k}) = \frac{1}{2}u_{ik} + \frac{1}{2}(u_{ik})^2 \tag{11}$$

Where u_{ik} is the membership degree of the *k*-th training vector to the *i*-th codebook vector. The objective is to minimize the above function under the following equality constraint,

$$\sum_{i=1}^{c} u_{ik} = 1, \quad \forall\, k \tag{12}$$

The membership degrees and the codebook vector values that solve the above minimization problem are given by the following theorems,

Theorem 1

If v_i are settled then u_{ik} that minimize J in eq (10), under the imperative in eq. (12), is presented as follows,

$$u_{ik} = \frac{c+2}{2}.\frac{1}{\sum_{j=1}^{c}\left(\frac{\|x_k - v_i\|}{\|x_k - v_j\|}\right)^2} - \frac{1}{2} \tag{13}$$

Notes

Proof

By eq. (12), the Lagranian of eq.(10) for a only one training vectorx_k is,

$$F(u_{ik},\lambda_k) = \sum_{i=1}^{c} f(u_{ik})\|x_k - v_i\|^2 - \lambda_k\left(\sum_{i=1}^{c} u_{ik} - 1\right)$$

Notes

The partial derivative of the Lagranian with respect to λ_k is,

$$\frac{\partial F(u_{ik},\lambda_k)}{\partial \lambda_k} = -\left(\sum_{i=1}^{c} u_{ik} - 1\right)$$

Equating the above derivative equal to zero and get the eq. (12).

$$\frac{\partial F(u_{ik},\lambda_k)}{\partial u_{ik}} = \frac{\partial f(u_{ik})}{\partial u_{ik}}\|x_k - v_i\|^2 - \lambda_k \quad (14)$$

Considering (u_{ik}), in eq. (11), the eq. (14) can be written as,

$$\frac{\partial F(u_{ik},\lambda_k)}{\partial u_{ik}} = \left(\frac{1}{2} + u_{ik}\right)\|x_k - v_i\|^2 - \lambda_k \quad (15)$$

Setting the above derivative equal to zero and illuminating as for u_{ik} we get the following equation,

$$u_{ik} = \frac{\lambda_k}{\|x_k - v_i\|^2} - \frac{1}{2} \quad (16)$$

Combining eqs (16) and (12) it follows that,

$$\sum_{j=1}^{c}\left(\frac{\lambda_k}{\left\|x_k - v_j\right\|^2} - \frac{1}{2}\right) = 1 \quad (17)$$

Fathoming the last mathematical statement concerning λ_k and substituting into eq. (16) we can without much of a stretch determine the eq. (13). This finishes the confirmation of the theorem 1

Theorem 2

In the event that the u_{ik} are settled, then the cluster centers v_i that minimize J in eq. (11) is given by the following mathematical equation.

$$v_i = \frac{\sum_{k=1}^{n} f(u_{ik})x_k}{\sum_{k=1}^{n} f(u_{ik})} \quad (18)$$

Proof

In perspective of eq. (10), setting the partial derivative ${\partial J}/{\partial v_i}$ equal to zero and tackling regarding v_i we can undoubtedly get the eq. (18). This finishes the verification of of theorem 2.

Substituting eq. (13) into the objective function in (10) we can easily obtain the following reformulating function,

$$R_J = R_{J1} + R_{J2} \quad (19)$$

$$R_{J1} = \frac{2+c}{4}\sum_{k=1}^{n}\sum_{i=1}^{c}\left\{\|x_k - v_i\|^2\left[\sum_{j=1}^{c}\frac{\|x_k - v_i\|^2}{\|x_k - v_j\|^2}\right]^{-1}\right\}$$

$$-\frac{1}{4}\sum_{k=1}^{n}\sum_{i=1}^{c}\|x_k - v_i\|^2 \Rightarrow$$

$$R_{J1} = c\frac{2+c}{4}\sum_{k=1}^{n}\left[\sum_{j=1}^{c}\frac{1}{\|x_k - v_j\|^2}\right]^{-1}$$

$$-\frac{1}{4}\sum_{k=1}^{n}\sum_{i=1}^{c}\|x_k - v_i\|^2 \tag{20}$$

Relationally,

$$R_{J2} = \frac{4-c^2}{8}\sum_{k=1}^{n}\left[\sum_{i=1}^{c}\frac{1}{\|x_k - v_i\|^2}\right]^{-1}$$

$$+\frac{1}{8}\sum_{k=1}^{n}\sum_{i=1}^{c}\|x_k - v_i\|^2 \tag{21}$$

Substituting (20) and (21) into (19), the reformulating function is novel as follows,

$$R_J = K_1\sum_{k=1}^{n}\left[\sum_{i=1}^{c}\frac{1}{\|x_k - v_i\|^2}\right]^{-1} - K_2\sum_{k=1}^{n}\sum_{i=1}^{c}\|x_k - v_i\|^2 \tag{22}$$

Where

$$K_1 = \frac{(2+c)^2}{8} \quad and \quad K_2 = \frac{1}{8} \tag{23}$$

By minimizing the reformulating function in (22) with respect to the codebook vectors, the gradient-descent based learning rule for the *i*-th codebook vector is given as,

$$v_i(t+1) = v_i(t) - a(t)\sum_{k=1}^{n} f\big(u_{ik}(t)\big)x_k \tag{24}$$

Where $f(u_{ik})$ is given in eqn. (11), and $a(t)$ is the learning rate parameter, which can be calculated as follows,

$$a(t) = a_0\left(1 - \frac{t}{t_{max}}\right) \tag{25}$$

Where a_0 the initial is value for the learning parameter, and t_{max} is the maximum number of iteration. Based on the above analysis, the proposed fuzzy learning vector quantization algorithm for speaker recognition given as follows,

Notes

Notes

The Novel vector quantization for speaker recognition

Randomly select initial values for the v_i.

Set values for the design parameters t_{max} and a_0.

For $t = 1\ to$ $\mathrm{t}_{\max}$

Using eqn (13) calculate the u_{ik}

Using eqn. (11) calculate $f(u_{ik})(1 \leq i \leq c, 1 \leq k \leq n)$.

For $i = 1\ to\ c$

Using eqn. (24) to update the codebook vectors.

Endfor

Endfor

End

NFVQ clustering and its minimum objective function J=0.073. Fig. 5 shows the plot of objective function by novel Fuzzy vector quantization.

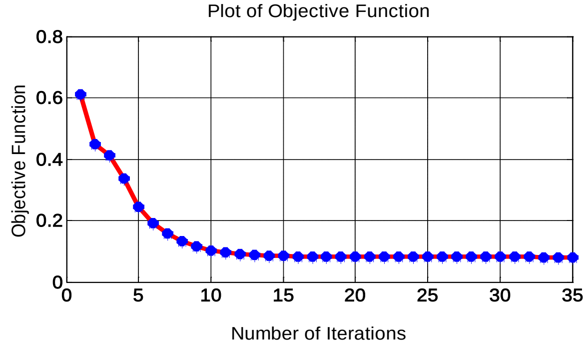

Fig. 5 : Plot of objective function of Novel Fuzzy Vector Quantization

Fig. 6 shows the plot of distortion of the speakers using NFVQ clustering.

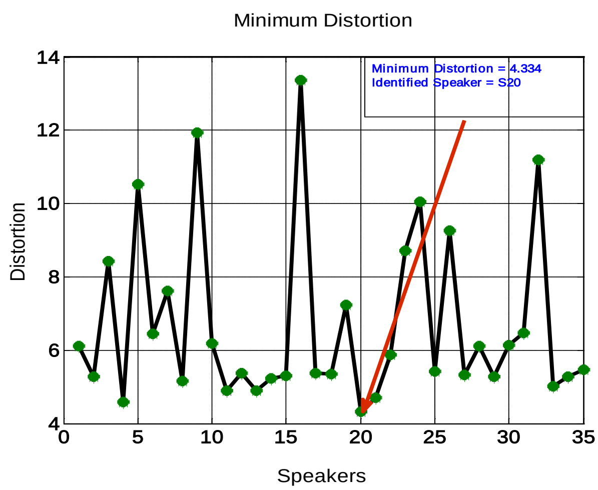

Fig. 6 : Plot of distortion measurement of NFVQ

Distortion of the voice data of the speakers during testing phase using NFVQ. D = 6.1162, 5.2875, 8.4223, 4.5901, 10.5284, 6.4421, 7.6078, 5.1621, 11.9275, 6.1786, 4.8972, 5.3811, 4.8904, 5.2311, 5.3065, 13.3695, 5.3833, 5.3433, 7.2431, **4.334**, 4.7034, 5.8735, 8.7089, 10.0453, 5.4298, 9.2664, 5.3304, 6.1129, 5.2765, 6.1359, 6.4721, 11.205, 5.0118, 5.2669, 5.4778.

In the following section we present the experimental results.

IV. Experimental Results

The proposed algorithm was implemented in MATLAB and results were compared with those of the Fuzzy c-Means and FVQ2 algorithms.

The proposed speaker recognition system efficiency is evaluated with the following design parameters $t_{max} = 35$ and $a_o = 0.5$ codebook size $= 256$. The experiment uses two sets of databases TIMIT and self-collected database.

a) Experimental Result

The performance of MFCC based classifier has been evaluated where each feature set was tested using TF, GF and Tukey Filter. A total 1000 utterances were put to test for 100 speakers. For the above cases, recognition accuracy has been calculated using the expression:

Percentage of Identification Accuracy=No of utterance correctly identified/Total No of utterance under test.

Table I shows the identification accuracies of TIMIT database for TF, GF and Tukey based filters and Fuzzy c-means, FVQ2 and NFVQ techniques respectively. It can be observed from this table that use of GF and NFVQ show significant improvement.

Table 1 : Speaker Recognition of TIMIT Database

Filters	Fuzzy c-Means Accuracy (%)	FVQ 2 Accuracy (%)	NFVQ Accuracy (%)
Triangular Filter	96.9	97	**97.2**
Gaussian Filter	98.1	98.3	**98.8**
Tukey Filter	97.3	97.5	**97.9**

It can be observed from this table that the combination of GF and NFVQ algorithms shows significant improvement up to 98.8%. In a real life situation, a biometric security system, which is usually imperfect, the characteristic curves of FRR and FAR intersect at a certain point called 'Equal Error Rate (EER). If one fixes a very low threshold value, then the system would exhibit very low FRR and very high FAR and accept all identity claims. Alternatively, if one fixes a very high threshold value, then the system would exhibit very high FRR and very low FAR and reject all identity claims. In this context, one could plot a curve called 'Receiver Operating Characteristic (ROC)', which involves FRR and FAR. ROC curve is a graphical indication of the system performance. Fig. 7 shows a typical EER curve.

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Fig. 7 : Plot of Equql Error Rate

Fig. 8 shows a typical DET curve showing the optimum detection cost for Fuzzy c-means clustering based speaker recognition system. In Fuzzy c-means clustering the EER is 6.5.

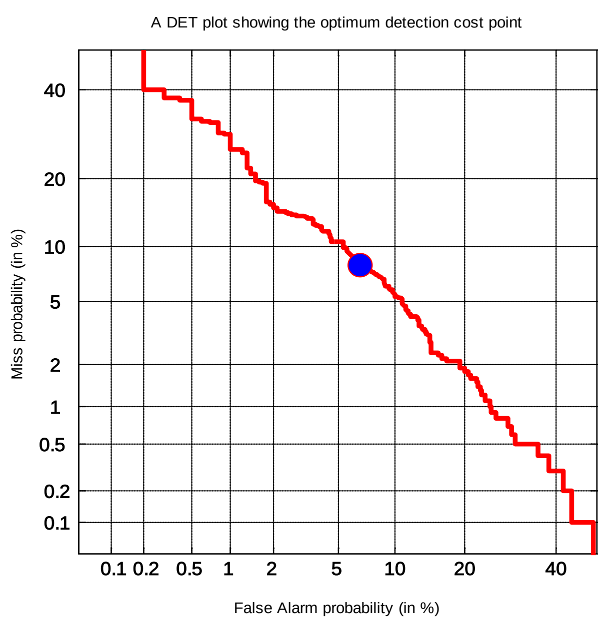

Fig. 8 : Plot of DET showing the optimum detection cost for Fuzzy c-means

Fig. 9 shows a typical DET Curves showing the optimum detection cost for FVQ2 clustering based speaker recognition system and EER is 6.1.

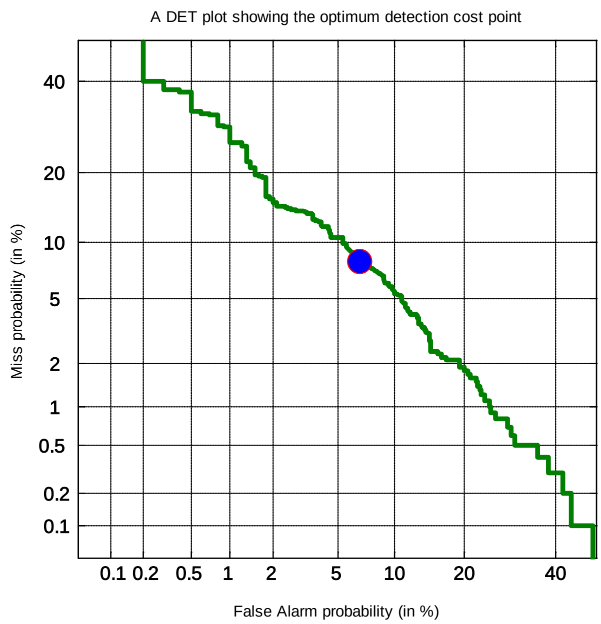

Fig. 9 : Plot of DET showing the optimum detection cost for FVQ2

Fig. 10 shows typical DET curve of optimum detection cost for NFVQ clustering based speaker recognition system and EER is 5.5. In this case the EER is minimum compare to fuzzy c-means and FVQ2. Proposed NFVQ algorithm gives the lower EER that is 5.6, FVQ2 algorithms gives medium EER performance that is 5.9. Finally, the Fuzzy c-means provided the highest EER of 6.1.

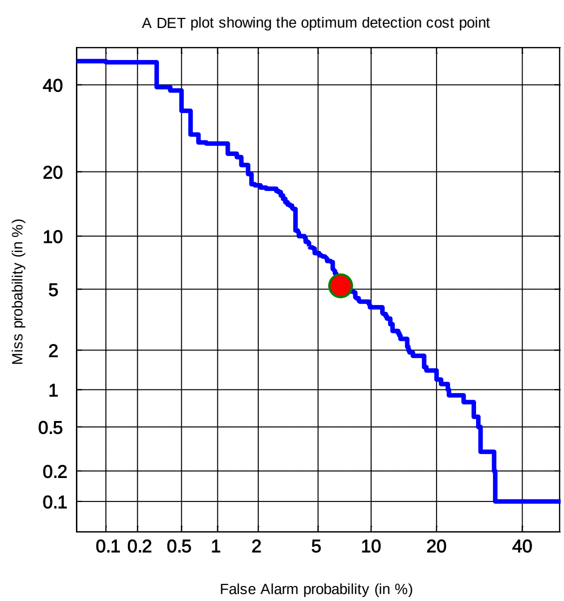

Fig. 10 : Plot of DET showing the optimum detection cost for NFVQ

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Fig. 11 shows a typical DET Curves showing the speaker recognition evaluation for NFVQ, FVQ2 and Fuzzy c-means clustering techniques.

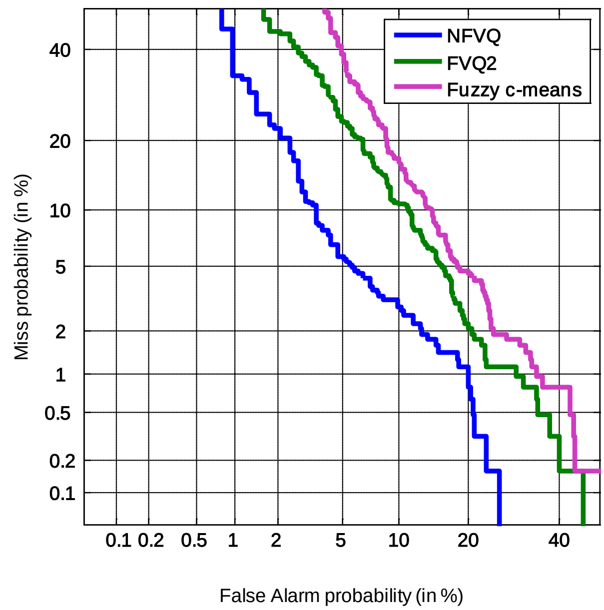

Fig. 11 : Plot of DET Curves for a speaker recognition evaluation

V. Conclusion and Future Work

This paper presented the evaluation of a NFVQ algorithm for speaker recognition. The calculation was intended to catch the favorable circumstances gave by Fuzzy choice making procedures, while keeping up the computational abilities accomplished by fresh crisp making procedures. This was accomplished by developing and reformulating a novel objective function for the well known fuzzy c-means. Several simulations were performed, in which the proposed algorithm was compared to other techniques found in the literature. The objective function is minimized and distortion of the new NFVQ approach is reduced when compared with the objective function and distortion of Fuzzy c-means, and FVQ2. The NFVQ clustering algorithm for speaker recognition is promising as it shows improvement compared to other methods. Equal Error Rate (EER) due to NFVQ is very small when compared to the EER due to Fuzzy c-means clustering and FVQ2 hence NFVQ algorithm for speaker recognition is better than the others. The aftereffect of this examination demonstrates that the calculation can be utilized as a solid instrument as a part of speaker recognition applications. The system performance and speaker recognition efficiency can be further improved by using systematic hierarchical database.

VI. Acknowledgment

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Improved Class of Ratio Estimators for Finite Population Variance

By Audu Ahmed, Adedayo Amos Adewara, Ran Vijay Kumar Singh
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Abstract- In this paper, we have suggested a class of improved ratio estimators for finite population variance. The proposed class of estimators is obtained by transforming both the sample variances of study and auxiliary variables. The MSE of the proposed estimators have been obtained and the conditions for their efficiency over some existing variance estimators have been established. The present family of finite variance estimator, having obtaining the optimal values of the constants, exhibit significant improvement over the estimators considered in the study. The empirical study is also conducted to corroborate the theoretical results and the results show that the proposed class of estimators is more efficient.

Keywords: efficiency, mean square error, ratio estimator, finite population, variance.

GJSFR-F Classification : *MSC 2010: 00A05*

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Improved Class of Ratio Estimators for Finite Population Variance

Audu Ahmed [α], Adedayo Amos Adewara [σ] & Ran Vijay Kumar Singh [ρ]

Abstract- In this paper, we have suggested a class of improved ratio estimators for finite population variance. The proposed class of estimators is obtained by transforming both the sample variances of study and auxiliary variables. The MSE of the proposed estimators have been obtained and the conditions for their efficiency over some existing variance estimators have been established. The present family of finite variance estimator, having obtaining the optimal values of the constants, exhibit significant improvement over the estimators considered in the study. The empirical study is also conducted to corroborate the theoretical results and the results show that the proposed class of estimators is more efficient.

Keywords: efficiency, mean square error, ratio estimator, finite population, variance.

I. Introduction

The variation of produce or yields in the manufacturing industries and pharmaceutical laboratories are sometime a matter of concern to researchers (Ahmed et al. [2]). The use of supplementary (auxiliary) information, being constant with unit (e.g. population mean, population standard deviation, e.t.c.) or unit free constant (e.g. Coefficient of variation, Kurtosis, e.t.c.), can enhance the efficiency at the estimation stage. In recent past, this concept has been utilized by several authors to improve the efficiency of ratio and product type estimators for estimating population mean as well population variance of study variable.

In this paper, an improved class of ratio estimators for estimating finite population variance has been proposed with objective to produce efficient estimators and their properties have established.

Let $\Omega = (1\ 2, 3 \ldots N)$ be a population of size N and Y, X be two real valued functions having values$(Y_i,\ X_i) \in \mathbb{R}^{+} > 0$ on the i^{th} unit of $U(1 \leq i \leq N)$. We assume positive correlation $\rho > 0$ between the study variable Y and auxiliary variable X. Let S_y^2 and S_x^2 be the finite population variance of Y and X respectively and s_y^2 and s_x^2 be respective sample variances based on the random sample of size n drawn without replacement.

Singh et al. [8] defined the general family of estimators for estimating finite population variance S_y^2 of the study variable Y as

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$$\eta = s_y^2\left[\frac{aS_x^2+b}{\alpha\left(as_x^2-b\right)+\left(1-\alpha\right)\left(aS_x^2-b\right)}\right] \tag{1.1}$$

where a and b are constants based on auxiliary variable X like coefficient of skewness, kurtosis and correlation coefficient etc. α is the constant that minimizes the mean square error (MSE) of the estimator. Table 1 shows some members of η-family for different values of a, b and α.

The MSEs/Variance of the estimators in Table 1 are given below:

$$Var\left(\eta_0\right) = \gamma S_y^2\left(\psi_{40}-1\right) \tag{1.2}$$

$$MSE\left(\eta_i\right) = \begin{cases} S_y^4\gamma\left[\left(\psi_{40}-1\right)+\left(\psi_{04}-1\right)-2\left(\psi_{22}-1\right)\right] & i=1 \\ S_y^4\gamma\left[\left(\psi_{40}-1\right)+h_i^2\left(\psi_{04}-1\right)-2h_i\left(\psi_{22}-1\right)\right] & i=2,3,4,5,6 \end{cases} \tag{1.3}$$

$$h_2 = \frac{S_x^2}{S_x^2-C_x},\ h_3 = \frac{S_x^2}{S_x^2-\beta_2(x)},\ h_4 = \frac{S_x^2\beta_2(x)}{S_x^2\beta_2(x)-C_x},\ h_5 = \frac{S_x^2C_x}{S_x^2C_x-\beta_2(x)},\ h_6 = \frac{S_x^2}{S_x^2+\beta_2(x)}$$

where

$$s_y^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(y_i-\bar{y}\right)^2,\ s_x^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i-\bar{x}\right)^2,\ S_y^2 = \frac{1}{N-1}\sum_{i=1}^{n}\left(Y_i-\bar{Y}\right)^2,\ S_x^2 = \frac{1}{N-1}\sum_{i=1}^{n}\left(X_i-\bar{X}\right)^2$$

$$\bar{X} = \frac{1}{N}\sum_{i=1}^{N}X_i,\quad \bar{Y} = \frac{1}{N}\sum_{i=1}^{N}Y_i,\quad \bar{x} = \frac{1}{n}\sum_{i=1}^{n}x_i,\quad \bar{y} = \frac{1}{n}\sum_{i=1}^{n}y_i$$

$$\gamma = \frac{1}{n},\ \psi_{rs} = \frac{\lambda_{rs}}{\lambda_{20}^{r/2}\lambda_{02}^{s/2}} \text{ and } \lambda_{rs} = \frac{1}{N-1}\sum_{i=1}^{N}\left(Y_i-\bar{Y}\right)^r\left(X_i-\bar{X}\right)^s$$

The MSE of η to second order approximation is given below:

$$MSE\left(\eta\right) = \gamma S_y^2\left[\left(\psi_{40}-1\right)+\alpha^2\theta^2\left(\psi_{04}-1\right)-2\alpha\theta\left(\psi_{22}-1\right)\right] \tag{1.4}$$

The $MSE\left(\eta\right)$ expression is minimized for the optimum values of α given by equation (1.5). This is obtained by partial differentiation of equation (1.4) with respect to α.

$$MSE\left(\eta\right)_{\min} = \gamma S_y^4\left[\left(\psi_{40}-1\right)-\frac{\left(\psi_{22}-1\right)^2}{\left(\psi_{04}-1\right)}\right] \tag{1.5}$$

Many other researchers including Kadilar and Cingi [6], Yadav and Kadilar [15], Singh and Solanki [10], Gupta and Shabir [3], Subramani and Kumarapandiyan [13], Singh and Vishwakarma [11], Singh, et al. [9], Sanaullah, et al. [7] and Solanki and Singh [12] have significantly contributed to the improvement of both ratio and product mean & variance estimators in sampling survey.

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II. Proposed Estimator

After studying the related finite population variance estimators stated in section 1 and motivated by the work of Yadav and Kadilar [16] and Adewara et al.[1] estimators for population mean in which the former (i.e. Yadav and Kadilar [16]) equals the latter (i.e. Adewara et al.[1]) when $k_1 = k_2 = k_3 = k_4 = k_5 = k_6 = 1$. Their estimators are defined as

$$\eta_1^* = k_1 \bar{y}^* \left(\frac{\bar{X}}{\bar{x}^*} \right),\ \eta_2^* = k_2 \bar{y}^* \left(\frac{\bar{x}^*}{\bar{X}} \right),\ \eta_3^* = k_3 \bar{y}^* \left(\frac{\bar{X} + C_x}{\bar{x}^* + C_x} \right),$$

$$\eta_4^* = k_4 \bar{y}^* \left(\frac{\bar{x}^* + C_x}{\bar{X} + C_x} \right),\ \eta_5^* = k_5 \bar{y}^* \left(\frac{\bar{X} + \rho}{\bar{x}^* + \rho} \right),\ \eta_6^* = k_6 \bar{y}^* \left(\frac{\bar{x}^* + \rho}{\bar{X} + \rho} \right)$$

Where $\bar{x}^*$ and $\bar{y}^*$ are the respective sample means of the auxiliary and study variables, having the relationship: (1) $\bar{X} = f\,\bar{x} + (1-f)\bar{x}^*$ (2) $\bar{Y} = f\,\bar{y} + (1-f)\bar{y}^*$ where $f = \frac{n}{N}$ is finite population correction, k_i, $i = 1,2,3,4,5,6,$ are real constants.

We proposed the following class of ratio estimator

$$\tau_1^* = k_1 s_y^{2*} \left(\frac{S_x^2}{s_x^{2*}} \right),\ \tau_2^* = k_2 s_y^{2*} \left(\frac{S_x^2 - C_x}{s_x^{2*} - C_x} \right),\ \tau_3^* = k_3 s_y^{2*} \left(\frac{S_x^2 - \beta_2(x)}{s_x^{2*} - \beta_2(x)} \right),\ \tau_4^* = k_4 s_y^{2*} \left(\frac{S_x^2 \beta_2(x) - C_x}{s_x^{2*} \beta_2(x) - C_x} \right),$$

$$\tau_5^* = k_5 s_y^{2*} \left(\frac{S_x^2 C_x - \beta_2(x)}{s_x^{2*} C_x - \beta_2(x)} \right),\ \tau_6^* = k_6 s_y^{2*} \left(\frac{S_x^2 + \beta_2(x)}{s_x^{2*} + \beta_2(x)} \right)$$

Where s_x^{2*} and s_y^{2*} are the respective sample finite variances of the auxiliary and study variables, having the relationship: (1) $S_x^2 = f\,s_x^2 + (1-f)s_x^{2*}$ (2) $S_y^2 = f\,s_y^2 + (1-f)s_y^{2*}$ with condition that $n < \frac{1}{2}N$.

In order to obtain the MSE, we defined $e_0 = \frac{s_y^2 - S_y^2}{S_y^2}$ and $e_1 = \frac{s_x^2 - S_x^2}{S_x^2}$ such that

$$\left. \begin{aligned} &E(e_0) = E(e_1) = 0,\ E(e_0^2) = \gamma(\psi_{40} - 1) \\ &E(e_1^2) = \gamma(\psi_{04} - 1),\ E(e_0 e_1) = \gamma(\psi_{22} - 1) \end{aligned} \right\} \tag{2.1}$$

Expressing τ_i^*, $i = 1,2,3,4,5,6,$ in terms of e_0 and e_1, we have

$$\tau_i^* = \left. \begin{matrix} k_i S_y^2 (1 - v e_0)(1 - v e_1)^{-1} & i = 1 \\ k_i S_y^2 (1 - v e_0)(1 - v h_i e_1)^{-1} & i = 2,3,4,5,6 \end{matrix} \right\} \tag{2.2}$$

now assume that $|ve_1| < 1$ and $|vh_i e_1| < 1$ so that $(1 - ve_1)^{-1}$ and $(1 - vh_i e_i)^{-1}$ are expandable. Expanding the right hand side of (2.2) up to second degree approximation,

1. A. A. Adewara, R. Singh, M. Kumar, Efficiency of some modified ratio and product estimators using known value of some population parameters, International Journal of Applied Science and Technology 2 (2) (2012) 76-79.

subtract S_y^2 from its both sides, square the both sides and taking expectation using the results in equation (2.1), we obtain the MSEs of the proposed estimators as:

$$MSE\left(\tau_i^*\right)=\begin{cases} S_y^4\left\{\gamma v^2\begin{bmatrix} k_i^2\left(\psi_{40}-1\right)+\left(3k_i^2-2k_i\right)\left(\psi_{04}-1\right) \\ -2\left(2k_i^2-k_i\right)\left(\psi_{22}-1\right)\end{bmatrix}+\left(k_i-1\right)^2\right\} & i=1 \\ S_y^4\left\{\gamma v^2\begin{bmatrix} k_i^2\left(\psi_{40}-1\right)+\left(3k_i^2-2k_i\right)h_i^2\left(\psi_{04}-1\right) \\ -2\left(2k_i^2-k_i\right)h_i\left(\psi_{22}-1\right)\end{bmatrix}+\left(k_i-1\right)^2\right\} & i=2,3,4,5,6 \end{cases} \tag{2.3}$$

The $MSE\left(\eta_i\right)$, $i=1,2,3,4,5,6$ expressions are minimized for the optimum values of k given by

$$k_i^*=\frac{v^2\gamma\left[\left(\psi_{04}-1\right)-\left(\psi_{22}-1\right)\right]+1}{v^2\gamma\left[3\left(\psi_{04}-1\right)-4\left(\psi_{22}-1\right)+\left(\psi_{40}-1\right)\right]+1}=\frac{A_1}{B_1},\qquad i=1 \tag{2.4}$$

$$k_i^*=\frac{v^2\gamma\left[h_i^2\left(\psi_{04}-1\right)-h_i\left(\psi_{22}-1\right)\right]+1}{v^2\gamma\left[3h_i^2\left(\psi_{04}-1\right)-4h_i\left(\psi_{22}-1\right)+\left(\psi_{40}-1\right)\right]+1}=\frac{A_2}{B_2},\qquad i=2,3,4,5,6 \tag{2.5}$$

where $v=\dfrac{n}{N-n}$

Replacing k_i by k_i^*, $i=1,2,3,4,5,6$ in equation (2.3), we obtain the minimum MSE as

$$MSE_{\min}\left(\tau_i^*\right)=\begin{cases} S_y^4\left(1-\dfrac{A_1^2}{B_1}\right), & i=1 \\ S_y^4\left(1-\dfrac{A_2^2}{B_2}\right), & i=2,3,4,5,6 \end{cases} \tag{2.6}$$

III. Efficiency Comparisons

In this section efficiencies of the proposed estimators are compared with efficiencies of some estimators in the literature

The $\tau_i^*-family$ of estimators of the population variance is more efficient than η_0 if,

$$MSE_{\min}\left(\tau_i^*\right)<Var\left(\eta_0\right)\quad i=1,2,3,4,5,6$$

$$\left.\begin{array}{ll} \left(1-\dfrac{A_1^2}{B_1}\right)<\gamma\left[\left(\psi_{40}-1\right)\right] & i=1 \\ \left(1-\dfrac{A_2^2}{B_2}\right)<\gamma\left[\left(\psi_{40}-1\right)\right] & i=2,3,4,5,6 \end{array}\right\} \tag{3.1}$$

The $\tau_i^*-family$ of estimators of the population variance is more efficient than $\eta_i-family$ if,

Notes

$$MSE_{\min}\left(\tau_i^*\right) < MSE\left(\eta_i\right) \quad i = 1,2,3,4,5,6$$

$$\left.\begin{array}{ll} \left(1-\dfrac{A_1^2}{B_1}\right) < \gamma\left[\left(\psi_{40}-1\right)+\left(\psi_{04}-1\right)-2\left(\psi_{22}-1\right)\right] & i=1 \\ \left(1-\dfrac{A_2^2}{B_2}\right) < \gamma\left[\left(\psi_{40}-1\right)+h_i^2\left(\psi_{04}-1\right)-2h_i\left(\psi_{22}-1\right)\right] & i=2,3,4,5,6 \end{array}\right\} \quad (3.2)$$

The $\tau_i^* - family$ of estimators of the population variance is more efficient than η if,

$$MSE_{\min}\left(\tau_i^*\right) < MSE_{\min}\left(\eta\right) \quad i = 1,2,3,4,5,6$$

$$\left.\begin{array}{ll} \left(1-\dfrac{A_1^2}{B_1}\right) < \gamma\left[\left(\psi_{40}-1\right)-\left(\psi_{04}-1\right)\right] & i=1 \\ \left(1-\dfrac{A_2^2}{B_2}\right) < \gamma\left[\left(\psi_{40}-1\right)-\left(\psi_{04}-1\right)\right] & i=2,3,4,5,6 \end{array}\right\} \quad (3.3)$$

When conditions (3.1), (3.2) and (3.3) are satisfied, we can conclude that the family of proposed class of estimators is more efficient than the $\eta - family$ estimator.

IV. Empirical Study

In order to investigate the merits of the proposed estimators, we have considered the following two real populations given as:

Data 1: Subramani and Kumarapandiyan [13]

$$N = 49,\ n = 20,\ \bar{Y} = 116.1633,\ \bar{X} = 98.6765,\ \rho = 0.6904,\ S_y = 98.8286$$

$$S_x = 102.9709,\ C_y = 0.8508,\ C_x = 1.0435,\ \psi_{40} = 4.9245,\ \psi_{04} = 5.9878,\ \psi_{22} = 4.6977$$

Data 2: Subramani and Kumarapandiyan [13]

$$N = 80,\ n = 20,\ \bar{Y} = 51.8264,\ \bar{X} = 11.2646,\ \rho = 0.9413,\ S_y = 18.3569$$

$$S_x = 8.4563,\ C_y = 0.3542,\ C_x = 0.7507,\ \psi_{40} = 2.2667,\ \psi_{04} = 2.8664,\ \psi_{22} = 2.2209$$

The numerical demonstration to justify the appropriateness of the suggested class of variance estimator has been conducted using the two data sets. Table 2 and Table 3 show the mean square errors of proposed estimators and that of some existing estimators and their percentage relative efficiencies of different estimators with respect to s_y^2 respectively.

V. Conclusion

From section 3, the theoretical conditions obtained for the efficiencies of the proposed estimators supported by the numerical illustration in section 4 given in the Table 2 and 3. From the result, we infer that the suggested variance estimators produce a better estimate of finite population variance than the existing estimators in the sense of having higher percentage relative efficiency which implies lesser mean square error.

Ref

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Notes

Table 1 : Some Member of η-family for different values of a, b and α

Estimator	a	b	α
$\eta_0 = s_y^2$ Sample variance	1	0	0
$\eta_1 = s_y^2\left(\frac{S_x^2}{s_x^2}\right)$ Isaki [4]	1	0	1
$\eta_2 = s_y^2\left(\frac{S_x^2 - C_x}{s_x^2 - C_x}\right)$ Kadilar and Cingi [5]	1	C_x	1
$\eta_3 = s_y^2\left(\frac{S_x^2 - \beta_2(x)}{s_x^2 - \beta_2(x)}\right)$	1	$\beta_2(x)$	1
$\eta_4 = s_y^2\left(\frac{S_x^2\beta_2(x) - C_x}{s_x^2\beta_2(x) - C_x}\right)$	$\beta_2(x)$	C_x	1
$\eta_5 = s_y^2\left(\frac{S_x^2 C_x - \beta_2(x)}{s_x^2 C_x - \beta_2(x)}\right)$	C_x	$\beta_2(x)$	1
$\eta_6 = s_y^2\left(\frac{S_x^2 + \beta_2(x)}{s_x^2 + \beta_2(x)}\right)$ Upadhyaya and Singh [14]	1	$-\beta_2(x)$	1

Table 2 : MSE of different estimators

Estimator	Data 1	Data 2	Estimator	Data 1	Data 2
η_0	187190	7191.859	η_{opt}	5643700	2657.427
$\eta - family$ **(Singh et al. [7] estimators)**			$\tau^* - family$ (Newly proposed estimators)		
η_1	723531	3924.948	τ_1^*	3053941	429.9105
η_2	723652	4003.906	τ_2^*	3054289	438.3024
η_3	724227	4249.508	τ_3^*	3055940	464.3576
η_4	723551	3952.035	τ_4^*	3053999	432.7903
η_5	724198	4372.12	τ_5^*	3055857	477.3385
η_6	722837	3658.1	τ_6^*	3051947	401.5034

Ref

14. L.N. Upadhyaya, H. P. Singh, An estimator for population variance that utilizes the kurtosis of an auxiliary variable in sample surveys, Vikram Mathematical Journal, 19 (1999) 14-17.

Table 3 : PRE of different estimators with respect to s_y^2

Estimator	Data 1	Data 2	Estimator	Data 1	Data 2
η_0	100.00	100.00	η_{opt}	331.6813	270.6324
$\eta-family$ (Singh et al. [7] estimators)			$\tau^*-family$ (Newly proposed estimators)		
η_1	258.7184	183.2345	τ_1^*	612.949	1672.873
η_2	258.6751	179.6211	τ_2^*	612.8791	1640.844
η_3	258.4697	169.2398	τ_3^*	612.5479	1548.776
η_4	258.7112	181.9786	τ_4^*	612.9373	1661.742
η_5	258.4801	164.4934	τ_5^*	612.5646	1506.658
η_6	258.9668	196.5956	τ_6^*	613.3494	1791.233

Notes

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New Generalization of Angular Displacement with Product of Certain Special Functions in A Shaft

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Abstract- The object of this paper is to establish a new generalization of angular displacement in a shaft with a product of certain special function. A main result based upon the H-function of several complex variables, I-function of one variable, general polynomial of several variables, which provide unification and extension of numerous results in theory of special function. The special cases of the main result (which are also sufficiently general in nature and are of interested in themselves) have also been given.

Keywords: *partial differential equation for angular displacement, series representation of multivariable H-function, I-function, general polynomial.*

GJSFR-F Classification : MSC 2010: 33C60, 93C20, 49C10

Ref

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New Generalization of Angular Displacement with Product of Certain Special Functions in A Shaft

Ashok Singh Shekhawat [α] & Sunil Kumar Sharma [σ]

Abstract- The object of this paper is to establish a new generalization of angular displacement in a shaft with a product of certain special function. A main result based upon the H-function of several complex variables, I-function of one variable, general polynomial of several variables, which provide unification and extension of numerous results in theory of special function. The special cases of the main result (which are also sufficiently general in nature and are of interested in themselves) have also been given.

Keywords: partial differential equation for angular displacement, series representation of multivariable H-function, I-function, general polynomial.

I. Introduction

The importance of the H-function, I-function are realized by scientists, engineers and statisticians (caputo [16], Glockle and Nonnenmacher [17], Mainardi et al. [18], Hifer[19] etc) due to vast potential of their applications in diversified fields of science and engineering such as rheology, diffusion in porous media, fluid flow, turbulence, propagation of seismic waves etc.

In the view of importance and popularity of the I-function and H-function a large number of integral formulas involving these functions have been developed by many authors.

The series representation for the **H-function** of several complex variables [3], is as follows:

$$H[z_1,..,z_k] = H^{0,\lambda:(u',v');...;(u^K,v^K)}_{A,C:(B',D');...;(B^K,D^K)}\left[\begin{matrix}[(a):\theta';...;\theta^{(K)}]:[b':\varphi'];...;[b^{(k)}:\varphi^{(K)}]\\ [(c):\psi';...;\psi^{(K)}]:[d':\delta'];...;[d^{(K)}:\delta^{(K)}]\end{matrix};z_1,...,z_K\right]$$

$$= \sum_{e_l=0}^{u^{(l)}} \sum_{f_l=0}^{\infty} \phi_1\phi_2 \frac{\prod_{l=1}^{K}(z_l)^{U_l}(-1)^{\sum_{l=1}^{K}(f_l)}}{\prod_{l=1}^{K}\left(\delta_{(e_l)}^{(l)} f_l!\right)} \quad ...(1.1)$$

where

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$$\phi_1 = \frac{\prod_{j=1}^{\lambda}\Gamma\left(1-a_j+\sum_{l=1}^{K}\theta_j^{(l)}U_l\right)}{\prod_{j=\lambda+1}^{A}\Gamma\left(a_j-\sum_{l=1}^{K}\theta_j^{(l)}U_l\right)\prod_{j=1}^{C}\Gamma\left(1-c_j+\sum_{l=1}^{K}\psi_j^{(l)}U_l\right)} \quad \text{...(1.2)}$$

$$\phi_2 = \frac{\prod_{j=1}^{u^{(l)}}\Gamma\left(d_j^{(l)}-\delta_j^{(l)}U_l\right)\prod_{j=1}^{v^{(l)}}\Gamma\left(1-b_j^{(l)}+\varphi_j^{(l)}U_l\right)}{\prod_{j=v^{(l)}+1}^{B^{(l)}}\Gamma\left(b_j-\varphi_j^{(l)}U_l\right)\prod_{j=u^{(l)}+1}^{D^{(l)}}\Gamma\left(1-d_j+\delta_j^{(l)}U_l\right)} \quad \text{...(1.3)}$$

where

$$U_l = \frac{d_{e_l}^{(l)}+f_l}{\delta_{e_l}^{(l)}} \quad \forall\, l = 1,\dots K \quad \text{...(1.4)}$$

For the converges conditions and other detail of H-function of several complex variables, see [3].

The **I-function**, which is more general the Fox's H-function [5], defined by V.P. Sexena [4], by means of the following Mellin-Barns type contour integral:

$$I[z] = I_{p_i,q_i;r}^{m,n}[z] = I_{p_i,q_i;r}^{m,n}\left[z\left|\begin{matrix}(a_j,\alpha_j)_{1,n}:(a_{ji},\alpha_{ji})_{n+1,p_i}\\(b_j,\beta_j)_{1,m}:(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right.\right] = \frac{1}{2\pi i}\int_L \Theta(\zeta)z^{\zeta}\,d\zeta \quad \text{...(1.5)}$$

where

$$\Theta(\zeta) = \frac{\prod_{j=1}^{m}\Gamma(b_j-\beta_j\zeta)\prod_{j=1}^{n}\Gamma(1-a_j+\alpha_j\zeta)}{\sum_{i=1}^{r}\left\{\prod_{j=m+1}^{q_i}\Gamma(1-b_{ji}+\beta_{ji}\zeta)\prod_{j=n+1}^{p_i}\Gamma(a_{ji}-\alpha_{ji}\zeta)\right\}} \quad \text{...(1.6)}$$

p_i, q_i $(i=1,\dots,r), m, n$ are integer satisfying $0 \le n \le p_i, 0 \le m \le q_i,$ $\alpha_j, \beta_j, \alpha_{ji}, \beta_{ji}$ are real and positive and a_j, b_j, a_{ji}, b_{ji} are complex numbers. L is suitable contour of the Mellin-Barnes type running from $\gamma - i\alpha$ to $\gamma + i\alpha$ (γ is real) in the complex ζ-plane. Detail regarding existence conditions and various parametric restriction of I-function, we may refer [4].

Remark: For $r = 1$, [4] reduce to the Fox's H-function

$$I_{p_i,q_i;1}^{m,n}\left[z\left|\begin{matrix}(a_j,\alpha_j)_{1,n}:(a_{ji},\alpha_{ji})_{n+1,p_i}\\(b_j,\beta_j)_{1,m}:(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right.\right] = H_{p,q}^{m,n}\left[z\left|\begin{matrix}(a_j,\alpha_j)_{1,n}:(a_j,\alpha_j)_{n+1,p}\\(b_j,\beta_j)_{1,m}:(b_j,\beta_j)_{m+1,q}\end{matrix}\right.\right] \quad \text{... (1.7)}$$

The multidimensional analogue of a general class of polynomials $S_n^m(x)$ is defined by [6],

$$S_n^{m_1,\dots,m_s}(x_1,\dots,x_s) = \sum_{k_1,\dots,k_s}^{m_1k_1+\dots+m_sk_s\le n}(-n)_{m_1k_1+\dots+m_sk_s}.\mathcal{A}(n:k_1,\dots,k_s)\,\frac{x_1^{k_1}}{k_1!}\cdots\frac{x_s^{k_s}}{k_s!} \quad \text{... (1.8)}$$

where $m_1,\dots,m_s$ are arbitrary positive integers, $n = 0,1,2\dots$ and the coefficients constant $\mathcal{A}(n:k_1,\dots,k_s)$, $k_i \ge 0$, $i = 1,\dots,s$ are arbitrary constants, real and complex. The order of highest degree of variables $x_1,\dots,x_s$ of the multivariable polynomial (1.8) can be written as [7],

Ref

5. C. Fox., The G and H-functions as Symmetrical Fourier kernels, Tran.Amer.Math.Soc.98 (1961), 359-429.

$$\mathbb{O}\left(S_n^{m_1,\dots,m_s}(x_1,\dots,x_s)\right)=\mathbb{O}\left(x_1^{[n/m_1]},\dots,x_s^{[n/m_s]}\right) \quad \dots(1.9)$$

where $[x]$ denotes the greatest integer $\leq x$.

Remarks

1. For $m_i=1, i=1,\dots,s$ and $\mathcal{A}(n:k_1,\dots,k_s)=(1+\alpha_1+n_1)_{k_1}\dots(1+\alpha_s+n_s)_{k_s}$, the multivariable polynomial reduces to a multivariable Bessel polynomial [8].
2. For $m_i=2, \sigma_i=1, i=1,\dots,s$, $\mathcal{A}(n:k_1,\dots,k_s)=1$ and replacing $tx_1\to\frac{1}{2(tx_1)^2}, tx_i\to\frac{tx_j}{2(tx_1)^2}, j=1,\dots,s$, the multivariable polynomial reduces to a multivariable Hermite polynomial [9].
3. For the case $r=1$ the multivariable polynomial (1.8) would give rise to the general class polynomials $S_n^m(x)$ denoted by Srivastava [10].

The following result is required in our present investigation, which is due to [Gradshteyn and Ryzhik 1965, p.375, eq. (2)]

$$\int_0^{\mu} cos\left(\frac{\pi\varepsilon x}{\mu}\right)\left(sin\frac{\pi x}{2\mu}\right)^{2\varepsilon-\sigma-1}\left(cos\frac{\pi x}{2\mu}\right)^{\sigma-1}dx=\frac{\mu.\,2^{2\varepsilon-\sigma}\Gamma\left(\frac{2\varepsilon-\sigma}{2}\right)\Gamma(\sigma)}{\sqrt{\pi}\,\Gamma\left(\frac{1-2\varepsilon+\sigma}{2}\right)\Gamma(2\varepsilon)} \quad \dots(1.10)$$

Provided $2\varepsilon>Re\,(\sigma)>0$.

Also by Churchill [11],

$$\Psi(x,t)=\frac{1}{2}a_0+\sum_{\tau=1}^{\infty}a_\tau\left(cos\frac{\pi x\tau}{\mu}\right)\left(cos\frac{\pi\tau Rt}{\mu}\right) \quad \dots(1.11)$$

where a_τ $(\tau=0,1,2\dots)$ are the coefficients in the Fourier cosine series for f(x) in the interval (o,μ).

II. Statement of the Problem and Governing Equations

Here the continuous system and the equation of motion has been explained through a simple diagram it was earlier developed. There is the x-axis of the rod which is also called the side axis and is loaded with distributed moment. At the normal location of the rod, the angular displacement takes place is known as xt and at the side view of the rod it can be pointed out is to be said point p. The motion of point can be seen in a large view, point p is present on to the shaft. Its radial position is r and if it got displaced, it reaches to the point p prime and this angle is called θ (theta).

It also can be assumed that the point p moves in a same plane and it does not move towards the axial direction. Presently, the equation of motion for this type continuous system in which, the mass and stiffness both are distributed in a systematic way.

Ref

10. H.M. Sriavastava, A Contour Integral Involving Fox's H-function, Indian J. Math. 14(1972), 1-6.

Notes

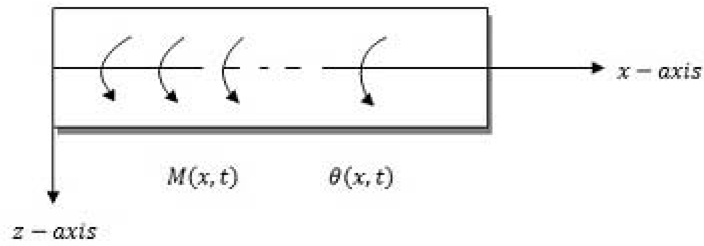

Figure 2.1

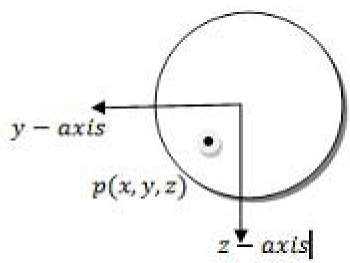

Figure 2.2

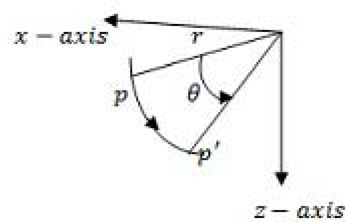

Figure 2.3

As an example of an application of one and multivariable special functions in applied mathematics, we shall consider the problem of determining the angular displacement. The solution of boundary value problem involving the partial differential equation of angular displacement in a shaft is given by

$$\frac{\partial^2\theta}{\partial t^2} = c^2 \frac{\partial^2\theta}{\partial x^2} \qquad ...(2.1)$$

Let us consider the problem determine the angular displacement or twist $\theta(x,t)$ in a shaft of circular section with its axis along the x-axis. The ends $x = 0$ to $x = \mu$ of the shaft are free, the $\theta(x,t)$ due to initial twist must satisfy the boundary value (2.1). Also the boundary conditions are

$$\frac{\partial}{\partial x}\theta(0,t) = 0\,, \frac{\partial}{\partial x}\theta(\mu,t) = 0, \frac{\partial}{\partial t}\theta(x,0) = 0 \qquad ...(2.2)$$

and $\theta(x,0) = f(x)$, where c is a constant.

$$\text{Let } f(x) = \left(sin\frac{\pi x}{2\mu}\right)^{2\delta-\lambda-1} \left(cos\frac{\pi x}{2\mu}\right)^{\lambda-1} S^{m_1,\dots,m_s}_{n_1,\dots,n_s}\begin{bmatrix} t_1\left(tan\frac{\pi x}{2\mu}\right)^{2k_1} \\ \vdots \\ t_s\left(tan\frac{\pi x}{2\mu}\right)^{2k_s}\end{bmatrix}.H\begin{bmatrix} y_1\left(tan\frac{\pi x}{2\mu}\right)^{2v_1} \\ \vdots \\ t_K\left(tan\frac{\pi x}{2\mu}\right)^{2v_K}\end{bmatrix}$$

$$\times I^{m,n}_{p_i,q_i;r}\left[z\left(tan\frac{\pi x}{2\mu}\right)^{2h}\right] \qquad ...(2.3)$$

Notes

III. The Main Integral Formula

The main result to be established here as follows:

$$\int_0^{\mu}\left(cos\frac{\pi\delta x}{\mu}\right)\left(sin\frac{\pi x}{2\mu}\right)^{2\delta-\lambda-1}\left(cos\frac{\pi x}{2\mu}\right)^{\lambda-1} S_{n_1,\dots,n_s}^{m_1,\dots,m_s}\begin{bmatrix} t_1\left(tan\frac{\pi x}{2\mu}\right)^{2k_1} \\ \vdots \\ t_s\left(tan\frac{\pi x}{2\mu}\right)^{2k_s}\end{bmatrix}.H\begin{bmatrix} y_1\left(tan\frac{\pi x}{2\mu}\right)^{2v_1} \\ \vdots \\ t_K\left(tan\frac{\pi x}{2\mu}\right)^{2v_K}\end{bmatrix}$$

$$\times I_{p_i,q_i;r}^{m,n}\left[z\left(tan\frac{\pi x}{2\mu}\right)^{2h}\right]$$

$$=\frac{\mu}{\sqrt{2\delta}}2^{\delta\lambda-\lambda+2\sum_{i=1}^{s}\alpha_i k_i+2\sum_{l=1}^{K}V_l U_l}\sum_{e_l=1}^{U_l}\sum_{f_l=0}^{\infty}\phi_1\phi_2\,.\frac{\prod_{l=1}^{K}(y_l)^{V_l}(-1)^{\sum_{l=1}^{K}(f_l)}}{\prod_{l=1}^{K}\left\{\delta_{(e_l)}^{(l)}f_l!\right\}}\sum_{\alpha_i=0}^{[n_1/m_1]}\cdots\sum_{\alpha_s=0}^{[n_s/m_s]}$$

$$\frac{(-1)_{m_1\alpha_1}}{\alpha_1!}\cdots\frac{(-1)_{m_s\alpha_s}}{\alpha_s!}B[n_1\alpha_1;\dots;n_s\alpha_s].\left[t_1^{\alpha_1},\dots,t_s^{\alpha_s}\right].I_{p_i+2,q_i+1:r}^{m+1,n+1}$$

$$\left[z2^{2h}\left|\begin{matrix}\left(1-\delta+\frac{\lambda}{2}-\sum_{i=1}^{s}\alpha_i k_i-\sum_{l=1}^{K}V_l U_l\,;h\right),(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i} \\ \left(\lambda-2\sum_{i=1}^{s}\alpha_i k_i-2\sum_{l=1}^{K}V_l U_l\,;2h\right),(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i},\left(\frac{1}{2}-\delta+\frac{\lambda}{2}-\sum_{i=1}^{s}\alpha_i k_i-\sum_{l=1}^{K}V_l U_l\,;h\right)\end{matrix}\right.\right]$$

...(3.1)

where $k_i>0\ (i=1,\dots,s),h>0,\Re\left(\lambda-2k\left(\frac{\beta_j}{b_j}\right)\right)>0, j=1,\dots m\,,m$ is an arbitrary positive integer and the coefficient $B[n_1\alpha_1,\dots,n_s\alpha_s]$ are arbitrary constant, real or complex.

Proof. To prove the integral formula (3.1), we express the first multivariable H-function in series form by (1.1) and I-function in terms of Mellin-Barnes type of contour integral by (1.5) and the generalized polynomial given by (1.8), then interchanging the order summation and integration, which is justified due to the absolute converges of integral involved in the process. Evaluate the integral with help of (1.10), after straight calculation we finally arrive at (3.1).

IV. Solution of the Problem

In this section we shell study the solution of the problem. So the solution of the problem posed to be obtained is

$$\Phi(x,t)=\frac{2^{2\tau-\lambda+2\sum_{i=1}^{s}\alpha_i k_i+2\sum_{l=1}^{K}V_l U_l+1}}{\sqrt{\pi}\sqrt{2\tau}}\sum_{e_l=1}^{U_l}\sum_{f_l=0}^{\infty}\phi_1\phi_2\,.\frac{\prod_{l=1}^{K}(y_l)^{V_l}(-1)^{\sum_{l=1}^{K}(f_l)}}{\prod_{l=1}^{K}\left\{\delta_{(e_l)}^{(l)}f_l!\right\}}$$

$$\sum_{\alpha_i=0}^{[n_1/m_1]}\cdots\sum_{\alpha_s=0}^{[n_s/m_s]}\frac{(-1)_{m_1\alpha_1}}{\alpha_1!}\cdots\frac{(-1)_{m_s\alpha_s}}{\alpha_s!}B[n_1\alpha_1,\dots,n_s\alpha_s].\left[t_1^{\alpha_1},\dots,t_s^{\alpha_s}\right].I_{p_i+2,q_i+1:r}^{m+1,n+1}$$

$$\left[z2^{2h} \left| \begin{matrix} \left(1-\tau+\frac{\lambda}{2}-\sum_{i=1}^{s}\alpha_i k_i-\sum_{l=1}^{K}V_l U_l\,;h\right),(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i} \\ \left(\tau-2\sum_{i=1}^{s}\alpha_i k_i-2\sum_{l=1}^{K}V_l U_l\,;2h\right),(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i},\left(1-\tau+\frac{\lambda}{2}-\sum_{i=1}^{s}\alpha_i k_i-\sum_{l=1}^{K}V_l U_l\,;h\right) \end{matrix} \right. \right]$$

$$\times\left(cos\frac{\pi x\tau}{\mu}\right)\left(cos\frac{\pi\tau Rt}{\mu}\right) \qquad ...(4.1)$$

Notes

where $k_i > 0\ (i = 1, \dots, s), h > 0, \Re\left(\lambda - 2k\left(\frac{\beta_j}{b_j}\right)\right) > 0, j = 1, \dots m\,, m$ is an arbitrary positive integer and the coefficient $B[n_1\alpha_1; \dots; n_s\alpha_s]$ are arbitrary constant, real or complex.

Proof. By eq. (1.11)

$$\Psi(x,t) = \frac{1}{2}a_0 + \sum_{\tau=1}^{\infty} a_\tau \left(cos\frac{\pi x\tau}{\mu}\right)\left(cos\frac{\pi\tau Rt}{\mu}\right)$$

where $a_\tau\ (\tau = 0,1,2 \dots)$ are the coefficients in the Fourier cosine series for f(x) in the interval (o, μ).

If $t = 0$, then from (1.11) and (2.3), we have

$$\left(sin\frac{\pi x}{2\mu}\right)^{2\delta-\lambda-1}\left(cos\frac{\pi x}{2\mu}\right)^{\lambda-1} S_{n_1,\dots,n_s}^{m_1,\dots,m_s}\begin{bmatrix} t_1\left(tan\frac{\pi x}{2\mu}\right)^{2k_1} \\ \vdots \\ t_s\left(tan\frac{\pi x}{2\mu}\right)^{2k_s}\end{bmatrix}.H\begin{bmatrix} y_1\left(tan\frac{\pi x}{2\mu}\right)^{2v_1} \\ \vdots \\ t_K\left(tan\frac{\pi x}{2\mu}\right)^{2v_K}\end{bmatrix}$$

$$\times I_{p_i,q_i;r}^{m,n}\left[z\left(tan\frac{\pi x}{2\mu}\right)^{2h}\right] = \frac{1}{2}a_0 + \sum_{\tau=1}^{\infty} a_\tau\left(cos\frac{\pi x\tau}{\mu}\right) \qquad ...(4.2)$$

Now multiplying (4.1) both the sides by $\left(cos\frac{\pi\delta x}{\mu}\right)$ and integrating with respect to x from 0 to μ, we get

$$\int_0^\mu \left(cos\frac{\pi\delta x}{\mu}\right)\left(sin\frac{\pi x}{2\mu}\right)^{2\delta-\lambda-1}\left(cos\frac{\pi x}{2\mu}\right)^{\lambda-1} S_{n_1,\dots,n_s}^{m_1,\dots,m_s}\begin{bmatrix} t_1\left(tan\frac{\pi x}{2\mu}\right)^{2k_1} \\ \vdots \\ t_s\left(tan\frac{\pi x}{2\mu}\right)^{2k_s}\end{bmatrix}.H\begin{bmatrix} y_1\left(tan\frac{\pi x}{2\mu}\right)^{2v_1} \\ \vdots \\ t_K\left(tan\frac{\pi x}{2\mu}\right)^{2v_K}\end{bmatrix}$$

$$\times I_{p_i,q_i;r}^{m,n}\left[z\left(tan\frac{\pi x}{2\mu}\right)^{2h}\right]dx = \frac{1}{2}a_0\int_0^\mu\left(cos\frac{\pi\delta x}{\mu}\right)dx + \sum_{\tau=1}^{\infty} a_\tau \int_0^\mu \left(cos\frac{\pi x\tau}{\mu}\right)\left(cos\frac{\pi\delta x}{\mu}\right)dx \qquad ...(4.3)$$

Now using result (3.1) along with orthogonal property of the cosine function, we obtain

$$a_\tau = \frac{2^{2\tau-\lambda+2\sum_{i=1}^{s}\alpha_i k_i+2\sum_{l=1}^{K}V_l U_l+1}}{\sqrt{\pi}\sqrt{2\tau}}\sum_{e_l=1}^{U_l}\sum_{f_l=0}^{\infty}\phi_1\phi_2\,.\frac{\prod_{l=1}^{K}(y_l)^{V_l}(-1)^{\sum_{l=1}^{K}(f_l)}}{\prod_{l=1}^{K}\left\{\delta_{(e_l)}^{(l)}f_l!\right\}}$$

$$\sum_{\alpha_i=0}^{[n_1/m_1]}\cdots\sum_{\alpha_s=0}^{[n_s/m_s]}\frac{(-1)_{m_1\alpha_1}}{\alpha_1!}\cdots\frac{(-1)_{m_s\alpha_s}}{\alpha_s!}B[n_1\alpha_1;\dots;n_s\alpha_s].\left[t_1^{\alpha_1},\dots,t_s^{\alpha_s}\right].I_{p_i+2,q_i+1:r}^{m+1,n+1}$$

$$\left[z2^{2h} \left| \begin{matrix} \left(1-\tau+\frac{\lambda}{2}-\sum_{i=1}^{s}\alpha_i k_i - \sum_{l=1}^{K} V_l U_l \,;h\right), (a_j,\alpha_j)_{1,n}; (a_{ji},\alpha_{ji})_{n+1,p_i} \\ \left(\tau - 2\sum_{i=1}^{s}\alpha_i k_i - 2\sum_{l=1}^{K} V_l U_l \,;2h\right), (b_j,\beta_j)_{1,m}; (b_{ji},\beta_{ji})_{m+1,q_i}, \left(1-\tau+\frac{\lambda}{2}-\sum_{i=1}^{s}\alpha_i k_i - \sum_{l=1}^{K} V_l U_l \,;h\right) \end{matrix} \right. \right] \qquad \ldots(4.4)$$

Now with help of (1.11) and (4.4), the solution (4.1) is obtained.

V. Special Cases

(A) If we set $S_{n_i}^{m_i} = 1, (i = 1, \ldots s)$ and $r = 1$ in (3.1), then the results reduces to the result given by chaurasia and Gupta [12].

(B) For $r = 1$, taking $s = 2$ and $k_i \to 0$ in (3.1), we find the known result concluded by Chaurasia and Godika [13].

(C) Taking $\bar{H}$-function place of I-function and H-function of series form to unity, then we can easily obtained the known results given by Chaurasia and Shakhawat [14].

(D) For $r = 2$ in multivariable H-function and $S_{n_i}^{m_i} = 1, (i = 1, \ldots s)$ and taking series representation of Fox's H-function place of I-function, we find the known result obtained by Mittal and Gupta [15]

VI. Conculsion

In view of the generality of I-function and H-function of several complex variables, on specializing the various parameters, involved there in, we can obtain from our results, several Results involving a remarkably wide variety of useful function, which are expressible in terms of the H-function, $\bar{H}$-function, I-function of one variable and their special cases. Thus the results presented in this paper would at once yield a very large number of results involving a large variety of special function occurring in the problems of science, engineering and mathematical physics etc. Also results deduced in the present paper may provide better result of Angular displacement in a shaft and boundary value problem of some simpler functions.

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Notes

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Ducting in Extended Plates of Variable Thickness

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Abstract- The paper deals with the spread of its own waves on the visco elastic plate with variable thickness. Basic relations of the classical theory of plates of variable thickness obtained on the basis of the principles of virtual displacements. The spectral problem, which is not self ad joint. Built for the task biorthogonality conditions, based on the Lagrange formula. The numerical solution of the spectral tasks performed on the computer software system based on the method of orthogonal shooting S.K. Godunov in combination with the method of Muller.

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GJSFR-F Classification : *MSC 2010: 00A06*

DUCTINGINEXTENDEDPLATESOFVARIABLETHICKNESS

Ducting in Extended Plates of Variable Thickness

Ismail Ibrahimovich Safarov [α], Maqsud Sharipovich Akhmedov [σ] & Zafar Ihterovich Boltaev [ρ]

Annotation- The paper deals with the spread of its own waves on the visco elastic plate with variable thickness. Basic relations of the classical theory of plates of variable thickness obtained on the basis of the principles of virtual displacements. The spectral problem, which is not self ad joint. Built for the task biorthogonality conditions, based on the Lagrange formula. The numerical solution of the spectral tasks performed on the computer software system based on the method of orthogonal shooting S.K. Godunov in combination with the method of Muller.

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I. Introduction

Known [1,2,3] that normal wave deformable layer (Lamb wave) is not orthogonal thickness, i.e. the integral of the scalar product of vectors of displacements of two different waves, considered as a function of position perpendicular to the surface layer is not zero. They are also not orthogonal conjugate wave, which is obtained from a consideration of the dual problem. This introduces additional difficulties in solving practical problems. This article is based conjugate spectral biorthogonality objectives and conditions for the problem.

a) *The mathematical formulation of the problem*

We consider the visco elastic waveguide as an infinite axial x_1 variable thickness (Figure 1).

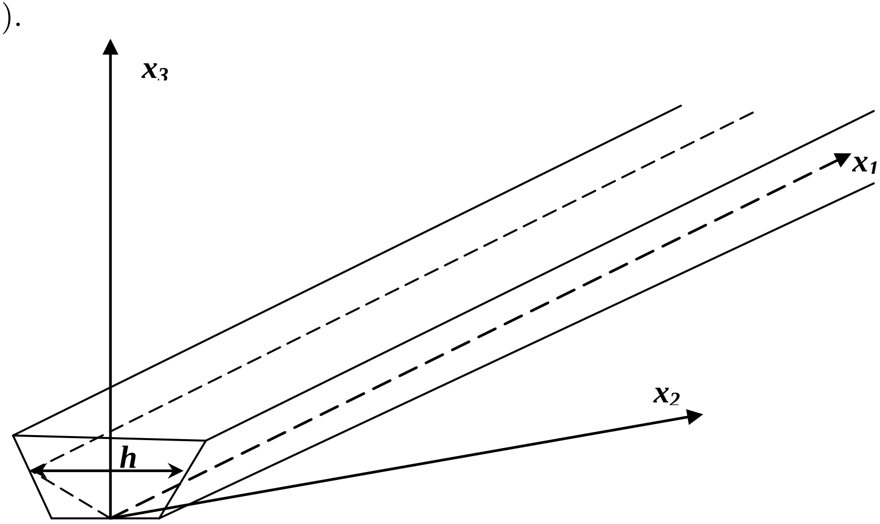

Figure 1 : Design scheme: plates of variable thickness

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Basic relations of the classical theory of plates of variable thickness can be obtained on the basis of the principles of virtual displacements. The variation equation problem visco elasticity theory in three-dimensional statement has the form

$$\iiint_V (\sigma_{ij}\delta\varepsilon_{ij} + \rho u_i \delta u_{i\ \ i})dx_3 dx_2 dx_1 = 0 \qquad (i = 1,2,3;\ j = 1,2,3) \tag{1}$$

where ρ- material density; u_i – displacement components; σ_{ij} and ε_{ij} – components of the stress tensor and strain; h- plate thickness; V – the volume occupied by the body. In accordance with the hypotheses of Kirchhoff - Love

$$\sigma_{12} = \sigma_{23} = \sigma_{33} = 0, \qquad u_i = -x_3\frac{\partial w}{\partial x_i},\ w(x_3,t) = w. \tag{2}$$

Neglecting in (1) the members of which take into account the inertia of rotation normal to the middle plane, will have the following variation equation:

$$\int_S ds \int_{-\frac{h}{2}}^{\frac{h}{2}} (\sigma_{11}\delta\varepsilon_{11} + 2\tau_{12}\delta\varepsilon_{12} + \tau_{22}\delta\varepsilon_{22})dx_3 + \int_S ds \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho\frac{\partial^2 w}{\partial t^2}\delta w dx_3 = 0 \tag{3}$$

Based on the geometric relationships and relations of the generalized Hooke's law, taking into account the kinematic hypotheses (2), the expressions for the components of the strain and stress tensor has the form

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - x_3\frac{\partial^2 w}{\partial x_i \partial x_j}, \qquad i,j = 1,2;$$

$$\sigma_{11} = \frac{\tilde{E}}{1-\nu}(\varepsilon_{11} + \nu\varepsilon_{22}); \quad \sigma_{22} = \frac{\tilde{E}}{1-\nu}(\varepsilon_{22} + \nu\varepsilon_{11}); \tag{4}$$

$$\sigma_{12} = \frac{\tilde{E}}{1+\nu}\varepsilon_{12},$$

$$\tilde{E}_n\varphi(t) = E_{0n}\left[\varphi(t) - \int_0^t R_{En}(t-\tau)\varphi(t)d\tau\right], \tag{5,a}$$

where $\varphi(t)$– arbitrary function of time; ν– Poisson's ratio; $R_{En}(t-\tau)$– the core of relaxation; E_{01}– instantaneous modulus of elasticity; We accept the integral terms in (5, a) small, then the function $\varphi(t) = \psi(t)e^{-i\omega_R t}$, where $\psi(t)$- slowly varying function of time, ω_R- real constant. The [7], we replace of (5,a) approximate species

$$\bar{E}_n\varphi = E_{0j}\left[1 - \Gamma_j^C(\omega_R) - i\Gamma_j^S(\omega_R)\right]\varphi, \tag{5,б}$$

where $\Gamma_n^C(\omega_R) = \int_0^\infty R_{En}(\tau)\cos\omega_R\tau\, d\tau$, $\Gamma_n^S(\omega_R) = \int_0^\infty R_{En}(\tau)\sin\omega_R\tau\, d\tau$, respectively, cosine and sine Fourier transforms relaxation kernel material. As an example, the visco elastic material take three parametric relaxation nucleus $R_{En}(t) = A_n e^{-\beta_n t}/t^{1-\alpha_n}$. Here

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A_n, α_n, β_n- parameters relaxation nucleus. On the effect of the function $R_{En}(t-\tau)$ superimposed usual requirements inerrability, continuity (except $t=\tau$), signs - certainty and monotony:

$$R_{En}\rangle 0\ ,\ \frac{dR_{En}}{dt}\leq 0,\ 0\langle \int_0^\infty R_{En}(t)dt\langle 1.$$

Notes

Introducing the notation for points

$$M_{11}=\overline{D}\left(\frac{\partial^2 w}{\partial x_1^2}+v\frac{\partial^2 w}{\partial x_2^2}\right); M_{22}=\overline{D}\left(\frac{\partial^2 w}{\partial x_2^2}+v\frac{\partial^2 w}{\partial x_1^2}\right);$$

$$M_{12}=\overline{D}(1-v)\frac{\partial^2 w}{\partial x_1\,\partial x_2}\qquad \overline{D}=\frac{\overline{E}h^3}{12\left(1-v^2\right)}.$$

When $R_{En}(t-\tau)$=0, then $D=\dfrac{Eh^3}{12\left(1-v^2\right)}$. Here E is the modulus of elasticity.

Integrating (3) in the strip thickness leads to the following form

$$\int_5\left(M_{11}\frac{\partial^2\delta w}{\partial x_1^2}+2M_{12}\frac{\partial^2\delta w}{\partial x_1\partial x_2}+M_{22}\frac{\partial^2\delta w}{\partial x_2^2}\right)ds-\int_s\rho h\frac{\partial^2 w}{\partial t^2}\delta w ds=0. \qquad (6)$$

Integrating twice by parts and alignment to zero, the coefficients of variation δ w inside the body and on its boundary, and we obtain the following differential equation

$$\frac{\partial^2 M_{11}}{\partial x^2}+2\frac{\partial^2 M_{12}}{\partial x_1\partial x_2}+\frac{\partial^2 M_{22}}{\partial x_2^2}+\rho h\ddot{w}=0,\ \left(\ddot{w}=\partial^2\ w/\partial t^2\right) \qquad (7)$$

with natural boundary conditions:

$$\begin{cases}\dfrac{\partial w}{\partial x_2}=0\\ w=0;\ x_2=0:l_2\end{cases} \qquad (8)$$

$$\begin{cases}\dfrac{\partial w}{\partial x_1}=0 & x_1=0;\ l_1\\ w=0\ ; \end{cases}\ . \qquad (9)$$

The main alternative boundary conditions to them

$$\begin{cases}M_{22}=0\\ \dfrac{\partial M_{22}}{\partial x_2}+2\dfrac{\partial M_{12}}{\partial x_1}=0;\ x_2=0\ ;\quad l_2\end{cases} \qquad (10)$$

$$\begin{cases} M_{11} = 0 \\ \dfrac{\partial M_{11}}{\partial x_1} + 2\dfrac{\partial M_{12}}{\partial x_2} = 0\,;\; x_1 = 0 : l_1. \end{cases} \tag{11}$$

For, we construct a spectral problem by entering the following change of variables

$$w = W; \quad \varphi = \frac{\partial W}{\partial x_2};$$

$$M = \left(\frac{\partial^2 W}{\partial x_1^2} + \frac{\partial^2 W}{\partial x_2^2}\right); \quad Q = \frac{\partial M_{22}}{\partial x_2} + \frac{\partial M_{12}}{\partial x_1} \;. \tag{12}$$

Substituting (12) into (7) we obtain the differential equation of the system relatively sparse on the first derivatives x_2 :

$$\frac{\partial Q}{\partial x_2} + \frac{\partial^2 M}{\partial x_1{}^2} + \overline{D}'(1-\nu)\frac{\partial^2 \varphi}{\partial x_1{}^2} + \rho h \frac{\partial^2 W}{\partial t^2} = 0\,;$$

$$\frac{\partial M}{\partial x_2} - Q - \overline{D}''(1-\nu)\frac{\partial^2 W}{\partial x_1{}^2} = 0\,;$$

$$\overline{D}\frac{\partial \varphi}{\partial x_2} - M + \overline{D}\frac{\partial^2 W}{\partial x_1{}^2} = 0\,; \tag{13}$$

$$\frac{\partial W}{\partial x_2} - \varphi = 0:$$

and alternative boundary conditions $x_2 = 0$: $x_2 = l_2$:

$$\varphi = 0 \text{ or } M - \overline{D}(1-\nu)\frac{\partial^2 M}{\partial x_1^2} = 0; \tag{14}$$

$$W = 0 \text{ or } Q + \overline{D}(1-\nu)\frac{\partial^2 \varphi}{\partial x_1^2} = 0.$$

and $x_1 = 0$, $x_1 = l_1$,

$$\varphi = 0 \text{ or } M - \overline{D}(1-\nu)\frac{\partial^2 M}{\partial x_1^2} = 0;$$

$$W = 0 \text{ or } Q + \overline{D}(1-\nu)\frac{\partial^2 \varphi}{\partial x_1^2} = 0 \tag{15}$$

Now consider the infinite along the axis x_1 band with an arbitrary thickness changes $h = h(x_2)$. We seek a solution of problem (13) - (15) in the for

Notes

$$\left(Q, M, \varphi, W\right)^T = \left(\overline{Q}, \overline{M}, \overline{\varphi}, \overline{W}\right)^T e^{i(\alpha x_1 - \omega t)} \tag{16}$$

Describing the harmonic plane waves propagating along the axis x_1. Here $\left(\overline{Q}, \overline{M}, \overline{\varphi}, \overline{W}\right)^T$ – complex amplitude - function; k – wave number; C ($C = C_R + iC_i$)– complex phase velocity; ω – complex frequency.

To clarify their physical meaning, consider two cases:

1) $k = k_R$; $C = C_R + iC_i$, $(\omega_R = \omega_I + i\omega_I)$ then the solution of differential equations (13) has the form of a sine wave at x_1, whose amplitude decays over time;
2) $k = k_R + ik_I$; $C = C_R$, Then at each point x_1 fluctuations established, but x_1 attenuated.

In both cases, the imaginary part k_I or C_I characterized by the intensity of the dissipative processes. Substituting (16) in (17), we obtain a system of first order differential equations solved for the derivative

$$\begin{aligned}
&\overline{Q}' - \alpha^2 \overline{M} - \alpha^2 \overline{D}'(1-\nu)\overline{\varphi} - \rho h \omega^2 \overline{W} = 0;\\
&\overline{M}' - \overline{Q} + \alpha^2 \overline{D}'(1-\nu)\overline{W} = 0;\\
&\overline{\varphi}' - \frac{1}{\overline{D}}\overline{M} - \alpha^2 \overline{W} = 0;\\
&\overline{W}' - \overline{\varphi} = 0
\end{aligned} \tag{17}$$

with boundary conditions at the ends of the band $x_2=0$, l_2, one of the four types

а. swivel bearing: $\overline{W} = \overline{M} = 0;$ (18)

б. sliding clamp: $\overline{Q} = \overline{\varphi} = 0$ (19)

в. anchorage: $\overline{W} = \overline{\varphi} = 0$ (20)

г. free edge: $\begin{cases} \overline{M} + \alpha^2 \overline{D}\,(1-\nu)\overline{W} = 0 \\ \overline{Q} - \alpha^2\,(1-\nu)\,\overline{D}\,\overline{\varphi} = 0 \end{cases}$ (21)

Thus, the spectral formulated task (17) and (21) the parameter α^2, describes the propagation of flexural waves in planar waveguide made as a band with an arbitrary coordinate on the thickness change x_2. It is shown that the spectral parameter α^2 It takes complex values (in the case of $R_{En}(t-\tau) \neq 0$) If $R_{En}(t-\tau) = 0$, whereas the spectral parameter α^2 It takes only real values. Transform this system (17). We have

$$\overline{Q}' = \overline{M}'' + \overline{D}''(1-\nu)\alpha^2 \overline{W} + \overline{D}'(1-\nu)\alpha^2 \overline{\varphi}$$

From whence

$$M'' + \overline{D}''(1-\nu)\alpha^2 \overline{W} - \alpha^2 \overline{M} - \rho h w^2 \overline{W} = 0$$

Moreover

$$\overline{W} - \frac{1}{\overline{D}}\overline{M} - \alpha^2 \overline{W} = 0.$$

Thus, the conversion system is of the form

$$\begin{cases} \overline{M}'' - \alpha^2 \overline{M} - \left(\rho h \omega^2 - \overline{D}''(1-\nu)\alpha^2\right) \overline{W} = 0 \\ W'' - \alpha^2 \overline{W} - \dfrac{1}{\overline{D}} \overline{M} = 0 \end{cases} \tag{22}$$

The boundary conditions (18) - (21) in alternating $\overline{W}, \overline{M}$ it has the form:

a. swivel bearing: $\overline{W} = \overline{M} = 0;$ (23)

б. sliding clamp: $\overline{W}' = \overline{M}' - \alpha^2 \overline{D}'(1-\nu)\overline{W} = 0;$ (24)

в. anchorage: $\overline{W} = \overline{W}' = 0$ (25)

г. free edge:
$$\begin{aligned} &\overline{M}' + \alpha^2 \overline{D}(1-\nu)\overline{W} = 0 \\ &\overline{M}' - \alpha^2 (1-\nu)(D\overline{W})' = 0 \end{aligned} \tag{26}$$

at $x_2=0$ or $x_2=+l_2$

Let $\overline{M}$ and $\overline{W}$ some own functions of the system (22) - (26) may have a complex meaning. Multiply the equation system (22) to function $\widehat{M}$ and $\widehat{W}$, complex conjugate to $\overline{M}$ and $\overline{W}$. Identical converting the first equation, we integrate the resulting equality x_2 and composed of the following linear combination

$$\begin{aligned} &\int_0^{l_2} \overline{M}''\widehat{W} dx_2 - \alpha^2(1-\nu)\int_0^{l_2} (\overline{D}\,\overline{W})''\widehat{W} dx_2 + \alpha^2(1-\nu)\int_0^{l_2} (\overline{D}\,\overline{W})''\widehat{W} dx_2 - \\ &- \alpha^2 \int_0^{l_2} \overline{M}\widehat{W} dx_2 - \omega^2 \int_0^{l_2} \rho h \overline{W}\,\widehat{W} dx_2 - \alpha^2(1-\nu)\int_0^{l_2} \overline{D}''\overline{W}\widehat{W} dx_2 + \\ &+ \int_0^{l_2} \overline{W}''\widehat{M} dx_2 - \alpha^2 \int_0^{l_2} \overline{W}\widehat{M} dx_2 - \int_0^{l_2} \frac{\overline{M}\widehat{M}}{\overline{D}} dx_2 = 0 \end{aligned} \tag{27}$$

Integrating (27) by parts,

$$\begin{aligned} &\left[\overline{M}' - \alpha^2(1-\nu)(D\overline{W}')\right]\widehat{W}\Big|_0^{l_2} - \int_0^{l_2} [\overline{M}'\widehat{W}' + \widehat{M}'\overline{W}']dx_2 + \\ &+ \alpha^2(1-\nu)\int_0^{l_2} \overline{D}\,\overline{W}'\widehat{W}' dx_2 - \alpha^2 \int_0^{l_2} [\overline{M}\widehat{W}' + \widehat{M}\overline{W}]dx_2 + \\ &+ 2\alpha^2(1-\nu)\int_0^{l_2} \overline{D}''\overline{W}\,\widehat{W} dx_2 - \omega^2 \int_0^{l_2} \rho h \overline{W}\widehat{W} dx_2 - \int_0^{l_2} \frac{\overline{M}\widehat{M}}{\overline{D}} dx_2 + \overline{W}'\widehat{M}\Big|_0^{l_2} + \\ &+ 2\alpha^2(1-\nu)\int_0^{l_2} \overline{D}'\overline{W}'\widehat{W} dx_2 + \alpha^2(1-\nu)\int_0^{l_2} \overline{D}'\overline{W}\widehat{W}' dx_2 + \alpha^2(1-\nu)\int_0^{l_2} D\overline{W}''\widehat{W} dx_2 = 0 \end{aligned}$$

or

Notes

$$\left[\overline{M}'-\alpha^2(1-\nu)(\overline{D}\overline{W})'\right]\widehat{W}\Big|_0^{l_2}+\left[\widehat{M}+\alpha^2(1-\nu)\overline{D}\widehat{W}\right]\overline{W}'\Big|_0^{l_2}$$

$$-\int_0^{l_2}\left(\overline{M}'\widehat{W}'+\widehat{M}'\overline{W}'\right)dx_2-\alpha^2\int_0^{l_2}\left(\overline{W}\widehat{M}+\widehat{W}\overline{M}\right)dx_2- \tag{28}$$

$$-\int_0^{l_2}\frac{\overline{M}\widehat{M}}{\overline{D}}dx_2-\omega^2\int_0^{l_2}\rho h\overline{W}\widehat{W}dx_2-2\alpha^2(1-\nu)\int_0^{e_2}\overline{D}''\widehat{W}\overline{W}dx_2+\alpha^2(1-\nu)\int_0^{e_2}\overline{D}'\left(\widehat{W}\overline{W}\right)'dx_2=0.$$

It is easy to make sure that is the integral terms of (28) vanish at any combination of the boundary conditions (23) - (26). It should also be noted that all the functions under the integral valid at $R_{En}(t-\tau)$=0. The expressing α^2 (28) We find that

$$\alpha^2=\frac{\int_0^{l_2}(\overline{M}'\widehat{W}'+\widehat{M}'\overline{W}')\,dx_2+\int_0^{l_2}\frac{\overline{M}\widehat{M}}{\overline{D}}dx_2+\omega^2\int_0^{l_2}\rho h\overline{W}\widehat{W}dx_2}{\int_0^{l_2}(\overline{M}\widehat{W}+\widehat{M}\overline{W})\,dx_2-2(1-\nu)\int_0^{l_2}\overline{D}''\overline{W}\,\widehat{W}dx_2-(1-\nu)\int_0^{l_2}\overline{D}'(\overline{W}\widehat{W})'dx_2}\text{ - real number.}$$

Thus (with $R_{En}(t-\tau)$=0), It is shown that the square of the wave number for own endless strip of varying thickness is valid for any combination of boundary conditions. If $R_{En}(t-\tau)\neq 0$, then α^2 It is a complex value for any combination of boundary conditions.

II. Adjoin Spectral Problem, Orthogonality Condition

The resulting spectral problem (17) - (21) is not self-adjoin. Built for her adjoin problem using this Lagrange formula [16]

$$\int_0^l L(U)\cdot V^*dx=Z(U,V^*)\Big|_0^l-\int_0^l L^*(V^*)\cdot Udx, \tag{29}$$

where L and L* - direct and adjoin linear differential operators; U and V* - arbitrary decisions of relevant boundary value problems.

In our case

$$L=\begin{bmatrix}\frac{\partial}{\partial x_2} & -\alpha^2 & -\alpha^2\overline{D}'(1-\nu) & -\rho h\omega^2\\ -1 & \frac{\partial}{\partial x_2} & 0 & -\alpha^2\overline{D}'(1-\nu)\\ 0 & \frac{1}{\overline{D}} & \frac{\partial}{\partial x_2} & -\alpha^2\\ 0 & 0 & -1 & \frac{\partial}{\partial x_2}\end{bmatrix}, \tag{30}$$

on the left-hand side of equation (29) will be as follows

16. M. A. Neumark, Linear differential operators. Publishing Moscow, 1969. - 526 p.

$$\int_0^{l_2}\Big[\overline{Q}'\overline{Q}^{\bullet} - \alpha^2\overline{M}\,\overline{Q}^{\bullet} - \alpha^2\overline{D}'(1-\nu)\overline{\varphi}\overline{Q}^{\bullet} - \rho h\omega^2\overline{W}\overline{Q}^{\bullet} + \overline{M}'\overline{M}^{\bullet} - \overline{Q}\,\overline{M}^{\bullet} +$$

$$+\alpha^2\overline{D}'(1-\nu)\overline{W}\,\overline{M}^{\bullet} + \overline{\varphi}'\overline{\varphi}^{\bullet} - \frac{1}{\overline{D}}\overline{M}\overline{\varphi}^{\bullet} - \alpha^2\overline{W}\overline{\varphi}^{\bullet} + \overline{W}'\overline{W}^{\bullet} - \overline{\varphi}\overline{W}^{\bullet}\Big]dx_2 = 0 \qquad (31)$$

or, integrating parts

$$\Big[\overline{Q}\overline{Q}^{\bullet} + \overline{M}\,\overline{M}^{\bullet} + \overline{\varphi}\overline{\varphi}^{\bullet} + \overline{W}\overline{W}^{\bullet}\Big]\Big|_0^{l_2} - \int_0^{l_2}\Big[(\overline{Q}^{\bullet\prime} + \overline{M}^{\bullet})\overline{Q} + (\overline{M}^{\bullet\prime} + \alpha^2\overline{Q}^{\bullet} +$$

$$+\frac{1}{\overline{D}}\overline{\varphi}^{\bullet})\overline{M} + (\overline{\varphi}^{\bullet\prime} + \overline{W}^{\bullet} + \alpha^2\overline{D}'(1-\nu)\overline{Q}^{\bullet})\overline{\varphi} + (\overline{W}^{\bullet\prime} + \alpha^2\overline{\varphi}^{\bullet} - \qquad (32)$$

$$-\alpha^2\overline{D}'(1-\nu)\overline{M}^{\bullet} + \rho h\omega^2\overline{Q}^{\bullet})\overline{W}\Big]dx_2 = 0$$

Thus the conjugate (30) - (32), the system has the form

$$\begin{cases} \overline{Q}^{\bullet\prime} + \overline{M}^{\bullet} = 0 \\ \overline{M}^{\bullet\prime} + \alpha^2\overline{Q}^{\bullet} + \dfrac{1}{D}\overline{\varphi}^{\bullet} = 0 \\ \overline{\varphi}^{\bullet\prime} + \overline{W}^{\bullet} + \alpha^2\overline{D}'(1-\nu)\overline{Q}^{\bullet} = 0 \\ \overline{W}^{\bullet\prime} - \alpha^2\overline{D}'(1-\nu)\overline{M}^{\bullet} + \alpha^2\overline{\varphi}^{\bullet} + \rho h\omega^2\overline{Q}^{\bullet} = 0 \end{cases} \qquad (33)$$

Moreover, we get the conjugate boundary conditions of equality to zero is integral members $Z(U,V^*)\Big|_0^{l_2}$ expression in (32):

a. swivel bearing: $\overline{\varphi}^{\bullet} = \overline{Q}^{\bullet} = 0, \qquad x_2 = 0,\ l_2$ (34)

б. sliding clamp: $\overline{W}^{\bullet} = \overline{M}^{\bullet} = 0, \qquad x_2 = 0,\ l_2$ (35)

в. anchorage: $\overline{M}^{\bullet} = \overline{Q}^{\bullet} = 0, \qquad x_2 = 0,\ l_2$ (36)

г. free edge: $\begin{cases} \overline{\varphi}^{\bullet} + \alpha^2\overline{D}(1-\nu)\overline{Q}^{\bullet} = 0, \\ \overline{W}^{\bullet} - \alpha^2\overline{D}(1-\nu)\overline{M}^{\bullet} = 0, \quad x_2 = 0,\ l_2 \end{cases}$

For conditions biorthogonality solutions once again use the Lagrange formula (29) in the form

$$\int_0^{l}\Big[L(U)V^{\bullet} + L^{\bullet}(V^{\bullet})U\Big]dx = Z(U,V^{\bullet})\Big|_0^{l_2}, \qquad (37)$$

that leads to the consideration of the following integral

$$\int_0^{l_2}\Big[\overline{Q}_i'\,\overline{Q}_j^{\bullet} - \alpha_i^2\overline{M}_i\overline{Q}_j^{\bullet} - \alpha_i^2\overline{D}'(1-\nu)\overline{\varphi}_i\overline{Q}_j^{\bullet} - \rho h\omega^2\overline{W}_i\overline{Q}_j^{\bullet} + \overline{M}_i{}'\,\overline{M}_j^{\bullet} -$$

Notes

$$-\overline{Q}_i\overline{M}^{\bullet}_j+\alpha_i^2\overline{D}'(1-\nu)\overline{W}_i\overline{M}^{\bullet}_j+\overline{\varphi}_i'\overline{\varphi}^{\bullet}_j-\frac{1}{\overline{D}}\overline{M}_i\overline{\varphi}^{\bullet}_j-\alpha^2{}_i\ \overline{W}_i\ \overline{\varphi}^{\bullet}_j+$$

$$+\overline{W}_i'\ \overline{W}^{\bullet}_j-\overline{\varphi}_i\ \overline{W}^{\bullet}_j+\overline{Q}^{\bullet\prime}_j\overline{Q}_i+\overline{M}^{\bullet}_j\overline{Q}_i+\overline{M}^{\bullet\prime}_j\ \overline{M}_i+\alpha^2{}_j\overline{M}_i\overline{Q}^{\bullet}_j+ \tag{38}$$

$$+\frac{1}{\overline{D}}\overline{M}_i\overline{\varphi}^{\bullet}_j+\overline{\varphi}_i\overline{\varphi}^{\bullet\prime}_j+\overline{W}^{\bullet}_j\overline{\varphi}_i+\alpha_j^2D'(1-\nu)Q^{\bullet}_j\overline{\varphi}_i+\overline{W}_i\overline{W}^{\bullet}_j{}'+$$

$$+\alpha_j^2\overline{W}_i\overline{\varphi}^{\bullet}_j-\alpha_j^2\overline{D}'(1-\nu)\overline{W}_i\overline{M}^{\bullet}_j+\rho h\omega^2\overline{Q}^{\bullet}_j\overline{W}_i]dx_2=0,$$

where $(\overline{Q}_i\ ,\overline{M}_{i,}\overline{\varphi}_i,\overline{W}_i)^T$ - own form, corresponding to the Eigen value α_i original spectral problem; $(\overline{Q}^{\bullet}_j,\overline{M}^{\bullet}_j,\overline{\varphi}^{\bullet}_j,\overline{W}^{\bullet}_j)^T$ - own form, corresponding to the Eigen value α_j adjoin.

Integrating (38) by parts

$$\left(\alpha_i^2-\alpha_j^2\right)\left[\int_0^{l_2}\left[-\overline{M}_i\overline{Q}^{\bullet}_j-\overline{D}'(1-\nu)\overline{Q}^{\bullet}_j\overline{\varphi}_i+\overline{D}'(1-\nu)\overline{W}_i\overline{M}^{\bullet}_j-\overline{W}_i\overline{\varphi}^{\bullet}_j\right]dx_2+\right. \tag{39}$$

$$+\left[\overline{D}(1-\nu)\overline{Q}^{\bullet}_j\overline{\varphi}_i-\overline{D}(1-\nu)\overline{W}_i\overline{M}^{\bullet}_j\ \right]_0^{l_2}=0,$$

where to i≠j we have the condition biorthogonality forms:

$$\int_0^{l_2}\left[\overline{M}_i+\overline{D}'(1-\nu)\overline{\varphi}_i)\overline{Q}^{\bullet}_j+\overline{W}_i(\overline{\varphi}^{\bullet}_j=\overline{D}'(1-\nu)\overline{M}^{\bullet}_j)\right]dx_2+$$

$$+\overline{D}(1-\nu)\left[\overline{W}_i\overline{M}^{\bullet}_j-\overline{Q}^{\bullet}_j\overline{\varphi}_i\right]\Big|_0^{l_2}=\delta_{ij} \tag{40}$$

The expression $\overline{W}_i\overline{M}^{\bullet}_j-\overline{Q}^{\bullet}_j\overline{\varphi}_i$ zero, if the border is set to any of the conditions (18) - (21) in addition to the conditions of the free edge.

III. Fixed Problem for a Semi-Infinite Strip of Variable Thickness.

Consider a semi-infinite axial x_1 lane variable section, wherein at the end (x_1 =0) harmonic set time exposure of one of two types of:

$$W=f_W(x_2)e^{i\omega t},\ \ M_{11}=f_W(x_2)e^{i\omega t},\quad \mathrm{x}_1=0 \tag{41}$$

or

$$\varphi_1=f_\varphi(\mathrm{x}_2)\mathrm{e}^{i\omega t},\qquad Q_1=f_Q(x_2)e^{i\omega t},\quad x_1=0 \tag{42}$$

where

$$\varphi_1=\frac{\partial W}{\partial x_1},\quad Q_1=\overline{D}\left[\frac{\partial^3W}{\partial x_1^3}+2(1-\nu)\frac{\partial^3W}{\partial x_1\partial x_2^2}\right] \tag{43}$$

Notes

Transform the boundary conditions (41) so that they contain only selected our variables $W,\varphi,\ M$ and Q

$$W = f_w(x_2)e^{i\omega t},\quad \overline{D}\left(\frac{\partial^2 W}{\partial x_1^2}+\nu\frac{\partial^2 W}{\partial x_2^2}\right)=f_M(x_2)e^{i\omega t},\ x_1=0,$$

$$\frac{\partial W}{\partial x_1}=f_\varphi(x_2)e^{i\omega t},\ \overline{D}\left(\frac{\partial^3 W}{\partial x_1^3}+2(1-\nu)\frac{\partial^3 W}{\partial x_1\partial x_2^2}\right)=f_Q(x_2)e^{i\omega t},\quad x_1=0,$$

or

$$W = f_w(x_2)e^{i\omega t},\quad \overline{D}\left(\frac{\partial^2 W}{\partial x_1^2}+\nu\frac{\partial^2 W}{\partial x_2^2}\right)-\overline{D}(1-\nu)\frac{\partial^2 W}{\partial x_2^2}=f_M(x_2)e^{i\omega t},\ x_1=0,$$

$$\frac{\partial W}{\partial x_1}=f_\varphi(x_2)e^{i\omega t},\ \frac{\partial}{\partial x_1}\left[\overline{D}\left(\frac{\partial^2 W}{\partial x_1^2}+\frac{\partial^2 W}{\partial x_2^2}\right)\right]+\overline{D}(1-\nu)\frac{\partial^2}{\partial x_2^2}\left(\frac{\partial W}{\partial x_1}\right)=f_Q(x_2)e^{i\omega t},\quad x_1=0,$$

Of finally

$$W = f_w(x_2)e^{i\omega t},\ M=\left[f_M(x_2)+\overline{D}(1-\nu)f_w''(x_2)\right]e^{i\omega t},\ x_1=0 \tag{44}$$

$$\frac{\partial W}{\partial x_1}=f_\varphi(x_2)e^{i\omega t},\quad \frac{\partial M}{\partial x_1}=\left[f_Q(x_2)-\overline{D}(1-\nu)f_\varphi''(x_2)\right]e^{i\omega t},\quad x_1=0. \tag{45}$$

Assume that the desired solution of the no stationary problem can be expanded in a series in Eigen functions of the solution of the spectral problem. In the case of constant thickness it is evident, and in general, the question remains open.
The solution of the stationary problem (17) - (21) (41) - (42) will seek a

$$\begin{pmatrix} W \\ \varphi \\ M \\ Q \end{pmatrix} = \sum_{k=1}^{N} a_k \begin{pmatrix} \overline{W}_k(x_2) \\ \overline{\varphi}_k(x_2) \\ \overline{M}_k(x_2) \\ \overline{Q}_k(x_2) \end{pmatrix} e^{-i(\alpha_k x_1-\omega t)} \tag{46}$$

where $\overline{W}_k,\ \overline{\varphi}_k,\ \overline{\theta}_k,\ \overline{M}_k$ - biorthonormal own forms of the spectral problem (17) – (21).

The representation (46) gives us the solution to the problem of non-stationary wave in the far field, i.e., where it has faded not propagating modes. The number of propagating modes used N course for each specific frequency ω, since the cutoff frequency is greater than the other ω.

Consider two cases of excitation of stationary waves in the band:

а) $f_w=0$ – antisymmetric relative x_1;

б) $f_\varphi=0$ – symmetric.

In the case of antisymmetric excitation, substituting (46) into (44) and expressing $f_M(x_2)$, obtain

$$f_M(x_2)=\sum_{k=1}^{N}\alpha_k\overline{M}_k(x_2). \tag{47}$$

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Value biorthogonality (40) gives expression to determine the unknown coefficients

$$a_k = \int_0^{e_2} f_M(x_2)\overline{Q}_k^{\bullet}(x_2)dx_2 .$$

In the case of a symmetrical excitation $f_Q(x_2)$ We obtain rearranging (46) to (45) in the following form

Notes

$$f_Q(x_2) = \sum_{k=1}^{N}(-i\alpha_k a_k \overline{M}_k(x_2)) \tag{48}$$

Biorthogonality ratio (30) gives

$$a_k = \frac{i}{\alpha_k}\int_0^{l_2} f_Q(x_2)\overline{Q}_k^{\bullet} dx_2 \tag{49}$$

IV. Testing Software System and Study the Properties of Propagation of Flexural Waves in a Band of Variable Thickness

Testing program was carried out on the task of distributing the flexural waves in a plate of constant thickness. Consider the floor plate of constant thickness infinitely satisfying Kirchhoff-Love hypotheses, with supported long edges (Figure 1).

at end face $x_1=0$ specified:

$$w=f_1(x_2)e^{i\omega t}, \qquad M_{11}=f_2(x_2)e^{i\omega t} \tag{50}$$

Spread along the axis x_1 flexural wave is described by the differential control system (13)

$$\begin{cases} Q' + \dfrac{\partial^2 M}{\partial x_1^2} + \rho h w'' = 0\left(w'' = \partial^2 w/\partial t^2\right), \\ M' - \theta = 0 , \\ \varphi' - \dfrac{1}{\overline{D}}M + \dfrac{\partial^2 w}{\partial x_1^2} = 0 \\ w' - \varphi = 0, \quad h = h_0 \end{cases} \tag{51}$$

with boundary conditions of the form (15)

$$w=0, M - \overline{D}(1-\nu)\frac{\partial^2 w}{\partial {x^2}_1} = 0, \quad x_2 = 0, \ \pi \quad . \tag{52}$$

Introducing the desired motion vector in the form of

$$\begin{pmatrix} Q \\ M \\ \varphi \\ w \end{pmatrix} = \begin{pmatrix} \overline{Q} \\ \overline{M} \\ \overline{\varphi} \\ \overline{W} \end{pmatrix} e^{-i(a_k x_1 - \omega t)} \tag{53}$$

Go to the spectral problem

$$\begin{cases} \overline{Q}' - \alpha^2 \overline{M} - \omega^2 \rho h \overline{W} = 0, \\ \overline{M}' - \overline{Q} = 0, \\ \overline{\varphi}' - \dfrac{1}{\overline{\overline{D}}} \overline{M} - \alpha^2 \overline{W} = 0, \\ \overline{W}' - \overline{\varphi} = 0, \end{cases} \tag{54}$$

with the boundary conditions

$$W = 0,\ \overline{M} + \overline{D}(1-\nu)\alpha^2 \overline{W} = 0,\ x_2 = 0,\ \pi$$

or

$$\overline{W} = 0,\ \overline{M} = 0,\ x_2 = 0, \quad \pi\,. \tag{55}$$

Rewrite the system (54) as follows

$$\begin{cases} \overline{W}'' - \dfrac{1}{\overline{\overline{D}}} \overline{M} - \alpha^2 \overline{W} = 0, \\ \overline{M}'' - \alpha^2 \overline{M} - \omega^2 \rho h \overline{W} = 0, \end{cases} \tag{56}$$

and $\overline{W} = 0,\ \overline{M} = 0,\ x_2 = 0, \quad \pi.$

We seek the solution of (56) in the form

$$\begin{aligned} \overline{W} &= a_w \sin n x_2, \\ M &= a_M \sin n x_2,\ (\ n = 1, 2, \ldots) \end{aligned} \tag{57}$$

satisfying the boundary conditions (55).

We obtain an algebraic homogeneous system

$$-n^2 a_W - \alpha^2 a_W - \frac{1}{\overline{\overline{D}}} a_M = 0 \quad ;$$

$$-n^2 a_M - \alpha^2 a_M - \omega^2 \rho h a_w = 0\ . \tag{58}$$

For the existence of a nontrivial solution, which is necessary to require the vanishing of its determinant

$$\det \begin{vmatrix} n^2 + \alpha^2 & \dfrac{1}{\overline{\overline{D}}} \\ \omega^2 \rho h & n^2 + \alpha^2 \end{vmatrix} = 0,$$

or

$$\alpha_{1,2}{}^2 + n^2 = \pm \omega \sqrt{\frac{\rho h}{\overline{\overline{D}}_k}}, \tag{59,a}$$

Notes

Notes

where, when $R_{En}(t-\tau)=0$

$$\alpha_{1,2}{}^{2} = -n^{2} \pm \omega\sqrt{\frac{\rho h}{D}}. \tag{59,b}$$

Ownership of constant thickness strip bending vibrations are of the form

$$\overline{W}_n{}^{1,2} = \sin n x_2;$$

$$M_n{}^{1,2} = \pm\omega\sqrt{\rho h \overline{D}} \sin n x_2; \tag{60}$$

$$\varphi_n{}^{1,2} = n \cos n x_2;$$

$$Q_n{}^{1,2} = \mp\omega\sqrt{\rho h \overline{D}} \cos n x_2.$$

We construct the solution of the problem adjoin to (54)-(55)

$$\begin{cases} \overline{W}^{\bullet\prime} + \alpha^2\overline{\varphi}^{\bullet} + \rho h\omega^2\overline{Q}* = 0, \\ \overline{\varphi}^{\bullet\prime} + W^{\bullet} = 0 \\ \overline{M}*' + \frac{1}{\overline{D}}\overline{\varphi}^{\bullet} + \alpha^2\overline{Q}^{\bullet} = 0, \\ \overline{Q}^{\bullet\prime} + \overline{M}^{\bullet} = 0 \end{cases} \tag{61}$$

and

$$\overline{\varphi}* = 0, \quad \overline{Q}* = 0, \quad x_2 = 0, \quad \pi. \tag{62}$$

Transforming (61) - (62) we obtain the following system of first order differential equations

$$\begin{cases} \overline{\varphi}^{\bullet\prime\prime} - \alpha^2\overline{\varphi}^{\bullet} - \rho h\omega^2\overline{Q}^{\bullet} = 0 \\ \overline{Q}^{\bullet\prime\prime} - \alpha^2\overline{Q}^{\bullet} - \frac{1}{\overline{D}}\overline{\varphi}^{\bullet} = 0, \end{cases}$$

$$x_2 = 0, \pi: \ \overline{\varphi}^{\bullet} = 0, \overline{Q}^{\bullet} = 0. \tag{63}$$

The solution of (63) in the form

$$\overline{\varphi}^{\bullet} = a_{\varphi}^{\bullet} \sin n x_2, \quad \overline{Q}^{\bullet} = a_{Q}^{\bullet} \sin n x_2 \tag{64}$$

From whence $\alpha_{1,2}^2$ It has the same form (59a), own forms of vibrations are of the form:

$$
\begin{aligned}
\overline{\varphi}_n^{\bullet 1,2} &= \pm\omega\sqrt{ph\overline{D}}\,\sin nx_2, \\
\overline{Q}_n^{\bullet 1,2} &= \sin n\,x_2, \\
\overline{W}_n^{\bullet 1,2} &= \pm n\omega\sqrt{ph\overline{D}}\,\cos nx_2 \\
\overline{M}_n^{\bullet 1,2} &= n\cos n\,x_2.
\end{aligned}
\tag{65}
$$

Notes

For biorthogonality conditions direct solutions and the adjoin problem is necessary to consider the following equation

$$
\begin{aligned}
&\int_0^{\pi}\Big[\overline{Q}_i'\overline{Q}_j^{\bullet} - \alpha_i^{\,2}\overline{M}_i\,\overline{Q}_j^{\bullet} - \omega^2\rho h\overline{W}_i\overline{Q}_j^{\bullet} + \overline{Q}_j^{\bullet\,\prime}\,\overline{Q}_i + \overline{M}_j^{\bullet}\overline{Q}_i + \overline{M}_i^{\,\prime}\,\overline{M}_j^{\bullet} - \overline{Q}_i\overline{M}_j^{\bullet} + \\
&+\overline{M}_j^{\bullet\,\prime}\,\overline{M}_i + \frac{1}{\overline{D}}\overline{\varphi}_j^{\bullet}\overline{M}_i + \alpha_j^2\overline{Q}_j^{\bullet}\overline{M}_i + \overline{\varphi}_i^{\,\prime}\,\overline{\varphi}_j^{\bullet} - \frac{1}{\overline{D}}\overline{M}_i\,\overline{\varphi}_j^{\bullet} - \alpha_i^{\,2}\overline{W}_i\overline{\varphi}_j^{\bullet} + \overline{\varphi}_j^{\bullet\,\prime}\,\overline{\varphi}_i + \\
&+\overline{W}_j^{\bullet}\overline{\varphi}_i + \overline{W}_i^{\,\prime}\,\overline{W}_j^{\bullet} - \overline{\varphi}_i\overline{W}_j^{\bullet} + \overline{W}_j^{\bullet\,\prime}\,\overline{W}_i + \alpha^2\overline{\varphi}_j^{\bullet}\overline{W}_i + \rho h\omega^2\overline{Q}_j^{\bullet}\overline{W}_i\Big]dx_2 = \delta_{ij}
\end{aligned}
\tag{66}
$$

where $\overline{Q}_i$, $\overline{M}_i$, $\overline{\varphi}_i$, $\overline{W}_i$ – own form for the direct problem, the corresponding Eigen values α_i, and $Q_j^{\bullet}$, $\overline{M}_j^{\bullet}$, $\overline{\varphi}_j^{\bullet}$, $\overline{W}_j^{\bullet}$ - own form of the dual problem, the corresponding Eigen value α_j. Integrating by parts in (66), using the boundary conditions (55) and (62) we obtain the desired condition:

$$
\int_0^{\pi}\Big[\overline{M}_i\overline{Q}_j^{\bullet} + \overline{W}_i\overline{\varphi}_j^{\bullet}\Big]dx_2 = \delta_{ij}\,. \tag{67}
$$

We now verify biorthogonality received their own forms (60) and (65) using the condition biorthogonality (67)

$$
\begin{aligned}
&\int_0^{\pi}\Big[\sqrt{\rho h\overline{D}}\,Sin(ix_2)\cdot Sin(jx_2) + \sqrt{\rho h\overline{D}}\,Sin(ix_2)Sin(jx_2)\Big]dx_2 = \\
&= 2\sqrt{\rho h\overline{D}}\int_0^{\pi} Sin(ix_2)Sin(jx_2)dx_2 = \pi\sqrt{\rho h\overline{D}}\,\delta_{ij}
\end{aligned}
$$

The normalized adjoin eigenvector on $\pi\sqrt{\rho h\overline{D}}$, we have a system of eigenvectors satisfying the condition (67).

We now obtain the solution of the problem of the distribution of the stationary wave in the semi-infinite strip of constant thickness. Suppose that at the border $x_1=0$ set the following stationary disturbance:

$$
\begin{aligned}
w &= \overline{W}e^{i\omega t} = b_W\sin(nx_2)e^{i\omega t}, \\
M &= \overline{M}e^{i\omega t} = b_M\sin(nx_2)e^{i\omega t}, \qquad x_1 = 0
\end{aligned}
\tag{68}
$$

We seek a solution of a problem

$$
w(x_1,x_2,t) = \sum_{k=1}^{\infty} a_k W_k\ ,\ M(x_1,x_2,t) = \sum_{k=1}^{\infty} a_k M_k \tag{69}
$$

Notes

where

$$W_k = \overline{W}(x_{21})e^{-i(a_k x_l - \omega t)}, \quad M_k = \overline{M}(x_2)e^{-i(a_k x_l - \omega t)},$$

a $\overline{W}_k$ u $\overline{M}_k$ - own form (60), corresponding to α_k

It is evident from the band at the end face at x_1=0 decision (69) must satisfy the boundary conditions (68)

$$b_W \sin(nx_2)e^{i\omega t} = \sum_{k=1}^{\infty} a_k \overline{W}_k(x_2)e^{i\omega t},$$

$$b_M \sin(nx_2)e^{i\omega t} = \sum_{k=1}^{\infty} a_k \overline{M}_k(x_2)e^{i\omega t},$$

or go to the amplitude values

$$b_w \sin(nx_2)e^{i\omega t} = \sum_{k=1}^{\infty} a_k \overline{W}_k,$$

$$b_M \sin(nx_2)e^{i\omega t} = \sum_{k=1}^{\infty} a_k \overline{M}_k, \tag{70}$$

Consider the following integral

$$\begin{aligned}\int_0^\pi \left[\overline{MQ}_j^\bullet + \overline{W}\overline{\varphi}_j^\bullet\right]dx_2 &= \int_0^\pi \left[\sum_{k=1}^{\infty} a_k \overline{M}_k \overline{Q}_j^\bullet + \sum_{k=1}^{\infty} a_k \overline{W}_k \overline{\varphi}_j^\bullet\right]dx_2 = \\ &= \sum_{k=1}^{\infty} a_k \int_0^\pi \left[\overline{M}_k \overline{Q}_j^\bullet + \overline{W}_k \overline{\varphi}_j^\bullet\right]dx_2 = a_j.\end{aligned} \tag{71}$$

On the other hand on the edge $x_1 = 0$ the same integral as follows

$$\int_0^\pi \left[b_M \sin(nx_2)\overline{Q}_j^\bullet + b_W \sin(nx_2)\overline{\varphi}_j^\bullet\right]dx_2 \tag{72}$$

Substituting in (72) from the normalized own form (65) we obtain

$$\begin{aligned}&\int_0^\pi \left[b_M \sin(nx_2)\frac{1}{\pi\sqrt{\rho h\overline{D}}}\sin(jx_2) \pm b_w \sin(nx_2)\frac{\omega}{\pi}\sin(jx_2)\right]dx_2 = \\ &= \left[\frac{b_M}{\pi\sqrt{\rho h\overline{D}}} \pm \frac{b_w\omega}{\pi}\right]\cdot \int_0^\pi \left[\sin(nx_2)\cdot\sin(jx_2)\right]dx_2\end{aligned} \tag{73}$$

From a comparison of the formulas (71) and (73) it is clear that under such boundary conditions is excited only "n" - Single private form:

$$a_j^{\pm} = \delta_{nj}\left[\frac{b_M}{2\sqrt{\rho h\overline{D}}} \pm \frac{b_w\omega}{2}\right] \tag{74}$$

Thus, the solution of the no stationary problem for a half-strip of constant thickness has the form

$$\overline{W} = \left(a_n^+ \overline{W}_n^{\,+} + a_n^- \overline{W}_n^{\,-}\right) e^{-i(\alpha x_l - \omega t)}$$
$$\overline{M} = \left(a_n^- \overline{M}_n^{\,+} + a_n^- \overline{M}_n^{\,-}\right) e^{-i(\alpha x_l - \omega t)} \tag{75}$$

where

$\overline{W}_n^{\pm} = \pm \sin n x_2$, $\overline{M}_n^{\pm} = \pm \omega \sqrt{\rho h \overline{D}} \sin n x_2$, a $a_n^{\pm}$ - determined from the ratio (74).

Now suppose that the steady influence on the border of semi-infinite strip $x_1 = 0$ it has the form

$$w = f_w(x_2) e^{i\omega t}, \quad M = f_M(x_2) e^{i\omega t} \tag{76}$$

Let us expand the function f_w and f_M Fourier series of sinus in the interval $[0, \pi]$

$$f_w(x_2) = \sum_{k=1}^{\infty} B_w^k \sin(k x_2), \quad f_M(x_2) = \sum_{k=1}^{\infty} b_M^k \sin(k x_2) \tag{77}$$

Using the results of the previous problem, we find that the solution can be represented as a Fourier series:

$$W = \sum_{k=1}^{\infty} \left(a_k^+ W_k^{\,+} + a_k^- W_k^{\,-}\right) e^{-i(\alpha x_l - \omega t)}$$

$$M = \sum_{k=1}^{\infty} \left(a_k^+ M_k^+ + a_k^- M_k^-\right) e^{-i(\alpha x_l - \omega t)} \tag{78}$$

where $a_k^{\pm} = \dfrac{b_M^k}{2\sqrt{\rho h \overline{D}}} \pm \dfrac{b_w^k \omega}{2}$, $a \overline{W}_k^{\,\pm}$, $\overline{M}_k^{\,\pm}$ determined from the ratio (60).

V. Numerical Results and Analysis

The numerical solution of spectral problems carried out by computer software system based on the method of orthogonal shooting S.K. Godunov [4] combined with the method of Muller. The results obtained in testing with the same software package analytically up to 4-5 mark frequency range from 0.01 to 100. Hereinafter, the entire analysis is conducted in dimensionless variables, in which the density of the material ρ, half the width of the waveguide l_2 and E modulus taken to be unity, and the parameters of relaxation kernel $A = 0{,}048$; $\beta = 0{,}05$; $\alpha = 0{,}1$.

Ref

4. SK Godunov, On the numerical solution of boundary value problems for systems of linear ordinary differential equations. - Russian Mathematical Surveys, 1061, T.16, vol.3, 171-174 p.

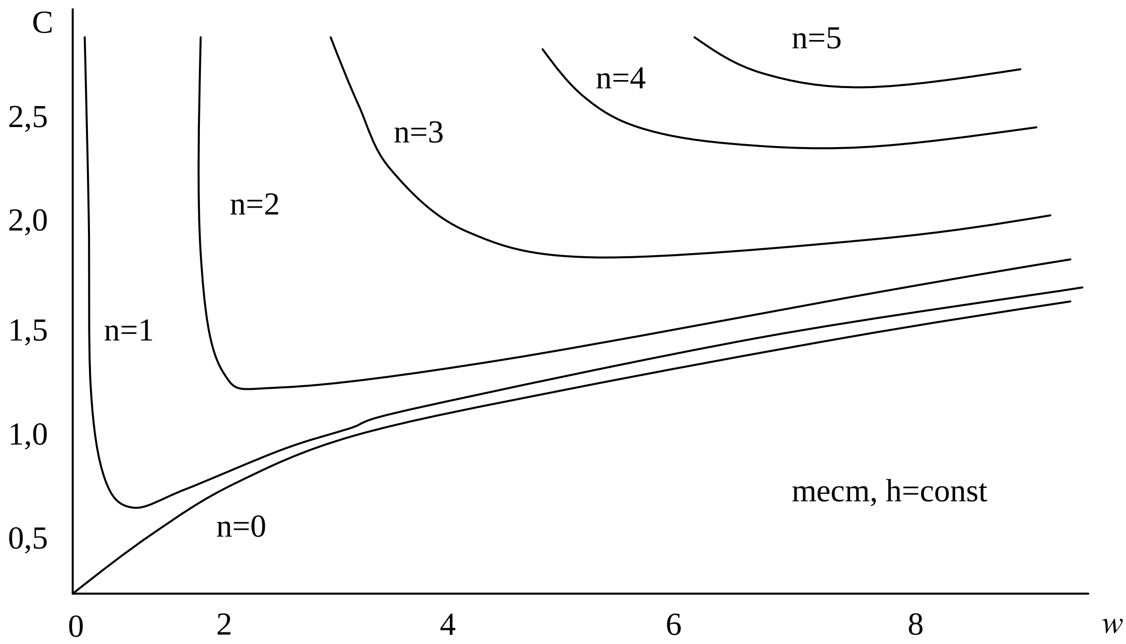

Figure 2 : The dependence of the phase velocity on frequency

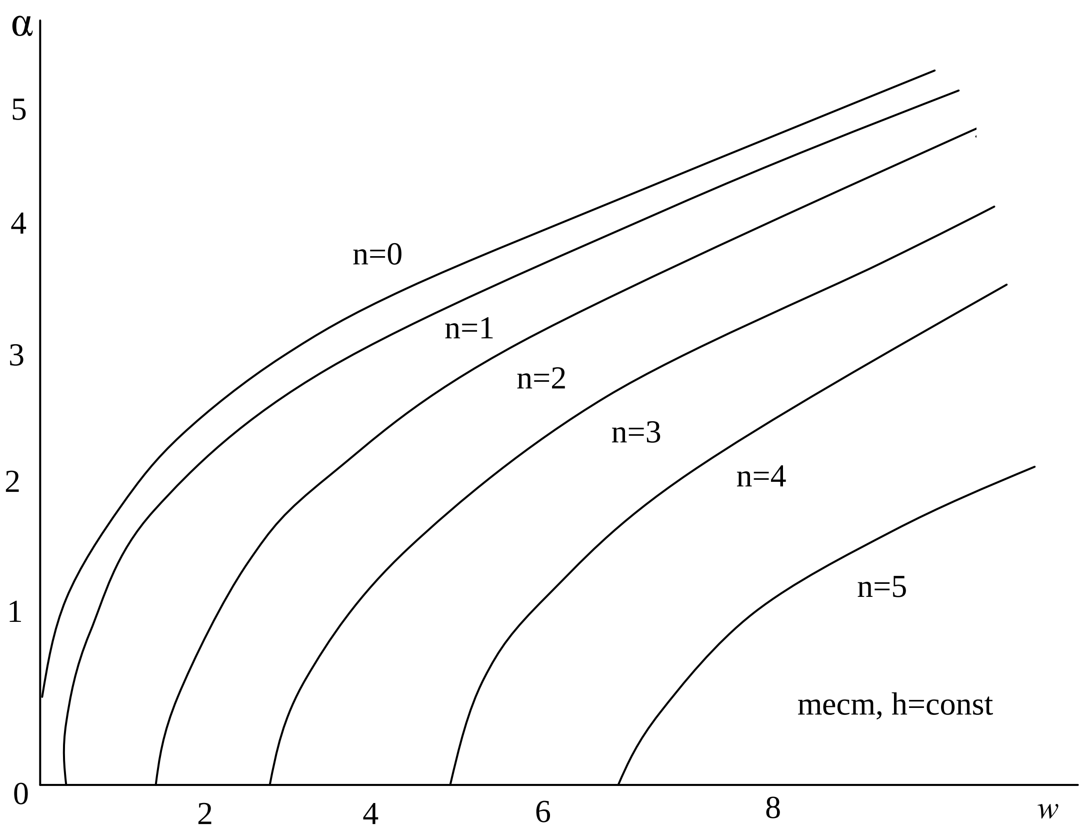

Figure 3 : The dependence of the frequency of the wave

The calculation results are obtained when A = 0. Figure 2 shows the spectral curves of the lower modes of oscillation of constant thickness plate, the corresponding n=0, 1, 2, 3, 4, 5 for Poisson's ratio υ=0,25. Analysis of the data shows that the range of applicability of the theory of Kirchhoff-Love to a plate of constant thickness is limited by the low frequency range. For example, for the first mode (h = 0), the range of application of the theory 0≤ω≤3 because of the unlimited growth of the phase propagation velocity with increasing frequency, for high frequencies $C_f C_s \sim\sqrt{\omega}$.

At high frequencies, where the wavelength is comparable or less than the fashion of strip thickness, there is, as is well known, localized in the faces of the Rayleigh wave band at a speed slower speed C_s, however, as is obvious, this formulation of the problem, in principle, does not allow to obtain this result. However, it should be noted that in the application of the theory of Kirchhoff - Love platinum constant thickness is obtained the correct conclusion about the growth of the number of propagating modes with increasing frequency that is well seen from the spectral curves of Figure 2 and Figure 3, which shows the dependence of the wave number α the frequency for the same modes of waves.

Figure 4 shows the obtained numerical form for the above modes of oscillations coincided with the same accuracy (4-5 decimal places) in the division of bandwidth by 90 equal segments.

Figure 5 illustrates the solution of the stationary problem: the amplitude of the excited oscillation modes linearly depends on the frequency ω.

We proceed to the propagation of flexural waves in a symmetric band Kirchhoff - Love of variable thickness. Let us first consider a waveguide with a linear thickness change, presented in Figure 6 and 7 which are free edges. Figure 8 shows the dispersion curves for the first mode, depending on the verge of tilt angle $\varphi/2$. Curve I corresponds to a strip of constant thickness $h_0=h_1$. Curve 2 corresponds to a waveguide with an angle of inclination of faces $\varphi/2 = \pi/4$ or $tg\ \varphi/2=1$ and curve 3 corresponds to a waveguide $tg\ \varphi/2=0{,}2$. The figure shows that, unlike the bands in the case of constant cross-section of the waveguide with a small tapered angle at the base of the wedge α (Curve 3) there exists a finite limit of the phase velocity of the fashion spread, and

$$\lim_{\omega\to\infty} \tilde{C}_f = 2C_s\ tg\frac{\varphi}{2}$$

where C_s -. The speed of shear waves, which coincides with the results of other studies [5,6,14,15] Thus, it is shown that -Lava Kirchhoff theory provides a wave propagating in the waveguide is tapered with a sufficiently small angle at the base of the wedge-speed, lower shear wave velocity and different from the Rayleigh wave velocity. Moreover, these waves from a frequency distributed without dispersion. This wave is called "wave Troyanovskiy - Safarov" [10, 12,13].

Figure 9 shows the waveform of the same frequency for $\omega=10$, from which it follows that the strip of constant thickness behaves like a rod while at the wedge-shaped strip there is a significant localization of waves in the area of acute viburnum, and the more, the smaller the angle φ. The above fact explains the Kirchhoff theory -Lava applicability for studying wave propagation in waveguides is tapered, as the frequency increases with decreasing length of one side of the wave modes, with different wave localizes with the sharp edge of the wedge so that the ratio of the wavelength and the effective thickness of the material is in the field of applicability of the theory. This statement is true, the smaller the angle at the base of the wedge.

It should also be noted that the numerical analysis of the dispersion equation (33) does not allow to show the presence of strictly limit the speed of wave propagation

Ref

5. V. T. Grinchenko, VV Myaleshka, harmonic oscillations and waves in elastic bodies. - Kiev: 1981, 284p.

modes, since the computer cannot handle infinitely large quantities. We can only speak about the numerical stability result in a large frequency range, which is confirmed by research. For example, when $tg\varphi/2=0{,}2$ value of the phase velocity of a measured without shear wave velocity at $\omega=3$ and $\omega=40$ It differs fifth sign that corresponds to the accuracy of calculations, resulting in test problem.

Notes

In the example $h_0 = 0{,}0001$, it certainly gives an increase of the phase velocity when the frequency increases further, since such a strong localization of the wave to the thin edge of the wedge, starts to affect the characteristic dimension - the thickness of the truncated wedge, and Kirchhoff hypothesis -Lava stops working. To solve the problem of acute wedge numerically is not possible, since the dispersion equation contains a term D^{-1}, and the thickness tends to zero flexural rigidity D behaves as a cube and the thickness goes to zero. This significantly increases the "rigidity" (ie the ratio between the small and large coefficient) system, increases dramatically the computing time and decreases the accuracy of the results. However, it is clear that you can trust the results obtained where the agreed parameters h_0 and α. We note also that the numerical experiment showed no significant dependence of the phase velocity of the first mode of the Poisson's ratio ν, and the fact that a family of dispersion curves with different apex angles of the wedge have a similarity property: the ratio of the phase velocity to the limit does not depend on the angle of the wedge φ. On the modes, starting from the second, the speed limit dependence on Poisson's ratio becomes noticeable - about 8.5% for the second mode when changing $0 \leq \nu \leq 0{,}5$. Generally, the limit speed increases with the stronger and the more the mode number.

Figure 10 shows the dispersion curves for the first modal wedge $tg\varphi/2=0{,}2$. The figure shows that the speed of the first mode (curve I) is equal to zero for $\omega=0$ and since the frequency $\omega=1$, virtually unchanged. The speed of the second mode (curve 2) is nonzero and finite for $\omega=0$ and stabilized at $\omega=3$. The rest of the modes (curve number matches the number of fashion) have a cut-off frequency, which can be easily determined from Figure 11 (the dependence of the wave number α of the frequency), and decreasing, stabilized (seen 3 and 4 modes) at top speed.

Figure 12 shows the evolution of the first waveform with the frequency ω for frequencies $\omega=0{,}5;\ 1;\ 5$ и 20. Pronounced localized form with increasing frequency. Figure 13-16 shows the own forms respectively for 2-4 modes of vibration for different frequencies: $\omega = 1,\ 2,\ 3$ and 4 (the number of grid points corresponds to the number form). And here there are localized forms in the area of thin wedge edge. Figure 16 gives an idea of the degree of localization of the forms at the frequency ω = 1, obviously, the lower the number of forms, the stronger it is localized at the edge of the wedge.

Figures 17-19 shows the spectral curves of the first three events in the case of the nonlinear dependence of the thickness of the strip from the coordinates x_2.

$$h(x_2) = h_0 + h{x_2}^p, \qquad 0\langle x_2 \leq 1,$$

where the parameter ρ It was assumed to be 1.5; 2; 2.5; 3 (curves 1, 2, 3, 4, respectively, curve "0" corresponds to p = 1 - linear relationship).

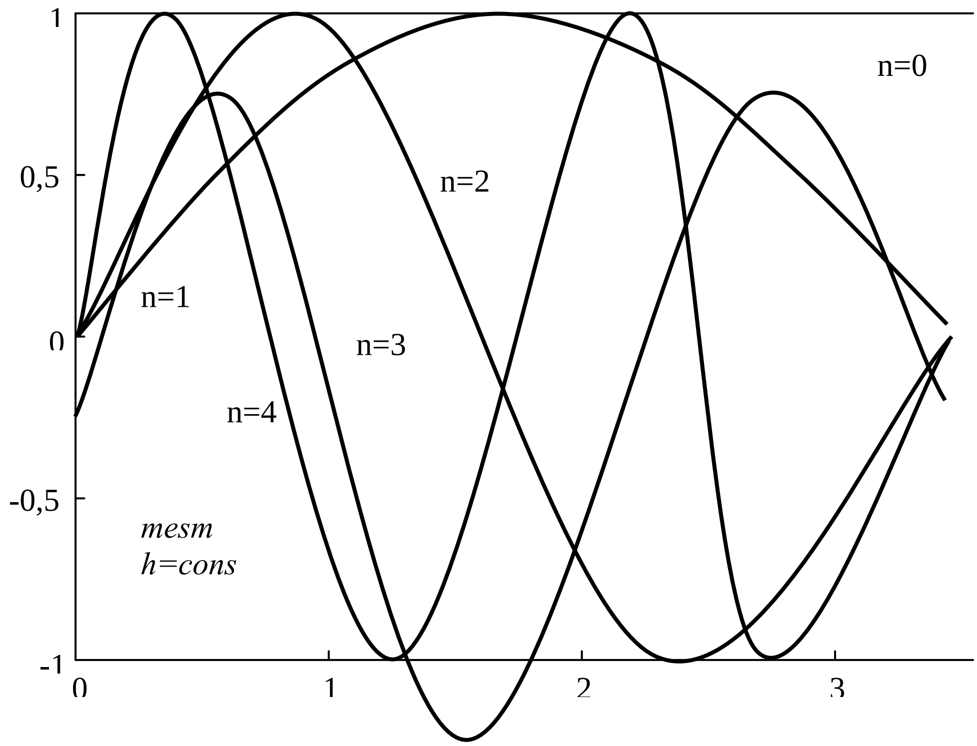

Figure 4 : Form for the higher oscillation modes

From the equation of "0" with the remaining curve shows that they are located on the horizontal high-frequency asymptote, monotonically to zero. The midrange is observed a characteristic peak which is shifted to lower frequencies with an increase in "p". In accordance with the charts of waveforms at ris.20-22 quicker and localization of motion near the edge of the waveguide.

Thus, it can be concluded that the phase velocity of the wave in the localized waveguide edge is defined as the frequency increases the rate of change of thickness in the vicinity of the sharp edge.

Figures 23-28 illustrate the solution of the stationary problem for a wedge-shaped waveguide with a linear change in the thickness of the coordinates x_2 depending on the location of the excitation zone, from which it is clear that the main contribution to the resulting solution brings a sharp edge excited waveguides. Analysis of figures 23-25 shows that, if aroused sharp edge of the wedge is raised mostly first oscillation mode, and ratio α_1 increases with increasing frequency.

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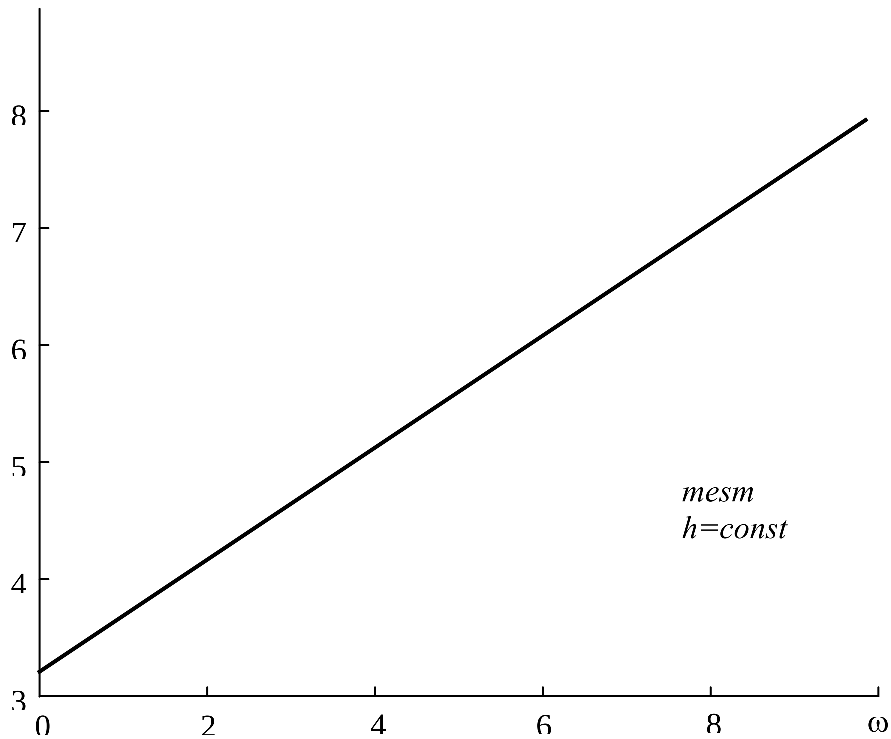

Figure 5 : The amplitude of the excited mode depending on the frequency

The amplitude of the remaining modes is not more than 5% from the first (ω = 10). Upon excitation of the central waveguide portion (fig.26 and fig.27) the amplitude of oscillation is 20-50 times lower than when excited sharp edge and decreases with increasing frequency. On Figure 28 shows the factors driving modes when the excitation zone does not capture any region, the center of the waveguide. The oscillation amplitude is also here oscillations 20-50 times less than in the first case. 26-28 of the drawings can be made and another conclusion that in this case the entire frequency range can be divided into zones, in which one of the modes propagates mainly. For example, in the case of Figure 25:

$0 \leq \omega \leq 2$ *I* fashion; $2 \leq \omega \leq 5$ *II* fashion; $5 \leq \omega \leq 10$ *III* fashion, t. i.

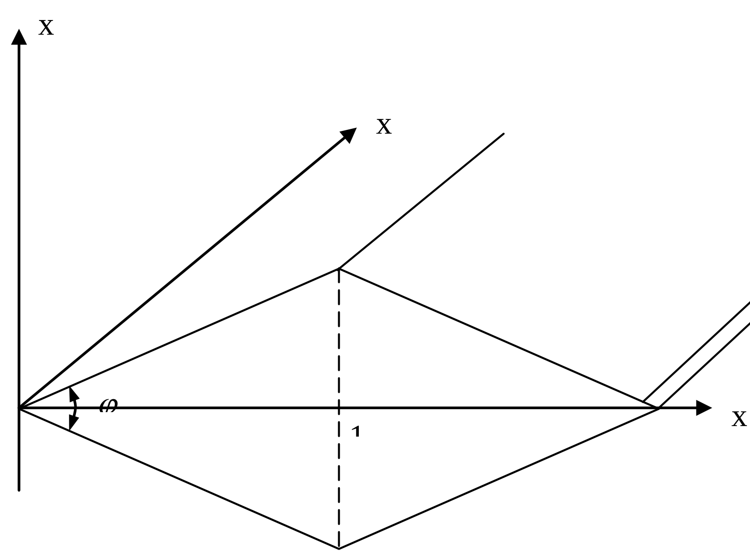

Figure 6 : The settlement scheme

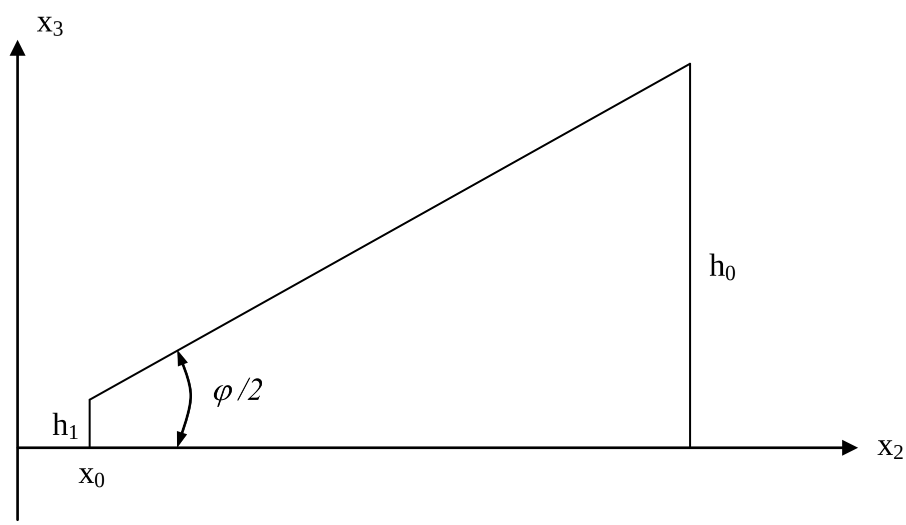

Figure 7 : The settlement scheme

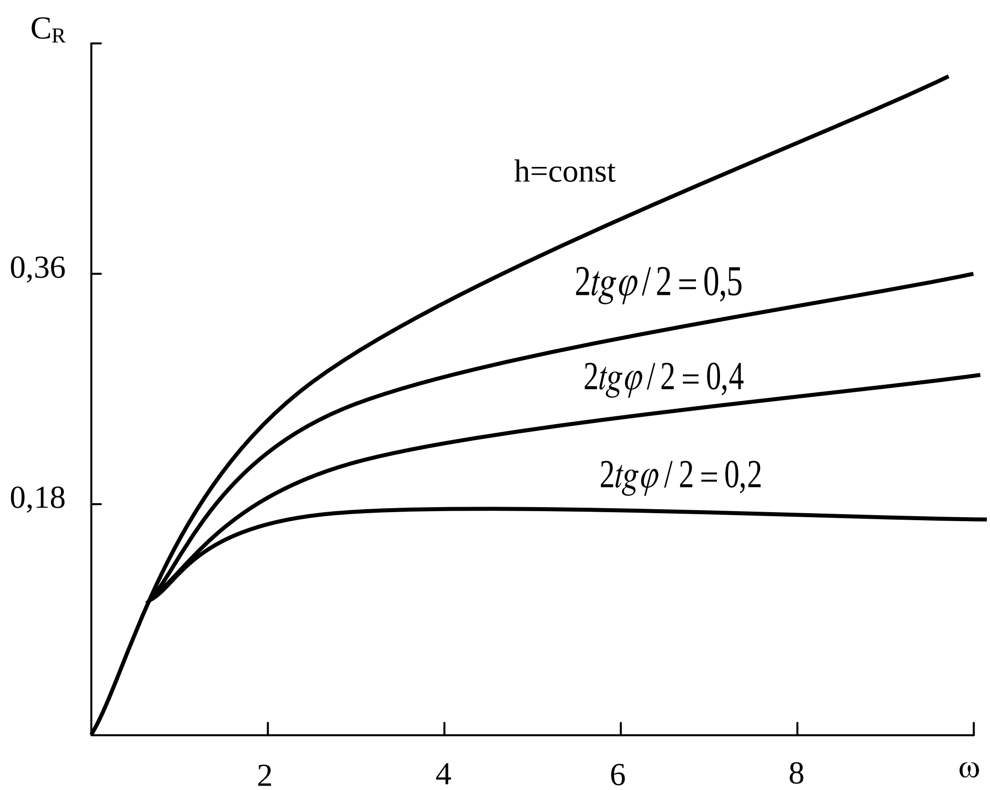

Figure 8 : The dependence of the real and imaginary parts of the phase velocity on frequency

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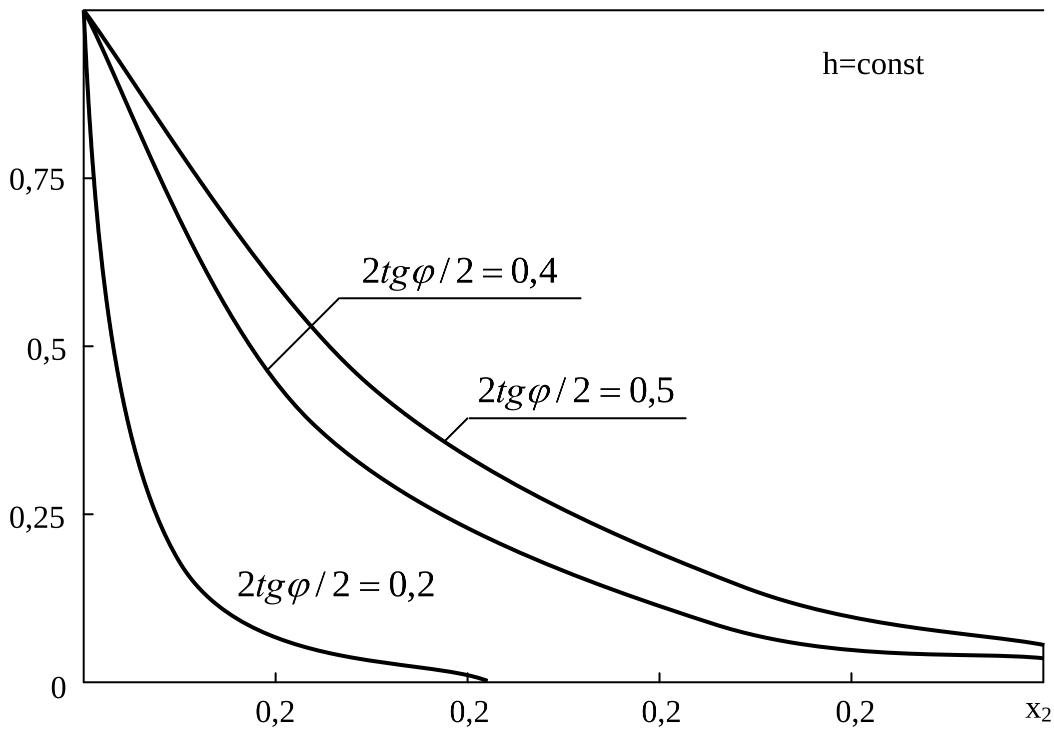

Figure 9 : The forms of the coordinate fluctuations X_2

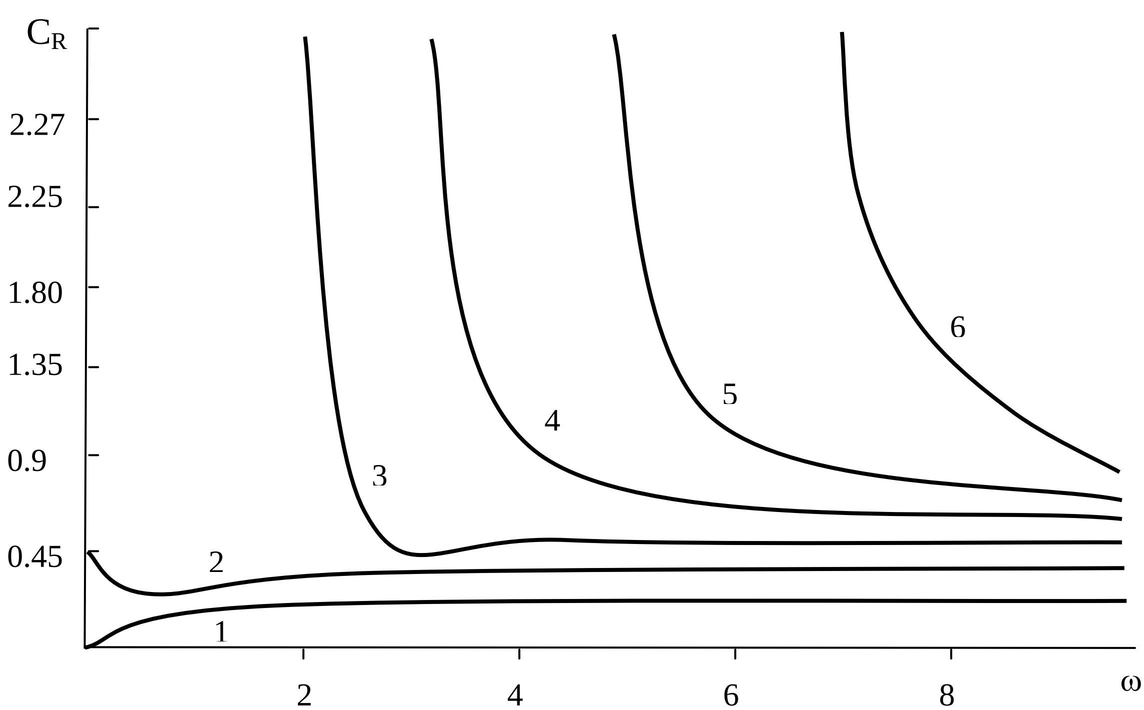

Figure 10 : Dependence of the real and imaginary parts of the phase velocity on frequency

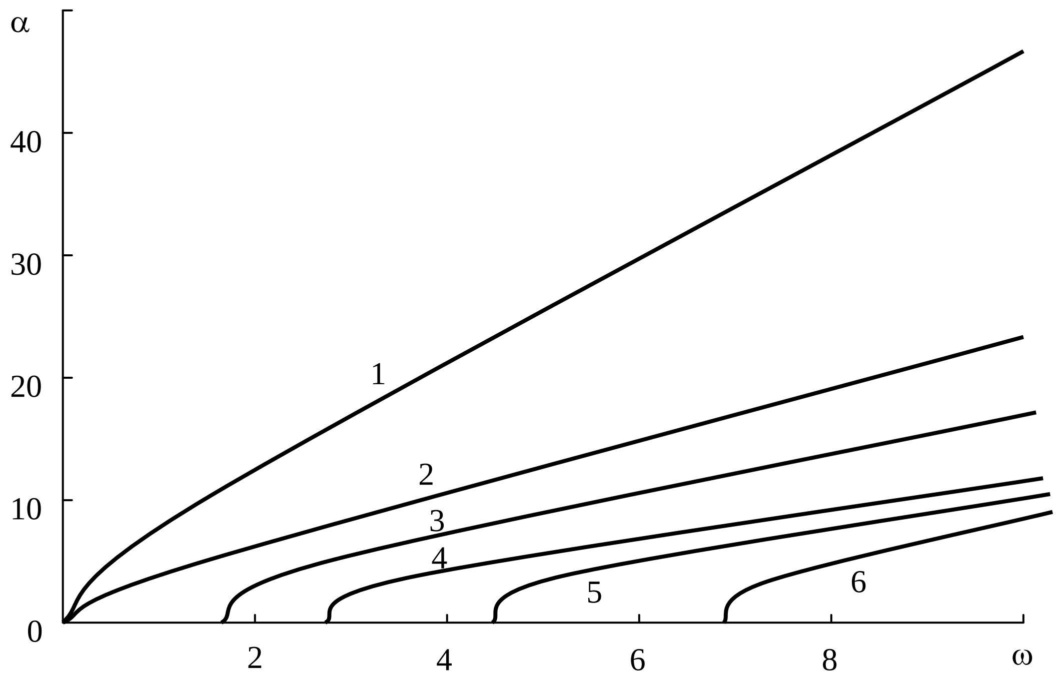

Figure 11 : Dependence of the frequency of the wave

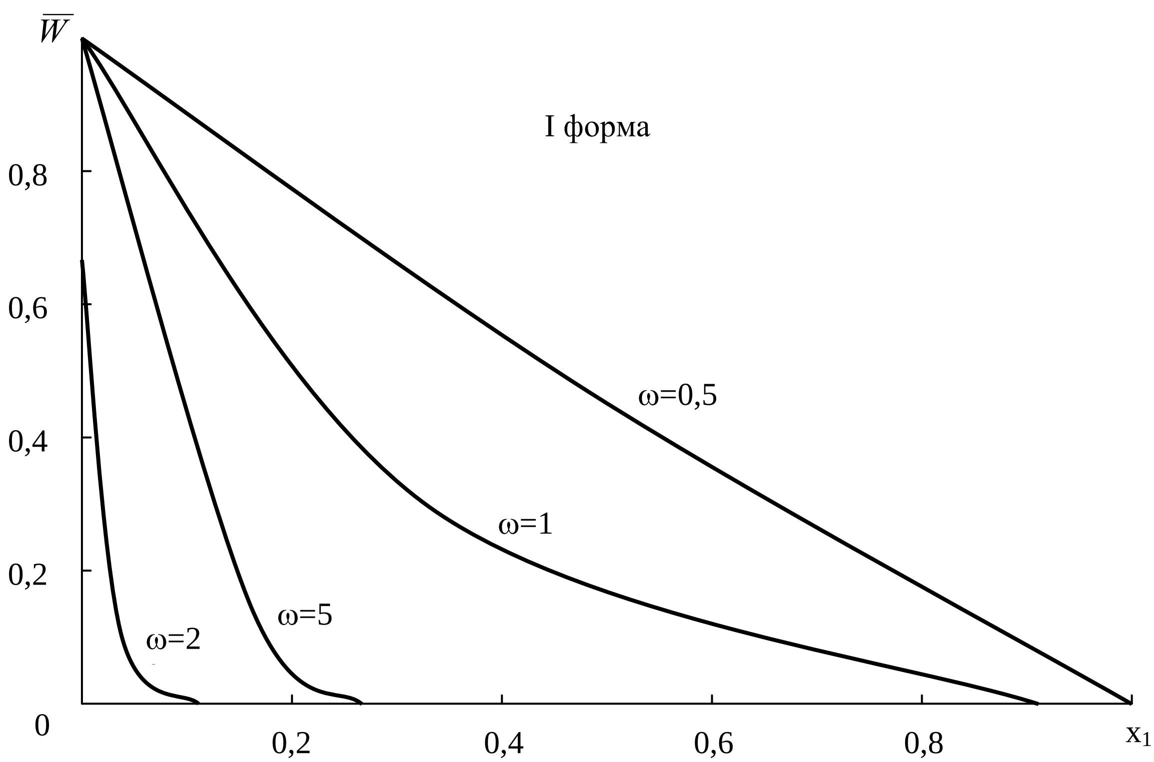

Figure 12 : Changing the shape of the coordinate fluctuations X_1

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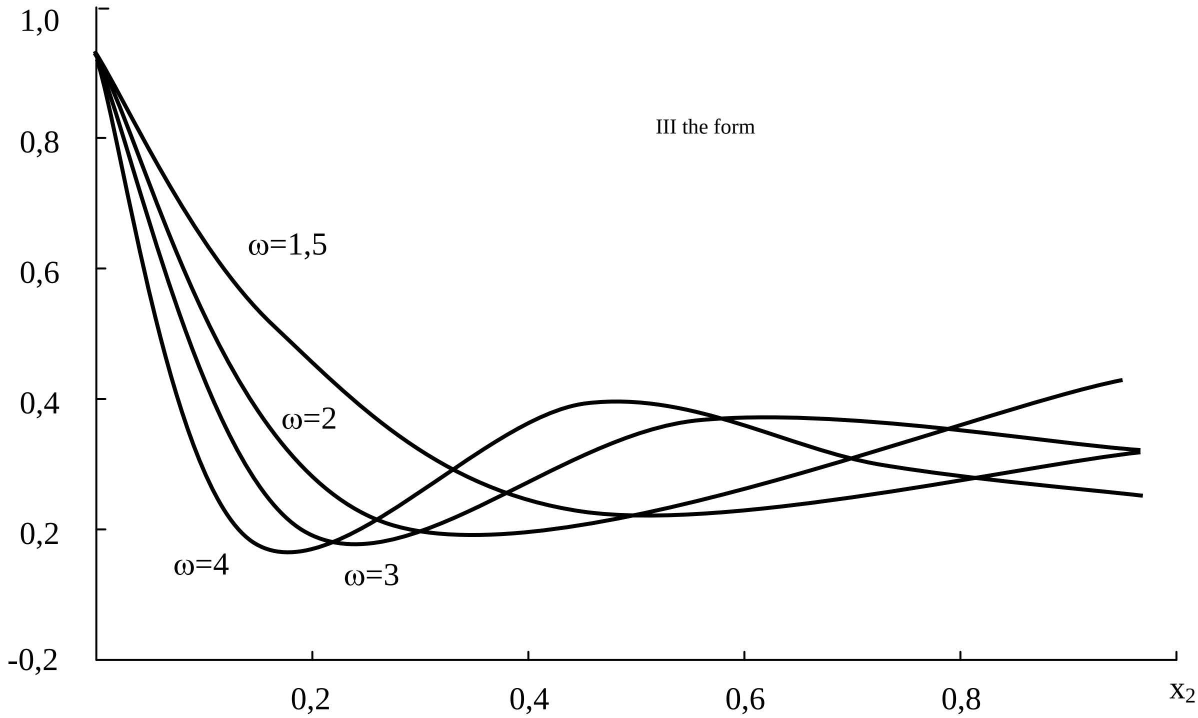

Figure 13 : Changing the shape of the coordinate fluctuations X_2

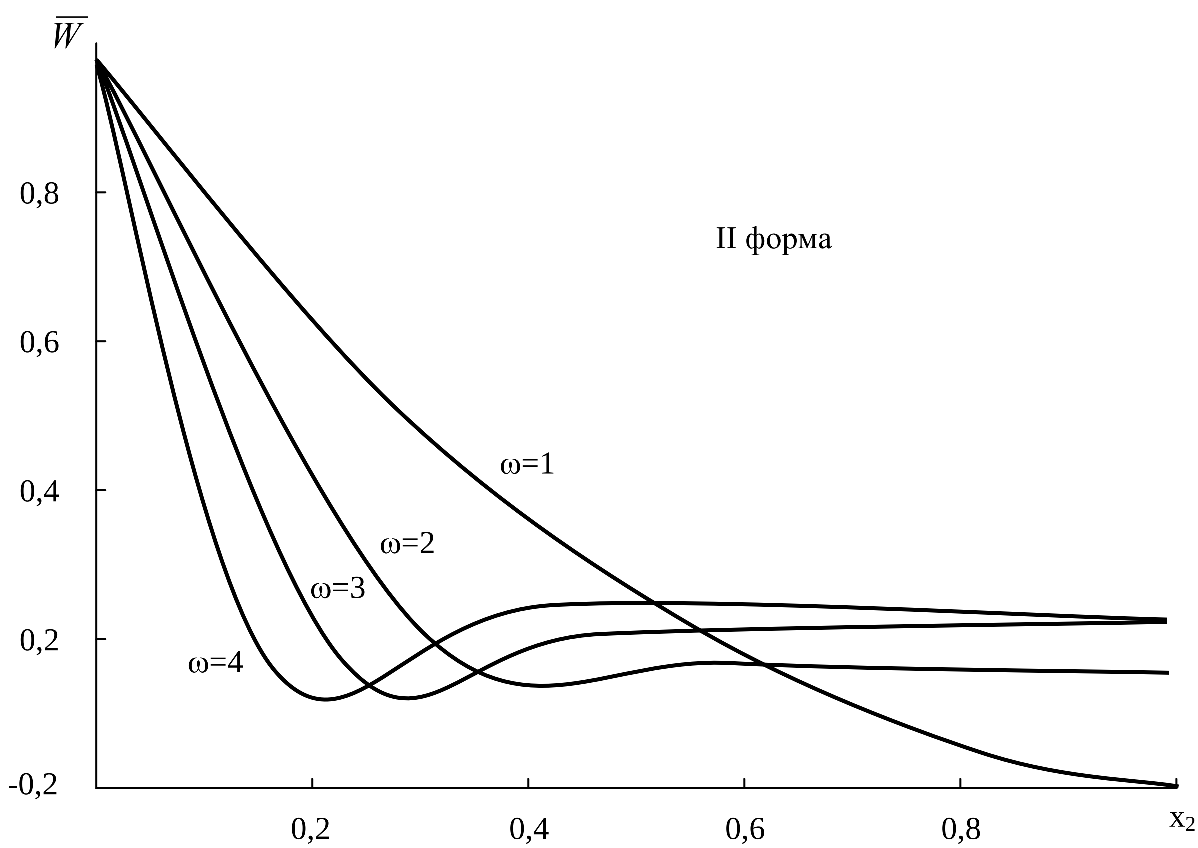

Figure 14 : Changing the shape of the coordinate fluctuations

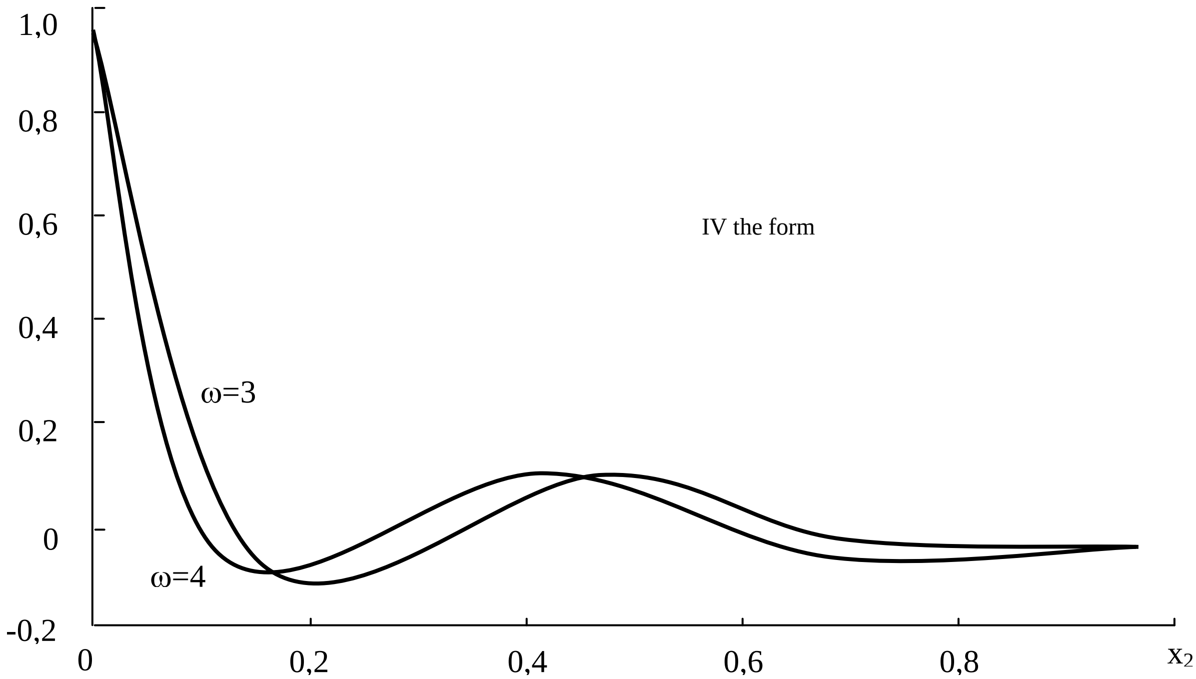

Figure 15 : Changing the shape of the coordinate fluctuations

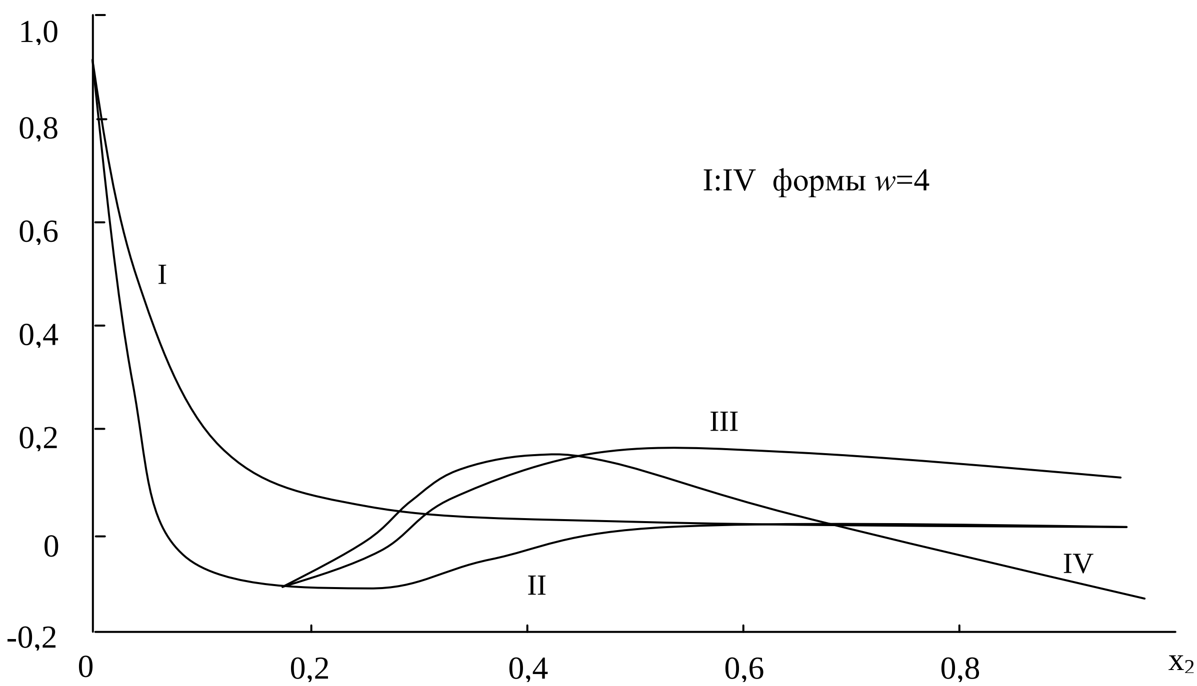

Figure 16 : Changing the shape of the coordinate fluctuations

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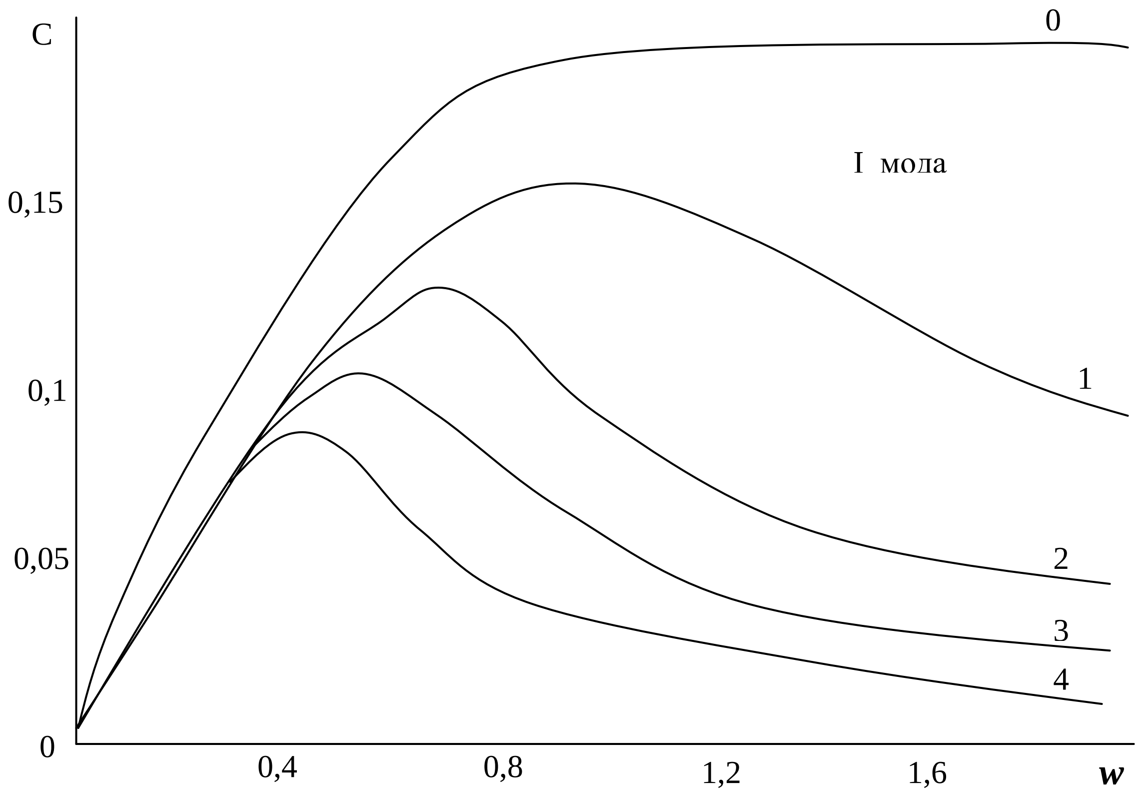

Figure 17 : Changing the phase velocity as a function of frequency

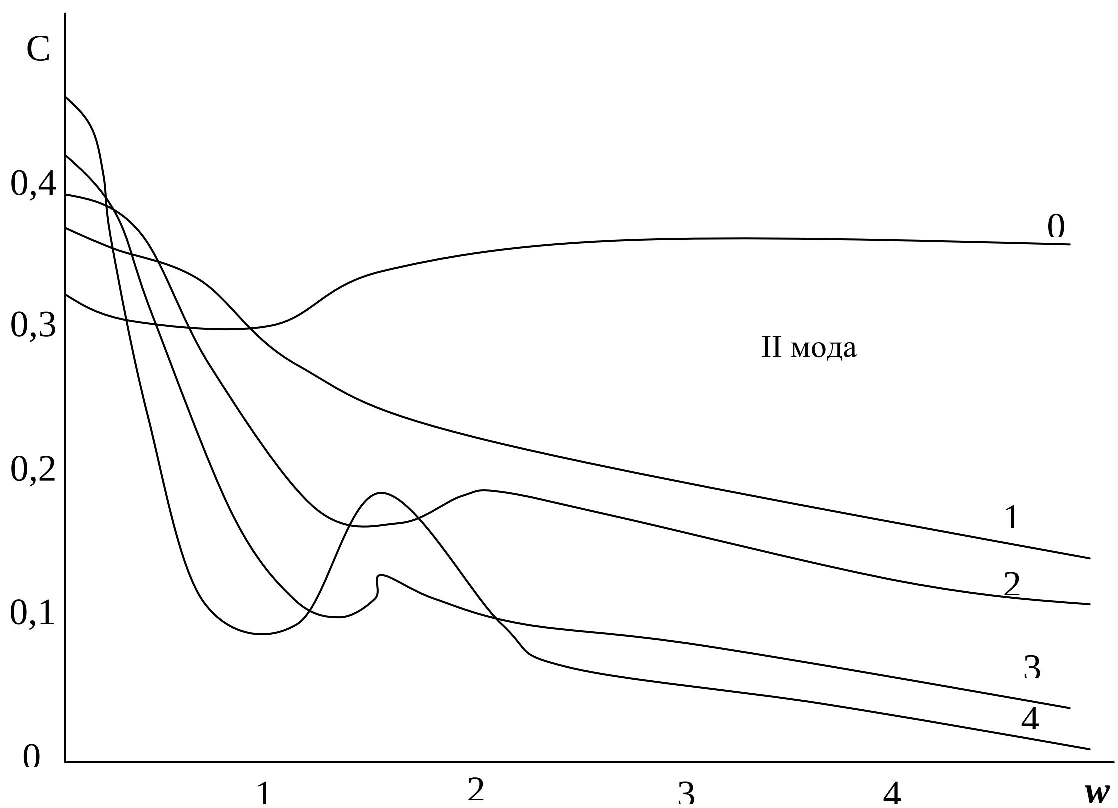

Figure 18 : Changing the phase velocity as a function of frequency

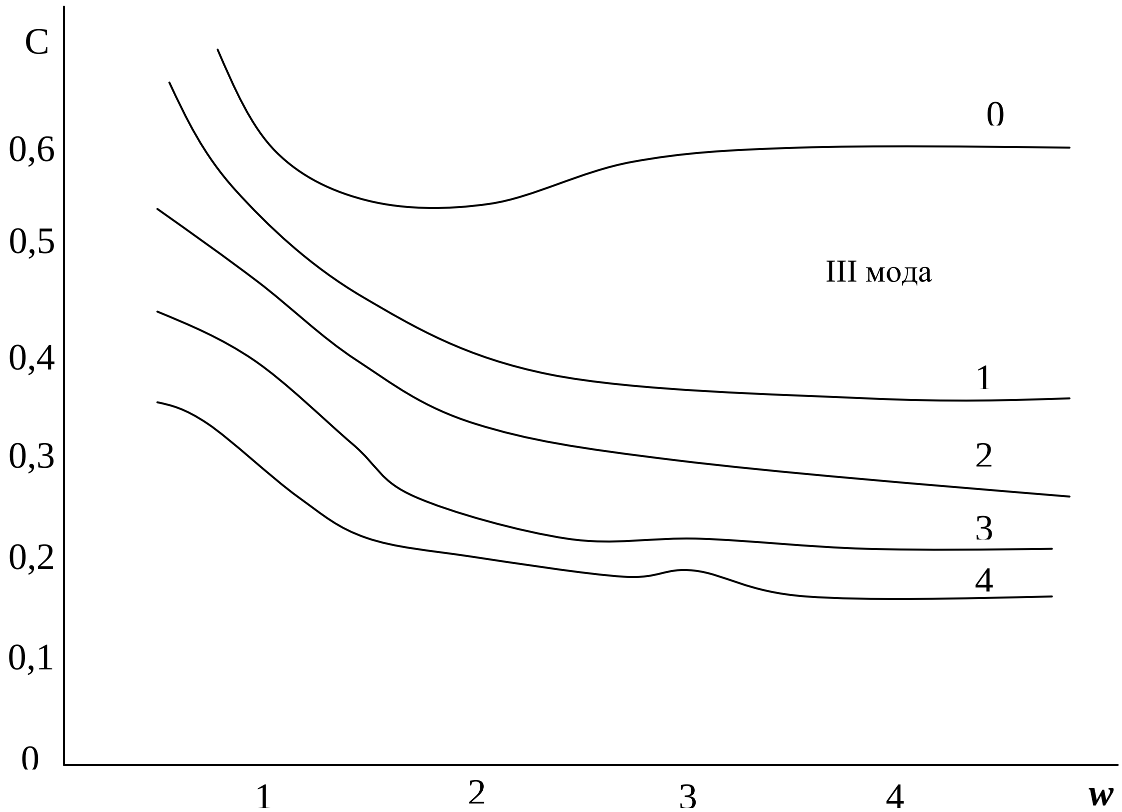

Figure 19 : Changing the phase velocity as a function of frequency

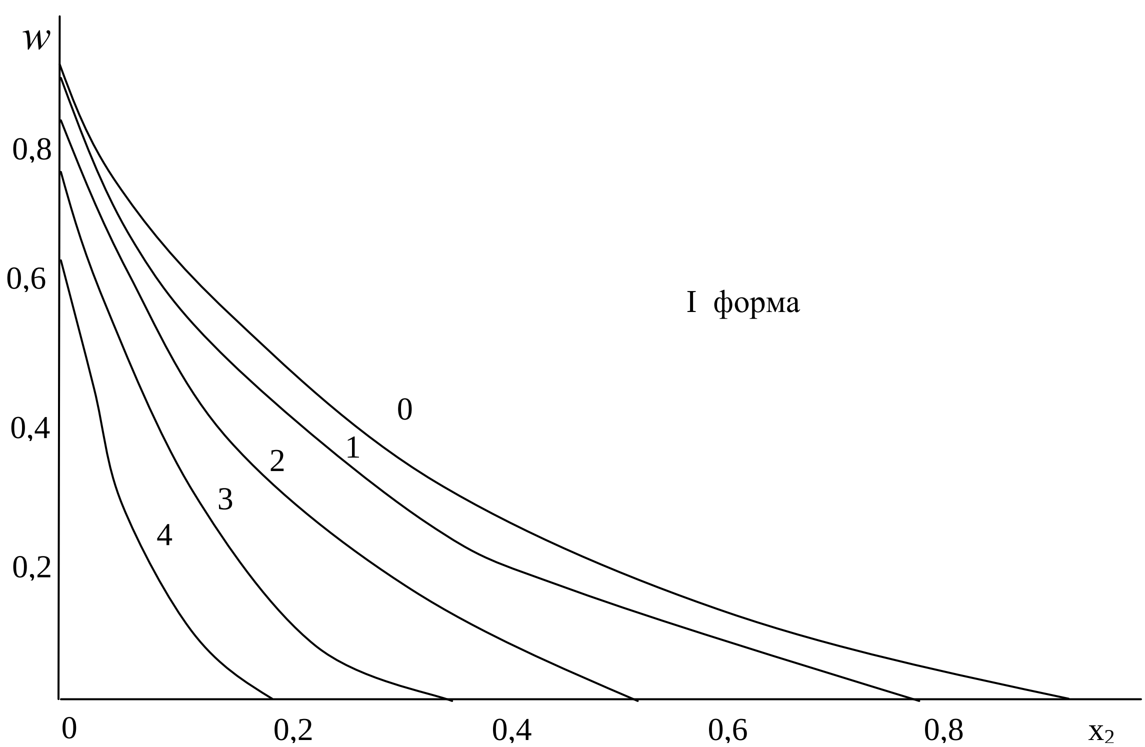

Figure 20 : Changing the shape of the coordinate fluctuations

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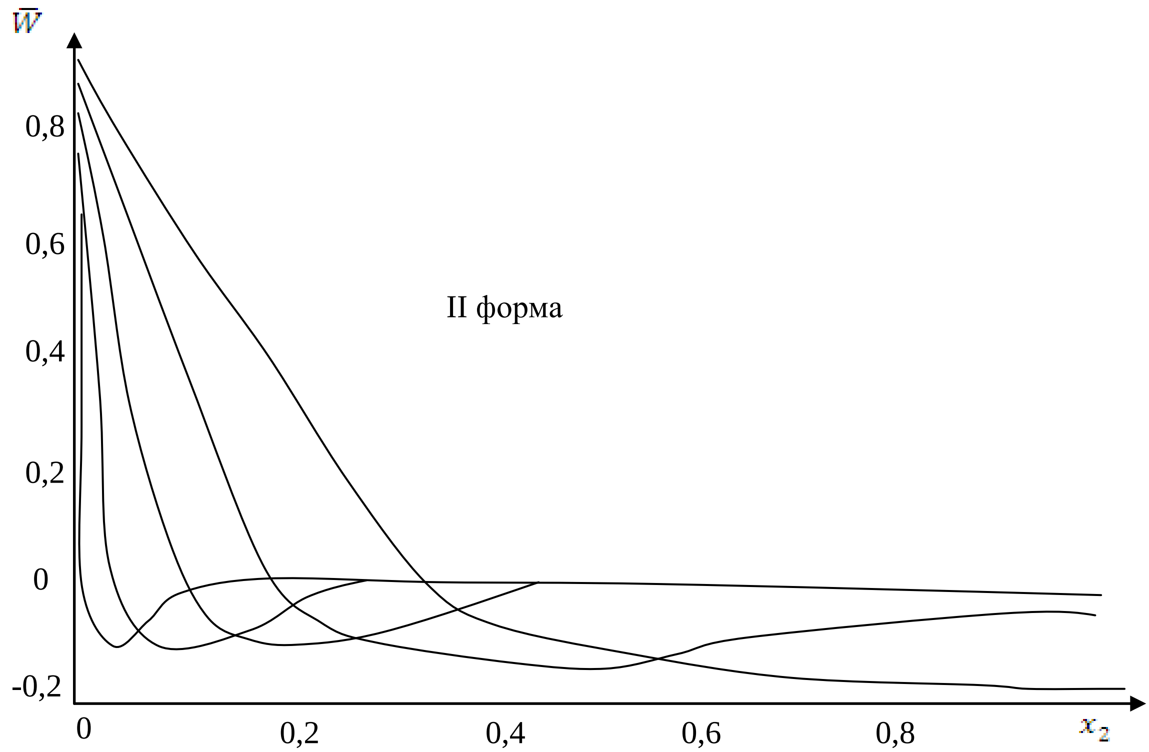

Figure 21 : Changing the shape of the coordinate fluctuations

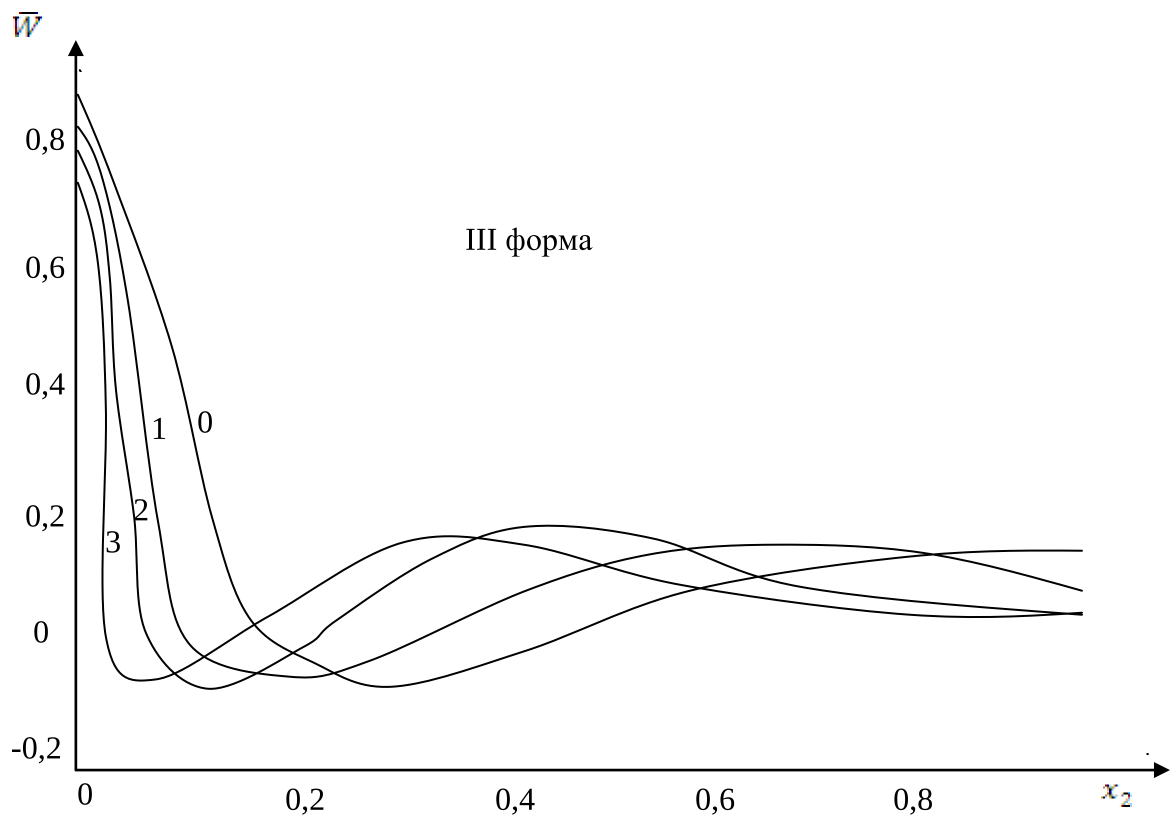

Figure 22 : Changing the shape of the coordinate fluctuations

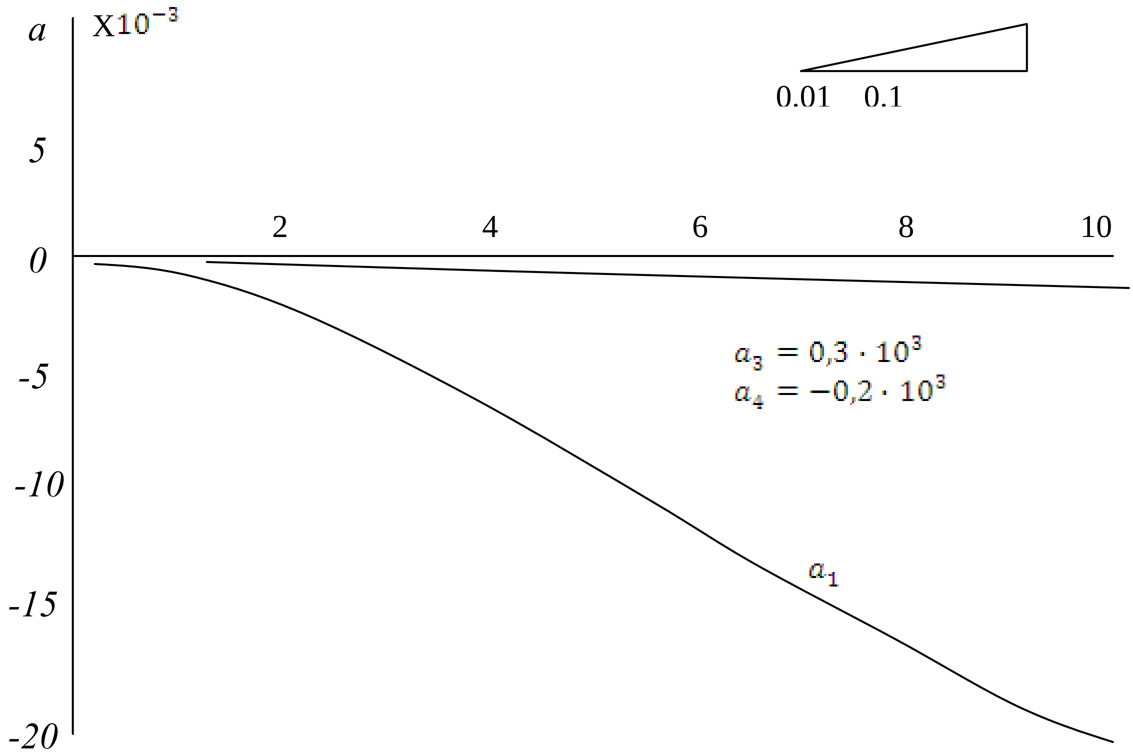

Figure 23 : The change factor a depending on the frequency

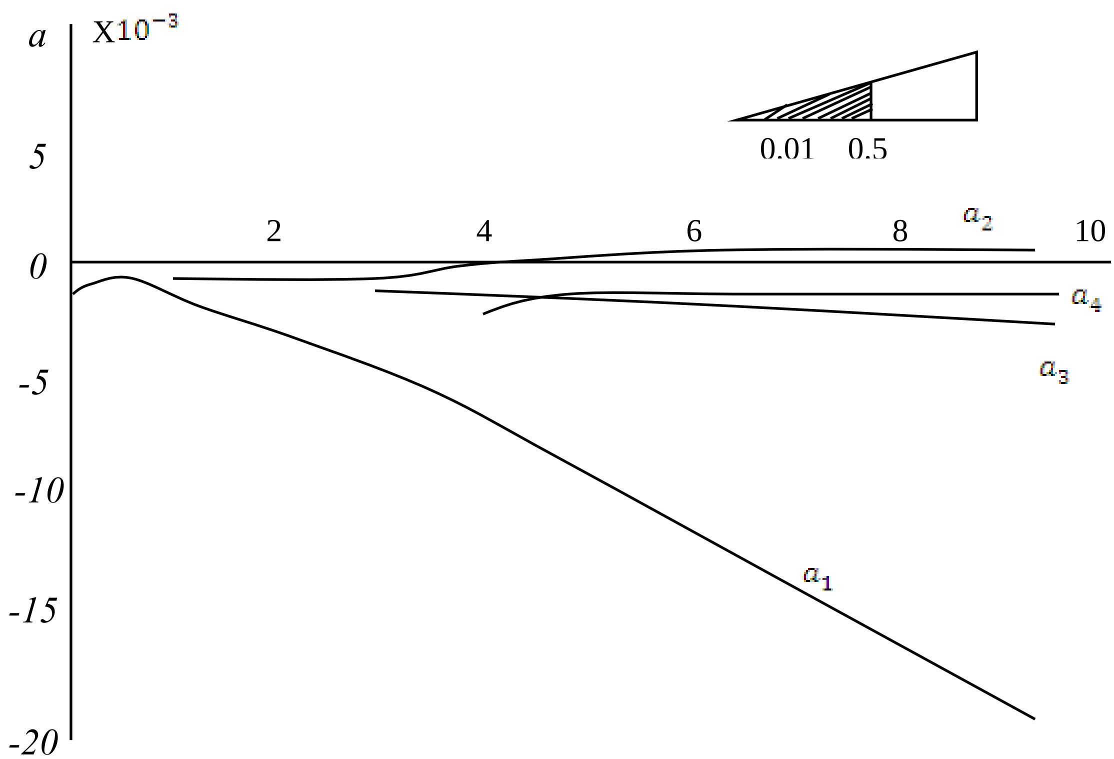

Figure 24 : The change factor a depending on the frequency

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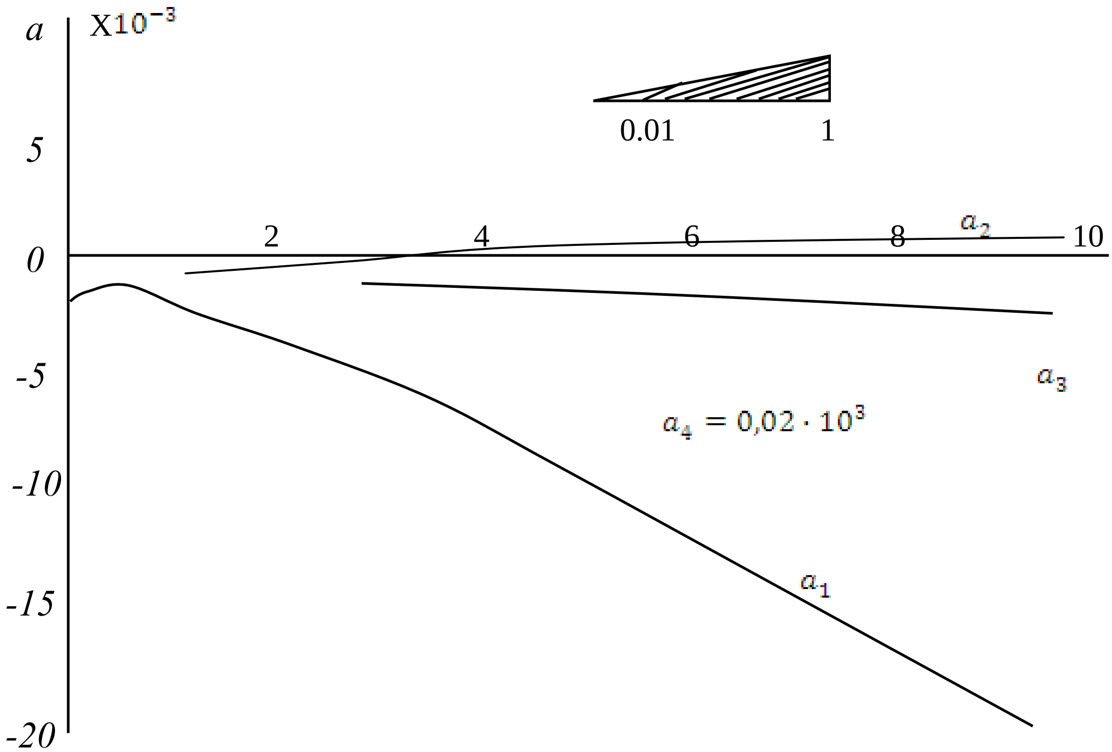

Figure 25 : The change factor a depending on the frequency

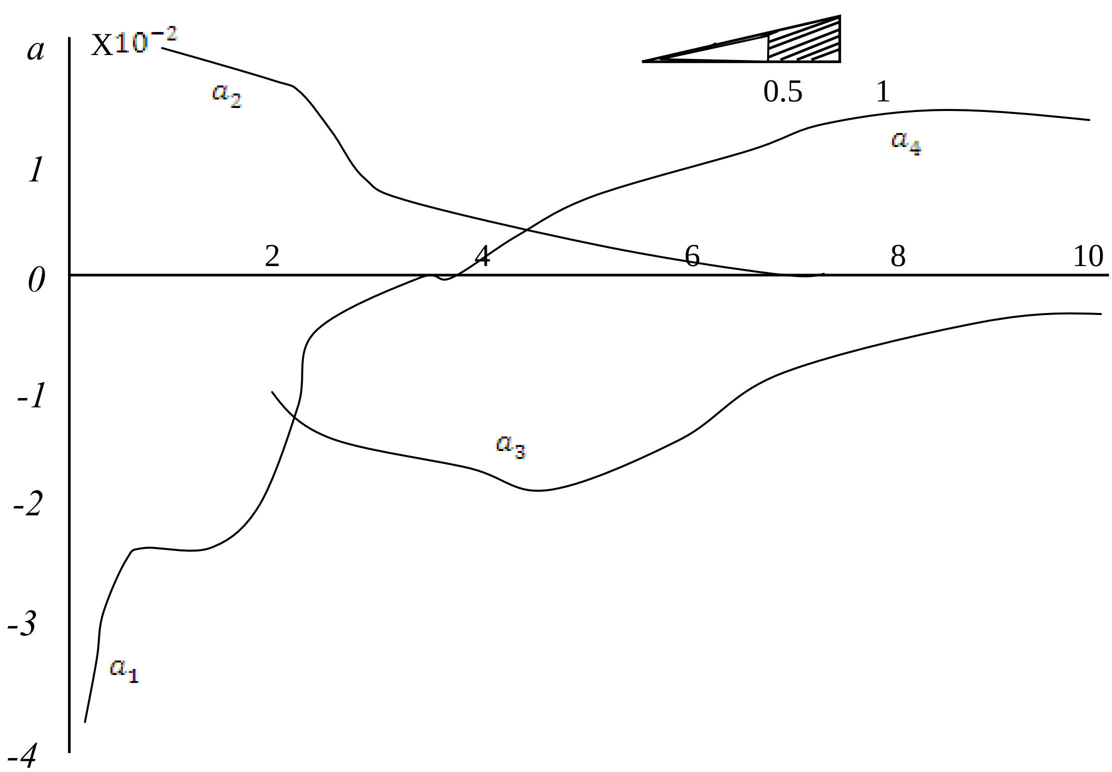

Figure 26 : The change factor a depending on the frequency

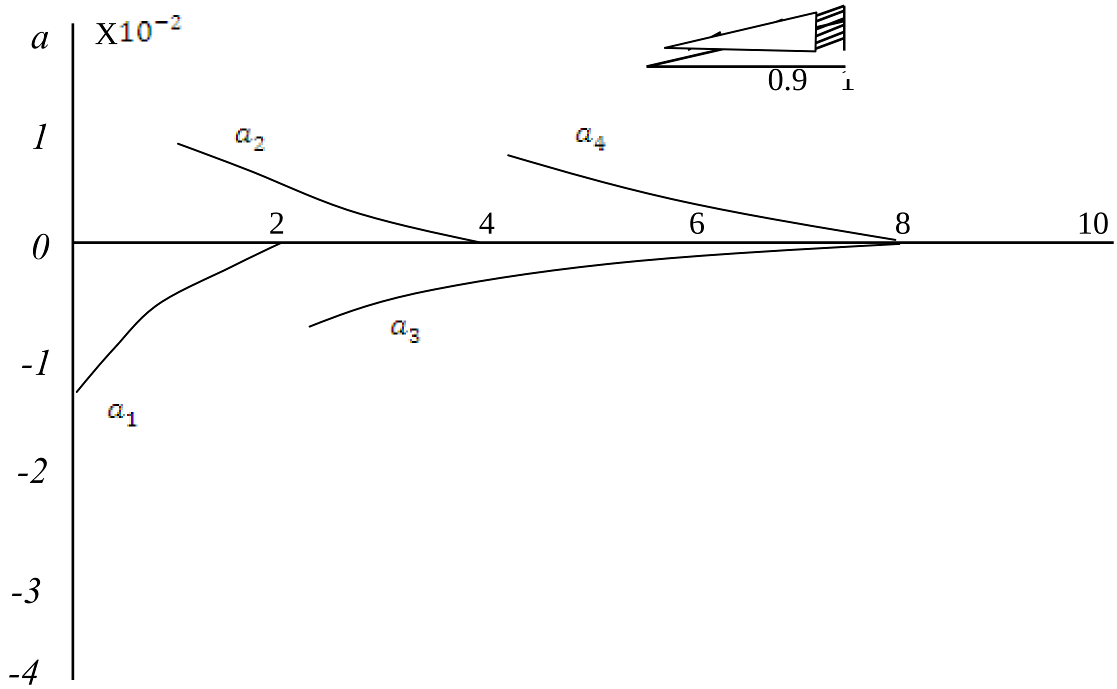

Figure 27 : The change factor *a* depending on the frequency

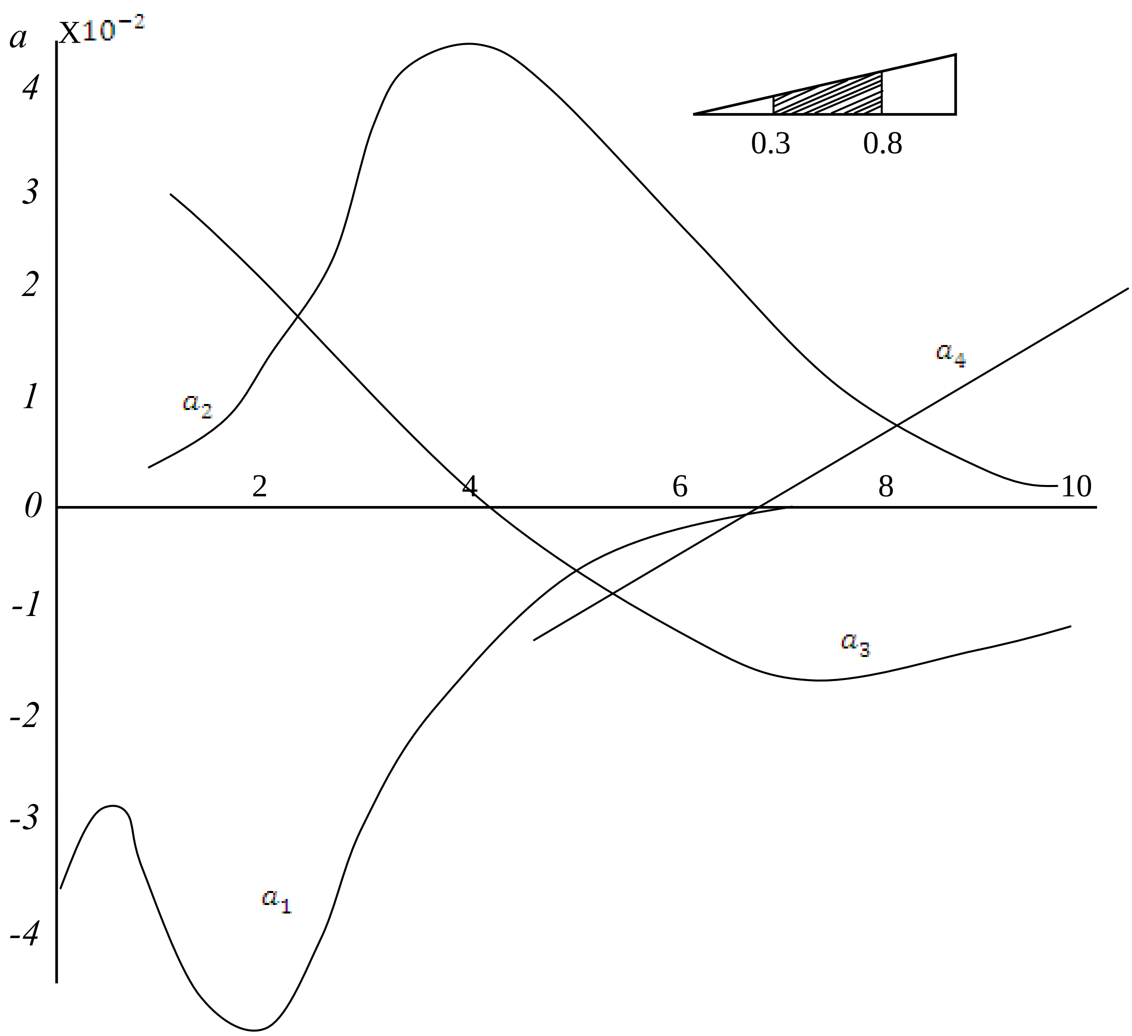

Figure 28 : The change factor *a* depending on the frequency

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On the basis of these results the following conclusions:

- On the basis of the variation equations of elasticity theory, the mathematical formulation of the problem of wave propagation in the extended plates of variable thickness. A system of differential equations with the appropriate boundary conditions.
- Showing that the square of the wave number for own endless bands of variable thickness in any combination of the action of the boundary conditions.
- Obtained spectral problem is not self-adjoin. Built conjugate problem for her. Coupling system consists of ordinary differential equations with the appropriate boundary conditions. With the help of the Lagrange formula obtained conditions biorthogonality forms. The problem is solved numerically by the method of orthogonal shooting S.K. Godunov in conjunction with the method of Muller.
- Analysis of the data shows that the region with the imaginary theory of Kirchhoff-Love to the plate of constant thickness is limited by the low frequency range. At high frequencies, when wavelength comparable to fashion or less than the thickness of the plate theory Kirchhoff -Love does not yield reliable results.
- For the phase velocity of propagation modes in the band of variable thickness, there is final repartition unlike the constant cross-section strip.

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Two Curious Summation Formulae in the Monograph of Salahuddin Et Al

By Salahuddin & R. K. Khola
Mewar University, India

Abstract- In this paper we have developed two summation formulae which are unique with the help of contiguous relation.

Keywords: *contiguous relation, summation formulae, prudnikov et al.*

GJSFR-F Classification : *33C05, 33C20, 33C60*

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Strictly as per the compliance and regulations of :

Ref
2. Prudnikov, A.P., Brychkov, Yu. A. and Marichev, O.I.; Integral and Series Vol 3: More Special Functions, Nauka, Moscow, 2003.

Two Curious Summation Formulae in the Monograph of Salahuddin Et Al

Salahuddin [α] & R. K. Khola [σ]

Abstract- In this paper we have developed two summation formulae which are unique with the help of contiguous relation.

Keywords: *contiguous relation, summation formulae, prudnikov et al.*

I. Introduction and Results Required

Special functions are very useful in Mathematical Analysis, Applied Mathematics, Physical sciences and so many other branches of science and engineering. The uses of summation formulae are spectacular. So many scientists are involved to develop summation formulae in the field of Hypergeometric function. Contiguous relations are also useful with summation formulae.

Prudnikov et al[2,p.414] developed the following seven summation formulae

$$ {}_2F_1\left[\begin{array}{cc} a, & -a \;; \\ c & ; \end{array} \frac{1}{2}\right] = \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c}}\left[\frac{1}{\Gamma(\frac{c+a+1}{2})\,\Gamma(\frac{c-a}{2})} + \frac{1}{\Gamma(\frac{c+a}{2})\,\Gamma(\frac{c-a+1}{2})}\right] \tag{1} $$

$$ {}_2F_1\left[\begin{array}{cc} a, & 1-a \;; \\ c & ; \end{array} \frac{1}{2}\right] = \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-1}}\left[\frac{1}{\Gamma(\frac{c+a}{2})\,\Gamma(\frac{c-a+1}{2})}\right] \tag{2} $$

$$ {}_2F_1\left[\begin{array}{cc} a, & 2-a \;; \\ c & ; \end{array} \frac{1}{2}\right] = \frac{\sqrt{\pi}\,\Gamma(c)}{(a-1)\;2^{c-2}}\left[\frac{1}{\Gamma(\frac{c+a-2}{2})\,\Gamma(\frac{c-a+1}{2})} - \frac{1}{\Gamma(\frac{c+a-1}{2})\,\Gamma(\frac{c-a}{2})}\right] \tag{3} $$

$$ {}_2F_1\left[\begin{array}{cc} a, & 3-a \;; \\ c & ; \end{array} \frac{1}{2}\right] = \frac{\sqrt{\pi}\,\Gamma(c)}{(a-1)(a-2)\;2^{c-3}}\left[\frac{(c-2)}{\Gamma(\frac{c+a-2}{2})\,\Gamma(\frac{c-a+1}{2})} - \frac{2}{\Gamma(\frac{c+a-3}{2})\,\Gamma(\frac{c-a}{2})}\right] \tag{4} $$

$$ {}_2F_1\left[\begin{array}{cc} a, & 4-a \;; \\ c & ; \end{array} \frac{1}{2}\right] = \frac{\sqrt{\pi}\,\Gamma(c)}{(1-a)(2-a)(3-a)\;2^{c-4}}\left[\frac{(a-2c+3)}{\Gamma(\frac{c+a-4}{2})\,\Gamma(\frac{c-a+1}{2})} + \frac{(a+2c-7)}{\Gamma(\frac{c+a-3}{2})\,\Gamma(\frac{c-a}{2})}\right] \tag{5} $$

$$ {}_2F_1\left[\begin{array}{cc} a, & 5-a \;; \\ c & ; \end{array} \frac{1}{2}\right] = \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-5}\{\prod_{\gamma=1}^{4}(\gamma-a)\}}\left[\frac{\{2(c-2)(c-4)-(a-1)(a-4)\}}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-4}{2})} + \frac{(12-4c)}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-5}{2})}\right] \tag{6} $$

$$ {}_2F_1\left[\begin{array}{cc} a, & 6-a \;; \\ c & ; \end{array} \frac{1}{2}\right] = $$

$$ = \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-6}\{\prod_{\delta=1}^{5}(\delta-a)\}}\left[\frac{(4c^2+2ac-a^2-a-34c+62)}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-5}{2})} - \frac{(4c^2-2ac-a^2+13a-22c+20)}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-6}{2})}\right] \tag{7} $$

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The contiguous relation is defined as Abramowitz et al[1,p.558]

$$b\ {}_2F_1\left[\begin{matrix} a, & b+1 & ; & \\ c & & ; & z\end{matrix}\right] = (b-c+1)\ {}_2F_1\left[\begin{matrix} a, & b & ; & \\ c & & ; & z\end{matrix}\right] + (c-1)\ {}_2F_1\left[\begin{matrix} a, & b & ; & \\ c-1 & & ; & z\end{matrix}\right] \tag{8}$$

Salahuddin et al[3,4,5] derived the following fifteen summation formulae

$${}_2F_1\left[\begin{matrix} a, & 7-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] =$$

$$= \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-7}\{\prod_{\varsigma=1}^{6}(\varsigma-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-6}{2})}(-3a^2c+12a^2+21ac-84a+4c^3-48c^2+158c-120)+$$

$$+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-7}{2})}(2a^2-14a-8c^2+64c-108)\Big] \tag{9}$$

$${}_2F_1\left[\begin{matrix} a, & 8-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] =$$

$$= \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-8}\{\prod_{\xi=1}^{7}(\xi-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-7}{2})}(-a^3-4a^2c+30a^2+4ac^2-4ac-107a+8c^3-124c^2+576c-762)+$$

$$+\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-8}{2})}(-a^3+4a^2c-6a^2+4ac^2-68ac+181a-8c^3+92c^2-288c+210)\Big] \tag{10}$$

$${}_2F_1\left[\begin{matrix} a, & 9-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] =$$

$$= \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-9}\{\prod_{\varpi=1}^{8}(\varpi-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-8}{2})}(a^4-18a^3-8a^2c^2+80a^2c-85a^2+72ac^2-720ac+1494a+8c^4-$$

$$-160c^3+1056c^2-2560c+1680)+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-9}{2})}(8a^2c-40a^2-72ac+360a-16c^3+240c^2-1072c+1360)\Big] \tag{11}$$

$${}_2F_1\left[\begin{matrix} a, & 10-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] =$$

$$= \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-10}\{\prod_{\upsilon=1}^{9}(\upsilon-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-10}{2})}(-a^4-4a^3c+42a^3+12a^2c^2-72a^2c-107a^2+8ac^3-252ac^2+$$

$$+1772ac-3054a-16c^4+312c^3-2000c^2+4704c-3024)+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-9}{2})}(a^4-4a^3c+2a^3-12a^2c^2+192a^2c-$$

$$-553a^2+8ac^3-12ac^2-868ac+3406a+16c^4-392c^3+3320c^2-11224c+12264)\Big] \tag{12}$$

$${}_2F_1\left[\begin{matrix} a, & 11-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] =$$

$$= \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-11}\{\prod_{\varphi=1}^{10}(\varphi-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-10}{2})}(5a^4c-30a^4-110a^3c+660a^3-20a^2c^3+360a^2c^2-1305a^2c-$$

$$-810a^2+220ac^3-3960ac^2+21010ac-31020a+16c^5-480c^4+5240c^3-25200c^2+50544c-30240)+$$

$$+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-11}{2})}(-2a^4+44a^3+24a^2c^2-288a^2c+530a^2-264ac^2+3168ac-8492a-32c^4+768c^3-6352c^2+$$

$$+20928c-22320)\Big] \tag{13}$$

$${}_2F_1\left[\begin{matrix} a, & 12-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] =$$

Ref

4. Salahuddin, Khola, R. K.; New hypergeometric summation formulae arising from the summation formulae of Prudnikov, South Asian Journal of Mathematics, 4(2014),192-196.

Notes

$$= \frac{\sqrt{\pi}\ \Gamma(c)}{2^{c-12}\{\prod_{\chi=1}^{11}(\chi-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a-12}{2})}(a^5-6a^4c+9a^4-12a^3c^2+300a^3c-1103a^3+32a^2c^3-$$
$$-408a^2c^2+46a^2c+6351a^2+16ac^4-800ac^3+10364ac^2-46852ac+62182a-32c^5+944c^4-10112c^3+47656c^2-$$
$$-93776c+55440)+\frac{1}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a-11}{2})}(a^5+6a^4c-69a^4-12a^3c^2+12a^3c+769a^3-32a^2c^3+840a^2c^2-$$
$$-5662a^2c+8301a^2+16ac^4-32ac^3-4612ac^2+42380ac-96002a+32c^5-1136c^4+15104c^3-$$
$$-92536c^2+255392c-245640)\Big] \tag{14}$$

$${}_2F_1\left[\begin{array}{lll} a, & 13-a & ; \\ c & & ; \end{array} \frac{1}{2}\right] =$$

$$= \frac{\sqrt{\pi}\ \Gamma(c)}{2^{c-13}\{\prod_{\beta=1}^{12}(\beta-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a-12}{2})}(-a^6+39a^5+18a^4c^2-252a^4c+275a^4-468a^3c^2+6552a^3c$$
$$-18135a^3-48a^2c^4+1344a^2c^3-9834a^2c^2+5964a^2c+74246a^2+624ac^4-17472ac^3+167388ac^2-631176ac$$
$$+752856a+32c^6-1344c^5+21824c^4-172032c^3+674384c^2-1187424c+665280)+$$
$$+\frac{1}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a-13}{2})}(-12a^4c+84a^4+312a^3c-2184a^3+64a^2c^3-1344a^2c^2$$
$$+6620a^2c-2436a^2-832ac^3+17472ac^2-112424ac+216216a-64c^5+2240c^4-29312c^3+176512c^2-$$
$$-478752c+453600)\Big] \tag{15}$$

$${}_2F_1\left[\begin{array}{lll} a, & 14-a & ; \\ c & & ; \end{array} \frac{1}{2}\right] =$$

$$= \frac{\sqrt{\pi}\ \Gamma(c)}{2^{c-14}\{\prod_{\gamma=1}^{13}(\gamma-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a-14}{2})}(a^6+6a^5c-87a^5-24a^4c^2+150a^4c+925a^4-32a^3c^3+1392a^3c^2$$
$$-12706a^3c+24615a^3+80a^2c^4-1728a^2c^3+5368a^2c^2+58986a^2c-242486a^2+32ac^5-2320ac^4+47328ac^3$$
$$-391568ac^2+1344076ac-1496568a-64c^6+2656c^5-42560c^4+330752c^3-1278144c^2+2222160c-1235520)+$$
$$+\frac{1}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a-13}{2})}(-a^6+6a^5c-$$
$$-3a^5+24a^4c^2-570a^4c+2225a^4-32a^3c^3+48a^3c^2+7454a^3c-39225a^3-80a^2c^4+3072a^2c^3-35608a^2c^2+133626a^2c-$$
$$-68104a^2+32ac^5-80ac^4-19872ac^3+313808ac^2-1676564ac+2856228a+64c^6-3104c^5+59360c^4-566848c^3+$$
$$+2810304c^2-6724560c+5897520)\Big] \tag{16}$$

$${}_2F_1\left[\begin{array}{lll} a, & 15-a & ; \\ c & & ; \end{array} \frac{1}{2}\right] =$$

$$= \frac{\sqrt{\pi}\ \Gamma(c)}{2^{c-15}\{\prod_{\varepsilon=1}^{14}(\varepsilon-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a-14}{2})}(-7a^6c+56a^6+315a^5c-2520a^5+56a^4c^3-1344a^4c^2+5103a^4c+$$
$$+16520a^4-1680a^3c^3+40320a^3c^2-271215a^3c+449400a^3-112a^2c^5+4480a^2c^4-54040a^2c^3+150080a^2c^2+845824a^2c-$$
$$-3383296a^2+1680ac^5-67200ac^4+999600ac^3-6787200ac^2+20482140ac-21070560a+64c^7-3584c^6+80864c^5-$$
$$-940800c^4+5987520c^3-20296192c^2+32464368c-17297280)+\frac{1}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a-15}{2})}(2a^6-90a^5-48a^4c^2+768a^4c-$$
$$-1474a^4+1440a^3c^2-23040a^3c+77970a^3+160a^2c^4-5120a^2c^3+46640a^2c^2-90880a^2c-226192a^2-2400ac^4+$$
$$+76800ac^3-861600ac^2+3955200ac-6138120a-128c^6+6144c^5-116160c^4+1095680c^3-5363584c^2+$$
$$+12679168c-11009376)\Big] \tag{17}$$

$${}_2F_1\left[\begin{array}{lll} a, & 16-a & ; \\ c & & ; \end{array} \frac{1}{2}\right] =$$

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$$= \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-16}\{\prod_{\zeta=1}^{15}(\zeta-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-16}{2})}(-a^7+8a^6c-12a^6+24a^5c^2-792a^5c+3710a^5-80a^4c^3+1080a^4c^2+$$

$$+6280a^4c-66600a^4-80a^3c^4+5280a^3c^3-85480a^3c^2+435480a^3c-458929a^3+192a^2c^5-6240a^2c^4+45200a^2c^3+$$

$$+271560a^2c^2-3746640a^2c+8942052a^2+64ac^6-6336ac^5+186000ac^4-2408160ac^3+15005072ac^2-42553152ac+$$

$$+41722740a-128c^7+7104c^6-158720c^5+1827360c^4-11505152c^3+38596416c^2-61194240c+32432400)+$$

$$+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-15}{2})}(-a^7-8a^6c+124a^6+24a^5c^2-24a^5c-2818a^5+80a^4c^3-3000a^4c^2+26360a^4c-40760a^4-80a^3c^4+$$

$$+160a^3c^3+45080a^3c^2-534760a^3c+1499471a^3-192a^2c^5+10080a^2c^4-175760a^2c^3+1189560a^2c^2-2226480a^2c-$$

$$-2760884a^2+64ac^6-192ac^5-75120ac^4+1782560ac^3-16394608ac^2+65703616ac-93008652a+128c^7-8128c^6+$$

$$+210944c^5-2878240c^4+22080512c^3-94015552c^2+202146816c-165145680)\Big] \tag{18}$$

$${}_2F_1\left[\begin{matrix} a, & 17-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] =$$

$$= \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-17}\{\prod_{\vartheta=1}^{16}(\vartheta-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-16}{2})}(a^8-68a^7-32a^6c^2+576a^6c-638a^6+1632a^5c^2-29376a^5c+101320a^5+$$

$$+160a^4c^4-5760a^4c^3+44640a^4c^2+129600a^4c-1341071a^4-5440a^3c^4+195840a^3c^3-2303840a^3c^2+$$

$$+9743040a^3c-9832052a^3-256a^2c^6+13824a^2c^5-246560a^2c^4+1411200a^2c^3+4297408a^2c^2-64103040a^2c+$$

$$+143207628a^2+4352ac^6-235008ac^5+4977600ac^4-52289280ac^3+282566656ac^2-727036416ac+670152240a+$$

$$+128c^8-9216c^7+275456c^6-4423680c^5+41249792c^4-224907264c^3+683065344c^2-1014128640c+518918400+$$

$$+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-17}{2})}(16a^6c-144a^6-816a^5c+7344a^5-160a^4c^3+4320a^4c^2-22480a^4c-30960a^4+5440a^3c^3-$$

$$-146880a^3c^2+1157360a^3c-2484720a^3+384a^2c^5-17280a^2c^4+247840a^2c^3-1092960a^2c^2-1901760a^2c+$$

$$+15669504a^2-6528ac^5+293760ac^4-4999360ac^3+39804480ac^2-146267456ac+194890176a-256c^7+16128c^6-$$

$$-414976c^5+5610240c^4-42628864c^3+179788032c^2-383195904c+310867200)\Big] \tag{19}$$

$${}_2F_1\left[\begin{matrix} a, & 18-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] =$$

$$= \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-18}\{\prod_{\eta=1}^{17}(\eta-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-18}{2})}(-a^8-8a^7c+148a^7+40a^6c^2-256a^6c-3362a^6+80a^5c^3-4440a^5c^2+$$

$$+49664a^5c-103400a^5-240a^4c^4+5520a^4c^3+18760a^4c^2-849520a^4c+3240271a^4-192a^3c^5+17760a^3c^4-440560a^3c^3+$$

$$+4091160a^3c^2-12923320a^3c+3622852a^3+448a^2c^6-20352a^2c^5+253360a^2c^4+576240a^2c^3-31091248a^2c^2+$$

$$+192701168a^2c-344444908a^2+128ac^7-16576ac^6+660032ac^5-12228640ac^4+118499872ac^3-604789504ac^2+$$

$$+1488844864ac-1324543920a-256c^8+18304c^7-542976c^6+8650240c^5-79993344c^4+432549376c^3-1303568384c^2+$$

$$+1923025920c-980179200)+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-17}{2})}(a^8-8a^7c+4a^7-40a^6c^2+1264a^6c-6214a^6+80a^5c^3-120a^5c^2-$$

$$-32416a^5c+213904a^5+240a^4c^4-12720a^4c^3+186440a^4c^2-743120a^4c-456391a^4-192a^3c^5+480a^3c^4+216080a^3c^3-$$

$$-4278120a^3c^2+27569480a^3c-52277444a^3-448a^2c^6+30720a^2c^5-745840a^2c^4+7817520a^2c^3-30345632a^2c^2-$$

$$-19224224a^2c+253516684a^2+128ac^7-448ac^6-259264ac^5+8556320ac^4-118218848ac^3+813195488ac^2-$$

$$-2692403360ac+3335839536a+256c^8-20608c^7+696192c^6-12817024c^5+139638144c^4-913535872c^3+3463541888c^2-$$

$$-6848013696c+5284782720)\Big] \tag{20}$$

$${}_2F_1\left[\begin{matrix} a, & 19-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] =$$

Notes

$$
\begin{aligned}
&= \frac{\sqrt{\pi}\ \Gamma(c)}{2^{c-19}\{\prod_{\lambda=1}^{18}(\lambda-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a-18}{2})}(9a^8c-90a^8-684a^7c+6840a^7-120a^6c^3+3600a^6c^2-14046a^6c-99540a^6+\\
&+6840a^5c^3-205200a^5c^2+1664856a^5c-2968560a^5+432a^4c^5-21600a^4c^4+277080a^4c^3+327600a^4c^2-20793831a^4c+\\
&+70898310a^4-16416a^3c^5+820800a^3c^4-14644440a^3c^3+111013200a^3c^2-315518940a^3c+131909400a^3-576a^2c^7+\\
&+40320a^2c^6-992880a^2c^5+9324000a^2c^4+4429536a^2c^3-636886080a^2c^2+3695816316a^2c-6211091160a^2+10944ac^7-\\
&-766080ac^6+21827808ac^5-325310400ac^4+2707726176ac^3-12394025280ac^2+28254838896ac-23908836960a+256c^9-\\
&-23040c^8+880512c^7-18627840c^6+238347264c^5-1891123200c^4+9158978048c^3-25507261440c^2+35661692160c-\\
&-17643225600)+\frac{1}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a-19}{2})}(-2a^8+152a^7+80a^6c^2-1600a^6c+3148a^6-4560a^5c^2+91200a^5c-371488a^5-\\
&-480a^4c^4+19200a^4c^3-185680a^4c^2-126400a^4c+4559182a^4+18240a^3c^4-729600a^3c^3+9799440a^3c^2-50068800a^3c+\\
&+73373288a^3+896a^2c^6-53760a^2c^5+1107680a^2c^4-8467200a^2c^3-743936a^2c^2+274718720a^2c-822056088a^2-\\
&-17024ac^6+1021440ac^5-24338240ac^4+292569600ac^3-1853708096ac^2+5798641920ac-6885423072a-\\
&-512c^8+40960c^7-1374464c^6+25123840c^5-271685888c^4+1764075520c^3-6639757056c^2+13042437120c-\\
&-10013310720)\Big] \qquad (21)
\end{aligned}
$$

$$
{}_2F_1\left[\begin{matrix} a, & 20-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right]=
$$

$$
\begin{aligned}
&= \frac{\sqrt{\pi}\ \Gamma(c)}{2^{c-20}\{\prod_{\Upsilon=1}^{19}(\Upsilon-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a-20}{2})}(33522128640+47215599696a+14182895460a^2+345040520a^3-\\
&-140133105a^4+962073a^5+330750a^6-9330a^7+15a^8+a^9-67958134272c-57343402272ac-9605975576a^2c+\\
&+295428296a^3c+58846422a^4c-2100880a^5c-32820a^6c+1640a^7c-10a^8c+48842214912c^2+\\
&+25998562336ac^2+2187966784a^2c^2-168954152a^3c^2-6101120a^4c^2+380720a^5c^2-2240a^6c^2-40a^7c^2-17641896960c^3-\\
&-5917427456ac^3-182014144a^2c^3+27821280a^3c^3-9440a^4c^3-19680a^5c^3+160a^6c^3+3666323456c^4+750095264ac^4-\\
&-1895280a^2c^4-1926160a^3c^4+23280a^4c^4+240a^5c^4-465172736c^5-54369728ac^5+1155616a^2c^5+55104a^3c^5-672a^4c^5+\\
&+36595328c^6+2174144ac^6-61824a^2c^6-448a^3c^6-1740800c^7-41984ac^7+1024a^2c^7+45824c^8+256ac^8-512c^9)+\\
&+\frac{1}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a-19}{2})}(-190253266560-131460917904a-15315714660a^2+1718684120a^3+100625805a^4-10839927a^5+\\
&+135450a^6+7470a^7-195a^8+a^9+258458522112c+117489033888ac+5199265016a^2c-1259577944a^3c+961578a^4c+\\
&+3256720a^5c-84780a^6c+40a^7c+10a^8c-139931759232c^2-40815588704ac^2+198370336a^2c^2+283436248a^3c^2-\\
&-7330880a^4c^2-224080a^5c^2+7840a^6c^2-40a^7c^2+40472263680c^3+7213462784ac^3-274206656a^2c^3-26053920a^3c^3+\\
&+1017440a^4c^3-480a^5c^3-160a^6c^3-6993636736c^4-700147936ac^4+42392880a^2c^4+896240a^3c^4-47280a^4c^4+240a^5c^4+\\
&+757008896c^5+36475712ac^5-2849056a^2c^5+1344a^3c^5+672a^4c^5-51764608c^6-836416ac^6+88704a^2c^6-448a^3c^6+\\
&+2170880c^7-1024ac^7-1024a^2c^7-50944c^8+256ac^8+512c^9)\Big] \qquad (22)
\end{aligned}
$$

$$
{}_2F_1\left[\begin{matrix} a, & 21-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right]=
$$

$$
\begin{aligned}
&= \frac{\sqrt{\pi}\ \Gamma(c)}{2^{c-21}\{\prod_{\Psi=1}^{20}(\Psi-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a-20}{2})}(670442572800+946321185600a+284169369024a^2+4885689900a^3-\\
&-3333875180a^4+41694345a^5+10037727a^6-381150a^7+1230a^8+105a^9-a^{10}-1394694005760c-1198379286720ac-\\
&-203053089360a^2c+8433107760a^3c+1530533620a^4c-70408800a^5c-1146200a^6c+92400a^7c-1100a^8c+\\
&+1048586614272c^2+578478838560ac^2+49539606520a^2c^2-4805882760a^3c^2-177714670a^4c^2+15397200a^5c^2-\\
&-141500a^6c^2-4200a^7c^2+50a^8c^2-404078540800c^3-143591669760ac^3-4354528640a^2c^3+902932800a^3c^3-\\
&-2094400a^4c^3-1108800a^5c^3+17600a^6c^3+91700259840c^4+20464187520ac^4-122473120a^2c^4-77439600a^3c^4+
\end{aligned}
$$

Notes

$$+1402800a^4c^4+25200a^5c^4-400a^6c^4-13092907520c^5-1739633280ac^5+50240960a^2c^5+3104640a^3c^5-73920a^4c^5+$$
$$+1209103616c^6+87071040ac^6-3652320a^2c^6-47040a^3c^6+1120a^4c^6-72089600c^7-2365440ac^7+112640a^2c^7+$$
$$+2677760c^8+26880ac^8-1280a^2c^8-56320c^9+512c^{10})+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-21}{2})}(362387520000+268742591040a+$$
$$+41471452880a^2-1867829040a^3-305673060a^4+14303520a^5+225720a^6-18480a^7+220a^8-494250063360c-$$
$$-247867413696ac-19713479280a^2c+1984361232a^3c+69962284a^4c-6179040a^5c+56920a^6c+1680a^7c-20a^8c+$$
$$+268936121344c^2+89644203264ac^2+2511762176a^2c^2-547968960a^3c^2+1404480a^4c^2+665280a^5c^2-10560a^6c^2-$$
$$-78226625536c^3-16719935232ac^3+112602112a^2c^3+62139840a^3c^3-1126720a^4c^3-20160a^5c^3+320a^6c^3+$$
$$+13598953984c^4+1756191360ac^4-51029440a^2c^4-3104640a^3c^4+73920a^4c^4-1480941056c^5-104786304ac^5+$$
$$+4397120a^2c^5+56448a^3c^5-1344a^4c^5+101871616c^6+3311616ac^6-157696a^2c^6-4296704c^7-43008ac^7+2048a^2c^7+$$
$$+101376c^8-1024c^9)\Big] \tag{23}$$

II. Main Summation Formulae

$${}_2F_1\left[\begin{array}{ccc} a, & 22-a & ; \\ c & & ; \end{array} \frac{1}{2}\right]=$$

$$=\frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-22}\{\prod\limits_{\Xi=1}^{21}(\Xi-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-22}{2})}(-1279935820800-1868233671360a-628352859744a^2-$$
$$-34417212780a^3+5753119700a^4+134236095a^5-20700687a^6+312270a^7+8730a^8-225a^9+a^{10}+2668809669120c+$$
$$+2417863186656ac+489345655848a^2c-219480864a^3c-3388493178a^4c+53042458a^5c+4970700a^6c-142820a^7c+$$
$$+390a^8c+10a^9c-2014029186048c^2-1197461040576ac^2-138256171792a^2c^2+5911683120a^3c^2+583619652a^4c^2-$$
$$-22165920a^5c^2-176120a^6c^2+10800a^7c^2-60a^8c^2+779711413248c^3+306554335232ac^3+17513420736a^2c^3-$$
$$-1499242976a^3c^3-32128320a^4c^3+2199680a^5c^3-13440a^6c^3-160a^7c^3-177857647616c^4-45404661120ac^4-$$
$$-823493664a^2c^4+154420560a^3c^4-467600a^4c^4-75600a^5c^4+560a^6c^4+25531683072c^5+4062167872ac^5-$$
$$-27880608a^2c^5-7519456a^3c^5+86688a^4c^5+672a^5c^5-2370643968c^6-219093504ac^6+4625152a^2c^6+161280a^3c^6-$$
$$-1792a^4c^6+142098432c^7+6765568ac^7-178176a^2c^7-1024a^3c^7-5305344c^8-103680ac^8+2304a^2c^8+112128c^9+$$
$$+512ac^9-1024c^{10})+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-21}{2})}(7610141548800+5664039006240a+862754799384a^2-50618670580a^3-$$
$$-7467040370a^4+438809595a^5+6355587a^6-793890a^7+14040a^8-5a^9-a^{10}-10762094073600c-5490993903456ac-$$
$$-428082370072a^2c+53697863232a^3c+1803933278a^4c-214732742a^5c+2793980a^6c+100060a^7c-2370a^8c+10a^9c+$$
$$+6167102701056c^2+2126343680320ac^2+53972779984a^2c^2-16286791024a^3c^2+92193948a^4c^2+28580160a^5c^2-$$
$$-673960a^6c^2+240a^7c^2+60a^8c^2-1923629552128c^3-434298545536ac^3+5057543680a^2c^3+2145900064a^3c^3-$$
$$-52633280a^4c^3-1200640a^5c^3+38080a^6c^3-160a^7c^3+366720941312c^4+51431687104ac^4-1928215296a^2c^4-$$
$$-133374640a^3c^4+4718000a^4c^4-1680a^5c^4-560a^6c^4-45108419328c^5-3603513536ac^5+200868192a^2c^5+$$
$$+3361568a^3c^5-160608a^4c^5+672a^5c^5+3654604800c^6+142266880ac^6-10065664a^2c^6+3584a^3c^6+1792a^4c^6-$$
$$-193800192c^7-2561024ac^7+245760a^2c^7-1024a^3c^7+6471168c^8-2304ac^8-2304a^2c^8-123392c^9+512ac^9+1024c^{10})\Big]$$

$${}_2F_1\left[\begin{array}{ccc} a, & 23-a & ; \\ c & & ; \end{array} \frac{1}{2}\right]= \tag{24}$$

$$=\frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-23}\{\prod\limits_{\Omega=1}^{22}(\Omega-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-22}{2})}(-28158588057600-41169473009280a-13846175136288a^2-$$
$$-688985043120a^3+149373094200a^4+2886127860a^5-648260844a^6+12963720a^7+382800a^8-15180a^9+132a^{10}+$$
$$+60062080780800c+55217638146528ac+11293160726232a^2c-82022959260a^3c-93645815450a^4c+1995539777a^5c+$$

Notes

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$$+178029929a^6c - 6909430a^7c + 31460a^8c + 1265a^9c - 11a^10c - 47111453908992c^2 - 28820579344128ac^2 -$$
$$-3392485386048a^2c^2 + 183098051136a^3c^2 + 17810133264a^4c^2 - 867081600a^5c^2 - 6985440a^6c^2 + 728640a^7c^2 - 7920a^8c^2 +$$
$$+19258856466432c^3 + 7922696165824ac^3 + 461791289168a^2c^3 - 49584004624a^3c^3 - 1091526436a^4c^3 + 105693280a^5c^3 -$$
$$-988680a^6c^3 - 20240a^7c^3 + 220a^8c^3 - 4723308327936c^4 - 1289002826496ac^4 - 22320173568a^2c^4 + 5914856640a^3c^4 -$$
$$-30824640a^4c^4 - 5100480a^5c^4 + 73920a^6c^4 + 745452131072c^5 + 130485126464ac^5 - 1364040832a^2c^5 - 359725520a^3c^5 +$$
$$+6190800a^4c^5 + 85008a^5c^5 - 1232a^6c^5 - 78371758080c^6 - 8302221312ac^6 + 235834368a^2c^6 + 10881024a^3c^6 -$$
$$-236544a^4c^6 + 5546010624c^7 + 322674176ac^7 - 12539648a^2c^7 - 129536a^3c^7 + 2816a^4c^7 - 260941824c^8 - 6994944ac^8 +$$
$$+304128a^2c^8 + 7822848c^9 + 64768ac^9 - 2816a^2c^9 - 135168c^{10} + 1024c^{11}) + \frac{1}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a-23}{2})}(-14558535129600-$$
$$-11503844032320a - 2137013714928a^2 + 23993967080a^3 + 17223845140a^4 - 382057830a^5 - 32189094a^6 + 1259940a^7 -$$
$$-5760a^8 - 230a^9 + 2a^{10} + 20652447375360c + 11426734414848ac + 1249606186752a^2c - 70894693632a^3c -$$
$$-6453736128a^4c + 319011840a^5c + 2486400a^6c - 264960a^7c + 2880a^8c - 11881425202176c^2 - 4559948772992ac^2 -$$
$$-249358186400a^2c^2 + 27700361696a^3c^2 + 584111304a^4c^2 - 57805440a^5c^2 + 541520a^6c^2 + 11040a^7c^2 - 120a^8c^2 +$$
$$+3722781351936c^3 + 967717718016ac^3 + 15372177408a^2c^3 - 4341281280a^3c^3 + 23278080a^4c^3 + 3709440a^5c^3 -$$
$$-53760a^6c^3 - 713155826176c^4 - 120677707136ac^4 + 1319899392a^2c^4 + 327847520a^3c^4 - 5645920a^4c^4 - 77280a^5c^4 +$$
$$+1120a^6c^4 + 88159518720c^5 + 9128189952ac^5 - 260370432a^2c^5 - 11870208a^3c^5 + 258048a^4c^5 - 7178121216c^6 -$$
$$-411665408ac^6 + 16002560a^2c^6 + 164864a^3c^6 - 3584a^4c^6 + 382500864c^7 + 10174464ac^7 - 442368a^2c^7 - 12831744c^8 -$$
$$-105984ac^8 + 4608a^2c^8 + 245760c^9 - 2048c^{10})\Big] \tag{25}$$

III. Derivation of the Main Formulae

Involving the contiguous relation (8) and the formula of Salahuddin et al(23), one can established the result(24) and on the same way result(25) can be established.

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Notes

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Inventory Management for Irregular Shipment of Goods in Distribution Center

By Jun Usuki, Hitoshi Takeda & Masatoshi Kitaoka
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Abstract- The shipping amount of commodity goods (Foods, confectionery, dairy products, such as public cosmetic pharmaceutical products) changes irregularly at the distribution center dealing with the general consumer goods. Because the shipment time and the amount of the shipment are irregular, the demand forecast becomes very difficult. For this, the inventory control becomes difficult, too. It cannot be applied to the shipment of the commodity by the conventional inventory control methods. This paper proposes the method for inventory control by cumulative flow curve method. It proposed the method of deciding the order quantity of the inventory control by the cumulative flow curve. Here, it proposes three methods. 1) Power method, 2) Polynomial method and 3) Revised Holt's linear method that forecasts data with trends that is a kind of exponential smoothing method. This paper compares the economics of the conventional method, which is managed by the experienced and three new proposed methods. And, the effectiveness of the proposal method is verified from the numerical calculations.

Keywords: *inventory control, supply chain management, distribution center, forecasting method.*

GJSFR-F Classification : *MSC 2010: 00A05*

INVENTORYMANAGEMENTFORIRREGULARSHIPMENTOFGOODSINDISTRIBUTIONCENTER

Strictly as per the compliance and regulations of :

Ref

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Inventory Management for Irregular Shipment of Goods in Distribution Center

Jun Usuki [α], Hitoshi Takeda [σ] & Masatoshi Kitaoka [ρ]

Abstract- The shipping amount of commodity goods (Foods, confectionery, dairy products, such as public cosmetic pharmaceutical products) changes irregularly at the distribution center dealing with the general consumer goods. Because the shipment time and the amount of the shipment are irregular, the demand forecast becomes very difficult. For this, the inventory control becomes difficult, too. It cannot be applied to the shipment of the commodity by the conventional inventory control methods. This paper proposes the method for inventory control by cumulative flow curve method. It proposed the method of deciding the order quantity of the inventory control by the cumulative flow curve. Here, it proposes three methods. 1) Power method, 2) Polynomial method and 3) Revised Holt's linear method that forecasts data with trends that is a kind of exponential smoothing method. This paper compares the economics of the conventional method, which is managed by the experienced and three new proposed methods. And, the effectiveness of the proposal method is verified from the numerical calculations.

Keywords: inventory control, supply chain management, distribution center, forecasting method.

I. Introduction

The distribution center only ship merchandise after they receive direct orders from customers, who may be wholesalers, supermarkets or large-scale stores. The wholesalers then ship merchandise to retailers, who in turn sell the merchandise to consumers. The supermarkets and other large-scale stores, on the other hand, sell the merchandise directly to consumers. Generally, the sale of merchandise directly to ultimate consumers fluctuates widely according to the characteristics of the merchandise. For this reason, the amounts shipped from distribution center to their customers and the times of shipment also vary greatly. The commodity goods (Foods, confectionery, dairy products, such as public cosmetic pharmaceutical products) do irregular changes at the distribution center dealing with the general consumer goods. Therefore, the forecast of the shipment time and the shipment volume are difficult. It cannot be applied to the shipment of the commodity by the conventional inventory management techniques. For this reason, we have proposed a method of performing with the cumulative flow graph[1][2]. The irregular shipment characteristics of goods, we have proposed a theoretical formula that determines the order timing and order quantity. The cumulative inflow corresponds to the order quantity in inventory management. Here, it proposes three methods. 1) Power method, 2) Polynomial method[3] and 3) Revised Holt's linear method that forecasts data with trends that is a kind of exponential smoothing method. This paper compares the economics of the conventional

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method, which is managed by the experienced and three new proposed methods. And, the effectiveness of the proposal method is verified for the real distribution center.

II. Cumulative Curve

Distribution center ship merchandise in direct response to orders from customers, be they wholesalers, supermarkets or other large-scale stores. The merchandise shipped from the distribution centers to wholesalers is then shipped by the wholesalers to retailers, as shown in Figure 1. Then the merchandise is sold by retailers to consumers. The merchandise shipped from distribution centers to supermarkets and other large-scale stores is sold directly to consumers at the supermarkets and those large-scale stores. Movement of warehouses and distribution center can be expressed by the cumulative flow curve. The cumulative flow curve is composed of the inflow, the outflow, and the processing line of the distribution center. Movement of the product can be represented by the inflow and outflow as shown in Figure 2[4][5]. Cumulative flow curve is a graph obtained by the cumulative value of the inflow and outflow of goods. Vertical interval of the cumulative line becomes the inventory volume and horizontal line becomes the lead time[6].

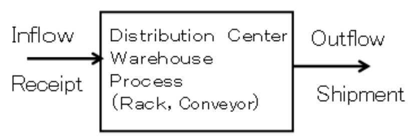

Fig. 1 : Relation between inflow and outflow in distribution center

Figure 2 shows a state in which store them in stock products in distribution centers there. Figure 2 shows cumulative flow curve by orders from customers, and are shipping the goods from distribution centers. Figure 2 also shows that the inventory quantity Qa is increasingly July and August. And, inventory quantity Qb has become less in October. Figure 3 shows the inventory quantity calculated from the inflow and outflow of the cumulative flow curve. Inventory levels in Figure 3 are changed to ship the order from the customer on a monthly. In the distribution center, processing time of machine transport number of forklift, man-hours of loading and unloading work, such as conveyor come determined by the amount of stock[7].

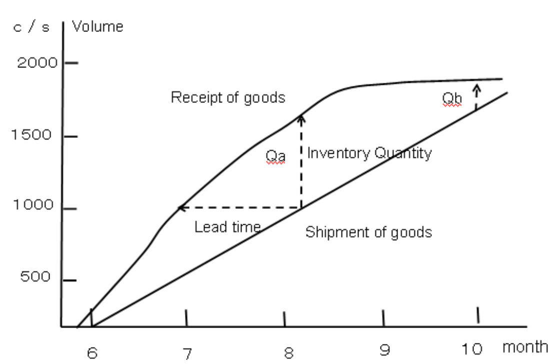

Fig. 2 : Cumulative flow curve at distribution center

Ref

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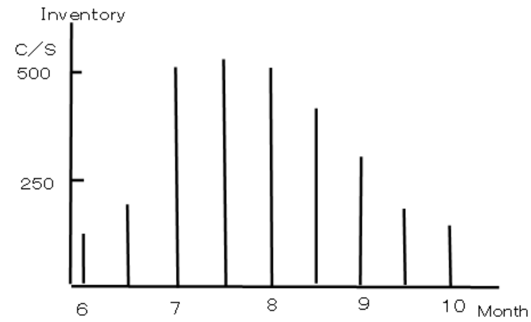

Fig. 3 : Amount of inventory to be changed every month

III. Characteristic of Shipment Data

The amount and the time of the products shipped from the distribution center to the supermarket, the large-scale retail stores, and the wholesale make a big change. The reason for change is because the direct product to ship is intended for consumer food, confectionery, dairy products, cosmetics, and pharmaceuticals. In the distribution center, product management, inventory management, shipment planning, delivery vehicle planning, labor management is difficult as compared to the production plant for the change of the product sales of shipping to large. Production planning and inventory control in the production plant can easily control for planning and management since the fluctuation is little[3]. There are various patterns in shipping time by customer. In one pattern, the time of shipments is concentrated at the beginning of month, whereas shipment time focuses at the end of a month in another pattern. In still another type of patterns, shipments are made almost every day. The shipment amount patterns also vary in a similar way to the shipping time patterns: large volume may be concentrated at the beginning of a month or at the end of month, or they can be almost same every day or differ every day. The shipping pattern can be classified as follows.

1) There are shipments every day, shipments to the normal distribution.
2) Shipment shall be concentrated at the beginning of a month.
3) There is a ship at the end of the month.
4) Those to be shipped to day irregular.
5) There is a shipping amount on the day of at the beginning of a month, shipment shall disappear from the end of the month.
6) There are shipments amount at the end of the month, that there is no ship of a beginning month.
7) The various combinations of shipping these patterns exist. Inventory management of goods with shipping such properties is difficult and is not available, such as reorder point method in the past. Therefore, the method for inventory control, shipment management, storage management using the cumulative flow curves in the distribution center is proposed.

IV. Relationship of Cumulative Inflow and Cumulative Outflow

Cumulative flow curve is made from the cumulative outflow and cumulative inflow as shown in Figure 4. In Figure 4, the top line of the graph represents the cumulative inflow, the lower line indicates the cumulative outflow. The vertical axis in figure 4 shows the amount of stock. The horizontal axis shows the date of arrival of goods. R1, R2, and R3 are the volume of inventories.

Ref

3. Li J., Kitaoka M., and Suzuki S, 2004, *Times series Analysis of Shipments and receipts for Distribution Center using GREY Theory and EIQ Table ICLS,246-252.*

Q1, Q2, and Q3 are the amounts of arrival of goods that the distribution center ordered the supplier.
The t1 of the horizontal axis represents the day of order quantity Q1 is in-stock.

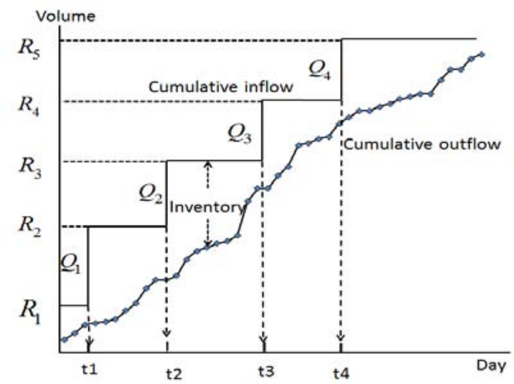

Fig. 4 : The cumulative inflow and outflow curve and the inventory

The following relations are approved between volume of inventories R and amount Q of the order.

$$R_2 = R_1 + Q_1 \; ; R_3 = R_2 + Q_2$$
$$R_4 = R_3 + Q_3 \; ; R_5 = R_4 + Q_4 \qquad (1)$$

The difference between the two graphs of the upper cumulative inflow and lower cumulative outflow is to the volume of inventories. The inventory control in the distribution center becomes a problem of deciding amount Q of the order.

V. Cumulative flow Curve of Shipping Characteristics

a) Cumulative shipments characteristics shipment characteristics

The data must be cumulative in order to draw the cumulative flow curves of the product for the irregular shipment characteristics. Cumulative flow curve can be controlled by the amount of quantity and timing of arrival[8]. However, it is not possible to control the shipment quantity and shipment time. Because this is an order from the customer, it is not possible to control freely. It is necessary to examine the pattern at the shipment amount and the shipment time according to the product. Three shipments of the kind of product and patterns at the shipment time are shown here as a typical example.

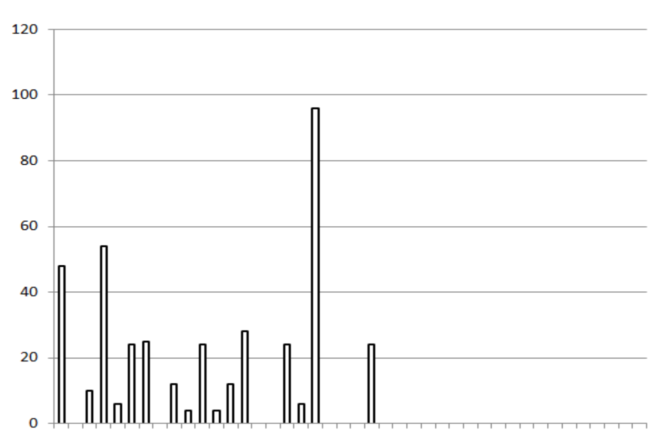

Fig. 5 : Shipping is concentrated in the beginning of the month (Item 1)

Ref

8. Abraham B. and Ledolter J, 2005, *Statistical Methods for Forecasting* (Wiley Inter-Science).

1) Figure 5 is a product it ships at the beginning of the month, and without the shipment until the end of the month. The shipment of the cumulative flow curve is shown in figure 6[8][9][10].
2) Figure 7 is a product it ships at the end of the month, and without the shipment until the end of the month. The shipment of the cumulative flow curve is shown in figure 8.
3) Figure 9 is a product regularly shipped. The shipment of the cumulative flow curve is shown in figure 10.

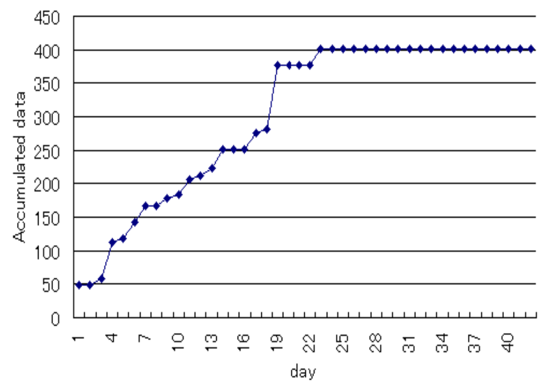

Fig. 6 : Cumulative flow curve for figure 3

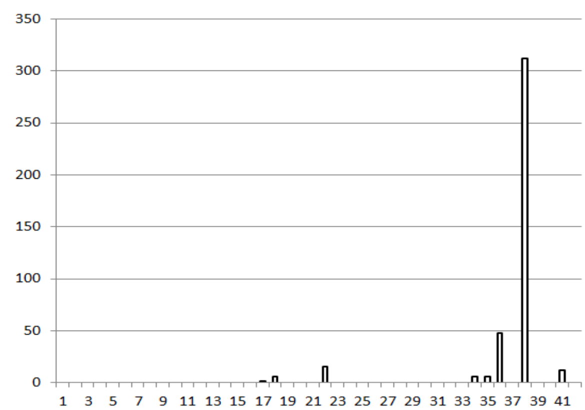

Fig. 7 : Shipping is concentrated in the end of the month (Item 2)

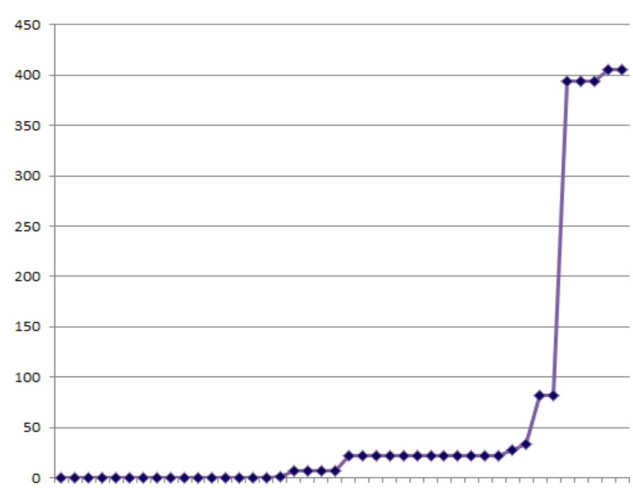

Fig. 8 : Cumulative flow curve for figure 5

Ref

9. Vollmann T, Berry W, Whybark D and Jacobs F, 2005, *Manufacturing Planning Systems for Supply Chain Management* (McGraw-Hill).

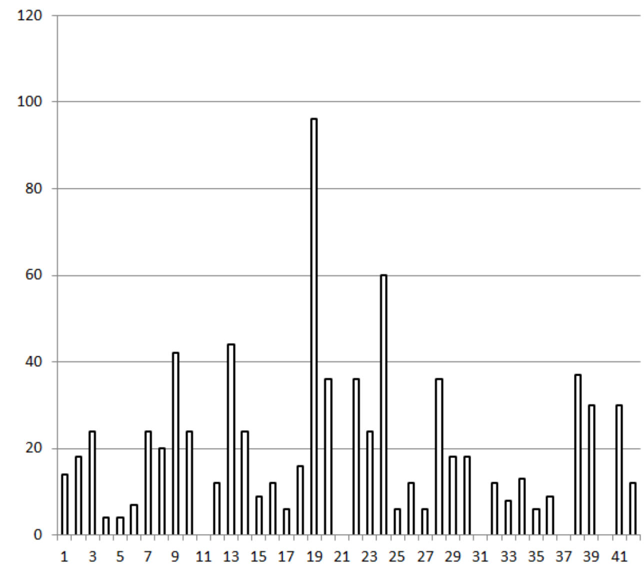

Fig. 9 : The product is evenly shipped (Item 3)

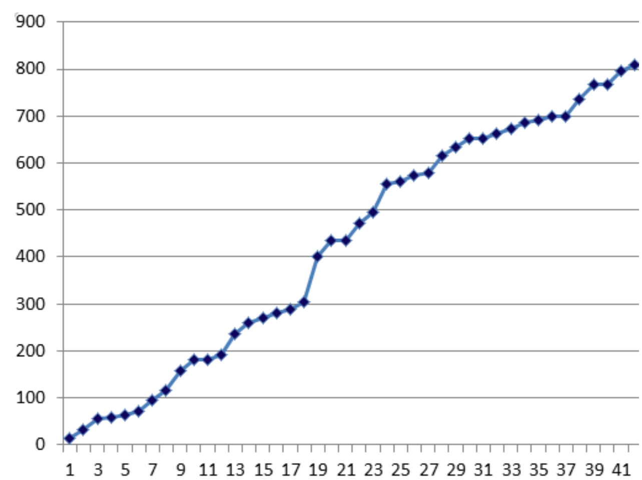

Fig. 10 : Cumulative flow curve for fig 7

b) Inventory control that uses cumulative flow curve in distribution center

The current state of the inventory control that uses cumulative flow curve at the distribution center is examined here. Figure 11 is cumulative flow curve of item I4. The volume of inventories at this time is shown in figure 12, and the total of the volume of inventories becomes 12549. Cumulative flow curve of item 5 is similarly shown to figure 13. The volume of inventories is shown in figure 14. The total of the volume of inventories becomes 16507. The judgment whether this volume of inventories is a lot or is little cannot be judged because there is no technique of a scientific inventory management. However, because the shipment time and the shipment are irregular. It becomes possible to stock a lot of inventory in the distribution center[8][9][10].

Ref

10. Donald W., 2003, Inventory Control Management (John Wiley&Sons).

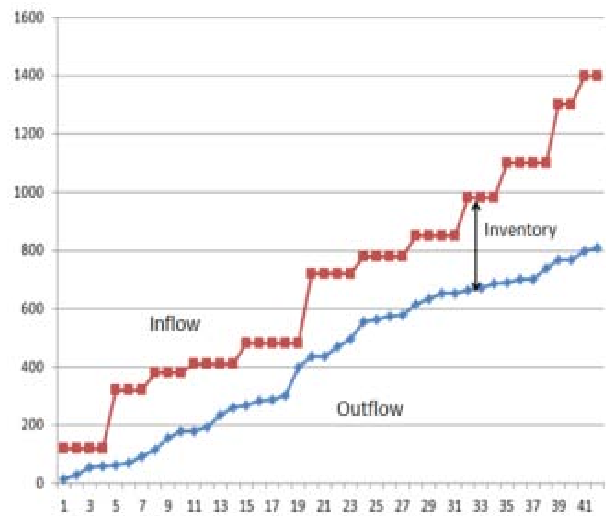

Fig. 11 : Cumulative flow curve of Item I4

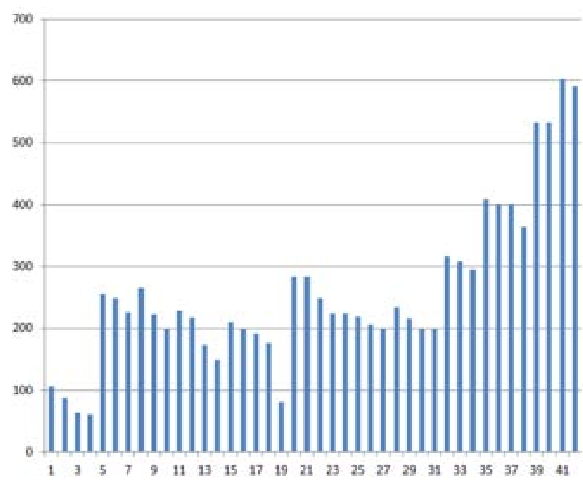

Fig. 12 : Volume of inventories of Item I4

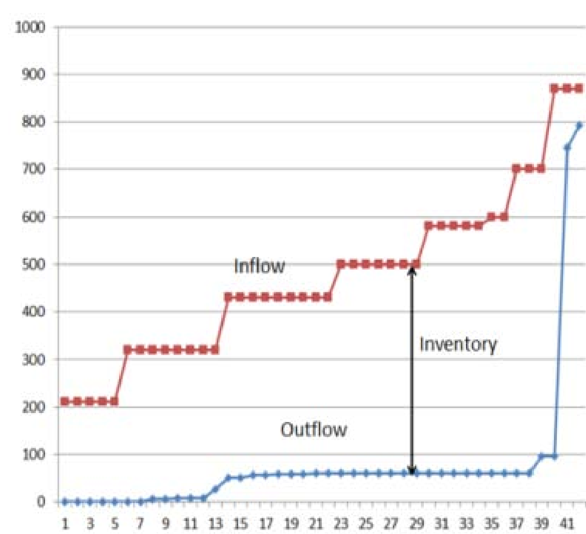

Fig. 13 : Cumulative flow curve of Item I5

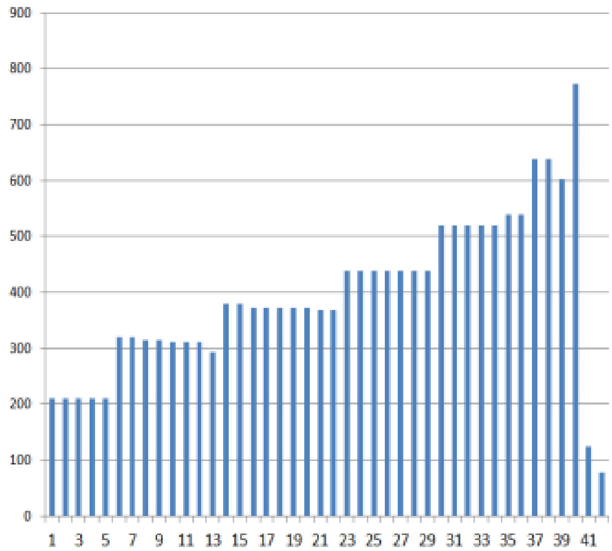

Fig. 14 : Inventory volume of Item I5

c) *Inventory control for irregular demand*

The formula of Economic Order Quantity is used for the inventory control. The EOQ calculation is the most important analysis of inventory control. We start with a basic model that makes a number of assumptions.

1) The quantity demanded is known exactly and continuous and is constant over time;
2) All costs are already known exactly and do not vary;
3) No shortage is allowed, and lead time is zero.

However, the formula of the EOQ cannot be applied to irregular shipment time and the shipment. The EOQ formula becomes the assumption that the amount of the demand is constant succession. However, the amount of the order and the order time in distribution center are irregular. For this reason, the EOQ equation prerequisites are greatly different. Therefore the formula of the EOQ cannot be applied. The inventory control of such irregular demand is generated in the inventory control of the spare parts of the mechanical equipment. The failure of the mechanical equipment occurs only unusually. It becomes so-called intermittent demand. The order quantity is calculated by using the shortage probability also for such inventory control on the assumption of normal distribution. However the distribution center shipping time is irregular, and shipment volume is also irregular. Moreover, the order for a lot of amounts is irregularly generated in the distribution center. For this reason it cannot be solved by conventional inventory control techniques. Here it is necessary to the new inventory control techniques.

VI. Cumulative Inflow Curve and Volume of Inventories

a) *Decision of cumulative inflow curve*

The inventory control is carried at the distribution center is a method by a past experience and intuition. In Figure 11 is the cumulative flow curve for item I4 and the Figure 12 is the volume of inventories. Figure 13 is the cumulative flow curve for item I5 and the Figure 14 is the volume of inventories. It is necessary to ship the amount (Outflow) of cumulative outflow by the order from the customer. Then, we can control at the distribution center is only an amount of the cumulative inflow from the vendor. The cumulative inflow (Inflow) Q shown in the upper row of figure 4 shows the amount of received of goods from the vendor. The problem is the method of deciding amount

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(Inflow) of the cumulative inflow curve in figure 4. It is necessary to decide the method how the time of received of goods and the amount of cumulative inflow. The volume of inventories can be reduced by efficiently doing the amount of received of goods and time of received of goods. The relation of the cumulative inflow amount ¿ amount of the cumulative outflow is approved from figure 4. The volume of inventories becomes the difference between the amount of cumulative inflow and the amount of the cumulative outflow. It proposes a new method of controlling the amount of the cumulative inflow to reduce the volume of inventories. When the difference is reduced, the volume of inventories is minimized. To reduce the volume of inventories, it proposes a new method here about time and inlet flow Q of the order to the amount of the accumulation inflow, that is, the vendor. When this difference is reduced, the volume of inventories is minimized. To reduce the volume of inventories, it proposes a new method here about time and inlet flow Q of the order to the amount of the accumulation inflow, that is, the vendor[11].

b) Inventory control by cumulative flow graph

Cumulative flow graph puts out the order with t01 as shown in Figure 15. The commodity arrives to tR1. The lead time at this time becomes tR1-t01. However, the commodity handled here is a general consumption material. Therefore, it will be delivered the next day when ordering. Therefore, it is thought that there is no lead time. Amount Q of the order is assumed to take the periodic reordering method. The reorder cycle is assumed to be TC. Amount Q of the order will be decided with forecasting of the actual data. The forecast period becomes TC. The forecasting value is calculated for actual data of Figure 15. At this time, the safety stock SS is put in the forecasting value or the safety stock is separately installed.

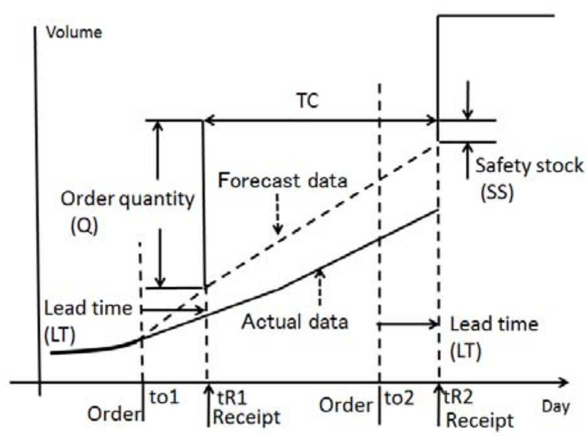

Fig. 15 : Order quantity of accumulative

In this research paper, the method of demand forecasting to validate the two methods according to the Holts exponential smoothing method and power approximation, polynomial approximation[11].

c) Order quantity of cumulative inflow by power and polynomial method

The forecast period is assumed to be TC. The forecast shows period p cycle TC by n. The predictive value becomes amount Q of the order. Amount Q of the order uses the forecast value by the exponential approximation and the polynomial approximation. The forecast time is assumed to be x. Stock shortage is caused when the forecast value is small. Therefore, the safety value Pσ of the upper bound is given to the forecast value. The expression of the forecast is calculated by using the past data. It forecasts p

Ref

11. Simichi D.Levi P, Kaminskty and Simichi E., 2008, *Designing and Managing the Supply Chain* (McGraw-Hill).

period ahead by the expression of the forecast of the past data of n period. As a result, the forecast by the power and the polynomial forecast becomes the next expression[8][9][10].

$$\hat{x}\ (t+p) = A(t+p)^{B} + p\sigma$$
$$\hat{x}\ (t+p) = A_1 + A_2(t+p) + A_3(t+p)x^2 + p\sigma \qquad (2)$$

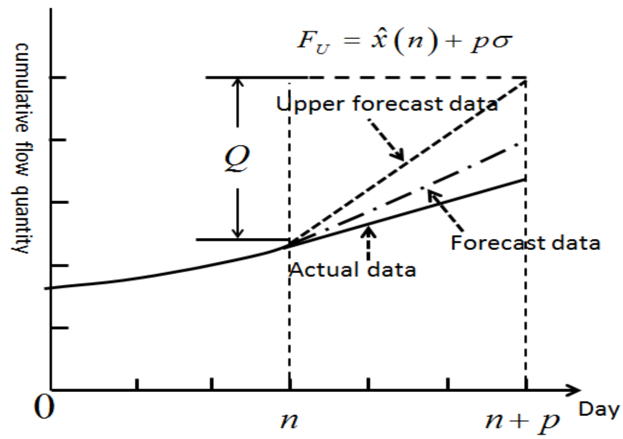

Fig. 16 : Forecast for order quantity

It applies to the commodity of Item3 shown in figure 9 to verify the proposed forecast method. The cumulative curve of Item3 becomes figure 10.

[Numerical Example 1]

(1) Power forecast method

It forecasts to the shipment data of eight days in the past by the Power method. The expression of the forecast becomes as follows.

$$\hat{x}\ (t) = 15.843 \cdot t^{0.9282} + p\sigma \qquad (3)$$

Order quantity adds and calculates safety stock Pσ in the expression of the forecast. As a result, order quantity became 82. Next, the prediction error occurs between a predictive value and actual value as shown in figure 17. Table 1 has shown the volume of inventories. The total of the volume of inventories becomes 158.

Table 1 : Inventory and order quantity with power method

week	8	9	10
Inventory	82	40	16
week	11	12	Total
Inventory	16	4	158

Ref

8. Abraham B. and Ledolter J, 2005, *Statistical Methods for Forecasting* (Wiley Inter-Science).

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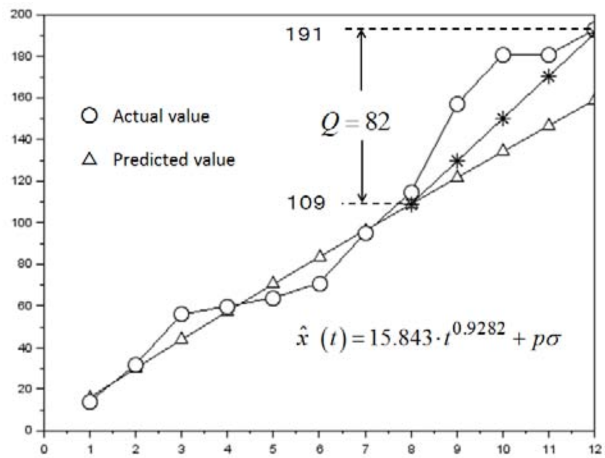

Fig. 17 : Power forecast of demand and order quantity

(2) Polynomial forecast method

In the same way as the power method to calculate it forecasts by polynomial to the shipment data of the past eight days. The expression of the forecast becomes as follows.

$$\hat{x}(t) = 8.5893 + 11.196 \cdot t + 0.1726 \cdot t^2 + p\sigma \tag{4}$$

Order quantity is calculated by adding the safety stock Pσ in the equation of prediction. As a result, order quantity became 108. Figure 17 has shown an expression of the forecast and actual data. Figure 17 has shown the relation between the safety stock and the volume of inventories when Order quantity is 100. Table 4 has shown the volume of inventories. The total volume of inventories becomes 218.

Table. 2 : Inventory and order quantity with polynomial method

week	8	9	10
Inventory	94	52	28
week	11	12	Total
Inventory	28	16	218

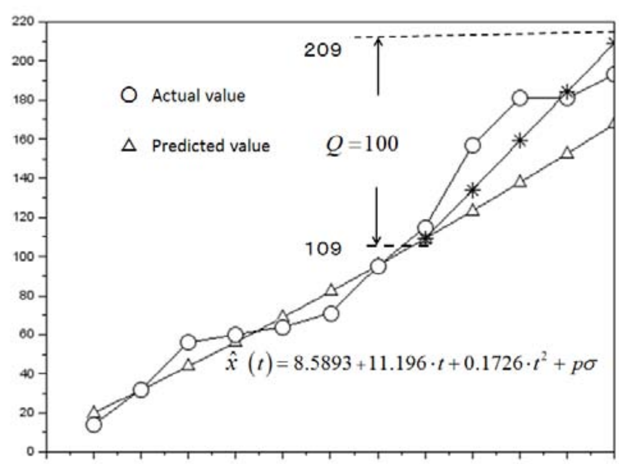

Fig. 18 : Polynomial forecast of demand and order quantity

d) Order quantity of cumulative inflow by Improved Holt's linear method

Linear exponential smoothing is found using two constants, α and β, and three equations. The equation is given as follow[8][9][10].

$$
\begin{aligned}
L_t &= \alpha \cdot Y_t + (1-\alpha)(L_{t-1} + b_{t-1}) \\
b_t &= \beta(L_t - L_{t-1}) + (1-\beta)b_{t-1} \\
Q_{t+k} &= L_t + b_t k + Z_t k
\end{aligned}
\tag{5}
$$

L_t denotes an estimate of the level and b_t denotes an estimate of slope of the time series at time t.

L_t adjusts for the trend of previous period. b_{t-1} by adding it to the last smoothed value, L_{t-1}. Equation b_t the updates the trend, which is expressed as the difference between the last two smoothed values.

The trend is modified by smoothing with β the trend in the last period ($L_t - L_{t-1}$), and adding that to the previous estimate of the trend multiplied by $(1-\beta)$. Finally Q_{t+k} is used to forecast ahead. This equation is different for Holt's linear method. The trend b_t is multiplied by the number of periods ahead to forecast k, and added to the base value of L_t [11].

[Numerical Example 2]

The predictor equation of Holt was calculated by using data here for eight weeks. Figure 19 shows the procedure for requesting the amount of the accumulation inflow from the forecast type of Holt of Item I1. The forecast value of t=8 and k=4 is calculated as α=0.8 and β=0.08. The forecast value of the seventh week becomes Q=67. In graph, triangle indicates the predicted value round is in the actual value. Table 7 shows the volume of inventories at this time.

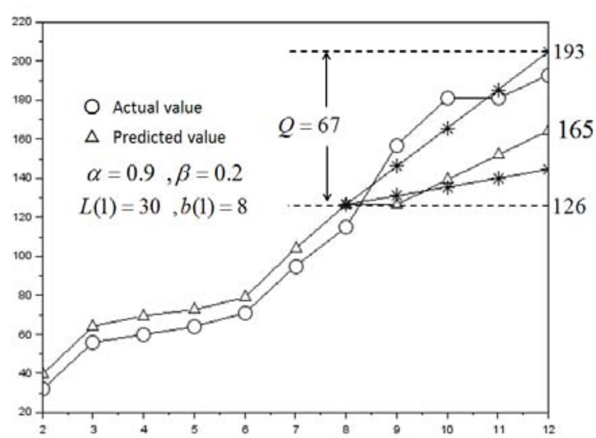

Fig. 19 : Forecast of demand and order quantity

Table. 3 : Inventory and order quantity with improved Holt's method

week	8	9	10
Inventory	78	36	12
week	11	12	Total
Inventory	12	0	138

Ref

11. Simichi D.Levi P, Kaminskty and Simichi E., 2008, *Designing and Managing the Supply Chain* (McGraw-Hill).

Ref

7. Makridakis S, Wheelwright S and Hyndman V, 1998, *Forecasting: Method and Applications* (John-Wiley and Son).

V. Comparison between Current method and Improvement Method

The volume of inventories of three proposed methods and current methods are compared. Table 4 is a method by the experience of the current method. The volume of inventories is examined for five week. The volume of inventories in the stock control managed in the distribution center experiencing becomes 1180. Compared to other methods of Table 5, the current management methods is clear that the amount of inventory is large. The improved Holt's method[7][8] is the most excellent clearly from Table 5. Because other forecasting methods are used power and straight line, the demand that fluctuates not adapted. For this, the prediction error grows. Power method is more excellent than the polynomial method because there are a lot of quantity safety stocks. The volume of inventories changes by setting the safety stock. Therefore, it is difficult to judge which of these two methods is excellent.

Table. 4 : Inventory quantity for current method

week	8	9	10
Inventory	275	233	209
week	11	12	Total
Inventory	239	227	1183

Table. 5 : Comparison between current method and proposed method

	Inventory quantity
Current method	1183
Power method	158
Polynomial method	218
Improved Holt's method	138

VI. Conclusion

Up to now, the method of theoretically computing the order quantity in the cumulative flow curve has not been proposed. This paper proposed the method of deciding the order quantity of the inventory control by the cumulative flow curve. Three proposed methods are shown in the calculation of the order quantity. The time and volumes of the shipments from the distribution center to the customers are irregular. With the cumulative flow curve, it is possible to inventory control of products that the irregular shipping. To provide useful forecasts of such irregular shipping times and volumes, a method utilizing the power, polynomial method and improved Holt's method are presented. The volume of inventories of three proposed methods and current methods are compared. By applying the cumulative flow curves, it is clarified that the inventory control is possible for irregular shipment products.

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Existence of Fixed Points of a Pair of Self Maps under Weak Generalized Geraghty Contractions in Complete Partially Ordered Partial b - Metric Spaces

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Abstract- We introduce the notion of (α, φ, β) - weak generalized Geraghty contractions in complete partially ordered partial b - metric spaces via triangular α-admissible mappings. We obtain sufficient conditions for the existence of fixed points of such maps in complete partially ordered partial b - metric spaces with coefficient $s \geq 1$, where φ is an altering distance function and $\beta \in \Omega$ where $\Omega = \{\beta : (0, \infty) \to [0, 1)$ satisfying $\beta(t_n) \to 1 \Rightarrow t_n \to 0\}$. Examples are provided to illustrate our results.

Keywords: fixed points, weak contraction, altering distance function, geraghty type contraction. coupled α – admissible, partial b - metric, complete partially ordered partial b - metric space.

GJSFR-F Classification : *MSC 2010: 30L05*

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9. Banach.S: *Sur les operations dans les ensembles abstraits et leur application oux equations integrals, Fundamenta Mathematicae, 3(1922), 133-181. 1.*

Existence of Fixed Points of a Pair of Self Maps under Weak Generalized Geraghty Contractions in Complete Partially Ordered Partial b - Metric Spaces

Vedula Perraju

Abstract- We introduce the notion of (α, φ, β) - weak generalized Geraghty contractions in complete partially ordered partial b - metric spaces via triangular α-admissible mappings. We obtain sufficient conditions for the existence of fixed points of such maps in complete partially ordered partial b - metric spaces with coefficient $s \geq 1$, where φ is an altering distance function and $\beta \in \Omega$ whe re $\Omega = \{\beta : (0, \infty) \to [0, 1)$ satisfying $\beta(t_n) \to 1 \Rightarrow t_n \to 0\}$. Examples are provided to illustrate our results.

Keywords: fixed points, weak contraction, altering distance function, geraghty type contraction. coupled α – admissible, partial b - metric, complete partially ordered partial b - metric space.

I. Introduction and Preliminaries

Most of the fixed point theorems in nonlinear analysis usually start with Banach [9] contraction principle. A huge amount of literature is witnessed on applications, generalizations and extensions of this principle carried out by several authors in different directions like weakening the hypothesis and considering different mappings. But all the generalizations may not be from this principle. In 1989, Bakktin [8] introduced the concept of a b - metric space as a generalization of a metric space. In 1993, Czerwik[11] extended many results related to the b - metric space. In 1994, Matthews [20] introduced the concept of partial metric space in which the self distance of any point of space may not be zero. In 1996, O'Neill [28] generalized the concept of partial metric space by admitting negative distances. In 2013, Shukla [35] generalized both the concepts of b - metric and partial metric space by introducing the notation of partial b - metric spaces. Many authors recently studied the existence of fixed points of self maps in different types of metric spaces [16,35,23,32,38]. Some authors [4,20,24,30,31] obtained some fixed point theorems in b - metric spaces . After that some authors proved $\alpha - \psi$ versions of certain fixed point theorems in different types of metric

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spaces [3,16]. Recently Samet et.al [29] and Jalal Hassanzadeasl [14] obtained fixed point theorems for $\alpha - \psi$ contractive mappings . Mustafa [24] gave a generalization of Banach contraction principle in complete ordered partial b - metric space by using the notion of a generalized $\alpha - \psi$ weakly contractive mapping. Babu et.al[5] proved coupled fixed point theorems by using (α, φ, β) - weak generalized Geraghty contraction. In 2012, Mohammad Mursaleen et.al [22] proved coupled fixed point theorems for α - ψ contractive type mappings in partially ordered metric spaces.
In this paper we extend the concepts of G. V. R. Babu. et.al.[6] to complete partially ordered partial b- metric space with coefficient $s \geq 1$ and obtain sufficient conditions for the existence of fixed points of weak generalized Geraghty contractions in a complete partially ordered partial b - metric space with coefficient $s \geq 1$. A supporting example is also given. Further an open problem is also given at the end of this paper.
Shukla [35] introduced the notation of a partial b - metric space as follows.

Definition 1.1. (S.Shukla [35]) Let X be a non empty set and let $s \geq 1$ be a given real number. A function $p : X \times X \to [0, \infty)$ is called a partial b - metric if for all $x, y, z \in X$ the following conditions are satisfied.

(i) $x = y$ if and only if $p(x, x) = p(x, y) = p(y, y)$

(ii) $p(x, x) \leq p(x, y)$

(iii) $p(x, y) = p(y, x)$

(iv) $p(x, y) \leq s\{p(x, z) + p(z, y)\} - p(z, z)$

The pair (X, p) is called a partial b - metric space.
The number $s \geq 1$ is called a coefficient of (X, p).

Definition 1.2. (Z.Mustafa.et.al.[24]) A sequence $\{x_n\}$ in a partial b - metric space (X, p) is said to be:

(i) convergent to a point $x \in X$ if $\lim_{n\to\infty} p(x_n, x) = p(x, x)$

(ii) a Cauchy sequence if $\lim_{n,m\to\infty} p(x_n, x_m)$ exists and is finite

(iii) a partial b - metric space (X, p) is said to be complete if every Cauchy sequence $\{x_n\}$ in X converges to a point $x \in X$ such that
$\lim_{n,m\to\infty} p(x_n, x_m) = \lim_{n\to\infty} p(x_n, x) = p(x, x)$.

Definition 1.3. (E.Karapinar. et.al [18]) Let $(X, \leq)$ be a partially ordered set. A sequence $\{x_n\} \in X$ is said to be non decreasing,
if $x_n \leq x_{n+1} \forall\, n \in \mathbb{N}$

Definition 1.4. (Z.Mustafa.et.al.[24]) A triple $(X, \leq, p)$ is called an ordered partial b - metric space if $(X, \leq)$ is a partially ordered set and p is a partial b - metric on X. In Sastry et.al[30], the notion of a partially ordered partial metric space is introduced.

Ref

14. Jalal Hassanzadeasl: *Common Fixed point theorems for α–φ cnotractivetype mappings, International Journal of analysis, (2013).*

For definiteness sake Sastry et.al[30](Definition 2.1) adopting the definition 1.1 for partial metric and defined the triple $(X, \leq, p)$ as partially ordered partial metric space.
A partially ordered partial metric space $(X, \leq, p)$ is said to be complete if every Cauchy sequence is convergent.

Notation: The following notation is used throughout this paper.
$(X, \leq, p)$ be a complete partially ordered partial b - metric space with coefficient $s \geq 1$ and we write it as X. Let $T : X \to X$ be a self map of X and let $Fix(T)$ denote the set of all fixed points of T . We denote $\Omega = \{\beta : (0, \infty) \to [0, 1)/\beta(t_n) \to 1 \Rightarrow t_n \to 0\}$, and that $\Phi_s = \{\varphi : [0, \infty) \to [0, \infty)/\varphi$ is non-decreasing, continuous, $\lim\limits_{t \to r^+} \varphi(t) < \frac{r}{s}$ and $\varphi(t) = 0 \Leftrightarrow t = 0\}$. We call the elements of Φ_s as altering distance functions.
Further, we use the following notation: for any sequences $\{a_n\}$ and $\{b_n\}$ in X with

$p_n = p(a_n, b_n) \neq 0$,

we write $\Delta_n = \frac{p(T(a_n),T(b_n))}{p_n}$ and $\Delta_n^{\varphi} = \frac{\varphi(sp(T(a_n),T(b_n)))}{\varphi(p_n)}$ $\forall$ n,

We denote the set of all real numbers by $\mathbb{R}$, the set of all nonnegative reals by $\mathbb{R}^+$ and the set of all natural numbers by $\mathbb{N}$.

Definition 1.5. (I.Beg and A.R.Butt [10]) Let $(X, \leq)$ be a partially ordered set and $S, T : X \to X$ be such that $Sx \leq TSx$ and $Tx \leq STx$ $\forall x \in X$. Then S and T are said to be weakly increasing mappings.

Definition 1.6. (B.Samet et.al.[29])Let $T : X \to X$ be a self map and $\alpha : X \times X \to \mathbb{R}$ be a function. Then T is said to be α - admissible if $\alpha(x, y) \geq 1 \Rightarrow \alpha(Tx, Ty) \geq 1$.

Definition 1.7. $(JalalHassanzadeasl.,$ [14]) Let $T, S : X \to X$, and let
$\alpha : X \times X \to [0, +\infty)$. We say that T, S are coupled $\alpha -$ admissible if
$\alpha(x, y) \geq 1 \Rightarrow \alpha(Tx, Sy) \geq 1$ and $\alpha(Sx, Ty) \geq 1$ for all $x, y \in X$

Definition 1.8. (E. Karapinar. et.al. [18]) An α - admissible map T is said to be triangular $\alpha -$ admissible if $\alpha(x, z) \geq 1$ and $\alpha(z, y) \geq 1 \Rightarrow \alpha(x, y) \geq 1$.

Lemma 1.9. *(E.Karapinar. et.al.[18]) Let $T : X \to X$ be triangular $\alpha -$ admissible map. Assume that there exists $x_1 \in X \ni \alpha(x_1, Tx_1) \geq 1$. Define the sequence $\{x_n\}$ by $x_{n+1} = Tx_n,\ n = 0, 1, 2, \ldots$. Then we have $\alpha(x_n, x_m) \geq 1$ for all $m, n \in \mathbb{N}$ with $n < m$.*

For more details and examples on $\alpha -$ admissible and coupled $\alpha -$ admissible maps, one can refer [17], [18] and [29].
The following lemma can be easily established.

Definition 1.10. (S. H. Cho. et.al.[13]) Let (X, d) be a metric space, and let $\alpha : X \times X \to \mathbb{R}$ be a function. A map $T : X \to X$ is called an $\alpha -$ Geraghty type contraction if there exists $\beta \in \Omega$ such that

$$\alpha(x, y)d(Tx, Ty) \leq \beta(d(x, y))d(x, y) \text{ for all } x, y \in X. \tag{1.10.1}$$

Ref

18. Karapinar.E, Kumam.E, Salimi.P: *On $\phi - \psi$ Meir-Keeler contractive mappings, Fixed Point Theory and Applications, 2013, 2013/1/94.*

Definition 1.11. (G. V. R. Babu. et.al.[6]) Let (X, d) be a metric space and $T : X \to X$ be a self map. If there exist $\alpha : X \times X \to \mathbb{R}, \varphi \in \Phi$ and $L \geq 0$ such that

$$\alpha(x, y)\varphi((d(Tx, Ty))) < \varphi((M(x, y))) + L.N(x, y) \tag{1.11.1}$$

for all $x, y \in X, x \neq y$ where

$M(x, y) = max\{d(x, y), d(x, Tx), d(y, Ty), \frac{1}{2}[d(x, Ty) + d(y, Tx)]\}$,

and $N(x, y) = min\{d(x, Tx), d(x, Ty), d(y, Tx)\}$, then we say that T is an almost generalized $\alpha -$ contractive map with respect to an altering distance function φ.

Definition 1.12. (G. V. R. Babu. et.al.[6]) Let (X, d) be a metric space let $T : X \to X$ be a self map. If $\exists\ \alpha : X \times X \to \mathbb{R}, \beta \in \Omega, \varphi \in \Phi$ and $L \geq 0 \ni \alpha(x, y)\varphi((d(Tx, Ty))) \leq$

$$\beta(\varphi(M(x, y)))\varphi(M(x, y)) + L.N(x, y) \tag{1.12.1}$$

for all $x, y \in X$, where

$M(x, y) = max\{d(x, y), d(x, Tx), d(y, Ty), \frac{1}{2}[d(x, Ty) + d(y, Tx)]\}$,

$N(x, y) = min\{d(x, Tx), d(x, Ty), d(y, Tx)\}$,

then we say that T is (α, φ, β) - weak generalized Geraghty contractive map.

The following theorems are established in (K.P.R,Sastry et.al.[32]).

Theorem 1.13. *(K.P.R,Sastry et.al.[32]) Let T be a self map on a complete partially ordered partial b - metric space $(X, \leq, p)$ with coefficient $s \geq 1$. Let $\alpha : X \times X \to \mathbb{R}$ be a continuous function and*

$\alpha(x, x) > 1\ \forall\ x \in X$.

Assume that there exists $\varphi \in \Phi_s$ such that

$$\alpha(x, y)\varphi(sp(Tx, Ty)) < \varphi(M(x, y)) \tag{1.13.1}$$

for all $x, y \in X, p(x, y) \neq 0$ where

$M(x, y) = max\{p(x, y), p(x, Tx), p(y, Ty), \frac{1}{2s}[p(x, Ty) + p(Tx, y)]\}$

Further, assume that

(i) T is triangular α - admissible, and

(ii) there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$

and set $x_n = Tx_{n-1}$ for $n = 1, 2, 3, \dots$.

(iii) for any two sequences $\{a_n\}$ and $\{b_n\}$ in X with $p_n = p(a_n, b_n) \neq 0$, we have that $\Delta_n^{\varphi} \to 1 \Rightarrow \varphi(p_n) \to 0$ as $n \to \infty$.

Then $\{x_n\}$ is a Cauchy sequence. And if $x_n \to z$ then z is a fixed point T in X. Further if y, z are fixed point of T in X, then either $\alpha(y, z) < 1$ or $y = z$.

Corollary 1.14. *(K.P.R,Sastry et.al.[32]) Let T be a self map on a complete partially ordered partial b - metric space X. Let $\alpha : X \times X \to \mathbb{R}$ be a continuous function. Assume that there exists $\varphi \in \Phi_s$ such that*

$$\alpha(x, y)\varphi(sp(Tx, Ty)) < \varphi(M(x, y)) \tag{1.14.1}$$

for all $x, y \in X, p(x, y) \neq 0$ where

$M(x, y) = max\{p(x, y), p(x, Tx), p(y, Ty), \frac{1}{2s}[p(x, Ty) + p(Tx, y)]\}$

Ref

6. Babu.G.V.R, Sarma.K.K.M, and Krishna.P.H: *Necessary and sufficient conditions for the existence of fixed points of weak generalized geraghty contractions international journal of mathematics and scientific computing (issn: 2231-5330), vol. 5, no. 1, 2015 38*

Further, assume that
(i)T is α - triangular admissible,
(ii) there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$
and set $x_n = Tx_{n-1}$ for $n = 1, 2, 3, \ldots.$
(iii)for any two sequences $\{a_n\}$ and $\{b_n\}$ of X with $p_n = p(a_n, b_n) \neq 0$, we have that

$$\Delta_n^{\varphi} \to 1 \Rightarrow \varphi(p_n) \to 0 \text{ as } n \to \infty \qquad (1.14.2)$$

Then the sequence $\{x_n\}$ is a Cauchy sequence. Suppose $\{x_n\}$ converges to z and $\alpha(z, z) > 1$. Then z is a fixed point of T in X.

Theorem 1.15. *(K.P.R,Sastry et.al.[32]) Let $(X, \leq, p)$ be a complete partially ordered partial b - metric space with coefficient $s \geq 1$,and $T : X \to X$ be a self map. Let $\alpha : X \times X \to \mathbb{R}$ be a continuous function and $\beta \in \Omega, \varphi \in \Phi_s$.*
Suppose the following conditions hold:
(i) T is (α, φ, β) - weak generalized Geraghty contraction map i.e.,

$$\alpha(x, y)\varphi(sp(Tx, Ty)) \leq \beta(\varphi(M(x, y)))\varphi(M(x, y)) \; \forall \, x, y \in X, p(x, y) \neq 0 \qquad (1.15.1)$$

where

$$M(x, y) = max\{p(x, y), p(x, Tx), p(y, Ty), \tfrac{1}{2s}[p(x, Ty) + p(Tx, y)]\}$$

(ii) T is triangular α - admissible, and
(iii) there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$
and set $x_n = Tx_{n-1}$ for $n = 1, 2, 3, \ldots$
Then $\{x_n\}$ is a Cauchy sequence. Suppose $\{x_n\}$ converges to x and $\alpha(x, x) > 1$. Then x is a fixed point of T in X

These results are extended to partially ordered partial b - metric space for a pair of self maps in the next session.

II. Main Result

In this section we continue our study to extend the concepts of G. V. R. Babu. et.al.[6] to complete partially ordered partial b- metric space with coefficient $s \geq 1$ and obtain sufficient conditions for the existence of fixed points of weak generalized Geraghty contractions in a complete partially ordered partial b- metric space with coefficient $s \geq 1$. A supporting example is given. Further an open problem is also given at the end of this paper.
We begin this section with the following definition.

Definition 2.1. (K.P.R,Sastry et.al.[30]) Suppose $(X, \leq)$ is a partially ordered set and p is a partial b - metric on X as in definition 1.1 with coefficient $s \geq 1$. Then we say that the triplet $(X, \leq, p)$ is a partially ordered partial b - metric space. Notions of convergence of a sequence and Cauchy sequence are as in definition 1.2. A partially ordered partial b - metric space $(X, \leq, p)$ is said to be complete if every Cauchy sequence in X is convergent .

Ref

6. Babu.G.V.R, Sarma.K.K.M, and Krishna.P.H: *Necessary and sufficient conditions for the existence of fixed points of weak generalized geraghty contractions international journal of mathematics and scientific computing (issn: 2231-5330), vol. 5, no. 1, 2015 38.*

Now we state the following useful lemmas, whose proofs can be found in Sastry. et. al.[30].

Lemma 2.2. *Let $(X, \leq, p)$ be a complete partially ordered partial b - metric space. Let $\{x_n\}$ be a sequence in X such that $\lim_{n\to\infty} p(x_n, x_{n+1}) = 0$. Suppose $\lim_{n\to\infty} x_n = x$ and $\lim_{n\to\infty} x_n = y$*

Then (a) $\lim_{n\to\infty} p(x_n, x) = \lim_{n\to\infty} p(x_n, y) = p(x, y)$ and hence $x = y$

(b) (i) $\{x_n\}$ is a Cauchy sequence $\Rightarrow \lim_{m,n\to\infty} p(x_m, x_n) = 0$.

(ii) $\{x_n\}$ is not a Cauchy sequence $\Rightarrow \exists\ \epsilon > 0$ and sequences $\{m_k\}, \{n_k\} \ni m_k > n_k > k \in N$; $p(x_{n_k}, x_{m_k}) > \epsilon$ and $p(x_{n_k}, x_{m_k-1}) \leq \epsilon$

Lemma 2.3. *(a) $p(x, y) = 0 \Rightarrow x = y$*

(b) $p(x_{n_k-1}, x_{m_k-1}) \neq 0$

Proof. : Suppose $p(x_{n_k-1}, x_{m_k-1}) = 0$

$\Rightarrow x_{n_k-1} = x_{m_k-1}$

$\Rightarrow Tx_{n_k-1} = Tx_{m_k-1}$

$\therefore x_{n_k} = x_{m_k}$

Now $\epsilon < p(x_{n_k}, x_{m_k}) = p(x_{n_k}, x_{n_k}) < p(x_{n_k}, x_{n_k-1})$

Allowing $k \to \infty$

$\epsilon \leq \lim_{k\to\infty} p(x_{n_k}, x_{n_k-1}) = 0$, a contradiction.

$\therefore p(x_{n_k-1}, x_{m_k-1}) \neq 0$

Lemma 2.4. *If $\varphi \in \Phi_s$ then*

(i) $\lim_{n\to\infty} \varphi^n(t) = 0 \quad \forall\ t > 0$

(ii) $\varphi(t) < \frac{t}{s}\ \forall\ t > 0$ where $s \geq 1$ is the coefficient of (X, p)

Definition 2.5. Let $(X, \leq, p)$ be a complete partially ordered partial b - metric space with coefficient $s \geq 1$. Let $T : X \to X$ be a self map. If there exist $\alpha : X \times X \to \mathbb{R}, \beta \in \Omega, \varphi \in \Phi_s$ such that

$$\alpha(x, y)\varphi(sp(Tx, Sy)) \leq \beta(\varphi(M(x, y)))\varphi(M(x, y)) \tag{2.5.1}$$

for all $x, y \in X$, where

$M(x, y) = max\{p(x, y), p(x, Tx), p(y, Ty), \frac{1}{2s}[p(x, Ty) + p(Tx, y)]\}$. Then we say that T is (α, φ, β) - weak generalized Geraghty contractive map.

Notation: The following notation is used throughout this paper. $(X, \leq, p)$ be a complete partially ordered partial b - metric space with coefficient $s \geq 1$ and we write it as X. Let $S, T : X \to X$ be a self map of X and let $Fix(T)$ denotes the set of all fixed points of S and T . We denote $\Omega = \{\beta : (0, \infty) \to [0, 1)/\beta(t_n) \to 1 \Rightarrow t_n \to 0\}$, and that $\Phi_s = \{\varphi : [0, \infty) \to [0, \infty)/\varphi$ is non-decreasing, continuous, $\lim_{t\to r^+} \varphi(t) < \frac{r}{s}$ and $\varphi(t) = 0 \Leftrightarrow t = 0\}$. We call the elements of Φ_s as altering distance functions.

Notes

Notes

Further, we use the following notation: for any sequences $\{a_n\}$ and $\{b_n\}$ in X with $p_n = p(a_n, b_n) \neq 0$,
we write $\Delta_n = \frac{p(S(a_n),T(b_n))}{p_n}$ and $\Delta_n^{\varphi} = \frac{\varphi(sp(S(a_n),T(b_n)))}{\varphi(p_n)}$ $\forall$ n,
We denote the set of all real numbers by $\mathbb{R}$, the set of all nonnegative reals by $\mathbb{R}^+$ and the set of all natural numbers by $\mathbb{N}$.

Now we state our first main result :

Theorem 2.6. *Let S, T be weakly increasing self maps on a complete partially ordered partial b - metric space $(X, \leq, p)$ with coefficient $s \geq 1$. Let $\alpha : X \times X \to \mathbb{R}$ be a continuous function and*
$\alpha(x, x) > 1 \ \forall \ x \in X$.
Assume that there exists $\varphi \in \Phi_s$ such that

$$\alpha(x, y)\varphi(sp(Tx, Sy)) < \varphi(M(x, y)) \tag{2.6.1}$$

for all $x, y \in X, p(x, y) \neq 0$ where
$M(x, y) = max\{p(x, y), p(x, Tx), p(y, Sy), \frac{1}{2s}[p(y, Tx) + p(Sy, x)]\}$
Further, assume that
(i) S, T are weakly increasing
(ii) S, T are coupled and triangular α - admissible,
(iii) there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$
and set $x_n = Tx_{n-1}$ for $n = 1, 2, 3, ...$.
(iv) for any two sequences $\{a_n\}$ and $\{b_n\}$ in X with $p_n = p(a_n, b_n) \neq 0$, we have that $\Delta_n^{\varphi} \to 1 \Rightarrow \varphi(p_n) \to 0$ as $n \to \infty$.
Then $\{x_n\}$ is a Cauchy sequence. And if $x_n \to z$ then z is a common fixed point T and S in X. Further if y, z are fixed point of T in X, then either $\alpha(y, z) < 1$ or $y = z$.

Proof. We first prove that any fixed point of T is also a fixed point of S and conversely.
Let x be a fixed point of T.
Then $Tx = x$
Now $M(x, x) = max\{p(x, x), p(Tx, x), p(Sx, x), \frac{1}{2}[p(Tx, x) + p(Sx, x)]\}$
$= p(Sx, x)$

$$\begin{aligned}\therefore \varphi(p(x, Sx)) &= \varphi(d(Tx, Sx)) \\ &\leq \alpha(x, x)\varphi(d(Tx, Sx)) \\ &< \varphi\ (M(x, x))) \\ &= \varphi\ (p(x, Sx)))\end{aligned}$$

a contradiction, if $p(x, Sx) \neq 0$
$\therefore p(x, Sx) = 0$
$\therefore$ by lemma 2.3 $Sx = x$
Similarly if $Sx = x$ then $Tx = x$.
Further we show that if T and S have a common fixed point then it is unique.
Let $Tx = Sx = x$ and $Ty = Sy = y$

Suppose $\alpha(x,y) < 1$ then there is nothing to prove.

To show that $x = y$. Suppose $x \neq y$

We have $M(x,y) = max\{p(x,y), p(Tx,x), p(Sy,y), \frac{1}{2}[p(Tx,y) + p(Sy,x)]\}$

$= p(x,y)$

$\therefore \varphi(p(x,y)) = \varphi(p(Tx,Sy))$

$\leq \alpha(x,y)\varphi(M(x,y))$

$= \alpha(x,y)\varphi\ (p(x,y)) < \varphi\ (p(x,y))$, a contradiction

$\therefore \varphi(p(x,y)) = 0 \Rightarrow p(x,y) = 0 \Rightarrow x = y$

Let $x_0 \in X$ and $x_{2n+1} = Tx_{2n}$;

$x_{2n+2} = Sx_{2n+1}$; $n = 0,1,2...$

For any n suppose $x_{n+1} = x_n$

Now $n = 2m$

$\Rightarrow x_{2m+1} = x_{2m}$

$\Rightarrow Tx_{2m} = x_{2m}$

$\Rightarrow x_n$ is a fixed point of T.

$n = 2m+1$

$\Rightarrow x_{2m+2} = x_{2m+1}$

$Sx_{2m+1} = x_{2m+1}$

$\Rightarrow x_n$ is a fixed point of S

$\therefore$ For any n if $x_{n+1} = x_n$ then x_n is a common fixed point of T and S.

Hence for any n,we suppose that $x_{n+1} \neq x_n$ for all $n \in \mathbb{N}$

Since S and T are weakly increasing,

$x_1 = Tx_0 \leq STx_0 = Sx_1 = x_2 \leq TSx_1 = Tx_2 = x_3$ ....

$\therefore x_1 \leq x_2 \leq x_3 \leq ...$ Thus $\{x_n\}$ is increasing.

Let $x_0 \in X$ be such that $\alpha(x_0, Tx_0) \geq 1$ by (iii). Without loss of generality, we assume that $x_n \neq x_{n+1}$ for all $n \in \mathbb{N}$. By using the α - admissibility of T, we have $\alpha(x_0,x_1) = \alpha(x_0,Tx_0) \geq 1 \Rightarrow \alpha(x_1,x_2) = \alpha(Tx_0,Sx_1) \geq 1$. Now, by mathematical induction, it is easy to see that $\alpha(x_n,x_{n+1}) \geq 1$ for all $n \in \mathbb{N}$.

Let n be even and by taking $x = x_{n-1}$ and $y = x_n$ in the inequality (2.6.1), and observing that $p(x_{n-1},x_n) \neq 0$ by lemma 2.3,

we get

$\varphi(p(x_n,x_{n+1}))$

$\leq \varphi(sp(x_n,x_{n+1}))$

$= \varphi(sp(Sx_{n-1},Tx_n))$

$\leq \alpha(x_{n-1},x_n)\varphi(sp(Sx_{n-1},Tx_n))$

$< \varphi(M(x_n,x_{n-1}))$ (2.6.2)

where

$M(x_n,x_{n-1})$

$= max\{p(x_{n-1},x_n), p(x_{n-1},Sx_{n-1}), p(x_n,Tx_n), \frac{1}{2s}[p(x_{n-1},Tx_n) + p(x_n,Sx_{n-1})]\}$

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$= max\{p(x_{n-1}, x_n), p(x_{n-1}, x_n), p(x_n, x_{n+1}), \frac{1}{2s}[p(x_{n-1}, x_{n+1}) + p(x_n, x_n)]\}$
$\leq max\{p(x_{n-1}, x_n), p(x_n, x_{n+1}), \frac{1}{2s}[sp(x_{n-1}, x_n) + sp(x_n, x_{n+1}) - p(x_n, x_n) + p(x_n, x_n)]\}$
$= max\{p(x_{n-1}, x_n), p(x_n, x_{n+1}), \frac{1}{2}[p(x_{n-1}, x_n) + p(x_n, x_{n+1})]\}$
$= max\{p(x_{n-1}, x_n), p(x_n, x_{n+1})\}$

If $max\{p(x_{n-1}, x_n), p(x_n, x_{n+1})\} = p(x_n, x_{n+1})$ for some $n \in \mathbb{N}$ (2.6.3)

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then from (2.6.2) and (2.6.3), we have

$\varphi(sp(x_n, x_{n+1})) < \varphi(M(x_{n-1}, x_n)) = \varphi(p(x_n, x_{n+1}))$, a contradiction.

Thus, we have $M(x_{n-1}, x_n) = max\{p(x_{n-1}, x_n), p(x_n, x_{n+1})\} = p(x_{n-1}, x_n)$ Similarly, Let n be odd and by taking $x = x_{n-1}$ and $y = x_n$ in the inequality (2.6.1), and observing that $p(x_{n-1}, x_n) \neq 0$ by lemma 2.3,

we get

$\varphi(p(x_n, x_{n+1}))$
$\leq \varphi(sp(x_n, x_{n+1}))$
$= \varphi(sp(Tx_{n-1}, Sx_n))$
$\leq \alpha(x_{n-1}, x_n)\varphi(sp(Tx_{n-1}, Sx_n))$
$< \varphi(M(x_{n-1}, x_n))$ (2.6.4)

where

$M(x_{n-1}, x_n)$
$= max\{p(x_{n-1}, x_n), p(x_{n-1}, Tx_{n-1}), p(x_n, Sx_n), \frac{1}{2s}[p(x_{n-1}, Sx_n) + p(x_n, Tx_{n-1})]\}$
$= max\{p(x_{n-1}, x_n), p(x_{n-1}, x_n), p(x_n, x_{n+1}), \frac{1}{2s}[p(x_{n-1}, x_{n+1}) + p(x_n, x_n)]\}$
$\leq max\{p(x_{n-1}, x_n), p(x_n, x_{n+1}), \frac{1}{2s}[sp(x_{n-1}, x_n) + sp(x_n, x_{n+1}) - p(x_n, x_n) + p(x_n, x_n)]\}$
$= max\{p(x_{n-1}, x_n), p(x_n, x_{n+1}), \frac{1}{2}[p(x_{n-1}, x_n) + p(x_n, x_{n+1})]\}$
$= max\{p(x_{n-1}, x_n), p(x_n, x_{n+1})\}$

If $max\{p(x_{n-1}, x_n), p(x_n, x_{n+1})\} = p(x_n, x_{n+1})$ for some $n \in \mathbb{N}$ (2.6.5)

then from (2.6.2) and (2.6.3), we have

$\varphi(sp(x_n, x_{n+1})) < \varphi(M(x_{n-1}, x_n)) = \varphi(p(x_n, x_{n+1}))$, a contradiction.

Thus, we have $M(x_{n-1}, x_n) = max\{p(x_{n-1}, x_n), p(x_n, x_{n+1})\} = p(x_{n-1}, x_n)$ for all $n \in \mathbb{N}$ and hence, $p(x_n, x_{n+1}) < p(x_{n-1}, x_n)$ for all $n \in \mathbb{N}$. (2.6.6)

Thus it follows that $\{p(x_n, x_{n+1})\}$ is a non-negative, decreasing sequence of real numbers. Suppose that $\lim_{n\to\infty} p(x_n, x_{n+1}) = r, r \geq 0$

Now we prove that $r = 0$.

Assume that $r > 0$.

Now by (2.6.2)

$\varphi(p(x_n, x_{n+1}))$
$\leq \varphi(sp(x_n, x_{n+1}))$
$< \varphi(M(x_{n-1}, x_n))$
$= \varphi(p(x_{n-1}, x_n))$ for all $n \in N$

On taking limits as $n \to \infty$,

we have,

$$\lim_{n\to\infty} \varphi(p(x_n, x_{n+1}))$$
$$\leq \lim_{n\to\infty} \varphi(sp(x_n, x_{n+1}))$$
$$\leq \lim_{n\to\infty} \varphi(p(x_{n-1}, x_n))$$
$$\Rightarrow \varphi(r) \leq \varphi(sr) \leq \varphi(r)$$
$$\Rightarrow \varphi(r) = \varphi(sr)$$

Let n be odd. By choosing $a_n = x_n, b_n = x_{n+1}$,

$$\Delta_n^{\varphi} = \frac{\varphi(sp(T(x_{n-1}), S(x_n))}{\varphi(p(x_{n-1}, x_n))}$$
$$\therefore \lim_{n\to\infty} \Delta_n^{\varphi}$$
$$= \lim_{n\to\infty} \frac{\varphi(sp(x_n, x_{n+1}))}{\varphi(p(x_{n-1}, x_n))}$$
$$= \frac{\lim_{n\to\infty} \varphi(sp(x_n, x_{n+1}))}{\lim_{n\to\infty} \varphi(p(x_{n-1}, x_n))}$$
$$= \frac{\varphi(s \lim_{n\to\infty} p(x_n, x_{n+1}))}{\varphi(\lim_{n\to\infty} p(x_{n-1}, x_n))}$$
$$= \frac{\varphi(sr)}{\varphi(r)}$$
$$= 1(\text{since } \varphi(r) = \varphi(sr)) \tag{2.6.7}$$

Hence by our assumption $\varphi(p_n) \to 0$ as $n \to \infty$ i.e.,

$$\lim_{n\to\infty} \varphi(p(x_{n-1}, x_n)) = 0$$
$$\Rightarrow \varphi(r) = 0$$

$\Rightarrow r = 0$, a contradiction for our assumption $r > 0$

Hence $r = 0$.

Similarly,

Let n be even. By choosing $a_n = x_n, b_n = x_{n+1}$,

$$\Delta_n^{\varphi} = \frac{\varphi(sp(S(x_{n-1}), T(x_n))}{\varphi(p(x_{n-1}, x_n))}$$
$$\therefore \lim_{n\to\infty} \Delta_n^{\varphi}$$
$$= \lim_{n\to\infty} \frac{\varphi(sp(x_n, x_{n+1}))}{\varphi(p(x_{n-1}, x_n))}$$
$$= \frac{\lim_{n\to\infty} \varphi(sp(x_n, x_{n+1}))}{\lim_{n\to\infty} \varphi(p(x_{n-1}, x_n))}$$
$$= \frac{\varphi(s \lim_{n\to\infty} p(x_n, x_{n+1}))}{\varphi(\lim_{n\to\infty} p(x_{n-1}, x_n))}$$
$$= \frac{\varphi(sr)}{\varphi(r)}$$
$$= 1(\text{since } \varphi(r) = \varphi(sr)) \tag{2.6.8}$$

Hence by our assumption $\varphi(p_n) \to 0$ as $n \to \infty$ i.e.,

$$\lim_{n\to\infty} \varphi(p(x_{n-1}, x_n)) = 0 \ \forall \ n \in \mathbb{N}$$
$$\Rightarrow \varphi(r) = 0$$

$\Rightarrow r = 0$, a contradiction for our assumption $r > 0$

Hence $r = 0$.

Now, we show that $\{x_n\}$ is a Cauchy sequence in X. Suppose that $\{x_n\}$ is not a Cauchy sequence. Then by Lemma 2.2(b), there exist some $\epsilon > 0$, and sub-sequences $\{x_{m_k}\}$

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and $\{x_{n_k}\}$ of $\{x_n\}$ with $m_k > n_k > k$ such that $p(x_{m_k}, x_{n_k}) \geq \epsilon$ and $p(x_{m_k-1}, x_{n_k}) < \epsilon$ and by lemma 1.9

We have Case(i): Let m_k is odd and n_k is even

$\therefore s\epsilon \leq sp(x_{m_k}, x_{n_k})$

$\Rightarrow \varphi(s\epsilon) \leq \varphi(sp(x_{m_k}, x_{n_k}))$

Notes

$= \varphi(sp(Tx_{m_k-1}, Sx_{n_k-1}))$

$\leq \alpha(x_{m_k-1}, x_{n_k-1})\ \varphi(sp(Tx_{m_k-1}, Sx_{n_k-1}))$

(by lemma 1.9, $\alpha(x_{m_k-1}, x_{n_k-1}) \geq 1$)

$< \varphi(M(x_{m_k-1}, x_{n_k-1}))$ (2.6.9)

where $M(x_{m_k-1}, x_{n_k-1})$

$= max[p(x_{m_k-1}, x_{n_k-1}),\ p(x_{n_k-1}, Sx_{n_k-1}),\ p(x_{m_k-1}, Tx_{m_k-1}),$

$\frac{1}{2s}\ \{p(x_{m_k-1}, Sx_{n_k-1}) + p(Tx_{m_k-1}, x_{n_k-1})\}]$

$= max[p(x_{m_k-1}, x_{n_k-1}),\ p(x_{n_k-1}, x_{n_k}),\ p(x_{m_k-1}, x_{m_k}), \frac{1}{2s}\ \{p(x_{m_k-1}, x_{n_k}) + p(x_{m_k}, x_{n_k-1})\}]$

$\leq max[p(x_{m_k-1}, x_{n_k-1}),\ p(x_{n_k-1}, x_{n_k}),\ p(x_{m_k-1}, x_{m_k}), \frac{1}{2s}\ \{sp(x_{m_k-1}, x_{n_k-1}) + sp(x_{n_k-1}, x_{n_k}) -$

$p(x_{n_k-1}, x_{n_k-1}) + sp(x_{m_k-1}, x_{n_k-1}) + sp(x_{m_k-1}, x_{m_k}) - p(x_{m_k-1}, x_{m_k-1})\}]$

$\leq max[p(x_{m_k-1}, x_{n_k-1}),\ p(x_{n_k-1}, x_{n_k}),\ p(x_{m_k-1}, x_{m_k}), \frac{1}{2s}\{2sp(x_{m_k-1}, x_{n_k-1}) + sp(x_{n_k-1}, x_{n_k}) +$

$sp(x_{m_k}, x_{m_k-1})\}]$

$\leq p(x_{m_k-1}, x_{n_k-1}) + p(x_{n_k-1}, x_{n_k}) + p(x_{m_k}, x_{m_k-1})$

$\leq sp(x_{m_k-1}, x_{n_k}) + sp(x_{n_k}, x_{n_k-1}) - p(x_{n_k}, x_{n_k}) + p(x_{n_k-1}, x_{n_k}) + p(x_{m_k}, x_{m_k-1})$

$\leq sp(x_{m_k-1}, x_{n_k}) + sp(x_{n_k}, x_{n_k-1}) + p(x_{n_k-1}, x_{n_k}) + p(x_{m_k}, x_{m_k-1})$

$\therefore \varphi(s\epsilon)$

$\leq \varphi(sp(x_{m_k}, x_{n_k}))$

$< \varphi(M(x_{m_k-1}, x_{n_k-1})$

$\leq \varphi(p(x_{m_k-1}, x_{n_k-1}) + p(x_{n_k-1}, x_{n_k}) + p(x_{m_k}, x_{m_k-1}))$

$\leq \varphi(sp(x_{m_k-1}, x_{n_k}) + sp(x_{n_k}, x_{n_k-1}) + p(x_{n_k-1}, x_{n_k}) + p(x_{m_k}, x_{m_k-1}))$ (2.6.10)

Allowing $k \to \infty$,

$\varphi(s\epsilon) \leq \lim_{k\to\infty} \varphi(sp(x_{m_k}, x_{n_k}))$

$\leq \lim_{k\to\infty} \varphi(M(x_{m_k-1}, x_{n_k-1})$

$\leq \lim_{k\to\infty} \varphi(p(x_{m_k-1}, x_{n_k-1})) \leq \varphi(s\epsilon)$

$\therefore \lim_{k\to\infty} \varphi(p(x_{m_k}, x_{n_k}))$

$= \lim_{k\to\infty} \varphi(M(x_{m_k-1}, x_{n_k-1}))$

$= \lim_{k\to\infty} \varphi(p(x_{m_k-1}, x_{n_k-1}))$

$= \varphi(s\epsilon)$ (2.6.11)

$\therefore \lim\limits_{n\to\infty} \Delta_n^{\varphi}$

$= \lim\limits_{n\to\infty} \frac{\varphi(sp(Tx_{m_k-1}, Sx_{n_k-1}))}{\varphi(p_n)}$

$= \frac{\lim\limits_{k\to\infty} \varphi(sp(x_{m_k}, x_{n_k}))}{\lim\limits_{k\to\infty} \varphi(p(x_{m_k-1}, x_{n_k-1}))}$

$= \frac{\varphi(s\epsilon)}{\varphi(s\epsilon)}$ (by (2.6.8)

$= 1.$ (2.6.12)

Hence by our assumption $\varphi(p(x_{m_k-1}, x_{n_k-1}))) \to 0$ as $k \to \infty$

i.e., $\varphi(s\epsilon) = 0$

$\Rightarrow s\epsilon = 0$, a contradiction.

Case(ii): Let m_k is odd and n_k is odd

$\therefore \varphi(sp(x_{m_k}, x_{n_k+1})) \leq \alpha(x_{m_k-1}, x_{n_k})\ \varphi(sp(Tx_{m_k-1}, Sx_{n_k}))$

$< \varphi(M(x_{m_k-1}, x_{n_k}))$

where $M(x_{m_k-1}, x_{n_k})$

$= max[p(x_{m_k-1}, x_{n_k}), p(x_{m_k-1}, Tx_{m_k-1}), p(x_{n_k}, Sx_{n_k}), \frac{1}{2s}\ [\{p(Tx_{m_k-1}, x_{n_k}) + p(x_{m_k-1}, Sx_{n_k})\}]$

$= max[p(x_{m_k-1}, x_{n_k}), p(x_{m_k-1}, x_{m_k}), p(x_{n_k}, x_{n_k+1}), \frac{1}{2s}\ [\{p(x_{m_k}, x_{n_k}) + p(x_{m_k-1}, x_{n_k+1})\}]$

$= p(x_{m_k-1}, x_{n_k})$ or $\frac{1}{2s}\ [\{p(x_{m_k}, x_{n_k}) + p(x_{m_k-1}, x_{n_k+1})\}]$

Suppose $M(x_{m_k-1}, x_{n_k}) = p(x_{m_k-1}, x_{n_k}) < \epsilon$

But $\epsilon \leq p(x_{m_k}, x_{n_k}) \leq sp(x_{m_k}, x_{n_k+1}) + sp(x_{n_k+1}, x_{n_k}) - p(x_{n_k+1}, x_{n_k+1})$

$\leq sp(x_{m_k}, x_{n_k+1}) + s\eta$ where $\eta > 0 \ni p(x_{n_k+1}, x_{n_k}) < \eta$ (2.6.13)

$\Rightarrow \epsilon - s\eta \leq sp(x_{m_k}, x_{n_k+1})$ (2.6.14)

Since φ is non decreasing

$\therefore \varphi(\epsilon - s\eta) \leq \varphi(sp(x_{m_k}, x_{n_k+1}))$

$< \varphi(p(x_{m_k-1}, x_{n_k})) < \varphi(\epsilon)$ (2.6.15)

As φ is continuous and $\eta \to 0$ as $k \to \infty$, we get

$\varphi(\epsilon) \leq \lim\limits_{k\to\infty} \varphi(sp(x_{m_k}, x_{n_k+1})) \leq \lim\limits_{k\to\infty} \varphi(p(x_{m_k-1}, x_{n_k})) \leq \varphi(\epsilon)$

$\therefore \lim\limits_{k\to\infty} \varphi(sp(x_{m_k}, x_{n_k+1})) = \varphi(\epsilon) = \lim\limits_{k\to\infty} \varphi(p(x_{m_k-1}, x_{n_k}))$

Suppose $M(x_{m_k-1}, x_{n_k}) = \frac{1}{2s}\ [\{p(x_{m_k}, x_{n_k}) + p(x_{m_k-1}, x_{n_k+1})\}]$

On the other hand

$p(x_{m_k}, x_{n_k}) + p(x_{m_k-1}, x_{n_k+1})$

$\leq sp(x_{m_k}, x_{n_k-1}) + sp(x_{n_k-1}, x_{n_k}) - p(x_{n_k-1}, x_{n_k-1}) + sp(x_{m_k+1}, x_{m_k})$

$+ sp(x_{m_k}, x_{n_k-1}) - p(x_{m_k}, x_{m_k})$

$\leq sp(x_{m_k}, x_{n_k-1}) + sp(x_{n_k-1}, x_{n_k}) + sp(x_{m_k}, x_{n_k-1}) + sp(x_{m_k+1}, x_{m_k})$

$\leq 2sp(x_{m_k}, x_{n_k-1}) + 2s\eta \leq 2s\epsilon + 2s\eta$

where $p(x_{m_k+1}, x_{m_k}) \leq \eta$ and $p(x_{n_k}, x_{n_k-1}) \leq \eta$ for some $\eta > 0$ for large k

$\therefore \frac{1}{2s}\ [\{p(x_{m_k}, x_{n_k}) + p(x_{m_k-1}, x_{n_k+1})\}] \leq \epsilon + \eta$(2.6.16)

Therefore,

$M(x_{m_k-1}, x_{n_k}) = \frac{1}{2s}\ [\{p(x_{m_k}, x_{n_k}) + p(x_{m_k-1}, x_{n_k+1})\}] \leq \epsilon + \eta$

$\therefore$ From (2.6.13), (2.6.15) and (2.6.16)

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$\varphi(\epsilon - s\eta) \leq \varphi(sp(x_{m_k}, x_{n_k+1}))$
$\leq \varphi(M(x_{m_k-1}, x_{n_k}))$
$\leq \varphi(\epsilon + \eta)$

As φ is continuous and $\eta \to 0$ as $k \to \infty$, we get

$\varphi(sp(x_{m_k}, x_{n_k+1})) = \varphi(\epsilon)$

$\therefore \lim\limits_{n\to\infty} \Delta_n^{\varphi}$

$= \lim\limits_{n\to\infty} \frac{\varphi(sp(Tx_{m_k-1}, Sx_{n_k}))}{\varphi(p_n)}$

$= \frac{\lim\limits_{k\to\infty} \varphi(sp(x_{m_k}, x_{n_k+1}))}{\lim\limits_{k\to\infty} \varphi(p(x_{m_k-1}, x_{n_k}))}$

$= \frac{\varphi(\epsilon)}{\varphi(\epsilon)}$ (by (2.7.8))

$= 1.$ (2.6.17)

Hence by our assumption $\varphi(p(x_{m_k-1}, x_{n_k-1}))) \to 0$ as $k \to \infty$

i.e., $\varphi(\epsilon) = 0$

$\Rightarrow \epsilon = 0$, a contradiction.

Similarly the other two cases can be discussed.

$\therefore \{x_n\}$ is a Cauchy sequence.

Since X is complete, there exists $z \in X$ such that $\lim\limits_{n\to\infty} x_n = z$. Now, we show that z is a fixed point of T . We consider

$\varphi(sp(x_{2n+1}, Tz))$

$= \varphi(sp(Sx_{2n}, Tz))$

$\leq \alpha(x_{2n}, z)\varphi(sp(Sx_{2n}, Tz))$(since α is continuous and $\alpha(z, z) > 1$)

$< \varphi(M(x_{2n}, z))$ (2.6.18)

But $M(x_{2n}, z)$

$= max\{p(x_{2n}, z), p(x_{2n}, x_{2n+1}), p(z, Tz), \frac{1}{2s}[p(x_{2n+1}, z) + p(x_{2n}, Tz)]\}$

$= p(z, Tz)$ for large n

$\therefore \varphi(sp(x_{2n+1}, Tz)) \leq \alpha(x_{2n}, z)\varphi(sp(Sx_{2n}, Tz)) < \varphi(p(z, Tz))$

Suppose $\varphi(sp(z, Tz)) \neq 0$

Dividing through out by $\varphi(sp(z, Tz))$

$\frac{\varphi(sp(x_{2n+1}, Tz))}{\varphi(sp(z,Tz))} \leq \alpha(x_{2n}, z)\{\frac{\varphi(sp(x_{2n}, Tz))}{\varphi(sp(z,Tz))}\} < \frac{\varphi(p(z,Tz))}{\varphi(sp(z,Tz))} \leq 1$

On letting $n \to \infty$,

we get $1 \leq \alpha(z, z) \leq 1 \Rightarrow \alpha(z, z) = 1$, a contridiction

$\therefore \varphi(sp(z, Tz)) = 0 \Rightarrow p(z, Tz) = 0$

Then by lemma 2.3, $z = Tz$.

$\therefore z$ is a fixed point of T in X.

Similarly $\varphi(sp(x_{2n}, Sz))$ is to be considered to show z is a fixed point of S in X.

Hence z is a common fixed point of S and T in X.

Suppose y, z and $y \neq z$ are common fixed points of S and T in X

Then $Ty = Sy = y, Tz = Sz = z$ (2.6.19)

Suppose $\alpha(y,z) < 1$ then there is nothing to prove.

Suppose $\alpha(y,z) \geq 1$ and $p(y,z) \neq 0$

Now $\varphi(sp(Sy,Tz))$

$\leq \alpha(y,z)\varphi(sp(Sy,Tz))$

$< \varphi(M(y,z))$ (2.6.20)

But $M(y,z)$

$= max\{p(y,z), p(y,Sy), p(z,Tz), \frac{1}{2s}[p(y,Tz) + p(Sy,z)]\}$

$= p(y,z)$

$\therefore \varphi(sp(Sy,Tz)) \leq \alpha(y,z)\varphi(sp(Sy,Tz)) < \varphi(p(y,z))$

which is a contridiction

$\therefore p(y,z) = 0$

Then by lemma 2.3, $y = z$.

The following corollary can be easily established.

Corollary 2.7. Let T be a self map on a complete partially ordered partial b - metric space X. Let $\alpha : X \times X \to R$ be a continuous function.
Assume that there exists $\varphi \in \Phi_s$ such that

$$\alpha(x,y)\varphi(sp(Tx,Ty)) < \varphi(M(x,y)) \tag{2.7.1}$$

for all $x, y \in X, p(x,y) \neq 0$ where

$M(x,y) = max\{p(x,y), p(x,Tx), p(y,Ty), \frac{1}{2s}[p(x,Ty) + p(Tx,y)]\}$

Further, assume that

(i)T is α - triangular admissible,

(ii) there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$
and set $x_n = Tx_{n-1}$ for $n = 1, 2, 3, ...$

(iii)for any two sequences $\{a_n\}$ and $\{b_n\}$ of X with $p_n = p(a_n, b_n) \neq 0$, we have that

$$\Delta_n^{\varphi} \to 1 \Rightarrow \varphi(p_n) \to 0 \text{ as } n \to \infty \tag{2.7.2}$$

Then the sequence $\{x_n\}$ is a Cauchy sequence. Suppose $\{x_n\}$ converges to z and $\alpha(z,z) > 1$. Then z is a fixed point of T in X.

In the following, we prove the existence of fixed points of (α, φ, β) - weak generalized Geraghty contraction type maps in a complete partially ordered partial b - metric space.

Theorem 2.8. Let $(X, \leq, p)$ be a complete partially ordered partial b - metric space with coefficient $s \geq 1$,and $S, T : X \to X$ are weakly increasing self maps on X. Let $\alpha : X \times X \to \mathbb{R}$ be a continuous function and $\beta \in \Omega, \varphi \in \Phi_s$.
Suppose the following conditions hold:

(i) S, T are (α, φ, β) - weak generalized Geraghty contraction map i.e.,

$$\alpha(x,y)\varphi(sp(Tx,Sy)) \leq \beta(\varphi(M(x,y)))\varphi(M(x,y)) \;\forall\; x, y \in X, p(x,y) \neq 0 \tag{2.8.1}$$

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where

$M(x,y) = max\{p(x,y), p(x,Tx), p(y,Sy), \frac{1}{2s}[p(x,Sy)+p(Tx,y)]\}$

(ii) S,T are coupled and triangular α - admissible,

(iii) there exists $x_0 \in X$ such that $\alpha(x_0,Tx_0) \geq 1$

Then $\{x_n\}$ is a Cauchy sequence. Suppose $\{x_n\}$ converges to x and $\alpha(x,x) > 1$. Then x is a fixed point of T in X.

Proof. As in theorem 2.6, let $x_0 \in X$ be such that $\alpha(x_0,Tx_0) \geq 1$ by (iii). If $x_n = x_{n+1}$ for some $n \in N$, then $x_n = Tx_n$ and hence x_n is a fixed point of T or S. Without loss of generality, we assume that $x_n \neq x_{n+1}$ for all $n \in \mathbb{N}$. By using the α - admissibility of T , we have $\alpha(x_0,x_1) = \alpha(x_0,Tx_0) \geq 1 \Rightarrow \alpha(x_1,x_2) = \alpha(Tx_0,Sx_1) \geq 1$. Now, by mathematical induction, it is easy to see that $\alpha(x_n,x_{n+1}) \geq 1$ for all $n \in \mathbb{N}$.

Let n be even and by taking $x = x_{n-1}$ and $y = x_n$ in the inequality (2.8.1), and observing that $p(x_{n-1},x_n) \neq 0$ by lemma 2.3,

we get

$\varphi(sp(x_n,x_{n+1}))$

$= \varphi(sp(Sx_{n-1},Tx_n))$

$\leq \alpha(x_{n-1},x_n)\varphi(sp(Sx_{n-1},Tx_n))$

$\leq \beta(\varphi(M(x_{n-1},x_n)))\varphi(M(x_{n-1},x_n))(\text{ since } p(x_n,x_{n+1}) \neq 0 \ \forall \ n)$

$< \varphi(M(x_{n-1},x_n)) \qquad (2.8.2)$

where

$M(x_{n-1},x_n)$

$= max\{p(x_{n-1},x_n), p(x_{n-1},Sx_{n-1}), p(x_n,Tx_n), \frac{1}{2s}[p(x_{n-1},Tx_n)+p(x_n,Sx_{n-1})]\}$

$= max\{p(x_{n-1},x_n), p(x_{n-1},x_n), p(x_n,x_{n+1}), \frac{1}{2s}[p(x_{n-1},x_{n+1})+p(x_n,x_n)]\}$

$= max\{p(x_{n-1},x_n), p(x_n,x_{n+1})\}$

If $max\{p(x_{n-1},x_n), p(x_n,x_{n+1})\} = p(x_n,x_{n+1})$ for some $n \in N$ (2.8.3)

then from (2.8.2) and (2.8.3), we have

$\varphi(sp(x_n,x_{n+1})) < \varphi(M(x_{n-1},x_n)) = \varphi(p(x_n,x_{n+1}))$, a contradiction.

Let n be odd and by taking $x = x_{n-1}$ and $y = x_n$ in the inequality (2.8.1), and observing that $p(x_{n-1},x_n) \neq 0$ by lemma 2.3,

we get

$\varphi(sp(x_n,x_{n+1}))$

$= \varphi(sp(Tx_{n-1},Sx_n))$

$\leq \alpha(x_{n-1},x_n)\varphi(sp(Tx_{n-1},Sx_n))$

$\leq \beta(\varphi(M(x_{n-1},x_n)))\varphi(M(x_{n-1},x_n))(\text{ since } p(x_n,x_{n+1}) \neq 0 \ \forall \ n)$

$< \varphi(M(x_{n-1},x_n)) \qquad (2.8.4)$

where

$M(x_{n-1}, x_n)$

$= max\{p(x_{n-1}, x_n), p(x_{n-1}, Tx_{n-1}), p(x_n, Sx_n), \frac{1}{2s}[p(x_{n-1}, Sx_n) + p(x_n, Tx_{n-1})]\}$

$= max\{p(x_{n-1}, x_n), p(x_{n-1}, x_n), p(x_n, x_{n+1}), \frac{1}{2s}[p(x_{n-1}, x_{n+1}) + p(x_n, x_n)]\}$

$= max\{p(x_{n-1}, x_n), p(x_n, x_{n+1})\}$

If $max\{p(x_{n-1}, x_n), p(x_n, x_{n+1})\} = p(x_n, x_{n+1})$ for some $n \in \mathbb{N}$ (2.8.5)

then from (2.8.4) and (2.8.5), we have

$\varphi(sp(x_n, x_{n+1})) < \varphi(M(x_{n-1}, x_n)) = \varphi(p(x_n, x_{n+1}))$, a contradiction.

Thus, we have $max\{p(x_{n-1}, x_n), p(x_n, x_{n+1})\} = p(x_{n-1}, x_n)$ for all $n \in N$ and hence, $p(x_n, x_{n+1}) < p(x_{n-1}, x_n)$ for all $n \in N$. (2.8.6)

Thus it follows that $\{p(x_n, x_{n+1})\}$ is a non-negative, decreasing sequence of real numbers. Suppose that $\lim_{n\to\infty} p(x_n, x_{n+1}) = r, r \geq 0$

Now we prove that $r = 0$.

Assume that $r > 0$.

We have

$\varphi(p(x_n, x_{n+1}))$

$\leq \varphi(sp(x_n, x_{n+1}))$

$\leq \beta(\varphi(p(x_{n-1}, x_n)))\varphi(p(x_{n-1}, x_n))$

$< \varphi(p(x_{n-1}, x_n))$

Allowing as $n \to \infty$

$\varphi(r) = \lim_{n\to\infty} \varphi(p(x_n, x_{n+1}))$

$\leq \liminf_{n\to\infty} \beta(\varphi(p(x_{n-1}, x_n)))\varphi(p(x_{n-1}, x_n))$

$\leq \limsup_{n\to\infty} \beta(\varphi(p(x_{n-1}, x_n)))\varphi(p(x_{n-1}, x_n))$

$\leq \lim_{n\to\infty} \varphi(p(x_{n-1}, x_n)) = \varphi(r)$

$\Rightarrow \varphi(r) \leq \liminf_{n\to\infty} \beta(\varphi(p(x_{n-1}, x_n)))\varphi(r)$

$\leq \limsup_{n\to\infty} \beta(\varphi(p(x_{n-1}, x_n)))\varphi(r)$

$\leq \varphi(r)$

$\Rightarrow \varphi(r) = 0$ or $\lim_{n\to\infty} \beta(\varphi(p(x_{n-1}, x_n))) = 1$

$\varphi(r) = 0$ or $\lim_{n\to\infty} \varphi(p(x_{n-1}, x_n)) = 0($ since $\beta \in \Omega)$

$\lim_{n\to\infty} \varphi(p(x_{n-1}, x_n)) = 0$

$\therefore \varphi(r) = 0 \Rightarrow r = 0$, a contradiction.

Hence $r = \lim_{n\to\infty} p(x_n, x_{n+1}) = 0$ (2.8.7)

Now, we show that $\{x_n\}$ is a Cauchy sequence in X. Suppose that $\{x_n\}$ is not a Cauchy sequence. Then by Lemma 2.2(b), there exist some $\epsilon > 0$, and sub-sequences $\{x_{m_k}\}$ and $\{x_{n_k}\}$ of $\{x_n\}$ with $m_k > n_k > k$ such that $p(x_{m_k}, x_{n_k}) \geq \epsilon$ and $p(x_{m_k-1}, x_{n_k}) < \epsilon$. Let m_k be odd and n_k be even

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$\therefore s\epsilon \leq sp(x_{m_k}, x_{n_k})$
$\Rightarrow \varphi(s\epsilon) \leq \varphi(sp(x_{m_k}, x_{n_k}))$
$= \varphi(sp(Tx_{m_k-1}, Sx_{n_k-1}))$
$\leq \alpha(x_{m_k-1}, x_{n_k-1})\ \varphi(sp(Tx_{m_k-1}, Sx_{n_k-1}))$
(by lemma 1.9 $\alpha(x_{m_k-1}, x_{n_k-1}) \geq 1$)
$\leq \beta(\varphi(M(x_{m_k-1}, x_{n_k-1})))\varphi(M(x_{m_k-1}, x_{n_k-1}))$
$< \varphi(M(x_{m_k-1}, x_{n_k-1}))$ (2.8.8)
where $M(x_{m_k-1}, x_{n_k-1})$
$= max[p(x_{m_k-1}, x_{n_k-1}),\ p(x_{n_k-1}, Sx_{n_k-1}),\ p(x_{m_k-1}, Tx_{m_k-1}),$
$\frac{1}{2s}\ \{p(x_{m_k-1}, Sx_{n_k-1}) + p(Tx_{m_k-1}, x_{n_k-1})\}]$
$= max[p(x_{m_k-1}, x_{n_k-1}),\ p(x_{n_k-1}, x_{n_k}),\ p(x_{m_k-1}, x_{m_k}), \frac{1}{2s}\ \{p(x_{m_k-1}, x_{n_k}) + p(x_{m_k}, x_{n_k-1})\}]$
$\leq max[p(x_{m_k-1}, x_{n_k-1}), p(x_{n_k-1}, x_{n_k}), p(x_{m_k-1}, x_{m_k}), \frac{1}{2s}\ \{sp(x_{m_k-1}, x_{n_k-1}) + sp(x_{n_k-1}, x_{n_k}) -$
$p(x_{n_k-1}, x_{n_k-1}) + sp(x_{m_k-1}, x_{n_k-1}) + sp(x_{m_k-1}, x_{m_k}) - p(x_{m_k-1}, x_{m_k-1})\}]$
$\leq max[p(x_{m_k-1}, x_{n_k-1}), p(x_{n_k-1}, x_{n_k}), p(x_{m_k-1}, x_{m_k}), \frac{1}{2s}\{2sp(x_{m_k-1}, x_{n_k-1}) + sp(x_{n_k-1}, x_{n_k}) +$
$sp(x_{m_k}, x_{m_k-1})\}]$
$\leq p(x_{m_k-1}, x_{n_k-1}) + p(x_{n_k-1}, x_{n_k}) + p(x_{m_k}, x_{m_k-1})$
$\leq sp(x_{m_k-1}, x_{n_k}) + sp(x_{n_k}, x_{n_k-1}) - p(x_{n_k}, x_{n_k}) + p(x_{n_k-1}, x_{n_k}) + p(x_{m_k}, x_{m_k-1})$
$\leq sp(x_{m_k-1}, x_{n_k}) + sp(x_{n_k}, x_{n_k-1}) + p(x_{n_k-1}, x_{n_k}) + p(x_{m_k}, x_{m_k-1})$
$\therefore \varphi(s\epsilon)$
$\leq \beta(\varphi(M(x_{m_k-1}, x_{n_k-1})))\varphi(M(x_{m_k-1}, x_{n_k-1}))$
$\leq \varphi(M(x_{m_k-1}, x_{n_k-1})$
$\leq \varphi(sp(x_{m_k-1}, x_{n_k}) + sp(x_{n_k}, x_{n_k-1}) + p(x_{n_k-1}, x_{n_k}) + p(x_{m_k}, x_{m_k-1}))$ (2.8.9)
Allowing $k \to \infty$, we get
$\varphi(s\epsilon)$
$\leq \liminf_{k\to\infty} \beta(\varphi(M(x_{m_k-1}, x_{n_k-1})))\ \varphi(M(x_{m_k-1}, x_{n_k-1})$
$\leq \limsup_{k\to\infty} \beta(\varphi(M(x_{m_k-1}, x_{n_k-1})))\ \varphi(M(x_{m_k-1}, x_{n_k-1})$
$\leq \lim_{k\to\infty}\ \varphi(sp(x_{m_k-1}, x_{n_k}) + sp(x_{n_k}, x_{n_k-1}) + p(x_{n_k-1}, x_{n_k}) + p(x_{m_k}, x_{m_k-1}))$
$\therefore \varphi(s\epsilon)$
$\leq \liminf_{k\to\infty} \beta(\varphi(M(x_{m_k-1}, x_{n_k-1})))\varphi(s\epsilon)$
$\leq \limsup_{k\to\infty} \beta(\varphi(M(x_{m_k-1}, x_{n_k-1})))\varphi(s\epsilon)$
$\leq \varphi(s\epsilon)$
$\therefore$ Either $\varphi(s\epsilon) = 0$ or $\lim_{k\to\infty} \beta(\varphi(M(x_{m_k-1}, x_{n_k-1}))) = 1$(since $\beta \in S$) (2.8.10)
$\Rightarrow \varphi(s\epsilon) = 0$
$\Rightarrow s\epsilon = 0$, a contradiction.
Let m_k be odd and n_k be odd
$\therefore \varphi(sp(x_{m_k}, x_{n_k+1})) \leq \alpha(x_{m_k-1}, x_{n_k})\ \varphi(sp(Tx_{m_k-1}, Sx_{n_k}))$
$\leq \beta(\varphi(M(x_{m_k-1}, x_{n_k})))\varphi(M(x_{m_k-1}, x_{n_k}))$
$< \varphi(M(x_{m_k-1}, x_{n_k}))$

where $M(x_{m_k-1}, x_{n_k})$

$= max[p(x_{m_k-1}, x_{n_k}), p(x_{m_k-1}, Tx_{m_k-1}), p(x_{n_k}, Sx_{n_k}), \frac{1}{2s}[\{p(Tx_{m_k-1}, x_{n_k}) + p(x_{m_k-1}, Sx_{n_k})\}]$

$= max[p(x_{m_k-1}, x_{n_k}), p(x_{m_k-1}, x_{m_k}), p(x_{n_k}, x_{n_k+1}), \frac{1}{2s}[\{p(x_{m_k}, x_{n_k}) + p(x_{m_k-1}, x_{n_k+1})\}]$

$= p(x_{m_k-1}, x_{n_k})$ or $\frac{1}{2s}[\{p(x_{m_k}, x_{n_k}) + p(x_{m_k-1}, x_{n_k+1})\}]$

Suppose $M(x_{m_k-1}, x_{n_k}) = p(x_{m_k-1}, x_{n_k}) < \epsilon$

But $\epsilon \leq p(x_{m_k}, x_{n_k}) \leq sp(x_{m_k}, x_{n_k+1}) + sp(x_{n_k+1}, x_{n_k}) - p(x_{n_k+1}, x_{n_k+1})$

$\leq sp(x_{m_k}, x_{n_k+1}) + s\eta$ where $\eta > 0 \ni p(x_{n_k+1}, x_{n_k}) < \eta$ (2.8.11)

$\Rightarrow \epsilon - s\eta \leq sp(x_{m_k}, x_{n_k+1})$ (2.8.12)

Since φ is non decreasing

$\therefore \varphi(\epsilon - s\eta) \leq \varphi(sp(x_{m_k}, x_{n_k+1}))$

$< \varphi(p(x_{m_k-1}, x_{n_k})) < \varphi(\epsilon)$ (2.8.13)

As φ is continuous and $\eta \to 0$ as $k \to \infty$, we get

$\varphi(\epsilon) \leq \lim_{k\to\infty} \varphi(sp(x_{m_k}, x_{n_k+1})) \leq \lim_{k\to\infty} \varphi(p(x_{m_k-1}, x_{n_k})) \leq \varphi(\epsilon)$

$\therefore \lim_{k\to\infty} \varphi(sp(x_{m_k}, x_{n_k+1})) = \varphi(\epsilon) = \lim_{k\to\infty} \varphi(p(x_{m_k-1}, x_{n_k}))$

Suppose $M(x_{m_k-1}, x_{n_k}) = \frac{1}{2s}[\{p(x_{m_k}, x_{n_k}) + p(x_{m_k-1}, x_{n_k+1})\}]$

On the other hand

$p(x_{m_k}, x_{n_k}) + p(x_{m_k-1}, x_{n_k+1})$

$\leq sp(x_{m_k}, x_{n_k-1}) + sp(x_{n_k-1}, x_{n_k}) - p(x_{n_k-1}, x_{n_k-1}) + sp(x_{m_k+1}, x_{m_k})$

$+ sp(x_{m_k}, x_{n_k-1}) - p(x_{m_k}, x_{m_k})$

$\leq sp(x_{m_k}, x_{n_k-1}) + sp(x_{n_k-1}, x_{n_k}) + sp(x_{m_k}, x_{n_k-1}) + sp(x_{m_k+1}, x_{m_k})$

$\leq 2sp(x_{m_k}, x_{n_k-1}) + 2s\eta \leq 2s\epsilon + 2s\eta$

where $p(x_{m_k+1}, x_{m_k}) \leq \eta$ and $p(x_{n_k}, x_{n_k-1}) \leq \eta$ for some $\eta > 0$ for large k

$\therefore \frac{1}{2s}[\{p(x_{m_k}, x_{n_k}) + p(x_{m_k-1}, x_{n_k+1})\}] \leq \epsilon + \eta$ (2.8.16)

Therefore,

$M(x_{m_k-1}, x_{n_k}) = \frac{1}{2s}[\{p(x_{m_k}, x_{n_k}) + p(x_{m_k-1}, x_{n_k+1})\}] \leq \epsilon + \eta$

$\therefore$ From (2.8.13), (2.8.15) and (2.8.16)

$\varphi(\epsilon - s\eta) \leq \varphi(sp(x_{m_k}, x_{n_k+1}))$

$\leq \varphi(M(x_{m_k-1}, x_{n_k}))$

$\leq \varphi(\epsilon + \eta)$

As φ is continuous and $\eta \to 0$ as $k \to \infty$, we get

$\varphi(sp(x_{m_k}, x_{n_k+1})) = \varphi(\epsilon)$

$\therefore \varphi(\epsilon)$

$\leq \beta(\varphi(M(x_{m_k-1}, x_{n_k-1})))\varphi(M(x_{m_k-1}, x_{n_k-1}))$

$\leq \varphi(M(x_{m_k-1}, x_{n_k-1})$

$\leq \varphi(sp(x_{m_k-1}, x_{n_k}) + sp(x_{n_k}, x_{n_k-1}) + p(x_{n_k-1}, x_{n_k}) + p(x_{m_k}, x_{m_k-1}))$ (2.8.17)

Notes

Notes

Allowing $k \to \infty$, we get

$\varphi(\epsilon)$

$\leq \liminf_{k\to\infty} \beta(\varphi(M(x_{m_k-1}, x_{n_k})))\ \varphi(M(x_{m_k-1}, x_{n_k})$

$\leq \limsup_{k\to\infty} \beta(\varphi(M(x_{m_k-1}, x_{n_k})))\ \varphi(M(x_{m_k-1}, x_{n_k})$

$\leq \lim_{k\to\infty}\ \varphi(sp(x_{m_k-1}, x_{n_k}) + sp(x_{n_k}, x_{n_k-1}) + p(x_{n_k-1}, x_{n_k}) + p(x_{m_k}, x_{m_k-1}))$

$\therefore \varphi(\epsilon)$

$\leq \liminf_{k\to\infty} \beta(\varphi(M(x_{m_k-1}, x_{n_k})))\varphi(\epsilon)$

$\leq \limsup_{k\to\infty} \beta(\varphi(M(x_{m_k-1}, x_{n_k})))\varphi(\epsilon)$

$\leq \varphi(\epsilon)$

$\therefore$ Either $\varphi(\epsilon) = 0$ or $\lim_{k\to\infty} \beta(\varphi(M(x_{m_k-1}, x_{n_k}))) = 1$(since $\beta \in S$) (2.8.18)

$\Rightarrow \varphi(\epsilon) = 0$

$\Rightarrow \epsilon = 0$, a contradiction.

Similarly we can discuss the other two cases.

$\therefore \{x_n\}$ is a Cauchy sequence.

Since X is complete, there exists $z \in X$ such that $\lim_{n\to\infty} x_n = z$. Now, we show that z is a fixed point of T . We consider

$\varphi(sp(x_{2n}, Tz))$

$= \varphi(sp(Sx_{2n-1}, Tz))$

$\leq \alpha(x_{2n-1}, z)\varphi(sp(Sx_{2n-1}, Tz))$(since α is continuous and $\alpha(z, z) > 1$)

$\leq \beta(\varphi(M(x_{2n-1}, z)))\varphi(M(x_{2n-1}, z))$ (2.8.19)

But $M(x_{2n-1}, z)$

$= max\{p(x_{2n-1}, z), p(x_{2n-1}, x_{2n}), p(z, Tz), \frac{1}{2s}[p(x_{2n}, z) + p(x_{2n-1}, Tz)]\}$

$= p(z, Tz)$ for large n

$\therefore \varphi(sp(x_{2n}, Tz)) \leq \beta(\varphi(p(z, Tz)))\varphi(p(z, Tz)) \leq \varphi(p(z, Tz))$

On letting $n \to \infty$,
we get

$\varphi(p(z, Tz)) \leq \varphi(sp(z, Tz)) \leq \beta(\varphi(p(z, Tz)))\varphi(p(z, Tz)) \leq \varphi(p(z, Tz))$

$\Rightarrow \varphi(p(z, Tz)) = 0$ or $\beta(\varphi(p(z, Tz))) = 1$

$\Rightarrow p(z, Tz) = 0$

Then by lemma 2.3, $z = Tz$.

$\therefore z$ is a fixed point of T in X. Further $\varphi(sp(x_{2n+1}, Sz))$

$= \varphi(sp(Tx_{2n}, Sz))$

$\leq \alpha(x_{2n}, z)\varphi(sp(Tx_{2n}, Sz))$(since α is continuous and $\alpha(z, z) > 1$)

$\leq \beta(\varphi(M(x_{2n}, z)))\varphi(M(x_{2n}, z))$ (2.8.20)

But $M(x_{2n}, z)$

$= max\{p(x_{2n}, z), p(x_{2n}, x_{2n+1}), p(z, Sz), \frac{1}{2s}[p(x_{2n+1}, z) + p(x_{2n}, Sz)]\}$

$= p(z, Sz)$ for large n

$\therefore \varphi(sp(x_{2n+1}, Sz)) \le \beta(\varphi(p(z, Sz)))\varphi(p(z, Sz)) \le \varphi(p(z, Sz))$
On letting $n \to \infty$,
we get
$\varphi(p(z, Sz)) \le \varphi(sp(z, Tz)) \le \beta(\varphi(p(z, Sz)))\varphi(p(z, Sz)) \le \varphi(p(z, Sz))$
$\Rightarrow \varphi(p(z, Tz)) = 0$ or $\beta(\varphi(p(z, Tz))) = 1$
$\Rightarrow p(z, Sz) = 0$
Then by lemma 2.3, $z = Sz$.
$\therefore$ z is a fixed point of S in X.
Hence$z$ is a common fixed point of S and Tin X

Now we give an example in support of theorem 2.8

Example 2.9. (K.P.R.Sastry et.al.[33]) Let $X = \{0, 1, \frac{1}{2}, \frac{1}{3}, ..., \frac{1}{10}\}$ with usual ordering.
Define

$$p(x, y) = \begin{cases} 0 \ if \ x = y \\ 1 \ if \ x \neq y \in \{0, 1\} \\ |x - y| \ if \ x, y \in \{0, \frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}\} \\ 4 \ otherwise \end{cases}$$

Clearly, $(X, \le, p)$ is a partially ordered partial b - metric space with coefficient $s = \frac{8}{3}$ (P.Kumam et.al [19])
Define $S, T : X \to X$ by
$T1 = T\frac{1}{3} = T\frac{1}{5} = T\frac{1}{7} = T\frac{1}{9} = 0$; $T0 = T\frac{1}{2} = T\frac{1}{4} = T\frac{1}{6} = T\frac{1}{8} = T\frac{1}{10} = \frac{1}{4}$

$\therefore A = \{0, \frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}\} \Rightarrow T(A) = \frac{1}{4}$
and
$B = \{1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}\} \Rightarrow T(B) = 0$
$\therefore T(X) = \{0, \frac{1}{4}\}$
and $Sx = \frac{1}{4} \ \forall \ x \in X \Rightarrow S(A) = S(B) = \frac{1}{4}$

$$\beta(t) = \begin{cases} \frac{1}{1+t} \ \text{ if } \ t \in (0, \infty) \\ 0 \ \text{ if } t = 0 \end{cases}$$

$\alpha(x, y) = 2 \ \forall \ x, y \in X$ and $\varphi(t) = 2t \ \forall \ t \ge 0$
For $x, y \in X$ and $p(x, y) \neq 0 \Rightarrow x \neq y$, then following are the cases
(i)For $x, y \in A \Rightarrow Sx = Ty = \frac{1}{4} \Rightarrow sp(Sx, Ty) = 0$
$\therefore \alpha(x, y)\varphi(sp(Sx, Ty)) \le \beta(\varphi(M(x, y)))\varphi(M(x, y))$
(ii)For $x, y \in B \Rightarrow Sx = \frac{1}{4}, Ty = 0 \Rightarrow sp(Sx, Ty) = (\frac{8}{3})(\frac{1}{4}) = \frac{2}{3} \Rightarrow \varphi(sp(Sx, Ty)) = \frac{4}{3}$
where $M(x, y) = 4 \Rightarrow \varphi(\beta(M(x, y)))\varphi(M(x, y)) = 4(\frac{4}{5}) = \frac{16}{5}$
$\therefore \alpha(x, y)\varphi(sp(Sx, Ty)) \le \beta(\varphi(M(x, y)))\varphi(M(x, y))$
(iii)For $x \in A, y \in B \Rightarrow Sx = \frac{1}{4}, Ty = 0 \Rightarrow sp(Sx, Ty) = (\frac{8}{3})(\frac{1}{4}) = \frac{2}{3}$
$\Rightarrow \varphi(sp(Sx, Ty)) = \frac{4}{3}$ where $M(x, y) = 4 \Rightarrow \varphi(\beta(M(x, y)))\varphi(M(x, y)) = 4(\frac{4}{5}) = \frac{16}{5}$

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$\therefore \alpha(x,y)\varphi(sp(Sx,Ty)) \leq \beta(\varphi(M(x,y)))\varphi(M(x,y))$

(iv)For $x \in A, y \in B \Rightarrow Tx = Sy = \frac{1}{4} \Rightarrow sp(Tx,Sy) = 0 \Rightarrow \varphi(sp(Tx,Sy)) = 0$

$\therefore \alpha(x,y)\varphi(sp(Tx,Sy)) \leq \beta(\varphi(M(x,y)))\varphi(M(x,y))$

Since $T(\frac{1}{4}) = S(\frac{1}{4}) = \frac{1}{4}$ and $\alpha(\frac{1}{4}, T\frac{1}{4}) = 2 > 1$

Therefore $\frac{1}{4} \in X$ is a fixed point.

The hypothesis and conclusions of 2.8 satisfied.

Notes

Open Problem: Are the theorems 2.6 and 2.8 true for partial b - metric spaces with coefficient $s \geq 1$ when conditions on α removed?

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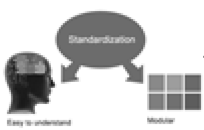

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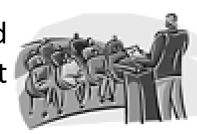

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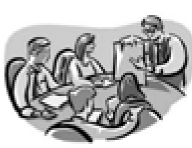

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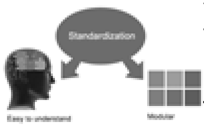

After nomination of your institution as "Institutional Fellow" and constantly functioning successfully for one year, we can consider giving recognition to your institute to function as Regional/Zonal office on our behalf.
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Note :

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3. Submission of Manuscripts,

4. Manuscript's Category,

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8. Use the Internet for help: An excellent start for your paper can be by using the Google. It is an excellent search engine, where you can have your doubts resolved. You may also read some answers for the frequent question how to write my research paper or find model research paper. From the internet library you can download books. If you have all required books make important reading selecting and analyzing the specified information. Then put together research paper sketch out.

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10. Bookmarks are useful: When you read any book or magazine, you generally use bookmarks, right! It is a good habit, which helps to not to lose your continuity. You should always use bookmarks while searching on Internet also, which will make your search easier.

11. Revise what you wrote: When you write anything, always read it, summarize it and then finalize it.

12. Make all efforts: Make all efforts to mention what you are going to write in your paper. That means always have a good start. Try to mention everything in introduction, that what is the need of a particular research paper. Polish your work by good skill of writing and always give an evaluator, what he wants.

13. Have backups: When you are going to do any important thing like making research paper, you should always have backup copies of it either in your computer or in paper. This will help you to not to lose any of your important.

14. Produce good diagrams of your own: Always try to include good charts or diagrams in your paper to improve quality. Using several and unnecessary diagrams will degrade the quality of your paper by creating "hotchpotch." So always, try to make and include those diagrams, which are made by your own to improve readability and understandability of your paper.

15. Use of direct quotes: When you do research relevant to literature, history or current affairs then use of quotes become essential but if study is relevant to science then use of quotes is not preferable.

16. Use proper verb tense: Use proper verb tenses in your paper. Use past tense, to present those events that happened. Use present tense to indicate events that are going on. Use future tense to indicate future happening events. Use of improper and wrong tenses will confuse the evaluator. Avoid the sentences that are incomplete.

17. Never use online paper: If you are getting any paper on Internet, then never use it as your research paper because it might be possible that evaluator has already seen it or maybe it is outdated version.

18. Pick a good study spot: To do your research studies always try to pick a spot, which is quiet. Every spot is not for studies. Spot that suits you choose it and proceed further.

19. Know what you know: Always try to know, what you know by making objectives. Else, you will be confused and cannot achieve your target.

20. Use good quality grammar: Always use a good quality grammar and use words that will throw positive impact on evaluator. Use of good quality grammar does not mean to use tough words, that for each word the evaluator has to go through dictionary. Do not start sentence with a conjunction. Do not fragment sentences. Eliminate one-word sentences. Ignore passive voice. Do not ever use a big word when a diminutive one would suffice. Verbs have to be in agreement with their subjects. Prepositions are not expressions to finish sentences with. It is incorrect to ever divide an infinitive. Avoid clichés like the disease. Also, always shun irritating alliteration. Use language that is simple and straight forward. put together a neat summary.

21. Arrangement of information: Each section of the main body should start with an opening sentence and there should be a changeover at the end of the section. Give only valid and powerful arguments to your topic. You may also maintain your arguments with records.

22. Never start in last minute: Always start at right time and give enough time to research work. Leaving everything to the last minute will degrade your paper and spoil your work.

23. Multitasking in research is not good: Doing several things at the same time proves bad habit in case of research activity. Research is an area, where everything has a particular time slot. Divide your research work in parts and do particular part in particular time slot.

24. Never copy others' work: Never copy others' work and give it your name because if evaluator has seen it anywhere you will be in trouble.

25. Take proper rest and food: No matter how many hours you spend for your research activity, if you are not taking care of your health then all your efforts will be in vain. For a quality research, study is must, and this can be done by taking proper rest and food.

26. Go for seminars: Attend seminars if the topic is relevant to your research area. Utilize all your resources.

27. Refresh your mind after intervals: Try to give rest to your mind by listening to soft music or by sleeping in intervals. This will also improve your memory.

28. Make colleagues: Always try to make colleagues. No matter how sharper or intelligent you are, if you make colleagues you can have several ideas, which will be helpful for your research.

29. Think technically: Always think technically. If anything happens, then search its reasons, its benefits, and demerits.

30. Think and then print: When you will go to print your paper, notice that tables are not be split, headings are not detached from their descriptions, and page sequence is maintained.

31. Adding unnecessary information: Do not add unnecessary information, like, I have used MS Excel to draw graph. Do not add irrelevant and inappropriate material. These all will create superfluous. Foreign terminology and phrases are not apropos. One should NEVER take a broad view. Analogy in script is like feathers on a snake. Not at all use a large word when a very small one would be sufficient. Use words properly, regardless of how others use them. Remove quotations. Puns are for kids, not grunt readers. Amplification is a billion times of inferior quality than sarcasm.

32. Never oversimplify everything: To add material in your research paper, never go for oversimplification. This will definitely irritate the evaluator. Be more or less specific. Also too, by no means, ever use rhythmic redundancies. Contractions aren't essential and shouldn't be there used. Comparisons are as terrible as clichés. Give up ampersands and abbreviations, and so on. Remove commas, that are, not necessary. Parenthetical words however should be together with this in commas. Understatement is all the time the complete best way to put onward earth-shaking thoughts. Give a detailed literary review.

33. Report concluded results: Use concluded results. From raw data, filter the results and then conclude your studies based on measurements and observations taken. Significant figures and appropriate number of decimal places should be used. Parenthetical remarks are prohibitive. Proofread carefully at final stage. In the end give outline to your arguments. Spot out perspectives of further study of this subject. Justify your conclusion by at the bottom of them with sufficient justifications and examples.

34. After conclusion: Once you have concluded your research, the next most important step is to present your findings. Presentation is extremely important as it is the definite medium though which your research is going to be in print to the rest of the crowd. Care should be taken to categorize your thoughts well and present them in a logical and neat manner. A good quality research paper format is essential because it serves to highlight your research paper and bring to light all necessary aspects in your research.

Informal Guidelines of Research Paper Writing

Key points to remember:

- Submit all work in its final form.
- Write your paper in the form, which is presented in the guidelines using the template.
- Please note the criterion for grading the final paper by peer-reviewers.

Final Points:

A purpose of organizing a research paper is to let people to interpret your effort selectively. The journal requires the following sections, submitted in the order listed, each section to start on a new page.

The introduction will be compiled from reference matter and will reflect the design processes or outline of basis that direct you to make study. As you will carry out the process of study, the method and process section will be constructed as like that. The result segment will show related statistics in nearly sequential order and will direct the reviewers next to the similar intellectual paths throughout the data that you took to carry out your study. The discussion section will provide understanding of the data and projections as to the implication of the results. The use of good quality references all through the paper will give the effort trustworthiness by representing an alertness of prior workings.

Writing a research paper is not an easy job no matter how trouble-free the actual research or concept. Practice, excellent preparation, and controlled record keeping are the only means to make straightforward the progression.

General style:

Specific editorial column necessities for compliance of a manuscript will always take over from directions in these general guidelines.

To make a paper clear

· Adhere to recommended page limits

Mistakes to evade

- Insertion a title at the foot of a page with the subsequent text on the next page
- Separating a table/chart or figure - impound each figure/table to a single page
- Submitting a manuscript with pages out of sequence

In every sections of your document

· Use standard writing style including articles ("a", "the," etc.)

· Keep on paying attention on the research topic of the paper

· Use paragraphs to split each significant point (excluding for the abstract)

· Align the primary line of each section

· Present your points in sound order

· Use present tense to report well accepted

· Use past tense to describe specific results

· Shun familiar wording, don't address the reviewer directly, and don't use slang, slang language, or superlatives

· Shun use of extra pictures - include only those figures essential to presenting results

Title Page:

Choose a revealing title. It should be short. It should not have non-standard acronyms or abbreviations. It should not exceed two printed lines. It should include the name(s) and address (es) of all authors.

Abstract:

The summary should be two hundred words or less. It should briefly and clearly explain the key findings reported in the manuscript--must have precise statistics. It should not have abnormal acronyms or abbreviations. It should be logical in itself. Shun citing references at this point.

An abstract is a brief distinct paragraph summary of finished work or work in development. In a minute or less a reviewer can be taught the foundation behind the study, common approach to the problem, relevant results, and significant conclusions or new questions.

Write your summary when your paper is completed because how can you write the summary of anything which is not yet written? Wealth of terminology is very essential in abstract. Yet, use comprehensive sentences and do not let go readability for briefness. You can maintain it succinct by phrasing sentences so that they provide more than lone rationale. The author can at this moment go straight to shortening the outcome. Sum up the study, with the subsequent elements in any summary. Try to maintain the initial two items to no more than one ruling each.

- Reason of the study - theory, overall issue, purpose
- Fundamental goal
- To the point depiction of the research
- Consequences, including definite statistics - if the consequences are quantitative in nature, account quantitative data; results of any numerical analysis should be reported
- Significant conclusions or questions that track from the research(es)

Approach:

- Single section, and succinct
- As a outline of job done, it is always written in past tense
- A conceptual should situate on its own, and not submit to any other part of the paper such as a form or table
- Center on shortening results - bound background information to a verdict or two, if completely necessary
- What you account in an conceptual must be regular with what you reported in the manuscript
- Exact spelling, clearness of sentences and phrases, and appropriate reporting of quantities (proper units, important statistics) are just as significant in an abstract as they are anywhere else

Introduction:

The **Introduction** should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable to comprehend and calculate the purpose of your study without having to submit to other works. The basis for the study should be offered. Give most important references but shun difficult to make a comprehensive appraisal of the topic. In the introduction, describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will have no attention in your result. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here. Following approach can create a valuable beginning:

- Explain the value (significance) of the study
- Shield the model - why did you employ this particular system or method? What is its compensation? You strength remark on its appropriateness from a abstract point of vision as well as point out sensible reasons for using it.
- Present a justification. Status your particular theory (es) or aim(s), and describe the logic that led you to choose them.
- Very for a short time explain the tentative propose and how it skilled the declared objectives.

Approach:

- Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done.
- Sort out your thoughts; manufacture one key point with every section. If you make the four points listed above, you will need a least of four paragraphs.

- Present surroundings information only as desirable in order hold up a situation. The reviewer does not desire to read the whole thing you know about a topic.
- Shape the theory/purpose specifically - do not take a broad view.
- As always, give awareness to spelling, simplicity and correctness of sentences and phrases.

Procedures (Methods and Materials):

This part is supposed to be the easiest to carve if you have good skills. A sound written Procedures segment allows a capable scientist to replacement your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt for the least amount of information that would permit another capable scientist to spare your outcome but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section. When a technique is used that has been well described in another object, mention the specific item describing a way but draw the basic principle while stating the situation. The purpose is to text all particular resources and broad procedures, so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step by step report of the whole thing you did, nor is a methods section a set of orders.

Materials:

- Explain materials individually only if the study is so complex that it saves liberty this way.
- Embrace particular materials, and any tools or provisions that are not frequently found in laboratories.
- Do not take in frequently found.
- If use of a definite type of tools.
- Materials may be reported in a part section or else they may be recognized along with your measures.

Methods:

- Report the method (not particulars of each process that engaged the same methodology)
- Describe the method entirely
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures
- Simplify - details how procedures were completed not how they were exclusively performed on a particular day.
- If well known procedures were used, account the procedure by name, possibly with reference, and that's all.

Approach:

- It is embarrassed or not possible to use vigorous voice when documenting methods with no using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result when script up the methods most authors use third person passive voice.
- Use standard style in this and in every other part of the paper - avoid familiar lists, and use full sentences.

What to keep away from

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings - save it for the argument.
- Leave out information that is immaterial to a third party.

Results:

The principle of a results segment is to present and demonstrate your conclusion. Create this part a entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Carry on to be to the point, by means of statistics and tables, if suitable, to present consequences most efficiently.You must obviously differentiate material that would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matter should not be submitted at all except requested by the instructor.

Content

- Sum up your conclusion in text and demonstrate them, if suitable, with figures and tables.
- In manuscript, explain each of your consequences, point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation an exacting study.
- Explain results of control experiments and comprise remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or in manuscript form.

What to stay away from

- Do not discuss or infer your outcome, report surroundings information, or try to explain anything.
- Not at all, take in raw data or intermediate calculations in a research manuscript.
- Do not present the similar data more than once.
- Manuscript should complement any figures or tables, not duplicate the identical information.
- Never confuse figures with tables - there is a difference.

Approach

- As forever, use past tense when you submit to your results, and put the whole thing in a reasonable order.
- Put figures and tables, appropriately numbered, in order at the end of the report
- If you desire, you may place your figures and tables properly within the text of your results part.

Figures and tables

- If you put figures and tables at the end of the details, make certain that they are visibly distinguished from any attach appendix materials, such as raw facts
- Despite of position, each figure must be numbered one after the other and complete with subtitle
- In spite of position, each table must be titled, numbered one after the other and complete with heading
- All figure and table must be adequately complete that it could situate on its own, divide from text

Discussion:

The Discussion is expected the trickiest segment to write and describe. A lot of papers submitted for journal are discarded based on problems with the Discussion. There is no head of state for how long a argument should be. Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implication of the study. The purpose here is to offer an understanding of your results and hold up for all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of result should be visibly described. Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved with prospect, and let it drop at that.

- Make a decision if each premise is supported, discarded, or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."
- Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work
- You may propose future guidelines, such as how the experiment might be personalized to accomplish a new idea.
- Give details all of your remarks as much as possible, focus on mechanisms.
- Make a decision if the tentative design sufficiently addressed the theory, and whether or not it was correctly restricted.
- Try to present substitute explanations if sensible alternatives be present.
- One research will not counter an overall question, so maintain the large picture in mind, where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

- When you refer to information, differentiate data generated by your own studies from available information
- Submit to work done by specific persons (including you) in past tense.
- Submit to generally acknowledged facts and main beliefs in present tense.

The Administration Rules

Please carefully note down following rules and regulation before submitting your Research Paper to Global Journals Inc. (US):

Segment Draft and Final Research Paper: You have to strictly follow the template of research paper. If it is not done your paper may get rejected.

- The **major constraint** is that you must independently make all content, tables, graphs, and facts that are offered in the paper. You must write each part of the paper wholly on your own. The Peer-reviewers need to identify your own perceptive of the concepts in your own terms. NEVER extract straight from any foundation, and never rephrase someone else's analysis.
- Do not give permission to anyone else to "PROOFREAD" your manuscript.
- Methods to avoid Plagiarism is applied by us on every paper, if found guilty, you will be blacklisted by all of our collaborated research groups, your institution will be informed for this and strict legal actions will be taken immediately.)
- To guard yourself and others from possible illegal use please do not permit anyone right to use to your paper and files.

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Topics	Grades		
	A-B	**C-D**	**E-F**
Abstract	Clear and concise with appropriate content, Correct format. 200 words or below	Unclear summary and no specific data, Incorrect form Above 200 words	No specific data with ambiguous information Above 250 words
Introduction	Containing all background details with clear goal and appropriate details, flow specification, no grammar and spelling mistake, well organized sentence and paragraph, reference cited	Unclear and confusing data, appropriate format, grammar and spelling errors with unorganized matter	Out of place depth and content, hazy format
Methods and Procedures	Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads	Difficult to comprehend with embarrassed text, too much explanation but completed	Incorrect and unorganized structure with hazy meaning
Result	Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake	Complete and embarrassed text, difficult to comprehend	Irregular format with wrong facts and figures
Discussion	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

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