Online ISSN : 2249-4626
Print ISSN : 0975-5896
DOI : 10.17406/GJSFR

Global Journal

of Science Frontier Research: F

Mathematics and Decision Science

Highlights

- Cube Root Transformation
- Household Survey Estimates
- Radial Teukolsky Equation
- Effect of Cherenkov Radiation

Discovering Thoughts, Inventing Future

VOLUME 16 ISSUE 5 VERSION 1.0

Global Journal of Science Frontier Research: F
Mathematics & Decision Sciences

Global Journal of Science Frontier Research: F
Mathematics & Decision Sciences
Volume 16 Issue 5 (Ver. 1.0)

Open Association of Research Society

Incorporation No.: 0423089
License No.: 42125/022010/1186
Registration No.: 430374
Import-Export Code: 1109007027
Employer Identification Number (EIN):
USA Tax ID: 98-0673427

Global Journals Inc.

*(A Delaware USA Incorporation with "Good Standing"; **Reg. Number: 0423089**)*

Sponsors: *Open Association of Research Society*
Open Scientific Standards

Publisher's Headquarters office

Global Journals® Headquarters
945th Concord Streets,
Framingham Massachusetts Pin: 01701,
United States of America
USA Toll Free: +001-888-839-7392
USA Toll Free Fax: +001-888-839-7392

Offset Typesetting

Global Journals Incorporated
2nd, Lansdowne, Lansdowne Rd., Croydon-Surrey,
Pin: CR9 2ER, United Kingdom

Packaging & Continental Dispatching

Global Journals
E-3130 Sudama Nagar, Near Gopur Square,
Indore, M.P., Pin:452009, India

Find a correspondence nodal officer near you

To find nodal officer of your country, please email us at *local@globaljournals.org*

eContacts

Press Inquiries: *press@globaljournals.org*
Investor Inquiries: *investors@globaljournals.org*
Technical Support: *technology@globaljournals.org*
Media & Releases: *media@globaljournals.org*

Pricing (Including by Air Parcel Charges):

For Authors:
22 USD (B/W) & 50 USD (Color)
Yearly Subscription (Personal & Institutional):
200 USD (B/W) & 250 USD (Color)

Integrated Editorial Board
(Computer Science, Engineering, Medical, Management, Natural Science, Social Science)

Dr. Bart Lambrecht
Director of Research in Accounting and Finance Professor of Finance
Lancaster University Management School
BA (Antwerp); MPhil, MA, PhD (Cambridge)

Dr. Carlos García Pont
Associate Professor of Marketing
IESE Business School, University of Navarra
Doctor of Philosophy (Management), Massachusetts Institute of Technology (MIT)
Master in Business Administration, IESE, University of Navarra
Degree in Industrial Engineering, Universitat Politècnica de Catalunya

Dr. Fotini Labropulu
Mathematics - Luther College
University of Regina Ph.D., M.Sc. in Mathematics
B.A. (Honors) in Mathematics
University of Windso

Dr. Lynn Lim
Reader in Business and Marketing
Roehampton University, London
BCom, PGDip, MBA (Distinction), PhD, FHEA

Dr. Mihaly Mezei
ASSOCIATE PROFESSOR
Department of Structural and Chemical Biology, Mount Sinai School of Medical Center
Ph.D., Etvs Lornd University
Postdoctoral Training,
New York University

Dr. Söhnke M. Bartram
Department of Accounting and Finance Lancaster University Management School Ph.D. (WHU Koblenz)
MBA/BBA (University of Saarbrücken)

Dr. Miguel Angel Ariño
Professor of Decision Sciences
IESE Business School
Barcelona, Spain (Universidad de Navarra)
CEIBS (China Europe International Business School).
Beijing, Shanghai and Shenzhen
Ph.D. in Mathematics
University of Barcelona
BA in Mathematics (Licenciatura)
University of Barcelona

Philip G. Moscoso
Technology and Operations Management
IESE Business School, University of Navarra
Ph.D in Industrial Engineering and Management, ETH Zurich
M.Sc. in Chemical Engineering, ETH Zurich

Dr. Sanjay Dixit, M.D.
Director, EP Laboratories, Philadelphia VA Medical Center
Cardiovascular Medicine - Cardiac Arrhythmia
Univ of Penn School of Medicine

Dr. Han-Xiang Deng
MD., Ph.D
Associate Professor and Research Department Division of Neuromuscular Medicine
Davee Department of Neurology and Clinical Neuroscience Northwestern University
Feinberg School of Medicine

Contents of the Issue

Global Journal of Science Frontier Research: F
Mathematics and Decision Sciences
Volume 16 Issue 5 Version 1.0 Year 2016
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

New Solutions of Radial Teukolsky Equation Via Transformation to Heuns Equation with the Application of Rational Polynomial of at Most Degree 2

By S. Akinbode & A. Anjorin
East Tennessee State University, United States

Abstract- The perturbation equation of masseless fields for Kerr-de Sitter geometry are written in form of seperable equations as in[19] called the Radial Teukolsky equation. The Radial Teukolsky equation is converted to General Heun's equation with singularities coinciding through some conuent process of one of five singularities. As in [17], [18] rational polynomials of at most degree two are introduced.

Keywords: *heun equation, teukolsky equation, type-d metrics, polynomial solutions.*

GJSFR-F Classification : *FOR Code : 37F10*

NEWSOLUTIONSOFRADIALTEUKOLSKYEQUATIONVIATRANSFORMATIONTOHEUNSEQUATIONWITHTHEAPPLICATIONOFRATIONALPOLYNOMIALOFATMOSTDEGREE2

Strictly as per the compliance and regulations of :

New Solutions of Radial Teukolsky Equation Via Transformation to Heuns Equation with the Application of Rational Polynomial of at Most Degree 2

S. Akinbode α & A. Anjorin σ

Abstract- The perturbation equation of masseless fields for Kerr-de Sitter geometry are written in form of seperable equations as in[19] called the Radial Teukolsky equation. The Radial Teukolsky equation is converted to General Heun's equation with singularities coinciding through some conuent process of one of five singularities. As in [17], [18] rational polynomials of at most degree two are introduced.

Keywords: *heun equation, teukolsky equation, type-d metrics, polynomial solutions.*

I. Introduction

Teukolsky equation are the consequences of perturbation equation fer Kerr-de Sitter geometry with the separability of angular and radial parts respectively. Carter[1] was the first to discover that the scalar wave function is separable. Other consideration is the $\frac{1}{2}$ spin electromagnetic field, gravitational perturbations and gravitino for the Kerr-de Sitter class of geometry.

The Teukolsky equation is applicable in he study of black holes in general. The solutions of the equation are in most cases expressed as series solutions of some specialized functions. This approach has been carried out by so many researchers say Teukolsky (1973), Breuer et all (1977), Frackerell and Crossman (1977), Leahy and Unruh (1979), Chakrabarti (1984), Siedel (1989), Suzuki et all (1989) just to mention but few. Although Teukolsky equation has five singular points one irregular with four regular points. By some confluent process, these singular points are reduced to four coinciding with the singular points of Heun's equation.

The objective of this work is to obtain polynomial solutions for the derived Tuekolsky equation through its conversion to Heun's equation through rational polynomials of degree at most 2. New solutions in terms of the rational polynomials are obtained.

The paper is organized as follows; The first section deals with the introduction of Teukolsky equation as described in [19]. The second section deals with the derivation of Teukolsky using the work of [19]. The third

Ref

1. B. Carter, Comm.Math. Phys 10, 280 1968.

Author α: *Department of Mathematics and Statistics, East Tennessee State University, TN, U.S.A. e-mail: akinbodes@etsu.edu*
Author σ: *Department of Mathematics Lagos State University, P.M. B 1087 Apapa Lagos Nigeria. e-mail: anjomathss@yahoo.com*

section has to do with the derivation of Radial Teukolsky and its conversion to Heun's equation. The fourth section has to do with Heun's differential equation and its transformation to hypergeometric differential equation via rational polynomials of at most degree two.

II. The Teukolsky Equation [19]

Tekolsky equation was derived using the Kerr(-Newman)-de Sitter geometries.

$$ds^2 = -p^2(\frac{dr^2}{\Delta_r} + \frac{d\theta^2}{\Delta_\theta}) - \frac{\Delta_\theta \sin^2\theta}{(1+\alpha)^2 p^2}[adt - (r^2+a^2)d\varphi]^2 + \frac{\Delta_r}{(1+\alpha)^2\rho^2}(dt - a\sin^2\theta d\varphi)^2, \quad (1)$$

where

$$\Delta_r = (r^2+a^2)(1 - \frac{a}{ar^2}r^2) - 2Mr + Q^2 = -\frac{\alpha}{a^2}(r-r_+)(r-r_-)(r-r'_+)(r-r'_-)$$
$$\Delta_\theta = 1 + a\cos^2\theta, \alpha = \frac{\Lambda a^2}{3}, \bar{\rho} = r + ia\cos\theta \text{ and } \rho^2 = \bar{\rho}\bar{\rho}, \quad (2)$$

where Λ is the cosmological constant, M is the mass of the black hole, Mr its radial momentum and Q its charge. The electromagnetic field due to the charge of the black hole was given by

$$A_\mu dx^\mu = -\frac{Qr}{(1+\alpha)^2\rho^2}(dt - a\sin^2\theta d\varphi). \quad (3)$$

In particular, the following vectors were adopted as the null tetrad,

$$\iota^\mu = (\frac{(1+\alpha)(r^2+a^2)}{\Delta_r}, 1, 0, \frac{a(1+\alpha)}{\Delta_r}),$$
$$n^\mu = \frac{1}{2\rho^2}((1+\alpha)(r^2+a^2), -\Delta_r, 0, a(1+\alpha)),$$
$$m^\mu = \frac{1}{\bar{p}\sqrt{2\Delta_\theta}}(ia(1+\alpha)\sin\theta, 0, \Delta_\theta, \frac{i(1+\alpha)}{\sin\theta})\bar{m}^\mu = m^{*\mu}. \quad (4)$$

It was assumed that the time and azimuthal dependence of the fields has the form $e^{-i(\omega t - m\varphi)}$,the tetrad components of derivatives and the electromagnetic field were

$$\iota^\mu = D_0, \; n^\mu\partial_\mu = \frac{\Delta_r}{2\bar{\rho}}D_0^\dagger, \; m^\mu\partial_\mu = \frac{\sqrt{\Delta_\theta}}{\sqrt{2}\bar{\rho}}L_0^\dagger,$$
$$m^\mu\partial_\mu = \frac{\sqrt{\Delta_\theta}}{\sqrt{2}\bar{p}^*}L_0, \; \iota^\mu A_\mu = -\frac{Qr}{\Delta_r}, \; n^\mu A_\mu = -\frac{Qr}{2\rho^2},$$
$$m^\mu A_\mu = \bar{m}^\mu A_\mu = 0, \quad (5)$$

where

$$D_n = \partial_r - \frac{\imath(1+\alpha)K}{\Delta_r} + n\frac{\partial_r\Delta_r}{\Delta_r}, \; D_n^\dagger = \partial_r + \frac{\imath(1+\alpha)K}{\Delta_r} + n\frac{\partial_r\Delta_r}{\Delta_r},$$

Notes

$$L_n = \partial_\theta + \frac{\imath(1+\alpha)H}{\Delta_\theta} + n\frac{\partial_\theta(\sqrt{\Delta_\theta}\sin\theta)}{\sqrt{\Delta_\theta}\sin\theta},$$

$$L_n^\dagger = \partial_\theta - \frac{\imath(1+\alpha)H}{\Delta_\theta} + n\frac{\partial_\theta(\sqrt{\Delta_\theta}\sin\theta)}{\sqrt{\Delta_\theta}\sin\theta}, \tag{6}$$

with $K = \omega(r^2+a^2) - am$ and $H = -a\omega\sin\theta + \frac{m}{\sin\theta}$.

Using the Newman-Penrose formalism it was shown that perturbation equation in the Kerr-de sitter geometry are separable for massless spin 0, $\frac{1}{2}$,1,$\frac{3}{2}$ and 2 fields. Similarly in the Kerr-Newman-de sitter space those for spin 0,$\frac{1}{2}$ fields are also separable. The separated equations for fields with spin s and charge e were given by

$$\begin{aligned}&[\sqrt{\Delta_\theta}L_{1-s}^\dagger\sqrt{\Delta_\theta}L_s\\ &-2(1+\alpha)(2s-1)a\omega\cos\theta - 2\alpha(s-1)(2s-1)\cos^2\theta + \lambda]S_s(\theta) = 0\\ &[\Delta_r D_1 D_s^\dagger + 2(1+\alpha)(2s-1)\imath\omega - \frac{2\alpha}{a^2}(s-1)(2s-1)\\ &+\frac{-2(1+\alpha)eQKr + \imath seQr\partial_r\Delta_r + e^2Q^2r^2}{\Delta_r} - 2\imath seQ - \lambda]R_s(r) = 0.\end{aligned} \tag{7}$$

Ref

19. Hisao Suzuki, Ehuchi Takasugi and Hiroshi Umetsu (1998),"Pertubations of Kerr-de Sitter Black Hole and Heun's Equation. EPHOU 98005 OU-HET-296.

III. Transformation of Teukolsky Equation to Heun's Equation [19]

It was shown in [19] that the Teukolsky equations can be transformed to the Heun's equation by factoring out a single regular singularity.

a) Radial Teukolsky equation

From (7), the radial teukolsky equation is explictly written by

$$\Big\{\Delta_r^{-s}\frac{d}{dr}\Delta_r^{s+1}\frac{d}{dr} + \frac{1}{\Delta_r}\Big[(1+\alpha)^2\Big(K - \frac{eQr}{1+\alpha}\Big)^2\Big(K - \frac{eQr}{1+\alpha}\Big)\frac{d\Delta_r}{dr}\Big]$$

$$+4\imath s(1+\alpha)\omega r - \frac{2\alpha}{a^2}(s+1)(2s+1)r^2 + 2s(1-\alpha) - s\imath seQ - \lambda\Big\}R = 0, \tag{8}$$

This equation has five regular singularities at $r^\pm, r'_\pm$ and ∞ which are assigned such that $r_\pm \to M \pm \sqrt{M^2-a^2-Q^2} = r^0_\pm$ and $r'_\pm \to \pm\frac{a}{\sqrt{\alpha}}$ in the limit $\alpha \to 0(\Lambda \to 0)$. By using the new vriable

$$z = \Big(\frac{r_+ - r'_-}{r_+ - r_-}\Big)\Big(\frac{r - r_-}{r - r'_-}\Big),$$

equation (8) becomes an equation which has regular singularities at $0, 1, z_r, z_\infty$ and ∞,

$$z_r = \Big(\frac{r_+ - r'_-}{r_+ - r_-}\Big)\Big(\frac{r'_+ - r_-}{r'_+ - r'_-}\Big),$$

$$z_\infty = \frac{r_+ - r'_-}{r+ - r_-}.$$

Again we can factor out the singularity at $z = z_\infty$ by the transformation as

$$Ra(z) = z^{B_1}(z-1)^{B_2}(z-z_r)^{B_3}(z-z_\infty)^{2s+1}g(z)$$
$$B_1 = \frac{1}{2}\Big\{-s \pm \imath\Big[\frac{2(1+\alpha)a^2(\omega(r_-^2+a^2) - am - \frac{eQr_-}{1+\alpha})}{\alpha(r'_+ - r_-)(r'_- - r_-)(r_+ - r_-)} - \imath s\Big]\Big\}$$

$$B_2 = \frac{1}{2}\Big\{ - s \pm \imath\Big[\frac{2(1+\alpha)a^2(\omega(r_+^2+a^2) - am - \frac{eQr_+}{1+\alpha})}{\alpha(r'_+ - r_+)(r'_- - r_+)(r_- - r_+)} - \imath s\Big]\Big\}$$

$$B_3 = \frac{1}{2}\Big\{ - s \pm \imath\Big[\frac{2(1+\alpha)a^2(\omega(r_+^2+a^2) - am - \frac{eQr'_+}{1+\alpha})}{\alpha(r'_- - r_+)(r'_- - r_+)(r_+ - r_+)} - \imath s\Big]\Big\}. \quad (9)$$

Then $g(z)$satisfies the Heun's equation as

$$\Big\{\frac{d^2}{dz^2} + \Big[\frac{2B_1+s+1}{z} + \frac{2B_2+s+1}{z-1} + \frac{2B_3+s+1}{z-z_r}\Big]\frac{d}{dz}$$

$$+\frac{\sigma_+\sigma_- z + \nu}{z(z-1)(z-z_r)}\Big\}g(z) = 0, \quad (10)$$

where

$$\sigma_\pm = B_1 + B_2 + B_3 + 2s$$

$$+\frac{1}{2}\Big\{ - s \pm \imath\Big[\frac{2(1+\alpha)a^2\Big(\omega(r_-'^2+a^2) - am - \frac{eQr'_-}{1+\alpha}\Big)}{(r_+ - r'_-)(r_- - r'_-)(r'+ - r'_-)} - \imath s\Big]\Big\}$$

$$\nu = \frac{2a^4(1+\alpha)^2(r_+ - r_+)^2(r_+ - r'_-)^2(r_- - r'_-)(r'_+ - r'_-)}{\alpha^2 D(r_+ - r\_)}$$

$$\Big\{ - \omega^2 r_-^3(r_+r_- - 2r_+r'_+ + r_-r'_+) + 2a\omega(a\omega - m)r_-(r_+r'_+ - r_-^2)$$

$$-a^2(a\omega - m)^2(2r_- - r_+ - r'_+)$$

$$+\frac{eQ}{1+\alpha}[\omega r_-^2(r_+r_- + r_-^2 - 3r_+r'_+ + r_-r'_+)$$

$$-a(a\omega - m)(r_+r_- - 3r_-^2 + r_+r'_+ + r_-r'_+)]$$

$$+\Big(\frac{eQ}{1+\alpha}\Big)^2 r_-(-r_-^2 + r_+r'_+)\Big\}$$

$$+\frac{2\imath sa^2(1+\alpha)\Big[\omega(r_-r'_- + a^2) - am - \frac{eQ}{1+\alpha}\frac{r_-+r'_-}{2}\Big]}{(r_+ - r_-)(r'_+ - r'_-)(r_- - r'-)}$$

$$+(s+1)(2s+1)\Big[\frac{2r_-'^2}{(r_+ - r_-)(r'_+ - r'_-)} - z_\infty\Big]$$

$$-2B_1(z_rB_2 + B_3) - (s+1)[(1+z_r)B_1 + z_rB_2 + B_3]$$

$$-\frac{a^2}{\alpha(r_+ - r_-)(r'_+ - r'_-)}[-\lambda - 2\imath seQ + 2s(1-\alpha)]. \quad (11)$$

Here D is the discriminant of

$$\Delta_r = 0$$

$$D = (r_+ - r_-)^2(r_+ - r'_+)^2(r_+ - r'_-)^2(r_- - r'_+)^2(r_- - r'_-)^2(r'_+ - r'_-)^2$$

$$= \frac{16a^{10}}{\alpha^5}\Big\{(1-\alpha)^3[M^2 - (1-\alpha)(a^2+Q^2)]$$

$$+\frac{\alpha}{a^2}[-27M^4 + 36(1-\alpha)M^2(a^2+Q^2)$$

Notes

$$-8(1-\alpha)^2(a^2+Q^2)^2] - \frac{16\alpha^2}{a^4}(a^2+Q^2)^3\Big\}. \quad (12)$$

The sign ambiguity in B_2 or B_3 are related to the boundary condition at the horizon or at the de Sitter horizon, respectively. We can either one of signs of B_1

IV. Heun's Equation to Hypergeometric Via Rational Polynomial Transformations

In this section, we transform the Heun's equation derived above to hypergeometric differential equation with three singularities and back again to the Heun's solutions with polynomial terms.

The hypergeometric equation has three regular singular points. Heun's equation has four regular points. The problem of conversion from Heun's equation to hypergeometric equation has been treated in the works of K.Kuiken[17]. The purpose of this work is to derive some forms solution to the Heun's equation via some rational transformation as stated earlier. The steps taken shall be conversion of Heun's function to the hypergeometric function then taken the derivatives, and through a push and pull back process we arrive back to a new Heun;s function different from the original Heun's function.

Every homogenous linear second order differential equation with four regular singularities can be transformed into (10) with the assumption that $2B_1+s+1=\gamma$, $2B_2+s+1=\delta$, $2B_3+s+1=\epsilon$, $\rho_\pm=\alpha\beta$, $\nu=q$, $z=t$ and $z_r=d$ as defined above, and read as

$$\frac{d^2u}{dt^2}+\left(\frac{\gamma}{t}+\frac{\delta}{t-1}+\frac{\epsilon}{t-1}\right)\frac{du}{dt}+\frac{\alpha\beta t-q}{t(t-1)(t-d)}u=0, \quad (13)$$

where $\{\alpha,\beta,\gamma,\delta,\epsilon,d,q\}(d\neq 0,1)$ are parameters, generally complex and arbitrary, linked by FUSCHAIN constraint $\alpha+\beta+1=\gamma+\delta+\epsilon$. This equation has four regular singular points at $\{0,1,a,\infty\}$, with the exponents of these singular being respectively, $\{0,1,-\gamma\},\{0,1,-\epsilon\}$ and $\{\alpha,\beta\}$. The equation (13) is called Heun's equation.

The Hypergeometric equation

$$z(1-z)\frac{d^2u}{dt^2}+[c-(a+b+1)z]\frac{dy}{dz}-aby=0, \quad (14)$$

has three regular singular points. in the above (13), it has been shown that these two equation above can be transformed to one another via six rational polynomial $z=R(t)$, where $R(t)=t^2, 1-t^2, (t-1)^2, 2t-t^2(2t-1)^2, 4t(1-t)$. The following parameter relations were deduced.

For the polynomial $R(t)=t^2$

- $\alpha+\beta=2(a+b), \alpha\beta=4ab, \gamma=-1+2c, \delta=1+a+b-c, \delta\epsilon=\delta, q=0$ and $d=-2$.

  For the polynomial $R(t)=1-t^2$

- $\alpha+\beta=2(a+b), \alpha\beta=4ab, \gamma=-1-2c+2a+2b, \delta=c, \epsilon=\delta, q=0$ and $d=12$.

  For the polynomial $R(t)=2t-t^2$

Ref

17. K. Kuiken, *Heun's equations and the Hypergeometric equations*, S. I. A. M. J. Math. Anal. 10(3), 655-657 (1979).

- $\alpha+\beta = 2(a+b), \alpha\beta = 4ab, \gamma = c, \delta = 1-2c+2a+2b, \epsilon = \delta = c, q = 4ab$ and $d = 2$.

  For the polynomial $R(t) = (2t-1)^2$

- $\alpha+\beta = 2(a+b), \alpha\beta = 4ab, \gamma = -1+a+b-c, \delta = \gamma, \delta = \epsilon = -1, q = 4ab$ and $d = \frac{1}{2}$.

  For the polynomial $R(t) = 4t(1-t)^2$

- $\alpha+\beta = 2(a+b), \alpha\beta = 4ab, \gamma = c, \delta = \gamma, \delta = 1-2c+2a+2b, q = 2ab$ and $d = \frac{1}{2}$.

Assuming $H(d, q, \alpha, \beta, \gamma, \delta, \epsilon; t) = R_s(t);\ \ s = 1 \ldots 14$ are solutions of the Radial Teukolsky in terms of Heun's with polynomial factor and ${}_2F_1(a, b; c; z = R(t))$ are representative forms of the solutions of (13) and (14) respective, together with parameters above relations can be established between these two forms via the polynomials data given above. We provide an answer to this in this paper. Indeed, we provide that the derivative of the solution of Heun's can be expressed in terms of another Heun's solution giving rise to new solutions of Teukolsky Radial equation.

V. Main Results

a) New Derived Solutions of Radial Teukolsky Equation

In this section we shall apply the relation above in obtaining the derive solutions via these polynomial transformations. let $D = \frac{d}{dt}$ be a differential operator. Since $D{}_2F_1(a, b; c; z = R(t)) = R'(t)\frac{ab}{c}{}_2F_1(a+1, b+1; c+1; z = R(t))$ and through a push and pull back processes we have the following possible solutions for the Teukolsky Radial equation;

1. For polynomial $R(t) = t^2$.

   [a] Using $c = \frac{(\gamma+1)}{2}$, we get

$$DH(-1, 0; \alpha, \beta, \gamma, \delta, \epsilon; t)$$

$$= \frac{\alpha\beta t}{\gamma+1}{}_2F_1\left(\frac{\beta+2}{2}, \frac{\alpha+2}{2}, \frac{\gamma+3}{2}; R(t) = t^2\right)$$

$$= \frac{\alpha\beta t}{\gamma+1}H\left(-1, 0; \alpha+2, \beta+2, \gamma+2, \frac{\alpha+\beta-\gamma+3}{2}, \frac{\alpha+\beta-\gamma+3}{2}; t\right)$$

$$= R_1(t). \tag{15}$$

   [b] Using $c = 1-\delta+a+b$, we get

$$DH(-1, 0; \alpha, \beta, \gamma, \delta, \epsilon; t)$$

$$= \frac{\alpha\beta t}{\alpha+\beta+2(1-\delta)}{}_2F_1\left(\frac{\beta+2}{2}, \frac{\alpha+2}{2}, \alpha+\beta-2(2-\delta); R(t) = t^2\right)$$

$$= \frac{\alpha\beta t}{\gamma+1}\times$$

$$H\left(-1, 0; \alpha+2, \beta+2, \gamma+2, \frac{\alpha+\beta-\gamma+3}{2}, \frac{\alpha+\beta-\gamma+3}{2}; t\right)$$

Notes

$$= \frac{\alpha\beta t}{\alpha+\beta+2(1-\delta)}$$

$$\times H(-1,0;\alpha+2,\beta+2,\alpha+\beta-2\delta+1,1+\delta,1+\delta;t) = R_2(t). \quad (16)$$

By changing δ to ϵ, similar expression can be obtained.

2. For the polynomial $R(t) = 1 - t^2$.

[a] Using $\delta = c$, we get

$$DH(-1,0;\alpha,\beta,\gamma,\delta,\epsilon;t)$$
$$= \frac{\alpha\beta t}{2\delta} {}_2F_1(\frac{\beta+2}{2},\frac{\alpha+2}{2};\delta+1;R(t)=t^2)$$
$$= \frac{\alpha\beta t}{2\delta} H(-1,0,\alpha+2,\beta+2,\alpha+\beta,-2\delta+3,\delta+1,\delta+1;t)$$
$$= R_3(t). \quad (17)$$

[b] $$DH(-1,0;\alpha,\beta,\gamma,\delta,\epsilon;t)$$
$$= \frac{\alpha\beta t}{2\delta} {}_2F_1(\frac{\beta+2}{2},\frac{\alpha+2}{2};\epsilon+1;R(t)=1-t^2)$$
$$= \frac{\alpha\beta t}{2\delta} H(-1,0,\alpha+2,\beta+2,\alpha+\beta,-2\epsilon+3,\epsilon+1,\epsilon+1;t)$$
$$= R_4(t). \quad (18)$$

[c] Using $c = 1-\gamma+2a+2b$ we obtain

$$DH(-1,0;\alpha,\beta,\gamma,\delta,\epsilon;t) =$$
$$-\frac{\alpha\beta t}{\alpha+\beta+2(1-\delta)}\times$$
$${}_2F_1(\frac{\beta+2}{2},\frac{\alpha+2}{2},\frac{\alpha+\beta-\gamma+3}{2};R(t)=1-t^2)$$
$$= \frac{\alpha\beta t}{\alpha+\beta+2(1-\delta)}\times$$
$$\mathrm{H}(-1,0;\alpha+2,\beta+2,\gamma+2,\frac{\alpha+\beta-\gamma+3}{2},\frac{\alpha+\beta-\gamma+3}{2})$$
$$= R_5(t). \quad (19)$$

3. For the polynomial $R(t) = 2t - 2t^2$.

[a] Using $c = 1-a+b-\gamma$ we obtain

$$DH(2,\alpha\beta,\alpha,\gamma,\delta,\epsilon;t) =$$
$$\frac{\alpha\beta(1-t)}{2\gamma} {}_2F_1(\frac{\beta+2}{2},\frac{\alpha+2}{2};\gamma+1;R(t)=2t-t^2)$$
$$= \frac{\alpha\beta(1-t)}{2\gamma}\times$$
$$H(2,(\beta+2)(\alpha+2);\alpha+2,\beta+2,\gamma+1,\alpha+\beta-2\gamma+3,\eta;t)$$
$$= R_6(t)$$

Notes

$$\text{where} \quad \eta = \frac{\alpha+\beta-2\gamma+3}{2}. \tag{20}$$

[b] $DH(2,\alpha\beta,\alpha,\gamma,\delta,\epsilon;t)$

$$= \frac{\alpha\beta(1-t)}{2\gamma} {}_2F_1\left(\frac{\beta+2}{2}, \frac{\alpha+2}{2}; \frac{\alpha+\beta+2(2-\gamma)}{2}; R(t)=2t-t^2\right)$$

$$= \frac{\alpha\beta(1-t)}{2\gamma} \times$$

$$H(2,(\alpha+2)(\beta+2);\alpha+2,\beta+2,\gamma+1,\alpha+\beta-2\gamma+3,\gamma+1;t)$$

$$= R_7(t). \tag{21}$$

By changing γ to ϵ, we can obtain similar expression.

[c] Using $c=\frac{(1+\delta)}{2}$, we get

$$DH(2,\alpha\beta,\alpha,\gamma,\delta,\epsilon;t)$$

$$= \frac{\alpha\beta(1-t)}{2\gamma} {}_2F_1\left(\frac{\beta+2}{2}, \frac{\alpha+2}{2}, \frac{\delta+2}{2}; R(t)=2t-t^2\right)$$

$$= \frac{\alpha\beta(1-t)}{2\gamma}$$

$$\times(2,(\alpha+2)(\beta+2);\alpha+2,\beta+2,\zeta,\delta+2,\zeta;t)$$

$$= R_8(t).$$

$$\text{where} \quad \zeta = \frac{\alpha+\beta-\delta+3}{2} \tag{22}$$

4. For polynomial $R(t)=(1-t^2)$

[a]. Using $c=\frac{(1+\delta)}{2}$, we obtain

$$DH(2,\alpha\beta;\alpha,\beta,\gamma,\delta,\epsilon;t)$$

$$= \frac{\alpha\beta(t-1)}{\delta+1} {}_2F_1\left(\frac{\beta+2}{2}, \frac{\alpha+2}{2}; \frac{\delta+3}{2}; R(t)=(t-1)^2\right)$$

$$\frac{\alpha\beta(t-1)}{\delta+1} \times H(2,(\alpha+2)(\beta+2);\alpha+2,\beta+2,\zeta,\delta+2,\zeta;t)$$

$$= R_9(t),$$

$$\text{where} \quad \zeta = \frac{\alpha+\beta-\delta+3}{2}. \tag{23}$$

[b]. Using $c=\frac{(1-\gamma+2a+2b)}{2}$, we get

$$DH(2,\alpha\beta;\alpha,\beta,\gamma,\delta,\epsilon;t)$$

$$= \frac{\alpha\beta(t-1)}{\alpha+\beta-\gamma+1} {}_2F_1\left(\frac{\beta+2}{2}, \frac{\alpha+2}{2}; \frac{\alpha+\beta-\gamma+3}{2} R(t)=(t-1)^2\right)$$

$$= \frac{\alpha\beta(t-1)}{\alpha+\beta-\gamma+1}$$

Notes

$$\times H(2,(\beta+2)(\alpha+2);\alpha+2,\beta+2,\frac{\gamma+3}{2},\alpha+\beta-2\gamma+3,\frac{\gamma+3}{2};t)$$
$$= R_{10}(t). \qquad (24)$$

$[c]$ By changing γ to ϵ in above, similar relation can be obtained.

5. For polynomial $R(t)=(2t-1)^2$.

$[a]$. Using $c=\frac{1+\epsilon}{2}=\frac{1+\delta}{2}$

$$DH(\frac{1}{2},\alpha\beta;\alpha,\beta,\gamma,\delta,\epsilon;t)$$
$$=\frac{2(2t-1)\alpha\beta}{(\epsilon+1)}\,{}_2F_1(\frac{\beta+2}{2},\frac{\alpha+2}{2};\frac{\epsilon+3}{2}R(t)=(2t-1)^2)$$
$$=\frac{2(2t-1)\alpha\beta}{(\epsilon+1)}$$
$$\times(\frac{1}{2},\frac{(\alpha+2)(\beta+2)}{2};\alpha+2,\beta+2,\frac{\alpha+\beta-\epsilon-1}{2},\epsilon+2;t)$$
$$=R_{11}(t). \qquad (25)$$

By changing ϵ to δ a similar expression can be obtained.

[b] Using $c=-1+a+b-\gamma$, we obtained

$$DH(\frac{1}{2},\alpha\beta;\alpha,\beta,\gamma,\delta,\epsilon;t)$$
$$=\frac{2(2t-1)\alpha\beta}{(\epsilon+1)}\,{}_2F_1(\frac{\beta+2}{2},\frac{\alpha+2}{2};\frac{\alpha+\beta-2\gamma}{2}R(t)=(2t-1)^2)$$
$$=\frac{2(2t-1)\alpha\beta}{(\epsilon+1)}$$
$$\times(\frac{1}{2},\frac{(\alpha+2)(\beta+2)}{2};\alpha+2,\beta+2,\mu,\mu,\tau;t)$$
$$=R_{12}(t). \qquad (26)$$

where $\tau=\alpha+\beta-2(\gamma-\frac{1}{2})$ and $\mu=\frac{\alpha+\beta+2(1-\gamma)}{2}$

6. For the polynomial $R(t)=4t(1-t)$

$[a]$ Using $c=\gamma$,we get

$$DH(\frac{1}{2},\alpha\beta;\alpha,\beta,\gamma,\delta,\epsilon;t)$$
$$=\frac{(1-2t)\alpha\beta}{\gamma}\,{}_2F_1(\frac{\beta+2}{2},\frac{\alpha+2}{2};\gamma+1;R(t)=4t(1-t))$$
$$=\frac{(1-2t)\alpha\beta}{\gamma}$$
$$\times H(\frac{1}{2},\frac{(\alpha+2)(\beta+2)}{2};\alpha+2,\beta+2,\gamma+1,\alpha+\beta-2\gamma+3;t)$$

Notes

$$= R_{13}(t). \tag{27}$$

[b] Using $c = \frac{(1-\epsilon+2a+2b)}{2}$

$$DH(\frac{1}{2}, \frac{\alpha\beta}{2}; \beta, \alpha, \gamma, \delta, \epsilon, ; t)$$

$$= \frac{(1-2t)\alpha\beta}{\alpha+\beta-\epsilon+1}{}_2F_1(\frac{\beta+2}{2}, \frac{\alpha+2}{2}; \frac{\alpha+\beta-\epsilon+3}{2}R(t) = 4t(1-t))$$
$$= \frac{(1-2t)\alpha\beta}{\alpha+\beta-\epsilon+1}$$

$$\times(\frac{1}{2}, \frac{(\alpha+2)(\beta+2)}{2}; \alpha+2, \beta+2, \gamma, \omega, \omega, \epsilon+2; t)$$

$$= R_{14}(t), \tag{28}$$

where

$$\omega = \frac{\alpha+\beta-\epsilon+3}{2}.$$

IV. Concluding Remarks and Suggestions

In this paper, we have shown that the solutions of the derived Radial Teukol-sky equation transformed to Heun's equation could be obtained in form of Heun's functions via polynomials of at most degree two transformations. The Heun's equation was initially compare with the hypergeometric differential equation with three singularities via the giving polynomials. Another results could be obtained if we apply the integral operator instead of the differential operator. Polynomials of higher degrees are being consider.

References Références Referencias

1. B. Carter, Comm.Math. Phys 10, 280 1968.
2. A. Ronveaux, *Heun's Differential equation* (Oxford University press, Oxford, 1995).
3. A. O. Smirnov, *Elliptic solutions and Heun's Equations*, C. R. M. Proceedings and Lecture notes 32, 287-305 (2002).
4. P. A. Clarkson and P. J. Oliver, J. Diff. Equations 124, 225-246(1996).
5. R. S. Maier, *Heun-to-hypergeometric transformations*, contribution to the conference of Foundations of Com-putational Mathematics 02 (2002); downloadable from http:// www.math.umn.edu/∼focm/c_/Maier.pdf
6. N. H. Christ and T. D. Lee, *Phys. Rev. D* 12 1606 (1975);
7. A. Ishkhanyan and K. A. Souminen, J. Phys. A: Math. Gen. 36, L81-L85 (2003).
8. G. Valent, *Heun functions versus elliptic functions*, International Conference on Differential Equations, Special Functions and Applications, Munich, 2005; [e-print math-ph/0512006].
9. V. Stanley Grossman, *Multivariate calculus, Linear Algebra, and differential equation* (Saunder college publishing, New York, 1995).
10. S. P. Tsarev, *An algorithm for complete enumeration of all factorizations of a linear ordinary differential operator*, Proceedings of the international symposium on Symbolic and algebraic computation, pp 226-231 (Switzerland, 1996).
11. M. Van Hoeij, Journal of Symbolic Computation 24 n 5, 537-561(1997).

Notes

Notes

12. R. K. Bhadari, A. Khare, J. Law, M. V. N. Murthy and D. Sen, J. Phys. A: Math. Gen. 30, 2557-2260 (1997).
13. M. Suzuki, E. Takasugi and H. Umetsu, Prog. Theor. Phys. 100, 491-505 (1998).
14. K. Takemura, Commun. Math. Phys. 235, 467-494 (2003); J. Non-linear Math. Phys. 11, 480-498 (2004).
15. P. Dorey, J. Suzuki and R. Tateo, J. Phys. A: Math. Gen. 37, 2047-2061 (2004).
16. M. N. Hounkonnou, A. Ronveaux and A. Anjorin, *Derivatives of Heun's function from some properties of hypergeometric equation*, Proceeding of International Workshop on Special Functions, Mar-seille (2007), *in press*; preprint; ICMPA-MPA/2007/21.
17. K. Kuiken, *Heun's equations and the Hypergeometric equations*, S. I. A. M. J. Math. Anal. 10(3), 655-657 (1979).
18. Anjorin A (2009). "On Derivative of solutions to Heun's Differential equation via Rational polynomial of Degree 2; 3; 4; 5; 6. Journal of the Association of Nigerian Mathematical Physics. Vol. 14, pp 289-292. Nigeria.
19. Hisao Suzuki, Ehuchi Takasugi and Hiroshi Umetsu (1998),"Pertubations of Kerr-de Sitter Black Hole and Heun's Equation. EPHOU 98005 OU-HET-296.

This page is intentionally left blank

Global Journal of Science Frontier Research: F
Mathematics and Decision Sciences
Volume 16 Issue 5 Version 1.0 Year 2016
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

On the Hazard Rate Functions of HIV/Aids using Weibull and Exponential Models

By O. D Ogunwale & O. Faweya
Ekiti State University

Abstract- This paper presents the hazard rate functions of Weibull and Exponential Distributions. Estimates of the hazard rates were obtained using simulated HIV/AIDs data, it was found that for exponential distribution the hazard rate is constant while that of the Weibull distribution increases as the time of infection increases.

Keywords: *hazard, exponential distribution, weibull distribution, infection time.*

GJSFR-F Classification : *FOR Code : 11D61*

ONTHEHAZARDRATEFUNCTIONSOFHIVAIDSUSINGWEIBULLANDEXPONENTIALMODELS

Strictly as per the compliance and regulations of :

Notes

On the Hazard Rate Functions of HIV/Aids using Weibull and Exponential Models

O. D Ogunwale [α] & O. Faweya [σ]

Abstract- This paper presents the hazard rate functions of Weibull and Exponential Distributions. Estimates of the hazard rates were obtained using simulated HIV/AIDs data, it was found that for exponential distribution the hazard rate is constant while that of the Weibull distribution increases as the time of infection increases.

Keywords: hazard, exponential distribution, weibull distribution, infection time.

I. Introduction

The hazard rate or mortality rates function ***h(t)*** is usually obtained from the survival function and is defined as the probability that an individual who is under observation at a time t has an event at time t. It represents the instantaneous event rate for an individual who has already survival to time t.

The hazard rate ***h(t),*** for a given distribution whose survival function is given by $\boldsymbol{S(t)}$ is defined as

$$h(t) = \frac{-\frac{d}{dt}S(t)}{S(t)} \qquad \text{------------------------------------} \quad (1)$$

II. Materials and Methods

The survival function of Weibull distribution is

$$\begin{aligned} S(t) &= P(T \geq t) \\ &= \int_t^\infty \lambda\gamma t^{\gamma-1} e^{-\lambda t^\gamma} dt \\ &= e^{-\lambda t^\gamma} \qquad \text{---} \quad (2) \end{aligned}$$

The hazard rate function is therefore

$$\begin{aligned} h(t) &= \frac{-\frac{d}{dt}e^{-\lambda t^\gamma}}{e^{-\lambda t^\gamma}} \\ &= \frac{\lambda\gamma t^{\gamma-1} e^{-\lambda t^\gamma}}{e^{-\lambda t^\gamma}} \\ &= \lambda\gamma t^\gamma \qquad \text{--} \quad (3) \end{aligned}$$

Author α σ: Deparment of Mathematical Sciences, Faculty of Science, Ekiti State University, Ado-Ekiti. e-mail: lanrefaweya@gmail.com

For exponential distribution $S(t)$ is obtained as follows

$$S(t) = \int_t^\infty \lambda e^{-\lambda t}\, dt$$

$$= e^{-\lambda t} \quad \text{---------------------------------------} \quad (4)$$

$$h(t) = \frac{-\frac{d}{dt}e^{-\lambda t}}{e^{-\lambda t}}$$

$$= \frac{\lambda e^{-\lambda t}}{e^{-\lambda t}}$$

$$= \lambda \quad \text{---------------------------------------} \quad (5)$$

Hence the hazard rate of exponential survival function is equal to the parameter and is always a constant.

In dynamics of epidemics such as HIV/AIDS the hazard rate is not constant. Hence the Weibull has assumption is preferred. In particular, the hazard rate increases as the period of infectiousness increases.

III. Results and Discussion

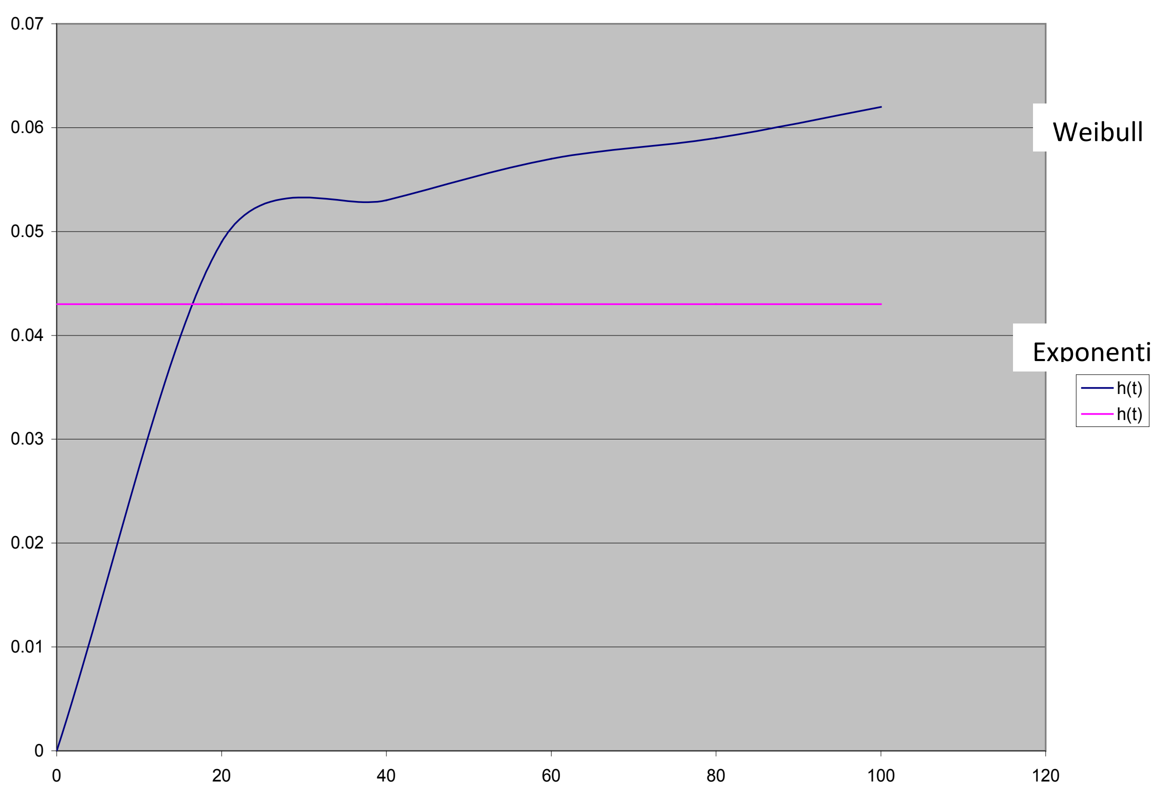
The Weibulll 7^{++} HIV/AIDS simulation for Weibull and Exponential Models with n = 500 yields for Weibull $\lambda = 0.0256$, $\gamma = 1.156$. For exponential $\lambda = 0.043$ With t (in months) = 0. 20, 40, 60, 80 and 100 yields the following tables For Weibull

T	0	20	40	60	80	100
h(t)	0	0.049	0.053	0.057	0.059	0.062

and for exponential

T	0	20	40	60	80	100
h(t)	0.043	0.043	0.043	0.043	0.043	0.043

The results are displayed in the following graph.



Notes

It could be observed from the graphical display that, the hazard rate using Exponential Survival Model is $\lambda = 0.043$ which is shown as a straight line with intercept 0.043 and is parallel to the t axis. Also looking at the display for Weibull's model, it increases sharply during the first 20 months of infectiousness and increases steadily from over 20 months.

Notes

IV. Conclusion

The hazard or mortality rate is the determiner of the number of years/months an individual diagnosed of HIV/AIDS or any other epidemic will survive before death.

The result obtained, for Exponential Function, show that the hazard rate is $\lambda = 0.043$ which is constant. This is not always the case with many epidemics like HIV/AIDS, for example, but the Weibull Model provided an in-depth information.

In the first 20 months of infection the failure rate increases sharply and this means that the immune system of an individual will breakdown devastatingly during this period this accounts for the fact that almost half of a cohort of infected individuals will die within the first 20 months approximately (Adeleke and Ogunwale 2013). The Weibull model is thus preferred to the Exponential Model because of this reason.

References Références Referencias

1. Anderson R.M, May R.M, (1991). Infectious Disease of Human: Dynamics and Control. Oxford U.P. Oxford
2. Anderson R.M, May R.M and Mclean A.R. (1988). Possible Demographic Consequences of AIDs in developing countries. Nature 332, 228-234.
3. Busenberg S, Cook K (1993). Vertically Transmitted Diseases: Models and Dynamics. Biomathematics 23, Spring, Berlin.
4. Odo Diemann and Heesterbek J.A.P (2000) Mathematical Epidemiology of Infectious Diseases.
5. Wiley series in Mathematical and Computational Biology. John Wiley & Sons, West Sub-Sex, England.

This page is intentionally left blank

Global Journal of Science Frontier Research: F
Mathematics and Decision Sciences
Volume 16 Issue 5 Version 1.0 Year 2016
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

A New Method for Estimating Smooth Regression Functions

By Eunji Lim & Annerys Matos
Kean University, United States

Abstract- We propose a new method for estimating a regression function from noisy data when the underlying function is known to satisfy a certain smoothness condition. The proposed method fits a function to the data set so that the roughness of the fitted function is minimized while ensuring that the sum of the absolute deviations of the fitted function from the data points does not exceed a certain limit. It is shown that the fitted function exists and can be computed by solving a quadratic program. Numerical results demonstrate that the proposed method generates more efficient estimates than its alternative in terms of the mean square error and the amount of time required to compute the fit.

Keywords: nonparametric regression, smoothing spline, quadratic programming, penalized least, squares regression.

GJSFR-F Classification : FOR Code : 62J05

ANEWMETHODFORESTIMATINGSMOOTHREGRESSIONFUNCTIONS

Strictly as per the compliance and regulations of :

Notes

A New Method for Estimating Smooth Regression Functions

Eunji Lim [α] & Annerys Matos [σ]

Abstract- We propose a new method for estimating a regression function from noisy data when the underlying function is known to satisfy a certain smoothness condition. The proposed method fits a function to the data set so that the roughness of the fitted function is minimized while ensuring that the sum of the absolute deviations of the fitted function from the data points does not exceed a certain limit. It is shown that the fitted function exists and can be computed by solving a quadratic program. Numerical results demonstrate that the proposed method generates more efficient estimates than its alternative in terms of the mean square error and the amount of time required to compute the fit.

Keywords: nonparametric regression, smoothing spline, quadratic programming, penalized least, squares regression.

I. Introduction

When estimating an unknown function from noisy data, the underlying function is often assumed to be smooth in the independent variables. For example, the price of a stock option and its second derivative are often assumed to be smooth in the underlying stock price over a domain of interest: see Section 3.1 of Lim and Attallah (2016) for an example. In this paper, we consider the problem of estimating a function $f_* : \mathbb{R} \to \mathbb{R}$ over a domain $[a, b)$ of interest, which is known to have a square integrable kth derivative $(k \geq 2)$, by observing a data set $((x_i, Y_{ij}) : 1 \leq i \leq n, 1 \leq j \leq m)$ satisfying

$$Y_{ij} = f_*(x_i) + \epsilon_{ij},$$

where the x_i's satisfy $a < x_1 < \cdots < x_n < b$, and the ϵ_{ij}'s are independent and identically distributed (iid) random variables with a mean of 0 and a variance of $\sigma^2 < \infty$.

One of the most popular approaches to estimating the underlying function f_* from the data set is to find a smooth function that is close to the data set by solving the following optimization problem:

$$\text{Minimize} \qquad (1/n)\sum_{i=1}^{n}\left(g(x_i) - \overline{Y}_i\right)^2 + \lambda \int_a^b \left\{g^{(k)}(x)\right\}^2 dx \tag{1}$$

over $g \in \mathcal{D}^k$, where $\overline{Y}_i = \sum_{j=1}^{n} Y_{ij}/m$ for $1 \leq i \leq n$, λ is a non-negative constant,

$$\mathcal{D}^k = \left\{ f : \mathbb{R} \to \mathbb{R} : f \text{ is } k \text{ times differentiable and } \int_a^b \{f^{(k)}(x)\}^2 dx < \infty \right\},$$

Author α σ: Kean University. e-mail: eunji96@gmail.com

and $g^{(k)}$ is the kth derivative of g. The term $(1/n)\sum_{i=1}^{n}(g(x_i)-\overline{Y}_i)^2$ in (1) measures the closeness of the fitted function g to the data set, and the term $\int_a^b \{g^{(k)}(x)\}^2 dx$ in (1) measures the "roughness" of the fitted function g. The parameter λ controls the trade-off between the closeness to the data set and the roughness of the fitted function.

Problem (1) appears to be an infinite–dimensional optimization problem at first glance, but the solution to (1) is known to be a piecewise polynomial function of degree $2k-1$ with knots $x_1,\ldots,x_n$ (Theorem 20.1 on page 412 of Györfi et al., 2002). Since the set of piecewise polynomial functions of degree $2k-1$ with knots $x_1,\ldots,x_n$ is finite dimensional, (1) can be reduced to an optimization problem over a finite-dimensional space. In fact, the solution to (1) can be obtained by solving a system of linear equations: see (20.6) on page 412 of Györfi et al. (2002) for details.

Notes

Despite the fact that the solution to (1) can be obtained easily, the performance of the solution to (1) is highly sensitive to the choice of λ. Several authors have proposed the method of cross-validation for choosing λ (Wahba and Wold, 1975). In the method of cross-validation, λ is chosen so that it minimizes the average squared error, which is defined by

$$\mathrm{CV}(\lambda)=\sum_{i=1}^{n}\left(\tilde{g}_\lambda^i(x_i)-\overline{Y}_i\right)^2/n$$

for $\lambda\geq 0$, where $\tilde{g}_\lambda^i$ is the solution to (1) with the ith data point, $(x_i,\overline{Y}_i)$, omitted. In order to select the right value of λ, one needs to find the minimizer of $\mathrm{CV}(\lambda)$ over $\lambda\geq 0$. $\mathrm{CV}(\lambda)$ is a nonlinear function in λ in general. Thus, it takes a significant amount of time in practice to find the minimizer of $\mathrm{CV}(\lambda)$.

To overcome the issue of selecting the right value of λ, the solution $\tilde{g}_n$ to the following alternative formulation is preferred in the numerical analysis community:

$$\text{Problem (A):}\quad \text{Minimize}\quad \int_a^b\left\{g^{(k)}(x)\right\}^2 dx$$

$$\text{subject to}\quad (1/n)\sum_{i=1}^{n}\left(g(x_i)-\overline{Y}_i\right)^2\leq u_0$$

for some constant u_0 over $g\in\mathcal{D}^k$, which was first proposed by Reinsch (1967). Problem (A) is preferred in the numerical analysis community because a good estimate of u_0 can be easily computed from the data set $((x_i,Y_{ij}):1\leq i\leq n, 1\leq j\leq m)$ by using $\sum_{i=1}^{n}S_i^2/(nm)$, where $S_i^2=\sum_{j=1}^{m}\left(Y_{ij}-\overline{Y}_i\right)^2/(m-1)$ for $1\leq i\leq n$: see page 151 of Lim and Attallah (2016) for details.

Recently, Lim and Attallah (2016) showed that the solution to Problem (A) can be computed by solving a convex program. Several efficient algorithms exist for solving convex programming problems, and they provide guaranteed convergence to the global solution; see the Lagrangian method on page 217 of Zangwill (1969) for an example of methods that solve convex programs. Thus, the formulation in Lim and Attallah (2016) enables one to compute the solution to Problem (A) with guaranteed convergence. However, the amount of time required to solve a convex program increases rapidly as $n\to\infty$, and hence, a computationally more efficient formulation is desired.

In this paper, we propose a new formulation that is designed for better computational efficiency. The new formulation replaces the constraint $(1/n)\sum_{i=1}^{n}(g(x_i)-\overline{Y}_i)^2\leq u_0$ in Problem (A) with $(1/n)\sum_{i=1}^{n}|g(x_i)-\overline{Y}_i|\leq g_0$ for some constant g_0. Thus, our proposed estimator is the solution $\hat{g}_n$ to the following optimization problem:

$$\text{Problem (B):}\quad \text{Minimize} \quad \int_a^b \left\{ g^{(k)}(x) \right\}^2 dx$$

$$\text{subject to} \quad (1/n) \sum_{i=1}^{n} \left| g(x_i) - \overline{Y}_i \right| \leq g_0$$

Notes

over $g \in \mathcal{D}^k$. Problem (B) can be further transformed into a quadratic program (see Proposition 1 of this paper), so the solution to Problem (B) can be obtained by solving a quadratic program. Quadratic programs are special cases of convex programs with quadratic objective functions and linear constraints, so they can be solved more efficiently than convex programs. Thus, the solution to our new formulation can be computed more efficiently than the solution to Problem (A). Moreover, the constant g_0 appearing in Problem (B) can be estimated from the data set $((x_i, Y_{ij}) : 1 \leq i \leq n, 1 \leq j \leq m)$ readily, so the performance of the proposed estimator $\hat{g}_n$ does not rely on any unnatural parameters; see Section 2.1 of this paper for a discussion of how to estimate g_0. Even though Problem (B) was motivated by the need for better computational efficiency, the numerical experiments in Section 3 show that the proposed estimator not only is computed faster than the solution to Problem (A), but also achieves better mean squared errors, and hence, is a better estimate of the underlying function f_*. Furthermore, the convergence of the proposed estimator to the true function f_* as $n \to \infty$ is demonstrated empirically in Section 3 by showing that the empirical integrated mean square error between $\hat{g}_n$ and f_* converges to 0 as $n \to \infty$. The numerical results in Section 3.2 show that our formulation is successfully applied to a problem of estimating the sensitivities of option prices as functions of the underlying stock price.

This paper is organized as follows. In Section 2.1, we describe how g_0 can be estimated from the data set in more detail. In Section 2.2, we prove that the solution to Problem (B) exists and can be obtained by solving a quadratic program. In Section 3, we compare the performance of the proposed estimator to that of the solution to Problem (A) through numerical experiments. Concluding remarks are included in Section 4.

II Problem Formulation

a) *How to Estimate g_0 from the Data Set?*

In this section, we provide a heuristic argument on how g_0 can be estimated from the data set $((x_i, Y_{ij}) : 1 \leq i \leq n, 1 \leq j \leq m)$. We start by noticing that $\sum_{j=1}^{m} \epsilon_{ij}/\sqrt{m}$ converges in distribution to $N(0, \sigma^2)$ as $m \to \infty$ by the weak law of large numbers, where $N(0, \sigma^2)$ denotes a normal random variable with a mean of 0 and a variance of σ^2. Hence, if we denote $\sum_{j=1}^{m} \epsilon_{ij}/m$ by $\bar{\epsilon}_i$ for $1 \leq i \leq n$, then $|\bar{\epsilon}_i|$ can be approximated by $\left|N(0, \sigma^2)\right|/\sqrt{m}$ for m sufficiently large. Therefore, the following approximation is possible:

$$\frac{1}{n} \sum_{i=1}^{n} \left| \overline{Y}_i - f_*(x_i) \right| = \frac{1}{n} \sum_{i=1}^{n} |\bar{\epsilon}_i| \approx \frac{1}{n\sqrt{m}} \sum_{i=1}^{n} \left| N(0, \sigma^2) \right|$$

for m sufficiently large. The symbol $\approx$ is used to express "approximate equality" informally. When the ϵ_i's are assumed to be normally distributed, $\sum_{i=1}^{n} \left|N(0, \sigma^2)\right|/n$ converges to $\mathbb{E}\,|\epsilon_{11}|$ as $n \to \infty$ by the strong law of large numbers. Thus, the following approximation is appropriate:

$$\frac{1}{n}\sum_{i=1}^{n}\left|\overline{Y}_i - f_*(x_i)\right| \approx \frac{\mathbb{E}\left|\epsilon_{11}\right|}{\sqrt{m}}$$

for n and m sufficiently large. Furthermore, $\mathbb{E}\left|\epsilon_{11}\right|$ can be estimated from the data set via $\sum_{i=1}^{n}\sum_{j=1}^{m}\left|Y_{ij} - \overline{Y}_i\right|/(mn)$, and hence, a good estimate of g_0 is

$$\sum_{i=1}^{n}\sum_{j=1}^{m}\left|Y_{ij} - \overline{Y}_i\right|/(m^{3/2}n). \tag{2}$$

Notes

b) Quadratic Programming Representation of the Proposed Formulation

In this section, we describe how the proposed estimator can be obtained by solving a quadratic program. To make this paper self-contained, we present some preliminary results.

A spline function with degree $r > 1$ with knots $x_1, \ldots, x_n$, where $a < x_1 < \ldots, < x_n < b$, is a function $s : [a, b) \to \mathbb{R}$ having the following two properties: (a) In each of the intervals $[a, x_1), [x_1, x_2), \ldots, [x_{n-1}, x_n), [x_n, b)$, $s(x)$ is given by some polynomial of degree r or less, and (b) $s(x)$ is $r-1$ times continuously differentiable on $[a, b)$. We denote the set of spline functions with degree r by $\mathcal{S}_r([a, b))$. $\mathcal{S}_r([a, b))$ can be spanned by a finite number of elements in $\mathcal{S}_r([a, b))$, so we introduce one of the bases for $\mathcal{S}_r([a, b))$, which is the set of B-splines; the B-splines have bounded supports and produce well–conditioned numerical settings. We need to introduce additional knots $x_{-r}, \ldots, x_0, x_{n+1}, \ldots, x_{n+r+1}$ so that

$$x_{-r} < x_{-r+1} < \cdots < x_0 < a < x_1 < \cdots < x_n < b < x_{n+1} < \cdots < x_{n+r+1}.$$

The B-spline $B_{i,r}$ of degree r is defined recursively by

$$B_{i,0}(x) = \begin{cases} 1, & \text{if } x_i \le x < x_{i+1} \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

for $i = -r, \ldots, n+r$ and $x \in \mathbb{R}$ and

$$B_{i,l}(x) = \frac{x - x_i}{x_{i+l} - x_i} B_{i,l-1}(x) + \frac{x_{i+l+1} - x}{x_{i+l+1} - x_{i+1}} B_{i+1,l-1}(x) \tag{4}$$

for $i = -r, \ldots, n+r-l$, $l = 1, \ldots, r$, and $x \in \mathbb{R}$. By Theorem 14.1 on page 262 of Györfi et al. (2002), $\{B_{i,r} : i = -r, \ldots, n\}$ restricted to $[a, b)$ is a basis of $\mathcal{S}_r([a, b))$. Proposition 1 below proves the existence of the solution to Problem (B) and describes how Problem (B) can be solved through a quadratic program.

Proposition 1 *Assume $2 \le k \le n$. There exists a solution $\hat{g}_n \in \mathcal{D}^k$ to Problem (B). Furthermore, $\hat{g}_n$ has the following representation:*

$$\hat{g}_n(x) = \sum_{i=-(2k-1)}^{n} \hat{c}_i B_{i,2k-1}(x)$$

for $x \in [a, b)$, where $\hat{c}_{-(2k-1)}, \ldots, \hat{c}_n, \hat{y}_1, \ldots, \hat{y}_n, \hat{p}_1, \ldots, \hat{p}_n, \hat{m}_1, \ldots, \hat{m}_n$ is the solution to the following quadratic program in the decision variables $c_{-(2k-1)}, \ldots, c_n, y_1, \ldots, y_n, p_1, \ldots, p_n, m_1, \ldots, m_n \in \mathbb{R}$:

Notes

$$\textit{Minimize} \quad \int_a^b \left(\sum_{i=-(2k-1)}^{n} c_i B_{i,2k-1}^{(k)}(x) \right)^2 dx$$

$$= \sum_{i=-(2k-1)}^{n} \sum_{j=-(2k-1)}^{n} c_i c_j \int_a^b B_{i,2k-1}^{(k)}(x) B_{j,2k-1}^{(k)}(x) dx$$

$$\textit{subject to} \quad \overline{Y}_i - y_i = p_i - m_i, \quad 1 \le i \le n, \tag{5}$$

$$\sum_{i=1}^{n} (p_i + m_i)/n \le g_0,$$

$$\sum_{i=-(2k-1)}^{n} c_i B_{i,2k-1}(X_j) = y_j, \quad j = 1, \ldots, n,$$

$$p_i, m_i \ge 0, \quad 1 \le i \le n.$$

Proof. The existence of the solution to Problem (B) is proven by an argument similar to the proof of Proposition 1 in Lim and Attallah (2016). Next, to show that the solution to (5) exists, we notice that (i) for any $c_{-(2k-1)}, \ldots, c_n$, the objective function of (5) is greater than or equal to 0, and (ii) Problem (5) has a feasible solution because there exist $\overline{c}_{-(2k-1)}, \ldots, \overline{c}_n$ satisfying

$$\sum_{i=-(2k-1)}^{n} \overline{c}_i B_{i,2k-1}(x_j) = \overline{Y}_j$$

for $1 \le j \le n$ by Lemmas 20.2 and 20.3 on pages 415 and 416 of Györfi et al. (2002). By Frank and Wolfe (1956), there exists a solution to (5).

Let $\hat{g}_n$ be a solution to Problem (B). Let $\hat{y}_i = \hat{g}_n(x_i)$ for $1 \le i \le n$. By Lemmas 20.2 and 20.3 on pages 415 and 416 of Györfi et al. (2002), there exists a unique solution $(\hat{c}_{-(2k-1)}, \ldots, \hat{c}_n)$ to the following linear system:

$$\begin{aligned} \sum_{i=-(2k-1)}^{n} \hat{c}_i B_{i,2k-1}(x_j) &= \hat{y}_j \text{ for } 1 \le j \le n, \\ \sum_{i=-(2k-1)}^{n} \hat{c}_i B_{i,2k-1}^{(l)}(a) &= 0, \\ \sum_{i=-(2k-1)}^{n} \hat{c}_i B_{i,2k-1}^{(l)}(b) &= 0. \end{aligned}$$

Let $\hat{p}_i = \max(\overline{Y}_i - \hat{y}_i, 0)$ and $\hat{m}_i = \max(\hat{y}_i - \overline{Y}_i, 0)$ for $1 \le i \le n$. We will show that $\hat{c}_{-(2k-1)}, \ldots, \hat{c}_n$, $\hat{y}_1, \ldots, \hat{y}_n, \hat{p}_1, \ldots, \hat{p}_n, \hat{m}_1, \ldots, \hat{m}_n$ is a solution to (5). Let $\overline{c}_{-(2k-1)}, \ldots, \overline{c}_n, \overline{y}_1, \ldots, \overline{y}_n, \overline{p}_1, \ldots, \overline{p}_n, \overline{m}_1$, $\ldots, \overline{m}_n$ be any feasible solution to (5). Without loss of generality, we may assume that either $\overline{p}_i = 0$ or $\overline{m}_i = 0$ for each $1 \le i \le n$. (Otherwise, we replace $\overline{p}_i$ with $\overline{p}_i - \min(\overline{p}_i, \overline{m}_i)$ and $\overline{m}_i$ with $\overline{m}_i - \min(\overline{p}_i, \overline{m}_i)$ for each $1 \le i \le n$.) We notice that $|\overline{Y}_i - \overline{y}_i| = \overline{p}_i + \overline{m}_i$ for each $1 \le i \le n$. So, if we define $\overline{g}_n : \mathbb{R} \to \mathbb{R}$ by

$$\bar{g}_n(x) = \sum_{i=-(2k-1)}^{n} \bar{c}_i B_{i,2k-1}(x)$$

for $x \in \mathbb{R}$, then $\bar{g}_n$ satisfies $(1/n)\sum_{i=1}^{n}\left|\bar{g}_n(x_i) - \overline{Y}_i\right| \le g_0$, and hence, is a feasible solution to Problem (B). Thus, $\int_a^b \left\{\hat{g}_n^{(k)}(x)\right\}^2 dx \le \int_a^b \left\{\bar{g}_n^{(k)}(x)\right\}^2 dx$, and hence, $\hat{c}_{-(2k-1)}, \ldots, \hat{c}_n, \hat{y}_1, \ldots, \hat{y}_n, \hat{p}_1, \ldots, \hat{p}_n, \hat{m}_1, \ldots, \hat{m}_n$ is a solution to (5).

Conversely, let $\tilde{c}_{-(2k-1)}, \ldots, \tilde{c}_n, \tilde{y}_1, \ldots, \tilde{y}_n, \tilde{p}_1, \ldots, \tilde{p}_n, \tilde{m}_1, \ldots, \tilde{m}_n$ be a solution to (5). We will show that $\tilde{g}_n : \mathbb{R} \to \mathbb{R}$ defined by

$$\tilde{g}_n(x) = \sum_{i=-(2k-1)}^{n} \tilde{c}_i B_{i,2k-1}(x)$$

for $x \in \mathbb{R}$ is a solution to Problem (B), or equivalently, $\int_a^b \left\{\tilde{g}_n^{(k)}(x)\right\}^2 dx \le \int_a^b \left\{\hat{g}_n^{(k)}(x)\right\}^2 dx$ for any solution $\hat{g}_n$ to Problem (B). First, we note that $\tilde{g}_n$ is a feasible solution to Problem (B) since we may assume that either $\tilde{p}_i = 0$ or $\tilde{m}_i = 0$ for each $1 \le i \le n$. For any solution $\hat{g}_n$ to Problem (B), let $\hat{y}_i = \hat{g}_n(X_i)$ for $1 \le i \le n$ and define the $\hat{c}_i$'s, $\hat{p}_i$'s, and $\hat{m}_i$'s as before. The $\hat{c}_i$'s, $\hat{y}_i$'s, $\hat{p}_i$'s, and $\hat{m}_i$'s form a feasible solution to (5), so it follows that $\int_a^b \left\{\tilde{g}_n^{(k)}(x)\right\}^2 dx \le \int_a^b \left\{\hat{g}_n^{(k)}(x)\right\}^2 dx$. □

Notes

III. Numerical Results

In this section, we compare the performance of the proposed estimator to that of the solution to Problem (A). In Section 3.1, we consider the case where f_* is given by a polynomial function of degree 5. In Section 3.2, f_* is the expected payoff of a certain equity-linked security. In both cases, we compute the empirical integrated mean square error and the amount of time required to compute the estimators.

All of the simulations are conducted on a 64-bit computer with an Intel(R) Core(TM) i7-6700K CPU at 4 GHz and a 32GB RAM. All of the simulations are programmed in MATLAB R2010a.

When solving Problems (A) and (B), one needs to evaluate the B-splines and the integration of the product of their kth derivatives. The B-splines can be evaluated recursively through Equations (3) and (4). The kth derivative of the B-spline can be evaluated recursively through the following relation: for a B-spline of degree r,

$$dB_{i,r}(x)/dx = (r/(x_{i+r} - x_i))B_{i,r-1}(x) - (r/(x_{i+r+1} - x_{i+1}))B_{i+1,r-1}(x) \qquad (6)$$

for $i = -r, \ldots, n$ and $x \in [a, b)$; see Lemma 14.6 on page 265 of Györfi et al. (2002). Next, we compute $\int_a^b B_{i,2k-1}^{(k)}(x) B_{j,2k-1}^{(k)}(x)dx$ by evaluating $B_{i,2k-1}^{(k)}$ using the recursion in (6) and by numerically evaluating the integration.

a) A Stylized Model

We consider the case where $f_*(x) = x(x-0.5)(x+0.5)(x-1.05)(x+1.05)$ for $x \in \mathbb{R}$, $x_i = i/n - 1/(2n)$ for $1 \le i \le n$, $Y_{ij} = f(x_i) + \epsilon_{ij}$ for $1 \le i \le n$ and $1 \le j \le m$, and the ϵ_{ij}'s are iid random variables, each of which is normally distributed with a mean of 0 and a variance of 4. The proposed estimator $\hat{g}_n$ is computed as the solution to Problem (B) with $m = 100$, $k = 4$, and g_0 estimated from (2).

The solution $\tilde{g}_n$ to Problem (A) is computed from Problem (A) with $m = 100$, $k = 4$, and u_0 estimated from $\sum_{i=1}^n S_i^2/(nm)$, where $S_i^2 = \sum_{j=1}^m (Y_{ij} - \overline{Y}_i)^2/(m-1)$ for $1 \le i \le n$. Both Problems (A) and (B) are solved with CVX, a package for specifying and solving convex programs (Grant and Boyd, 2014). To measure the accuracy of the proposed estimator, we compute the following empirical integrated mean square error (EIMSE):

Notes

$$\frac{1}{n}\sum_{i=1}^n \left(\hat{g}_n(x_i) - f_*(x_i)\right)^2 . \tag{7}$$

The EIMSE of $\tilde{g}_n$ is computed similarly by $\sum_{i=1}^n \left(\tilde{g}_n(x_i) - f_*(x_i)\right)^2/n$.

Table 1 reports the 95% confidence intervals of the EIMSE and the average amounts of time required to compute $\hat{g}_n$ and $\tilde{g}_n$, based on 300 iid replications, for a variety of n values. The proposed estimator produces lower EIMSE values and is computed in less time than $\tilde{g}_n$.

Table 1 : The 95% confidence intervals of the EIMSE and the amounts of computer time required to compute the proposed estimator and the solution to Problem (A) when $f_*(x) = x(x-0.5)(x+0.5)(x-1.05)(x+1.05)$.

	Solution to Problem (A)		Proposed Estimator	
n	EIMSE	Time (sec)	EIMSE	Time (sec)
10	0.0168 ± 0.0014	0.097	0.0156 ± 0.0013	0.094
20	0.0090 ± 0.0007	0.099	0.0081 ± 0.0007	0.097
40	0.0047 ± 0.0004	0.111	0.0041 ± 0.0003	0.107
80	0.0032 ± 0.0003	0.129	0.0023 ± 0.0002	0.125

Figure 1 displays the graphs of f_*, the $\overline{Y}_i$'s, $\hat{g}_n$, and $\tilde{g}_n$ on the left side and the graphs of $f_*^{(2)}$, $\hat{g}_n^{(2)}$, and $\tilde{g}_n^{(2)}$ on the right side for the case when $n = 40$. The proposed estimator appears to have a smoother second derivative than $\tilde{g}_n$.

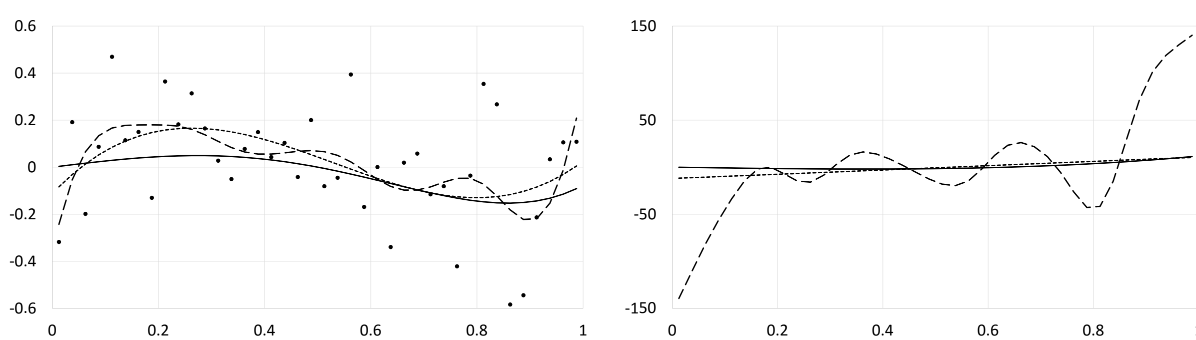


Figure 1 : The horizontal axis is x. On the left side, the solid line is $f_*(x)$, the dots are the $\overline{Y}_i$'s, the dotted line is $\hat{g}_n(x)$, and the dashed line is $\tilde{g}_n(x)$. On the right side, the solid line is $f_*^{(2)}(x)$, the dotted line is $\hat{g}_n^{(2)}(x)$, and the dashed line is $\tilde{g}_n^{(2)}(x)$.

b) Sensitivity Estimation of Option Prices

We consider the case where $f_*(x)$ is the expected payoff of a certain equity-linked security (ELS) when the underlying stock price is $x \geq 0$. The second derivative of f_* plays an important role when financial portfolio managers try to hedge the risks associated with the ELS; see Section 3.1 of Lim and Attallah (2016) for details. In particular, the second derivative of f_* is often assumed to be smooth over a domain of interest. The smoothness of the second derivative is particularly important because portfolio managers use the second derivative to make a decision on whether to buy or sell their ELS in order to hedge the risks, and a smooth second derivative suggests consistent selling strategies; see Section 3.1 of Lim and Attallah (2016) for a detailed explanation. One of the challenges is that there is no-closed form formula for f_* when the payoff structure of the ELS is complex, so simulation must be used to estimate f_* and its second derivative. Thus, the problem boils down to an estimation of the second derivative of f_* as a smooth function over a domain of interest. Since the roughness of a function $g : [a, b) \to \mathbb{R}$ is measured by $\int_a^b \left\{g^{(2)}(x)\right\}^2 dx$, the roughness of the second derivative of g is measured by $\int_a^b \left\{g^{(4)}(x)\right\}^2 dx$. Therefore, our proposed estimator is the solution to the following optimization problem:

$$\begin{aligned} &\text{Minimize} && \int_a^b \left\{g^{(4)}(x)\right\}^2 dx \\ &\text{subject to} && (1/n)\sum_{i=1}^{n} |g(x_i) - Y_i| \leq g_0, \end{aligned} \tag{8}$$

where g_0 can be estimated from (2).

We assume that the ELS is paid off in the following way: The ELS is issued at time 0 and matures at time $T = 365$. We denote the price of the underlying stock at time $t \in [0, T]$ by S_t. There are six days, $d_1, \ldots, d_6$, when early redemption is possible. On day d_i $(1 \leq i \leq 6)$, the ELS expires with a payoff of $\$r_i$ if S_{d_i}/S_0 exceeds some threshold b_i. Otherwise, the ELS does not expire until maturity. If there is no early redemption and S_t/S_0 does not drop below a lower limit b over the entire lifetime of the ELS, then the ELS expires with a payoff of \$1 at maturity. In the rest of the cases, the ELS expires with a payoff of $\$S_T/S_0$ at maturity.

We let $x_i = 90 + (20)(i/n) - (10/n)$ for $1 \leq i \leq n$. For each x_i, a sample path of a geometric Brownian motion is generated as a trajectory of the stock price between now and maturity, and the corresponding payoff of the ELS is computed. Y_{ij} is the payoff computed this way in the jth replication at x_i. The parameters are $d_1 = 61, d_2 = 122, d_3 = 182, d_4 = 243, d_5 = 304, d_6 = 365, b_1 = 0.9, b_2 = 0.9, b_3 = 0.85, b_4 = 0.85, b_5 = 0.8, b_6 = 0.8, r_1 = 1.05, r_2 = 1.10, r_3 = 1.15, r_4 = 1.20, r_5 = 1.25, r_6 = 1.30$, and $b = 0.7$. The remaining time until maturity is 60 days, the annual volatility is 30%, the annual risk–neutral interest rate is 5%, and the initial stock price at time 0 is \$125.

We set $m = 10$, so 10 sample paths for the geometric Brownian motion are generated at each x_i to compute $Y_{i1}, \ldots, Y_{i10}$ for $1 \leq i \leq n$. We compute $\overline{Y}_i = \sum_{j=1}^{10} Y_{ij}/10$ for $1 \leq i \leq n$ and use $(x_1, \overline{Y}_1), \ldots, (x_n, \overline{Y}_n)$ to compute the proposed estimator $\hat{g}_n$ by solving Problem (B) with g_0 estimated from (2). The solution $\tilde{g}_n$ to Problem (A) is computed from Problem (A) with $m = 10$, $k = 4$, and u_0 estimated from $\sum_{i=1}^{n} S_i^2/(nm)$, where $S_i^2 = \sum_{j=1}^{m}(Y_{ij} - \overline{Y}_i)^2/(m-1)$ for $1 \leq i \leq n$. Both Problems (A) and (B) are solved with CVX. To measure the accuracy of the proposed estimate,

Notes

Notes

Table 2 : The 95% confidence intervals of the EIMSE and the amounts of computer time required to compute the proposed estimator and the solution to Problem (A) when f_* is the expected payoff of the ELS.

	Solution to Problem (A)		Proposed Estimator	
n	EIMSE	Time (sec)	EIMSE	Time (sec)
10	0.0024 ± 0.0002	0.100	0.0024 ± 0.0002	0.098
20	0.0013 ± 0.0001	0.102	0.0012 ± 0.0001	0.096
40	0.0008 ± 0.0000	0.113	0.0005 ± 0.0000	0.104
80	0.0005 ± 0.0000	0.134	0.0003 ± 0.0000	0.129

we compute the following EIMSE between the underlying function and $\hat{g}_n$:

$$\frac{1}{n}\sum_{i=1}^{n}\left(\hat{g}_n(x_i) - f_*(x_i)\right)^2, \tag{9}$$

where $f_*(x_i)$ is estimated from the average of $2,000,000$ iid replications of Y_{ij} at each x_i. The EIMSE of $\tilde{g}_n$ is computed similarly by $\sum_{i=1}^{n}\left(\tilde{g}_n(x_i) - f_*(x_i)\right)^2/n$.

Table 2 reports the 95% confidence intervals of the EIMSE and the average amounts of time required to compute $\hat{g}_n$ and $\tilde{g}_n$, based on 300 iid replications, for a variety of n values. The proposed estimator produces lower IMSE values and is computed in less time than $\tilde{g}_n$.

Figure 2 displays the graphs of f_*, the $\overline{Y}_i$'s, $\hat{g}_n$, and $\tilde{g}_n$ on the left side and the graphs of $f_*^{(2)}$, $\hat{g}_n^{(2)}$, and $\tilde{g}_n^{(2)}$ on the right side for the case when $n = 40$. In the graphs of Figure 2, the proposed estimator shows a smoother second derivative than that of $\tilde{g}_n$. Considering the fact that our goal is to estimate the second derivative of f_* as a smooth function, our proposed estimator appears to have the desired property.

IV. Concluding Remarks

In this paper, we proposed a new method for estimating a smooth regression function f_*. The proposed estimator $\hat{g}_n$ of f_* is designed for better computational efficiency. Numerical results show that the proposed estimator is computed faster and achieves better mean square errors than its alternative. Furthermore, the proposed estimator shows smoother derivatives than its alternative, which is a desired property when estimating a smooth regression function. The convergence of the proposed estimator to f_* as the number of data points increases to infinity was demonstrated

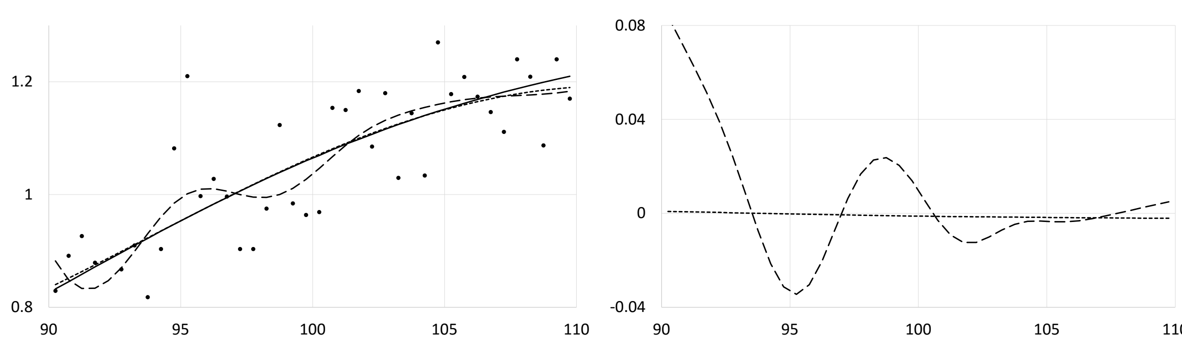


Figure 2 : The horizontal axis is x. On the left side, the solid line is $f_*(x)$, the dots are the $\overline{Y}_i$'s, the dotted line is $\hat{g}_n(x)$, and the dashed line is $\tilde{g}_n(x)$. On the right side, the dotted line is $\hat{g}_n^{(2)}(x)$, and the dashed line is $\tilde{g}_n^{(2)}(x)$.

empirically, so a promising future research topic involves proving the consistency of the proposed estimator theoretically.

References Références Referencias

1. Frank, M and Wolfe, P. (1956). An algorithm for quadratic programming. *Naval Research Logistics Quarterly*, 3, 95110.
2. Grant, M., and Boyd, S. (2014). CVX: Matlab software for disciplined convex programming, version 2.1. http://cvxr.com/cvx, March 2014.
3. Györfi, L., Kohler, M, Krzy˙zak, A., and Walk, H. (2002). *A distribution-free theory of nonparametric regression*. New York: Springer.
4. Lim, E., and Attallah, M. (2016). Estimation of smooth functions via convex programs. *International Journal of Statistics and Probability*, 5, 150-155.
5. Reinsch, C. H. (1967). Smoothing by spline functions. *Numer. Math., 10,* 177-183.
6. Wahba, G., and Wold, S. (1975). A completely automatic French curve: fitting spline functions by cross-validation. *Communications in Statistics,* 4, 1-17.
7. Zangwill, W. I. (1969). *Nonlinear programming: a unified approach.* New Jersey: Prentice-Hall Inc.

Notes

Global Journal of Science Frontier Research: F
Mathematics and Decision Sciences
Volume 16 Issue 5 Version 1.0 Year 2016
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Effect of Missingness Mechanism on Household Survey Estimates in Nigeria

By Faweya Olanrewaju & G. N. Amahia
Ekiti State University

Abstract- In this study, we employed three missingness mechanisms - MCAR, MAR and MNAR to investigate the effects of proportion of Missing data on descriptive and analytic statistics: Mean ($\bar{Y}$), Variance ($\sigma^2 y$), correlation coefficient ($\rho_{yx_1x_2}$), coefficient of variation (cv), skewness (sk) and Kurtosis (K) which are likely situation a researcher may encounter in the field when dealing with household surveys.

This study reveals that sometimes, missing data introduce systematic distortion in survey estimates and bias flows from missing data when the causes of the missing data are linked to the survey statistic measured.

Keywords: missing data, MCAR, MAR, MNAR, descriptive statistics.

GJSFR-F Classification : FOR Code : 70B15

EFFECTOFMISSINGNESSMECHANISMONHOUSEHOLDSURVEYESTIMATESINNIGERIA

Strictly as per the compliance and regulations of :

Effect of Missingness Mechanism on Household Survey Estimates in Nigeria

Faweya Olanrewaju [α] & G. N. Amahia [σ]

Abstract- In this study, we employed three missingness mechanisms - MCAR, MAR and MNAR to investigate the effects of proportion of Missing data on descriptive and analytic statistics: Mean ($\bar{y}$), Variance ($\sigma^2 y$), correlation coefficient ($\rho_{yx_1x_2}$), coefficient of variation (cv), skewness (sk) and Kurtosis (K) which are likely situation a researcher may encounter in the field when dealing with household surveys.

This study reveals that sometimes, missing data introduce systematic distortion in survey estimates and bias flows from missing data when the causes of the missing data are linked to the survey statistic measured.

Keywords: missing data, MCAR, MAR, MNAR, descriptive statistics.

I. Introduction

One of the greatest threats compromising the accuracy of most surveys estimate during design and analysis is the problem of missing data. This may occur when some individuals provide no information because of non-contact of refusal to respond (unit non-response) or when other individuals are contacted and provide some information, but fail to answer some of the questions (item non-response).

Unfortunately, unit and item non-response are often neglected or not properly handled during analysis, and this leads to bias in the estimate. Thus, this study focused on the detection and minimization of bias associated with missing data. Though missing data is a common problem in most research studies, yet no commonly agreed upon solution exists.

Consequently, researchers have developed a wide variety of approaches for handling missing data, however, no single approaches is without pitfalls. Thus, researchers facing a missing data problem should thoroughly investigate the sources of the missing data as well as the options for handling missing data under different missingness mechanism with different amount of missing data. Otherwise, when researchers use missing data techniques without considering the mechanism of the missingness, they run the risk of obtaining biased estimates and misleading conclusions. In such cases, analysis and publication of the data may be of dubious value and jeopardize credibility of the organization preparing the report (Little & Smith, 1983).

II. Missing Data Mechanism

a) *Missing Completely At Random*

The distribution of the missing values R is assumed independent of both the target variable Y and auxiliary variable X. Thus,

Author α: Department of mathematical sciences, ekiti state university, iworoko, ekiti state. e-mail: lanrefaweya@gmail.com
Author σ: Department of statistics, university of ibadan.

$$P(R/Y, X) = P(R) \quad (1)$$

b) Missing At Random (MAR)

In general, MAR occur when there is no direct relation between the target variable Y and response behaviour R and the same time there is a relation between the auxiliary variable and the response behaviour R. This is expressed as:

$$P(R/Y, X) = P(R\ /\ Y^0, X) \quad (2)$$

c) Missing Not At Random

Missing data Mechanism where missing values are assumed to be related to the unobserved dependent variable vector Y_i^m, in addition to the remaining observed values is called Missing not at Random (MNAR). This is expressed as:

$$P(R/Y, X) = P(R/\ Y^m\, Y^0\ X) \quad (3)$$

III. Non-Response in Survey

Non-response is the failure of a sample survey (or a census) to collect data for all items in the survey questionnaire from all the population units designated for data collection. Non-response can be manifested either as item or as non-response.

a) Unit Non-Response

This refers to outright failure of a sampled subject to participation in a study.

b) Item Non-Response

Item non-response occurs in any kind of multivariate study (e.g. a survey) in which a subject responds to some, but not all survey items.

IV. Method of Analysis

We employed three missingness mechanisms - MCAR, MAR, and MNAR to investigate the effects of proportion of Missing data on descriptive and analytic statistics (Mean($\bar{y}$)), Variance ($\sigma^2 y$), correlation coefficient ($\rho_{yx_1x_2}$), coefficient of variation (cv), skewness (sk) and Kurtosis (K) which are likely situation a researcher may encounter in the field when dealing with household surveys.

We denote by S the sample, Y = $(y_1, y_2, \dots, y_n)^T$ where y_i denote the value of targeted random variable for unit i. Let X_i, I = 1, 2, …, k be some auxiliary variable which is available for all i $\in S$. Let R = $(r_1, \dots, r_n)$ where r_i = 0 for unit that are observed and r_i= 1 for units that are missing. MCAR assumed distribution of missing values R to be independent of both targeted variable Y and auxiliary variable X_i, thus,

$$P(R/\ Y, X_i\ ) = P(R).$$

However, under MAR, there is no direct relationship between the targeted variable Y and the response behavior R and at the same time; there is no relationship between the auxiliary variable X_i and the response behaviour R. Thus, P(R/ Y, X_i) = P(R/ Y^0, X_i). In MNAR, missing values assumed to be related to unobserved dependent variable Y^m in addition to the remaining observed values Y^0 and this relationship cannot be explained by an auxiliary variable X_i. Thus, P(R/ Y^m, Y^0, X_i). A simple random sample of n = 100 households was selected from the record of survey data on "household income" from Akure North Local Government, Iju/ Ita- Ogbolu in Ondo

Notes

Notes

State to demonstrate the effect of missingness on descriptive an inferential statistics when different proportions of data are missing.

Three demographic variables; Y (Income N'000), Age (X_1) and year of schooling (X_2) were considered.

The variable Y was a combination of explanatory variables with added random components.

Then, differing amounts were deleted at random causing MCAR data, which had 0, 1, 5, 12, 23 and 44% missing data.

In MAR situation y become missing as follows: 0% for complete data set, 5% when $X_1 < 5$, 12% when $X_2 \geq 55$, 23% when $X_1 \leq 6$ and 44% when $X_1 \leq 6$ or $X_2 \geq 50$.

Sorting according to the actual y values in deleting the cases to give 6 different rate created MNAR data.

Table 1 : Table of Means, variance, correlation, skewness and kurtosis when different amounts of data are missing, under different assumption of missingness. The first row shows the mean, variances, correlation, skewness and kurtosis of the household income data when no data are missing. That is the data are complete

	Missing Completely At Random (MCAR)					
Missing	$\bar{y}$	$\bar{\sigma}^2{}_y$	$\rho y_1 x_2$	CV	S_k	K
0	13.814	46.577	0.946	49.62	0.217	2.616
1	13.754	46.691	0.946	49.68	0.238	2.633
5	13.682	48.02	0.943	50.68	0.258	2.580
12	13.371	44.688	0.952	49.90	0.135	2.470
23	14.288	45.84	0.997	46.98	0.071	2.419
44	14.260	48.077	0.995	48.83	-0.35	2.437

Table : The table cont

	Missing At Random (MAR)					
Missing	$\bar{y}$	$\bar{\sigma}^2{}_y$	$\rho y_1 x_2$	CV	S_k	K
0	13.814	46.577	0.946	49.62	0.217	2.616
1	13.794	47.014	0.945	49.71	0.228	2.596
5	14.396	41.933	0.944	44.50	0.294	2.659
12	14.210	40.639	0.916	47.29	0.243	2.797
23	16.244	32.17	0.910	34.95	0.358	2.980
44	16.371	23.216	0.844	29.95	0.582	2.925

Table : The table cont

	Missing Not At Random (MNAR)					
Missing	$\bar{y}$	$\bar{\sigma}^2{}_y$	$\rho y_1 x_2$	CV	S_k	K
0	13.814	46.577	0.946	49.62	0.217	2.616
1	13.880	46.608	0.496	49.21	0.198	2.623
5	13.045	36.965	0.944	45.84	0.005	2.204
12	12.22	30.331	0.936	44.52	-0.096	2.268
23	11.103	24.457	0.931	44.93	-0.089	2.348
44	9.07	17.48	0.916	44.09	-0.072	2.490

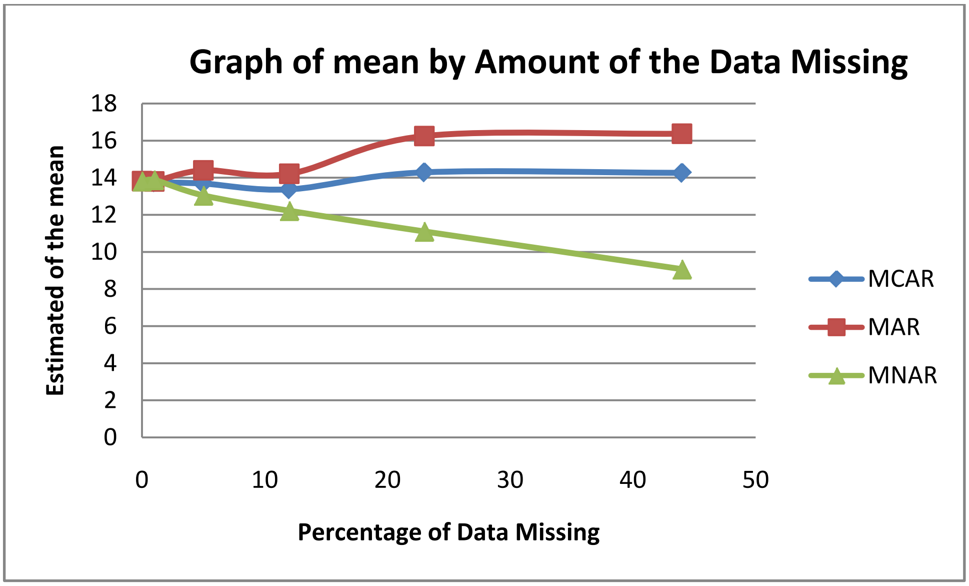


Figure 1 : Graph of Mean by Amount of the Data Missing

Comment: MCAR is approximately constant, while for MAR increases and MNAR decreases.

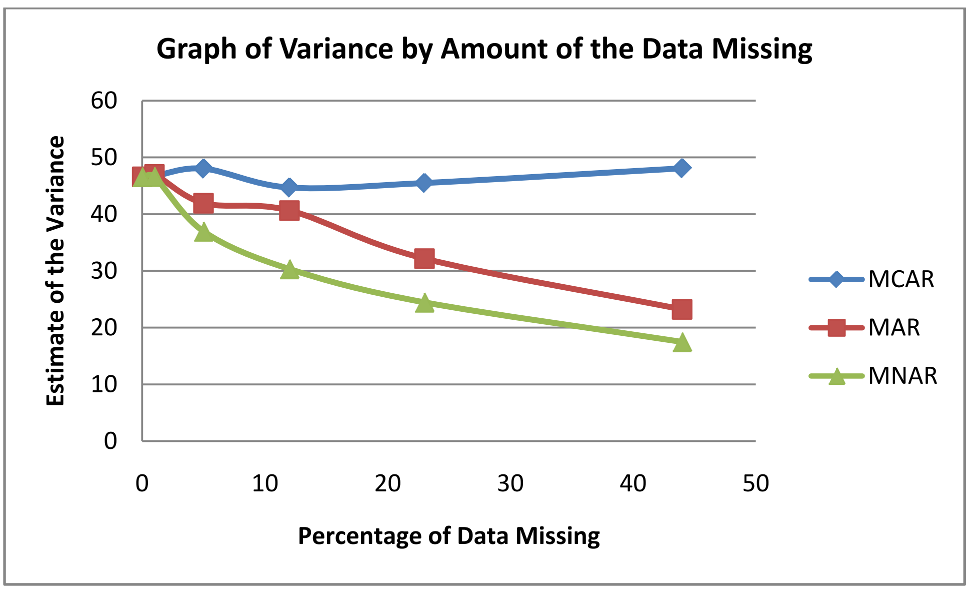


Figure 2 : Graph of Variance by Amount of the Data Missing

Comment: Under MCAR values for variances is approximately constant as proportion of missingness increases, while for MAR and MNAR decreases but MNAR is more drastical.

Notes

Notes

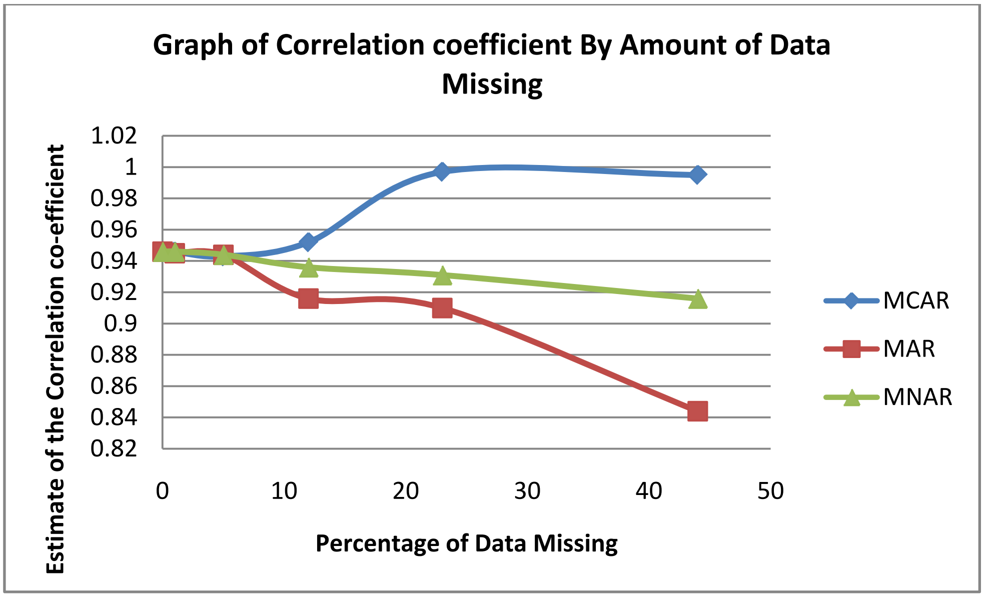


Figure 3 : Graph of Correlation Coefficient by Amount of the Data Missing

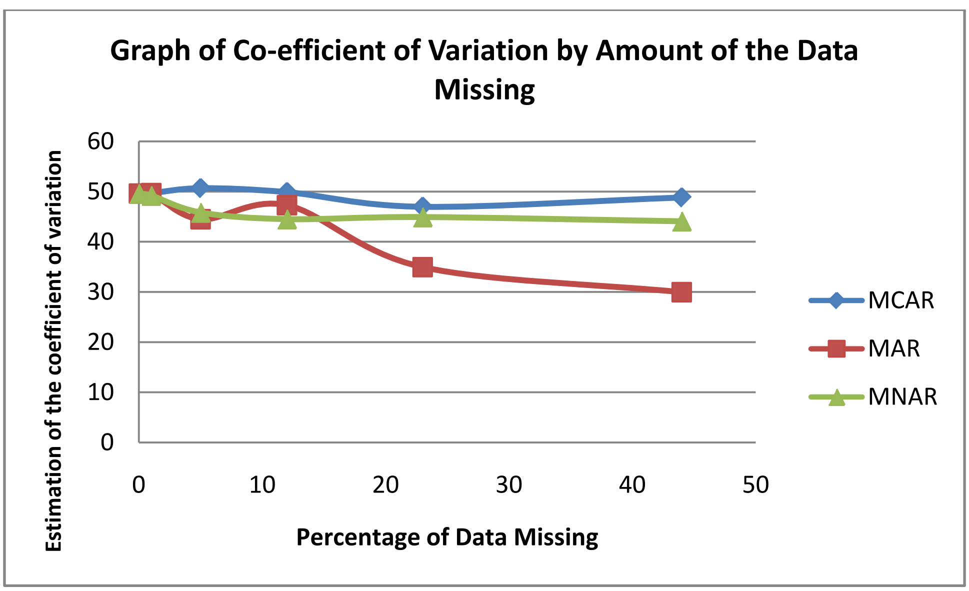


Figure 4 : Graph of Coefficient of Variation by Amount of the Data Missing

Comment: MCAR is approximately constant, while for MAR decreases.

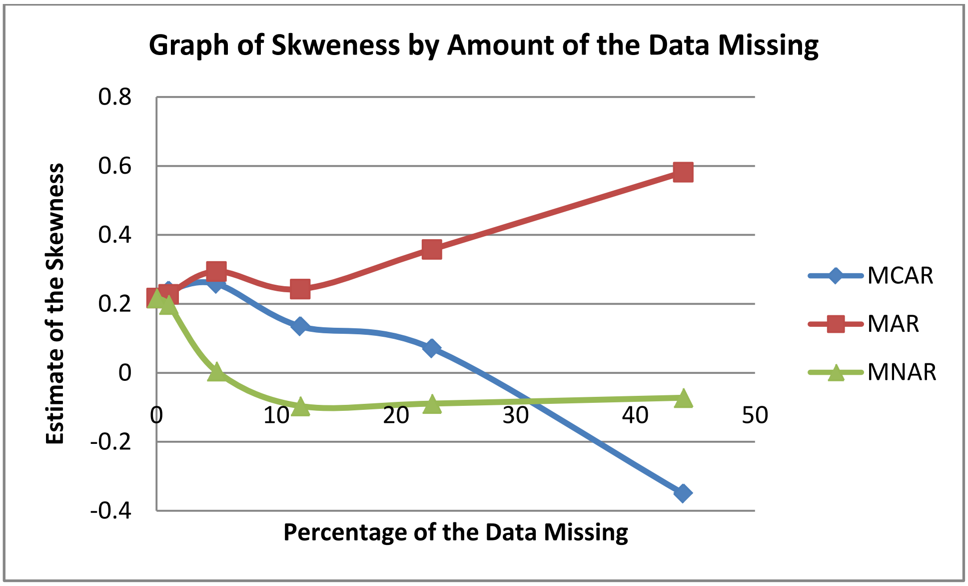


Figure 5 : Graph of Skewness by Amount of the Data Missing

Comment: MNAR formerly decreased but later constant as the proportion of missingness increases but MCAR increases while MAR decreases.

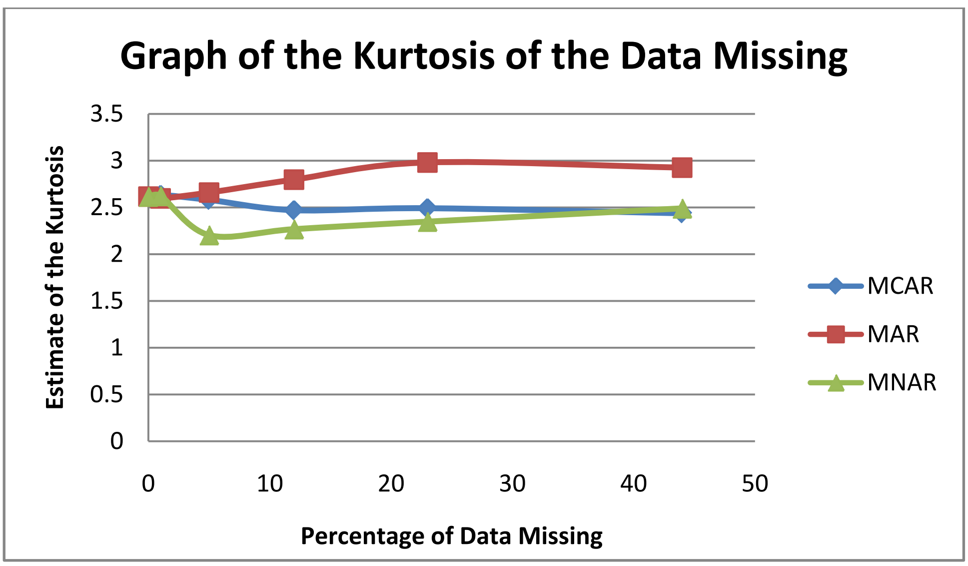


Figure 6 : Graph of Kurtosis by Amount of the Data Missing

Comment: MCAR and MNAR is approximately constant for kurtosis even as the proportional of missingness increases but MAR is a bit different.

V. Discussion of Results

Among all the parameters considered, the one where there was no major significance difference under the three mechanisms is Kurtosis, which is the degree of peakedness of the curve of the distribution of the variable under consideration. Thus from the study, it implies that as the sample size 'n' increases, the curve tends to normality irrespective of the nature of irrespectively of the nature missingness. In addition, this study revealed that sometimes, missing data introduce systematic distortion in survey estimates and bias flows from missing data when the causes of the missing data are linked to the survey statistics measured.

Notes

Notes

References Références Referencias

1. Amahia, G.N (2010) Factors, prevention and correction methods for non-response in sample surveys. Central Bank of Nigeria, Journal of Applied Statistics 1(1), pp 79-89, (Nigeria).
2. Arbuckle, J.L. (1996). Full information likelihood estimation in the presence of incomplete data. In G.A. Marcoulides and R.E. Schumaker (Eds.), Advance structural equation modeling. Mahwah, Nj: Lawrence Erlbaum.
3. Beale, E.M.L., & Little, R.J.A (1975). Missing values in multivariate analysis.Journal of the Royal Statistical Society, Series B, 37, 129- 146.
4. Cochran, W.(1968). "The Effective of Adjustment by Subclassification in Removing Bias in Observational studies." Biometrics, 24, pp. 295-313.
5. Cohen, J and Cohen, P, (1983). Missing data. In J Cohen and P. Cohen, Applied multiple regression:Correction analysis for the behavioral sciences (pp. 275-300). Hillsdale, NJ Erlbaum.
6. Cool, A.L. (2000). A review of methods for dealing with missing data. Paper presented at the Annual Meeting of the Southwest Educational Research Association, Dallas, TX.(ERIC Document ReproductioN Service No.ED 438 311).
7. Diggle P.J Testing for random dropouts in repeated measurement data.Biometrics,45 (1989) 1255-1258.
8. Diggle P.J., Heagerty P., Liang, K.Y. et al. Analysis of longitudinal Data. 2nd ed. Oxford University Press Inc.,(2002).
9. Dixon W. J BMDP Statistical software.Los Angeles. University of California Press,(1988).
10. Ender C.K., and Bandalos, D.L (2001). The relative performance of full information maximum likelihood estimation for missing data in structural equation models.Structural Equation Modeling, 8(3),430-457.
11. Groves, R Couper M. (1998); Non-response Interview Survey, New York Wiley.
12. Groves, R Singer. and Corning, A (2000); Leverage- Salience Theory of Survey Participation Description and an Illustration. Public Opinion Quarterly,64,299-308.
13. Groves, R. Cialdini R and Couper M (1992): "Understanding the Decision to participate in a Survey" The public Opinion Quaterly,54(4), 475-495.
14. Howell, D.C (2007) The analysis of missing data.Inoutwaite, W and Turner, S .Handbook of social science Methodology London: Sage. Return.
15. Kalton, G. and Flores-Cervantes, I (2003), "Weighting Methods" Journal of official statistics, 19,pp.81-97.
16. Kim, J.O., & Curry, J. (1977). The instrument of missing data in multivariate analysis Sociological Methods & Research, 6(2), 215-240.
17. Little R.J.A. A test of missing completely at random for multivariate data with missing values. Journal of American statistical Association, 83 (1988),1198-2002.
18. Little, J. & An, H., (2004). Robust likelihood-based analysis of multivariate data with missing values.Statisticasinica Vol. 14, pp 949-968.
19. Madlow, W.G., Nisselson, H., Olkin, I. (Eds.), 1983. Incomplete data in sample surveys.Report and Case Studies, vol.1. Academic Press, New York.
20. Quinten, A., Raaijmakers, W., 1999. Effectiveness of different missing data treatments in survey with Likert-type data: introducing the relative mean substitution approach. Educational and psychological Measurement 59 (5), 725-748.
21. Schafer J.L., Graham J. W. Missing data: Over view of the state of the art. Psychological Methods, 7 (2002),147:177.

22. Schafer, J (2002). Dealing with missing data.Research letter information mathematics science. Vol. 3, pp 153-160.
23. Tshering, S., Okazaki T and Endo S., March 2013: A Method to Identify Missing Data Mechanism in Incomplete Dataset. IJCSNS International journal of Computer Science and Network Security,Vol.13 No14 Page (14-21).
24. Utazi C.E., Onyeagu S.I. & Osuji G.A (2010). On the efficiency of some techniques for estimating covariance and correlation matrices from incomplete data. Journal of the Nigerian Statistical Association Vol. 22, 44-63.

Notes

Global Journal of Science Frontier Research: F
Mathematics and Decision Sciences
Volume 16 Issue 5 Version 1.0 Year 2016
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Effect of Cherenkov Radiation on Superluminal Free Spin-half Particles Motion in Spacetime

By Emmanuel D. K. Gazoya
National Nuclear Research Institute

Abstract- Conservation laws, consisting of the existence of quantities which do not change in time, independent of the dynamical evolution of a system, are crucial and vital for the construction of any dynamical system theory. The basic properties such as conservation of energy, momentum, angular momentum, charge, isospin, or generalization thereof are fundamental and must be guaranteed by a physical system, if it is to give a valid description of nature. One persistent objection against the concept of superluminal entities is based on the anticipation of fast energy loss which could be incurred under Vavilov-Cherenkov radiation, with the consequent prediction that no such particles could be detected. Yet presently, no theoretical or experimental explication exists which justifies this claim. Here we show, in the limit of a kinematically permissible and non-dispersive medium, that energy conservation is feasible. Corresponding to radiation intensities from large energy-momentum transfer, when the parameter k of the generalized linear velocity of the superluminal free spin-half field is made sufficiently large, Cherenkov cone becomes flattened at 90^o with direction of motion, bringing the radiated energy to merge with the circulating energy flow in the wave field of the particle.

Keywords: *cherenkov: radiation - cherenkov: photons - cherenkov: angle - cherenkov: flattened cone, spin, superluminal motion, Huygens' construction.*

GJSFR-F Classification : *FOR Code : 35Q85*

EFFECTOFCHERENKOVRADIATIONONSUPERLUMINALFREESPINHALFPARTICLESMOTIONINSPACETIME

Strictly as per the compliance and regulations of :

Notes

Effect of Cherenkov Radiation on Superluminal Free Spin-half Particles Motion in Spacetime

Emmanuel D. K. Gazoya

Abstract- Conservation laws, consisting of the existence of quantities which do not change in time, independent of the dynamical evolution of a system, are crucial and vital for the construction of any dynamical system theory. The basic properties such as conservation of energy, momentum, angular momentum, charge, isospin, or generalization thereof are fundamental and must be guaranteed by a physical system, if it is to give a valid description of nature. One persistent objection against the concept of superluminal entities is based on the anticipation of fast energy loss which could be incurred under Vavilov-Cherenkov radiation, with the consequent prediction that no such particles could be detected. Yet presently, no theoretical or experimental explication exists which justifies this claim. Here we show, in the limit of a kinematically permissible and non-dispersive medium, that energy conservation is feasible. Corresponding to radiation intensities from large energy-momentum transfer, when the parameter $\boldsymbol{k}$ of the generalized linear velocity of the superluminal free spin-half field is made sufficiently large, Cherenkov cone becomes flattened at $\mathbf{90^o}$ with direction of motion, bringing the radiated energy to merge with the circulating energy flow in the wave field of the particle.

Keywords: *cherenkov: radiation - cherenkov: photons - cherenkov: angle - cherenkov: flattened cone, spin, superluminal motion, Huygens' construction.*

I. Introduction

The blue light observed by (Cherenkov, 1934) in his experiments was originally given a theoretical explanation by (Frank and Tamm, 1937). They associated it with the radiation of a charge moving uniformly with a velocity greater than that of light in medium. Under the restriction of non-dispersive medium, they derived the radiation intensity confined to the surface of the so-called Cherenkov cone defined by

$$\text{Cos}\,\theta = \frac{c}{nv}\,, \tag{1.1}$$

where v is particle velocity and n the medium refractive index (which is the ratio of the phase velocity of light in the medium to its velocity in free space). In the presence of dispersion, n and $\cos\theta$ vary with ω continuously. As a result, the Cherenkov radiation fills continuous sequence of Cherenkov cones, relatively to different frequencies in the frequency regions where $n > 1$. In their derivations, Tamm and Frank suggested that the charge velocity was constant, and they disregarded the recoil effects in case of which equation (1.1) involves an additive term, given by[1]

Author: *National Nuclear Research Institute (NNRI) / ARC, Ghana Atomic Energy Commission, Accra, Box LG 80, Ghana.*
e-mail: gazoya2001@yahoo.com

1. (Katharina, 2014)

$$\cos\theta = \frac{1}{n\beta} + \frac{\hbar k}{2p}\left(1 - \frac{1}{n^2}\right). \quad (1.2)$$

Here, $\beta = v/c$, and p and $\hbar k$ are particle and photon momentum, respectively. Also disregarded in their calculations was the case of arbitrary emission angle produced by the use of properly engineered one dimensional metamaterials, where

$$\cos\theta = \frac{1}{n\beta} + \frac{n}{k_0}.\left(\frac{d\varphi}{n^2}\right). \quad (1.3)$$

Notes

This last equation originates from (Ginzburg, 1940) contribution, who evaluated the photon emission angle for an arbitrary energy loss of the initial charge, found the radiation intensity in the nonrelativistic approximation and showed that corrections to the Tamm–Frank formula are negligible in the visible and ultraviolet parts of the radiation spectrum. They further assumed smallness of the photon energy with respect to the energy of the initial charge. Since then, it is usually believed that the Vavilov–Cherenkov radiation for the fixed refractive index lies on the surface of the Cherenkov cone. As an early remark, it is clear from (1.1) that the emission angle θ relative to the direction of (linear) velocity depends only on particle linear constant velocity and the refractive index, but not on the coordinate x and hence, not on the particle trajectory. In his note (Vavilov, 1934) accompanying Cherenkov paper, Vavilov suggested that the radiation observed in Cherenkov experiments was due to the electron deceleration. This was further admitted by (Afanasiev, 2004) to be at least partly right since electrons were completely stopped in Cherenkov experiments(thorough discussion on this may be found in Cherenkov's Doctor of Science dissertation; (Cherenkov, 1944)), thus exhibiting deceleration.

Cherenkov radiation is frequently used in particle identification detectors (PID), (Alaeian, 2014; Jelle, 1955; Konrad, 1998; Leroy and Rancoita, 2014), a process which separates particles such as protons, electrons, muons, pions, etc at different velocities. A threshold measurement mechanism through which the number of particles at given velocities would be determined is set on a radiated angle equation in which the particle velocity exceeds c/n and photons are generated. If the particles pass through, for example, lucite or plexiglass, for which $n \approx 1.5$, only those with $v > 0.67c$ emit Cherenkov radiation and so can be detected as an optical signal. Particles with extreme relativistic energies can be detected in gas Cherenkov detectors where the refractive index n of the gas is just greater than 1. Another application is in the detection of ultra-high energy $\gamma - rays$ when they enter the top of the atmosphere. The high energy $\gamma - ray$ initiates an electron – photon cascade and, if the electron – positron pairs acquire velocities greater than the speed of light in air, optical Cherenkov radiation is emitted which can be detected by light detectors at sea-level (Longair, 1981; 1995; 1997). At present, evaluation of radiation intensities effect of Cherenkov photons on the superluminal motion of free fermions in spacetime remains to be determined. The aim of this observation is to analyze and ascertain where the context of very large energy-momentum transfer corresponding to radiation at an angle near to 90^o leads, as far as energy conservation is concerned in superluminal motion.

The plan of our examination is as follows. In Section 2, we review the theory of Vavilov-Cherenkov radiation, using John Peacock's approach (Dunlop et al., 1990;

1996). In Section 3, the true mechanism of spin is re-exposed in order to provide a basis for correctly understanding the interaction between the fermion wave field and the right-angled radiated photons energy plane, as explained in Section 4; there, we show that, in a kinematically permissible localized region of spacetime and in the absence of dispersion, the radiated energy (conventionally believed to be lost) is a constant of motion and could merge with the circulating energy flow of the field; so, in this condition, it contributes to the field which carries the particle.

Notes

II. Theory of Cherenkov Radiation

Geometrically, the angle of emission (Cherenkov angle) is derived by Huygens' construction (see Fig.1). The origin of the emission is best appreciated from the following two Liénard -Wiechert potentials expressions $A(r,t)$ and $\phi(r,t)$ which are given by:

$$A(r,t) = \frac{\mu_0}{4\pi r}\left[\frac{qv}{1-(\boldsymbol{v}.i_{obs})/c}\right]_{ret};$$

$$\phi(r,t) = \frac{1}{4\pi\varepsilon_0 r}\left[\frac{q}{1-(\boldsymbol{v}.\boldsymbol{i}_{obs})/c}\right]_{ret}, \tag{2.1}$$

where $\boldsymbol{i}_{obs}$ is the unit vector in the direction of observation from the moving charge q with velocity v at distance r, the subscript ret stands for retarded potential, and μ_0 and ε_0 are permeability and permittivity of space, respectively. In the case of a vacuum, one of the standard results of electromagnetic theory is that a charged particle moving at constant velocity v does not radiate electromagnetic radiation. Radiation is emitted in vacuum if the particle is accelerated. In a medium with a finite permittivity ε, or refractive index n, however, the potentials in (2.1) become singular along the cone, invalidating the denominators in this equation. Explicitly, we have:

$$1-\frac{1}{c}(\boldsymbol{v}.i_{obs}) = 0 \quad \Longrightarrow \quad \cos\theta = \frac{c}{nv}, \tag{2.2}$$

if one takes θ to be the angle between velocity direction and the unit vector $\boldsymbol{i}_{obs}$. Thus the usual rule that only accelerated charges radiate no longer applies.

The geometric representation of this process is that, because the particle moves superluminally through the medium, a 'shock wave' is created behind the particle. The wave front of the radiation propagates at a fixed angle with respect to the velocity vector of the particle because the wave fronts only add up coherently in this direction according to Huygens' construction. The geometry of this figure shows that the angle of the wave vector with respect to the direction of motion of the particle is $\cos\theta = c/nv$.

We will now determine the main features of Cherenkov radiation, analytically. Let us consider an electron moving along the positive x- axis at a constant velocity v. This motion corresponds to a current density $\boldsymbol{J}$ where

Notes

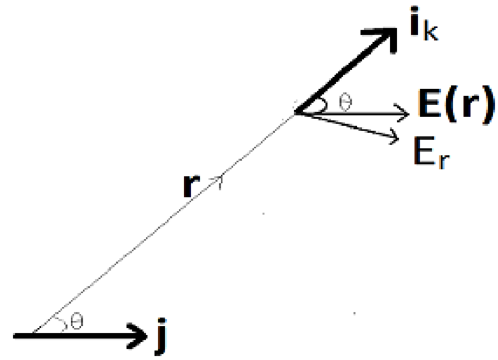


Fig. 1 : Illustrating the geometry used in the derivation of the expressions for Cherenkov angle and Cherenkov radiation

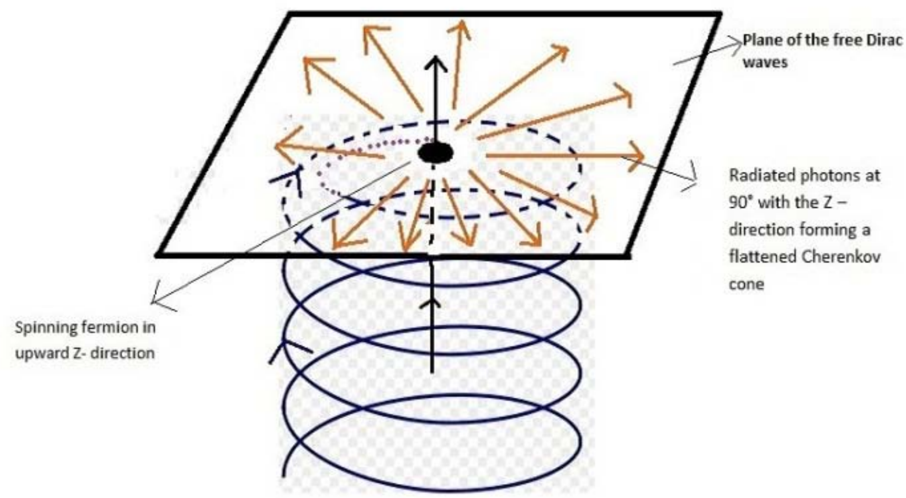


Fig. 2 : Sufficiently superluminal helical energy flow of a free fermion field for large parameter k. Photons are radiated at nearly 90^o where Cherenkov cone becomes flattened and coincides with the plane of the particle wave field . The flow is shown in the transverse direction.

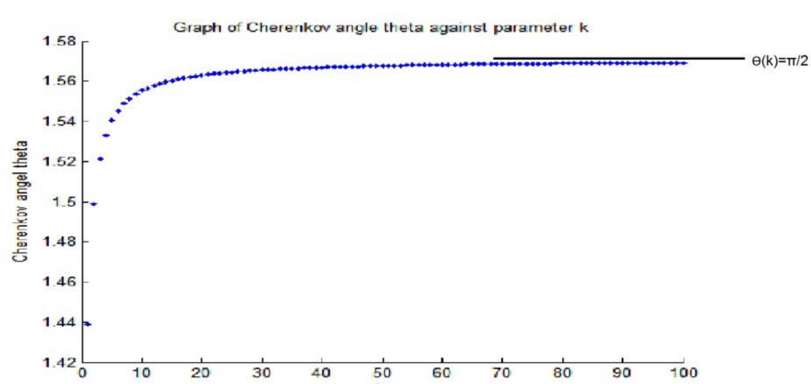


Fig. 3 : A graph illustrating the discrete evolution of Cherenkov angle θ to a limiting value, with respect to discrete parameter k

Notes

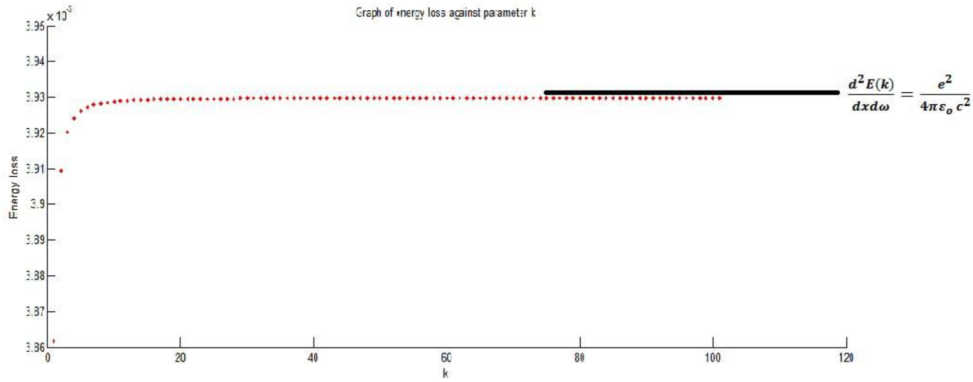


Fig. 4 : A graph illustrating the discrete evolution of the energy loss rate per unit path length and per frequency unit to a limiting value, with respect to discrete parameter k

$$\boldsymbol{J} = ev\delta(x - vt)\delta(y)\delta(z)i_x. \tag{2.3}$$

Taking the Fourier transform of this current density to find the frequency components $\boldsymbol{J}(\omega)$ corresponding to this motion, we have

$$\boldsymbol{J}(\omega) = \frac{1}{(2\pi)^{1/2}}\int \boldsymbol{J}\exp\,(i\omega t)\mathrm{d}t$$

$$= \frac{e}{(2\pi)^{1/2}}\delta(y)\delta(z)\exp\,(i\omega x/v)\boldsymbol{i}_x. \tag{2.4}$$

Equation (2.4) can be regarded as a representation of the motion of the electron by a line distribution of coherently oscillating currents. Our task is to work out the coherent emission, if any, from this distribution of oscillating currents. The quite cumbersome full treatments given in standard texts such as (Jackson, 1999) and (Clemmow and Dougherty, 1969) will not be used in this paper. Rather, we will adopt an approach developed by John Peacock in (Dunlop et al., 1990; 1996), different from the usual derivation which employs energy and momentum conservation.

First, let us review some of the standard results concerning the propagation of electromagnetic waves in a medium of permittivity ε, or refractive index $n = \varepsilon^{1/2}$. It is a standard result of classical electrodynamics that the flow of electromagnetic energy through a surface $\boldsymbol{dS}$ is given by the *Poynting vector flux*, $\boldsymbol{N}.\boldsymbol{dS} = (\boldsymbol{E} \times \boldsymbol{H}).\boldsymbol{dS}$. The electric and magnetic field strengths $\boldsymbol{E}$ and $\boldsymbol{H}$ are related to the electric flux density $\boldsymbol{D}$ and the magnetic flux density $\boldsymbol{B}$ by the constitutive relations

$$\boldsymbol{D} = \varepsilon\varepsilon_0\boldsymbol{E}, \qquad \boldsymbol{B} = \mu\mu_0\boldsymbol{H} \tag{2.5}$$

The energy density of the electromagnetic field in the medium is given by the standard formula

$$u = \int \boldsymbol{E}.d\boldsymbol{D} + \int \boldsymbol{H}.d\boldsymbol{B}. \tag{2.6}$$

If the medium has a constant real permittivity ε and permeability $\mu = 1$, the energy density in the medium is

$$u = \frac{1}{2}\varepsilon\varepsilon_0 E^2 + \frac{1}{2}\mu_0 H^2. \tag{2.7}$$

The speed of propagation of the waves is found from the dispersion relation $k^2 = \varepsilon\varepsilon_0\mu_0\,\omega^2$, that is, $c(\varepsilon) = \omega/k = (\varepsilon\varepsilon_0\mu_0)^{-1/2} = c/\varepsilon^{1/2}$. This demonstrates the well-known result that, in a linear medium, the refractive index n is $\varepsilon^{1/2}$. Another useful relation between the $\boldsymbol{E}$ and $\boldsymbol{B}$ fields in the electromagnetic wave, the ratio E/B, is $c/\varepsilon^{1/2} = c/n$. Substituting this result into the expression for the electric and magnetic field energies (2.7), it is found that these are equal. Thus, the total energy density in the wave is $u = \varepsilon\varepsilon_0 E^2$. Furthermore, the Poynting vector flux $\boldsymbol{E}\times\boldsymbol{H}$ is $\varepsilon^{1/2}\varepsilon_0 E^2 c = n\varepsilon_0 E^2 c$. This energy flow corresponds to the energy density of radiation in the wave $\varepsilon\varepsilon_0 E^2$ propagating at the velocity of light in the medium c/n. It follows that $N = n\varepsilon_0 E^2 c$, as expected.

Let us consider the expressions for the retarded values of the current which contributes to the vector potential at the point $\boldsymbol{r}$, (Fig.1). The expression for the vector potential $\boldsymbol{A}$ due to the current density $\boldsymbol{J}$ at distance r is

$$\boldsymbol{A}(\boldsymbol{r}) = \frac{\mu_0}{4\pi}\int d^3\boldsymbol{r}'\,\frac{\boldsymbol{J}(r',\ \boldsymbol{t}-|\boldsymbol{r}-r'|/c)}{|\boldsymbol{r}-r'|} = \frac{\mu_0}{4\pi}\int d^3\boldsymbol{r}'\,\frac{[\boldsymbol{J}]}{|\boldsymbol{r}-r'|}\,; \tag{2.8}$$

the square brackets refer to retarded potentials. Taking the time derivative,

$$\boldsymbol{E}(\boldsymbol{r}) = -\frac{\partial\boldsymbol{A}}{\partial t} = -\frac{\mu_0}{4\pi}\int d^3\boldsymbol{r}'\,\frac{[\dot{\boldsymbol{J}}]}{|\boldsymbol{r}-r'|}\,. \tag{2.9}$$

In the distant far field limit, the electric field component E_r of the radiation field is perpendicular to the radial vector $\boldsymbol{r}$ and so, as indicated in Fig.1, $\boldsymbol{E}(\boldsymbol{r}) = \boldsymbol{E}(\boldsymbol{r})\times\boldsymbol{i}_k$, i.e.,

$$|\boldsymbol{E}(\boldsymbol{r})| = \frac{\mu_0\sin\theta}{4\pi}\left|\int d^3\boldsymbol{r}'\,\frac{[\dot{\boldsymbol{j}}]}{|\boldsymbol{r}-r'|}\right|. \tag{2.10}$$

It should be observed that if we substitute $\int d^3\boldsymbol{r}'[\dot{\boldsymbol{j}}] = \boldsymbol{e}\ddot{\boldsymbol{r}}$ into (2.10), we obtain

$$E_{pc} = \frac{|\ddot{\boldsymbol{p}}|\sin\theta}{4\pi\varepsilon_0 c^2 r}; \tag{2.11}$$

this is the expression for the radiation of a point charge, and $\boldsymbol{p}$ is the electric dipole moment of the charge with respect to some origin.

We now evaluate the frequency spectrum of the radiation. First of all, we work out the total radiation rate by integrating the Poynting vector flux over a sphere at a large distance $\boldsymbol{r}$,

$$\left(\frac{dE}{dt}\right)_{rad} = \int_S nc\varepsilon_0 E_r^2\,dS$$

$$= \int_\Omega \frac{nc\varepsilon_0\sin^2\theta}{16\pi^2}\left|\int d^3\boldsymbol{r}'\,\frac{[\dot{j}]}{|\dot{\boldsymbol{r}}-r'|}\right|^2 r^2\,d\Omega \tag{2.12}$$

Assuming the size of the emitting region is much smaller than the distance to the point of observation, i.e., $L \ll r$, we can write $|\boldsymbol{r}-\boldsymbol{r}'| = r$ and then,

$$\left(\frac{dE}{dt}\right)_{rad} = \int \frac{n\sin^2\theta}{16\pi^2\varepsilon_0 c^3}\left|\int d^3\boldsymbol{r}'[\dot{\boldsymbol{j}}]\right|^2 d\Omega. \tag{2.13}$$

Notes

Next, we take the time integral of the radiation rate to find the total radiated energy,

$$E_{rad} = \int_{-\infty}^{\infty} \left(\frac{dE}{dt}\right)_{rad} dt$$

$$= \int_{-\infty}^{\infty} \int_{\Omega} \frac{n \sin^2\theta}{16\pi^2 \varepsilon_0 c^3} \left|\int d^3\boldsymbol{r}' [\dot{\boldsymbol{J}}]\right|^2 d\Omega dt. \quad (2.14)$$

Notes

Using Parseval's theorem to transform from an integral over time to an integral over frequency, and restricting our interest to positive frequencies only, we find:

$$E_{rad} = \int_0^{\infty} \int_{\Omega} \frac{n \sin^2\theta}{8\pi^2 \varepsilon_0 c^3} \left|\int d^3\boldsymbol{r}' [\dot{\boldsymbol{J}}(\omega)]\right|^2 d\Omega d\omega. \quad (2.15)$$

Let us now evaluate the volume integral $\int d^3\boldsymbol{r}' [\dot{\boldsymbol{J}}(\omega)]$. We take $\boldsymbol{R}$ to be the vector from the origin of the coordinate system to the observer, and x to be the position vector of the current element $\boldsymbol{J}(\omega) d^3\boldsymbol{r}'$ from the origin; so that $\boldsymbol{r}' = \boldsymbol{R} - \boldsymbol{x}$. Now the waves from the current element at x propagate outwards from the emitting region at velocity c/n with phase factor $\exp[i(\omega t - \boldsymbol{k}.\boldsymbol{r}')]$ and therefore, relative to the origin at O, the phase factor of the waves, which we need to find for the retarded value of $\dot{\boldsymbol{J}}(\omega)$, is

$$\exp\left[i\left(\omega t - \boldsymbol{k}.(\boldsymbol{R} - \boldsymbol{x})\right)\right] = \exp(-i\boldsymbol{k}.\boldsymbol{R}) \exp[i(\omega t + \boldsymbol{k}.\boldsymbol{x})]. \quad (2.16)$$

So, evaluating $[\dot{\boldsymbol{J}}(\omega)]$, we have

$$\left|\int d^3\boldsymbol{r}' [\dot{\boldsymbol{J}}(\omega)]\right| = \left|i\omega \int d^3\boldsymbol{r}' [\boldsymbol{J}(\omega)]\right|$$

or, by including the phase factor explicitly

$$\left|\int d^3\boldsymbol{r}' [\dot{\boldsymbol{J}}(\omega)]\right| = \left|\int d^3\boldsymbol{r}' \omega \boldsymbol{J}(\omega) \exp[i(\omega t + \boldsymbol{k}.\boldsymbol{x})]\right|. \quad (2.17)$$

Using (2.4) we obtain:

$$\left|\int d^3\boldsymbol{r}' [\dot{\boldsymbol{J}}(\omega)]\right| = \left|\frac{\omega e}{(2\pi)^{1/2}} \exp(i\omega t) \int \exp\left[i(\boldsymbol{k}.\boldsymbol{x} + \frac{\boldsymbol{\omega x}}{\boldsymbol{v}})\right] dx\right|$$

$$= \left|\frac{\omega e}{(2\pi)^{1/2}} \int \exp\left[i(\boldsymbol{k}.\boldsymbol{x} + \frac{\boldsymbol{\omega x}}{\boldsymbol{v}})\right] dx\right|. \quad (2.18)$$

This is the key integral in deciding whether or not the particle radiates. If the electron propagates in a vacuum, $\omega/k = c$ and we can write the exponent as

$$kx\left(\cos\theta + \frac{\omega}{kv}\right) = kx\left(\cos\theta + \frac{c}{v}\right) \quad (2.19)$$

In a vacuum, $c/v > 1$, and so this exponent is always greater than zero and hence the exponential integral over all x is always zero, assuming boundary limits vanish. This means that a particle moving at constant velocity in a vacuum does not radiate. However, if the medium has refractive index n, $\omega/k = c/n$, and then the exponent is zero if $\cos\theta = -c/nv$. This is the origin of the Cherenkov radiation phenomenon. The radiation is only coherent along the angle θ corresponding to the Cherenkov cone derived from Huygens' construction, i.e., given by (1.1).

We can therefore write down formally the energy spectrum using the relation (2.22) (below) of the average number of photons in a given state in the phase space, recalling that the radiation is only emitted at an angle $\cos\theta = c/nv$. But first, we need the equation which describes how the spectrum of radiation evolves towards the so called Bose-Einstein distribution (Einstein, 1905; 1915).

In the non-relativistic limit, this equation is known as the Kompaneets equation. It is written in terms of the occupation number of photons in phase space, because we need to include both spontaneous and induced processes in the calculation. Let us compare this approach with that involving the coefficients of emission and absorption of radiation. As a good reference for understanding the basic physics of spontaneous and induced processes, we present in the following Feynman's enunciation of the key rule for the emission and absorption of photons, which are spin-1 bosons (Feynman, Leighton and Sands, 1965; Feynman, 1972):

$$\textit{The probability that an atom will emit a photon in a particular final state is increased by a factor } (n+1) \textit{ if there are already } n \textit{ photons in that state.}$$

The statement is made in terms of probabilities rather than quantum mechanical amplitudes; in the latter case, the amplitude would be increased by a factor $\sqrt{n+1}$. We will use probabilities in our analysis. The number n will turn out to be the occupation number. To derive the Planck spectrum, consider an atom which can be in two states, an upper state 2 with energy $\hbar\omega$ greater than the lower state 1. N_1 is the number of atoms in the lower state and N_2 the number in the upper state. In thermodynamic equilibrium, the ratio of the numbers of atoms in these states is given by the Boltzmann relation,

$$\frac{N_2}{N_1} = \exp(-\Delta E/kt) = \exp\left(-\frac{\hbar\omega}{kT}\right), \tag{2.20}$$

where $\nabla E = \hbar\omega$ and the corresponding statistical weights g_2 and g_1 are assumed to be the same. When a photon of energy $\hbar\omega$ is absorbed, the atom is excited from state 1 to state 2 and, when a photon of the same energy is emitted from state 2, the atom de-excites from state 2 to state 1. In thermodynamic equilibrium, the rates for the emission and absorption of photons between the two levels must be exactly balanced. These rates are proportional to the product of the probability of the events occurring and the number of atoms present in the appropriate state. Suppose $\bar{n}$ is the average number of photons in a given state in the phase space of the photons with energy $\hbar\omega$. Then, the absorption rate of these photons by the atoms in the state 1 is $N_1\bar{n}\,p_{12}$, where p_{12} is the probability that the photon will be absorbed by an atom in state 1, which is then excited to state 2. According to the rule enunciated above by Feynman, the rate of emission of photons when the atom de-excites from state 2 to state 1 is $N_2(\bar{n}+1)p_{12}$. At the quantum mechanical level, the probabilities p_{12} and p_{21} are equal. This is because the matrix element for, say, the p_{12} transition is the complex conjugate of the transition p_{21} and, since the probabilities depend upon the square of the magnitude of the matrix elements, these must be equal. This is called the *principle of jump rate symmetry*. Therefore,

Notes

$$N_1\bar{n} = N_2(\bar{n}+1). \tag{2.21}$$

Solving for $\bar{n}$ and using (2.20), we obtain

$$\bar{n} = \frac{1}{e^{\hbar\omega/kT}-1} \tag{2.22}$$

as the required average number of photons in a given state in the phase space. Now from (2.15) and (2.22) we have

Notes

$$\frac{dE_{rad}}{d\omega} = \int_{\Omega} \frac{n\omega^2 e^2 \sin^2\theta}{16\pi^2\varepsilon_0 c^3}\left|\int \exp\left[ikx\left(\cos\theta + \frac{\omega}{kv}\right)\right]dx\right|^2 d\Omega$$

$$= \frac{n\omega^2 e^2}{16\pi^2\varepsilon_0 c^3}\left(1-\frac{c^2}{n^2v^2}\right)\int_{\Omega}\left|\int \exp\left[ikx\left(\cos\theta + \frac{\omega}{kv}\right)\right]dx\right|^2 d\Omega\ . \tag{2.23}$$

If one sets $k(\cos\theta + \omega/kv) = \alpha$, one gets

$$\int_{\Omega}\left|\int \exp\left[ikx\left(\cos\theta + \frac{\omega}{kv}\right)\right]dx\right|^2 d\Omega$$

$$= \int_{\theta}\left|\int \exp(i\alpha x)dx\right|^2 2\pi\sin\theta\, d\theta\ . \tag{2.24}$$

We will evaluate the line integral along a finite path length from $-L$ to L, avoiding however the use of contour integration at values of θ ranging from $-\infty$ to ∞. The integral should be taken over a small finite range of angles about $\theta = \cos^{-1}(c/nv)$ for which $(\cos\theta + \omega/kv)$ or $(-\cos\theta + \omega/kv)$ is close to zero. Integration therefore can be taken over all values of θ or $\alpha = k(-\cos\theta + \omega/kv)$ knowing that most of the integral is contributed by values of θ very close to $\cos^{-1}(c/nv)$; so that $d\alpha = d(k(-\cos\theta + \omega/kv)) = k\sin\theta\, d\theta$. Thence, with respect to α, the integral (2.24) becomes

$$\int_{\Omega}\left|\int \exp\left[ikx\left(\cos\theta + \frac{\omega}{kv}\right)\right]dx\right|^2 d\Omega$$

$$= 8\pi\int \frac{\sin^2\alpha L}{\alpha^2}\frac{d\alpha}{k}\,, \tag{2.25}$$

This is an improper integral to be taken over all values of α from $-\infty$ to ∞. Combining test for convergence methods for such integrals and integration by parts, (2.25) is evaluated as

$$8\pi\int \frac{\sin^2\alpha L}{\alpha^2}\frac{d\alpha}{k} = 8\pi^2\left(\frac{L}{k}\right) = \frac{8\pi^2 c}{n\omega}L. \tag{2.26}$$

It follows that the energy radiated per unit bandwidth is

$$\frac{du}{d\omega} = \frac{\omega e^2}{2\pi\varepsilon_0 c^2}\left(1-\frac{c^2}{n^2v^2}\right)L. \tag{2.27}$$

We obtain the energy loss rate per unit path length directly by dividing by $2L$. Thus, the energy loss rate per unit path length follows as

$$\frac{du(\omega)}{dx} = \frac{\omega e^2}{4\pi\varepsilon_0 c^2}\left(1-\frac{c^2}{n^2v^2}\right). \tag{2.28}$$

Finally, the energy loss rate per unit path length and per frequency unit is obtained :

$$\frac{d^2E}{dxd\omega} = \frac{e^2}{4\pi\varepsilon_0 c^2}\left(1 - \frac{c^2}{n^2v^2}\right), \tag{2.29}$$

where it should be recalled that $v$is the particle superluminal velocity in the medium. Equations (1.1) and (2.29) are valid for arbitrary dependence $n(\omega)$.

As an important side remark, notice from (2.29) that the energy loss rate is a constant of motion with respect to constant ultra-relativistic (i.e., superluminal) velocity v and the medium refractive index n. We will now investigate what this means in the special context of free spin-1/2 particles superluminal motion in space time. However, before coming to the application in our field of interest, let us capture the true mechanism underlying the spin phenomenon.

Notes

III. Understanding the True Mechanism of the Free Spin-1/2 Field

What is spin? This is a short but exact query which had been perfectly clarified in (Belinfante, 1939; Ohanian, 1984). As echo of these references, we say that persistent prevailing speculations would have the spin of the electron or of some other particle a mysterious internal process for which no concrete physical picture exists, and for which there is no classical analogue. Judging from arguments which surface in scientific criticisms and statements found in modem textbooks on atomic physics and quantum theory, it is surprising to observe that our understanding of spin (or the lack thereof) has not made any advance since the early years of quantum mechanics (Dirac, 1928). It is usually believed that the spin is a nonorbital, "internal," "intrinsic," or "inherent" angular momentum (these words being often incorrectly used as synonyms), and often treated as an irreducible entity that cannot be explained further. Sometimes, the speculation goes that the spin is a product of an (unspecified) internal structure of the electron, or arises in a natural way from Dirac's equation or from the analysis of the representations of the Lorentz group. The mathematical formalism of the Dirac equation and of group theory resort to the existence of the spin to achieve the conservation of angular momentum and to construct the generators of the rotation group, but when it comes to understanding the physical mechanism that produces the spin, no explication is given. This lack of a concrete picture of the spin leaves a grievous gap in our understanding of quantum mechanics, and hinder the derivation of applications therefrom. However, the solution of this problem has been at hand since (Belinfante, 1939) who, on the basis of an old calculation, was able to give the true (concrete) picture of the spin. He established that the spin could be regarded as due to a circulating flow of energy, or a momentum density, in the electron wave field. He stressed that this picture of the spin is valid not only for electrons, but also for photons, vector mesons, and gravitons, and in all cases the spin angular momentum is due to a circulating energy flow in the fields. Thus, in contradistinction to the common prejudice, the spin of the electron has a close classical analogue; It is an angular momentum of exactly the same kind as carried by the fields of a classical circularly polarized electromagnetic wave. Moreover, according to a demonstration by (Gordon, 1928), the magnetic moment of the electron is due to the circulating flow of charge in the electron wave field. Definitely, as a result, neither the spin nor the magnetic moment are internal properties of the electron and other particles: they have nothing to do with the internal structure of the electron, but only depend on the structure of its wave field.

Further, a comparison between calculations of angular momentum in the Dirac field and the electromagnetic fields shows that the spin of the electron is entirely analogous to the angular momentum carried by a classical circularly polarized wave (Ohanian, 1984). From a theoretical point [cf.(Greiner,2000)], Maxwell (electromagnetic) equations given by

$$curl\,\boldsymbol{E} + \frac{1}{c}\frac{\partial \boldsymbol{H}}{\partial t} = 0, \quad curl\,\boldsymbol{H} - \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} = \frac{4\pi}{c}\boldsymbol{j}$$

Notes

can be represented in the form of the Dirac equations (spinor equation)

$$\frac{1}{i}\sum_{r=0}^{3} \hat{\alpha}^r \frac{\partial}{\partial x^r}\psi = \frac{4\pi}{c}\phi.$$

This relates the physics of self-interaction of the field of the particle, where it is known that a moving electron generates an electromagnetic field. All of these put together corroborate the above observations.

Having clarified the mechanism of spin, it is noteworthy that the Dirac (free) field is a plane wave, and so the axis of rotation of a free Dirac particle (which coincides with the direction of the field linear phase velocity) is perpendicular to this plane, Fig.2. Thus, a spinning free fermion is carried by a circularly polarized electromagnetic plane wave; in other words, a spinning free fermion rolls helically in an electromagnetic plane wave.

IV. Conservation of Energy by Superluminal Free Spin-1/2 Particles under Cherenkov Radiation

Theoretically (Afanasiev et al., 1999) and experimentally (Stevens et al., 2001; Wahlstr and and Merlin, 2003) it has been shown that the inclusion of the medium dispersion (a case we will not however consider in this work) leads to the appearance of additional radiation intensity maxima (or striped-like structure) in the angular distribution of the radiation.

Let us consider a free spin-1/2 particle moving in a localized, kinematically permissible region of spacetime with superluminal generalized linear velocity component of parameter k, given by(Gazoya et al., 2015; 2016):

$$V_{\text{Sup}}(k) = \left[\cos^{-1}\left(\frac{1}{4}\right)\right] \times c \approx \left(\frac{21\pi}{50} + 2\pi k\right) \times c,$$

$$k = 0, 1, 2, \ldots \tag{4.1}$$

where c is the universal value of the speed of light in a vacuum. In the absence of dispersion, the Cherenkov angle expression (1.1), as a function of the parameter k takes the form

$$\cos\theta(k) = \frac{c}{nV_{\text{Sup}}(k)} = \frac{50}{n\pi}\left(\frac{1}{100k+21}\right), \tag{4.2}$$

that is,

$$\theta(k) = \arccos\left(\frac{50}{n\pi}\left(\frac{1}{100k+21}\right)\right). \tag{4.3}$$

Clearly, as the parameter k assumes large numerical values, the argument of the inverse cosine function in (4.3) tends to zero, this brings the direction of the radiated

Cherenkov photons to an angle near to 90^o with the direction of the particle linear velocity; at this point Cherenkov cone becomes flattened. Mathematically, we write

$$\lim_{k\to\infty} \theta(k) = \frac{\pi}{2} \ . \tag{4.4}$$

This situation corresponds to Cherenkov radiation at an angle of approximately 90^o of moving free spin-1/2 particles with 'sufficiently' superluminal linear velocity in spacetime (of refractive index $n = 1.000277$ taken at Standard Temperature and Pressure (STP), for example) (see Fig.2).

On the other hand, the expression (2.29) of the radiated energy loss rate per unit path length and per frequency unit in terms of the parameter k becomes

$$\frac{d^2E(k)}{dxd\omega} = \frac{e^2}{4\pi\varepsilon_0 c^2}\left[1 - \frac{2500}{n^2\pi^2}\left(\frac{1}{100k+21}\right)^2\right]. \tag{4.5}$$

Upon taking the limit as k assumes large numerical values in (4.5) we obtain

$$\lim_{k\to\infty}\left[\frac{d^2E(k)}{dxd\omega}\right] = \frac{e^2}{4\pi\varepsilon_0 c^2} = \text{const.} \tag{4.6}$$

Thus, in the particular case of large k, the radiated energy is still a constant of motion, independent of the medium refractive index $n(\omega)$, and so independent of frequency.

V. Discussions

In light of the true mechanism of the free spin-1/2 field exposed in Section 3 above, the question arises: Is the radiated energy in (4.6) really lost, as conventionally claimed so far? For sufficiently superluminal motion induced by large parameter k, the direction of the radiated photons tends near to 90^o with that of the plane wave spin-half field. There, the Cherenkov cone becomes flattened to a plane which in turn coincides with the plane wave field of the particle (see Fig. 2). As a result, the radiated energy could be regarded as merging with the planewave of the circulating energy flow which carries the particle. It could not be considered lost; it could rather contribute to the fermion wave field, or, precisely, 're-invested' in the fermion wave field. Clearly, in case this radiated energy (being a constant of motion) could really go into waste, this could not significantly affect the superluminal nature of the propagation. In Fig.3 and Fig.4, graphs of the parameter k against angle $\theta(k)$ and energy loss rate $d^2E(k)/dxd\omega$ are plotted. A limiting value is reached in each case by these functions as the parameter k takes on big values. Thus, clearly, the result of this demonstration completely contradicts the speculative anticipation of instant collapse of such superluminal particles due to fast energy loss under Cherenkov radiation.

VI. Conclusions

Theoretically it has been shown that, in the limit of a kinematically permissible medium with absence of dispersion, the superluminal motion of free spin-half particles could be a reliable dynamical system in conformity with one of nature's basic laws of conservation of energy. The other side-argument which anticipates and insists on fast energy loss that would bring this kind of systems to instant collapse in their dynamical evolution could not hold. The radiated energy, which is a constant of motion in this case, whether lost or re-invested (as shown) in the wave field of the particle, could not

Notes

significantly affect the superluminal nature of the propagation. Moreover, the larger the number the parameter k assumes in the quantization of the superluminal linear velocity, the less energy loss would be expected. In other words, the highly superluminal the propagation,the less the radiated energy wouldbe gone into waste.

Notes

References Références Referencias

1. Afanasiev, G. N., Kartavenko, V. G. and Magar, E. N. (1999a).Valvilov–Cherenkov radiation in dispersive medium. Physica B 269, 95–113. (doi:10.1016/S0921-4526(99)00078-2).
2. Afanasiev, G. N., Eliseev, S. M. and Stepanovsky, Y. P. (1999b). Semi-analytic treatment of the Vavilov–Cherenkov radiation. Phys. Scripta 60, 535–546. (doi:10.1238/Physica. Regular. 060a00535) See also Afanasiev (2004), ch 4.
3. Afanasiev, G. N., Zrelov, V. P. and Kartavenko, V. G. (2003). Fine structure of the Vavilov–Cherenkov radiation. Phys. Rev. E 68, 1–12. (doi:10.1103/PhysRev E.68.066501).
4. Afanasiev, G. N. (2004).Vavilov–Cherenkov and synchrotron radiation. In Fundamental Theories of Physics Series (ed. A. van der Merwe), vol. 142. Dordrecht: Kluwer.
5. Alaeian (2014). An Introduction to β-Ray Spectroscopy.Physics 241, Stanford University.
6. Belinfante, F. J. (1939). Physics 6, 887.
7. Bock, R.K. and Vasilescu, A. (1998). The particle detector briefbook. Springer 1998.
8. Cherenkov, P. A. (1934). Visible radiation induced by g rays in pure fluids. Dokl. Akad. Nauk SSSR 2,455–457.
9. Cherenkov, P. A. (1944). Radiation of electrons moving with the velocity exceeding that of light.Trudy FIAN 2, 3–61.
10. Clemmow, P. C. W. and Dougherty, J. P. (1969). Electrodynamics of Particles and Plasmas. Reading, Massachusetts: Addison-Wesley.
11. Dirac, P. A. M. (1928). The quantum theory of the electron, Part 1. Proc. Roy. Soc. (London). 1928; A117:610-624. Part two. Proc. Roy. Soc. (London). A118:351-361.
12. Dunlop, J. S. & Peacock, J. A. (1990). The redshift cut-off in the luminosity function of radio galaxies and quasars, Monthly Notices of the Royal Astronomical Society, 247,19–42.
13. Dunlop, J. S., Peacock, J. A., Spinrad, H. (1996). A 3.5-Gyr-old galaxy at redshift 1.55. Nature, 381, 581–584.
14. Einstein, A. (1905).Uber einen die Erzeugung und Verwandlung des Lichtesbet reffendenheuristischen Gesichtspunkt, Annalen der Physik, 322, 132–148.
15. Einstein, A. (1936). Lens-like action of a star by the deviation of light in the gravitational field, Science, 84, 506–507.
16. Feynman, R., Leighton, R. B., & Sands, M. L. (1965). Feynman Lectures on Physics. Redwood City, California: Addison-Wesley.
17. Feynman, R. P. (1972). Statistical mechanics: A set of lectures. Reading, Massachusetts: W. A. Benjamin.
18. Frank, I. M. and Tamm, I. E. (1937). Coherent radiation of fast electron in medium. Dokl. Akad. Nauk.
19. Gazoya, E. D. K., Oduro, F. T. and Prempeh, E. (2015). Investigation of superluminal motion of free spin-half particles in spacetime. Available: http://dx.doi.org/10.5539/- jmr.v7n4p12.

20. Gazoya, E. D. K., Oduro, F. T. and Prempeh, E. (2016). Superluminal motion of free spin-half particles in a fiber bundle formalism. DOI: 10.9734/BJAST/2016/26498. ISSN: 2231-0843, NLM ID: 101664541.
21. Ginzburg, V. L. (1940). Quantum theory of the light radiation from the electron moving uniformly inmedium. Zh. Eksp. Theor. Fiz. 10, 589–600.
22. Gordon, W. (1928). Phys. 50, 630.
23. Greiner, W. (2000). Relativistic quantum mechanics- wave equations (3rd ed.). Berlin, Heidelberg: Springer.
24. Jackson, J. D. (1999). Classical Electrodynamics. New York: John Wiley.
25. Jelle, J. V. (1955). Cerenkov radiation and Its applications. Br. J. Appl. Phys. 6, 277.
26. Katharina, M. (2014).Cherenkov radiation.
27. Konrad, K. (1998). Detectors for particle radiation (2^{nd} edition). Chap.1.
28. Leroy, C. and Rancoita, P. G. (2014). Principles of radiation interaction in matter and detection. World Scientific.
29. Longair, M. S. (1978). Radio astronomy and cosmology, in Observational Cosmology: 8th advanced course, Swiss Society of Astronomy and Astrophysics, Saas-Fee 1978, edsMaeder, A., Martinet, L., an Tammann, G., pp. 125–257. Geneva: Geneva Observatory Publications.
30. Longair, M. S. (1981a). High Energy Astrophysics, 1st edition. Cambridge: Cambridge University Press.
31. Longair, M. S. (1995). The physics of background radiation, in The Deep Universe, by Sandage, A.R., Kron, R.G. and Longair, M.S., edsBinggeli, B. andBuser, R., pp. 317–514, Saas - Fee Advanced Course 23. Berlin: Springer-Verlag.
32. Longair, M. S. (1997b). High Energy Astrophysics, Volume 1 (revised 2nd edition). Cambridge: Cambridge University Press.
33. Longair, M. S. (1997c). High Energy Astrophysics, Volume 2 (revised 2nd edition). Cam- bridge: Cambridge University Press.
34. Ohanian, H. C. (1984). What Is Spin?. New York.
35. Vavilov, S. I. (1934). On the possible reasons for the blue g luminescence of fluids. Dokl. Akad. Nauk SSSR 2, 457–461.
36. Wahlstrand, J. K. and Merlin, R. (2003). Cherenkov radiation emitted by ultra-fast laser pulses and the generation of coherent polaritons. Phys. Rev. B 68, 1–11. (doi:10.1103/PhysRevB.68. 054301).

Notes

Global Journal of Science Frontier Research: F
Mathematics and Decision Sciences
Volume 16 Issue 5 Version 1.0 Year 2016
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

The Distribution of Cube Root Transformation of the Error Component of the Multiplicative Time Series Model

By Dike A. O., Otuonye E. L. & Chikezie D. C.
Akanu Ibiam Federal Polytechnic

Abstract- In this paper, the probability density function (pdf), of the cube root transformation was derived from the n^{th} power transformation of the error component of the multiplicative time series model. The mean and variance of the cube root transformation were equally established. From the simulated results it was found that the cube root transformed error component was normal with unit mean and variance approximately $\frac{1}{9}$ times that of the original error before transformation. Furthermore, the Kolmogorov-Smirnov test for normality was used ascertain the effect of cube root transformation as regard to normalization, from the results of the test at p-value of 0.05, we accepted normality for σ values of 0.001 to 0.22. Hence, a successful transformation is achieved when $0 \leq \sigma \leq 0.22$ depending on the decimal places desired.

Keywords: power transformations, probability density function, error component, mean, variance, multiplicative time series.

GJSFR-F Classification : *FOR Code : 65H04*

THEDISTRIBUTIONOFCUBEROOTTRANSFORMATIONOFTHEERRORCOMPONENTOFTHEMULTIPLICATIVETIMESERIESMODEL

Strictly as per the compliance and regulations of :

The Distribution of Cube Root Transformation of the Error Component of the Multiplicative Time Series Model

Dike A. O. [α], Otuonye E. L. [σ] & Chikezie D. C. [ρ]

Abstract- In this paper, the probability density function (pdf), of the cube root transformation was derived from the $\mathbf{n^{th}}$ power transformation of the error component of the multiplicative time series model. The mean and variance of the cube root transformation were equally established. From the simulated results it was found that the cube root transformed error component was normal with unit mean and variance approximately $\frac{1}{9}$ times that of the original error before transformation. Furthermore, the Kolmogorov-Smirnov test for normality was used ascertain the effect of cube root transformation as regard to normalization, from the results of the test at p-value of 0.05, we accepted normality for $\boldsymbol{\sigma}$ values of 0.001 to 0.22. Hence, a successful transformation is achieved when $\mathbf{0 \leq \sigma \leq 0.22}$ depending on the decimal places desired.

Keywords: power transformations, probability density function, error component, mean, variance, multiplicative time series.

I. Introduction

The Gausian distribution (commonly called the normal distribution) is the best well known and most frequently used in probability distribution theory. It is widely used in natural and social sciences to represent real- valued random variables whose distributions are not known.

The normal distribution derived its usefulness from the central limit theorem. The central limit theorem states that the averages of random variables independently drawn from independent distributions converges in distribution to the normal, that is, becomes normally distributed when the number of random variables is sufficiently large. Physical quantities that are expected to be the sum of many independent processes (such as measurement errors) often have distributions that are nearly normal.

The probability density function (pdf) of the normal distribution is given in Uche(2003)as

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}\ e^{-\frac{1}{2}\left(\frac{x-1}{\sigma}\right)^2}, x \geq 0, \sigma^2 > 0 \qquad (1)$$

The error component e_t of the multiplicative time series model has a pdf $N(1, \sigma^2)$ where $e_t > 0$, Iwueze(2007) established the distribution of the left-truncated normal distribution and is given by

Author α: Department of Maths and Statistics, Akanu Ibiam Federal Polytechnic, Unwana, Afikpo, Nigeria. e-mail: dikeawa@gmail.com
Author σ ρ: Department of Statistics, Faculty of Biological and Physical Sciences, Abia State University, Uturu, Nigeria.

$$f(x) = \frac{e^{-\frac{1}{2}\left(\frac{x-1}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}\left[1-\Phi\left(\frac{-1}{\sigma}\right)\right]}, x \geq 0, \sigma^2 > 0 \quad (2)$$

With mean $E(X)$ and variance $Var(X)$ given by

$$E(X) = 1 + \frac{\sigma e^{-\frac{1}{2}\sigma^2}}{\sigma\sqrt{2\pi}\left[1-\Phi\left(\frac{-1}{\sigma}\right)\right]}, x \geq 0, \sigma^2 > 0 \quad (3)$$

and

$$Var(X) = \frac{\sigma^2}{2\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)}\left(\left[1 + P_r\left(\chi^2_{(1)} \leq \frac{1}{\sigma^2}\right)\right]\right) - \frac{\sigma e^{-\frac{1}{2}\sigma^2}}{\sqrt{2\pi}\left[1-\Phi\left(\frac{-1}{\sigma}\right)\right]} - \left[\frac{\sigma e^{-\frac{1}{2}\sigma^2}}{\sqrt{2\pi}\left[1-\Phi\left(\frac{-1}{\sigma}\right)\right]}\right]^2 \quad (4)$$

He examined some implications of truncating the $N(1, \sigma^2)$ to the left.

The truncated normal distribution have gained much acceptance in various fields of human endeavours, these include inventory management, regression analysis, operation management, time series analysis and so on. Johnson and Thomopoules (2004) considered the use of the left truncated distribution for improving achieved service levels. They presented the table of the cumulative distribution function of the left truncated normal distribution and derived the characteristic parameters of the distribution, and also presented the table of the partial expectation of the left truncated normal distribution A time series is a collection of ordered observation made sequentially in time. Examples abound in Sciences, Engineering, Economics etc and methods of analysing time series constitute a vital area in the field of Statistics.

According to Spiegel and Stephens (1999) the general time series model is always considered as a mixture of four major components, namely the Trend T_t, Seasonal movements S_t, cyclical movements C_t and irregular or Random Movements e_t. The general multiplicative time series model is given as

$$X_t = T_t S_t C_t e_t \quad (5)$$

In short term series the trend and cyclical components are merged to give the trend-cycle component; hence equation (5) can be rewritten as

$$X_t = M_t S_t e_t \quad (6)$$

where M_t is the trend cycle component and e_t is independent identically distributed (iid) normal errors with mean 1 and variance $\sigma^2 > 0$ $[e_t \sim N(1, \sigma^2)]$

Cox (2007) observed that the cube (X^3) transformation is a fairly strong transformation with a substantial effect on distribution shape. It is also used for reducing right skewness, and has the advantage that it can be applied to zero and non negative values. A similar property is possessed by any other root whose power is the inverse of an odd positive integer example 1/3,1/5, 1/7, etc. The cubic transformation is stronger than the square (X^2) transformation, though weaker than the logarithm transformation.

a) Data Transformation

According to Cox (2007) transformation is the replacement of a variable by a function of that variable, for example, replacing a variable x by the square root of x or the logarithm of x. In a stronger sense, a transformation is a replacement that changes

Notes

Notes

the shape of a distribution or relationship. Reasons for transformation include stabilizing variance, normalizing, reducing the effect of outliers, making a measurement scale more meaningful, and to linearize a relationship. For more references see Bartlett (1947) Box and Cox (1964), etc.

Many time series analysis assume normality and it is well known that variance stabilization implies normality of the series. The most popular and common transformation are the logarithm transformation and the power transformations (square, square root, inverse, inverse square, and inverse square root). It is important to note that, if we apply the cube root transformation on model (6), we still obtain a multiplicative time series model given by

$$Y_t = M_t^3 \, S_t^3 \, e_t^3 = M_t^* \quad S_t^* \quad e_t^* \tag{7}$$

Where $M_t^3 = M_t^*$, $S_t^3 = S_t^*$, $e_t^3 = e_t^*$

Several studies abound in statistical literature on effects of power transformations on the error component of a multiplicative time series model whose error component is classified under the characteristics given in (3). The sole aim of such studies is to establish the conditions for successful transformation. A successful transformation is achieved when the derivable statistical properties of a data set remain unchanged after transformation, there basic properties or assumptions of interest for the studies are (i) unit mean and (ii) constant variance. Also Nwosu et al (2010) studied the effects of inverse and square root transformation respectively on the error component of the same model and discovered that the inverse transform $Y = \frac{1}{e_t}$ can be assumed to be normally distributed with mean, one and the same variance provided, $\sigma <$ 0.07. Similarly Otuonye et al (2011) discovered that the square root transform; $\sqrt{e_t}$ can be assumed to be normally distributed with unit mean and variance $4\sigma^2$, for $\sigma_1 \leq 0.3$ where σ_1^2 is the variance of the original error component before transformation.

Ibeh et al (2013) studied the inverse square transformation of error the component of the multiplicative time series model, the results of the research showed that the basic assumptions of the error term of the multiplicative model which is normally distributed with mean 1 and finite variance can only be maintained if the standard deviation of the untransformed error term is less than or equal to 0.07 (σ ¡ 0.07). the study also revealed that the variance of the transformed of the error term is 4 times the variance of the untransformed for $\sigma \leq 0.07$.

Ajibade et al (2015) studied the distribution of the inverse square root transformed error component of the multiplication time series model and found out that the means are the same and variance $Var(e_t^*) \approx \frac{1}{4} Var(e_t)$ for $\sigma \leq 0$.

Dike et al (2016) generalized the power transformation by establishing the n^{th} power transformation; they showed that any power transformation can be derived from the formular.

In this paper the cube root transformation was carried out, the probability density function, the mean and variance of the distribution were established using the n^{th} power transformation given by Dike *et al*(2016) by substituting $n = \frac{1}{3}$

According to Osborne (2002) caution should be exercised in the transformation so that the basic structure of the original series is not altered.

To this end comparison would be made between the transformed and the untransformed series for their mean and variances respectively to know the condition under which the transformation would be successful.

Otuonye *et al* (2011) showed that the variance of the untransformed is 4 times the variance of the transformed series for $0<\sigma\leq 0.3$

In this cube root transformation we would demonstrate whether the transformation is normal with mean 1 and same variance. This would be done by simulating the original series for specified values σ and carry out the transformation.

The normality test of the cube root transformed series would be done using the kolmogorov- Smirnov normality test for varying values of σ.

To validate our finding simulation and practical example would be use to drive the research home.

Notes

b) The Probability Density Function (pdf) of the Cube Root Transformation

The pdf of the general equation of the nth power transformation given by Dike *et al* (2016) is

$$f(y) = \frac{\frac{1}{|n|}.y^{\frac{1}{n}-1}\, e^{-\frac{1}{2}\left(\frac{y^{\frac{1}{n}}-1}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}\left[1-\Phi\left(\frac{-1}{\sigma}\right)\right]} \tag{8}$$

Substituting $n = \frac{1}{3}$ in the General equation given in (8) yields the pdf of the cube root transformation given as

\

$$f(y) = \frac{\frac{1}{3}.y^{\frac{1}{3}-1}\, e^{-\frac{1}{2}\left(\frac{\frac{1}{3}-1}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}\left[1-\Phi\left(\frac{-1}{\sigma}\right)\right]} \tag{9}$$

From equation (9) we derive the moments (mean and variance) of the cube root transformation

c) The Mean for cube root transformation

Given

$$E(Y) = 1 + \frac{n\sigma}{\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}} + \frac{n(n-1)\sigma^2}{2!\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(-\frac{e^{-\frac{1}{2\sigma^2}}}{\sigma} + \frac{\sqrt{2\pi}}{2}\left[1 + P_r\left(\chi^2_{(1)} \leq \frac{1}{\sigma^2}\right)\right]\right) + \frac{2n(n-1)(n-2)\sigma^3}{3!\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]} \left(1+\frac{1}{2\sigma^2}\right) e^{-\frac{1}{2\sigma^2}} \tag{10}$$

For $n = \frac{1}{3}$

$$E(Y) = 1 + \frac{(\frac{1}{3})\sigma}{\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}} + \frac{(\frac{1}{3})(\frac{1}{3}-1)\sigma^2}{2!\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(-\frac{e^{-\frac{1}{2\sigma^2}}}{\sigma} + \frac{\sqrt{2\pi}}{2}\left[1 + P_r\left(\chi^2_{(1)} \leq \frac{1}{\sigma^2}\right)\right]\right) + \frac{2(\frac{1}{3})(\frac{1}{3}-1)(\frac{1}{3}-2)\sigma^3}{3!\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]} \left(1+\frac{1}{2\sigma^2}\right) e^{-\frac{1}{2\sigma^2}} \tag{11}$$

$$= 1 + \frac{\sigma}{3\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}} + \frac{(\frac{1}{3})(-\frac{2}{3})\sigma^2}{2\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(-\frac{e^{-\frac{1}{2\sigma^2}}}{\sigma} + \frac{\sqrt{2\pi}}{2}\left[1 + P_r\left(\chi^2_{(1)} \leq \frac{1}{\sigma^2}\right)\right]\right) + \frac{2(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})\sigma^3}{6\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]} \left(1+\frac{1}{2\sigma^2}\right) e^{-\frac{1}{2\sigma^2}} \tag{12}$$

Notes

$$= 1 + \frac{\sigma}{3\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}} - \frac{\sigma^2}{9\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(-\frac{e^{-\frac{1}{2\sigma^2}}}{\sigma} + \frac{\sqrt{2\pi}}{2}\left[1 + P_r\left(\chi^2_{(1)} \le \frac{1}{\sigma^2}\right)\right]\right) + \frac{10\sigma^3}{81\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}$$

$$\left(1 + \frac{1}{2\sigma^2}\right) e^{-\frac{1}{2\sigma^2}} \qquad (13)$$

$$= 1 + \frac{\sigma}{3\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}} + \frac{\sigma}{9\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}} - \frac{\sigma^2}{18\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(1 + P_r\left(\chi^2_{(1)} \le \frac{1}{\sigma^2}\right)\right) + \frac{10\sigma^3}{81\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}} +$$

$$\frac{5\sigma}{81\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}} \qquad (14)$$

$$= 1 + \frac{(27\sigma + 9\sigma + 5\sigma)}{81\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}} - \frac{\sigma^2}{18\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(1 + P_r\left(\chi^2_{(1)} \le \frac{1}{\sigma^2}\right)\right) + \frac{10\sigma^3}{81\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}} \qquad (15)$$

$$= 1 + \frac{41\sigma}{16\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}} - \frac{\sigma^2}{18\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(1 + P_r\left(\chi^2_{(1)} \le \frac{1}{\sigma^2}\right)\right) + \frac{10\sigma^3}{81\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}} \qquad (16)$$

$$\therefore E(Y) = 1 + \frac{41\sigma}{16\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}} - \frac{\sigma^2}{18\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(1 + P_r\left(\chi^2_{(1)} \le \frac{1}{\sigma^2}\right)\right) + \frac{10\sigma^3}{81\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}} \qquad (17)$$

d) Variance for cube root transformation

For $E(Y^2)$

Given

$$E(Y^2) = 1 + \frac{2n\sigma}{\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}} + \frac{2n(2n-1)\sigma^2}{2!\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(-\frac{e^{-\frac{1}{2\sigma^2}}}{\sigma} + \frac{\sqrt{2\pi}}{2}\left[1 + P_r\left(\chi^2_{(1)} \le \frac{1}{\sigma^2}\right)\right]\right) +$$

$$\frac{2(2n)(2n-1)(2n-2)\sigma^3}{3!\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(1 + \frac{1}{2\sigma^2}\right) e^{-\frac{1}{2\sigma^2}} \qquad (18)$$

For $n = \frac{1}{3}$

$$E(Y^2) = 1 + \frac{2\left(\frac{1}{3}\right)\sigma}{\sqrt{2\pi}\left[1-\Phi\left(\frac{-1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}} + \frac{2\left(\frac{1}{3}\right)\left[2\left(\frac{1}{3}\right)-1\right]\sigma^2}{2!\sqrt{2\pi}\left[1-\Phi\left(\frac{-1}{\sigma}\right)\right]}\left(-\frac{e^{-\frac{1}{2\sigma^2}}}{\sigma} + \frac{\sqrt{2\pi}}{2}\left[1 + P_r\left(\chi^2_{(1)} \le \frac{1}{\sigma^2}\right)\right]\right) + \frac{2\left[2\left(\frac{1}{3}\right)\right]\left[2\left(\frac{1}{3}\right)-1\right]\left[2\left(\frac{1}{3}\right)-2\right]\sigma^3}{3!\sqrt{2\pi}\left[1-\Phi\left(\frac{-1}{\sigma}\right)\right]}$$

$$\left(1 + \frac{1}{2\sigma^2}\right) e^{-\frac{1}{2\sigma^2}} \qquad (19)$$

$$= 1 + \frac{2\sigma}{3\sqrt{2\pi}\left[1-\Phi\left(\frac{-1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}} + \frac{\left(\frac{2}{3}\right)\left(\frac{-1}{3}\right)\sigma^2}{2\sqrt{2\pi}\left[\left[1-\Phi\left(\frac{-1}{\sigma}\right)\right]\right]}\left(-\frac{e^{-\frac{1}{2\sigma^2}}}{\sigma} + \frac{\sqrt{2\pi}}{2}\left[1 + P_r\left(\chi^2_{(1)} \le \frac{1}{\sigma^2}\right)\right]\right) + \frac{2\left(\frac{2}{3}\right)\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)\sigma^3}{6\sqrt{2\pi}\left[1-\Phi\left(\frac{-1}{\sigma}\right)\right]}$$

$$\left(1 + \frac{1}{2\sigma^2}\right) e^{-\frac{1}{2\sigma^2}} \qquad (20)$$

$$= 1 + \frac{2\sigma}{3\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}} - \frac{\sigma^2}{9\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(-\frac{e^{-\frac{1}{2\sigma^2}}}{\sigma} + \frac{\sqrt{2\pi}}{2}\left[1 + P_r\left(\chi^2_{(1)} \le \frac{1}{\sigma^2}\right)\right]\right) + \frac{8\sigma^3}{81\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}$$

$$\left(1 + \frac{1}{2\sigma^2}\right) e^{-\frac{1}{2\sigma^2}} \qquad (21)$$

$$= 1 + \frac{2\sigma}{3\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}} + \frac{\sigma}{9\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}} - \frac{\sigma^2}{18\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(1 + P_r\left(\chi^2_{(1)} \le \frac{1}{\sigma^2}\right)\right) + \frac{8\sigma^3}{81\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}} +$$

$$\frac{4\sigma}{81\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}} \qquad (22)$$

$$= 1 + \frac{(54+9+4)\sigma}{81\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}} - \frac{\sigma^2}{18\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(1 + P_r\left(\chi^2_{(1)} \le \frac{1}{\sigma^2}\right)\right) + \frac{8\sigma^3}{81\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}} \quad (23)$$

$$\therefore E(Y^2) = 1 + \frac{67\sigma}{81\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}} - \frac{\sigma^2}{18\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(1 + P_r\left(\chi^2_{(1)} \le \frac{1}{\sigma^2}\right)\right) + \frac{8\sigma^3}{81\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}} \quad (24)$$

Without any lost in generality, the subsequent terms in $E(Y^2)$ *and* $E(Y)$ with the factor $e^{-\frac{1}{2\sigma^2}}$ will decay fast to zero for values of $\sigma \ge 0$

$$E(Y^2) = 1 - \frac{\sigma^2}{18\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(1 + P_r\left(\chi^2_{(1)} \le \frac{1}{\sigma^2}\right)\right)$$

And

$$\therefore E(Y) = 1 - \frac{\sigma^2}{18\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(1 + P_r\left(\chi^2_{(1)} \le \frac{1}{\sigma^2}\right)\right)$$

$$\Rightarrow Var(Y) = E(Y^2) - [E(y)]^2$$

$$= \left[1 - \frac{\sigma^2}{18\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(1 + P_r\left(\chi^2_{(1)} \le \frac{1}{\sigma^2}\right)\right)\right] - \left[1 - \frac{\sigma^2}{18\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(1 + P_r\left(\chi^2_{(1)} \le \frac{1}{\sigma^2}\right)\right)\right]^2$$

$$\therefore Var(Y) = \left[\frac{\sigma^2}{18\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(1 + P_r\left(\chi^2_{(1)} \le \frac{1}{\sigma^2}\right)\right)\right] - \left[\frac{\sigma^2}{18\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(1 + P_r\left(\chi^2_{(1)} \le \frac{1}{\sigma^2}\right)\right)\right]^2 \quad (25)$$

II. Numerical Illustration

The graph forms of the probability density function of the cube root transformation for specified values of σ are given on figures 3.9 to 3.12. for want of space only graph of σ = 0.22 to 0.25 are shown here.

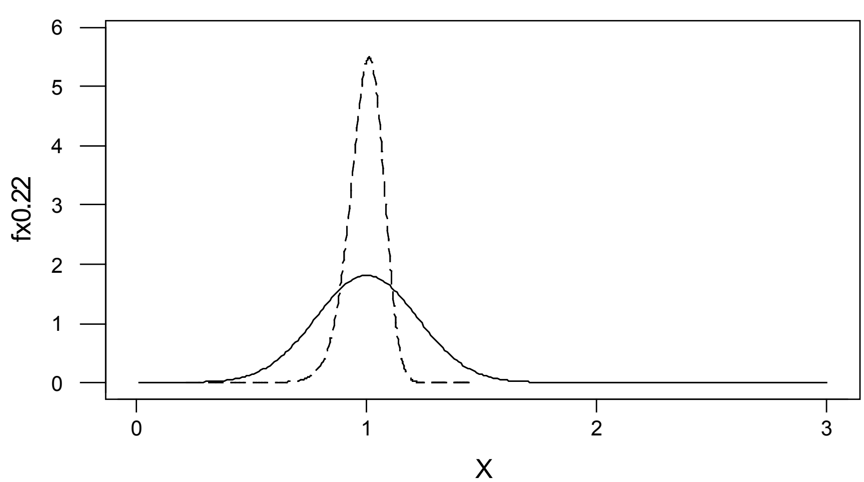


Figure 3.9 : Graph of $f(x)$ and $f(y)$ for $\sigma = 0.22$

Notes

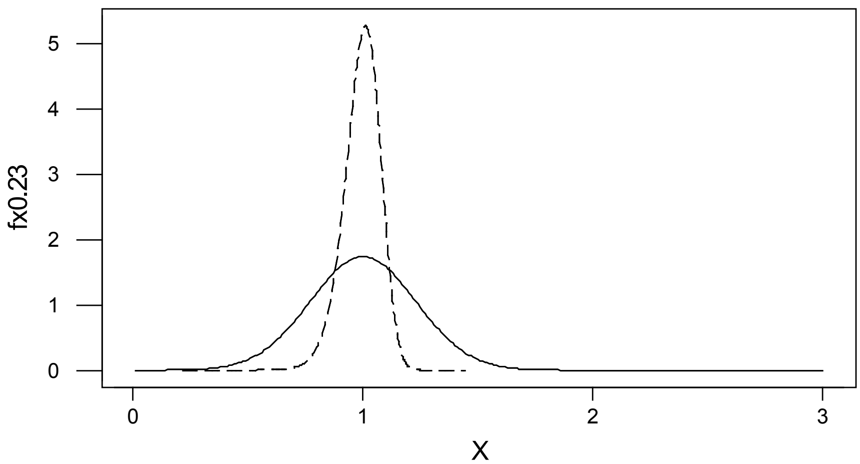


Figure 3.10 : Graph of $f(x)$ and $f(y)$ for $\sigma = 0.23$

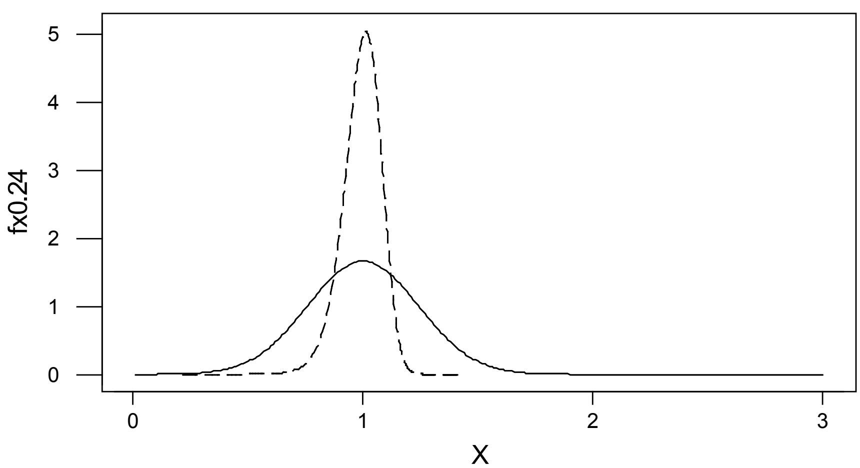


Figure 3.11 : Graph of $f(x)$ and $f(y)$ for $\sigma = 0.24$

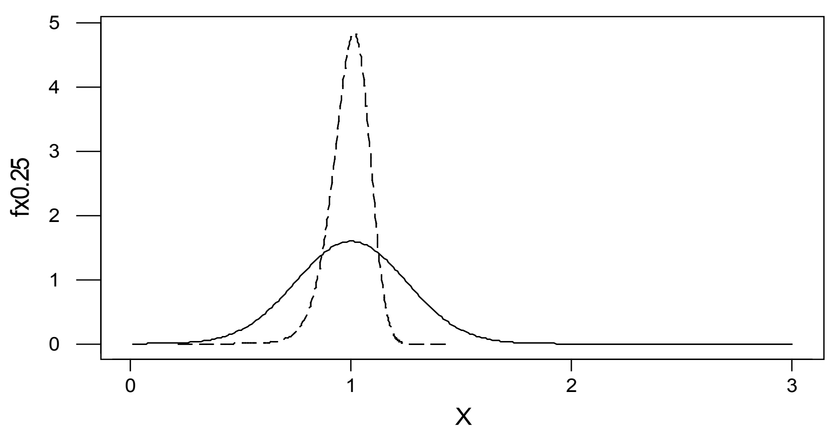


Figure 3.12 : Graph of $f(x)$ and $f(y)$ for $\sigma = 0.25$

The graphs of $f(x)$ and $f(y)$ look bell-shaped and there is none of the graphs where the two graphs coincide.

The numerical comparison of simulation between the cube root transformed distribution and the left-truncated $(1, \sigma^2)$ distribution for their means and variances for $0 < \sigma \leq 0.6$. the data is presented in table 1

Table 1 : Simulated values of E(X), E(Y), Var(X) and Var(Y)

S/no	sigma	E(X)	E(Y)	VAR(X)	VAR(Y)	var(x)/var(y)
1	0.001	1.00000	1.00000	0.000001	0.0000001	9.0000
2	0.002	1.00000	1.00000	0.000004	0.0000004	9.0000
3	0.003	1.00000	1.00000	0.000009	0.0000010	9.0000
4	0.004	1.00000	1.00000	0.000016	0.0000018	9.0000
5	0.005	1.00000	1.00000	0.000025	0.0000028	9.0000
6	0.006	1.00000	1.00000	0.000036	0.0000040	9.0000
7	0.007	1.00000	0.99999	0.000049	0.0000054	9.0000
8	0.008	1.00000	0.99999	0.000064	0.0000071	9.0001
9	0.009	1.00000	0.99999	0.000081	0.0000090	9.0001
10	0.010	1.00000	0.99999	0.000100	0.0000111	9.0001
11	0.011	1.00000	0.99999	0.000121	0.0000134	9.0001
12	0.012	1.00000	0.99998	0.000144	0.0000160	9.0001
13	0.013	1.00000	0.99998	0.000169	0.0000188	9.0002
14	0.014	1.00000	0.99998	0.000196	0.0000218	9.0002
15	0.015	1.00000	0.99997	0.000225	0.0000250	9.0002
16	0.016	1.00000	0.99997	0.000256	0.0000284	9.0003
17	0.017	1.00000	0.99997	0.000289	0.0000321	9.0003
18	0.018	1.00000	0.99996	0.000324	0.0000360	9.0003
19	0.019	1.00000	0.99996	0.000361	0.0000401	9.0004
20	0.020	1.00000	0.99996	0.000400	0.0000444	9.0004
21	0.021	1.00000	0.99995	0.000441	0.0000490	9.0004
22	0.022	1.00000	0.99995	0.000484	0.0000538	9.0005
23	0.023	1.00000	0.99994	0.000529	0.0000588	9.0005
24	0.024	1.00000	0.99994	0.000576	0.0000640	9.0006
25	0.025	1.00000	0.99993	0.000625	0.0000694	9.0006
26	0.026	1.00000	0.99992	0.000676	0.0000751	9.0007
27	0.027	1.00000	0.99992	0.000729	0.0000810	9.0007
28	0.028	1.00000	0.99991	0.000784	0.0000871	9.0008
29	0.029	1.00000	0.99991	0.000841	0.0000934	9.0008
30	0.030	1.00000	0.99990	0.000900	0.0001000	9.0009

Notes

Result of simulation of $\sigma = 0.001$ to 0.600, but for want of space only 0.001 to 0.030 are shown in table 1 above.

Depending on the level of accuracy needed, we have the following conditions for the means to be equal to 1.0 and variance of the left truncated $\boldsymbol{N(1, \sigma^2)}$ to be equal to 9 times the variance of the cube root transformed left truncated $\boldsymbol{N(1, \sigma^2)}$

Table 2 : Conditions for the means and variances to be equal

S/No	Decimal Places	E(x) = E(y)	Var(x) = 9*Var(y)
1	4	$0 \le \sigma \le 0.022$	$0 \le \sigma \le 0.007$
2	3	$0 \le \sigma \le 0.067$	$0 \le \sigma \le 0.021$
3	2	$0 \le \sigma \le 0.212$	$0 \le \sigma \le 0.070$
4	1	$0 \le \sigma \le 0.567$	$0 \le \sigma \le 0.221$

By adopting 1 decimal place, the interval where the left truncated $N(1, \sigma^2)$distribution and its cube root transformed counterpart have means equal to 1.0 and variance of the former equal to 9 times the variance of the later, that is given by $0 \le \sigma \le 0.221$.

a) Comparison of the means and standard deviations of the Simulated Series

In this section the summary o the simulated means, standard deviations, variances, ratios of standard deviations and variances of the untransformed and transformed distributions would be presented on table 1 to observe if there are departures from the earlier established results. The means, standard deviations, variances, ratios of standard deviations and variances of the of the simulated values for the untransformed and transformed distributions are presented in Table 2.

Also in this Section, we would test for the normality of the cube root transformed values using Kolmogorov-Smirnov Goodness-of-Fit test (One sample case).

Notes

b) The Test Statistic

The difference between the theoretical cumulative distribution function F(x) and the sample cumulative distribution $F^0(x)$ is measured by the statistic D, and it is the greatest vertical distance between F(x) and $F^0(x)$. For a two- sided test of the hypothesis, the null hypothesis and alternative is given by

$$H_o: F^0(x) = F(x) \ \forall x$$

$$H_1: F^0(x) \neq F(x) \text{ for at least one x}$$

The test statistic is D = Sup$\left|F^*(x) - F(x)\right|$

The null hypothesis is rejected at $\alpha = 0.05$ level of significance if the computed value of D exceeds the value read from a statistical table, and the sample size is n = 300. For ease of computations, Minitab software was used and the results summarized in table 2

Table 3 : Summary of Kolmogorov-Smirnov Test of Normality for the transformed series (For specified values of σ)

Σ	D^+	D^-	D	Approx p-value	α	Decision
0.001	0.043	0.033	0.043	0.072	0.05	Accept normality
0.005	0.031	0.037	0.037	0.15	,,	Accept normality
0.01	0.30	0.035	0.035	0.15	,,	Accept normality
0.05	0.023	0.033	0.033	0.15	,,	Accept normality
0.10	0.020	0.031	0.031	0.15	,,	Accept normality
0.15	0.023	0.033	0.033	0.15	,,	Accept normality
0.20	0.017	0.031	0.031	0.15	,,	Accept normality
0.21	0.018	0.031	0.031	0.15	,,	Accept normality
0.22	0.250	0.037	0.037	0.15	,,	Accept normality
0.23	0.028	0.054	0.054	<0.01	,,	Reject normality
0.24	0.022	0.060	0.060	<0.01	,,	Reject normality
0.25	0.027	0.049	0.049	<0.028	,,	Reject normality

From the summary of test of normality of the transformed series, we observe that the cube root transformed series is normal $0 < \sigma \leq 0.22$

III. Summary and Recommendations

The findings of the research work are summarised, conclusions were also drawn and suggestion for further research and recommendation using the n^{th} root transformation and to establish the cube root transformation.

a) Summary

The finding of the research are summarised as follows

(a) The probability density function of the cube root transformation derived from n^{th} root transformed error component of multiplicative time series model Dike et al (2016) and is given by

$$f(y) = \frac{\frac{1}{|3|} \cdot y^{\frac{1}{|3|}-1} e^{-\frac{1}{2}\left(\frac{y^{\frac{1}{|3|}}-1}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}\left[1 - \Phi\left(\frac{-1}{\sigma}\right)\right]}$$

The work went further by establishing the moments (mean and variance) of the cube root transformation .

(b) The graph forms of the probability density function were shown to be bell-shaped and symmetric for some values of σ.

(c) The mean and variance of the cube root transformation are in terms of the cumulative density function and the chi-square distribution with 1 degree of freedom (df). The mean given by

$$E(Y) = 1 + \frac{41\sigma}{16\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}} - \frac{\sigma^2}{18\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(1 + P_r\left(\chi^2_{(1)} \leq \frac{1}{\sigma^2}\right)\right) +$$

$$\frac{10\sigma^3}{81\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}}$$

also the variance is given by

(d) $Var(Y)=\left[\frac{\sigma^2}{18\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(1 + P_r\left(\chi^2_{(1)} \leq \frac{1}{\sigma^2}\right)\right)\right]-\left[\frac{\sigma^2}{18\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(1 + P_r\left(\chi^2_{(1)} \leq \frac{1}{\sigma^2}\right)\right)\right]^2$ Using simulated values it was found that the condition under which the mean and variance are equal as shown in table 4

Table 4 : Conditions for Successful transformation

S/No	Decimal Places	E(x) = E(y)	Var(x) = 9*Var(y)
1	4	$0 \leq \sigma \leq 0.022$	$0 \leq \sigma \leq 0.007$
2	3	$0 \leq \sigma \leq 0.067$	$0 \leq \sigma \leq 0.021$
3	2	$0 \leq \sigma \leq 0.212$	$0 \leq \sigma \leq 0.070$
4	1	$0 \leq \sigma \leq 0.567$	$0 \leq \sigma \leq 0.221$

(e) From the result in table 4 it was shown that the means of the error component of the original and the cube root transformed series is 1 and the variance of the original series is 9 times that of the transformed series for $0 < \sigma \leq 0.22$ depending on the decimal places desired.

(f) The test of normality using the Kolomogorov-Simirnov test shows in table 4.16 showed that the cube root transformed series is normal for $0 < \sigma \leq 0.22$

IV. Conclusion

In situation where the cube root is to be applied the following steps should be adopted for it to be successful and serve the need for which it was adopted. This would be achieved by ensuring that

Notes

(i) The model used to analyse the error component is multiplicative
(ii) It fits the transformation to be adopted
(iii) The untransformed and the transformed meet the conditions for normality. These measures will guarantee that data satisfy the assumption inherent in the statistical inference to be applied and ensure improved interpretation as expressed by Osborne(2002); who expressed that caution should be exercised on the choice of transformation to be used so that the fundamental structure of the series is not altered

Notes

References Références Referencias

1. Ajibade B. F, Nwosu C. R and Mbegdu J. I. (2015). The Distribution of the Inverse Square Root Transformed Error Component of The Multiplicative Time Series Model. *Journal of Modern Applied Statistical Methods.* Vol. 14. Issue 2. pp 171 – 199.
2. Akpanta A. C. and Iwueze I. S (2009). On Applying the Bartlett Transformation Methods to Time Series Data. Journal of Mathematical Sciences Vol. 20, No. 3. Pp 227 – 243.
3. Akpanta, A. C. and I. S. Iwueze (2009). On Applying the Bartlett Transformation Method to Time Series Data, *Journal of Mathematical Sciences*, Vol. 20, No. 3, pp. 227-243.
4. Bartlett, M. S. (1947). *The Use of Transformations,* Biometrics, 3, 39-52.
5. Box, G. E. P. and Cox, D. R. (1964). An Analysis of Transformations. J. Roy. Statistics Soc., B-26, 2 11-243, discussion 244-252.
6. Chartfield C. (1980). The Analysis of Time Series; An Introduction, Chapman & Hall, London.
7. Cox N. J. (2007). Transformations: an introduction, Durham, University of Durham. http//fmwww.be.edu
8. Dike A. O, Otuonye E. L., and Chikezie D. C (2016). The nth Power Transformation of the Error Component of the Multiplicative Time Series Model, British Journal of Mathematics and Computer Science. Science Domain International. ISSN: 2231 0851
9. Ibeh G. C and Nwosu C. R. (2013). Study on the Error Component of Multiplicative Time Series Model under Inverse Square Transformation. *American Journal of Mathematics and Statistics* (396): pp 362 – 374.
10. Iwueze Iheanyi S. (2007). Some Implications of Truncating the N (1,a2) Distribution to the left at Zero. Journal of Applied Sciences. 7(2) (2007) pp 189-195.
11. Iwueze I. S, Nwogu E.C., Ohakwe J and Ajaraogu J.C, (201 1). New Uses of Buys-Ballot Table. Applied Mathematics, 2011, 2. 633 — 645. DOT: 10.4236/am.2011.25084.
12. Iwueze, I. S., Nwogu, E. C. and Ajaraogu, J. C. (2011). Uses of the Buys-Ballot table in Time Series Analysis. Journal of Applied Mathematics, 2, 633 – 645.
13. Iwueze, I. S. and Nwogu, E. C. (2004). Buys-Ballot for Time Series Decomposition. *Global Journal of Mathematical Science, 3(2): 83 – 89.*
14. Nwosu C. R, Iwueze I. S. and Ohakwe J. (2010). Distribution of the Error Term of the Multiplicative Time Series Model Under Inverse Transformation. *Advances and Applications in Mathematical Sciences.* Volume 7, Issue 2, 2010, pp. 119 – 13.

15. Ohakwe, J., Dike, O. A. and Akpanta, A. C (2012). The Implication of Square Root Transformation on a Gamma Distribution Error Component of a Multiplicative Time Series Model, *Proceedings of African Regional Conference on Sustainable Development,* Vol. 6, No. 4, 2012, pp. 65 – 78.
16. Ohakwe J, Iwuoha O. and Otuonye E. L (2013). Condition for successful square Transformation in Time series modeling*. Journal of Applied Mathematics* Vol. 4. pp 680 – 687.
17. Okereke O. E. and Omekare C. O. (2010) on the properties of the truncated normal distribution and its square root transformation*. Journal of the Nigerian Statistical Association.* Vol. 22, 2010, pp37 – 43.
18. Osborne, J. (2002). Notes *on the use of data transformations*, J. Practice Practical Assessment, Research &Evaluation, 8(6).
19. Otuony E. L, Iwueze I.S. and Ohakwe J. (2011). The Effect of Square Root Transformation on the Error Component of the Multiplicative Time Series Model . *International Journal of Statistics and Systems.* Vol. 6, No. 4, 2011, pp. 461 - 476. ISSN: 0973 – 2675
20. Uche P. I. (2003). *Probability Theory and Applications*. Lagos Logman Publishers.
21. Wei, W.W. (1990). *Time Series Analysis*: *Multivariate Methods*. Addision-Wesley Publishing Company, Inc., Redwood City.

Notes

Global Journals Inc. (US) Guidelines Handbook 2016

www.GlobalJournals.org

Fellows

FELLOW OF ASSOCIATION OF RESEARCH SOCIETY IN SCIENCE (FARSS)

Global Journals Incorporate (USA) is accredited by Open Association of Research Society (OARS), U.S.A and in turn, awards "FARSS" title to individuals. The 'FARSS' title is accorded to a selected professional after the approval of the Editor-in-Chief/Editorial Board Members/Dean.

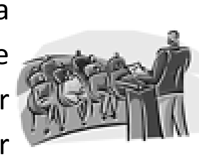
- The "FARSS" is a dignified title which is accorded to a person's name viz. Dr. John E. Hall, Ph.D., FARSS or William Walldroff, M.S., FARSS.

FARSS accrediting is an honor. It authenticates your research activities. After recognition as FARSB, you can add 'FARSS' title with your name as you use this recognition as additional suffix to your status. This will definitely enhance and add more value and repute to your name. You may use it on your professional Counseling Materials such as CV, Resume, and Visiting Card etc.

The following benefits can be availed by you only for next three years from the date of certification:

FARSS designated members are entitled to avail a 40% discount while publishing their research papers (of a single author) with Global Journals Incorporation (USA), if the same is accepted by Editorial Board/Peer Reviewers. If you are a main author or co-author in case of multiple authors, you will be entitled to avail discount of 10%.

Once FARSB title is accorded, the Fellow is authorized to organize a symposium/seminar/conference on behalf of Global Journal Incorporation (USA). The Fellow can also participate in conference/seminar/symposium organized by another institution as representative of Global Journal. In both the cases, it is mandatory for him to discuss with us and obtain our consent.

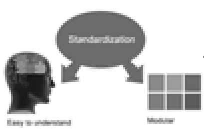


You may join as member of the Editorial Board of Global Journals Incorporation (USA) after successful completion of three years as Fellow and as Peer Reviewer. In addition, it is also desirable that you should organize seminar/symposium/conference at least once.

We shall provide you intimation regarding launching of e-version of journal of your stream time to time.This may be utilized in your library for the enrichment of knowledge of your students as well as it can also be helpful for the concerned faculty members.

The FARSS can go through standards of OARS. You can also play vital role if you have any suggestions so that proper amendment can take place to improve the same for the benefit of entire research community.

As FARSS, you will be given a renowned, secure and free professional email address with 100 GB of space e.g. johnhall@globaljournals.org. This will include Webmail, Spam Assassin, Email Forwarders,Auto-Responders, Email Delivery Route tracing, etc.



The FARSS will be eligible for a free application of standardization of their researches. Standardization of research will be subject to acceptability within stipulated norms as the next step after publishing in a journal. We shall depute a team of specialized research professionals who will render their services for elevating your researches to next higher level, which is worldwide open standardization.

The FARSS member can apply for grading and certification of standards of their educational and Institutional Degrees to Open Association of Research, Society U.S.A. Once you are designated as FARSS, you may send us a scanned copy of all of your credentials. OARS will verify, grade and certify them. This will be based on your academic records, quality of research papers published by you, and some more criteria. After certification of all your credentials by OARS, they will be published on your Fellow Profile link on website https://associationofresearch.org which will be helpful to upgrade the dignity.

The FARSS members can avail the benefits of free research podcasting in Global Research Radio with their research documents. After publishing the work, (including published elsewhere worldwide with proper authorization) you can upload your research paper with your recorded voice or you can utilize chargeable services of our professional RJs to record your paper in their voice on request.

The FARSS member also entitled to get the benefits of free research podcasting of their research documents through video clips. We can also streamline your conference videos and display your slides/ online slides and online research video clips at reasonable charges, on request.

The FARSS is eligible to earn from sales proceeds of his/her researches/reference/review Books or literature, while publishing with Global Journals. The FARSS can decide whether he/she would like to publish his/her research in a closed manner. In this case, whenever readers purchase that individual research paper for reading, maximum 60% of its profit earned as royalty by Global Journals, will be credited to his/her bank account. The entire entitled amount will be credited to his/her bank account exceeding limit of minimum fixed balance. There is no minimum time limit for collection. The FARSS member can decide its price and we can help in making the right decision.

The FARSS member is eligible to join as a paid peer reviewer at Global Journals Incorporation (USA) and can get remuneration of 15% of author fees, taken from the author of a respective paper. After reviewing 5 or more papers you can request to transfer the amount to your bank account.

MEMBER OF ASSOCIATION OF RESEARCH SOCIETY IN SCIENCE (MARSS)

The ' MARSS ' title is accorded to a selected professional after the approval of the Editor-in-Chief / Editorial Board Members/Dean.

The “MARSS” is a dignified ornament which is accorded to a person’s name viz. Dr. John E. Hall, Ph.D., MARSS or William Walldroff, M.S., MARSS.

MARSS accrediting is an honor. It authenticates your research activities. After becoming MARSS, you can add 'MARSS' title with your name as you use this recognition as additional suffix to your status. This will definitely enhance and add more value and repute to your name. You may use it on your professional Counseling Materials such as CV, Resume, Visiting Card and Name Plate etc.

The following benefitscan be availed by you only for next three years from the date of certification.

MARSS designated members are entitled to avail a 25% discount while publishing their research papers (of a single author) in Global Journals Inc., if the same is accepted by our Editorial Board and Peer Reviewers. If you are a main author or co-author of a group of authors, you will get discount of 10%.

As MARSS, you will be given a renowned, secure and free professional email address with 30 GB of space e.g. johnhall@globaljournals.org. This will include Webmail, Spam Assassin, Email Forwarders,Auto-Responders, Email Delivery Route tracing, etc.

We shall provide you intimation regarding launching of e-version of journal of your stream time to time.This may be utilized in your library for the enrichment of knowledge of your students as well as it can also be helpful for the concerned faculty members.

The MARSS member can apply for approval, grading and certification of standards of their educational and Institutional Degrees to Open Association of Research, Society U.S.A.

Once you are designated as MARSS, you may send us a scanned copy of all of your credentials. OARS will verify, grade and certify them. This will be based on your academic records, quality of research papers published by you, and some more criteria.

It is mandatory to read all terms and conditions carefully.

Auxiliary Memberships

Institutional Fellow of Global Journals Incorporation (USA)-OARS (USA)

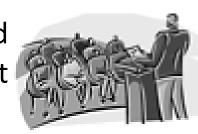
Global Journals Incorporation (USA) is accredited by Open Association of Research Society, U.S.A (OARS) and in turn, affiliates research institutions as “Institutional Fellow of Open Association of Research Society” (IFOARS).

The “FARSC” is a dignified title which is accorded to a person’s name viz. Dr. John E. Hall, Ph.D., FARSC or William Walldroff, M.S., FARSC.

The IFOARS institution is entitled to form a Board comprised of one Chairperson and three to five board members preferably from different streams. The Board will be recognized as “Institutional Board of Open Association of Research Society”-(IBOARS).

The Institute will be entitled to following benefits:

The IBOARS can initially review research papers of their institute and recommend them to publish with respective journal of Global Journals. It can also review the papers of other institutions after obtaining our consent. The second review will be done by peer reviewer of Global Journals Incorporation (USA) The Board is at liberty to appoint a peer reviewer with the approval of chairperson after consulting us.
The author fees of such paper may be waived off up to 40%.



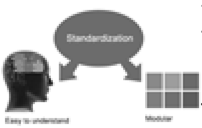
The Global Journals Incorporation (USA) at its discretion can also refer double blind peer reviewed paper at their end to the board for the verification and to get recommendation for final stage of acceptance of publication.

The IBOARS can organize symposium/seminar/conference in their country on behalf of Global Journals Incorporation (USA)-OARS (USA). The terms and conditions can be discussed separately.

The Board can also play vital role by exploring and giving valuable suggestions regarding the Standards of “Open Association of Research Society, U.S.A (OARS)” so that proper amendment can take place for the benefit of entire research community. We shall provide details of particular standard only on receipt of request from the Board.

The board members can also join us as Individual Fellow with 40% discount on total fees applicable to Individual Fellow. They will be entitled to avail all the benefits as declared. Please visit Individual Fellow-sub menu of GlobalJournals.org to have more relevant details.

We shall provide you intimation regarding launching of e-version of journal of your stream time to time. This may be utilized in your library for the enrichment of knowledge of your students as well as it can also be helpful for the concerned faculty members.



After nomination of your institution as "Institutional Fellow" and constantly functioning successfully for one year, we can consider giving recognition to your institute to function as Regional/Zonal office on our behalf.
The board can also take up the additional allied activities for betterment after our consultation.

The following entitlements are applicable to individual Fellows:

Open Association of Research Society, U.S.A (OARS) By-laws states that an individual Fellow may use the designations as applicable, or the corresponding initials. The Credentials of individual Fellow and Associate designations signify that the individual has gained knowledge of the fundamental concepts. One is magnanimous and proficient in an expertise course covering the professional code of conduct, and follows recognized standards of practice.

Open Association of Research Society (US)/ Global Journals Incorporation (USA), as described in Corporate Statements, are educational, research publishing and professional membership organizations. Achieving our individual Fellow or Associate status is based mainly on meeting stated educational research requirements.

Disbursement of 40% Royalty earned through Global Journals : Researcher = 50%, Peer Reviewer = 37.50%, Institution = 12.50% E.g. Out of 40%, the 20% benefit should be passed on to researcher, 15 % benefit towards remuneration should be given to a reviewer and remaining 5% is to be retained by the institution.

We shall provide print version of 12 issues of any three journals [as per your requirement] out of our 38 journals worth $ 2376 USD.

Other:

The individual Fellow and Associate designations accredited by Open Association of Research Society (US) credentials signify guarantees following achievements:

- The professional accredited with Fellow honor, is entitled to various benefits viz. name, fame, honor, regular flow of income, secured bright future, social status etc.

- In addition to above, if one is single author, then entitled to 40% discount on publishing research paper and can get 10%discount if one is co-author or main author among group of authors.
- The Fellow can organize symposium/seminar/conference on behalf of Global Journals Incorporation (USA) and he/she can also attend the same organized by other institutes on behalf of Global Journals.
- The Fellow can become member of Editorial Board Member after completing 3yrs.
- The Fellow can earn 60% of sales proceeds from the sale of reference/review books/literature/publishing of research paper.
- Fellow can also join as paid peer reviewer and earn 15% remuneration of author charges and can also get an opportunity to join as member of the Editorial Board of Global Journals Incorporation (USA)
- • This individual has learned the basic methods of applying those concepts and techniques to common challenging situations. This individual has further demonstrated an in–depth understanding of the application of suitable techniques to a particular area of research practice.

Note :

"

- In future, if the board feels the necessity to change any board member, the same can be done with the consent of the chairperson along with anyone board member without our approval.
- In case, the chairperson needs to be replaced then consent of 2/3rd board members are required and they are also required to jointly pass the resolution copy of which should be sent to us. In such case, it will be compulsory to obtain our approval before replacement.
- In case of "Difference of Opinion [if any]" among the Board members, our decision will be final and binding to everyone.

"

Process of submission of Research Paper

The Area or field of specialization may or may not be of any category as mentioned in 'Scope of Journal' menu of the GlobalJournals.org website. There are 37 Research Journal categorized with Six parental Journals GJCST, GJMR, GJRE, GJMBR, GJSFR, GJHSS. For Authors should prefer the mentioned categories. There are three widely used systems UDC, DDC and LCC. The details are available as 'Knowledge Abstract' at Home page. The major advantage of this coding is that, the research work will be exposed to and shared with all over the world as we are being abstracted and indexed worldwide.

The paper should be in proper format. The format can be downloaded from first page of 'Author Guideline' Menu. The Author is expected to follow the general rules as mentioned in this menu. The paper should be written in MS-Word Format (*.DOC,*.DOCX).

The Author can submit the paper either online or offline. The authors should prefer online submission.Online Submission: There are three ways to submit your paper:

(A) (I) First, register yourself using top right corner of Home page then Login. If you are already registered, then login using your username and password.

(II) Choose corresponding Journal.

(III) Click 'Submit Manuscript'. Fill required information and Upload the paper.

(B) If you are using Internet Explorer, then Direct Submission through Homepage is also available.

(C) If these two are not conveninet , and then email the paper directly to dean@globaljournals.org.

Offline Submission: Author can send the typed form of paper by Post. However, online submission should be preferred.

MANUSCRIPT STYLE INSTRUCTION **(Must be strictly followed)**

Page Size: 8.27" X 11'"

- Left Margin: 0.65
- Right Margin: 0.65
- Top Margin: 0.75
- Bottom Margin: 0.75
- Font type of all text should be Swis 721 Lt BT.
- Paper Title should be of Font Size 24 with one Column section.
- Author Name in Font Size of 11 with one column as of Title.
- Abstract Font size of 9 Bold, “Abstract” word in Italic Bold.
- Main Text: Font size 10 with justified two columns section
- Two Column with Equal Column with of 3.38 and Gaping of .2
- First Character must be three lines Drop capped.
- Paragraph before Spacing of 1 pt and After of 0 pt.
- Line Spacing of 1 pt
- Large Images must be in One Column
- Numbering of First Main Headings (Heading 1) must be in Roman Letters, Capital Letter, and Font Size of 10.
- Numbering of Second Main Headings (Heading 2) must be in Alphabets, Italic, and Font Size of 10.

You can use your own standard format also.
Author Guidelines:

1. General,

2. Ethical Guidelines,

3. Submission of Manuscripts,

4. Manuscript’s Category,

5. Structure and Format of Manuscript,

6. After Acceptance.

1. GENERAL

Before submitting your research paper, one is advised to go through the details as mentioned in following heads. It will be beneficial, while peer reviewer justify your paper for publication.

Scope

The Global Journals Inc. (US) welcome the submission of original paper, review paper, survey article relevant to the all the streams of Philosophy and knowledge. The Global Journals Inc. (US) is parental platform for Global Journal of Computer Science and Technology, Researches in Engineering, Medical Research, Science Frontier Research, Human Social Science, Management, and Business organization. The choice of specific field can be done otherwise as following in Abstracting and Indexing Page on this Website. As the all Global

Journals Inc. (US) are being abstracted and indexed (in process) by most of the reputed organizations. Topics of only narrow interest will not be accepted unless they have wider potential or consequences.

2. ETHICAL GUIDELINES

Authors should follow the ethical guidelines as mentioned below for publication of research paper and research activities.

Papers are accepted on strict understanding that the material in whole or in part has not been, nor is being, considered for publication elsewhere. If the paper once accepted by Global Journals Inc. (US) and Editorial Board, will become the copyright of the Global Journals Inc. (US).

Authorship: The authors and coauthors should have active contribution to conception design, analysis and interpretation of findings. They should critically review the contents and drafting of the paper. All should approve the final version of the paper before submission

The Global Journals Inc. (US) follows the definition of authorship set up by the Global Academy of Research and Development. According to the Global Academy of R&D authorship, criteria must be based on:

1) Substantial contributions to conception and acquisition of data, analysis and interpretation of the findings.

2) Drafting the paper and revising it critically regarding important academic content.

3) Final approval of the version of the paper to be published.

All authors should have been credited according to their appropriate contribution in research activity and preparing paper. Contributors who do not match the criteria as authors may be mentioned under Acknowledgement.

Acknowledgements: Contributors to the research other than authors credited should be mentioned under acknowledgement. The specifications of the source of funding for the research if appropriate can be included. Suppliers of resources may be mentioned along with address.

Appeal of Decision: The Editorial Board's decision on publication of the paper is final and cannot be appealed elsewhere.

Permissions: It is the author's responsibility to have prior permission if all or parts of earlier published illustrations are used in this paper.

Please mention proper reference and appropriate acknowledgements wherever expected.

If all or parts of previously published illustrations are used, permission must be taken from the copyright holder concerned. It is the author's responsibility to take these in writing.

Approval for reproduction/modification of any information (including figures and tables) published elsewhere must be obtained by the authors/copyright holders before submission of the manuscript. Contributors (Authors) are responsible for any copyright fee involved.

3. SUBMISSION OF MANUSCRIPTS

Manuscripts should be uploaded via this online submission page. The online submission is most efficient method for submission of papers, as it enables rapid distribution of manuscripts and consequently speeds up the review procedure. It also enables authors to know the status of their own manuscripts by emailing us. Complete instructions for submitting a paper is available below.

Manuscript submission is a systematic procedure and little preparation is required beyond having all parts of your manuscript in a given format and a computer with an Internet connection and a Web browser. Full help and instructions are provided on-screen. As an author, you will be prompted for login and manuscript details as Field of Paper and then to upload your manuscript file(s) according to the instructions.

To avoid postal delays, all transaction is preferred by e-mail. A finished manuscript submission is confirmed by e-mail immediately and your paper enters the editorial process with no postal delays. When a conclusion is made about the publication of your paper by our Editorial Board, revisions can be submitted online with the same procedure, with an occasion to view and respond to all comments.

Complete support for both authors and co-author is provided.

4. MANUSCRIPT'S CATEGORY

Based on potential and nature, the manuscript can be categorized under the following heads:

Original research paper: Such papers are reports of high-level significant original research work.

Review papers: These are concise, significant but helpful and decisive topics for young researchers.

Research articles: These are handled with small investigation and applications

Research letters: The letters are small and concise comments on previously published matters.

5.STRUCTURE AND FORMAT OF MANUSCRIPT

The recommended size of original research paper is less than seven thousand words, review papers fewer than seven thousands words also.Preparation of research paper or how to write research paper, are major hurdle, while writing manuscript. The research articles and research letters should be fewer than three thousand words, the structure original research paper; sometime review paper should be as follows:

Papers: These are reports of significant research (typically less than 7000 words equivalent, including tables, figures, references), and comprise:

(a)Title should be relevant and commensurate with the theme of the paper.

(b) A brief Summary, "Abstract" (less than 150 words) containing the major results and conclusions.

(c) Up to ten keywords, that precisely identifies the paper's subject, purpose, and focus.

(d) An Introduction, giving necessary background excluding subheadings; objectives must be clearly declared.

(e) Resources and techniques with sufficient complete experimental details (wherever possible by reference) to permit repetition; sources of information must be given and numerical methods must be specified by reference, unless non-standard.

(f) Results should be presented concisely, by well-designed tables and/or figures; the same data may not be used in both; suitable statistical data should be given. All data must be obtained with attention to numerical detail in the planning stage. As reproduced design has been recognized to be important to experiments for a considerable time, the Editor has decided that any paper that appears not to have adequate numerical treatments of the data will be returned un-refereed;

(g) Discussion should cover the implications and consequences, not just recapitulating the results; conclusions should be summarizing.

(h) Brief Acknowledgements.

(i) References in the proper form.

Authors should very cautiously consider the preparation of papers to ensure that they communicate efficiently. Papers are much more likely to be accepted, if they are cautiously designed and laid out, contain few or no errors, are summarizing, and be conventional to the approach and instructions. They will in addition, be published with much less delays than those that require much technical and editorial correction.

The Editorial Board reserves the right to make literary corrections and to make suggestions to improve briefness.

It is vital, that authors take care in submitting a manuscript that is written in simple language and adheres to published guidelines.

Format

Language: The language of publication is UK English. Authors, for whom English is a second language, must have their manuscript efficiently edited by an English-speaking person before submission to make sure that, the English is of high excellence. It is preferable, that manuscripts should be professionally edited.

Standard Usage, Abbreviations, and Units: Spelling and hyphenation should be conventional to The Concise Oxford English Dictionary. Statistics and measurements should at all times be given in figures, e.g. 16 min, except for when the number begins a sentence. When the number does not refer to a unit of measurement it should be spelt in full unless, it is 160 or greater.

Abbreviations supposed to be used carefully. The abbreviated name or expression is supposed to be cited in full at first usage, followed by the conventional abbreviation in parentheses.

Metric SI units are supposed to generally be used excluding where they conflict with current practice or are confusing. For illustration, 1.4 l rather than 1.4 × 10-3 m3, or 4 mm somewhat than 4 × 10-3 m. Chemical formula and solutions must identify the form used, e.g. anhydrous or hydrated, and the concentration must be in clearly defined units. Common species names should be followed by underlines at the first mention. For following use the generic name should be constricted to a single letter, if it is clear.

Structure

All manuscripts submitted to Global Journals Inc. (US), ought to include:

Title: The title page must carry an instructive title that reflects the content, a running title (less than 45 characters together with spaces), names of the authors and co-authors, and the place(s) wherever the work was carried out. The full postal address in addition with the e-mail address of related author must be given. Up to eleven keywords or very brief phrases have to be given to help data retrieval, mining and indexing.

Abstract, used in Original Papers and Reviews:

Optimizing Abstract for Search Engines

Many researchers searching for information online will use search engines such as Google, Yahoo or similar. By optimizing your paper for search engines, you will amplify the chance of someone finding it. This in turn will make it more likely to be viewed and/or cited in a further work. Global Journals Inc. (US) have compiled these guidelines to facilitate you to maximize the web-friendliness of the most public part of your paper.

Key Words

A major linchpin in research work for the writing research paper is the keyword search, which one will employ to find both library and Internet resources.

One must be persistent and creative in using keywords. An effective keyword search requires a strategy and planning a list of possible keywords and phrases to try.

Search engines for most searches, use Boolean searching, which is somewhat different from Internet searches. The Boolean search uses "operators," words (and, or, not, and near) that enable you to expand or narrow your affords. Tips for research paper while preparing research paper are very helpful guideline of research paper.

Choice of key words is first tool of tips to write research paper. Research paper writing is an art.A few tips for deciding as strategically as possible about keyword search:

- One should start brainstorming lists of possible keywords before even begin searching. Think about the most important concepts related to research work. Ask, "What words would a source have to include to be truly valuable in research paper?" Then consider synonyms for the important words.
- It may take the discovery of only one relevant paper to let steer in the right keyword direction because in most databases, the keywords under which a research paper is abstracted are listed with the paper.
- One should avoid outdated words.

Keywords are the key that opens a door to research work sources. Keyword searching is an art in which researcher's skills are bound to improve with experience and time.

Numerical Methods: Numerical methods used should be clear and, where appropriate, supported by references.

Acknowledgements: Please make these as concise as possible.

References

References follow the Harvard scheme of referencing. References in the text should cite the authors' names followed by the time of their publication, unless there are three or more authors when simply the first author's name is quoted followed by et al. unpublished work has to only be cited where necessary, and only in the text. Copies of references in press in other journals have to be supplied with submitted typescripts. It is necessary that all citations and references be carefully checked before submission, as mistakes or omissions will cause delays.

References to information on the World Wide Web can be given, but only if the information is available without charge to readers on an official site. Wikipedia and Similar websites are not allowed where anyone can change the information. Authors will be asked to make available electronic copies of the cited information for inclusion on the Global Journals Inc. (US) homepage at the judgment of the Editorial Board.

The Editorial Board and Global Journals Inc. (US) recommend that, citation of online-published papers and other material should be done via a DOI (digital object identifier). If an author cites anything, which does not have a DOI, they run the risk of the cited material not being noticeable.

The Editorial Board and Global Journals Inc. (US) recommend the use of a tool such as Reference Manager for reference management and formatting.

Tables, Figures and Figure Legends

Tables: Tables should be few in number, cautiously designed, uncrowned, and include only essential data. Each must have an Arabic number, e.g. Table 4, a self-explanatory caption and be on a separate sheet. Vertical lines should not be used.

Figures: Figures are supposed to be submitted as separate files. Always take in a citation in the text for each figure using Arabic numbers, e.g. Fig. 4. Artwork must be submitted online in electronic form by e-mailing them.

Preparation of Electronic Figures for Publication

Even though low quality images are sufficient for review purposes, print publication requires high quality images to prevent the final product being blurred or fuzzy. Submit (or e-mail) EPS (line art) or TIFF (halftone/photographs) files only. MS PowerPoint and Word Graphics are unsuitable for printed pictures. Do not use pixel-oriented software. Scans (TIFF only) should have a resolution of at least 350 dpi (halftone) or 700 to 1100 dpi (line drawings) in relation to the imitation size. Please give the data for figures in black and white or submit a Color Work Agreement Form. EPS files must be saved with fonts embedded (and with a TIFF preview, if possible).

For scanned images, the scanning resolution (at final image size) ought to be as follows to ensure good reproduction: line art: >650 dpi; halftones (including gel photographs) : >350 dpi; figures containing both halftone and line images: >650 dpi.

Color Charges: It is the rule of the Global Journals Inc. (US) for authors to pay the full cost for the reproduction of their color artwork. Hence, please note that, if there is color artwork in your manuscript when it is accepted for publication, we would require you to complete and return a color work agreement form before your paper can be published.

Figure Legends: Self-explanatory legends of all figures should be incorporated separately under the heading 'Legends to Figures'. In the full-text online edition of the journal, figure legends may possibly be truncated in abbreviated links to the full screen version. Therefore, the first 100 characters of any legend should notify the reader, about the key aspects of the figure.

6. AFTER ACCEPTANCE

Upon approval of a paper for publication, the manuscript will be forwarded to the dean, who is responsible for the publication of the Global Journals Inc. (US).

6.1 Proof Corrections

The corresponding author will receive an e-mail alert containing a link to a website or will be attached. A working e-mail address must therefore be provided for the related author.

Acrobat Reader will be required in order to read this file. This software can be downloaded

(Free of charge) from the following website:

www.adobe.com/products/acrobat/readstep2.html. This will facilitate the file to be opened, read on screen, and printed out in order for any corrections to be added. Further instructions will be sent with the proof.

Proofs must be returned to the dean at dean@globaljournals.org within three days of receipt.

As changes to proofs are costly, we inquire that you only correct typesetting errors. All illustrations are retained by the publisher. Please note that the authors are responsible for all statements made in their work, including changes made by the copy editor.

6.2 Early View of Global Journals Inc. (US) (Publication Prior to Print)

The Global Journals Inc. (US) are enclosed by our publishing's Early View service. Early View articles are complete full-text articles sent in advance of their publication. Early View articles are absolute and final. They have been completely reviewed, revised and edited for publication, and the authors' final corrections have been incorporated. Because they are in final form, no changes can be made after sending them. The nature of Early View articles means that they do not yet have volume, issue or page numbers, so Early View articles cannot be cited in the conventional way.

6.3 Author Services

Online production tracking is available for your article through Author Services. Author Services enables authors to track their article - once it has been accepted - through the production process to publication online and in print. Authors can check the status of their articles online and choose to receive automated e-mails at key stages of production. The authors will receive an e-mail with a unique link that enables them to register and have their article automatically added to the system. Please ensure that a complete e-mail address is provided when submitting the manuscript.

6.4 Author Material Archive Policy

Please note that if not specifically requested, publisher will dispose off hardcopy & electronic information submitted, after the two months of publication. If you require the return of any information submitted, please inform the Editorial Board or dean as soon as possible.

6.5 Offprint and Extra Copies

A PDF offprint of the online-published article will be provided free of charge to the related author, and may be distributed according to the Publisher's terms and conditions. Additional paper offprint may be ordered by emailing us at: editor@globaljournals.org .

Before start writing a good quality Computer Science Research Paper, let us first understand what is Computer Science Research Paper? So, Computer Science Research Paper is the paper which is written by professionals or scientists who are associated to Computer Science and Information Technology, or doing research study in these areas. If you are novel to this field then you can consult about this field from your supervisor or guide.

TECHNIQUES FOR WRITING A GOOD QUALITY RESEARCH PAPER:

1. Choosing the topic: In most cases, the topic is searched by the interest of author but it can be also suggested by the guides. You can have several topics and then you can judge that in which topic or subject you are finding yourself most comfortable. This can be done by asking several questions to yourself, like Will I be able to carry our search in this area? Will I find all necessary recourses to accomplish the search? Will I be able to find all information in this field area? If the answer of these types of questions will be "Yes" then you can choose that topic. In most of the cases, you may have to conduct the surveys and have to visit several places because this field is related to Computer Science and Information Technology. Also, you may have to do a lot of work to find all rise and falls regarding the various data of that subject. Sometimes, detailed information plays a vital role, instead of short information.

2. Evaluators are human: First thing to remember that evaluators are also human being. They are not only meant for rejecting a paper. They are here to evaluate your paper. So, present your Best.

3. Think Like Evaluators: If you are in a confusion or getting demotivated that your paper will be accepted by evaluators or not, then think and try to evaluate your paper like an Evaluator. Try to understand that what an evaluator wants in your research paper and automatically you will have your answer.

4. Make blueprints of paper: The outline is the plan or framework that will help you to arrange your thoughts. It will make your paper logical. But remember that all points of your outline must be related to the topic you have chosen.

5. Ask your Guides: If you are having any difficulty in your research, then do not hesitate to share your difficulty to your guide (if you have any). They will surely help you out and resolve your doubts. If you can't clarify what exactly you require for your work then ask the supervisor to help you with the alternative. He might also provide you the list of essential readings.

6. Use of computer is recommended: As you are doing research in the field of Computer Science, then this point is quite obvious.

7. Use right software: Always use good quality software packages. If you are not capable to judge good software then you can lose quality of your paper unknowingly. There are various software programs available to help you, which you can get through Internet.

8. Use the Internet for help: An excellent start for your paper can be by using the Google. It is an excellent search engine, where you can have your doubts resolved. You may also read some answers for the frequent question how to write my research paper or find model research paper. From the internet library you can download books. If you have all required books make important reading selecting and analyzing the specified information. Then put together research paper sketch out.

9. Use and get big pictures: Always use encyclopedias, Wikipedia to get pictures so that you can go into the depth.

10. Bookmarks are useful: When you read any book or magazine, you generally use bookmarks, right! It is a good habit, which helps to not to lose your continuity. You should always use bookmarks while searching on Internet also, which will make your search easier.

11. Revise what you wrote: When you write anything, always read it, summarize it and then finalize it.

12. Make all efforts: Make all efforts to mention what you are going to write in your paper. That means always have a good start. Try to mention everything in introduction, that what is the need of a particular research paper. Polish your work by good skill of writing and always give an evaluator, what he wants.

13. Have backups: When you are going to do any important thing like making research paper, you should always have backup copies of it either in your computer or in paper. This will help you to not to lose any of your important.

14. Produce good diagrams of your own: Always try to include good charts or diagrams in your paper to improve quality. Using several and unnecessary diagrams will degrade the quality of your paper by creating "hotchpotch." So always, try to make and include those diagrams, which are made by your own to improve readability and understandability of your paper.

15. Use of direct quotes: When you do research relevant to literature, history or current affairs then use of quotes become essential but if study is relevant to science then use of quotes is not preferable.

16. Use proper verb tense: Use proper verb tenses in your paper. Use past tense, to present those events that happened. Use present tense to indicate events that are going on. Use future tense to indicate future happening events. Use of improper and wrong tenses will confuse the evaluator. Avoid the sentences that are incomplete.

17. Never use online paper: If you are getting any paper on Internet, then never use it as your research paper because it might be possible that evaluator has already seen it or maybe it is outdated version.

18. Pick a good study spot: To do your research studies always try to pick a spot, which is quiet. Every spot is not for studies. Spot that suits you choose it and proceed further.

19. Know what you know: Always try to know, what you know by making objectives. Else, you will be confused and cannot achieve your target.

20. Use good quality grammar: Always use a good quality grammar and use words that will throw positive impact on evaluator. Use of good quality grammar does not mean to use tough words, that for each word the evaluator has to go through dictionary. Do not start sentence with a conjunction. Do not fragment sentences. Eliminate one-word sentences. Ignore passive voice. Do not ever use a big word when a diminutive one would suffice. Verbs have to be in agreement with their subjects. Prepositions are not expressions to finish sentences with. It is incorrect to ever divide an infinitive. Avoid clichés like the disease. Also, always shun irritating alliteration. Use language that is simple and straight forward. put together a neat summary.

21. Arrangement of information: Each section of the main body should start with an opening sentence and there should be a changeover at the end of the section. Give only valid and powerful arguments to your topic. You may also maintain your arguments with records.

22. Never start in last minute: Always start at right time and give enough time to research work. Leaving everything to the last minute will degrade your paper and spoil your work.

23. Multitasking in research is not good: Doing several things at the same time proves bad habit in case of research activity. Research is an area, where everything has a particular time slot. Divide your research work in parts and do particular part in particular time slot.

24. Never copy others' work: Never copy others' work and give it your name because if evaluator has seen it anywhere you will be in trouble.

25. Take proper rest and food: No matter how many hours you spend for your research activity, if you are not taking care of your health then all your efforts will be in vain. For a quality research, study is must, and this can be done by taking proper rest and food.

26. Go for seminars: Attend seminars if the topic is relevant to your research area. Utilize all your resources.

27. Refresh your mind after intervals: Try to give rest to your mind by listening to soft music or by sleeping in intervals. This will also improve your memory.

28. Make colleagues: Always try to make colleagues. No matter how sharper or intelligent you are, if you make colleagues you can have several ideas, which will be helpful for your research.

29. Think technically: Always think technically. If anything happens, then search its reasons, its benefits, and demerits.

30. Think and then print: When you will go to print your paper, notice that tables are not be split, headings are not detached from their descriptions, and page sequence is maintained.

31. Adding unnecessary information: Do not add unnecessary information, like, I have used MS Excel to draw graph. Do not add irrelevant and inappropriate material. These all will create superfluous. Foreign terminology and phrases are not apropos. One should NEVER take a broad view. Analogy in script is like feathers on a snake. Not at all use a large word when a very small one would be sufficient. Use words properly, regardless of how others use them. Remove quotations. Puns are for kids, not grunt readers. Amplification is a billion times of inferior quality than sarcasm.

32. Never oversimplify everything: To add material in your research paper, never go for oversimplification. This will definitely irritate the evaluator. Be more or less specific. Also too, by no means, ever use rhythmic redundancies. Contractions aren't essential and shouldn't be there used. Comparisons are as terrible as clichés. Give up ampersands and abbreviations, and so on. Remove commas, that are, not necessary. Parenthetical words however should be together with this in commas. Understatement is all the time the complete best way to put onward earth-shaking thoughts. Give a detailed literary review.

33. Report concluded results: Use concluded results. From raw data, filter the results and then conclude your studies based on measurements and observations taken. Significant figures and appropriate number of decimal places should be used. Parenthetical remarks are prohibitive. Proofread carefully at final stage. In the end give outline to your arguments. Spot out perspectives of further study of this subject. Justify your conclusion by at the bottom of them with sufficient justifications and examples.

34. After conclusion: Once you have concluded your research, the next most important step is to present your findings. Presentation is extremely important as it is the definite medium though which your research is going to be in print to the rest of the crowd. Care should be taken to categorize your thoughts well and present them in a logical and neat manner. A good quality research paper format is essential because it serves to highlight your research paper and bring to light all necessary aspects in your research.

Informal Guidelines of Research Paper Writing

Key points to remember:

- Submit all work in its final form.
- Write your paper in the form, which is presented in the guidelines using the template.
- Please note the criterion for grading the final paper by peer-reviewers.

Final Points:

A purpose of organizing a research paper is to let people to interpret your effort selectively. The journal requires the following sections, submitted in the order listed, each section to start on a new page.

The introduction will be compiled from reference matter and will reflect the design processes or outline of basis that direct you to make study. As you will carry out the process of study, the method and process section will be constructed as like that. The result segment will show related statistics in nearly sequential order and will direct the reviewers next to the similar intellectual paths throughout the data that you took to carry out your study. The discussion section will provide understanding of the data and projections as to the implication of the results. The use of good quality references all through the paper will give the effort trustworthiness by representing an alertness of prior workings.

Writing a research paper is not an easy job no matter how trouble-free the actual research or concept. Practice, excellent preparation, and controlled record keeping are the only means to make straightforward the progression.

General style:

Specific editorial column necessities for compliance of a manuscript will always take over from directions in these general guidelines.

To make a paper clear

· Adhere to recommended page limits

Mistakes to evade

- Insertion a title at the foot of a page with the subsequent text on the next page
- Separating a table/chart or figure - impound each figure/table to a single page
- Submitting a manuscript with pages out of sequence

In every sections of your document

· Use standard writing style including articles ("a", "the," etc.)

· Keep on paying attention on the research topic of the paper

· Use paragraphs to split each significant point (excluding for the abstract)

· Align the primary line of each section

· Present your points in sound order

· Use present tense to report well accepted

· Use past tense to describe specific results

· Shun familiar wording, don't address the reviewer directly, and don't use slang, slang language, or superlatives

· Shun use of extra pictures - include only those figures essential to presenting results

Title Page:

Choose a revealing title. It should be short. It should not have non-standard acronyms or abbreviations. It should not exceed two printed lines. It should include the name(s) and address (es) of all authors.

Abstract:

The summary should be two hundred words or less. It should briefly and clearly explain the key findings reported in the manuscript--must have precise statistics. It should not have abnormal acronyms or abbreviations. It should be logical in itself. Shun citing references at this point.

An abstract is a brief distinct paragraph summary of finished work or work in development. In a minute or less a reviewer can be taught the foundation behind the study, common approach to the problem, relevant results, and significant conclusions or new questions.

Write your summary when your paper is completed because how can you write the summary of anything which is not yet written? Wealth of terminology is very essential in abstract. Yet, use comprehensive sentences and do not let go readability for briefness. You can maintain it succinct by phrasing sentences so that they provide more than lone rationale. The author can at this moment go straight to shortening the outcome. Sum up the study, with the subsequent elements in any summary. Try to maintain the initial two items to no more than one ruling each.

- Reason of the study - theory, overall issue, purpose
- Fundamental goal
- To the point depiction of the research
- Consequences, including definite statistics - if the consequences are quantitative in nature, account quantitative data; results of any numerical analysis should be reported
- Significant conclusions or questions that track from the research(es)

Approach:

- Single section, and succinct
- As a outline of job done, it is always written in past tense
- A conceptual should situate on its own, and not submit to any other part of the paper such as a form or table
- Center on shortening results - bound background information to a verdict or two, if completely necessary
- What you account in an conceptual must be regular with what you reported in the manuscript
- Exact spelling, clearness of sentences and phrases, and appropriate reporting of quantities (proper units, important statistics) are just as significant in an abstract as they are anywhere else

Introduction:

The **Introduction** should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable to comprehend and calculate the purpose of your study without having to submit to other works. The basis for the study should be offered. Give most important references but shun difficult to make a comprehensive appraisal of the topic. In the introduction, describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will have no attention in your result. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here. Following approach can create a valuable beginning:

- Explain the value (significance) of the study
- Shield the model - why did you employ this particular system or method? What is its compensation? You strength remark on its appropriateness from a abstract point of vision as well as point out sensible reasons for using it.
- Present a justification. Status your particular theory (es) or aim(s), and describe the logic that led you to choose them.
- Very for a short time explain the tentative propose and how it skilled the declared objectives.

Approach:

- Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done.
- Sort out your thoughts; manufacture one key point with every section. If you make the four points listed above, you will need a least of four paragraphs.

- Present surroundings information only as desirable in order hold up a situation. The reviewer does not desire to read the whole thing you know about a topic.
- Shape the theory/purpose specifically - do not take a broad view.
- As always, give awareness to spelling, simplicity and correctness of sentences and phrases.

Procedures (Methods and Materials):

This part is supposed to be the easiest to carve if you have good skills. A sound written Procedures segment allows a capable scientist to replacement your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt for the least amount of information that would permit another capable scientist to spare your outcome but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section. When a technique is used that has been well described in another object, mention the specific item describing a way but draw the basic principle while stating the situation. The purpose is to text all particular resources and broad procedures, so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step by step report of the whole thing you did, nor is a methods section a set of orders.

Materials:

- Explain materials individually only if the study is so complex that it saves liberty this way.
- Embrace particular materials, and any tools or provisions that are not frequently found in laboratories.
- Do not take in frequently found.
- If use of a definite type of tools.
- Materials may be reported in a part section or else they may be recognized along with your measures.

Methods:

- Report the method (not particulars of each process that engaged the same methodology)
- Describe the method entirely
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures
- Simplify - details how procedures were completed not how they were exclusively performed on a particular day.
- If well known procedures were used, account the procedure by name, possibly with reference, and that's all.

Approach:

- It is embarrassed or not possible to use vigorous voice when documenting methods with no using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result when script up the methods most authors use third person passive voice.
- Use standard style in this and in every other part of the paper - avoid familiar lists, and use full sentences.

What to keep away from

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings - save it for the argument.
- Leave out information that is immaterial to a third party.

Results:

The principle of a results segment is to present and demonstrate your conclusion. Create this part a entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Carry on to be to the point, by means of statistics and tables, if suitable, to present consequences most efficiently.You must obviously differentiate material that would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matter should not be submitted at all except requested by the instructor.

Content

- Sum up your conclusion in text and demonstrate them, if suitable, with figures and tables.
- In manuscript, explain each of your consequences, point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation an exacting study.
- Explain results of control experiments and comprise remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or in manuscript form.

What to stay away from

- Do not discuss or infer your outcome, report surroundings information, or try to explain anything.
- Not at all, take in raw data or intermediate calculations in a research manuscript.
- Do not present the similar data more than once.
- Manuscript should complement any figures or tables, not duplicate the identical information.
- Never confuse figures with tables - there is a difference.

Approach

- As forever, use past tense when you submit to your results, and put the whole thing in a reasonable order.
- Put figures and tables, appropriately numbered, in order at the end of the report
- If you desire, you may place your figures and tables properly within the text of your results part.

Figures and tables

- If you put figures and tables at the end of the details, make certain that they are visibly distinguished from any attach appendix materials, such as raw facts
- Despite of position, each figure must be numbered one after the other and complete with subtitle
- In spite of position, each table must be titled, numbered one after the other and complete with heading
- All figure and table must be adequately complete that it could situate on its own, divide from text

Discussion:

The Discussion is expected the trickiest segment to write and describe. A lot of papers submitted for journal are discarded based on problems with the Discussion. There is no head of state for how long a argument should be. Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implication of the study. The purpose here is to offer an understanding of your results and hold up for all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of result should be visibly described. Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved with prospect, and let it drop at that.

- Make a decision if each premise is supported, discarded, or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."
- Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work
- You may propose future guidelines, such as how the experiment might be personalized to accomplish a new idea.
- Give details all of your remarks as much as possible, focus on mechanisms.
- Make a decision if the tentative design sufficiently addressed the theory, and whether or not it was correctly restricted.
- Try to present substitute explanations if sensible alternatives be present.
- One research will not counter an overall question, so maintain the large picture in mind, where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

- When you refer to information, differentiate data generated by your own studies from available information
- Submit to work done by specific persons (including you) in past tense.
- Submit to generally acknowledged facts and main beliefs in present tense.

The Administration Rules

Please carefully note down following rules and regulation before submitting your Research Paper to Global Journals Inc. (US):

Segment Draft and Final Research Paper: You have to strictly follow the template of research paper. If it is not done your paper may get rejected.

- The **major constraint** is that you must independently make all content, tables, graphs, and facts that are offered in the paper. You must write each part of the paper wholly on your own. The Peer-reviewers need to identify your own perceptive of the concepts in your own terms. NEVER extract straight from any foundation, and never rephrase someone else's analysis.

- Do not give permission to anyone else to "PROOFREAD" your manuscript.

- Methods to avoid Plagiarism is applied by us on every paper, if found guilty, you will be blacklisted by all of our collaborated research groups, your institution will be informed for this and strict legal actions will be taken immediately.)
- To guard yourself and others from possible illegal use please do not permit anyone right to use to your paper and files.

Criterion for Grading a Research Paper (Compilation) by Global Journals Inc. (US)

Please note that following table is only a Grading of "Paper Compilation" and not on "Performed/Stated Research" whose grading solely depends on Individual Assigned Peer Reviewer and Editorial Board Member. These can be available only on request and after decision of Paper. This report will be the property of Global Journals Inc. (US).

Topics	Grades		
	A-B	**C-D**	**E-F**
Abstract	Clear and concise with appropriate content, Correct format. 200 words or below	Unclear summary and no specific data, Incorrect form Above 200 words	No specific data with ambiguous information Above 250 words
Introduction	Containing all background details with clear goal and appropriate details, flow specification, no grammar and spelling mistake, well organized sentence and paragraph, reference cited	Unclear and confusing data, appropriate format, grammar and spelling errors with unorganized matter	Out of place depth and content, hazy format
Methods and Procedures	Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads	Difficult to comprehend with embarrassed text, too much explanation but completed	Incorrect and unorganized structure with hazy meaning
Result	Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake	Complete and embarrassed text, difficult to comprehend	Irregular format with wrong facts and figures
Discussion	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

Index

save our planet

Global Journal of Science Frontier Research

Visit us on the Web at www.GlobalJournals.org | www.JournalofScience.org
or email us at helpdesk@globaljournals.org

ISSN 9755896