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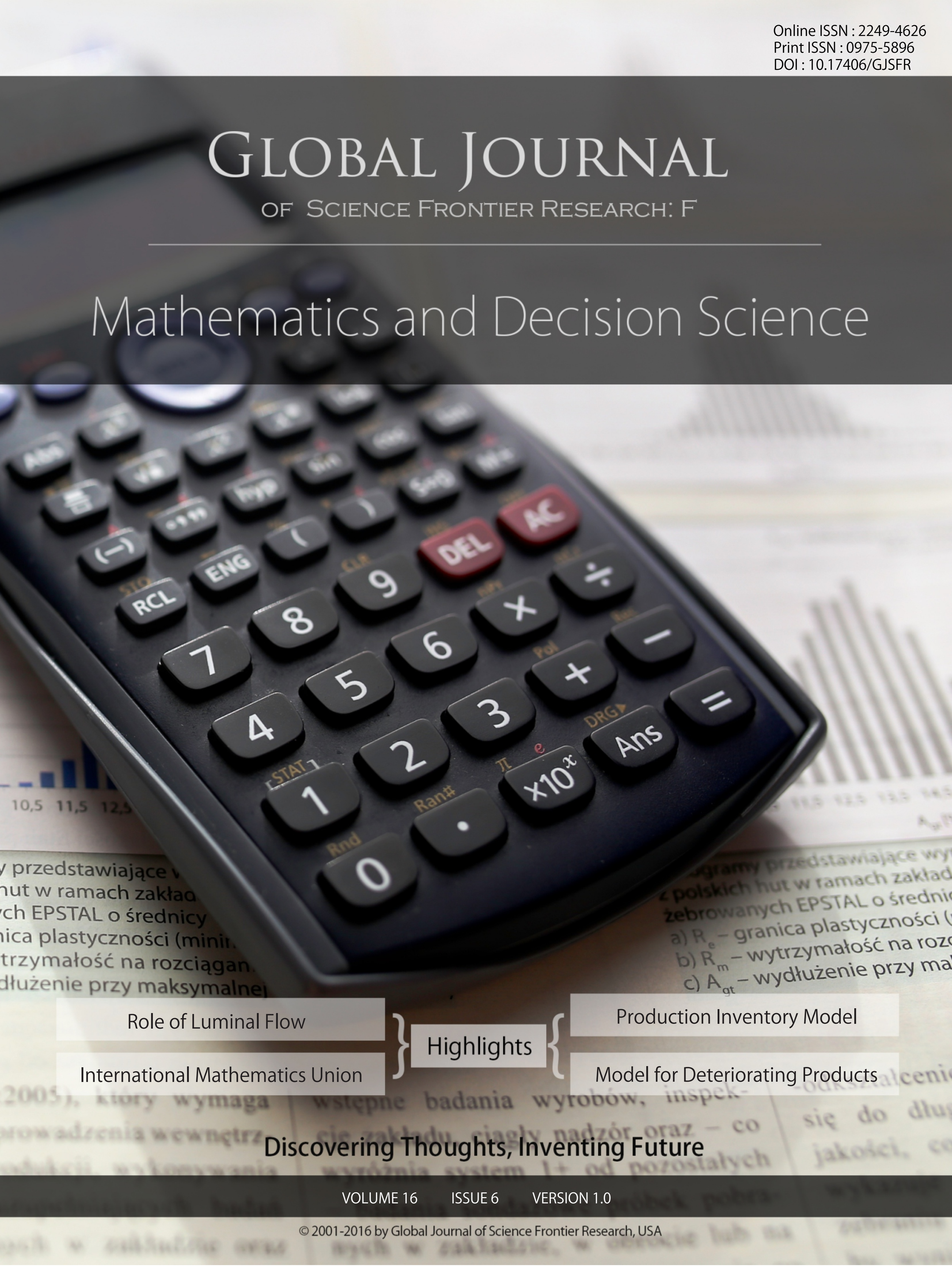

Global Journal of Science Frontier Research: F

Mathematics and Decision Science

Highlights

- Role of Luminal Flow
- International Mathematics Union
- Production Inventory Model
- Model for Deteriorating Products

Discovering Thoughts, Inventing Future

VOLUME 16 ISSUE 6 VERSION 1.0

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Mathematics & Decision Sciences

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An Open Letter to the International Mathematics Union on the Errors in the 1982 and 1990 Fields Medal Awards

By C. Y. Lo

Applied and Pure Research Institute, United States

Abstract- Since Einstein's theory is proven to be not self-consistent and incomplete, the positive mass theorem of S. T. Yau, which implied Einstein's theory would be consistent and stable is clearly incorrect. Apparently, Yau did not know that the Einstein equation does not have any bounded dynamic solution because he has never attempted to obtain one. Consequently, Yau was not aware that his assumption has already excluded the dynamic solutions. This error of Yau is due to that he does not understand physics. This error in the Fields Medal Awards is due to that those mathematicians responsible for the awards do not understand physics, and have a blind faith on Einstein. Thus, to remove the erroneous influence of Yau, it is desirable for the International Mathematics Union to correct her mistakes.

Keywords: *stable solution; dynamic solution, $E = mc^2$, Repulsive gravitation.*

GJSFR-F Classification: *MSC 2010: 00A05*

ANOPENLETTERTOTHEINTERNATIONALMATHEMATICSUNIONONTHEERRORSINTHE1982AND1990FIELDSMEDALAWARDS

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An Open Letter to the International Mathematics Union on the Errors in the 1982 and 1990 Fields Medal Awards

C. Y. Lo

Abstract- Since Einstein's theory is proven to be not self-consistent and incomplete, the positive mass theorem of S. T. Yau, which implied Einstein's theory would be consistent and stable is clearly incorrect. Apparently, Yau did not know that the Einstein equation does not have any bounded dynamic solution because he has never attempted to obtain one. Consequently, Yau was not aware that his assumption has already excluded the dynamic solutions. This error of Yau is due to that he does not understand physics. This error in the Fields Medal Awards is due to that those mathematicians responsible for the awards do not understand physics, and have a blind faith on Einstein. Thus, to remove the erroneous influence of Yau, it is desirable for the International Mathematics Union to correct her mistakes.

Keywords: stable solution; dynamic solution, $E = mc^2$, Repulsive gravitation.

Dear Dr. Holden:

I am writing to the International Mathematics Union because I have found some errors in the 1982 and 1990 awards to S. T. Yau and E. Witten.

Because of these, I went to Toronto to participate the Fields Medal Symposium, November 1-5, 2016. I met Dr. Ian Hambleton, Director of the Fields Institute for Research in Mathematical Sciences at Toronto, Canada. He recommends that I should discuss the errors in the Fields Medal award with the International Mathematical Union. He also provides the information with the names that I should talk to on such a subject matter. This is why I write this email to you.

I went to participate in the Fields Medal Symposium because they claim that they will carry out research and formulate problems of mutual interest. They also claim that their mission is to provide a supportive and stimulating environment for mathematics innovation and education. In their registration form, there are spaces for "Comments or Other Special Needs". It is there that I have written down my needs as follows:

"I have concern about the 1982 and 1990 awards for the fields Medal to S. T. Yau and E. Witten. The reason is that the positive mass theorem or positive energy theorem is actually invalid and misleading. The mistake is due to that they implicitly used an invalid assumption because they do not understand general relativity. I have published papers on this issue. This theorem has a wide spread erroneous influence in physics and thus stops progress of general relativity for at least 13 years. It is time for the fields Medal Institute to rectify this error without further delay. "

It turns out their position is that since they did not give the awards, it is better for the International Mathematical Union, who actually gave the awards, to rectify the errors. Prof. Peter C. Sarnak also confirms this.

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Helge Holden, Secretary, The International Mathematical Union, Secretariat, Markgrafenstr. 32, D-10117 Berlin, Germany.
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Therefore, I bring the issue to you. As the IMU Secretary, your duty is to handle suggestions and corrections. On the errors of Yau and Witten in the Fields Medal, I have written two papers. They are:

1. C. Y. Lo, On Contributions of S. T. Yau in Mathematics and Physics Related to General Relativity, GJSFR Volume 13-A Issue 4 Version 1.0 (August 2013).
2. C. Y. Lo, The Errors in the Fields Medals, 1982 to S. T. Yau, and 1990 to E. Witten, GJSFR Vol. 13-F, Issue 11, Version 1.0, 111-125 (2014).

These two papers are attached herewith for your perusal.

Their major error is due to inadequate understanding of physics, in particular general relativity. As A. Gullstrand, the Chairman of the Nobel Committee for Physics (1922-1929) suspected, the Einstein equation actually has no bounded dynamic solutions. 1) Yau apparently did not know this since he has never tried to obtain a dynamic solution, and thus he failed to see that his assumption in his theorem implies the exclusion of dynamic solutions. Consequently, he erroneously claimed that the Einstein equation has bounded physical solutions. Such a claim has misled the physicists for at least 13 years and thus also prevented the rectification of general relativity and the progress of physics.

In fact, many incorrectly believed that the positive mass theorem demonstrates that Einstein's theory is consistent and stable. However, now we know that Einstein's theory is not self-consistent and also incomplete [1, 2]. *This error in the Fields Medal award is due to that those mathematicians responsible do not understand physics, and have a blind faith on Einstein.*

It was not until in 1995 that I proved that there is no dynamic solution for the Einstein equation and published it in Astrophysical Journal [3]. Meanwhile, Christodoulou and Klainerman wrote a book [4] that invalidly claimed they have constructed dynamic solutions for the Einstein equation. Their mistake is actually very elementary; they simply did not proved the existence of a dynamic solution [5, 6] although they claimed they have.

One might wonder how such an elementary mistake in mathematics can happen to a professor of mathematics of the Princeton University. One can simply note that Christodoulou obtained his Ph. D. Degree under Professor John A. Wheeler. Wheeler el al. wrote a book, "Gravitation" [7] with many mistakes in mathematics and physics, including their misinterpretation of Einstein's equivalence principle. Their errors, which remain in their book unchanged since its publication, show that they are actually incompetent even in calculus since they invalidly claimed that their eq. (35.44) has bounded dynamic solutions [8]. Wald [9] also claimed that the Einstein equation has dynamic solutions to the second order but failed to provide one. It seems that they have drawn their false confidence from the invalid positive mass theorem.

Consequently, the 1993 Nobel Committee for Physics believed these errors and thus made erroneous claim in general relativity [10]. However, when Professor P. Morrison of MIT went to Princeton and questioned J. A. Taylor for the justification of their calculation on gravitational radiation, Taylor could not give an answer to Morrison [10]. Note that Yum=Tong Siu of Harvard University agreed to award Christodoulou the 2011 Shaw Prize in mathematics although he does not understand general relativity and nonlinear mathematics. A problem is that physicists are generally not very competent in pure mathematics, 2) and most pure mathematicians also do not understand physics.

For instance, Ludwig D. Faddeev, the Chairman of Fields Medal Committee has mistaken the misleading theorem of Witten as "another beautiful result of Witten – proof of the positive energy in Einstein's theory of gravitation". Michael Atiyah believed

Ref

4. D. Christodoulou & S. Klainerman, *The Global Nonlinear Stability of the Minkowski Space* (Princeton. Univ. Press, 1993).

that the positive mass theorem solved a formidable problem, 'leading in part to Yau's Fields Medal at the Warsaw Congress'. Note also that some famous mathematicians did not treat an award seriously. For instance, Yum-Tong Siu of Harvard University agreed to award Christodoulou the 2011 Shaw Prize in mathematics although he does not understand non-linear mathematics. Moreover, applied mathematician, 't Hooft, a Nobel Laureate actually did not understand physics such as Newtonian Mechanics and special relativity as shown in his Nobel speech because he considered the inertial mass of an electron includes also its electric energy [11-13].

Moreover, E. Witten was inappropriately awarded the 2016 APS Medal for exceptional Achievements in Research for mathematical physics without the necessary experiment supports. APS also failed to see his errors in general relativity [14]. The fact is, however, that his mathematics is at most half-braked since his under graduate degree is in history. This is evident since in addition to his errors on the Einstein equation, he does not understand even Einstein's equivalence principle because it requires adequate understanding of mathematical analysis. Moreover, since the current string theory has derived the invalid dynamic Einstein equation, it is clearly invalid.

In short, because of the errors in the Fields Medal, many errors in physics are mistaken as valid. Historically, even D. Hilbert had made errors in general relativity [15], but he did not continue his error. Moreover, Einstein's formula $E = mc^2$ is actually in conflict with the Einstein equation [1] and has been proven invalid for the electromagnetic energy by three experiments. Einstein also over-looked the repulsive gravitation [16]. Thus, in fact, errors in general relativity is rather common. However, we still consider Einstein a genius because he opened three important theories with accurate predictions.

I believe that your institute is a responsible one and thus you will take the information provided seriously and rectify the errors accordingly. Otherwise, the reputation of the Fields Medal would be known as questionable. Thank you for your kind attention. I am looking forward to hearing from you.

Sincerely yours,

C. Y. Lo

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Endnotes

1. Einstein equation does not have any dynamic solution because it violates the principle of causality [17].
2. For instance, according to APS Editorial Director, Daniel Kulp, none of the editors of APS has a degree in pure mathematics. In particular, Dr. Eric J. Weinberg, Editor of Physical Review D, did not understand even Einstein's equivalence principle as Pauli [18, 19] because of their inadequacy in pure mathematics [8].

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Bianchi Type V Universe Filled with Combination of Perfect Fluid and Scalar Field Coupled with Electromagnetic Fields in $f(R,T)$ Theory of Gravity

By D T Solanke & T M Karade
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Abstract- In $f(R,T)$ theory of gravity, we have studied the combination of perfect fluid and scalar field interacting with electromagnetic fields in Bianchi type V space-time, by considering the general cases $f(R,T) = f_1(R) + \lambda f_2(T)$, $f(R,T) = f_1(R)f_2(T)$ and $f(R)$ theory and its particular cases $f(R,T) = R + \lambda T$, $f(R,T) = RT$ and $f(R) = R$. It is observed that, even though the line element of space-time are distinct, the convergent and isotropic solution of metric functions can be evolved in each case along with the components of vector potential, corresponding to suitable integrable function in particular cases.

GJSFR-F Classification: MSC: 85

BIANCHITYPEVUNIVERSEFILLEDWITHCOMBINATIONOFPERFECTFLUIDANDSCALARFIELDCOUPLEDWITHELECTROMAGNETICFIELDSINFRTTHEORYOFGRAVITY

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Bianchi Type V Universe Filled with Combination of Perfect Fluid and Scalar Field Coupled with Electromagnetic Fields in $f(R,T)$ Theory of Gravity

D T Solanke [α] & T M Karade [σ]

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I. Introduction

Cosmological data from wide range of source have indicated that our universe is undergoing an accelerating expansion [2-8]. To explain this fact, two alternative theories are proposed: one concept of dark energy and other the amendment of general relativity leading to $f(R)$ and $f(R,T)$ theories [7, 10, 12] where R stands for Ricci scalar $R=g^{ij}R_{ij}$, R_{ij} being Ricci tensor and T stands for trace of energy momentum tensor and $T=g^{ij}T_{ij}$,T_{ij} being energy momentum tensor derived from Lagrangian L_m . The field equations of $f(R,T)$ theories due to Harko [10] are deduced by varying the action

$$s=\int f(R,T)\sqrt{-g}d^4x+\int L_m\sqrt{-g}d^4x \tag{1}$$

Where L_m is lagrangian and the other symbols have their usual meaning. Energy momentum tensor is given by

$$T_{ij}=L_m g_{ij}-2\frac{\delta L_m}{\delta g^{ij}} \tag{2}$$

Varying the action (1) with respect to g^{ij} which yields as

$$\delta s=\frac{1}{2\chi}\int\left\{f_R(R,T)\frac{\delta R}{\delta g^{ij}}+f_T(R,T)\frac{\delta T}{\delta g^{ij}}+\frac{f(R,T)}{\sqrt{-g}}\frac{\delta(\sqrt{-g})}{\delta g^{ij}}+\frac{2\chi}{\sqrt{-g}}\left(\frac{\delta(L_m\sqrt{-g})}{\delta g^{ij}}\right)\right\}\sqrt{-g}\,d^4x \tag{3}$$

We define $\theta_{ij}=g^{\alpha\beta}\frac{\delta T_{\alpha\beta}}{\delta g^{ij}}$ (4)

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By defining the generalized kronecker symbol $\frac{\delta g^{\alpha\beta}}{\delta g^{ij}} = \delta_i^\alpha \delta_j^\beta$ we can reduce

$$\frac{\delta g^{\alpha\beta}}{\delta g^{ij}} T_{\alpha\beta} = \delta_i^\alpha \delta_j^\beta T_{\alpha\beta} = g^{p\alpha} g_{pi} g^{q\beta} g_{qj} T_{\alpha\beta} = T_{ij}$$

Using above equations we can write

$$\frac{\delta T}{\delta g^{ij}} = \frac{\delta(g^{\alpha\beta} T_{\alpha\beta})}{\delta g^{ij}} = \frac{\delta g^{\alpha\beta}}{\delta g^{ij}} T_{\alpha\beta} + g^{\alpha\beta} \frac{\delta T_{\alpha\beta}}{\delta g^{ij}} = T_{ij} + \theta_{ij}$$

Integrating (3) we can obtain

$$f_R(R,T)R_{ij} - \tfrac{1}{2} f(R,T) g_{ij} + \left(g_{ij}\Box - \nabla_i\nabla_j\right) f_R(R,T) = \chi T_{ij} - f_T(R,T)\left[T_{ij} + \theta_{ij}\right] \qquad (5)$$

This can be further rewritten as

$$f_R(R,T)G_{ij} + \frac{1}{2}[f_R(R,T)R - f(R,T)]g_{ij} + (g_{ij}\Box - \nabla_i\nabla_j) f_R(R,T) =$$
$$\chi T_{ij} - f_T(R,T)\left[T_{ij} + \theta_{ij}\right] \qquad (6)$$

where $G_{ij} = R_{ij} - \frac{1}{2} R g_{ij}$

Taking trace of (5) we obtain

$$\Box f_R(R,T) = \tfrac{2}{3} f(R,T) - \tfrac{1}{3} f_R(R,T)R + \tfrac{\chi}{3} T - \tfrac{1}{3} f_T(R,T)[T+\theta] \qquad (7)$$

Using (7) the equation (6) can be organized in the form

$$G_j^i = \frac{1}{f_R(R,T)}\left[g^{mi}\nabla_m\nabla_j f_R(R,T)\right] - \frac{1}{6f_R(R,T)}[f_R(R,T)R + f(R,T)]g_j^i + \frac{\chi}{f_R(R,T)}\left[T_j^i - \frac{1}{3} T g_j^i\right] +$$
$$\frac{1}{3}\frac{f_T(R,T)}{f_R(R,T)}[T+\theta]g_j^i - \frac{f_T(R,T)}{f_R(R,T)}\left[T_j^i + \theta_j^i\right] \qquad (8)$$

Let us now calculate the tensor θ_{ij} .Varying (2) with respect to metric tensor g^{ij} and using the definition (4) we obtain

$$\theta_{ij} = -T_{ij} + 2\left[\frac{\delta L_m}{\delta g^{ij}} - g^{\alpha\beta}\frac{\delta^2 L_m}{\delta g^{ij}\delta g^{\alpha\beta}}\right] \qquad (9)$$

With this background, in this paper we discover the Bianchi type V space-time with combination of perfect fluid and scalar field interacting with electromagnetic one.

II. Matter field Lagrangian L_m

The electromagnetic field tensor is given by

$$F_{ij} = \frac{\partial V_i}{\partial x^j} - \frac{\partial V_j}{\partial x^i},$$

where V_i is electromagnetic four potential.

The aforesaid the matter Lagrangian L_m can be expressed as

$$L_m = \left[\frac{1}{4} F_{\eta\tau} F^{\eta\tau} - \frac{1}{2}\varphi_{,\eta}\varphi^{,\eta}\psi\right] \qquad (10)$$

where $\psi = \psi(I)$, $\quad I = V_i V^i$

Notes

The function ψ characterizes the interaction between the scalar φ and electromagnetic field [1].
Then the matter tensor in (2) can conveniently be expressed in the mixed form

$$T_j^i = \left(F_\alpha^i F_j^\alpha + \frac{1}{4} g_j^i F_{\alpha\beta} F^{\alpha\beta}\right) - \left[\frac{1}{2}\psi g_j^i - \dot{\psi} V^i V_j\right] \varphi_{,\eta} \varphi^{,\eta} + \psi \varphi^{,i} \varphi_{,j} \tag{11}$$

Similarly (9) is written as

$$\theta_j^i = -T_j^i - (\psi - I\dot{\psi})\varphi^{,i}\varphi_{,j} + I\ddot{\psi}\varphi_{,\eta}\varphi^{,\eta} V^i V_j \tag{12}$$

The equations (11) and (12), after contraction yield

$$T = -(\psi - I\dot{\psi})\varphi_{,\eta}\varphi^{,\eta} \tag{13}$$

$$\theta = I^2 \ddot{\psi} \varphi_{,\eta}\varphi^{,\eta} \tag{14}$$

III. Bianchi Type V Space-Time

The Bianchi type V metric of the space-time is specified by

$$ds^2 = dt^2 - A^2 dx^2 - e^{2\alpha x}(B^2 dy^2 + C^2 dz^2), \tag{15}$$

where A, B and C are functions of cosmic time t and α is non-zero constant
For this metric the non-vanishing components of the Ricci tensors are

$$R_1^1 = \frac{-2\alpha^2}{A^2} + \frac{\ddot{A}}{A} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC}, \quad R_2^2 = \frac{-2\alpha^2}{A^2} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC}, \quad R_3^3 = \frac{-2\alpha^2}{A^2} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC},$$

$$R_4^4 = \frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C}, \quad R_4^1 = \frac{2\alpha\dot{A}}{A^3} - \frac{\alpha\dot{B}}{A^2 B} - \frac{\alpha\dot{C}}{A^2 C}, \quad R_1^4 = -\frac{2\alpha\dot{A}}{A^3} + \frac{\alpha\dot{B}}{A^2 B} + \frac{\alpha\dot{C}}{A^2 C},$$

and the non-vanishing components of the Einstein tensors are

$$G_1^1 = \frac{\alpha^2}{A^2} - \frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} - \frac{\dot{B}\dot{C}}{BC}, \quad G_2^2 = \frac{\alpha^2}{A^2} - \frac{\ddot{A}}{A} - \frac{\ddot{C}}{C} - \frac{\dot{A}\dot{C}}{AC}, \quad G_3^3 = \frac{\alpha^2}{A^2} - \frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} - \frac{\dot{A}\dot{B}}{AB},$$

$$G_4^4 = \frac{3\alpha^2}{A^2} - \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{B}\dot{C}}{BC} - \frac{\dot{A}\dot{C}}{AC}, \quad G_4^1 = \frac{2\alpha\dot{A}}{A^3} - \frac{\alpha\dot{B}}{A^2 B} - \frac{\alpha\dot{C}}{A^2 C}, \quad G_1^4 = -2\alpha\frac{\dot{A}}{A} + \alpha\frac{\dot{B}}{B} + \alpha\frac{\dot{C}}{C}$$

a) *Electromagnetic field tensor F_{ij}*

To achieve the compatibility with the non-static space time (15), we assume the electromagnetic vector potential in the form

$$\mathrm{V_i} = [\mathrm{u(x)V_1(t), V_2(t), V_3(t), V_4(t)}] \tag{16}$$

$$I = V_i V^i = -\left[\frac{u^2 V_1{}^2}{A^2} + \frac{V_2{}^2}{B^2} e^{-2\alpha x} + \frac{V_3{}^2}{C^2} e^{-2\alpha x} - V_4{}^2\right] \tag{17}$$

Then it easy to deduce

$$\mathrm{F_{14} = u\dot{V}_1 \qquad F_{24} = \dot{V}_2 \qquad F_{34} = \dot{V}_3} \tag{18}$$

$$F_{ij} F^{ij} = -2\left[\frac{u^2 \dot{V}_1{}^2}{A^2} + \frac{\dot{V}_2{}^2}{B^2} e^{-2\alpha x} + \frac{\dot{V}_3{}^2}{C^2} e^{-2\alpha x}\right] \tag{19}$$

$$\varphi_i \varphi^i = \dot{\varphi}^2 \tag{20}$$

In reference to the above quantities at our disposal and space-time (15), the components of T_j^i in (11) assume the following values

Ref

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$$T_1^1 = \frac{1}{2}u^2\frac{\dot{V_1}^2}{A^2} - \frac{1}{2}\frac{\dot{V_2}^2}{B^2}e^{-2\alpha x} - \frac{1}{2}\frac{\dot{V_3}^2}{C^2}e^{-2\alpha x} - \frac{1}{2}\psi\dot{\varphi}^2 - \dot{\psi}\dot{\varphi}^2u^2\frac{V_1^2}{A^2} \tag{21a}$$

$$T_2^1 = \frac{u\dot{V}_1\dot{V}_2}{A^2} - \dot{\psi}\dot{\varphi}^2\frac{uV_1V_2}{A^2} \tag{21b}$$

$$T_3^1 = \frac{u\dot{V}_1\dot{V}_3}{A^2} - \dot{\psi}\dot{\varphi}^2\frac{uV_1V_3}{A^2} \tag{21c}$$

$$T_2^2 = -\frac{1}{2}u^2\frac{\dot{V_1}^2}{A^2} + \frac{1}{2}\frac{\dot{V_2}^2}{B^2}e^{-2\alpha x} - \frac{1}{2}\frac{\dot{V_3}^2}{C^2}e^{-2\alpha x} - \frac{1}{2}\psi\dot{\varphi}^2 - \dot{\psi}\dot{\varphi}^2\frac{V_2^2}{B^2}e^{-2\alpha x} \tag{21d}$$

$$T_3^2 = \frac{\dot{V}_2\dot{V}_3}{B^2}e^{-2\alpha x} - \dot{\psi}\dot{\varphi}^2\frac{V_2V_3}{B^2}e^{-2\alpha x} \tag{21e}$$

$$T_3^3 = -\frac{1}{2}u^2\frac{\dot{V_1}^2}{A^2} - \frac{1}{2}\frac{\dot{V_2}^2}{B^2}e^{-2\alpha x} + \frac{1}{2}\frac{\dot{V_3}^2}{C^2}e^{-2\alpha x} - \frac{1}{2}\psi\dot{\varphi}^2 - \dot{\psi}\dot{\varphi}^2\frac{V_3^2}{C^2}e^{-2\alpha x} \tag{21f}$$

$$T_4^4 = \frac{1}{2}u^2\frac{\dot{V_1}^2}{A^2} + \frac{1}{2}\frac{\dot{V_2}^2}{B^2}e^{-2\alpha x} + \frac{1}{2}\frac{\dot{V_3}^2}{C^2}e^{-2\alpha x} + \frac{1}{2}\psi\dot{\varphi}^2 + \dot{\psi}\dot{\varphi}^2{V_4}^2 \tag{21g}$$

$$T = -(\psi - I\dot{\psi})\dot{\varphi}^2 \tag{21h}$$

Similarly the components of tensor θ_j^i from (12) can assume the following values

$$\theta_1^1 = -T_1^1 - I\ddot{\psi}\dot{\varphi}^2u^2\frac{V_1^2}{A^2} \tag{22a}$$

$$\theta_2^1 = -T_2^1 - I\ddot{\psi}\dot{\varphi}^2u\frac{V_1V_2}{A^2} \tag{22b}$$

$$\theta_3^1 = -T_3^1 - I\ddot{\psi}\dot{\varphi}^2u\frac{V_1V_3}{A^2} \tag{22c}$$

$$\theta_2^2 = -T_2^2 - I\ddot{\psi}\dot{\varphi}^2\frac{V_2^2}{B^2}e^{-2\alpha x} \tag{22d}$$

$$\theta_3^2 = -T_3^2 - I\ddot{\psi}\dot{\varphi}^2\frac{V_2V_3}{B^2}e^{-2\alpha x} \tag{22e}$$

$$\theta_3^3 = -T_3^3 - I\ddot{\psi}\dot{\varphi}^2\frac{V_3^2}{C^2}e^{-2\alpha x} \tag{22f}$$

$$\theta_4^4 = -T_4^4 - \psi\dot{\varphi}^2 + I\dot{\psi}\dot{\varphi}^2 + I\ddot{\psi}\dot{\varphi}^2{V_4}^2 \tag{22g}$$

$$\theta = I^2\ddot{\psi}\dot{\varphi}^2 \tag{22h}$$

Variation of Lagrangian in (10) with respect to the electromagnetic field gives us

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^j}\left(\sqrt{-g}F^{ij}\right) - (\varphi_{,j}\varphi^{,j})\dot{\psi}V^i = 0$$

$$\text{for } i = 1, j = 4 \Rightarrow \left(\frac{\dot{V}_1}{V_1}\right)^{\cdot} + \frac{\dot{V_1}^2}{{V_1}^2} + \frac{\dot{V}_1}{V_1}\left[\frac{\dot{B}}{B} + \frac{\dot{C}}{C} - \frac{\dot{A}}{A}\right] = \dot{\psi}\dot{\varphi}^2 \tag{23a}$$

$$\text{for } i = 2, j = 4 \Rightarrow \left(\frac{\dot{V}_2}{V_2}\right)^{\cdot} + \frac{\dot{V_2}^2}{{V_2}^2} + \frac{\dot{V}_2}{V_2}\left[\frac{\dot{C}}{C} + \frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right] = \dot{\psi}\dot{\varphi}^2 \tag{23b}$$

$$\text{for } i = 3, j = 4 \Rightarrow \left(\frac{\dot{V}_3}{V_3}\right)^{\cdot} + \frac{\dot{V_3}^2}{{V_3}^2} + \frac{\dot{V}_3}{V_3}\left[\frac{\dot{A}}{A} + \frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right] = \dot{\psi}\dot{\varphi}^2 \tag{23c}$$

$$\text{for } i = 4, j = 1 \Rightarrow u(x) = k_1e^{-2\alpha x} \tag{23d}$$

Notes

for $i = 4, j = 4 \Rightarrow V_4 = 0$ (23e)

where k_1 is constant of integration.

IV. Combination of Perfect Fluid and Scalar Field Coupled with Electromagnetic Field

Notes

Energy momentum tensor for perfect fluid is given by

$$T_j^i = (\rho + p)u_j u^i - p\delta_j^i \tag{24}$$

where $g_{ij} u^i u^j = 1$

$$T_1^1 = T_2^2 = T_3^3 = -p \ , T_4^4 = \rho$$

$$T_j^i = 0 \text{ for } i \neq j \tag{25}$$

We take combination of perfect fluid and scalar field coupled with electromagnetic field as

$$T_j^i = T_j^i(PF) + T_j^i(EF) \tag{26}$$

By using (25), (26) the equation (21) reduces to

$$T_1^1 = -p + \frac{1}{2}u^2\frac{\dot{V_1}^2}{A^2} - \frac{1}{2}\frac{\dot{V_2}^2}{B^2}e^{-2\alpha x} - \frac{1}{2}\frac{\dot{V_3}^2}{C^2}e^{-2\alpha x} - \frac{1}{2}\psi\dot{\varphi}^2 - \dot{\psi}\dot{\varphi}^2u^2\frac{V_1^2}{A^2} \tag{27a}$$

$$T_2^1 = \frac{u\dot{V_1}\dot{V_2}}{A^2} - \dot{\psi}\dot{\varphi}^2\frac{uV_1V_2}{A^2} \tag{27b}$$

$$T_3^1 = \frac{u\dot{V_1}\dot{V_3}}{A^2} - \dot{\psi}\dot{\varphi}^2\frac{uV_1V_3}{A^2} \tag{27c}$$

$$T_2^2 = -p - \frac{1}{2}u^2\frac{\dot{V_1}^2}{A^2} + \frac{1}{2}\frac{\dot{V_2}^2}{B^2}e^{-2\alpha x} - \frac{1}{2}\frac{\dot{V_3}^2}{C^2}e^{-2\alpha x} - \frac{1}{2}\psi\dot{\varphi}^2 - \dot{\psi}\dot{\varphi}^2\frac{V_2^2}{B^2}e^{-2\alpha x} \tag{27d}$$

$$T_3^2 = \frac{\dot{V_2}\dot{V_3}}{B^2}e^{-2\alpha x} - \dot{\psi}\dot{\varphi}^2\frac{V_2V_3}{B^2}e^{-2\alpha x} \tag{27e}$$

$$T_3^3 = -p - \frac{1}{2}u^2\frac{\dot{V_1}^2}{A^2} - \frac{1}{2}\frac{\dot{V_2}^2}{B^2}e^{-2\alpha x} + \frac{1}{2}\frac{\dot{V_3}^2}{C^2}e^{-2\alpha x} - \frac{1}{2}\psi\dot{\varphi}^2 - \dot{\psi}\dot{\varphi}^2\frac{V_3^2}{C^2}e^{-2\alpha x} \tag{27f}$$

$$T_4^4 = \rho + \frac{1}{2}u^2\frac{\dot{V_1}^2}{A^2} + \frac{1}{2}\frac{\dot{V_2}^2}{B^2}e^{-2\alpha x} + \frac{1}{2}\frac{\dot{V_3}^2}{C^2}e^{-2\alpha x} + \frac{1}{2}\psi\dot{\varphi}^2 + \dot{\psi}\dot{\varphi}^2V_4^2 \tag{27g}$$

$$T = -3p + \rho - (\psi - I\dot{\psi})\dot{\varphi}^2 \tag{27h}$$

By using (25), (26) the equation (22) reduces to

$$\theta_1^1 = -T_1^1 - p - I\ddot{\psi}\dot{\varphi}^2u^2\frac{V_1^2}{A^2} \tag{28a}$$

$$\theta_2^1 = -T_2^1 - I\ddot{\psi}\dot{\varphi}^2u\frac{V_1V_2}{A^2} \tag{28b}$$

$$\theta_3^1 = -T_3^1 - I\ddot{\psi}\dot{\varphi}^2u\frac{V_1V_3}{A^2} \tag{28c}$$

$$\theta_2^2 = -T_2^2 - p - I\ddot{\psi}\dot{\varphi}^2\frac{V_2^2}{B^2}e^{-2\alpha x} \tag{28d}$$

$$\theta_3^2 = -T_3^2 - I\ddot{\psi}\dot{\varphi}^2\frac{V_2V_3}{B^2}e^{-2\alpha x} \tag{28e}$$

$$\theta_3^3 = -T_3^3 - p - I\ddot{\psi}\dot{\varphi}^2 \frac{{V_3}^2}{C^2} e^{-2\alpha x} \tag{28f}$$

$$\theta_4^4 = -T_4^4 + \rho - \psi\dot{\varphi}^2 + I\dot{\psi}\dot{\varphi}^2 + I\ddot{\psi}\dot{\varphi}^2 {V_4}^2 \tag{28g}$$

$$\theta = I^2\ddot{\psi}\dot{\varphi}^2 \tag{28h}$$

Since the expression of the Einstein tensor in (8) is complicated, the solution of the Einstein's field equation in general cannot be obtained. With this reality we take recourse to the particular cases of the function $f(R,T)$ and there upon try to obtain the solution.

Notes

V. Sub Case $f(R,T) = f_1(R) + \lambda f_2(T)$

In this case we follow the notations

$$f_R(R,T) = \frac{\partial f(R,T)}{\partial R} = \dot{f}_1(R) , \qquad f_T(R,T) = \frac{\partial f(R,T)}{\partial T} = \lambda \dot{f}_2(T)$$

Then (8) reduces to the form

$$G_j^i = \frac{1}{\dot{f}_1(R)}\left[g^{mi}\nabla_m\nabla_j \dot{f}_1(R)\right] - \frac{1}{6\dot{f}_1(R)}\left[\dot{f}_1(R)R + f_1(R) + \lambda f_2(T)\right]g_j^i + \frac{\chi}{\dot{f}_1(R)}\left[T_j^i - \frac{1}{3}T g_j^i\right] + \frac{\lambda}{3}\frac{\dot{f}_2(T)}{\dot{f}_1(R)}[T+\theta]g_j^i - \frac{\lambda \dot{f}_2(T)}{\dot{f}_1(R)}\left[T_j^i + \theta_j^i\right] \tag{29}$$

Since for the space-time (15), $G_2^1 = 0, G_3^1 = 0, G_3^2 = 0$, by using (27) and (28), the field equations (29) yield

$$\frac{\dot{V}_1\dot{V}_2}{V_1V_2} = \dot{\psi}\dot{\varphi}^2 - \frac{\lambda}{\chi}\dot{f}_2(T)I\ddot{\psi}\dot{\varphi}^2 \tag{30a}$$

$$\frac{\dot{V}_1\dot{V}_3}{V_1V_3} = \dot{\psi}\dot{\varphi}^2 - \frac{\lambda}{\chi}\dot{f}_2(T)I\ddot{\psi}\dot{\varphi}^2 \tag{30b}$$

$$\frac{\dot{V}_2\dot{V}_3}{V_2V_3} = \dot{\psi}\dot{\varphi}^2 - \frac{\lambda}{\chi}\dot{f}_2(T)I\ddot{\psi}\dot{\varphi}^2 \tag{30c}$$

From (30) we can write

$$\frac{\dot{V}_1\dot{V}_2}{V_1V_2} = \frac{\dot{V}_2\dot{V}_3}{V_2V_3} = \frac{\dot{V}_1\dot{V}_3}{V_1V_3} = \dot{\psi}\dot{\varphi}^2 - \frac{\lambda}{\chi}\dot{f}_2(T)I\ddot{\psi}\dot{\varphi}^2 \tag{31}$$

or $\frac{\dot{V}_2}{V_2} = \frac{\dot{V}_3}{V_3} \equiv \frac{\dot{h}_3}{h_3}$, say (32)

where h_3 is some unknown function of t
Inserting (32) in (31) we get

$$\left(\frac{\dot{h}_3}{h_3}\right)^2 = \left(\frac{\dot{h}_3}{h_3}\right)^2 = \left(\frac{\dot{h}_3}{h_3}\right)^2 = \dot{\psi}\dot{\varphi}^2 - \frac{\lambda}{\chi}\dot{f}_2(T)I\ddot{\psi}\dot{\varphi}^2 \tag{33}$$

Upon the integration of equation (32), yield

$$V_1 = k_{38}h_3 \qquad V_2 = k_{39}h_3 \qquad V_3 = k_{40}h_3 \tag{34}$$

where $k's$ are constants of integration.

Now our plan is to express the components of T_j^i in (27) in terms of T_4^4. For this we consider the expression

$$\frac{u^2\dot{V_1}^2}{A^2}+\frac{\dot{V_2}^2}{B^2}e^{-2\alpha x}+\frac{\dot{V_3}^2}{C^2}e^{-2\alpha x}=\left[\frac{u^2{v_1}^2}{A^2}+\frac{{V_2}^2}{B^2}e^{-2\alpha x}+\frac{{V_3}^2}{C^2}e^{-2\alpha x}\right]\left(\frac{\dot{h}_3}{h_3}\right)^2 \quad \text{by (32)}$$

$$=-I\left(\frac{\dot{h}_3}{h_3}\right)^2 \quad \text{by (17) and (23e)}$$

$$=\frac{\lambda}{\chi}\dot{f}_2(T)I^2\ddot{\psi}\dot{\varphi}^2-I\dot{\psi}\dot{\varphi}^2 \quad \text{by (33) (35)}$$

Notes

We attempt to express the components of T_j^i in (27) in terms of T_4^4 by using (32), (33) and (35)

$$T_4^4=\rho+\frac{1}{2}\psi\dot{\varphi}^2-\frac{1}{2}I\dot{\psi}\dot{\varphi}^2+\frac{1}{2}\frac{\lambda}{\chi}\dot{f}_2(T)I^2\ddot{\psi}\dot{\varphi}^2 \tag{36a}$$

$$T_1^1=-T_4^4+\rho-p-\frac{\lambda}{\chi}\dot{f}_2(T)I\ddot{\psi}\dot{\varphi}^2u^2\frac{{V_1}^2}{A^2} \tag{36b}$$

$$T_2^1=-\frac{\lambda}{\chi}\dot{f}_2(T)I\ddot{\psi}\dot{\varphi}^2u\frac{V_1V_2}{A^2} \tag{36c}$$

$$T_3^1=-\frac{\lambda}{\chi}\dot{f}_2(T)I\ddot{\psi}\dot{\varphi}^2u\frac{V_1V_3}{A^2} \tag{36d}$$

$$T_2^2=-T_4^4+\rho-p-\frac{\lambda}{\chi}\dot{f}_2(T)I\ddot{\psi}\dot{\varphi}^2\frac{{V_2}^2}{B^2}e^{-2\alpha x} \tag{36e}$$

$$T_3^2=-\frac{\lambda}{\chi}\dot{f}_2(T)I\ddot{\psi}\dot{\varphi}^2\frac{V_2V_3}{A^2}e^{-2\alpha x} \tag{36f}$$

$$T_3^3=-T_4^4+\rho-p-\frac{\lambda}{\chi}\dot{f}_2(T)I\ddot{\psi}\dot{\varphi}^2\frac{{V_3}^2}{C^2}e^{-2\alpha x} \tag{36g}$$

$$T=-3p+\rho-(\psi-I\dot{\psi})\dot{\varphi}^2 \quad (36f)$$

We consider the non-vanishing components of Einstein tensor G_1^1, G_2^2, G_3^3 from (29)

$$\frac{\alpha^2}{A^2}-\frac{\ddot{B}}{B}-\frac{\ddot{C}}{C}-\frac{\dot{B}\dot{C}}{BC}=-\frac{\dot{A}}{A}\frac{\ddot{f}_1(R)}{\dot{f}_1(R)}\frac{dR}{dt}-\frac{1}{6\dot{f}_1(R)}\left[\dot{f}_1(R)R+f_1(R)+\lambda f_2(T)\right]+\frac{\chi}{\dot{f}_1(R)}\left[T_1^1-\frac{1}{3}T\right]$$
$$+\frac{\lambda}{3}\frac{\dot{f}_2(T)}{\dot{f}_1(R)}[T+\theta]-\frac{\lambda\dot{f}_2(T)}{\dot{f}_1(R)}[T_1^1+\theta_1^1] \tag{37a}$$

$$\frac{\alpha^2}{A^2}-\frac{\ddot{C}}{C}-\frac{\ddot{A}}{A}-\frac{\dot{C}\dot{A}}{CA}=-\frac{\dot{B}}{B}\frac{\ddot{f}_1(R)}{\dot{f}_1(R)}\frac{dR}{dt}-\frac{1}{6\dot{f}_1(R)}\left[\dot{f}_1(R)R+f_1(R)+\lambda f_2(T)\right]+\frac{\chi}{\dot{f}_1(R)}\left[T_2^2-\frac{1}{3}T\right]$$
$$+\frac{\lambda}{3}\frac{\dot{f}_2(T)}{\dot{f}_1(R)}[T+\theta]-\frac{\lambda\dot{f}_2(T)}{\dot{f}_1(R)}[T_2^2+\theta_2^2] \tag{37b}$$

$$\frac{\alpha^2}{A^2}-\frac{\ddot{A}}{A}-\frac{\ddot{B}}{B}-\frac{\dot{A}\dot{B}}{AB}=-\frac{\dot{C}}{C}\frac{\ddot{f}_1(R)}{\dot{f}_1(R)}\frac{dR}{dt}-\frac{1}{6\dot{f}_1(R)}\left[\dot{f}_1(R)R+f_1(R)+\lambda f_2(T)\right]+\frac{\chi}{\dot{f}_1(R)}\left[T_3^3-\frac{1}{3}T\right]$$
$$+\frac{\lambda}{3}\frac{\dot{f}_2(T)}{\dot{f}_1(R)}[T+\theta]-\frac{\lambda\dot{f}_2(T)}{\dot{f}_1(R)}[T_3^3+\theta_3^3] \tag{37c}$$

Subtracting (37b) from (37a), (37c) from (37b) and (37a) from (37c) we get

$$\frac{\ddot{A}}{A}-\frac{\ddot{B}}{B}+\frac{\dot{C}}{C}\left[\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right]+\left[\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right]\frac{\ddot{f}_1(R)}{\dot{f}_1(R)}\frac{dR}{dt}=\frac{\chi}{\dot{f}_1(R)}[T_1^1-T_2^2]+\frac{\lambda\dot{f}_2(T)}{\dot{f}_1(R)}[(T_2^2+\theta_2^2)-(T_1^1+\theta_1^1)] \tag{38a}$$

$$\frac{\ddot{B}}{B}-\frac{\ddot{C}}{C}+\frac{\dot{A}}{A}\left[\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right]+\left[\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right]\frac{\ddot{f}_1(R)}{\dot{f}_1(R)}\frac{dR}{dt}=\frac{\chi}{\dot{f}_1(R)}[T_2^2-T_3^3]+\frac{\lambda\dot{f}_2(T)}{\dot{f}_1(R)}[(T_3^3+\theta_3^3)-(T_2^2+\theta_2^2)] \tag{38b}$$

$$\frac{\ddot{C}}{C}-\frac{\ddot{A}}{A}+\frac{\dot{B}}{B}\left[\frac{\dot{C}}{C}-\frac{\dot{A}}{A}\right]+\left[\frac{\dot{C}}{C}-\frac{\dot{A}}{A}\right]\frac{\ddot{f}_1(R)}{\dot{f}_1(R)}\frac{dR}{dt}=\frac{\chi}{\dot{f}_1(R)}[T_3^3-T_1^1]+\frac{\lambda\dot{f}_2(T)}{\dot{f}_1(R)}[(T_1^1+\theta_1^1)-(T_3^3+\theta_3^3)] \tag{38c}$$

With the help of (28) and (36) the equation (38) reduces to

$$\frac{\ddot{A}}{A}-\frac{\ddot{B}}{B}+\frac{\dot{C}}{C}\left[\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right]+\left[\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right]\frac{\ddot{f}_1(R)}{\dot{f}_1(R)}\frac{dR}{dt}=0 \tag{39a}$$

$$\frac{\ddot{B}}{B}-\frac{\ddot{C}}{C}+\frac{\dot{A}}{A}\left[\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right]+\left[\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right]\frac{\ddot{f}_1(R)}{\dot{f}_1(R)}\frac{dR}{dt}=0 \tag{39b}$$

$$\frac{\ddot{C}}{C}-\frac{\ddot{A}}{A}+\frac{\dot{B}}{B}\left[\frac{\dot{C}}{C}-\frac{\dot{A}}{A}\right]+\left[\frac{\dot{C}}{C}-\frac{\dot{A}}{A}\right]\frac{\ddot{f}_1(R)}{\dot{f}_1(R)}\frac{dR}{dt}=0 \tag{39c}$$

Integrating (39) we get

$$\frac{A}{B}=k_{42}exp\left\{k_{41}\int\frac{1}{ABC\dot{f}_1(R)}dt\right\} \tag{40a}$$

$$\frac{B}{C}=k_{44}exp\left\{k_{43}\int\frac{1}{ABC\dot{f}_1(R)}dt\right\} \tag{40b}$$

$$\frac{C}{A}=k_{46}exp\left\{k_{45}\int\frac{1}{ABC\dot{f}_1(R)}dt\right\} \tag{40c}$$

Where$k's$ are constants of integration, with the condition that $k_{42}k_{44}k_{46}=1$ and

$$k_{41}+k_{43}+k_{45}=0$$

From (40) we can express the values of A, B, C explicitly as

$$A=(ABC)^{\frac{1}{3}}k_{47}exp\left\{k_{48}\int\frac{1}{ABC\dot{f}_1(R)}dt\right\} \tag{41a}$$

$$B=(ABC)^{\frac{1}{3}}k_{49}exp\left\{k_{50}\int\frac{1}{ABC\dot{f}_1(R)}dt\right\} \tag{41b}$$

$$C=(ABC)^{\frac{1}{3}}k_{51}exp\left\{k_{52}\int\frac{1}{ABC\dot{f}_1(R)}dt\right\} \tag{41c}$$

where $k's$ are constants of integration.
Using (32) we can rewrite the equations (23) as

$$\left(\frac{\dot{h}_3}{h_3}\right)^{\cdot}+\frac{\dot{h}_3{}^2}{h_3{}^2}+\frac{\dot{h}_3}{h_3}\left[\frac{\dot{C}}{C}+\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right]=\dot{\psi}\dot{\varphi}^2 \tag{42a}$$

$$\left(\frac{\dot{h}_3}{h_3}\right)^{\cdot}+\frac{\dot{h}_3{}^2}{h_3{}^2}+\frac{\dot{h}_3}{h_3}\left[\frac{\dot{A}}{A}+\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right]=\dot{\psi}\dot{\varphi}^2 \tag{42b}$$

$$\left(\frac{\dot{h}_3}{h_3}\right)^{\cdot}+\frac{\dot{h}_3{}^2}{h_3{}^2}+\frac{\dot{h}_3}{h_3}\left[\frac{\dot{B}}{B}+\frac{\dot{C}}{C}-\frac{\dot{A}}{A}\right]=\dot{\psi}\dot{\varphi}^2 \tag{42c}$$

These equations further imply

$$\frac{\dot{C}}{C}+\frac{\dot{A}}{A}-\frac{\dot{B}}{B}=\frac{\dot{A}}{A}+\frac{\dot{B}}{B}-\frac{\dot{C}}{C}=\frac{\dot{B}}{B}+\frac{\dot{C}}{C}-\frac{\dot{A}}{A}$$

or

$$\frac{A^{\cdot}}{A}=\frac{B^{\cdot}}{B}=\frac{C^{\cdot}}{C} \tag{43}$$

Inserting (43) in (42) yield

Notes

$$\left(\frac{\dot{h}_3}{h_3}\right)^{\cdot} + \frac{\dot{h}_3{}^2}{h_3{}^2} + \frac{\dot{h}_3}{h_3}\left[\frac{\dot{A}}{A}\right] = \dot{\psi}\dot{\varphi}^2 \tag{44}$$

But from (33) we have
$$\dot{\psi}\dot{\varphi}^2 = \left(\frac{\dot{h}_3}{h_3}\right)^2 + \frac{\lambda}{\chi}\dot{f}_2(T)I\ddot{\psi}\dot{\varphi}^2 \tag{45}$$

Inserting (45) in (44) we have

$$\left(\frac{\dot{h}_3}{h_3}\right)^{\cdot} + \frac{\dot{h}_3}{h_3}\left[\frac{\dot{A}}{A}\right] = \frac{\lambda}{\chi}\dot{f}_2(T)I\ddot{\psi}\dot{\varphi}^2 \tag{46}$$

If we confine to the linearity of ψ (i.e. $\psi = k_{53}I + k_{54}$) we can have the perfect solution of (46)

$$h_3 = k_{56}exp\left\{k_{55}\int \frac{1}{A}dt\right\} \tag{47}$$

With the help of (47) equations (34) reduces to

$$V_1 = k_{57}exp\left\{k_{55}\int \frac{1}{A}dt\right\} \tag{48a}$$

$$V_2 = k_{58}exp\left\{k_{55}\int \frac{1}{A}dt\right\} \tag{48b}$$

$$V_3 = k_{59}exp\left\{k_{55}\int \frac{1}{A}dt\right\} \tag{48c}$$

Where $k's$ are constants of integration.

VI. Combination of Perfect Fluid and Electromagnetic Field in $f(R,T) = f_1(R)f_2(T)$

In this case we follow the notations

$$f_R(R,T) = \frac{\partial f(R,T)}{\partial R} = \dot{f}_1(R)f_2(T)\ , \quad \mathrm{f_T(R,T)} = \frac{\partial f(R,T)}{\partial T} = \mathrm{f_1(R)\dot{f}_2(T)}$$

Then the field equation (8) reduces to

$$G_j^{\mu} = \frac{1}{\dot{f}_1(R)f_2(T)}\left[g^{\mu i}\nabla_{\mu}\nabla_{j}\dot{f}_1(R)f_2(T)\right] - \frac{1}{6\dot{f}_1(R)f_2(T)}\left[\dot{f}_1(R)f_2(T)\mathrm{R} + \mathrm{f_1(R)f_2(T)}\right]g_j^{\mu} + \frac{\chi}{\dot{f}_1(R)f_2(T)}\left[T_j^{\mu} - \frac{1}{3}Tg_j^{\mu}\right] + \frac{1}{3}\frac{\mathrm{f_1(R)\dot{f}_2(T)}}{\dot{f}_1(R)f_2(T)}[T+\theta]g_j^{\mu} - \frac{\mathrm{f_1(R)\dot{f}_2(T)}}{\dot{f}_1(R)f_2(T)}\left[T_j^{\mu} + \theta_j^{\mu}\right] \tag{49}$$

Since for the space-time (15), $G_2^1 = 0, G_3^1 = 0, G_3^2 = 0$, from (49) and by using (27) and (28)

$$\frac{\dot{V}_1\dot{V}_2}{V_1V_2} = \dot{\psi}\,\dot{\varphi}^2 - \frac{\mathrm{f_1(R)\dot{f}_2(T)}}{\chi}I\ddot{\psi}\dot{\varphi}^2 \tag{50a}$$

$$\frac{\dot{V}_1\dot{V}_3}{V_1V_3} = \dot{\psi}\,\dot{\varphi}^2 - \frac{\mathrm{f_1(R)\dot{f}_2(T)}}{\chi}I\ddot{\psi}\dot{\varphi}^2 \tag{50b}$$

$$\frac{\dot{V}_2\dot{V}_3}{V_2V_3} = \dot{\psi}\,\dot{\varphi}^2 - \frac{\mathrm{f_1(R)\dot{f}_2(T)}}{\chi}I\ddot{\psi}\dot{\varphi}^2 \tag{50c}$$

From (50) we can write

$$\frac{\dot{V}_1\dot{V}_2}{V_1V_2} = \frac{\dot{V}_2\dot{V}_3}{V_2V_3} = \frac{\dot{V}_1\dot{V}_3}{V_1V_3} = \dot{\psi}\,\dot{\varphi}^2 - \frac{\mathrm{f_1(R)\dot{f}_2(T)}}{\chi}I\ddot{\psi}\dot{\varphi}^2 \tag{51}$$

Further we can rewrite it as

$$\frac{\dot{V}_1}{V_1} = \frac{\dot{V}_2}{V_2} = \frac{\dot{V}_3}{V_3} \equiv \frac{\dot{h}_8}{h_8}, \text{ say} \tag{52}$$

Where h_8 is some unknown function of t
Inserting (52) in (51) yields

$$\left(\frac{\mathrm{h}_8}{h_8}\right)^2 = \left(\frac{\dot{h}_8}{h_8}\right)^2 = \left(\frac{\dot{h}_8}{h_8}\right)^2 = \dot{\psi}\,\dot{\varphi}^2 - \frac{\mathrm{f}_1(\mathrm{R})\dot{\mathrm{f}}_2(\mathrm{T})}{\chi} I\ddot{\psi}\dot{\varphi}^2 \tag{53}$$

Integrating (52) we get

$$V_1 = m_{28}h_8 \qquad V_2 = m_{29}h_8 \qquad V_3 = m_{30}h_8 \tag{54}$$

Where m_{28}, m_{29}, m_{30} are constants of integration.

Now our plan is to express the components of T_j^i in (27) in terms of T_4^4 . For this we consider the expression

$$\frac{u^2\dot{V_1}^2}{A^2} + \frac{\dot{V_2}^2}{B^2}e^{-2\alpha x} + \frac{\dot{V_3}^2}{C^2}e^{-2\alpha x} = \left[\frac{u^2{V_1}^2}{A^2} + \frac{{V_2}^2}{B^2}e^{-2\alpha x} + \frac{{V_3}^2}{C^2}e^{-2\alpha x}\right]\left(\frac{\dot{h}_8}{h_8}\right)^2 \quad \text{by} \quad (52)$$

$$= -I\left(\frac{\dot{h}_8}{h_8}\right)^2 \quad \text{by (17) and (23e)}$$

$$= \frac{f_1(R)\dot{f}_2(T)}{\chi} I^2\ddot{\psi}\dot{\varphi}^2 - I\dot{\psi}\dot{\varphi}^2 \qquad \text{by} \qquad (53) \tag{55}$$

We attempt to express the components of T_j^i in (27) in terms of T_4^4 by using (52), (53) and (55)

$$T_4^4 = \rho + \frac{1}{2}\psi\dot{\varphi}^2 - \frac{1}{2}I\dot{\psi}\dot{\varphi}^2 + \frac{1}{2}\frac{f_1(R)\dot{f}_2(T)}{\chi}I^2\ddot{\psi}\dot{\varphi}^2 \tag{56a}$$

$$T_1^1 = -T_4^4 + \rho - p - \frac{f_1(R)\dot{f}_2(T)}{\chi}I\ddot{\psi}\dot{\varphi}^2u^2\frac{{V_1}^2}{A^2} \tag{56b}$$

$$T_2^1 = -\frac{f_1(R)\dot{f}_2(T)}{\chi}I\ddot{\psi}\dot{\varphi}^2u\frac{V_1V_2}{A^2} \tag{56c}$$

$$T_3^1 = -\frac{f_1(R)\dot{f}_2(T)}{\chi}I\ddot{\psi}\dot{\varphi}^2u\frac{V_1V_3}{A^2} \tag{56d}$$

$$T_2^2 = -T_4^4 + \rho - p - \frac{f_1(R)\dot{f}_2(T)}{\chi}I\ddot{\psi}\dot{\varphi}^2\frac{{V_2}^2}{B^2}e^{-2\alpha x} \tag{56e}$$

$$T_3^2 = -\frac{f_1(R)\dot{f}_2(T)}{\chi}I\ddot{\psi}\dot{\varphi}^2\frac{V_2V_3}{A^2}e^{-2\alpha x} \tag{56f}$$

$$T_3^3 = -T_4^4 + \rho - p - \frac{f_1(R)\dot{f}_2(T)}{\chi}I\ddot{\psi}\dot{\varphi}^2\frac{{V_3}^2}{C^2}e^{-2\alpha x} \tag{56g}$$

$$T = -3p + \rho - (\psi - I\dot{\psi})\dot{\varphi}^2 \tag{56h}$$

We consider the non-vanishing components of Einstein tensor G_1^1, G_2^2, G_3^3 from (49)

$$\frac{\alpha^2}{A^2} - \frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} - \frac{\dot{B}\dot{C}}{BC} = -\frac{1}{A^2}\left[\frac{\ddot{f}_2(T)}{f_2(T)}\left(\frac{dT}{dx}\right)^2 + \frac{\dot{f}_2(T)}{f_2(T)}\frac{d^2T}{dx^2}\right] - \frac{\dot{A}}{A}\left[\frac{\ddot{f}_1(R)}{\dot{f}_1(R)}\frac{dR}{dt} + \frac{\dot{f}_2(T)}{f_2(T)}\frac{dT}{dt}\right] - \frac{1}{6\dot{f}_1(R)f_2(T)} \times$$

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$$\left[\dot f_1(R)f_2(T)\mathrm{R}+\mathrm{f}_1(\mathrm{R})\mathrm{f}_2(\mathrm{T})\right]+\frac{\chi}{\dot f_1(R)f_2(T)}\left[T_1^1-\frac{1}{3}T\right]+\frac{1}{3}\frac{\mathrm{f}_1(\mathrm{R})\dot{\mathrm{f}}_2(\mathrm{T})}{\dot f_1(R)f_2(T)}[T+\theta]-\frac{\mathrm{f}_1(\mathrm{R})\dot{\mathrm{f}}_2(\mathrm{T})}{\dot f_1(R)f_2(T)}[T_1^1+\theta_1^1] \tag{57a}$$

$$\frac{\alpha^2}{A^2}-\frac{\ddot C}{C}-\frac{\ddot A}{A}-\frac{\dot C\dot A}{CA}=\frac{\alpha}{A^2}\left[\frac{\dot f_2(T)}{f_2(T)}\frac{dT}{dx}\right]-\frac{\dot B}{B}\left[\frac{\ddot f_1(R)}{\dot f_1(R)}\frac{dR}{dt}+\frac{\dot f_2(T)}{f_2(T)}\frac{dT}{dt}\right]-\frac{1}{6\dot f_1(R)f_2(T)}\left[\dot f_1(R)f_2(T)\mathrm{R}+\mathrm{f}_1(\mathrm{R})\mathrm{f}_2(\mathrm{T})\right]$$

$$+\frac{\chi}{\dot f_1(R)f_2(T)}\left[T_2^2-\frac{1}{3}T\right]+\frac{1}{3}\frac{\mathrm{f}_1(\mathrm{R})\dot{\mathrm{f}}_2(\mathrm{T})}{\dot f_1(R)f_2(T)}[T+\theta]-\frac{\mathrm{f}_1(\mathrm{R})\dot{\mathrm{f}}_2(\mathrm{T})}{\dot f_1(R)f_2(T)}[T_2^2+\theta_2^2] \tag{57b}$$

$$\frac{\alpha^2}{A^2}-\frac{\ddot A}{A}-\frac{\ddot B}{B}-\frac{\dot A\dot B}{AB}=\frac{\alpha}{A^2}\left[\frac{\dot f_2(T)}{f_2(T)}\frac{dT}{dx}\right]-\frac{\dot C}{C}\left[\frac{\ddot f_1(R)}{\dot f_1(R)}\frac{dR}{dt}+\frac{\dot f_2(T)}{f_2(T)}\frac{dT}{dt}\right]-\frac{1}{6\dot f_1(R)f_2(T)}\left[\dot f_1(R)f_2(T)\mathrm{R}+\mathrm{f}_1(\mathrm{R})\mathrm{f}_2(\mathrm{T})\right]$$

$$+\frac{\chi}{\dot f_1(R)f_2(T)}\left[T_3^3-\frac{1}{3}T\right]+\frac{1}{3}\frac{\mathrm{f}_1(\mathrm{R})\dot{\mathrm{f}}_2(\mathrm{T})}{\dot f_1(R)f_2(T)}[T+\theta]-\frac{\mathrm{f}_1(\mathrm{R})\dot{\mathrm{f}}_2(\mathrm{T})}{\dot f_1(R)f_2(T)}[T_3^3+\theta_3^3] \tag{57c}$$

Subtracting (57b) from (57a), (57c) from (57b) and (57a) from (57c) we obtain

$$\frac{\ddot A}{A}-\frac{\ddot B}{B}+\frac{\dot C}{C}\left[\frac{\dot A}{A}-\frac{\dot B}{B}\right]+\left[\frac{\dot A}{A}-\frac{\dot B}{B}\right]\left[\frac{\ddot f_1(R)}{\dot f_1(R)}\frac{dR}{dt}+\frac{\dot f_2(T)}{f_2(T)}\frac{dT}{dt}\right]=-\frac{1}{A^2}\frac{\ddot f_2(T)}{f_2(T)}\left(\frac{dT}{dx}\right)^2-\frac{1}{A^2}\frac{\dot f_2(T)}{f_2(T)}\frac{d^2T}{dx^2}-\frac{\alpha}{A^2}\frac{\dot f_2(T)}{f_2(T)}\frac{dT}{dx}$$

$$\frac{\chi}{\dot f_1(R)f_2(T)}[T_1^1-T_2^2]+\frac{\mathrm{f}_1(\mathrm{R})\dot{\mathrm{f}}_2(\mathrm{T})}{\dot f_1(R)f_2(T)}[(T_2^2+\theta_2^2)-(T_1^1+\theta_1^1)] \tag{58a}$$

$$\frac{\ddot B}{B}-\frac{\ddot C}{C}+\frac{\dot A}{A}\left[\frac{\dot B}{B}-\frac{\dot C}{C}\right]+\left[\frac{\dot B}{B}-\frac{\dot C}{C}\right]\left[\frac{\ddot f_1(R)}{\dot f_1(R)}\frac{dR}{dt}+\frac{\dot f_2(T)}{f_2(T)}\frac{dT}{dt}\right]=\frac{\chi}{\dot f_1(R)f_2(T)}[T_2^2-T_3^3]+$$

$$\frac{\mathrm{f}_1(\mathrm{R})\dot{\mathrm{f}}_2(\mathrm{T})}{\dot f_1(R)f_2(T)}[(T_3^3+\theta_3^3)-(T_2^2+\theta_2^2)] \tag{58b}$$

$$\frac{\ddot C}{C}-\frac{\ddot A}{A}+\frac{\dot B}{B}\left[\frac{\dot C}{C}-\frac{\dot A}{A}\right]+\left[\frac{\dot C}{C}-\frac{\dot A}{A}\right]\left[\frac{\ddot f_1(R)}{\dot f_1(R)}\frac{dR}{dt}+\frac{\dot f_2(T)}{f_2(T)}\frac{dT}{dt}\right]=\frac{1}{A^2}\frac{\ddot f_2(T)}{f_2(T)}\left(\frac{dT}{dx}\right)^2+\frac{1}{A^2}\frac{\dot f_2(T)}{f_2(T)}\frac{d^2T}{dx^2}+\frac{\alpha}{A^2}\frac{\dot f_2(T)}{f_2(T)}\frac{dT}{dx}$$

$$+\frac{\chi}{\dot f_1(R)f_2(T)}[T_3^3-T_1^1]+\frac{\mathrm{f}_1(\mathrm{R})\dot{\mathrm{f}}_2(\mathrm{T})}{\dot f_1(R)f_2(T)}[(T_1^1+\theta_1^1)-(T_3^3+\theta_3^3)] \tag{58c}$$

Using (56) and (28) the equations (58) reduces to

$$\frac{\ddot A}{A}-\frac{\ddot B}{B}+\frac{\dot C}{C}\left[\frac{\dot A}{A}-\frac{\dot B}{B}\right]+\left[\frac{\dot A}{A}-\frac{\dot B}{B}\right]\left[\frac{\ddot f_1(R)}{\dot f_1(R)}\frac{dR}{dt}+\frac{\dot f_2(T)}{f_2(T)}\frac{dT}{dt}\right]=-\frac{1}{A^2}\frac{\ddot f_2(T)}{f_2(T)}\left(\frac{dT}{dx}\right)^2-\frac{1}{A^2}\frac{\dot f_2(T)}{f_2(T)}\frac{d^2T}{dx^2}-\frac{\alpha}{A^2}\frac{\dot f_2(T)}{f_2(T)}\frac{dT}{dx} \tag{59a}$$

$$\frac{\ddot B}{B}-\frac{\ddot C}{C}+\frac{\dot A}{A}\left[\frac{\dot B}{B}-\frac{\dot C}{C}\right]+\left[\frac{\dot B}{B}-\frac{\dot C}{C}\right]\left[\frac{\ddot f_1(R)}{\dot f_1(R)}\frac{dR}{dt}+\frac{\dot f_2(T)}{f_2(T)}\frac{dT}{dt}\right]=0 \tag{59b}$$

$$\frac{\ddot C}{C}-\frac{\ddot A}{A}+\frac{\dot B}{B}\left[\frac{\dot C}{C}-\frac{\dot A}{A}\right]+\left[\frac{\dot C}{C}-\frac{\dot A}{A}\right]\left[\frac{\ddot f_1(R)}{\dot f_1(R)}\frac{dR}{dt}+\frac{\dot f_2(T)}{f_2(T)}\frac{dT}{dt}\right]=\frac{1}{A^2}\frac{\ddot f_2(T)}{f_2(T)}\left(\frac{dT}{dx}\right)^2+\frac{1}{A^2}\frac{\dot f_2(T)}{f_2(T)}\frac{d^2T}{dx^2}+\frac{\alpha}{A^2}\frac{\dot f_2(T)}{f_2(T)}\frac{dT}{dx} \tag{59c}$$

If we take the condition that $A=m_{31}C$ (60)

where m_{31} non-zero positive constant. Then we obtain

$$\frac{1}{A^2}\frac{\ddot f_2(T)}{f_2(T)}\left(\frac{dT}{dx}\right)^2+\frac{1}{A^2}\frac{\dot f_2(T)}{f_2(T)}\frac{d^2T}{dx^2}+\frac{\alpha}{A^2}\frac{\dot f_2(T)}{f_2(T)}\frac{dT}{dx}=0$$

And above equations reduces to

$$\frac{\ddot C}{C}-\frac{\ddot B}{B}+\frac{\dot C}{C}\left[\frac{\dot C}{C}-\frac{\dot B}{B}\right]+\left[\frac{\dot C}{C}-\frac{\dot B}{B}\right]\left[\frac{\ddot f_1(R)}{\dot f_1(R)}\frac{dR}{dt}+\frac{\dot f_2 T)}{f_2(T)}\frac{dT}{dt}\right]=0 \tag{61a}$$

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$$\frac{\ddot{B}}{B}-\frac{\ddot{C}}{C}+\frac{\dot{C}}{C}\left[\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right]+\left[\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right]\left[\frac{\ddot{f}_1(R)}{\dot{f}_1(R)}\frac{dR}{dt}+\frac{\dot{f}_2T)}{f_2(T)}\frac{dT}{dt}\right]=0 \tag{61b}$$

Integrating (61) we get

$$\frac{B}{C}=m_{33}exp\left\{m_{32}\int\frac{1}{BC^2\dot{f}_1(R)f_2(T)}dt\right\} \tag{61a}$$

$$\frac{C}{B}=m_{35}exp\left\{m_{34}\int\frac{1}{BC^2\dot{f}_1(R)f_2(T)}dt\right\} \tag{61b}$$

Where $m's$ are constants of integration with the condition that $m_{33}m_{35}=1$ and $m_{32}+m_{34}=0$

We can express explicitly as

$$C=(BC^2)^{\frac{1}{3}}m_{36}exp\left\{m_{37}\int\frac{1}{BC^2\dot{f}_1(R)f_2(T)}dt\right\} \tag{62a}$$

$$B=(BC^2)^{\frac{1}{3}}m_{38}exp\left\{m_{39}\int\frac{1}{BC^2\dot{f}_1(R)f_2(T)}dt\right\} \tag{62b}$$

From (60) and (62a) we obtain

$$A=(BC^2)^{\frac{1}{3}}m_{40}exp\left\{m_{41}\int\frac{1}{BC^2\dot{f}_1(R)f_2(T)}dt\right\} \tag{62c}$$

By converting C in to A we get

$$A=(ABC)^{\frac{1}{3}}m_{42}exp\left\{m_{43}\int\frac{1}{ABC\dot{f}_1(R)f_2(T)}dt\right\} \tag{63a}$$

$$B=(ABC)^{\frac{1}{3}}m_{44}exp\left\{m_{45}\int\frac{1}{ABC\dot{f}_1(R)f_2(T)}dt\right\} \tag{63b}$$

$$C=(ABC)^{\frac{1}{3}}m_{46}exp\left\{m_{47}\int\frac{1}{ABC\dot{f}_1(R)f_2(T)}dt\right\} \tag{63c}$$

Where $m's$ are constant of integration.

Adjusting the constants in (41) and (63), the line element (15) assumes an isotropic form and hence we can generalize the results in the form of the following theorem.

Theorem 1: In $f(R,T)$theory of gravity the Bianchi type V space-time, filled with combination of perfect fluid and scalar field coupled with electromagnetic field, admits isotropy for the functional form $f(R,T)=f_1(R)+\lambda f_2(T)$, $f(R,T)=f_1(R)f_2(T)$

With the help of (52) we can rewrite the equation (23) as

$$\left(\frac{\dot{h}_8}{h_8}\right)^{\cdot}+\frac{\dot{h}_8{}^2}{h_8{}^2}+\frac{\dot{h}_8}{h_8}\left[\frac{\dot{C}}{C}+\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right]=\dot{\psi}\dot{\varphi}^2 \tag{64a}$$

$$\left(\frac{\dot{h}_8}{h_8}\right)^{\cdot}+\frac{\dot{h}_8{}^2}{h_8{}^2}+\frac{\dot{h}_8}{h_8}\left[\frac{\dot{A}}{A}+\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right]=\dot{\psi}\dot{\varphi}^2 \tag{64b}$$

$$\left(\frac{\dot{h}_8}{h_8}\right)^{\cdot}+\frac{\dot{h}_8{}^2}{h_8{}^2}+\frac{\dot{h}_8}{h_8}\left[\frac{\dot{B}}{B}+\frac{\dot{C}}{C}-\frac{\dot{A}}{A}\right]=\dot{\psi}\dot{\varphi}^2 \tag{64c}$$

Further these equations imply that

$$\frac{\dot{C}}{C}+\frac{\dot{A}}{A}-\frac{\dot{B}}{B}=\frac{\dot{A}}{A}+\frac{\dot{B}}{B}-\frac{\dot{C}}{C}=\frac{\dot{B}}{B}+\frac{\dot{C}}{C}-\frac{\dot{A}}{A}$$

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Or
$$\frac{\dot{A}}{A}=\frac{\dot{B}}{B}=\frac{\dot{C}}{C} \tag{65}$$

Inserting (65) in (64) we get

$$\left(\frac{\dot{h}_8}{h_8}\right)^{\cdot}+\frac{\dot{h}_8{}^2}{h_8{}^2}+\frac{\dot{h}_8}{h_8}\left[\frac{\dot{A}}{A}\right]=\dot{\psi}\dot{\varphi}^2 \tag{66}$$

But from (53) we have

$$\dot{\psi}\,\dot{\varphi}^2=\left(\frac{\dot{h}_8}{h_8}\right)^2+\frac{\mathrm{f}_1(\mathrm{R})\dot{\mathrm{f}}_2(\mathrm{T})}{\chi}I\ddot{\psi}\dot{\varphi}^2 \tag{67}$$

Inserting (67) in (66) we have

$$\left(\frac{\dot{h}_8}{h_8}\right)^{\cdot}+\frac{\dot{h}_8}{h_8}\left[\frac{\dot{A}}{A}\right]=\frac{\mathrm{f}_1(\mathrm{R})\dot{\mathrm{f}}_2(\mathrm{T})}{\chi}I\ddot{\psi}\dot{\varphi}^2 \tag{68}$$

If $\ddot{\psi}=0$ or $\psi=m_{37}I+m_{38}$ then equation (68) has perfect solution and its solution is

$$h_8=m_{49}exp\left\{m_{48}\int\frac{1}{A}dt\right\} \tag{69}$$

With the help of (69) the equation (54) convert in to

$$V_1=m_{50}exp\left\{m_{48}\int\frac{1}{A}dt\right\} \tag{70a}$$

$$V_2=m_{51}exp\left\{m_{48}\int\frac{1}{A}dt\right\} \tag{70b}$$

$$V_3=m_{52}exp\left\{m_{48}\int\frac{1}{A}dt\right\} \tag{70c}$$

where $m's$ are constants of integration.

Adjusting the constants in (48) and (70), the vector potential assumes the following form

$$V_i=[V_1,V_1,V_1,0]$$

Hence we generalize the result in the form of following theorem

Theorem 2: In $f(R,T)$ theory of gravity, the Bianchi type V space-time filled with combination of perfect fluid and scalar field coupled with electromagnetic field, admits the vector potential in the form $V_i=[V_1,V_1,V_1,0]$ for the functional form $f(R,T)=f_1(R)+\lambda f_2(T)$ and $f(R,T)=f_1(R)f_2(T)$.

VII. Sub Case $f(R,T)=F(R)$

In this case we follow the notations $f_R(R,T)=\frac{\partial f(R)}{\partial R}=\dot{f}(R)$, $f_T(R,T)=\frac{\partial f(R)}{\partial T}=0$

In this case equation (5) reduces to

$$G_j^i=\frac{1}{\dot{f}(R)}\left[g^{im}\nabla_m\nabla_j\dot{f}(R)\right]-\frac{1}{6\dot{f}(R)}\left[\dot{f}(R)R+f(R)\right]g_j^i+\frac{\chi}{\dot{f}(R)}\left[T_j^i-\frac{1}{3}Tg_j^i\right] \tag{71}$$

The computation for this case easily follows from those of the earlier case (section 5) by mere substitution of $f_1(R)=f(R)$, $\lambda=0$ or $f_2(T)=0$

We get the result as follows

$$A = (ABC)^{\frac{1}{3}} l_{53} exp\left\{l_{54}\int \frac{1}{ABC} dt\right\} \quad (72a)$$

$$B = (ABC)^{\frac{1}{3}} l_{55} exp\left\{l_{56}\int \frac{1}{ABC} dt\right\} \quad (72b)$$

$$C = (ABC)^{\frac{1}{3}} l_{57} exp\left\{l_{58}\int \frac{1}{ABC} dt\right\} \quad (72c)$$

Where $l's$ are constants of integration.

$$V_1 = l_{63} exp\left\{l_{61}\int \frac{1}{A} dt\right\} \quad (73a)$$

$$V_2 = l_{64} exp\left\{l_{61}\int \frac{1}{A} dt\right\} \quad (73b)$$

$$V_3 = l_{65} exp\left\{l_{61}\int \frac{1}{A} dt\right\} \quad (73c)$$

where $l's$ be constant of integration.

From section 5, 6 and 7 we observe that the result remain intact for $f(R,T) = f_1(R) + \lambda f_2(T)$, $f(R,T) = f_1(R) f_2(T)$ and $f(R,T) = f(R)$ only differ in constants of integration. Hence the equations (72) and (73) admit the theorem 1 and 2.

VIII. Sub Case $f(R,T) = R + \lambda T$

In this case we follow the notations $f_R(R,T) = \frac{\partial f(R,T)}{\partial R} = 1$, $f_T(R,T) = \frac{\partial f(R,T)}{\partial T} = \lambda$
In this case the field equation (5) reduces to

$$G_j^i = \chi T_j^i - \lambda\left[T_j^i + \theta_j^i\right] + \frac{\lambda}{2} T g_j^i \quad (74)$$

The consideration of this sub case follows from (section 5) $f(R,T) = f_1(R) + \lambda f_2(T)$ by taking $f_1(R) = R$ and $f_2(T) = T$.
We get the result as follows

$$A = (ABC)^{\frac{1}{3}} l_{53} exp\left\{l_{54}\int \frac{1}{ABC} dt\right\} \quad (75a)$$

$$B = (ABC)^{\frac{1}{3}} l_{55} exp\left\{l_{56}\int \frac{1}{ABC} dt\right\} \quad (75b)$$

$$C = (ABC)^{\frac{1}{3}} l_{57} exp\left\{l_{58}\int \frac{1}{ABC} dt\right\} \quad (75c)$$

Where $l's$ are constants of integration.

$$V_1 = l_{63} exp\left\{l_{61}\int \frac{1}{A} dt\right\} \quad (76a)$$

$$V_2 = l_{64} exp\left\{l_{61}\int \frac{1}{A} dt\right\} \quad (76b)$$

$$V_3 = l_{65} exp\left\{l_{61}\int \frac{1}{A} dt\right\} \quad (76c)$$

where $l's$ be constant of integration.

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From section 5, 6 and 8 we observe that the result remain intact for $f(R,T) = f_1(R) + \lambda f_2(T)$, $f(R,T) = f_1(R)f_2(T)$ and $f(R,T) = R + \lambda T$ only differ in constants of integration. Hence the equations (75) and (76) admit the theorem 1 and 2.

IX. Sub Case $f(R,T) = RT$

In this case we follow the notations

$$f_R(R,T) = \frac{\partial f(R,T)}{\partial R} = T \ , \quad f_T(R,T) = \frac{\partial f(R,T)}{\partial T} = R$$

Then the field equation (8) reduces to

$$G_j^i = \frac{1}{T}\left[g^{mi}\nabla_m \nabla_j T\right] - \frac{1}{3}Rg_j^i + \frac{\chi}{T}\left[T_j^i - \frac{1}{3}Tg_j^i\right] + \frac{R}{3T}[T+\theta]g_j^i - \frac{R}{T}\left[T_j^i + \theta_j^i\right] \tag{77}$$

The computation for this case easily follows from those of the earlier case (section 6) by mere substitution of $f_1(R) = R$, and $f_2(T) = T$
We get the result as follows

$$A = (ABC)^{\frac{1}{3}} n_{42} exp\left\{n_{43}\int \frac{1}{ABCT}dt\right\} \tag{78a}$$

$$B = (ABC)^{\frac{1}{3}} n_{44} exp\left\{n_{45}\int \frac{1}{ABCT}dt\right\} \tag{78b}$$

$$C = (ABC)^{\frac{1}{3}} n_{46} exp\left\{n_{47}\int \frac{1}{ABCT}dt\right\} \tag{78c}$$

Where $n's$ are constant of integration.

$$V_1 = n_{52} exp\left\{n_{50}\int \frac{1}{A}dt\right\} \tag{79a}$$

$$V_2 = n_{53} exp\left\{n_{50}\int \frac{1}{A}dt\right\} \tag{79b}$$

$$V_3 = n_{54} exp\left\{n_{50}\int \frac{1}{A}dt\right\} \tag{79c}$$

Where $n's$are constants of integration.

From section 5, 6 and 9 we observe that the result remain intact for $f(R,T) = f_1(R) + \lambda f_2(T)$, $f(R,T) = f_1(R)f_2(T)$ and $f(R,T) = RT$ only differ in constants of integration. Hence the equations (75) and (76) admit the theorem 1 and 2.

X. Conclusion

1) In the present paper we have considered the Bianchi type V model in combination of perfect fluid and scalar field coupled with electromagnetic field in sub cases of $f(R,T)$ theory of gravity models

(i) $f(R,T) = f_1(R) + \lambda f_2(T)$
(ii) $f(R,T) = f(R)$
(iii) $f(R,T) = R + \lambda T$
(iv) $f(R,T) = R$
(v) $f(R,T) = f_1(R)f_2(T)$
(vi) $f(R,T) = RT$

We have derived the gravitational field equations corresponding to the general and particular cases of $f(R,T)$ theory of gravity.

2) It is observed that, even though the cases of $f(R,T)$ theory are distinct, the convergent, non-singular, isotropic solutions can be evolved in each case along with the components vector potential.
3) From finding of the $f(R,T)$ and $f(R)$ theory, general and particular cases, in this paper we believe firmly that the results of $f(R,T)$ and $f(R)$ depends on only R and not on T
4) From different cases of $f(R,T)$ we observe that the results remain intact only differ in constants of integration.

Notes

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Using Spin, Twist and Dial Homeomorphisms to Generate Homeotopy Groups

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Introduction- In this paper we consider some problems concerned with the isotopy classification of homeomorphisms of multiply punctured compact 2-manifolds, i.e., manifolds of the form $X - \cup_{i=1}^{m} \mathring{D}_i - P$where X is a closed 2-manifold, $\{D_i : 1 \leq i \leq m\}$ is a family of disjoint discs in X and P $= \{p_{m+1}, \ldots, p_n\}$ is a finite subset of X disjoint from each D_i. Inparticular we will show that various homeotopy groups for these manifolds are generated by the isotopy class of three types of homeomorphisms. In the case X is the two sphere we will give a complete presentation of these homeotopy groups. Parts of the material in this paper have been considered in [5], [6] and [7], but this is the first time that a full treatment of this topic, including detailed illustrations of the isotopies involved, has been submitted for publication. For an alternate approach to the material in this paper see (for example) [2], and [].

GJSFR-F Classification: *MSC 2010: 37E30*

USINGSPINTWISTANDDIALHOMEOMORPHISMSTOGENERATEHOMEOTOPYGROUPS

Strictly as per the compliance and regulations of :

Using Spin, Twist and Dial Homeomorphisms to Generate Homeotopy Groups

David Sprows

I. Introduction

In this paper we consider some problems concerned with the isotopy classification of homeomorphisms of multiply punctured compact 2-manifolds, i.e., manifolds of the form $X - \cup_{i=1}^{m} \mathring{D}_i - P$where X is a closed 2-manifold, $\{D_i : 1 \leq i \leq m\}$ is a family of disjoint discs in X and $P = \{p_{m+1}, \ldots, p_n\}$ is a finite subset of X disjoint from each D_i. Inparticular we will show that various homeotopy groups for these manifolds are generated by the isotopy class of three types of homeomorphisms. In the case X is the two sphere we will give a complete presentation of these homeotopy groups. Parts of the material in this paper have been considered in [5], [6] and [7], but this is the first time that a full treatment of this topic, including detailed illustrations of the isotopies involved, has been submitted for publication. For an alternate approach to the material in this paper see (for example) [2], and [].

Definitions and Notation

Let x be a 2-manifold (connected, triangulated).
Let $A \subset X$ and $B \subset X$.
$G(X)$ = group of all homeomorphisms of X onto itself with the compact-open topology.
$G_0(X)$ = arc component of 1_X in $G(X)$. Clearly $G_0(X)$ is normal in $G(X)$.
$H(X) = G(X)/G_0(X)$ = the homeotopy group of X.
$G(X,A) = \{g \in G(X) : g/A \in G(A)\}$.
$G_0(X,A)$ = arc component of 1_X in $G(X, A)$.
$H(X,A) = G(X,A)/G_0(X,A)$ = the homeotopy group of (X, A).
$G`(X,A) = \{g \in G(X) : g / A = 1_A\}$.
$G`_0(X,A)$ = arc component of $1_X$ in $G`(X,A)$.
$H`(X,A) = G`(X,A)/G`_0(X,A)$
$H^*(X,A) = (G`(X,A) \cap G_0(X))/G`_0(X,A)$.
$G(X,A,B) = \{g \in G(X) : g / A \in G(A),\ g / B \in G(B)\}$.
$G_0(X,A,B)$ = arc component of 1_X in $G(X,A,B)$.
$H(X,A,B) = G(X,A,B)/G_0(X,A,B)$.
$\mathcal{S}_n$= symmetric group on n letters.

Remarks:

1) $f \in G_0(X)$iff f is isotopic to 1_X (denoted $f \simeq 1_X$).
2) $f \in G_0(X, A)$ iff f is isotopic to 1_X by an isotopy which keeps A invariant.

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5. D. Sprows, "Homeotopy groups of compact manifolds" Fundamenta Mathemaiticae, 90 (1975) 99-103.

3) $f \epsilon G`_0(X, A)$ iff f is isotopic to 1_X by an isotopy which is fixed on A (denoted $f = 1_X$ (rel A)).
4) If A is a finite set, then $G_0(X,A) = G`_0(X,A)$ and if also $A = A`\cup A``$, then $G_0(X, A`, A``) = G_0(X,A)$.

II. Homeotopy Groups

In the fpllowing we develop some of the basic tools used in studying homeotopy groups of multiply punctured compact manifolds. We also obtain a presentation for $H^*(S^2, F_n)$where F_n is a finite subset of S^2.

Lemma1: Let X be a compact 2-manifold, $F = \{p_i : 1<i<n\}$a finite subset of X.

Define k: $H(X, F) \rightarrow H(X) \times H(F)$ by $k(fG_0(X, F)) = (fG_0(X), (f/F)G_0(F)) = (fG_0(X), f/F)$.Then k is an epimorphism with kernel $H^*(X, F) = (G`(X, F)\cap G_0(X))/G`_0(X, F)$.Therefore, we have the exact sequence $0 \rightarrow H^*(X, F) \rightarrow H(X, F) \xrightarrow{k} H(X) \times \$_n \rightarrow 0$ where we have identified H(F) with $\$_n$.

Proof. The map k is clearly well-defined and a homomorphism. Let $[gG_0(X), \alpha] \epsilon H(X) \times \$_n$. Suppose $g(p_i) = y_i$. By the homogeneity of X for finite subsets of X we can assume $g(p_i) = p_i$. By the remark following Lemma 1.2 (or homogeneity again), we can find a homeomorphism $h \epsilon G_0(X)$ with $h/F = \alpha$. So k is an epimorphism.

Now $fG_0(X,F) \epsilon$ ker k if and only if $f \epsilon G_0(X)$ and $f/F = 1_F$. That is $f \epsilon G_0(X)\cap G`(X, F)$. But for finite sets, $G_0(X,F) = G`_0(X,F)$. Thus,
ker $f = (G_0(X)\cap G`(X, F))/G`_0(X, F) = H^*(X, F)$.

Remark 1. By Lemma 1.1, $H(X,F) \cong H(X-F)$ so we also have the short exact sequence $0 \rightarrow H^*(X, F) \rightarrow H(X, F) \rightarrow H(X) \times \$_n \rightarrow 0$.

Remark 2. Similarly, if $\{D_i : 1\leq i\leq n\}$ is a family of disjoint discs in a closed 2-manifold X, then by Theorem 1.6, we have the short exact sequence

$$0 \rightarrow H^*(X, F) \rightarrow H(X - \cup_{i=1}^{n} \mathring{D}_i) \xrightarrow{k1} H(X) \times \$_n \rightarrow 0.$$

Lemma 2 Let X be a closed 2-manifold, $\{D_i : 1\leq i\leq m\}$ a family of disjoint discs in X with $p_i \epsilon \mathring{D}_i$. Let $\{p_i: m+1\leq i\leq m+r\}$be a set of points in X disjoint from $\cup_{i=1}^{m} D_i$.Consider the composition $\phi : H(X- \cup_{i=1}^{m} \mathring{D}_i, \cup_{j=m+1}^{m+r} p_j) \xrightarrow{k1} H(X- \cup_{i=1}^{m} \mathring{D}_i) \times \$_n \xrightarrow{(\psi, 1)} H(X, \cup_{i=1}^{m} p_i) \times \$_n \xrightarrow{(k2,1)} H(X) \times \$_m \times \$_r$ where k_1 is as in Remark 2 after Lemma 3.1, ψ is as in Theorem 1.6 and k_2 is as in Lemma 3.1. Then ϕ is an epimorphism with kernal$\cong H^*(X,\cup_{i=1}^{m+r} p_i)$.

Proof.ϕ is an epimorphism, since k_1, k_2, and ψ are epimorphisms. We have $fG_0(X- \cup_{i=1}^{m} \mathring{D}_i, \cup_{m+1}^{m+r} p_j)$ in the kernal of ϕ provided 1) $f_C \epsilon G_0 (X)$, 2) $f(\partial D_i) = \partial D_i$, $1\leq i\leq m$, and 3) $f(p_i) = p_i$, $m+1\leq i\leq m+r$. Now by Corollary 1.8 $H(X- \cup_{i=1}^{n} \mathring{D}_i,\cup_{m+1}^{m+r} p_j) \cong H(X- \cup_{1}^{m} p_i, \cup_{m+1}^{m+r} p_j)$and the image of ker ϕ under this isomorphism is $H^*(X, \cup_{i=1}^{m+r} p_i)$. This follows since $G_0(X, \cup_{1}^{m} p_i, \cup_{m+1}^{m+r} p_i) = G_0(X, \cup_{1}^{m+r} p_i)$.

Remarks.

1) Note that we have the short exact sequence $0 \rightarrow H^*(X, \cup_{1}^{m+r} p_i) \rightarrow H(X - \cup_{i=1}^{m} \mathring{D}_i - \cup_{m+1}^{m+r} p_i) \rightarrow H(X) \times \$_m \times \$_r \rightarrow 0$.

Notes

2) Since H(X) is known for X a closed 2-manifold (see Sections 2 and 6 of [1]), Lemmas 1 and 2 show that once we determine H*(X,F) for any finite set F and any closed 2-manifold X, the homeotopy group of any multiply punctured compact 2-manifold will be determined up to a group extension.
3) In the following we will determine a presentation for $H^*(S^2, F)$ for any finite subset F of S^2.

Lemma.3 Let X be a 2-manifold and let $x_0 \in \mathring{X}$. Then

1) G(X) is a fibre bundle over $\mathring{X}$ with fibre $G(X, x_0)$
2) $i_* : \pi_i(\mathring{X}, x_0) \cong \pi_i(X, x_0)\ \forall i$.

Proof. 1) is given in Lemma 4.10 of [3].
2) is given in Lemma 4.11 of [3].

Remarks.

1) The homotopy sequence of this fibration together with the isomorphism in 2) yields $\rightarrow \pi_1(G(X, x_0),1_X) \xrightarrow{i^*} \pi_1(G(X),1_X) \xrightarrow{p^*} \pi_1(X, x_0) \xrightarrow{d^*} \pi_0(G(X, x_0), 1_X) \xrightarrow{i^*} \pi_0(G(X),1_X) \rightarrow 0$.

   The map $i_* : \pi_0(G(X, x_0),1_X) \rightarrow \pi_0(G(X),1_X)$ is onto because any homeomorphism of X is isotopic to one that fixes x_0.

2) $\pi_0(G(X, x_0),1_X) \cong H(X, x_0)$ and $\pi_0(G(X),1_X) \cong H(X)$ and making these replacements above we get the homeotopy sequence of X

$$\rightarrow \pi_1(G(X, x_0),1_X) \xrightarrow{i^*} \pi_1(G(X),1_X) \xrightarrow{p^*} \pi_1(X, x_0) \xrightarrow{d^*} \pi_0(H(X, x_0), 1_X) \xrightarrow{i^*} H(X) \rightarrow 0.$$

3) Let X be a closed 2-manifold. Let $F_{n+1} = \{p_0, \ldots, p_n\}$ be a set of n+1 points in X and let $F_n = \{p_0, \ldots, p_{n-1}\} = F_{n+1} - p_n$. Consider the homeotopy sequence of X-F_n where we use p_n for the base point of X-$F_{n,}$ the last few terms are $\ldots \rightarrow \pi_1(X\text{-}F_n, p_n) \xrightarrow{d^*} H(X\text{-}F_n, p_n) \rightarrow H(X\text{-}F_n) \rightarrow 0$.

If, in this sequence, we replace $H(X\text{-}F_n, p_n)$ by a subgroup L which still contains im $d_* = \ker i_*$ and replace $H(X\text{-}F_n)$ by i_*L then the new sequence will also be exact. Now, since $G_0(X, F_{n+1}) = G'_0(X, F_{n+1})$, we have that $H^*(X, F_{n+1})$ is a subgroup of $H(X, F_{n+1})$. The restriction map $h \rightarrow h/X\text{-}F_n$ defines an isomorphism of $H^*(X, F_{n+1})$ with a subgroup L of $H(X\text{-}F_n, p_n)$.

Now, a homeomorphism g of X-F_n represents an element of L if and only if its unique extension $\bar{g}$toXx sends p_i to itself, $0 \leq i \leq n$, and is isotopic to 1_X (all p_i being allowed to move). $g \in G(X\text{-}F_n)$ represents an element of $\ker i_*$ provided its extension $\bar{g}$ sends each p_i to itself and is isotopic to 1_X (all p_n being allowed to move). Hence, $\ker i_* \subset$ L.

$f \in G(X\text{-}F_n)$ represents an element of i_*L provided $\bar{f}$ takes each p_i to itself $0 \leq i \leq n$-1 and $\bar{f}$ is isotopic to 1_X (all p_i being allowed to move). Thus, i_*L can be identified with $H^*(X, F_n)$.

Now in the homeotopy sequence of X-F_n with p_n as base point, we replace $H(X\text{-}F_n, p_n)$ by its subgroup L and $H(X\text{-}F_n)$ by i_*L. Next, we replace L by its isomorph $H^*(X, F_{n+1})$ and i_*L by its isomorph $H(X, F_n)$ getting the exact sequence

$$\ldots \rightarrow \pi_1(X\text{-}F_n,p_n) \xrightarrow{d} H^*(X,F_{n+1}) \xrightarrow{e} H^*(X, F_n) \rightarrow 0.$$

4) Note if $[\tau]$ $\pi_1(X\text{-}F_n,p_n)$ where τ is a loop in X-F_n based at p_n, then $d([\tau])$ $H^*(X, F_{n+1})$ is determined as follows:

   Let H_t be an isotopy of X beginning at 1_X which drags the point p_n about the loop τ while leaving each p_i, $0 \leq i \leq n$-1 fixed; then $d([\tau])$ is represented by the

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1. D.B.A. Epstein, "Curves on 2-manifolds and isotopies" Acta. Math. 115 (1966) 83-107.

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homeomorphism H_1. Thus, the isotopy class (rel F_{n+1}) of a homeomorphism h represents $d([\tau])$, provided $h = 1_X$ (relF_n) and the isotopy H_t from 1_X to h is such that $H_t(p_n) = \tau$. Note: The map e, on the level of homeomorphisms, takes each h to itself.

Lemma 4 Let $F_n = \{p_0, \ldots, p_{n-1}\}$ denote a set of n points in S^2. It $n \geq 3$, then $\pi_1(G(S^2\text{-}F_n), 1) = 0$.

Proof. This is stated in Theorem 3.1 of [1]. See also page 303 of [3].

Remark. Let $p_0, p_1, \ldots, p_n, \ldots$ be a sequence of distinct points in S^2 and let $F_n = \{p_0, \ldots, p_{n-1}\}$. Then, in view of Lemma 3.4 we have $0 \to \pi_1(S^2\text{-}F_n, p_n) \to H^*(S^2\text{-}F_n) \to 0$ is exact for $n \geq 3$.

Lemma 3.5 Given an exact sequence $0 \to A \xrightarrow{f} B \xrightarrow{g} C \to 0$. Let $\{a_i, \ldots, a_n\}$ generate A and $\{c_1, \ldots, c_m\}$ generate C. Suppose $b_1, \ldots, b_r$ are elements of B satisfying $f(a_i) = b_i$, 1 i r and $b`_1, \ldots, b`_m$ in B satisfy $g(b`_i) = c_i$, $1 \leq i \leq m$, then $\{b_1, \ldots, b_r, b`_1, \ldots, b`_m\}$ generates B.

Proof. Let $b \in B$. Then $g(b) \in C$, i.e., $g(b) = w_1(c_i)$ where $w_1(c_i)$ is a word in $c_1, \ldots, c_m$. Thus, $g(b) = w_1(g(b`_i)) = g(w_1(b`_i))$. So $g(bw_1(b`_i))^{-1}) = 1_C$. Now, $bw_1(b`_i)^{-1} \in \ker g = \mathrm{lm}\ f \Rightarrow bw_1(b`_i)^{-1} = f(w_2(a_i))$ where $w_2(a_i)$ is a word in $a_1, \ldots, a_r$. Hence, $bw_1(b`_i)^{-1} = w_2(f(a_i)) = w_2(b_i)$. So $b = w_1(b`_i)w_2(b_i)$.

Next, we define "twist homeomorphisms". These will be used to obtain generators for $H^*(S^2, F_n)$.

Let $P = p_0, p_1, \ldots$ be a sequence of points in the interior of a disc D which converge to $p_\infty \in \mathring{D}$. Let α be an oriented simple closed curve in $\mathring{D}$-P and let D` denote the closure of that component of D-α which forms an open disc. We define the "twist homeomorphism of D supported by a``, denoted h_α, as follows:

Let A denote a collar neighborhood of α in the disc D` with $A \cap P = \emptyset$ (see Figure 1). Let e: $S`\times I \to A$ be a homeomorphism with $e/S`\times I = \alpha$ where $S`\times I$ is oriented as in Figure 2. Define g: $S`\times I \to S`\times I$ by $g(x,t) = (x-t,t)$ where $S` = R/Z$ (see Figure 2). Finally, we define h_α: $(D, P) \to (D, P)$ by $h_\alpha(x) = ege^{-1}(x)$ for $x \in A$ and $h_\alpha(x) = x$ for $x \in D\text{-}A$.

Now, suppose $D \subset \mathring{X}$ where X is a 2-manifold. Since h_α is the identity on ∂D we can extend h_α to X by the identity. This new homeomorphism will also be denoted h_α and called the twist homeomorphism of X supported by α.

Remark 1. Different choices for the annulus A, satisfying the above conditions, result in different choices for h_α. However, each of these possibilities for h_α are ambient isotopic (rel P) to one another by the regular neighborhood theorem (applied to ∂D` in D`-P). Since we are interested only in the isotopy classes (rel P) of homeomorphisms, we will abuse the notation slightly and use h_α to denote any homeomorphism which results from the above process.

Remark 2. In [6] twist homeomorphisms (or "c-homeomorphisms") are defined for any simple closed curve B in a closed 2-manifold X as follows: Let e`: $S`\times I \to A`$ be a regular neighborhood of B in X with $e`/S` \times æ = B$ and define g`: $A` \to A`$ as indicated in Figure 3. Define the "c-homeomorphism corresponding to B" to be the homeomorphism obtained by conjugating g` by e` and extending by the identity. In the case B bounds a disc D`, this definition yields our definition of $h_B$ provided we add the condition $e`(S` \times 0) \subset D`$. Without this condition it is possible for two different embeddingse`: $S`\times I \to A`$ and e``: $S`\times I \to A``$ to yield two non-isotopic "c-homeomorphisms", namely a homeomorphism and its inverse. Thus, the isotopy class of a "c-homeomorphism" is not uniquely determined by the curve B.

Notes

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Remark 3. With the notational abuse mentioned in Remark 1 we can write $h_{\alpha}^{-1} = (h_\alpha)^{-1}$.

Remark 4. For any n, h_α is defined in X-F_n where $F_n = \{p_0, \dots , p_{n-1}\}$ to be the restriction of h_α to X-F_n.

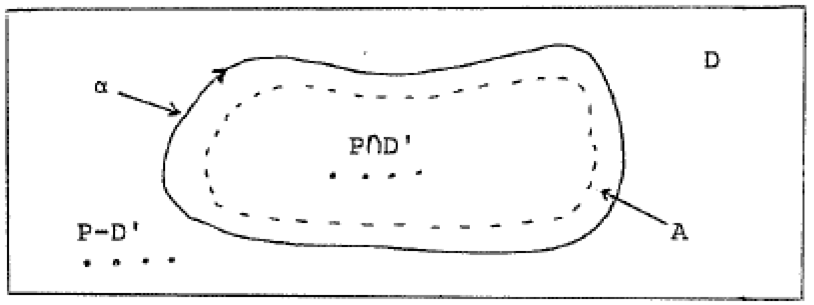

Figure 1

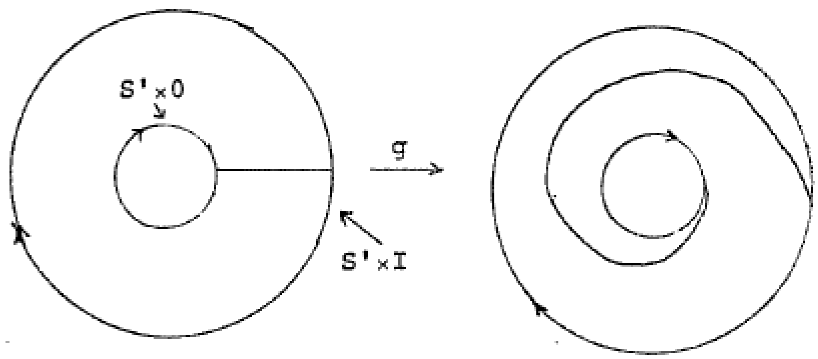

Figure 2

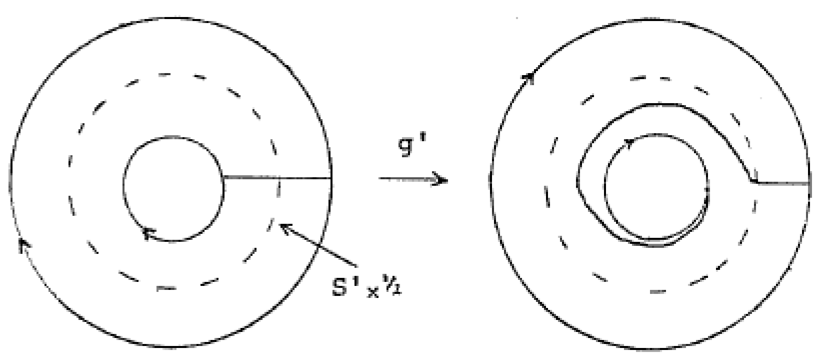

Figure 3

The following two lemmas are versions of lemmas given in [7]. The proofs follow almost immediately from the definitions.

Lemma 6 Let x be a 2-manifold and let F be a finite set of points in $\mathring{X}$. Let D be a disc in $\mathring{X}$ with $F \subset \mathring{D}$ and let α be a simple closed curve in $\mathring{D}$-F. Let f: (X, F) -> (X, F) be a homeomorphism; then $h_{f\circ\alpha} \simeq f h_\alpha f^{-1}$(rel F).

Proof. If e : S`×I → A is the embedding used to define $h_\alpha$, then $f\circ e$ : S`×I → f(A) can be used to define $h_{f\circ\alpha}$. Hence, $h_{f\circ\alpha}(x) = (f\circ e)g(f\circ e)^{-1}(x)$ on f(A) and $h_{f\circ\alpha}(x) = x$ elsewhere. That is, $h_{f\circ\alpha}(x) = f\circ h_\alpha \circ f^{-1}(x)$ on f(A) and $h_{f\circ\alpha}(x) = x$ elsewhere. But $f\circ h_\alpha\circ f^{-1}(x) = x$ for x ˜ϵ f(A). Thus, $h_{f\circ\alpha} = f\circ h_\alpha\circ f^{-1}$everywhere on X.

Ref

7. D. Sprows, "Boundary fixed homeomorphisms of 2-manifolds with boundary" Global Journal of Science Frontier Research 11 (2011) 13-15.

Lemma 7 Let F, D, and X be as in Lemma 3.6. Let α, $\beta \in \mathring{D}$-F be simple closed curves with $\alpha \simeq \beta$ (rel F), i.e., α is ambient isotopic to β (rel F) in X. Then $h_\alpha \simeq h_\beta$(rel F).

Proof. Let $H_t : X \to X$ be the ambient isotopy with $H_1\alpha = \beta$, then by Lemma 3.6 $h_\alpha \simeq H_1 h_\alpha H_1^{-1}$. Thus, $F_t = H_t h_\alpha H_t^{-1}$. is an isotopy between h_α and h_β.

We now concentrate on S^2. Let P = $p_0, \ldots, p_n, \ldots$ be a sequence of points in S^2 converging to a point p. Let i¡j and let α_{ij} be the simple closed curve in S^2 given in Figure 4, i.e., α_{ij} is oriented in a clockwise direction about p and encloses the points p_i and p_k for $k \geq j$ as indicated. Note that for pictorial purposes S^2 appears as a disc with q as its boundary, but on S^2, q is identified to a single point. We let a_{ij} denote the homeomorphism $h_{\alpha ij}$ where to define $h_{\alpha ij}$we take a disc about P not containing the point q.

Notes

Lemma 8 Let $n \geq 3$ and let $F_n = \{p_0, \ldots, p_{n-1}\} \subset P$. Consider the short exact sequence $0 \to \pi_1(S^2\text{-}F_n,\ p_n) \xrightarrow{d} H^*(S^2,\ F_{n+1}) \xrightarrow{e} H^*(S^2,\ F_n) \to 0$ given in the remark following Lemma 3.4. If a_{ij} is defined as above, then the image of d is generated by $\{\bar{a}_{in} : 1 \leq i \leq n\}$ where $\bar{a}_{in}$= isotopy class of a_{in} in $H^*(S^2, F_{n+1})$.

Proof. The loops α_{in} can be deformed slightly to yield loops B_{in} passing thru p_n such that the homotopy classes $[B_{in}^{-1}]$, $1 \leq i < n$, generate $\pi_1(S^2\text{-}F_n,\ p_n)$. Let b_{in}^{-1} be the homeomorphism which dials the point p_n once around B_{in}^{-1} as indicated in Figure 5. The identity is clearly isotopic to b_{in}^{-1} (relF_n) by an isotopyH_t with the property that $H_t(p_n) = B_{in}^{-1}$. Thus, as indicated in Remark 4 following Lemma 3, we have $d[B_{in}^{-1}] = b_{in}^{-1}$. On the other hand,$b_{in}^{-1} = \bar{a}_{in}$, i.e., $b_{in}^{-1} \simeq a_{in}^{-1}$ keeping F_{n+1} fixed. To see this just note that on the disc about p_i bounded by B_{in}^{-1} we have b_{in}^{-1} restricted to the boundary of this disc is the identity. Hence, b_{in}^{-1} restricted to this can be isotope to the identity on this disc by an isotopy which keeps the boundary of the disc fixed. Extending this isotopyby the identity to all of S^2 we see b_{in}^{-1} is isotopic to a_{in} (relF_{n+1}). Note if we denote the isotopy from b_{in}^{-1} to a_{in} by I_t, then $I_{1/2}$ is given in Figure 6.

Theorem 9 For $n \geq 3$, $H^*(S^2,\ F_{n+1})$ is generated by$\cup_{k=3}^{n} \{\bar{a}_{ik} : 1 \leq i < k\}$where $\bar{a}_{ik}$is the isotopy class of a_{ik} in $H^*(S^2, F_{n+1})$.

Proof. Recall the short exact sequence $0 \to H^*(S^2, F_{n+1}) \to H^*(S^2, F_n) \xrightarrow{k} H(S^2) \times \mathscr{S}_n \to 0$ of Lemma 3.1. By Theorem 3.1(c) of [1], $H^*(S^2, F_3) \simeq H(S^2) \times \mathscr{S}_3$. Hence, $H^*(S^2, F_3) = 0$. In particular, letting n=3 in the sequence $0 \to \pi_1(S^2 - F_n, p_n) \xrightarrow{d} H^*(S^2, F_{n+1}) \xrightarrow{e} H^*(S^2, F_n) \to 0$ we have $\pi_1(S^2 - F_3,\ p_3) \cong H^*(S^2,\ F_4)$. Thus, by Lemma 3.8, $H^*(S^2,\ F_4)$ is generated by $\{\bar{a}_{13}, \bar{a}_{23}\}$. Inductively, assume ${}^{n-1}\cup_{k=3} \{\bar{a}_{ik} G_0(S^2,\ F_n) : 1 \leq i < k\}$generates $H^*(S^2,\ F_n)$. Now, $e(a_{ik} G_0(S^2,\ F_{n+l})) = a_{ik} G_0(S^2,\ F_n)$. Hence, $\cup_{k=3}^{n-1} \{e(\bar{a}_{ik}) : l \leq i < k\}$ generates $H^*(S^2,\ F_n)$. By Lemma 3.8, $\{\bar{a}_{ln}, \ldots, \bar{a}_{n\text{-}ln}\}$ generates the image of d. The Theorem now follows by applying Lemma 3.5.

Notation: We let $G_n = \cup_{k=3}^{n} \{\bar{a}_{ik} : 1 \leq i < k\} \subset H^*(S^2, F_{n+1})$.

Remark. G_n is a generating subset of $H^*(S^2, F_{n+1})$ by Theorem 9. The remainder of this chapter will be devoted to finding a complete set of relations among these generators.

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Lemma 10 As in Lemma 3.5, let $0 \to A \xrightarrow{f} B \xrightarrow{g} C \to 0$ be a short exact sequence of groups. Let $\{a_1,\ldots,a_r\}$ generate A and $\{c_1,\ldots, c_r\}$ generate C. Let $b_i = f(a_i)$, $1 \leq i \leq r$ and $g(b_1') = c_i$, $1 \leq i \leq m$; then (i) B has a presentation with generators $\{b_1,\ldots, b_r, b_1',\ldots, b_m'\}$ in which ever relation has the form (1) $w(b_i) = w'(b_i')$, (2) $b_j' b_i b_j'^{-1}$, (3) $b_j'^{-1} b_i b_j' = w(b_i)$where $w(b_i)$ denotes a word in $b_1,\ldots, b_r$and $w'(b_i')$ denotes a word in $b_1',\ldots, b_m'$. (ii) Moreover, if the exact sequence splits, i.e., if there exists a homomorphism $k: C \to B$ with $g \circ k = 1_C$ and if we suppose that $b_i' = k(c_i)$ for $1 \leq i \leq m$, then every relation of form (1) can be expressed as (1.1) $w(b_i) = 1$ and (1.2) $w'(b_i') = 1$.

Proof. Since f (A) is normal in B we have that relations of the form (2) $b_j' b_i' b_j'^{-1} = w(b_i)$and (3) $b_j'^{-1} b_j' b_i' = w(b_i)$ exist for $1 \leq i \leq r$ and $1 \leq j \leq m$. Using relations of form (2) and (3) any other relator can be rewritten in the form $w(b_i)w'(b_i')$ and then transposed into form (1). Thus, part (i) of the lemma holds.

Part (ii) of Lemma 10 follows immediately from part (i), since if the sequence splits, then $f(A) \cap k(C) = \{1_B\}$, so any relation $w(b_i) = w'(b_i')$ reduces to $w(b_i) = 1$ and $w'(b_i') = 1$.

Remark. Suppose that for a given k and j we have that a relation of the form $b_k = b_j' w(b_i) b_j'^{-1}$ is a consequence of the relations of type (1) and (2) of Lemma 3.10. Then it follows that $b_j'^{-1} b_k b_j' = w(b_i)$. Hence, the corresponding relation of type (3) can be dropped from the given presentation.

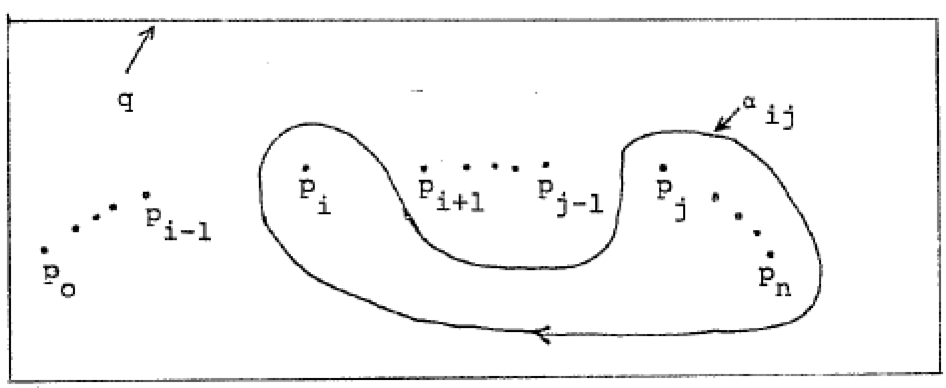

Figure 4

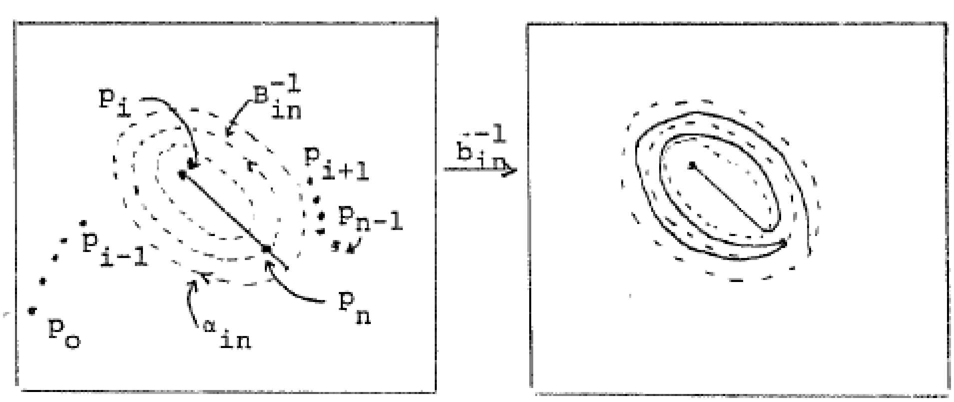

Figure 5

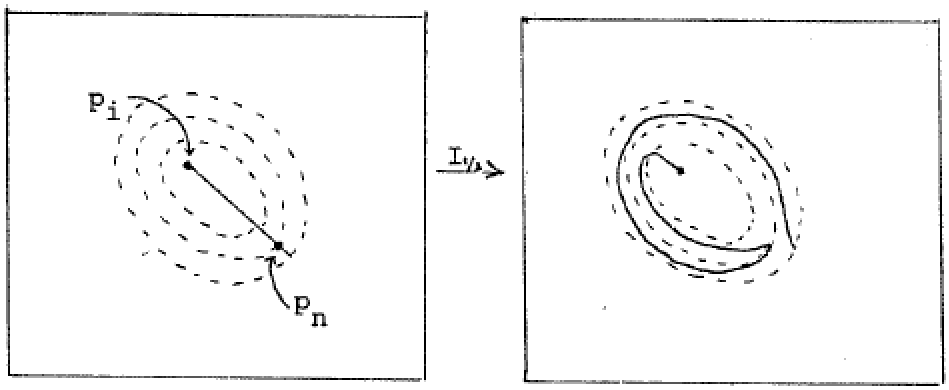

Figure 6

Remark 1. We will apply Lemma 10 to the short exact sequence $0 \to \pi_1(S^2\text{-}F_n, P_n) \xrightarrow{d} H^*(S^2, F_{n+1}) \xrightarrow{e} H^*(S^2, F_n) \to 0$. In the proof of Theorem 9 it is shown that the set $G_{n-1} = \cup_{k=3}^{n-1} \{\bar{a}_{ik} : 1 \leq i \leq k\}$ maps onto a generating set for $H^*(S^2, F_n)$ and the set $G_n - G_{n-1} = \{\bar{a}_{in}: 1 \leq i < n\}$ generates the image of d. Hence by part (i) of Lemma 3.10, $H^*(S^2, F_{n+1})$ has a presentation using these generators in which every relation has the form (1) $w(\bar{a}_{ik}) = w`(\bar{a}_{in})$, (2) $\bar{a}_{ik}\bar{a}_{rn}\bar{a}_{ik}^{-1} = w`(\bar{a}_{jn})$, (3) $\bar{a}_{ik}^{-1}\, \bar{a}_{rn}\bar{a}_{ik} = w`(\bar{a}_{jn})$ where in (1), (2), (3) we assume k<n.

Remark 2. The Technique for determining relations of types 2 and 3 is based on Lemmas 3.6 and 3.7. We will describe the approach for type 2 relations. Suppose $a_{ik}(\alpha_{rn}) = \gamma$. Then by Lemma 3.6, and the fact that a_{rn} is the twist homeomorphism corresponding to α_{rn}, we have $h_\gamma \simeq a_{ik}a_{rn}a_{ik}^{-1}$ (relF_{n+1}). Suppose also we can find a product of homeomorphisms $w`(a_{jn})$ such that $w`(a_{jn})(\alpha_{rn}) = \gamma` \simeq \gamma$ (rel$F_{n+1}$), then by Lemma 3.6 we have $h_{\gamma} \simeq w`(a_{jn})a_{rn}w`(a_{ik})^{-1}$ (rel $F_{n+1}$). Now $h_\gamma \simeq h_\gamma$, (rel$F_{n+1}$) by Lemma 3.7. Hence $a_{ik}a_{rn}a_{ik}^{-1} \simeq w`(a_{jn})a_{rn}w`(a_{ik})^{-1}$and therefore $\bar{a}_{ik}\bar{a}_{rn}\bar{a}_{ik}^{-1} \simeq w`(\bar{a}_{jn})\bar{a}_{rn}w`(\bar{a}_{ik})^{-1} = w``(\bar{a}_{jn})$ is the desired relation of type 2. To adapt this technique to yield type 3 relations, just replace a_{ik} with a_{ik}^{-1}.

Remark 3. The next lemma is needed in the proof of Lemma 3.12, which in turn supplies the basis for applying the technique in Remark 2.

Lemma 11 Let D_n be a disc and $D_1,\ldots,D_{n-1}$ a family of disjoint discs in $\mathring{D}_n$. Let $B = \cup_{i=1}^{n-1} D_i$. Let r_i be an arc from ∂D_i to ∂D_{i+1}, $1 \leq i \leq n\text{-}1$, as indicated in Figure 3.7, i.e. the r_i are disjoint arcs which have only their end points in common with $B \cup \partial D_m$. Let $g: D_n \to D_n$ be a homeomorphism which is fixed on $B \cup \partial D_n$ and is such that $g(\cup_{i=1}^{n} r_i) = \cup_{i=1}^{n} r_i$, then $g \simeq 1$ (rel $B \cup \partial D_n$).

Proof. Let $S_n = D_n - \mathring{B}$ and let $g` = g/S_n$. We will show that $g` \simeq 1$ (rel∂S_n) and then conclude $g \simeq 1$ (rel$\partial D_n \cup B$) by extending the isotopy for g` over each disc $D_i$ by the identity. For each i, $g`/r_i$ is a homeomorphism which fixes the end points of r_i, thus it will be isotopic to the identity on $r_i$by an isotopy of $r_i$which keeps the end points fixed. Such an isotopy can be covered by an isotopy of S_n which is the identity off a small relative neighborhood of r_i (mod r_i) and is the identity on ∂S_n. Combining these isotopies we get g` is isotopic (rel∂S_n) to a homeomorphism which fixes r_i for each i.

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Thus without loss of generality we can assume $g`/\cup r_i$ is the identity. Let D` be the disc formed by cutting S_n along $\cup_{i=1}^{n} r_i$. Let $p : D` \rightarrow S_n$ be the identification map. Define $h : D` \rightarrow D`$ by $h/\partial D` = 1$ and $h/D` - \partial D` = p^{-1}g`p$. By Lemma 1.4 $h \simeq 1$ (rel$\partial D`$). Let $G`_t: D` \rightarrow D`$ be an isotopy with $G`o = h$ and $G`, =1$ where $G`_t$ is a homeomorphism which fixes $\partial D`$ for each t. Let $G_t : S_n \rightarrow S_n$ be given by $G_t(x) = x$ if x is in $\cup r_i \cup \partial S_n$ and $G_t(x) = pG_t`p^{-1}(x)$ otherwise. Then $G_o = g`$ and $G_l = 1$ and also $G_t$ fixes $\partial S_n$ for all t. This $G_t$ is the desired isotopy between g` and the identity on S_n.

Lemma 12 Let D be a disc, $I = \{i_1,\ldots,i_p\}$, $J = \{j_1,\ldots, j_q\}$, $K = \{k_1,\ldots, k_r\}$ finite sets in $\mathring{D}$. Let γ_i, γ_j, γ_k,γ_{ij}, γ_{ik}, γ_{jk} and γ_{ijk} be the simple closed curves given in Figure 3.8 and let h_i, h_j, h_k, h_{ij}, h_{ik}, h_{jk}, and h_{ijk} be the corresponding twist homeomorphisms, then $h_{ik} \simeq h_i h_j h_k h_{ijk} h_{ik}^{-1} h_{jk}^{-1}$ (rel$\partial D \cup I \cup J \cup K$).

Moreover this isotopy can be taken to be fixed on suitably small discs D_1, D_2, D_3 in $\mathring{D}$ about I, J, K respectively.

Proof. Let D_1, D_2, D_3 be discs about I, J, K as in Figure 3.9 and let r_1, r_2, r_3 be arcs as given in Figure 3.9. Let $B = \cup_{i=1}^{3} D_i \cup \partial D$. By Lemma 3.11 it suffices to show we can isotope $h_{ik}(\cup_{i=1}^{3} r_i)$ to $h_i h_j h_k h_{ijk} h_{ik}^{-1} h_{jk}^{-1} (\cup_{i=1}^{3} r_i)$ (rel B). Figure 10 gives $h_{ik}(r_1)$ and Figure 12 gives $h_i h_j h_k h_{ijk} h_{ik}^{-1} h_{jk}^{-1}(r_1) = h_i h_j h_{jk}^{-1}(r_1)$. Clearly the curves in Figures 10 and 12 are isotopic (rel B). Figure 13 gives $h_{ik}(r_2)$. The curve γ given in Figure 16 is isotopic (rel B) to $h_{jk}^{-1} h_{ij}^{-1}(r_2)$ and hence the curve given in Figure 17 is isotopic (rel B) to $h_i h_j h_k h_{ijk} h_{ik}^{-1} h_{jk}^{-1}$. But clearly the curves in Figures 13 and.17 are isotopic.

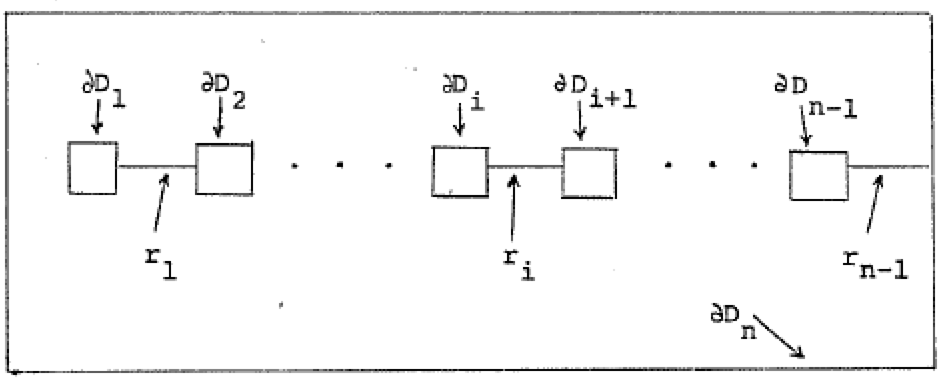

Figure 7

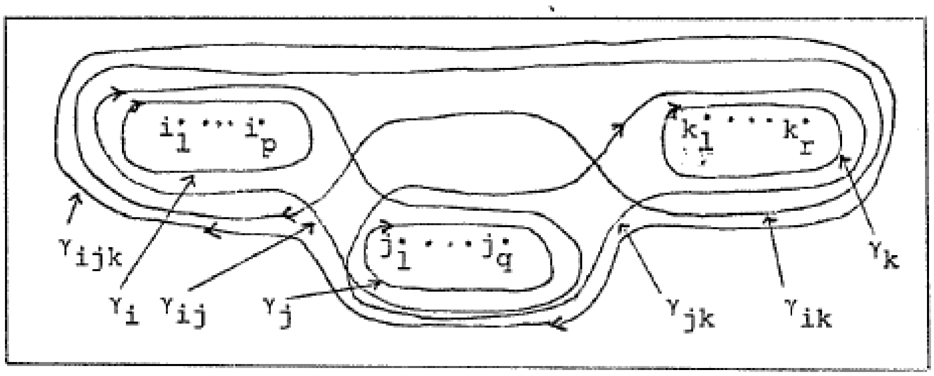

Figure 8

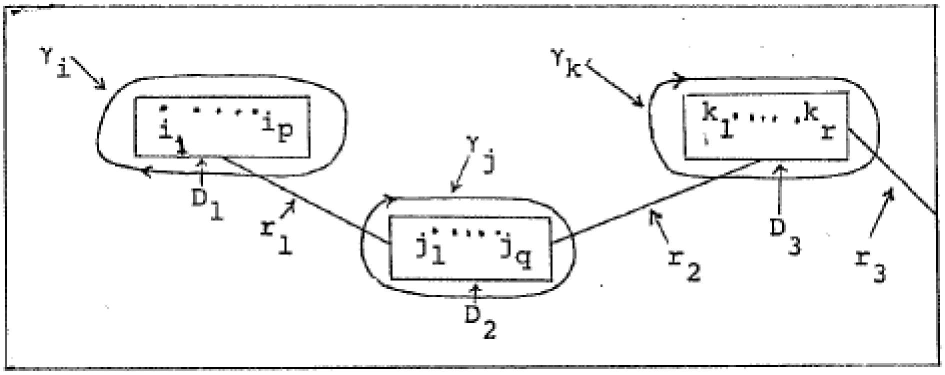

Figure 9

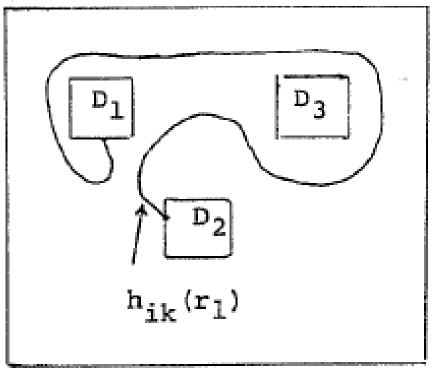

Figure 10

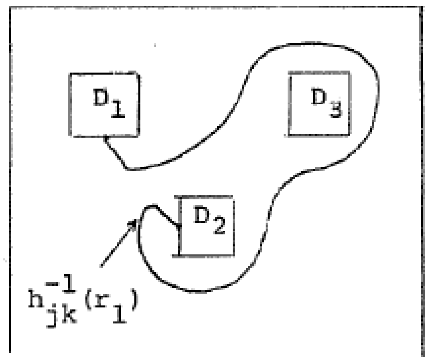

Figure11

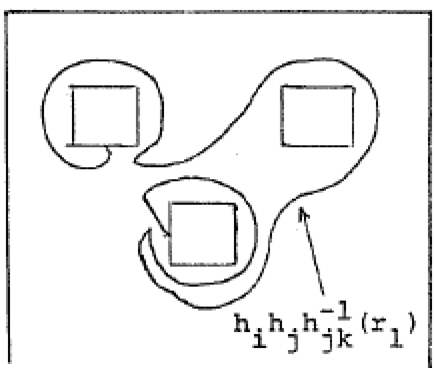

Figure 12

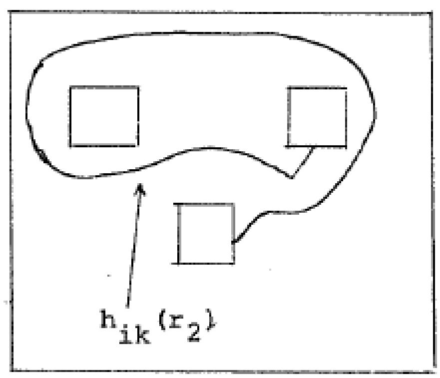

Figure 13

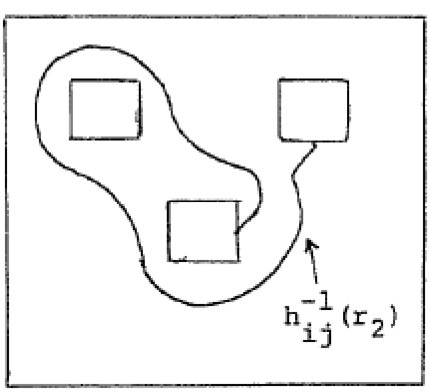

Figure14

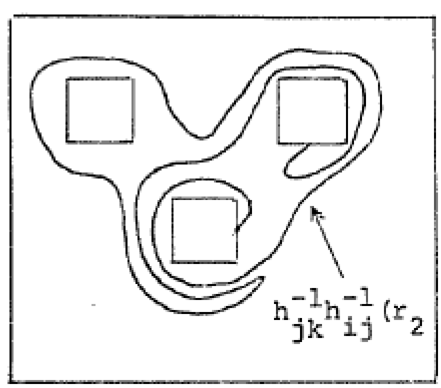

Figure 15

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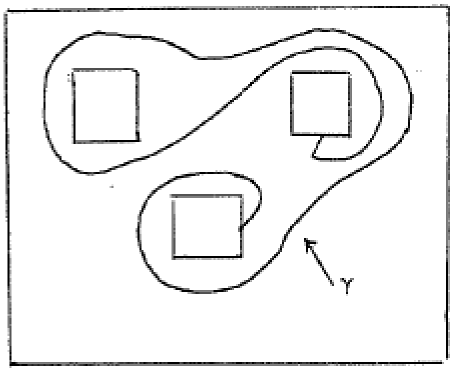

Figure 16

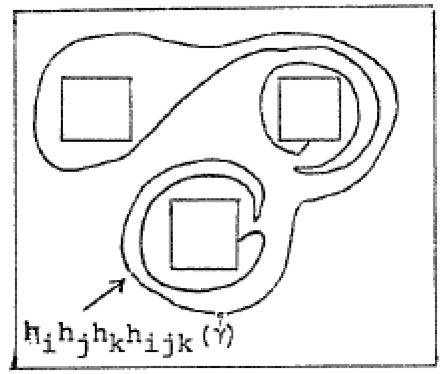

Figure 17

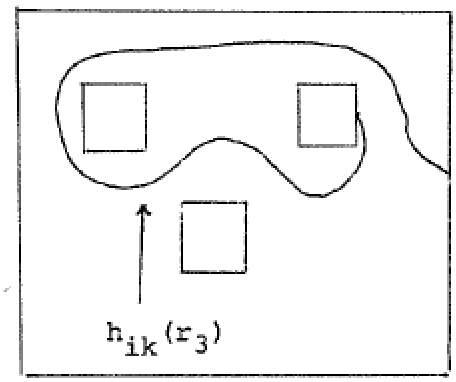

Figure 18

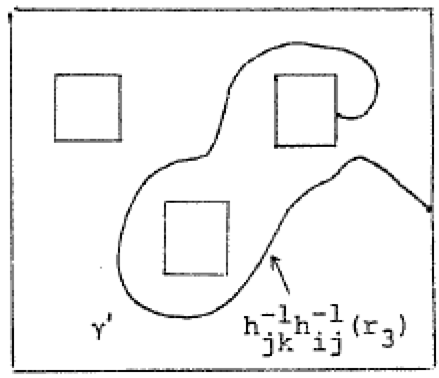

Figure 19

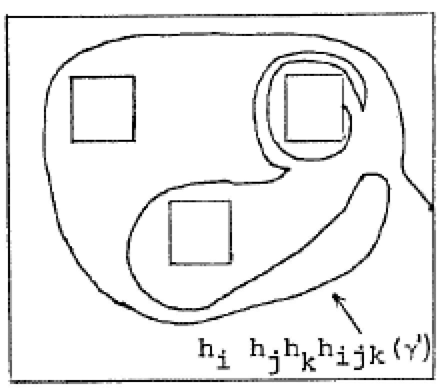

Figure 20

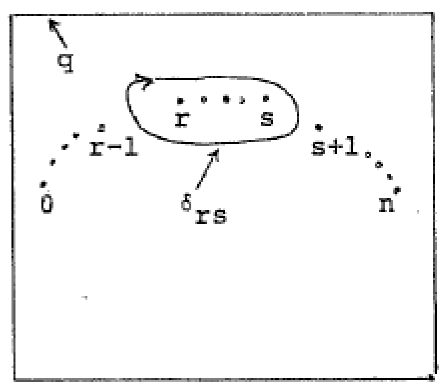

Figure 21

Finally the fact that the curves in Figures 18 and 20 are isotopic (rel B) implies that $h_{ik}(r_3)$ is isotopic (rel B) to $h_i h_j h_k h_{ijk} h_{ik}^{-1} h_{jk}^{-1}(r_3)$. Combining these isotopies we have the desired result.

Remark. In the above, if I, J, or K is singleton, then the corresponding twist homeomorphism h_i, h_j or h_k is isotopic to the identity and hence can be dropped from the statement of the lemma.

Remark 2. In Lemma 12, the homeomorphisms h_i, h_j or h_k commute with every homeomorphism in the lemma.

Remark 3. If the curves given in Lemma 12 are in the interior of a 2-manifold X, then we can consider the corresponding twist homeomorphisms as defined on X. In particular if F is a finite subset of X° and $L = I \cup J \cup K$ is a subset of F with F-L outside the disc bounded by γ_{ijk}, then we have $h_{ik} \simeq h_i h_j h_k h_{ijk} h_{ik}^{-1} h_{jk}^{-1}$ (rel F). Moreover this isotopy can be taken to be fixed on a family of disjoint discs in $^{\circ}$X where each disc contains a single point of F in its interior. In the following we will sometimes refer to the statement in this remark as Lemma 12 (a).

Lemma 13 Let r<s and let δ_{rs} be the simple closed curve in S^2 given in Figure 3.21 and let C_{rs} be the corresponding twist homeomorphism, then $C_{rs} \simeq a_{rr+1} a_{s+ls+2}^{s-r} a_{ss+l}^{-1} a_{s-ls+l}^{-1} \dots a_{rs+l}^{-1}$ (relF_{n+1}). Moreover this isotopy can be taken to be fixed a suitably small discs about each p_i in F_{n+l}.

Proof. Induct on p = s-r. If p=1, then $c_{s-ls} \simeq a_{s-ls} a_{s+ls+2} a_{ss+l}^{-1} a_{s-ls+l}^{-1}$ (relF_{n+1}) follows from Lemma 12 (a) by letting $\{i_1,\dots,i_p\} = \{s\text{-}l\}$, $\{j_1,\dots,j_q\} = \{s+l,\dots, n\}$ and $\{k_1,\dots, k_r\} = \{s\}$. See Figure 3.22. Now by induction assume (1) $c_{r+ls} \simeq a_{r+lr+2} a_{s+ls+2}^{s-r-l} a_{ss+l}^{-1} a_{s-ls+l}^{-1} \dots a_{r-ls+l}^{-1}$ (relF_{n+1}) claim (2) $c_{rs} \simeq a_{rr+1} a_{s+ls+2} a_{r-lr+2}^{-1} c_{r+ls} a_{rs+l}^{-1}$. Note that once (2) is established, substituting (1) into (2) gives the desired result. Letting $\{i_1,\dots,i_p\} = \{r\}$, $\{j_1,\dots,j_q\} = \{s+l,\dots, n\}$, $\{k_1,\dots, k_r\} = \{r+l,\dots, s\}$ in Lemma 3.12(a), we have (3) $c_{rs} \simeq a_{s+ls+2} c_{r+ls} a_{rr+1} a^{-1}_{r+lr+2} a^{-1}_{rs+l}$. See Figure 3.23. Keeping in mind which of these homeomorphisms commute, (2) now follows.

Lemma 14 Let $n \geq m \geq 3$ and $l \leq r < m$ and i<k.

(1) If i=r or $i<r\geq k$, then $a_{ik} a_{rm} a_{ik}^{-1} \simeq a_{rm}$ (relF_{n+l}).

(2) If i>r, then $a_{ik} a_{rm} a_{ik}^{-1} \simeq (a_{im}(a_{km} \dots a_{m-lm})) a_{rm} (a_{im}(a_{km} \dots a_{m-lm}))^{-1}$ (relF_{n+l}).

(3) If i<r<k, then $a_{ik} a_{rm} a_{ik}^{-1} \simeq ((a_{km} \dots a_{m-lm}) a_{im}) a_{rm} ((a_{km} \dots a_{m-lm}) a_{im})^{-1}$ (relF_{n+l}).

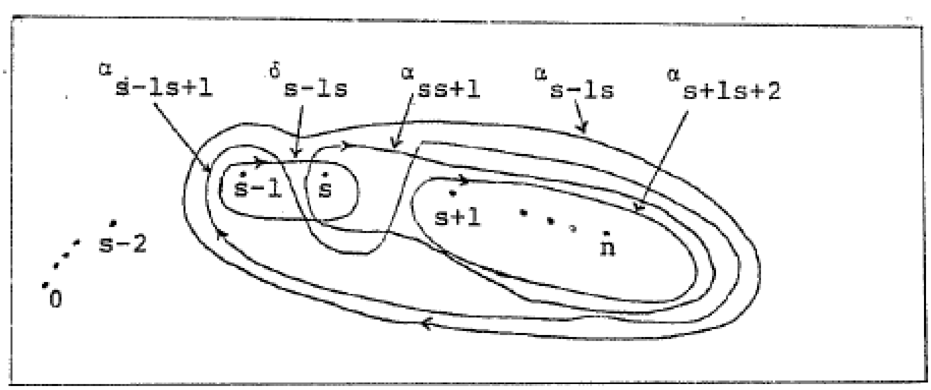

Figure 22

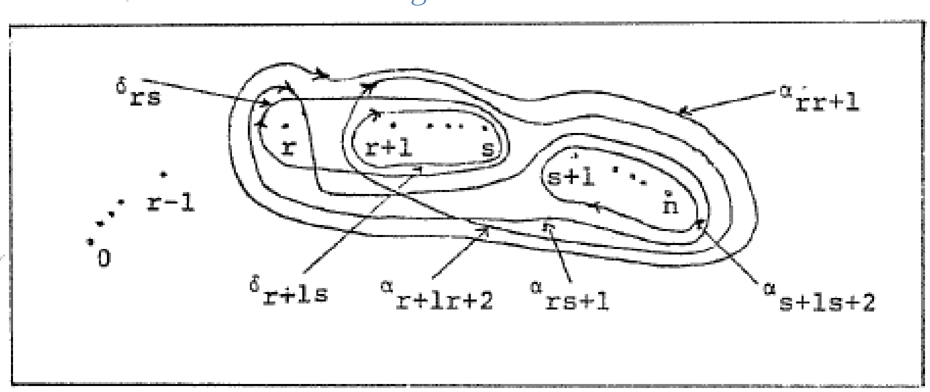

Figure 23

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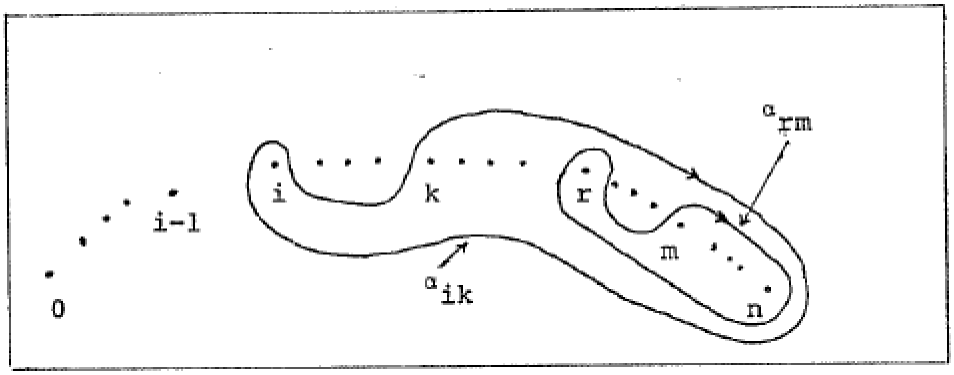

Figure 24

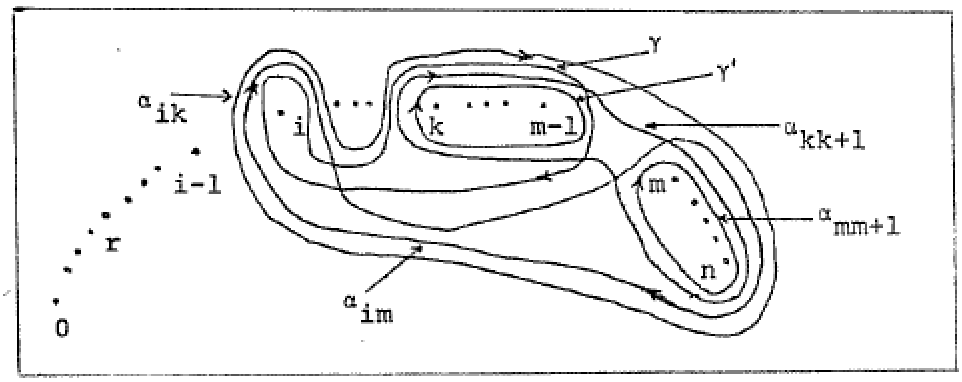

Figure 25

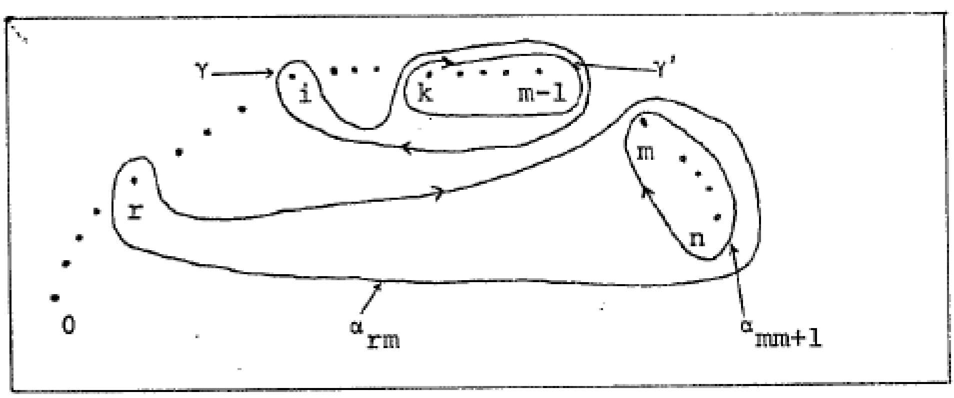

Figure 26

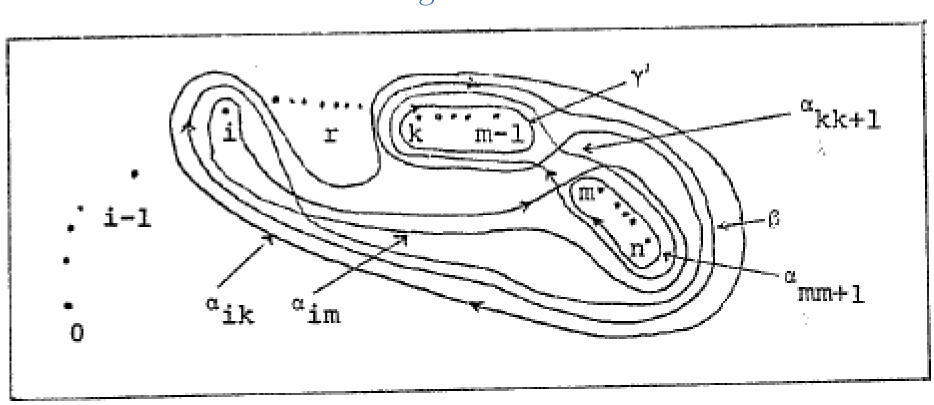

Figure 27

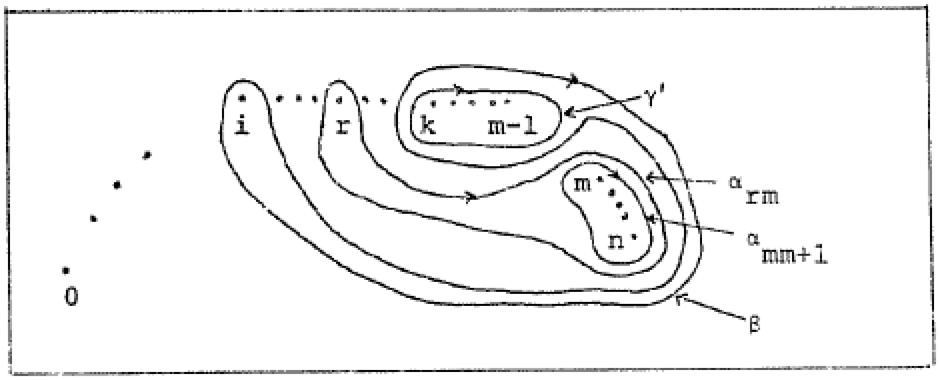

Figure 28

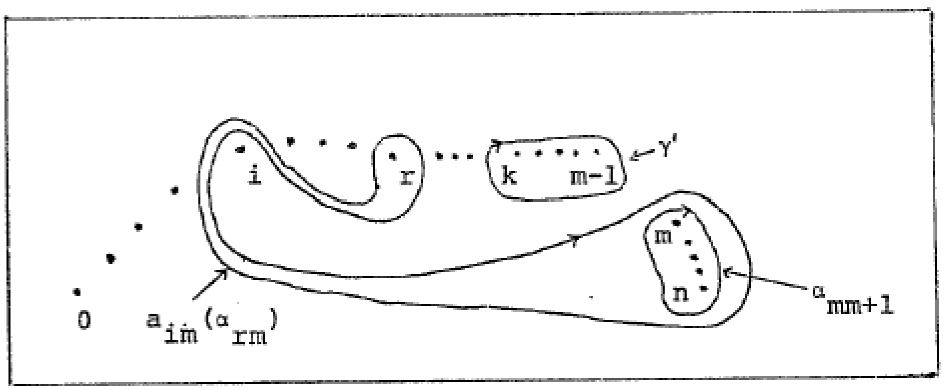

Figure 29

Moreover all the isotopes in Lemma 14 can be taken to be fixed on a family of disjoint discs $D_o,\ldots,D_n$ with p_i in $\mathring{D}_I$ and to be equal to the identity outside a disc about F_n.

Proof. Let $B = \cup_{i=o}^{n} D_i\ (S - \mathring{D}`)$ where D` is a disc containing F_n. In the proof we use "$h \simeq g$" to denote "$h \simeq g$ (rel B)".

(1) If i=r or $i<r\geq k$, then α_{ik}can be takento be disjoint from α_{rm}. This means that the corresponding twist homeomorphisms commute. See Figure 3.24.

(2) Let i>r. By Lemma 12 (a) we have $h_\gamma \simeq h_{\gamma`} a_{mm+1} a_{ik} a_{kk+1}^{-1} a_{im}^{-1}$ where γ and γ` are are given Figure 25. Keeping in mind the commuting properties of the above homeomorphisms we can solve for a_{ik} to yield $a_{ik} \simeq a_{im} a_{kk+1} a_{ik} h_\gamma h_{\gamma`}^{-1} a_{mm+1}^{-1}$. In particular, $a_{ik}(\alpha_{rm}) \simeq a_{im} a_{kk+1} a_{ik} h_\gamma h_{\gamma`}^{-1} a_{mm+1}^{-1}(\alpha_{rm})$. But $h_\gamma, h_\gamma$and a_{mm+1} are all the identity when restricted to α_{rm}, since the corresponding curves are disjoint from $\alpha_{rm}$when i>r. See Figure 3.26. Thus $a_{ik}(\alpha_{rm}) \simeq a_{im} a_{kk+1}(\alpha_{rm})$.

Now by Lemma 3.13, letting r=k and s=m-1 we have $h_{\gamma`} \simeq c_{km-l} \simeq a_{kk+1} a_{mm+1}^{k-m+1} a_{m-lm}^{-1} \cdots a_{km}^{-1}$. Again keeping in mind which of the homeomorphisms commute we can solve for a_{kk+1} to yield $a_{kk+1}\ (a_{km}\ldots a_{m-lm})\ a_{mm+1}^{m-k+1}\ c_{km-l}$. Now c_{km-l} and a_{mm+1} are both the identity when restricted to α_{rm}, so $a_{kk+1}(\alpha_{rm}) \simeq (a_{km}\ldots a_{m-lm})\ (\alpha_{rm})$. Thus $a_{ik}(\alpha_{rm}) \simeq a_{im} a_{kk+1}(\alpha_{rm}) \simeq a_{im}(a_{km}\ldots a_{m-lm})(\alpha_{rm})$.

Notes

Part (2) now follows from Lemmas 3.6 and 3.7 as indicated in Remark 2 following Lemma 3.10.

(3) Let i<r<k. By Lemma 12(a) we have $a_{im} \simeq a_{ik} h_{\gamma`} a_{mm+1} a_{kk+1}^{-1} h_{\beta}^{-1}$ where γ` and β are given in Figure 3.27. As before we can solve for a_{ik} to obtain $a_{ik} a_{kk+1} a_{im} a_{mm+1}^{-1} h_{\gamma`}^{-1} h_{\beta}$. Now $a_{mm+1}$, $h_{\gamma`}$, h_{β} are all the identity on a_{rm} when i<r<k, see Figure 3.28. Thus $a_{ik}(\alpha_{rm}) \simeq a_{kk+1} a_{im}(\alpha_{rm})$. Now as in the proof of part (2), $a_{kk+1} \simeq (a_{km} \ldots a_{m\text{-}lm}) a_{mm+1}^{m-k+1} c_{rm\text{-}l}$.

Since $c_{km\text{-}l}$ and a_{mm+1} are the identity on $a_{im}(\alpha_{rm})$, see Figure 3.29, we have $a_{ik}(\alpha_{rm}) \simeq (a_{km} \ldots a_{m\text{-}lm})\, a_{im}(\alpha_{rm})$. (3) now follows from Lemmas 3.6 and 3.7 as indicated above.

Theorem 15 Let $n \geq 3$, so that $H^*(S^2, F_{n+1})$ is generated by $G_n = {}^{n}\cup_{k=3} \{\bar{a}_{ik} : l \leq i \leq k\}$. In terms of these generators, $H^*(S^2, F_{n+1})$ has a presentation in which a complete set of relations is given as follows:

1. If $p \leq q$ and i=r or if p<q and $i<r \geq p$, then $\bar{a}_{ip}\bar{a}_{rq}\bar{a}^{-1}_{ip} = \bar{a}_{rq}$.
2. If p<q and i>r, then $\bar{a}_{ip}\bar{a}_{rq}\bar{a}_{ip}^{-1} = (\bar{a}_{iq}(\bar{a}_{pq} \ldots \bar{a}_{q\text{-}lq}))\bar{a}_{rq}(\bar{a}_{iq}(\bar{a}_{pq} \ldots \bar{a}_{q\text{-}lq}))^{-1}$.
3. If p<q and i<r<p, then $\bar{a}_{ip}\bar{a}_{rq}\bar{a}_{ip}^{-1} = ((\bar{a}_{pq} \ldots \bar{a}_{q\text{-}lq})\bar{a}_{iq})\bar{a}_{rq}((\bar{a}_{pq} \ldots \bar{a}_{q\text{-}lq})\bar{a}_{iq})^{-1}$.

Proof. Theorem 3.9 shows G_n generates $H^*(S^2, F_{n+1})$. We prove the present theorem by induction on n, beginning with n=3. For n = 3, $H^*(S^2, F_4) = \pi_1(S^2\text{-}F_3, P_4)$ = the free group on 2 generators, as seen in the proof of Theorem 3.9. Moreover, $H^*(S^2, F_4)$ is generated by $\bar{a}_{13}$and $\bar{a}_{23}$, hence the present theorem is true in this case.

Assume Theorem 15 for n-1.

Let $\bar{\bar{a}}_{ip}$denote the equivalence class of a_{ip} in $H^*(S^2, F_n)$, to distinguish it from $\bar{a}_{ip}$, the equivalence class of a_{ip} in $H^*(S^2, F_{n+1})$. By the induction assumption $G`_{n\text{-}l} = \cup_{k=3}^{n-1}$ $\{\bar{\bar{a}}_{ik} : l \leq i < k\}$ generates $H^*(S^2, F_n)$ with relations as in 1, 2, and 3 with ā replaced by $\bar{\bar{a}}$. If we then replace $\bar{\bar{a}}$ by ā, these relations hold in $H^*(S^2, F_{n+l})$ by Lemma 3.14. Hence the map k: $H^*(S^2, F_n) \to H^*(S^2, F_{n+l})$ defined by $k(\bar{\bar{a}}_{ik}) = \bar{a}_{ik}$ defines a homomorphism which splits the short exact sequence

$$o \to \pi_1(S^2\text{-}F_n, p_n) \to H^*(S^2, F_{n+l}) \to H^*(S^2, F_n) \to o.$$

Therefore by part (ii) of Lemma 3.10 $H^*(S^2, F_{n+l})$ has a presentation in terms of the generators G˙ in which every relation has the form (1.1) $w(\bar{a}_{rn}) = 1$, (1.2) $w`(\bar{a}_{ip}) = 1$, (2) $\bar{a}_{ip}\bar{a}_{rn}\bar{a}_{ip}^{-1} = w(\bar{a}_{rn})$ and (3) $\bar{a}_{ip}^{-1}\bar{a}_{rn}\bar{a}_{ip} = w(\bar{a}_{rn})$ where $\bar{a}_{rn}$is in $G_n - G_{n\text{-}l}$ and $\bar{a}_{ip}$ is in $G_{n\text{-}l}$. There can be no nontrivial relations of the form (1.1) since the subgroup generated by $G_n - G_{n\text{-}l}$, i.e. by $\{\bar{a}_{rn} : l \leq r < n\}$, is $d(\pi_1(S^2\text{-}F_n, p_n))$ which is free.

Since k is a monomorphism, $w`(\bar{\bar{a}}_{ip}) = l$ if and only if $w`(\bar{a}_{ip}) = l$. Hence by induction all relations of the form (1.2) are consequences of those of types 1, 2, and 3 given in the statement of the theorem.

Letting q = n, another application of Lemma 3.14 shows that the relations of type 1, 2, and 3 given in the statement of the present theorem supply the necessary relations of type (2) from Lemma 3.10. Finally we note that by 1 in Theorem 3.15, $\bar{a}_{ip}$ commutes with the elements $\bar{a}_{iq}, \bar{a}_{pq}, \ldots, \bar{a}_{q\text{-}lq}$ when p<q. Hence conjugating 2 and 3 by $\bar{a}_{ip}^{-1}$ - $(\bar{a}_{iq}(\bar{a}_{pq} \ldots \bar{a}_{q\text{-}lq}))^{-1}$ and using the above commuting properties we see that relations of type (3) from Lemma 10 are a consequence of 1, 2, and 3 as given in the theorem.

Notes

Corollary 16 $H^*(S^2, F_{n+1})$ made abelian is the free abelian group on 2+3+…+(n-1) = (n(n-1)/2) -1 generators.

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Notes

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On the Role of Luminal Flow and Interstrut Distance in Modelling Drug Transport from Half-Embedded Drug-Eluting Stent

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Abstract- A model for investigating the transport of drug from a half-embedded drug-eluting stent (DES) is developed. Keeping the relevance of the physiological situation in view, the luminal drug transport is considered as an unsteady convection-diffusion process, while the drug transport within the arterial tissue is supposed to commence as a diffusion process. The Marker and Cell (MAC) method has been used to handle numerically the governing equations of motion for the luminal flow and the drug transport through the lumen and the tissue. The effects of quantities of significance such as Reynolds number (Re), Womersley number (α) and interstrut distance on the transport of drug through both the lumen and the tissue are quantitatively investigated. Our simulation predicts that the mean concentration of drug increases with the decreases of Reynolds number and with an increase in the Womersley number.

Keywords: *embedded drug-eluting stent; flow pulsatility; interstrut distance; convection; womersley number.*

GJSFR-F Classification: *MSC 2010: 00A05*

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Akash Pradip Mandal [α] & Prashanta Kumar Mandal [σ]

Abstract- A model for investigating the transport of drug from a half-embedded drug-eluting stent (DES) is developed. Keeping the relevance of the physiological situation in view, the luminal drug transport is considered as an unsteady convection-diffusion process, while the drug transport within the arterial tissue is supposed to commence as a diffusion process. The Marker and Cell (MAC) method has been used to handle numerically the governing equations of motion for the luminal flow and the drug transport through the lumen and the tissue. The effects of quantities of significance such as Reynolds number (Re), Womersley number (α) and interstrut distance on the transport of drug through both the lumen and the tissue are quantitatively investigated. Our simulation predicts that the mean concentration of drug increases with the decreases of Reynolds number and with an increase in the Womersley number. The present results also predict a single peak profile of drug concentration when the pair of struts are placed one-half strut width and also as the interstrut distance increases, distinct peaks form over each strut.

Keywords: embedded drug-eluting stent; flow pulsatility; interstrut distance; convection; womersley number.

I. Introduction

Atherosclerosis is a disease that affects coronary, carotid and other peripheral arteries in the body. Coronary artery disease (CAD) pertains to a blockage or narrowing of coronary arteries. Once detected as such, there are a number of interventional ways to alleviate a stenosis in a coronary artery. Drug-eluting stents have drastically reduced the rate of in-stent restenosis compared to bare-metal stents (BMS^s) viz. 8.9% after eight months compared to 36.6% for BMS in the same study [35] and have since become the most choice for treatment of coronary arteries afflicted with advanced atherosclerotic lesions [34]. The success of DES is usually associated with the effective local delivery of potent therapeutics to the target site with programmed pharmocokinetics. The local drug concentrations achieved are directly correlated with the biological effects and local toxicity, and establishing the optimum dose to be delivered to the tissue remains a challenge in DES design and manufacturing [3, 11, 32]. The association of tissue prolapse (i.e. the deflection of the tissue between the struts of the stent) and in-stent thrombosis has also been reported in several studies [30, 27, 9]. Some researchers [13, 16, 5] are on the opinion that platelet activation, endothelium injury and inadequate antiplatelet therapy plays an important role for in-stent thrombosis. A number of experimental and numerical studies on DES have been carried out in recent past to address the issues like its efficiency and safety [4, 18, 2, 6, 21, 28, 29, 31, 22].

McGinty et al. [20] have included the layer structure of the arterial wall into their 1D studies. Zunino et al. [36] opined that model based on dimension reduction enable a comprehensive

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geometrical and physical description of stenting at affordable computational costs. Kolachalama et al. [14, 15] studied the impact of luminal flow on the drug transport from DES and they concluded that the use of DES requires a complex calculus that balances vascular and stent geometry as well as luminal flow. They showed that the flow imposes recirculation zones distal and proximal to the stent strut that extends the coverage of tissue absorption of eluted drug and induce asymmetry in drug distribution within the arterial tissue. Balakrishnan et al. [1] studied the coupled computational fluid dynamics and drug transport model in an idealised stented artery to predict drug deposition in single and overlapping DES and showed that the drug deposition appeared not only beneath regions of arterial contact with the strut but also beneath stagnant drug pools created by the separation flow. Hwang et al. [12] investigated the behaviour of hydrophobic and hydrophilic drugs as a convection-diffusion process and showed that the asymmetric strut distribution has a higher effect on hydrophilic drug distribution in comparison with hydrophobic one. Gogineni et al. [7] compared various stent geometries in order to determine most hemodynamically favourable stent. Models with multiple struts have also been developed to study the impact of different strut configurations and diffusivities on arterial drug distribution [2, 3, 8, 24, 25, 33]. Moreover, as the impact of flow pulsatility on arterial drug uptake is being increasingly characterised, several factors associated with the pulsatile nature of blood flow on arterial drug deposition have not been fully understood [26].

In an effort to better understand the validity and applicability of model assumptions, the present *in silico* investigation deals with the transport of drug from a half-embedded DES. This model is believed to be an extension of our previous model [19]. Here, we study the importance of flow pulsatility and interstrut distance on the drug transport within the framework of coupled computational fluid dynamics and mass transfer model. The governing equations of motion of unsteady momentum and mass transfer are successfully solved numerically by MAC method primarily introduced by Harlow and Welch [10]. The primary objective of the present study is to explore the effects of essential issues like Reynolds number, flow pulsatility, interstrut distance as well on the wall shear stress, the luminal drug concentration and the drug concentration within the arterial tissue by using relatively simple finite difference scheme in rather complex geometries. The novelty of the study lies in the inclusion of strut embedment and flow pulsatility on the unsteady distribution of both the mass and momentum transport. In this investigation, embedment of the strut in the arterial wall is assumed to be 50% only as the effect of embedment reveals only the change in magnitude of drug concentration by keeping the overall behaviour and the conclusions drawn unaltered [2].

II. Materials and Methods

a) *Model geometry*

The computational domain consists of a long axial arterial section of length L idealised as a rectangle where the embedded struts are assumed to be circular. The arterial wall thickness is taken to be 5 times of strut diameter (d) and the lumen to be 15-fold wide (cf. Figure 1). The size of strut and interstrut distance are adapted from the geometry introduced by Mongrain et al. [24]. We define a therapeutic zone for drug delivery consisting of a pair of half-embedded struts together with the upstream length (z_0) and the downstream length (z_l). All the simulation results are for this domain.

b) *Dimensionless governing equations and boundary conditions*

The streaming blood is assumed to be unsteady and Newtonian past a two-dimensional axi-symmetric stented artery and is thus governed by the time-dependent Navier-Stokes equations whose forms are as

Ref

12. Hwang CW, Wu D, Edelman ER (2001) Physiological transport forces govern drug distribution for stent-based delivery. Circulation 104:600–605

Notes

$$\frac{\partial w}{\partial t} + \left[\frac{\partial}{\partial r}(uw) + \frac{\partial}{\partial z}(w^2) + \frac{uw}{r}\right]\epsilon^8 = -\frac{\partial p}{\partial z} + \frac{\epsilon^7}{Re}\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \epsilon^2\frac{\partial^2 w}{\partial z^2}\right), \quad (1)$$

$$\frac{\partial u}{\partial t} + \left[\frac{\partial}{\partial r}(u^2) + \frac{\partial}{\partial z}(uw) + \frac{u^2}{r}\right]\epsilon^8 = -\frac{1}{\epsilon^2}\frac{\partial p}{\partial r} + \frac{\epsilon^7}{Re}\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \epsilon^2\frac{\partial^2 u}{\partial z^2} - \frac{u}{r^2}\right), \quad (2)$$

$$r\frac{\partial w}{\partial z} + \frac{\partial}{\partial r}(ur) = 0, \quad (3)$$

where r and z are the dimensionless coordinates scaled with respect to the radius of the unstented arterial lumen (a) and strut diameter (d) respectively while z-axis is located along the symmetry axis of the artery. The nondimensional components of the velocity along the axial and the radial directions are denoted by w and u respectively. The Reynolds number (Re) and the scaling parameter ϵ may be defined as $Re = \frac{\rho U_0 a}{\mu}$, $\epsilon = \frac{a}{d}$ where U_0 is the centreline velocity of the Poiseuille flow, ρ and μ are the density and the viscosity of the flowing blood respectively.

The unsteady convection-diffusion equation representing the transport of drug in the lumen may be written as

$$\frac{\partial c_f}{\partial t} + \left[\frac{\partial}{\partial r}(uc_f) + \frac{\partial}{\partial z}(wc_f) + \frac{uc_f}{r}\right]\epsilon^8 = \frac{\epsilon^7}{Pe_f}\left(\frac{\partial^2 c_f}{\partial r^2} + \frac{1}{r}\frac{\partial c_f}{\partial r} + \epsilon^2\frac{\partial^2 c_f}{\partial z^2}\right), \quad (4)$$

in which the luminal Peclet number $Pe_f = \frac{aU_0}{D_f}$; D_f, the drug diffusion coefficient in blood. Here c_f denotes the dimensionless luminal drug concentration.

Drug transport within the tissue has been represented by the unsteady diffusion process as

$$\frac{\partial c_t}{\partial t} = \frac{\epsilon^7}{Pe_t}\left(\frac{\partial^2 c_t}{\partial r^2} + \frac{1}{r}\frac{\partial c_t}{\partial r} + \epsilon^2\frac{\partial^2 c_t}{\partial z^2}\right), \quad (5)$$

where c_t is the dimensionless concentration of drug in the tissue and Pe_t $(= \frac{aU_0}{D_t})$ stands for the Peclet number in the tissue. Here D_t is the drug diffusion coefficient in the tissue.

In the lumen, symmetry boundary conditions were applied at the flow centreline:

$$\frac{\partial w}{\partial r} = 0 = u, \frac{\partial c_f}{\partial r} = 0 \quad \text{on} \quad r = 0. \quad (6)$$

No slip boundary condition have been imposed on lumen-strut (Γ_{bs}) and lumen-tissue (Γ_{bw}) interfaces:

$$w = 0 = u \quad \text{on} \quad r = R_b \ (= \Gamma_{bw} \cup \Gamma_{bs}). \quad (7)$$

Continuity of flux has been maintained at the lumen-tissue interface (Γ_{bw}) and release of drug from the struts has been simulated as a Dirichlet boundary condition with a drug concentration of unity at strut surfaces (Γ_{bs} and Γ_{ws}):

$$D_{f/t}\frac{\partial c_f}{\partial r} = \frac{\partial c_t}{\partial r} \quad \text{on} \quad \Gamma_{bw}, \quad (8)$$

$$c_f = 1 \quad \text{on} \quad \Gamma_{bs}, \quad (9)$$

$$c_t = 1 \quad \text{on} \quad \Gamma_{ws}, \quad (10)$$

where $D_{f/t}$ stands for the ratio of D_f and D_t.

A pulsatile velocity profile is imposed at the luminal inlet ($\Gamma_{b,in}$) [17]

$$w = (1 - \frac{r^2}{{R_b}^2})\bar{w}(t), u = 0 \quad \text{on} \quad \Gamma_{b,in}, \tag{11}$$

where $\bar{w}(t) = [1 + kcos(\epsilon^9 \frac{\alpha^2}{Re} t)]$, k is the dimensionless amplitude and $\alpha(= \sqrt{\frac{d^2\omega\rho}{\mu}})$ is the Womersley number in which ω is the frequency. Also, drug concentration was set to zero at the luminal inlet i.e.

$$c_f = 0 \quad \text{on} \quad \Gamma_{b,in}. \tag{12}$$

At the luminal outlet ($\Gamma_{b,out}$), zero velocity and concentration gradients are assumed:

$$\frac{\partial w}{\partial z} = 0 = \frac{\partial u}{\partial z} \quad \text{and} \quad \frac{\partial c_f}{\partial z} = 0 \text{ on } \Gamma_{b,out}. \tag{13}$$

In the tissue, symmetry boundary condition of drug transport are applied on the proximal ($\Gamma_{w,in}$) and the distal ($\Gamma_{w,out}$) walls:

$$\frac{\partial c_t}{\partial z} = 0 \quad \text{on} \quad \Gamma_{w,in} \text{ and } \Gamma_{w,out}. \tag{14}$$

A proper boundary condition at the perivascular wall (Γ_{wu}) is not apparent. Some researchers modelled it as a zero flux boundary condition [1, 19, 26], while some opted for a zero concentration boundary condition assuming the adventitia as a perfect sink [2, 15]. So instead of modelling a particular one, we investigate the effects of both the adventitial boundary conditions in this study:

$$\frac{\partial c_t}{\partial r} = 0 \quad \text{or} \quad c_t = 0 \text{ on } \Gamma_{wu}. \tag{15}$$

c) *Solution procedure*

In order to avoid interpolation error while discretising the governing equations, we transform the irregular (stented) domain into rectangular ones by making use of the following radial coordinate transformation:

$$\eta = \frac{r}{R_b} \quad , \quad \xi = 1 + \frac{r - R_{tl}}{R_{tu} - R_{tl}} = 1 + \frac{r - R_{tl}}{R}, \tag{16}$$

so that the arterial lumen transforms into a finite nondimensional rectangular domain as $[0, L] \times [0, 1]$, while the tissue domain into $[0, L] \times [1, 2]$. Here $R_b = \Gamma_{bw} \cup \Gamma_{bs}$, $R_{tl} = \Gamma_{bw} \cup \Gamma_{ws}$, $R_{tu} = \Gamma_{wu}$ and $R = R_{tu} - R_{tl}$. The transformed governing equations along with the set of boundary conditions are solved numerically by finite difference scheme in staggered grids. In this type of grid alignment, the velocities, the pressure and the drug concentrations are calculated in different locations of the control volume [cf. Figure 2]. The discretisation of the time-derivative terms is based on first order accurate two-level forward time-differentiating formula, while those for the convective terms in the momentum equations are accorded with a hybrid formula, consisting of central differencing and second order upwinding. The diffusive terms are, however, discretised by second order accurate central difference formula. The discretised equations are then solved by MAC method primarily introduced by Harlow and Welch [10]. Each run takes about 9 hours of CPU time in Debian Lenny 64-bit OS on a desktop of intel (R) core (TM) i5 with 8 GB RAM, for steady state computation. No standard package has been used in the present computations. However, our in-house code using FORTRAN language is based on the following computational sequences:

Ref

10. Harlow FH, Welch JE (1965) Numerical calculation of time-273 dependent viscous incompressible flow of fluid with free surface. Physics of Fluids 8:2182

Stage 1:

(i) $w^n_{i+\frac{1}{2},j}$, $u^n_{i,j+\frac{1}{2}}$, $c^n_{f_{i,j}}$ and $c^n_{t_{i,k}}$ are initialised at each cell (i,j). This is done either from result of the previous cycle or from the prescribed initial conditions.

(ii) Time step δt calculated from stability criteria.

(iii) The Poisson equation for pressure is solved to get the intermediate pressure-field $p^*_{i,j}$ using velocities $w^n_{i+\frac{1}{2},j}$, $u^n_{i,j+\frac{1}{2}}$ of the n^{th} time step.

(iv) The momentum equations are solved to get intermediate velocities $w^*_{i+\frac{1}{2},j}$, $u^*_{i,j+\frac{1}{2}}$ in an explicit manner using the previously known velocities and pressure.

Stage 2:

(v) The maximum cell divergence of velocity-field is calculated and checked for its tolerance. If the tolerance limit is satisfied, then drug transport equations are solved to get drug concentrations $c^{n+1}_{f_{i,j}}$ and $c^{n+1}_{t_{i,k}}$ in an explicit manner and steady-state convergence is checked for whether to stop calculation. If the maximum divergence of the velocity-field is found to be greater than the tolerance limit at any cell in absolute sense, go to step (vi).

(vi) The pressure at each cell is corrected to obtain $p^n_{i,j}$ and consequently the velocities at each cell are adjusted to get $w^{n+1}_{i+\frac{1}{2},j}$ and $u^{n+1}_{i,j+\frac{1}{2}}$. Then step (v) is again performed. This completes the necessary calculations for advancing flow-field through one cycle in time.

The process is to be repeated until steady-state convergence is achieved. Intersested readers are referred to Mandal et al. [19] for detailed discussion on method of solution, pressure-velocity correction and numerical stability.

III. Result and Discussion

For the purpose of numerical computation of the desired quantities of major physiological significance, the computational domain has been confined with a finite non-dimensional arterial length of 15. For this computational domain, solutions are computed through the generation of staggered grids with a size of 301×81 for both the lumen and the tissue regions. The simulation concerning the grid independence study was performed for the purpose of examining the error associated with the grid sizes used and is depicted in Figure 3. One may notice from this figure that the profiles concerning three distinct grid sizes almost overlap with one another for $Re = 500$ and $\alpha = 1$ in case of pulsatile inlet profile of the flowing blood. Thus the grid independence study in the present context of numerical simulation has its own importance to establish the correctness of the results obtained.

The variations of the dimensionless wall shear stress (WSS) distribution over the entire arterial segment having a pair of half–embedded DES^s in its lumen for different Reynolds numbers are exhibited in Figure 4. The flow separation points are observed when the WSS changes its sign. The reattachment points occur when the WSS changes its sign again. The curves in this figure show several flow separation zones from proximal and distal to the struts, where the latter is significantly larger than the former. One may note that the downstream separation length increases with increasing Reynolds numbers and the WSS gets its maximum value at the maximum height of the strut which also increases with increasing Reynolds numbers. Our simulations predict that these recirculation zones create pockets of stagnant drug–laden blood that allow drug accumulation at lumen-tissue interface and eventually filtration into the arterial wall. This observation is in conformity to that of Balakrishnan et al. [1] though they studied the transport of drug eluted from single and overlapping well-apposed DES^s in which the cross-section of the strut is square in shape.

Ref

1. Balakrishnan B, Tzafriri AR, Seifert P, Groothuis A, Rogers C, Edelman ER (2005) Strut position, blood flow, and drug deposition implications for single and overlapping drug-eluting stents. Circulation 111:2958–2965.

Figure 5(a) exhibits the profiles for the dimensionless drug concentration through the depth of the arterial tissue at $Pe_t = 1000, \alpha = 1$ and $Re = 500$ for different times. Here, drug concentration does increase with increasing time and thereafter, attains steady state for zero flux perivascular boundary condition. It is also to be observed that the penetration depth of drug within the tissue increases with increasing time. However, if one assumes zero concentration boundary condition at the perivascular end, the mean concentration appears to be much lower as compared to zero flux perivascular boundary condition [cf. Figure 5(b)].

Figures 6(a) and 6(b) show how the flow pulsatility affects the respective drug concentrations in both the lumen and the tissue at a depth of 1.5 strut radius for $Pe_f = 100, Pe_t = 1000, Re = 500$ and $\alpha = 1$. Both the figures reveal asymmetry between regions distal and proximal to the strut in which the degree of asymmetry appears to be much higher in the luminal concentration profile than that of the concentration profile within the tissue. The above observations may be argued in the sense that the asymmetry in the lumen is due to the convective nature of the luminal drug transport, however asymmetry in the tissue is due to stagnant drug–laden blood at the recirculation regions that allows drug accumulation and finally filtration beneath these regions. One more interesting observation is to be noted that the concentration of drug in both the lumen and the tissue is higher in case of usual parabolic inlet profile than that of the pulsatile one. Thus flow pulsatility significantly affects the transport of drug eluted from DES.

Figure 7(a) shows a change in Re of the flow alters the distribution of drug in the lumen while Figure 7(b) depicts that the mean drug uptake within the tissue is significantly affected by Re. Our simulation demonstrates that a decrease in concentration in the lumen with an increasing Re i.e. an inverse relationship between drug concentration and Re is revealed. A similar pattern is also observed in the tissue [cf. figure 7(b)]. The above observations may be justified in the sense that as Re increases, recirculation regions adjacent to the struts increase, which in turn, dilute the surface concentration of the pooling drug and decrease its contribution to the total drug deposition. Our observations are in good agreement with those of Kolachalama et al. [15] and O'Brien et al. [26].

Axial variation of drug concentration eluted from a pair of half–embedded DES^s with varying flow pulsatility (Womersley number) in the lumen is displaced in Figure 8(a). Our results reveal a complex interplay between Womersley number and arterial drug distribution. Evidently, an increase in vessel Womersley number via a change in the inlet flow profile is to increase the deposition in the lumen. It may also be noted that the mean concentration in the tissue does increase with increasing Womersley number [cf. Figure 8(b)], therefore, highlighting the importance of pulsatile inlet on stent-based drug delivery.

Figure 9 shows the concentration profile at a height 1.5 strut radius depending on interstrut distance for $Pe_f = 100, Pet = 1000, Re = 500$ and $\alpha = 1$. A single peak profile is noted when the struts are placed one strut radius apart. As the interstrut distance increases, the peak concentration falls and distinct peaks over each strut are observed. The spatial patterns for drug concentration [cf. Figures 10(a,b)] clearly establish our findings further. All these observations are in conformity to the findings of Balakrishnan et al. [1].

IV. Conclusion and Future Work

In this numerical investigation, we propose a two-dimensional axi-symmetric model of drug transport and luminal flow in presence of a pair of half-embedded DES^s. The finite-difference method used in this paper allows us to numerically solve the governing equations both in the lumen and the tissue. The present results predict a single peak profile when the struts

Ref

15. Kolachalama VB, Tzafriri AR, Arifin DY, Edelman ER (2009) Luminal flow patterns dictate arterial drug deposition in stent-based delivery. Journal of Controlled Release 133:24–30

are placed one strut radius apart and also as the interstrut distance increases, distinct peaks form over each strut. This study also highlights the facts that the mean concentration of drug within the tissue to be higher if one assumes the zero flux boundary condition instead of sink condition at the perivascular end. Another important observations may also be noted that as the Reynolds number increases, the drug concentration both in the lumen and the tissue decreases; however, a reverse trend is observed while changing the Womersley number. Though experimental studies can provide information on release kinetics and other histological information, computational studies can provide detailed predictions of the drug distribution over time. With the rapid ascent of stent–based drug delivery in the treatment of vascular disease, many important issues concerning drug delivery and its retention in the arterial tissue need to be addressed. The work presented in this paper only consider free drug as part of the governing equations. In reality, specific binding to tissue ultrastructural elements determines long term drug retention. As the target zone for stent-based delivery is atheroscelorетic plaque [23], future direction for this work may also include different compositions of the plaque with varying diffusivity together with the time-dependent release kinetics.

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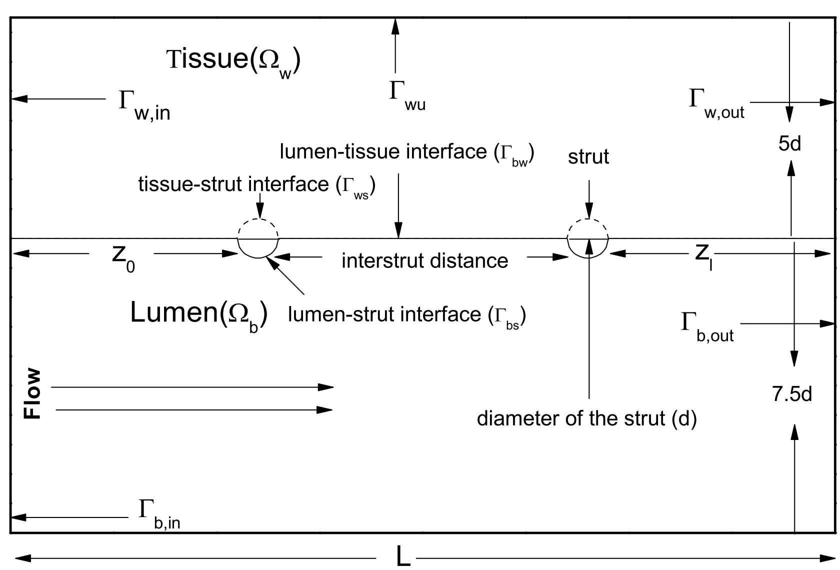

Figure 1: Schematic diagram of the stented artery

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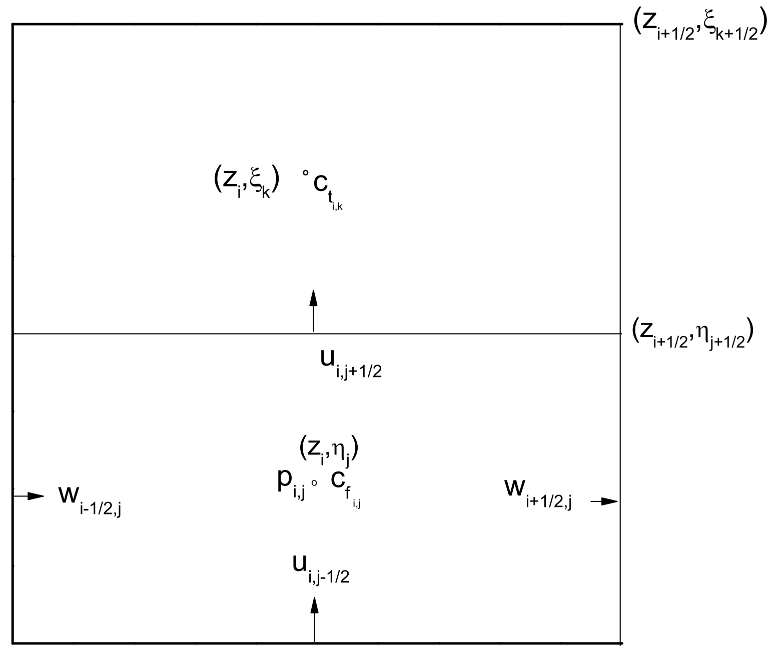

Figure 2: A typical combined MAC cell for lumen and tissue.

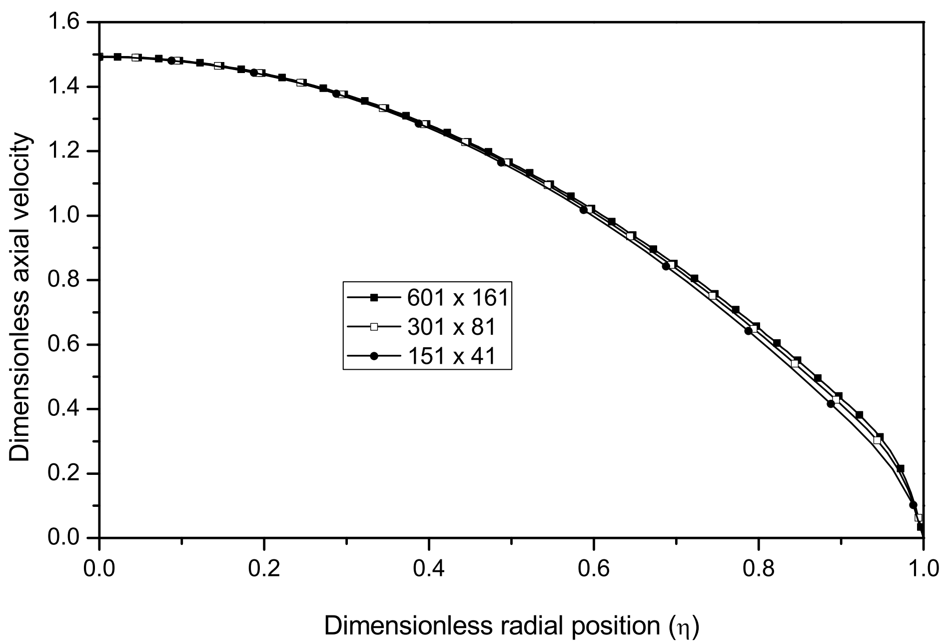

Figure 3: Dimensionless axial velocity profile at z=3.5 for different grid sizes (Re=500, α=1).

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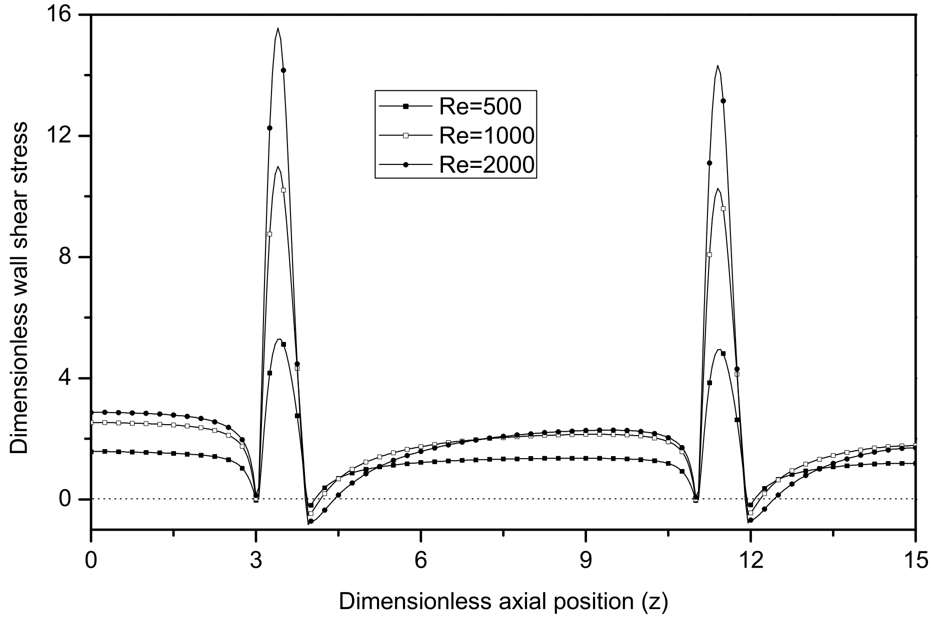

Figure 4: Distribution of dimensionless wall shear stress for different Reynlods numbers (α=1, Pe_f=100, Pe_t=1000).

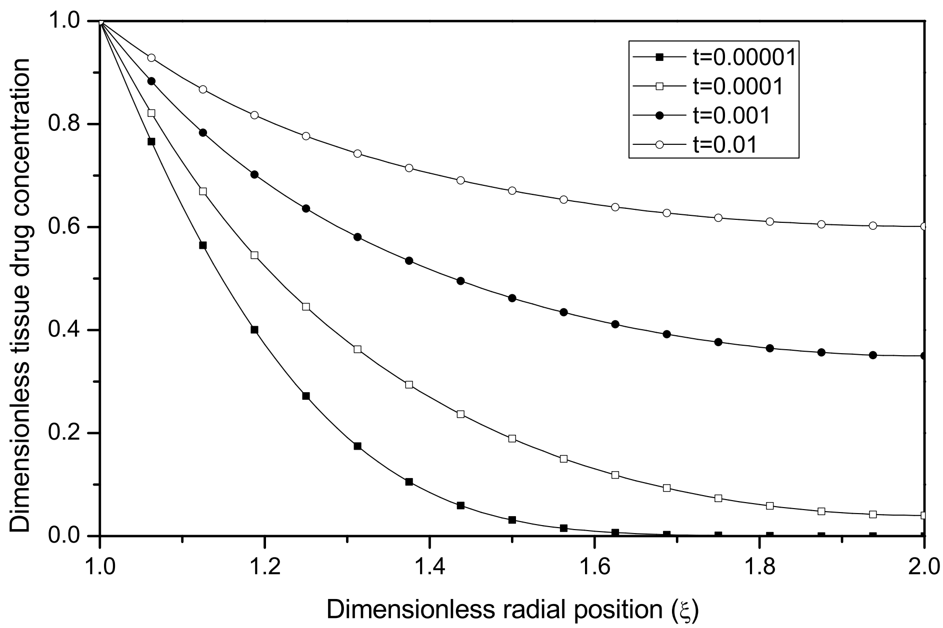

Figure 5(a): Transmural variation of dimensionless tissue drug concentration for different times (Pe_f=100, Pe_t=1000, Re=500, α=1).

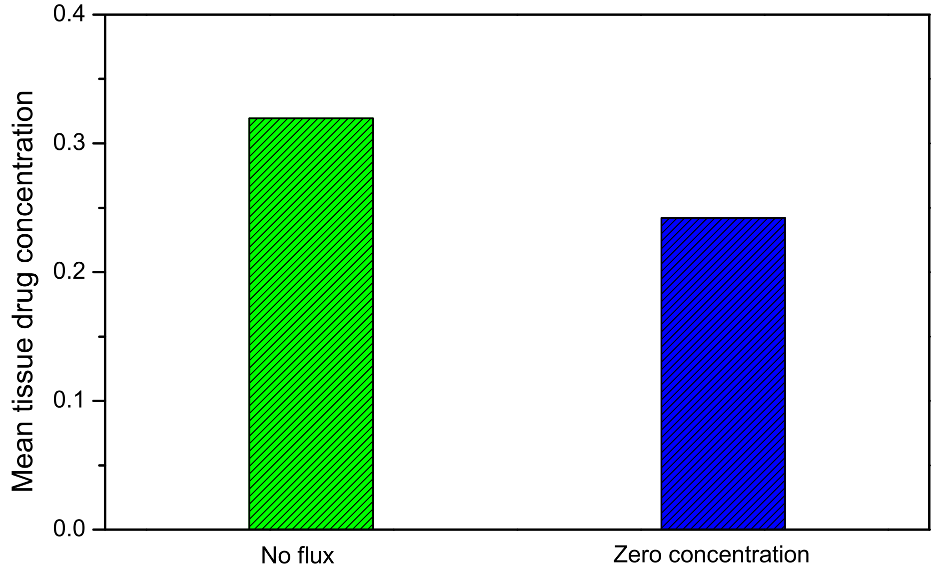

Figure 5(b): Mean concentration of drug within the tissue for different perivascular boundary conditions (Re=500, Pe_f=100, Pe_t=1000, α=1)

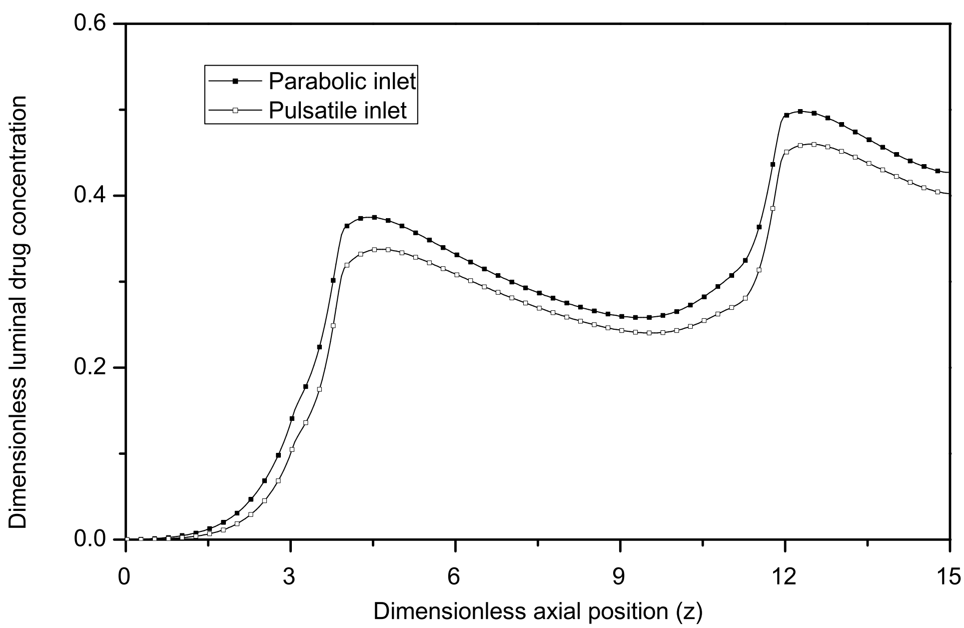

Figure 6(a): Axial variation of luminal drug concentration at a depth of 1.5 strut radius from mural interface for different types of inlet flow (Pe_f=100, Pe_t=1000, Re=500, α=1)

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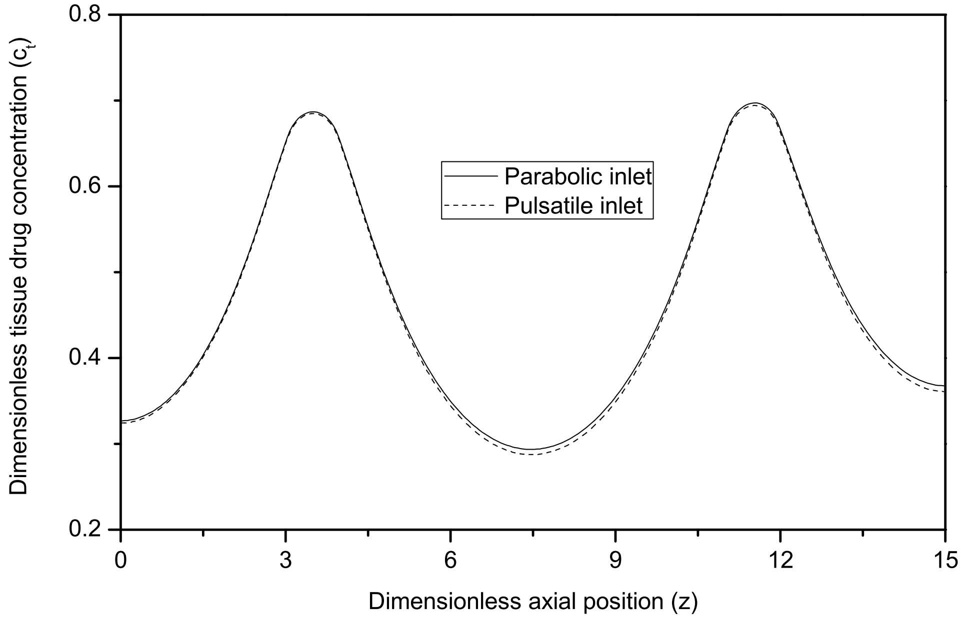

Figure 6(b): Axial variation of tissue drug concentration at a height of 1.5 strut radius from mural interface for different types of inlet flow (Pe_f=100, Pe_t=1000, Re=500, α=1)

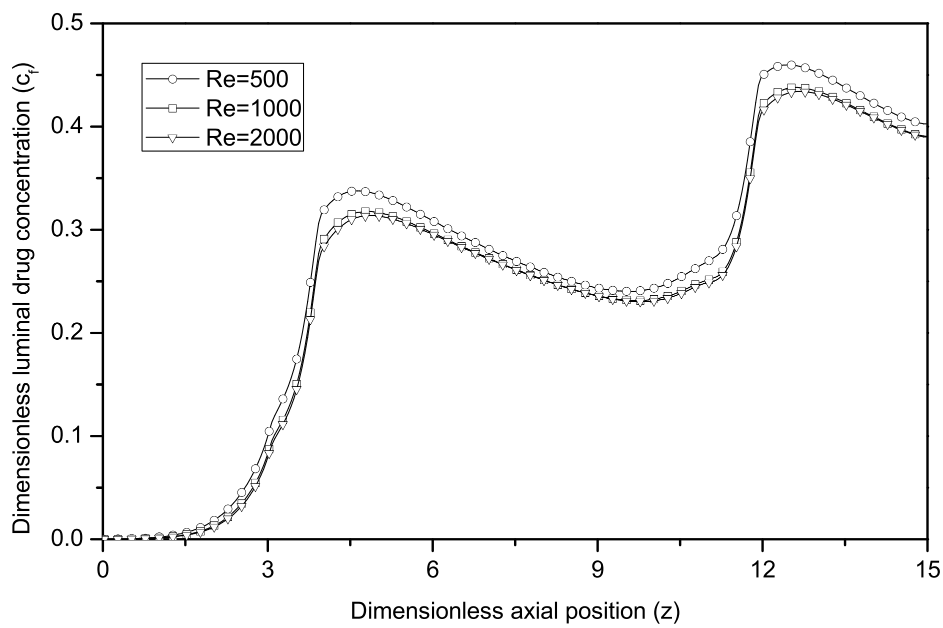

Figure 7(a): Axial variation of luminal drug concentration at a depth of 1.5 strut radius for different Reynolds numbers (Pef=100, Pet=1000, α=1)

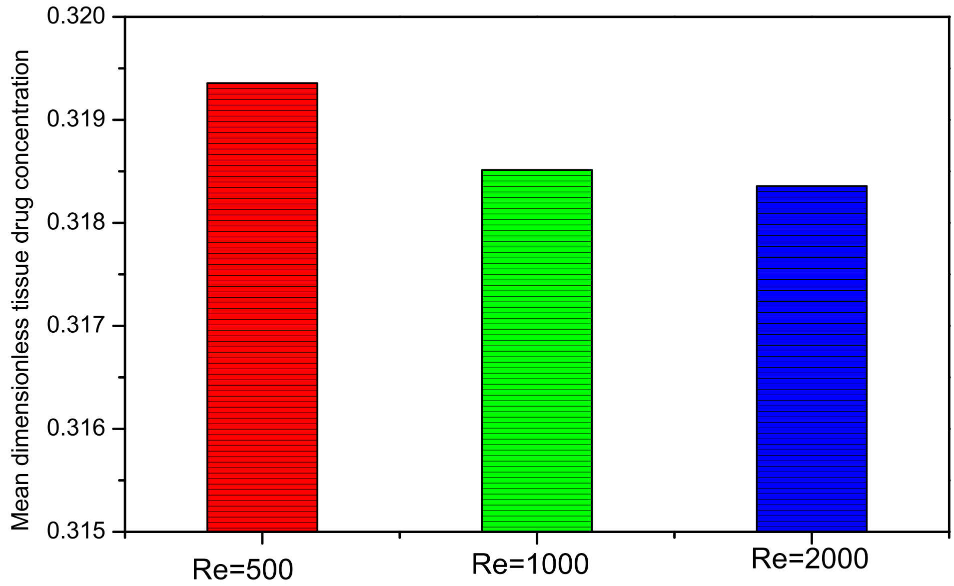

Figure 7(b): Mean drug concentration in the tissue for different Reynolds numbers (Pe_f=100, Pe_t=1000, α=1)

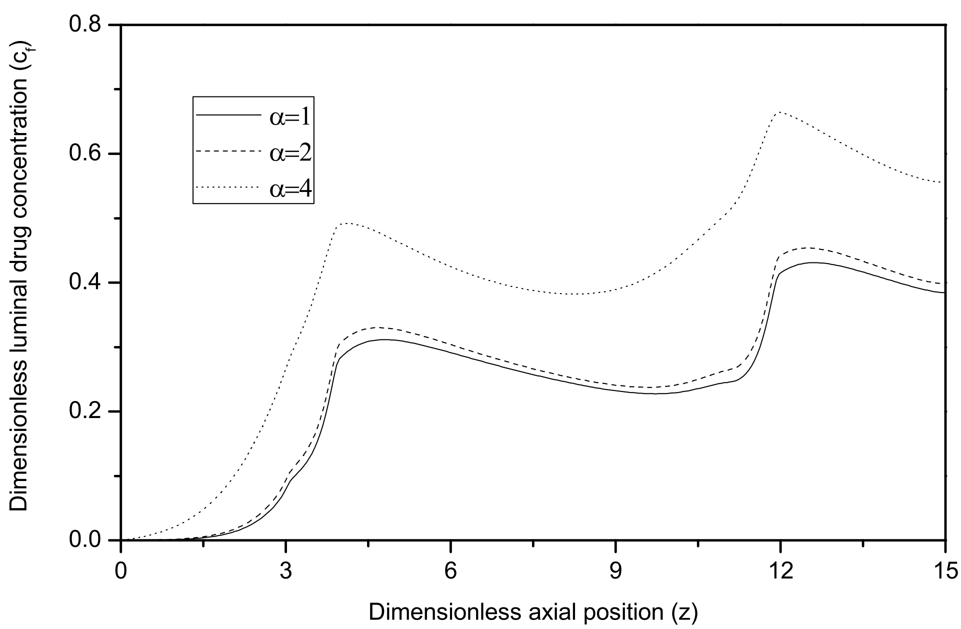

Figure 8(a): Axial variation of luminal drug concentration at a depth of 1.5 strut radius for different Womersley numbers (Re=500, Pe_f=100, Pe_t=1000)

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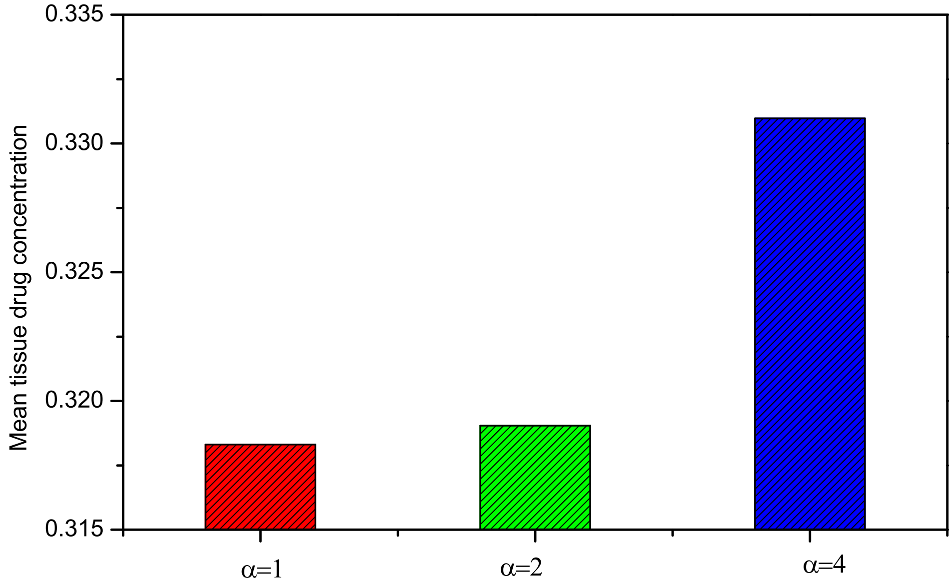


Figure 8(b): Mean tissue drug concentration in the tissue for different Womersley numbers (Pe_f=100, Pe_t=1000, Re=500)

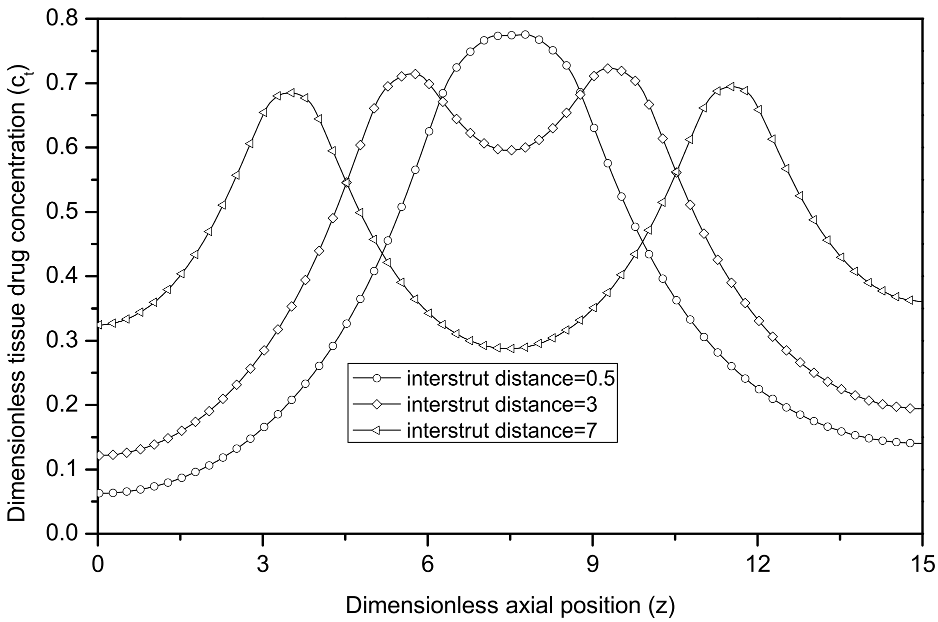
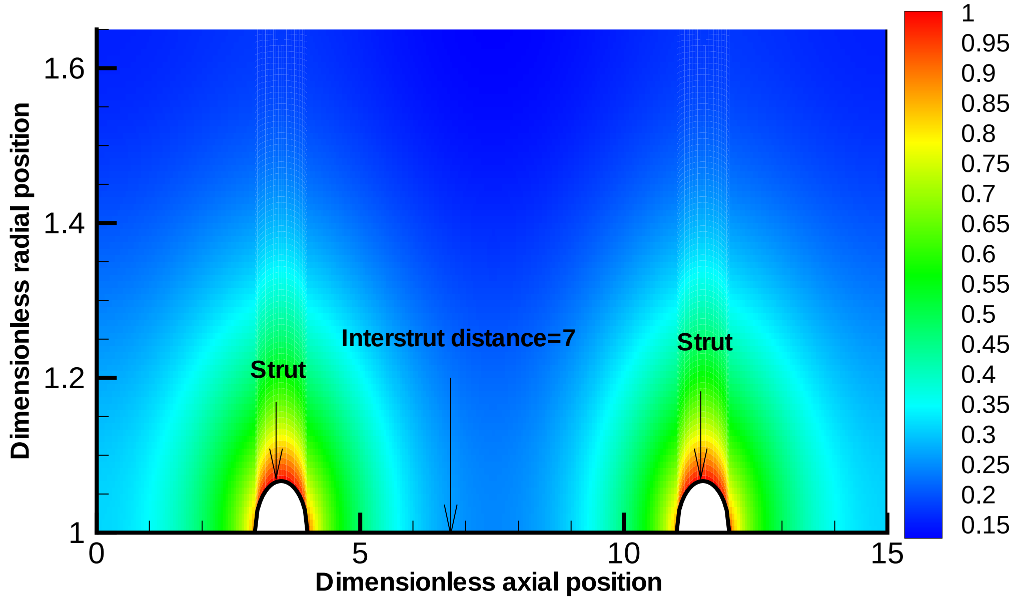
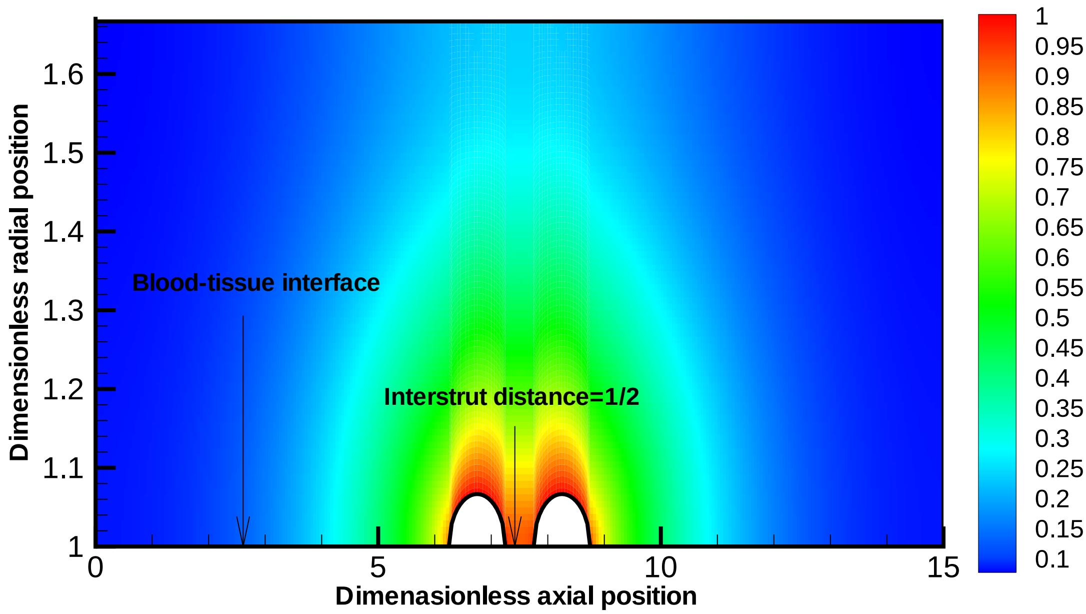


Figure 9: Axial variation of tissue drug concentration at a height of 1.5 strut radius in the tissue (Pe_f=100, Pe_t=1000, Re=500, α=1)



(a)



(b)

Figure 10: Visual representation of drug concentration in the tissue, (a) interstrut distance=7, (b) interstrut distance=1/2 (Re=500, Pe_f =100, Pe_t =1000, α=1)

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Note on the Properties of Trapezoid

By M. P. Chaudhary & Getachew Abiye Salilew
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Abstract- Gedefa and Chaudhary [2] introduce a new technique for identification of the nature of triangle. In this paper, we extended and applied this technique for the trapezoid.

Keywords: *trapezoid, triangle.*

GJSFR-F Classification: *MSC 2010: 51A15, 51M20, 51M30.*

Ref

2. GedefaNegassa Feyissa and M. P. Chaudhary, *On Identification of the Nature of Triangle by New Approach*, International Research Journal of Pure Algebra-5(9), 2015, 138-140.

Note on the Properties of Trapezoid

M. P. Chaudhary α & Getachew Abiye Salilew σ

Abstract- Gedefa and Chaudhary [2] introduce a new technique for identification of the nature of triangle. In this paper, we extended and applied this technique for the trapezoid.

Keywords: trapezoid, triangle.

I. Introduction

Trapezoid is one of the interested areas in geometry, for the researcher since ancient time. But major contributions have been made on it during 19^{th} centuries, which were initiated since 17^{th} century. On this topic several contributions have been already done by the mathematician [4- 13] , but we believe that our approach for dealing current problems described in this article is different than others. Triangle is the simplest polygon with three edges and three vertices. It is one of the basic shapes in geometry. In Euclidean geometry any three points, when non- collinear, determine a unique triangle and a unique plane. The basic elements of any triangle are its sides and angles. Triangles are classified depending on relative sizes of their elements [1, 3].In Euclidean geometry, a convex quadrilateral with at least one pair of parallel sides is referred to as a trapezoid in American and Canadian English but as a trapezium in English outside North America. A trapezium in Proclus' sense is a quadrilateral having one pair of its opposite sides parallel.

In this study we consider a trapezoid which has only one pair of parallel sides. The parallel sides are called the bases, while the other sides are called the legs or lateral sides. The larger base side of a trapezoid used as simply the base of a trapezoid. When the legs have the same length and the base angles have the same measure then the trapezoid is acute angle trapezoid. If the two adjacent angles are right angle, then the trapezoid is a right angle trapezoid. If the trapezoid has no sides of equal measures, it is called a scalene trapezoid. In this paper, we presented acute angle trapezoid, right angle trapezoid and obtuse angle trapezoid which obtained from acute angle triangle, right angle triangle and obtuse angle triangle respectively.

II. Main Result

Theorem 1: Find necessary conditions, which enable to identify the nature of a trapezoid.

Proof: Our approach to derive necessary conditions, for identification the nature of a trapezoid, motivated by the recent work [2], and also by using known results.

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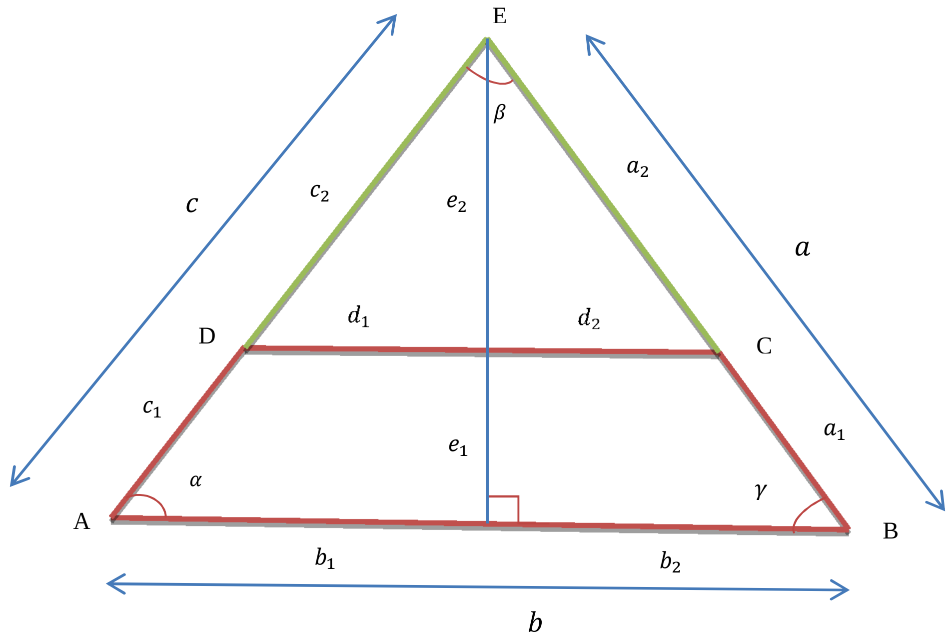


Figure 1

From the above figure-1, let $\mathrm{AB} = \mathrm{b} = \mathrm{b}_1 + \mathrm{b}_2, b_1 \neq b_2$
Let us assume $A\ B$ is parallel to $D\ C$ and assume that $'a'$ is a longest side in the ΔABE. Let $'e'$ be the length of the unique perpendicular segment from E to $A\ B$
From, ΔABE, we have the following conditions.

$B\ E = a = a_1 + a_2$; $A\ B = b = b_1 + b_2, b_1 \neq b_2$; $A\ E = c = c_1 + c_2$; $D\ C = d = d_1 + d_2$; $e = e_1 + e_2$, and clearly $b > d$.

$$\alpha \geq \beta, \alpha \geq \gamma;\ b_1 = c\cos\alpha;\ \ e = c\sin\alpha;\ \ A\ B = b = b_1 + b_2;\ \ b_2 = b - b_1.$$

Using the work done in [2] we have the following.

$$a^2 = b^2 + c^2 - 2b\ c\cos\alpha \tag{1}$$

Lemma 1: ΔABE is similar to ΔDCE. That is, ΔABE~ΔDCE.

Proof. Since AB ∥ DC, then ∠EDC = α, ∠DCE = γ.
Thus, ΔABE~ΔDCE, by angle - angle - angle similarity theorem.
Therefore, by definition of similarity of triangles we have: $\frac{A\ B}{D\ C} = \frac{B\ E}{C\ E} = \frac{A\ E}{D\ E}$
Equivalently, we have

$$\frac{b}{d} = \frac{a}{a_2} = \frac{c}{c_2} \tag{2}$$

From this relation we get

$$\frac{b}{d} = \frac{c}{c_2} \text{ and } \frac{b}{d} = \frac{a}{a_2} \Longrightarrow \frac{b}{c} = \frac{d}{c_2} \text{ and } \frac{b}{a} = \frac{d}{a_2}$$

$$\frac{b}{c_1+c_2} = \frac{d}{c_2} \text{ and } \frac{b}{a_1+a_2} = \frac{d}{a_2} \Longrightarrow d\ c_1 + d\ c_2 = bc_2 \text{ and } d\ a_1 + d\ a_2 = ba_2$$

$$\Longrightarrow \mathrm{c}_2 = \frac{\mathrm{d\ c}_1}{\mathrm{b-d}} \text{ and } \mathrm{a}_2 = \frac{\mathrm{d\ a}_1}{\mathrm{b-d}} \tag{3}$$

Similarly as equation (1), we have the following result from ΔDCE.

$$a_2{}^2 = c_2{}^2 + d^2 - 2dc_2\cos\alpha\,. \tag{4}$$

Using (3) in to (4) we have

Notes

2. GedefaNegassa Feyissa and M. P. Chaudhary, *On Identification of the Nature of Triangle by New Approach*, International Research Journal of Pure Algebra-5(9), 2015, 138-140.

$$a_2^{\,2} = c_2^{\,2} + d^2 - 2dc_2 \cos\alpha \Longrightarrow \left(\frac{da_1}{b-d}\right)^2 = \left(\frac{dc_1}{b-d}\right)^2 + d^2 - 2d\frac{dc_1}{b-d}\cos\alpha$$

$$\Longrightarrow \frac{a_1^{\,2}}{(b-d)^2} = \frac{c_1^{\,2}}{(b-d)^2} + 1 - \frac{2c_1}{b-d}\cos\alpha$$

$$\Longrightarrow a_1^{\,2} = c_1^{\,2} + (b-d)^2 - 2c_1(b-d)\cos\alpha \qquad (5)$$

Which is the general equation of any trapezoid.

For $\alpha = 90^0$ we get from equations (4) and (5), $a_2^{\,2} = c_2^{\,2} + d^2$ and $a_1^{\,2} = c_1^{\,2} + (b-d)^2$ respectively. Thus using [2], we have the following conditions from ΔDCE.

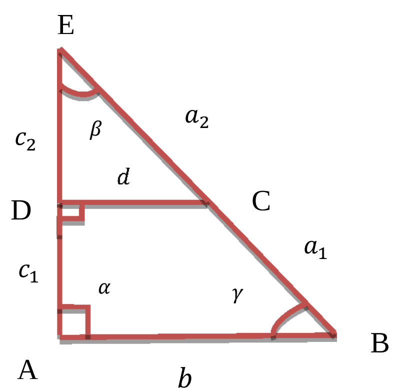
[A]. If $a_2^{\,2} = c_2^{\,2} + d^2$ then $\alpha = 90^0$.
[B]. If $a_2^{\,2} < c_2^{\,2} + d^2$ then α is an acute angle.
[C]. If $a_2^{\,2} > c_2^{\,2} + d^2$ then α is an obtuse angle.

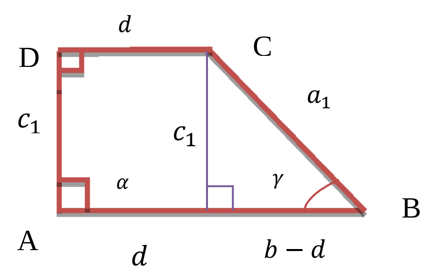
Equivalently we have three conditions for the trapezoid $A\ B\ C\ D$

[D]. If $a_1^{\,2} = c_1^{\,2} + (b-d)^2$ then $\alpha = 90^0$.
[E]. If $a_1^{\,2} < c_1^{\,2} + (b-d)^2$ then α is an acute angle.
[F]. If $a_1^{\,2} > c_1^{\,2} + (b-d)^2$ then α is an obtuse angle.

Let us give the name of a trapezoid based on the larger angle that lies on the larger base side from the two parallel sides of a trapezoid. Here α and γ are angles that lie on the larger base side of a trapezoid $A\ B\ C.D$ Angles α and γ are also called base angles for a trapezoid and $\alpha \geq \gamma$. So, based on the larger measure of angle α from larger base side of a trapezoid we have the following.

[I]. For $\alpha = 90^0$, the trapezoid is called right angle trapezoid.

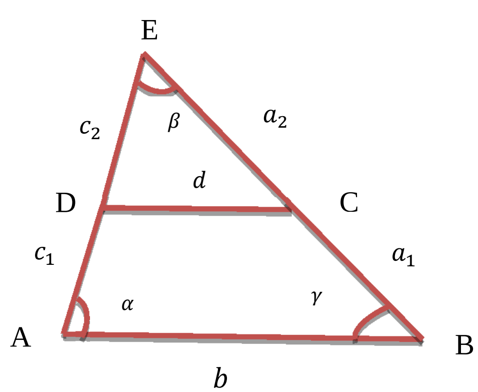


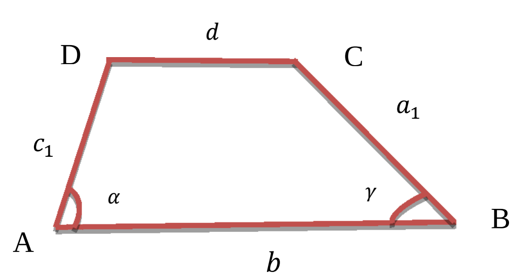


$a_1^{\,2} = c_1^{\,2} + (b-d)^2;\ 0 < \gamma < \alpha = 90^0$

Figure 2

[II]. For $0 < \alpha < 90^0$, the trapezoid is called an acute angle trapezoid.





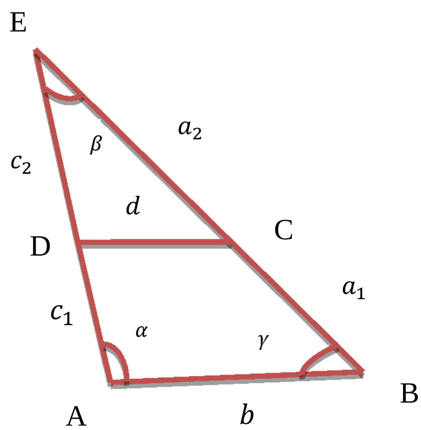
$a_1^{\,2} < c_1^{\,2} + (b-d)^2;\ 0 < \gamma \leq \alpha < 90^0$

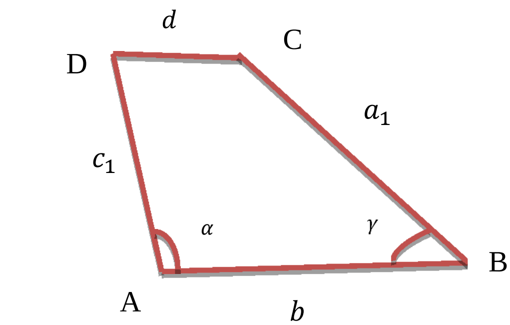
Figure 3

Ref

2. GedefaNegassa Feyissa and M. P. Chaudhary, *On Identification of the Nature of Triangle by New Approach*, International Research Journal of Pure Algebra-5(9), 2015, 138-140.

[III]. For $90^0 < \alpha < 180^0$, the trapezoid is called an obtuse angle trapezoid.





$a_1{}^2 > c_1{}^2 + (b-d)^2; 0 < \gamma < 90^0;\ 90^0 < \alpha < 180^0$

Figure 4

Now we have the following observations from figure 2 to 4.

Corollary 1: For a right angle trapezoid ABCD the acute angle $\gamma = 45^0 \Leftrightarrow c_1 = b - d$.

Proof. Consider the right angle trapezoid ABCD in figure 2.

$$\tan\gamma = \frac{c_1}{b-d} \Leftrightarrow 1 = \frac{c_1}{b-d} \Leftrightarrow c_1 = b - d; \angle BCD = 135^0$$

Similarly $\gamma = 30^0 \Leftrightarrow c_1 = \frac{\sqrt{3}}{3}(b-d)$ and $\gamma = 60^0 \Leftrightarrow c_1 = \sqrt{3}(b-d)$.

Corollary 2: For a right angle trapezoid, ABCD, BD $= \sqrt{2\mathrm{bd}} \Leftrightarrow a_1 = d$.

Proof. Consider the right angle trapezoid ABCD in figure 2.

$a_1{}^2 = c_1{}^2 + (b-d)^2 = c_1{}^2 + b^2 - 2b\ d + d^2$, because $A\ B\ C\ D$ is a right angle trapezoid.

Let BD $= \sqrt{2\mathrm{bd}}$ then we get $c_1{}^2 + b^2 = (\mathrm{BD})^2 = 2b\ d$

$\Longrightarrow a_1{}^2 = c_1{}^2 + b^2 - 2b\ d + d^2; \Longrightarrow a_1{}^2 = 2b\ d - 2b\ d + d^2$, because $c_1{}^2 + b^2 = (\mathrm{BD})^2$.

$\Longrightarrow a_1{}^2 = d^2 \Leftrightarrow a_1 = d$; because all sides must be positive.

Let $a_1 = d$

$\Rightarrow a_1{}^2 = c_1{}^2 + b^2 - 2b\ d + d^2; \Rightarrow 0 = c_1{}^2 + b^2 - 2b\ d \Rightarrow c_1{}^2 + b^2 = 2b\ d = (\mathrm{BD})^2; \Rightarrow \mathrm{BD} = \sqrt{2\mathrm{bd}}$

Similarly, AC $= \sqrt{2\mathrm{bd}} \Leftrightarrow a_1 = b; a_1{}^2 = d^2 + 2b(b-d) \Leftrightarrow c_1 = b$

If the two legs (lateral sides) of the trapezoid are equal then the trapezoid is called isosceles trapezoid.

Corollary 3: An acute angle trapezoid ABCD is an isosceles trapezoid $\Leftrightarrow 0 < \gamma = \alpha < 90^0$.

Proof. Consider an acute angle trapezoid ABCD in figure 3.

Let $a_1 = c_1$ (trapezoid ABCD is an isosceles trapezoid).

$\Rightarrow \sin\alpha = \frac{h}{c_1}$ and $\sin\gamma = \frac{h}{a_1}$; $\Rightarrow h = c_1\sin\alpha$ and $h = a_1\sin\gamma; \Rightarrow h = c_1\sin\alpha = a_1\sin\gamma$

$\Longrightarrow \sin\alpha = \sin\gamma$, because $a_1 = c_1; \Longrightarrow \gamma = \alpha$

Therefore, $0 < \gamma = \alpha < 90^0$.

Let, $\gamma = \alpha. \Longrightarrow c_1\sin\alpha = a_1\sin\gamma; \Longrightarrow a_1 = c_1$.

Similarly when $0 < \gamma < \alpha < 90^0$ an acute angle trapezoid ABCD, is not isosceles trapezoid.

Corollary 4: An isosceles trapezoid ABCD is equilateral trapezoid (i.e. $a_1 = c_1 = d$), whenever $b = c_1(1 + 2\cos\alpha)$.

Notes

Proof. Let the trapezoid ABCD in figure 3 is an isosceles trapezoid.

$$\Longrightarrow a_1 = c_1$$

From equation (5), we get

$a_1{}^2 = c_1{}^2 + (b-d)^2 - 2c_1(b-d)\cos\alpha.$

$\Longrightarrow 0 = (b-d)^2 - 2c_1(b-d)\cos\alpha$, because $a_1 = c_1$

$\Longrightarrow b - d = 2c_1\cos\alpha; \Longrightarrow c_1 + 2c_1\cos\alpha - d = 2c_1\cos\alpha; \Longrightarrow c_1 = d.$

Notes

III. Conclusion

We have found the following conditions from our main result.

(i). If, $a_1{}^2 = c_1{}^2 + (b-d)^2$, then trapezoid ABCD is right angle trapezoid.

(ii). If, $a_1{}^2 < c_1{}^2 + (b-d)^2$, then trapezoid ABCD is an acute angle trapezoid.

(iii). If, $a_1{}^2 > c_1{}^2 + (b-d)^2$, then trapezoid ABCD is an obtuse angle trapezoid.

IV. Acknowledgement

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A Production Inventory Model for Deteriorating Products with Multiple Market and Selling Price dependent demand

By Pinky Saxena, S. R Singh & Isha Sangal

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Abstract- This paper presents an inventory model for deteriorating items that have a single manufacturer but multiple market demands for a finite planning horizon. For the market, different selling seasons are considered. It is a production inventory model, which has a demand rate dependent on the selling price. Here, we have presented a solution-search procedure to find the optimal replenishment policy for raw material and optimal production time. The model is illustrated using a numerical example. Further; sensitivity analysis is also performed to check the stability of the model.

Keywords: *production, deterioration, multiple market, price dependent demand.*

GJSFR-F Classification: *MSC 2010: 91B26*

APRODUCTIONINVENTORYMODELFORDETERIORATINGPRODUCTSWITHMULTIPLEMARKETANDSELLINGPRICEDEPENDENTDEMAND

Strictly as per the compliance and regulations of :

Notes

A Production Inventory Model for Deteriorating Products with Multiple Market and Selling Price dependent demand

Pinky Saxena [α], S. R Singh [σ] & Isha Sangal [ρ]

Abstract- This paper presents an inventory model for deteriorating items that have a single manufacturer but multiple market demands for a finite planning horizon. For the market, different selling seasons are considered. It is a production inventory model, which has a demand rate dependent on the selling price. Here, we have presented a solution-search procedure to find the optimal replenishment policy for raw material and optimal production time. The model is illustrated using a numerical example. Further; sensitivity analysis is also performed to check the stability of the model.

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I. Introduction

To consider the effect of deterioration in the development of inventory models is an essential need of inventory modeling. Ignoring the effect of deterioration misleads the results. Ghare and Schrader (1963) were the first to introduce the deterioration in inventory modelling. They developed this model with constant rate of demand and deterioration. Mishra (1975) presented production lot-size model for a system having deteriorating inventory. Kang and Kim (1983) came forward with a study on the price and production level of the deteriorating inventory system. Wee (1993) defined the deterioration of products as "decay, damage, spoilage, evaporation, obsolescence, pilferage, loss of utility or loss of marginal value of a commodity that results in decreasing usefulness". The deterioration occurs in most of the physical products such as medicines, food products, dairy products and volatile products, etc. Goyal and Giri (2001) proposed an inventory model for recent trends in deteriorating inventory modeling.

In today's global market, there are so many opportunities for a vendor or manufacturer. It is not necessary that it will deal with a single market. It can deal in many markets for a single production run. Khouja (2001) presented the effect of large order quantities on expected profit in the single-period model. This model is developed with multiple market demand. In this model, he gave the example of a garment industry which sells its product in different markets. It is profitable for the manufacturer but very complicated also. This is because in different markets the demand rate will also be different. An Economic Order Quantity (EOQ) model was

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presented by Covert and Philip (1973) for items that have time-dependent demand and deterioration as per Wei-bull distribution. Giri *et al.* (1996) put forward an inventory model for deteriorating items that have a demand rate dependent on the stock. Yang and Wee (2002) presented a production inventory model with multiple-buyer for a deteriorating item. Wu *et al.* (2006) proposed, for non-instantaneous deteriorating items that have stock-dependent demand and partial backlogging, an optimal-replenishment policy.

In this paper, we propose and develop a production inventory model for deteriorating items that have multiple markets and a demand rate dependent on the selling price. This model is illustrated using a numerical example and sensitivity analysis is also performed to show the model's stability.

Notes

II. Assumptions and Notations

a) Assumptions

1. A single product is from a single manufacturer.
2. The model is for multiple market demand.
3. Demand rate is a function of the selling price.
4. Production rate is constant and is greater than the demand of all the markets.
5. The products being considered are deteriorating in nature.
6. The rate of deterioration is assumed to be a linear function of time.
7. No, shortages are allowed.
8. Planning horizon is considered finite.
9. No replacement or repair of deteriorated units is done during a given cycle.
10. It is assumed that the manufacturer arranges the fix quantity of raw material at a fix interval of time. The quantity of finished products produced by the manufacturer is enough to meet the occurring demand and deterioration in all the markets.

b) Notations

A : Production rate.

θ : Deterioration parameter.

α_k : Demand parameter in k^{th} time interval.

β : Demand parameter.

p : selling price per unit time.

T : production period.

K_0: setup cost

h_f : Holding cost per unit for finished products.

$I_i(t)$: Inventory level at any time t in i^{th} interval where (i=1,2,3……n).

d_f: Deterioration cost per unit for finished products.

d_r: Deterioration cost per unit for raw material inventory.

h_r: Holding cost per unit for raw material.

μ: Ordering cost per order.

q_r: Delivery lot size of raw material.

n_r: Number of deliveries of raw material in time T.

III. Model Description and Analysis

The model is developed with a piecewise constant function in demand. It is assumed that the optimal production run time T lies in the interval $[T_{m-1}, T_m]$. Let us

Notes

divide the interval $[T_{m-1}, T_m]$ into two parts. Here $I_m^-(t)$ represents the inventory level during $t \in [T_{m-1}, T]$ and $I_m^+(t)$ represents inventory during $t \in [T, T_m]$.

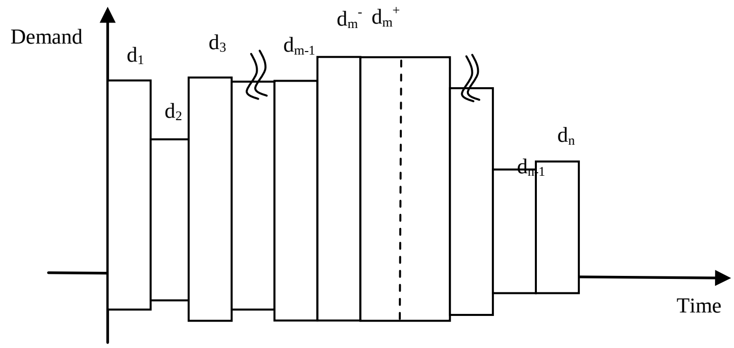

Fig. 1: Manufacture's time v/s demand

The inventory time behavior of the finished products at any time t is given by the following differential equations:

$$\frac{dI_k(t)}{dt} + \theta t I_k(t) = A - \frac{\alpha_k}{p^\beta} \; ; T_{k-1} \le t \le T_k \tag{1}$$

$$\frac{dI_m^-(t)}{dt} + \theta t I_m^-(t) = A - \frac{\alpha_m}{p^\beta} \; ; T_{m-1} \le t \le T \tag{2}$$

$$\frac{dI_m^+(t)}{dt} + \theta t I_m^+(t) = -\frac{\alpha_m}{p^\beta} \; ; T \le t \le T_m \tag{3}$$

$$\frac{dI_j(t)}{dt} + \theta t I_j(t) = -\frac{\alpha_j}{p^\beta} \; ; T_{j-1} \le t \le T_j \tag{4}$$

With boundary conditions:

$$I_1(0)=0,\ I_k(T_{k-1})=I_{k-1}(T_{k-1}), I_m^-(T_{m-1})=I_{m-1}(T_{m-1}),$$

$$I_n(T_n)=0,\ I_{j-1}(T_{j-1})=I_j(T_j \tag{5}$$

The solution of these equations is given by (see appendix A):

$$I_k(t) = (A - \frac{\alpha_k}{p^\beta})(t + \frac{\theta}{6}t^3)e^{-\frac{\theta t^2}{2}} + \sum_{i=1}^{k} \frac{(\alpha_i - \alpha_{i-1})}{p^\beta}(T_{i-1} + \frac{\theta}{6}T_{i-1}^3)e^{-\frac{\theta t^2}{2}}$$

$$T_{k-1} \le t \le T_k \qquad \text{for k=1,2,3...........(m-1)} \tag{6}$$

$$I_m^-(t) = (A - \frac{\alpha_m}{p^\beta})(t + \frac{\theta}{6}t^3)e^{-\frac{\theta t^2}{2}} + \sum_{i=1}^{m} \frac{(\alpha_i - \alpha_{i-1})}{p^\beta}(T_{i-1} + \frac{\theta}{6}T_{i-1}^3)e^{-\frac{\theta t^2}{2}}$$

$$T_{m-1} \le t \le T \tag{7}$$

$$I_m^+(t) = -\frac{\alpha_m}{p^\beta}(t + \frac{\theta}{6}t^3)e^{-\frac{\theta t^2}{2}} + \frac{\alpha_n}{p^\beta}(T_n + \frac{\theta}{6}T_n^3)e^{-\frac{\theta t^2}{2}} - \sum_{i=m+1}^{n} \frac{(\alpha_i - \alpha_{i-1})}{p^\beta}(T_{i-1} + \frac{\theta}{6}T_{i-1}^3)e^{-\frac{\theta t^2}{2}}$$

$$T \le t \le T_m \tag{8}$$

$$I_j(t) = -\frac{\alpha_j}{p^\beta}(t+\frac{\theta}{6}t^3)e^{-\frac{\theta t^2}{2}} + \frac{\alpha_n}{p^\beta}(T_n+\frac{\theta}{6}T_n^3)e^{-\frac{\theta t^2}{2}} - \sum_{i=j+1}^{m}\frac{(\alpha_i-\alpha_{i-1})}{p^\beta}(T_{i-1}+\frac{\theta}{6}T_{i-1}^3)e^{-\frac{\theta t^2}{2}}$$

$$T_{j-1} \le t \le T_j \qquad \text{for j=(m+1), (m+2)……….n} \tag{9}$$

Now the total inventory during $[T_{k-1}, T_k]$ will be calculated as follows:

$$I_k = \int_{T_{k-1}}^{T_k} I_k(t)\,dt$$

$$I_k = (A - \frac{\alpha_k}{p^\beta})(\frac{T_k^2 - T_{k-1}^2}{2} - \frac{\theta}{12}(T_k^4 - T_{k-1}^4))$$

$$+\sum_{i=1}^{k}\frac{(\alpha_i-\alpha_{i-1})}{p^\beta}(T_{i-1}+\frac{\theta}{6}T_{i-1}^3)((T_k - T_{k-1}) - \frac{\theta}{6}(T_k^3 - T_{k-1}^3)) \tag{10}$$

The total inventory during $[T_{m-1}, T]$ will be:

$$I_m^- = \int_{T_{m-1}}^{T} I_m^-(t)\,dt$$

$$I_m^- = (A - \frac{\alpha_m}{p^\beta})(\frac{T_m^2 - T_{m-1}^2}{2} - \frac{\theta}{12}(T_m^4 - T_{m-1}^4))$$

$$+\sum_{i=1}^{m}\frac{(\alpha_i-\alpha_{i-1})}{p^\beta}(T_{i-1}+\frac{\theta}{6}T_{i-1}^3)((T_m - T_{m-1}) - \frac{\theta}{6}(T_m^3 - T_{m-1}^3)) \tag{11}$$

The total inventory during $[T, T_m]$ will be:

$$I_m^+ = \int_{T}^{T_m} I_m^+(t)\,dt$$

$$I_m^+ = [-\frac{\alpha_m}{p^\beta}(\frac{T_m^2 - T^2}{2} - \frac{\theta}{12}(T_m^4 - T^4))$$

$$+\frac{\alpha_n}{p^\beta}(T_n + \frac{\theta}{6}T_n^3)((T_m - T) - \frac{\theta}{6}(T_m^3 - T^3))$$

$$-\sum_{i=m+1}^{n}\frac{(\alpha_i-\alpha_{i-1})}{p^\beta}(T_{i-1}+\frac{\theta}{6}T_{i-1}^3)((T_m - T) - \frac{\theta}{6}(T_m^3 - T^3))] \tag{12}$$

Similarly, the total inventory during $[T_{j-1}, T_j]$ for all j=(m+1)………n:

$$I_j = [-\frac{\alpha_j}{p^\beta}(\frac{T_j^2 - T_{j-1}^2}{2} - \frac{\theta}{12}(T_j^4 - T_{j-1}^4))$$

$$+\frac{\alpha_n}{p^\beta}(T_n + \frac{\theta}{6}T_n^3)((T_j - T_{j-1}) - \frac{\theta}{6}(T_j^3 - T_{j-1}^3))$$

Notes

$$-\sum_{i=j+1}^{n}\frac{(\alpha_i-\alpha_{i-1})}{p^{\beta}}(T_{i-1}+\frac{\theta}{6}T_{i-1}^{\ 3})((T_j-T_{j-1})-\frac{\theta}{6}(T_j^3-T_{j-1}^{\ 3}))] \tag{13}$$

The holding cost of the system for finished products is calculated as follows:

$$H.C_f = h_f(\sum_{k=1}^{m-1} I_k + I_m^{\ -} + I_m^{\ +} + \sum_{j=m+1}^{n} I_j) \tag{14}$$

Notes

The deterioration cost for the system of finished products is given by:

$$D.C_f = d_f(AT - \sum_{k=1}^{n}\frac{\alpha_k}{p^{\beta}}(T_k - T_{k-1})) \tag{15}$$

Set up cost for the finished product is computed as follows:

$$S.C_f = K_0 \tag{16}$$

Now, the total cost for the finished product can be calculated as follows:

$$T.C_f = H.C_f + D.C_f + S.C_f \tag{17}$$

a) Manufacturer's raw material inventory model

In this model, the inventory is replenished at t=0. During $[0,\ T/n_r]$, the inventory depletes as a result of the combined effect of demand and deterioration. At $t=T/n_r$, the inventory level becomes zero and is again replenished. The differential equation that governs the transition of the system is represented as follows:

$$\frac{dI_r(t)}{dt} + \theta_r t I_r(t) = -f_r\ ;0 \le t \le T/n_r \tag{18}$$

With boundary condition $I_r(T/n_r)=0$.
The solution of equation (18) will be:

$$I_r(t) = f_r\{(\frac{T}{n_r}-t)+(\frac{\theta r}{6}(\frac{T}{n_r})^3 - t^3)\}e^{-\theta_r\frac{t^2}{2}};0 \le t \le T/n_r \tag{19}$$

We know that $I_r(0) = q$

$$q = f_r\{(\frac{T}{n_r}) + \frac{\theta_r}{6}(\frac{T}{n_r})^3\} \tag{20}$$

Now, the holding cost of the system for carrying the raw material will be:

$$H.C_r = n_r h_r \int_0^{T/n_r} I_r(t)\,dt$$

$$H.C_r = nh_r[f_r\{\frac{T^2}{2n_r^{\ 2}} + \frac{\theta_r}{8}(\frac{T}{n_r})^4 - f_r\frac{\theta_r}{24}(\frac{T}{n_r})^4\}] \tag{21}$$

The cost of deteriorated units of raw material:

$$D.C_r = d_r(n_r q - f_r T) \tag{22}$$

Now, the total cost of raw material inventory can be calculated as follows:

$$T.C_r = H.C_r + D.C_r + O.C_r \tag{23}$$

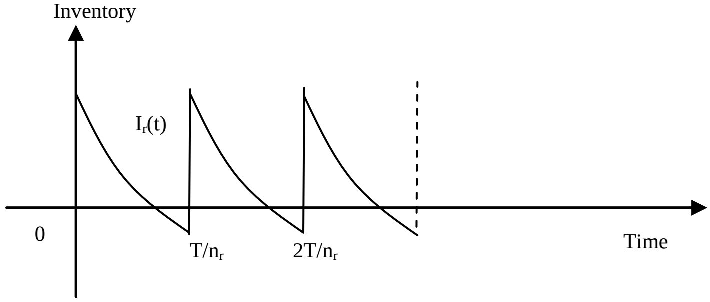

Fig. 2: Raw material inventory system

Then, the integrated total cost for whole the system will be as follows:

$$T.C. = T.C_f + T.C_r \tag{24}$$

After putting the values in equation (24), we observe that T.C. for whole the system is a function of n_r and T, where n_r is a discrete variable and T is a continuous variable.

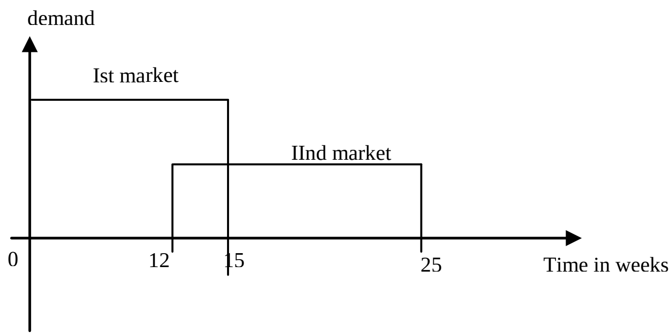

Fig. 3: Time v/s demand in each the market

IV. Solution Procedure

Step 1: Here n_r is a discrete variable.

Step 2: Solve the equation $\frac{\partial TC}{\partial T} = 0$ for all possible values of n_r starting from n_r=1.

Step 3: Put all these values of T and n_r in equation (24) and find the value of T.C.

Step 4: Repeat the step 2-3 until the optimal value of T.C. is found.

V. Numerical Example

Here we have discussed only the two market situation. The demand rate is taken as a function of selling price. The value of demand parameter α_k for market one and market two is 1500 units and 1200 units respectively. The selling rate is Rs 8 per unit. The value for the deterioration parameter for finished products and raw material are 0.01 and 0.015 respectively. The season for selling items in market one is considered from 1-15 weeks and for second market it is 12-25 weeks. The setup cost and ordering cost for production and ordering the raw material is given by Rs 750 and Rs 500 respectively. The production cost and purchasing cost of raw material is Rs 5 and Rs 2 per unit respectively. The holding cost h_f and h_r are 0.25 and 0.2 per unit per time. The production rate is 500 units/ week.

Here n=3, T_1=5, T_2=17, T_3=25

By applying the above mentioned solution procedure, we find the following values:

T*=21 weeks, nr=5.008, T.C. = Rs 852.012

Notes

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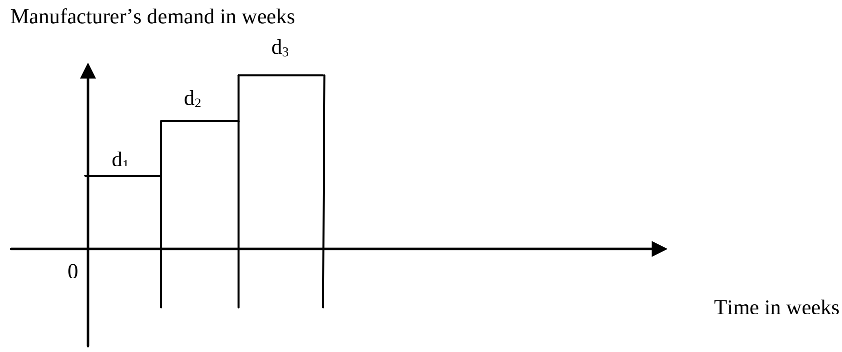

Fig. 4: Time v/s demand in each the market

VI. Sensitivity Analysis

For the given numerical illustration, investigations have been performed to study the impact of changes of different parameters on the number of deliveries of raw material, production period along with the total cost of the system.

Table 1: Sensitivity analysis for production rate A

% variation in A	A	nr	T	T.C.
-20%	400	5.2	16.5	720.85
-15%	425	5.168	17.32	745.72
-10%	450	5.16	18.45	775.81
-5%	475	5.09	19.2	822.01
0%	500	5.008	21	852.012
5%	525	4.89	21.8	878.92
10%	550	4.72	22.71	912.65
15%	575	4.61	23.5	934.37
20%	600	4.45	24	961.41

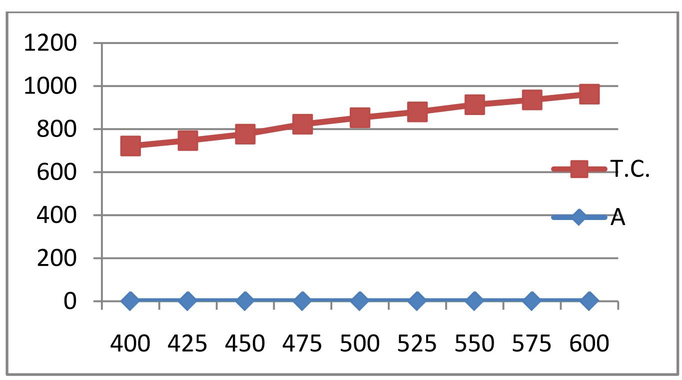

Fig. 5: Variation of total cost T.C. with respect to production rate A

Table 2: Sensitivity analysis for deterioration parameter θ

% variation in θ	θ	nr	T	T.C.
-20%	0.012	5.008	13.67	828.52
-15%	0.01275	5.008	15.52	839.716
-10%	0.0135	5.008	17.81	843.82
-5%	0.01425	5.008	19.2	847.79
0%	0.015	5.008	21	852.012

5%	0.01575	5.008	23.03	856.92
10%	0.0165	5.008	25.57	861.13
15%	0.01725	5.008	27.73	864.99
20%	0.018	5.008	29.89	867.321

Table 3: Sensitivity analysis for holding cost of finished products h_f

% variation in h_f	h_f	n_r	T	T.C.
-20%	0.2	6.712	25.7172	751.816
-15%	0.2125	6.381	24.875	782.71
-10%	0.225	5.862	23.981	813.48
-5%	0.2375	5.418	22.112	831.18
0%	0.25	5.008	21	852.012
5%	0.2625	4.67	19.987	878.93
10%	0.275	4.219	18.753	891.25
15%	0.2875	3.892	17.625	913.21
20%	0.3	3.416	16.512	934.48

Table 4: Sensitivity analysis for holding cost of raw material h_r

% variation in h_r	h_r	n_r	T	T.C.
-20%	0.16	5.008	21	839.78
-15%	0.17	5.008	21	842.14
-10%	0.18	5.008	21	845.32
-5%	0.19	5.008	21	849.516
0%	0.2	5.008	21	852.012
5%	0.21	5.008	21	855.221
10%	0.22	5.008	21	858.31
15%	0.23	5.008	21	861.54
20%	0.24	5.008	21	864.715

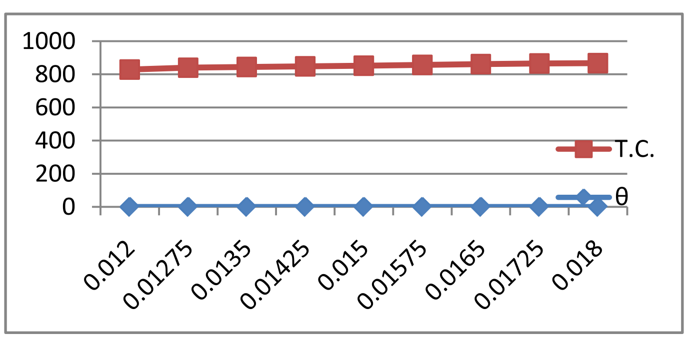

Fig. 6: Variation in total cost T.C. w.r.t deterioration parameter θ

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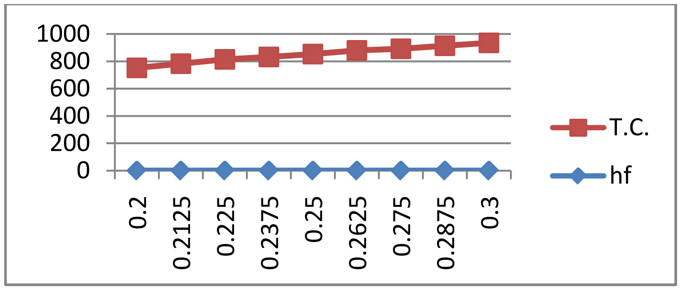

Fig. 7: Variation in total cost. T.C with respect to holding cost h_f

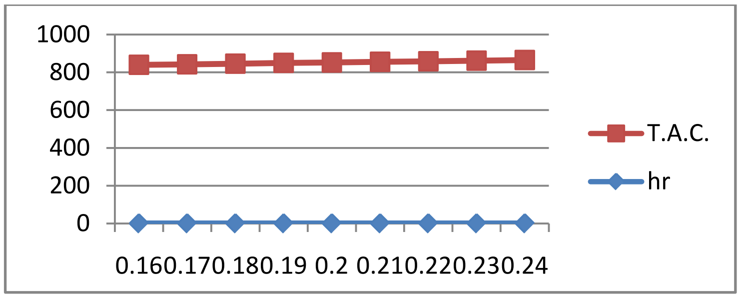

Fig. 8: Variation in total cost T.C. with respect to holding cost hr

VII. Concluding Remarks

A sensitivity analysis is performed with respect to different system parameters to check the stability of the model.

1. From table1, it is observed that as the value of the production parameter 'A' increases, it also results in increase in the total cost of the system.
2. Table 2 shows the variation in deterioration parameter 'θ'. It is shown in this table that an increment in deterioration parameter 'θ' results also in an increment in T.C. of the system.
3. In table 3 and table 4, the variation in holding cost h_f and h_r are shown. It is observed from these tables that with the increment in holding parameters, the T.C. of the system increases in both the cases.

VIII. Conclusion

In this chapter we have presented an inventory model for deteriorating products. In this model, the manufacturer produces the items at a single location and sells it in different markets having different demands and selling seasons. With the help of a mathematical model, we calculate the different associated costs and provide a method to reduce the total cost of the system. This is done by deriving the optimal production time and replenishes time for the raw material. Further; performing a sensitivity analysis helps to check the stability of the model. With the sensitivity analysis, the model is found to be quite stable. For further research, time value of money and different permissible conditions can be applied for the extension of this model.

Appendix A:

The solution of equation (1) is given by:

$$I_k(t) = (A - \frac{\alpha_k}{p^\beta})(t + \frac{\theta}{6}t^3)e^{-\frac{\theta t^2}{2}} + c_k e^{-\frac{\theta t^2}{2}} \qquad \text{for k=1,2.............(m-1)}$$

if k=1, from $I_1(0)=0$,We get $c_1=0$

Now from the boundary condition,
we get ,$I_k(T_{k-1}) = I_{k-1}(T_{k-1})$

$$(c_k - c_{k-1}) = \frac{1}{p^{\beta}}(T_{k-1} + \frac{\theta}{6}T_{k-1}{}^3)(\alpha_k - \alpha_{k-1})$$

From this equation we get:

$$c_2 - c_1 = \frac{1}{p^{\beta}}(\alpha_2 - \alpha_1)(T_1 + \frac{\theta}{6}T_1^3)$$

$$c_3 - c_2 = \frac{1}{p^{\beta}}(\alpha_3 - \alpha_2)(T_2 + \frac{\theta}{6}T_2^3)$$

$$c_4 - c_3 = \frac{1}{p^{\beta}}(\alpha_4 - \alpha_3)(T_3 + \frac{\theta}{6}T_3^3)$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$c_k - c_{k-1} = \frac{1}{p^{\beta}}(\alpha_k - \alpha_{k-1})(T_{k-1} + \frac{\theta}{6}T_{k-1}{}^3)$$

Summing up all these equations gives us:

$$c_k - c_1 = \sum_{i=2}^{k}\frac{1}{p^{\beta}}(\alpha_i - \alpha_{i-1})(T_{i-1} + \frac{\theta}{6}T_{i-1}{}^3)$$

Since c_1 is zero so:

$$c_k = \sum_{i=1}^{k}\frac{1}{p^{\beta}}(\alpha_i - \alpha_{i-1})(T_{i-1} + \frac{\theta}{6}T_{i-1}{}^3)$$

Then the solution of equation (1) will be:

$$I_k(t) = (A - \frac{\alpha_k}{p^{\beta}})(t + \frac{\theta}{6}t^3)e^{-\frac{\theta t^2}{2}} + \sum_{i=1}^{k}\frac{1}{p^{\beta}}(\alpha_i - \alpha_{i-1})(T_{i-1} + \frac{\theta}{6}T_{i-1}{}^3)e^{-\frac{\theta t^2}{2}}$$

$$T_{k-1} \le t \le T_k$$

for k=1,2,……………(m-1)
Let k=m, then solution of equation (2) will be:

$$I_m^{-}(t) = (A - \frac{\alpha_m}{p^{\beta}})(t + \frac{\theta}{6}t^3)e^{-\frac{\theta t^2}{2}} + \sum_{i=1}^{m}\frac{1}{p^{\beta}}(\alpha_i - \alpha_{i-1})(T_{i-1} + \frac{\theta}{6}T_{i-1}{}^3)e^{-\frac{\theta t^2}{2}}$$

$$T_{m-1} \le t \le T$$

$$I_j(t) = -\frac{\alpha_j}{p^{\beta}}(t + \frac{\theta}{6}t^3)e^{-\frac{\theta t^2}{2}} + \frac{\alpha_n}{p^{\beta}}(T_n + \frac{\theta}{6}T_n{}^3)e^{-\theta\frac{t^2}{2}}$$

$$-\sum_{i=j+1}^{n}\frac{1}{p^{\beta}}(\alpha_i - \alpha_{i-1})(T_{i-1} + \frac{\theta}{6}T_{i-1}{}^3)e^{-\frac{\theta t^2}{2}}$$

for j=m+1,m+2……n $\qquad T_{j-1} \le t \le T_j$

Notes

$$I_m^{+}(t) = -\frac{\alpha_m}{p^{\beta}}(t+\frac{\theta}{6}t^3)e^{-\frac{\theta t^2}{2}} + \frac{\alpha_n}{p^{\beta}}(T_n+\frac{\theta}{6}T_n^{\ 3})e^{-\theta\frac{t^2}{2}}$$

$$-\sum_{i=m+1}^{n}\frac{1}{p^{\beta}}(\alpha_i-\alpha_{i-1})(T_{i-1}+\frac{\theta}{6}T_{i-1}^{\ 3})e^{-\frac{\theta t^2}{2}}$$

$$T \le t \le T_m$$

Notes

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Pseudo Ricci-Symmetric $(LCS)_n$ -Manifolds

By N. S. Ravikumar & K. Nagana Gouda

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Abstract- The present paper deals with the study of pseudo Ricci symmetric properties on (LCS)n-manifolds. Here we study generalized pseudo Riccisymmetric, almost pseudo Ricci-symmetric and semi pseudo Ricci-symmetric (LCS)n-manifolds and obtained some interesting results.

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PSEUDORICCISYMMETRICLCSMANIFOLDS

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Pseudo Ricci-Symmetric $(LCS)_n$ -Manifolds

N. S. Ravikumar α & K. Nagana Gouda σ

Abstract- The present paper deals with the study of pseudo Ricci symmetric properties on (LCS)n-manifolds. Here we study generalized pseudo Riccisymmetric, almost pseudo Ricci-symmetric and semi pseudo Ricci-symmetric (LCS)n-manifolds and obtained some interesting results.

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I. Introduction

The study of Riemann symmetric manifolds began with the work of Cartan [5]. According to Cartan, a Riemannian manifold is said to be locally symmetric if its curvature tensor R satisfies the relation $DR = 0$, where D is the covariant differentiation operator with respect to the metric tensor g. During the last five decades the notion of locally symmetric manifolds has been weakened by many authors in several ways to a different extent such as recurrent manifold [27], semi symmetric manifold [22], pseudo symmetric manifold [6, 14] etc.,

If the Ricci tensor S of type $(0, 2)$ in a Riemannian manifold M satisfies the relation $DS = 0$, then S is said to be Ricci symmetric. The notion of Ricci symmetry has been studied extensively by many authors in several ways to a different extent viz., Ricci recurrent manifold [18], Ricci semi symmetric manifold [22], Ricci pseudo symmetric manifold [7, 13], weakly Ricci symmetric manifold [23].

A $(2n + 1)$-dimensional non-flat Riemannian manifold M is said to be pseudo Ricci symmetric if its Ricci tensor S of type $(0, 2)$ is not identically zero and satisfies the relation

$$(D_X S)(Y, Z) = 2A(X)S(Y, Z) + A(Y)S(X, Z) + A(Z)S(X, Y),$$

for any vector fields X, Y and Z, where A is a nowhere vanishing 1-form on M. The pseudo Ricci symmetric manifolds have also been studied by Arslan et. al [1], De and Mazumder [9], De et. al [11] and many others.

The present paper is organized as follows: In Section 2 we give the definitions and some preliminary results that will be needed thereafter. In Section 3 we discuss generalized pseudo-Ricci symmetric $(LCS)_n$-manifold and it is shown that the sum of $2A$, B and C is always nonzero. Section 4 is devoted to the study of almost pseudo Ricci-symmetric $(LCS)_n$-manifold and obtain that the sum $3A + B$ is nowhere zero. In section 5 we consider semi pseudo Ricci-symmetric $(LCS)_n$-manifold and proved that the 1-form A is always non zero.

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II. PRELIMINARIES

The notion of Lorentzian concircular structure manifolds (briefly $(LCS)_n$-manifolds) was introduced by A.A. Shaikh [20] in 2003. An n-dimensional Lorentzian manifold M is a smooth connected paracompact Hausdorff manifold with a Lorentzian metric g, that is, M admits a smooth symmetric tensor field g of type $(0,2)$ such that for each point $p \in M$, the tensor $g_p : T_pM \times T_pM \to R$ is a non-degenerate inner product of signature $(-,+,.....,+)$, where T_pM denotes the tangent vector space of M at p and R is the real number space.

Definition 2.1. In a Lorentzian manifold (M,g), a vector field P defined by

$$g(X,P) = A(X),$$

for any vector field $X \in \chi(M)$ is said to be a concircular vector field if

$$(\nabla_X A)(Y) = \alpha[g(X,Y) + \omega(X)A(Y)],$$

where α is a non-zero scalar function, A is a 1-form and ω is a closed 1-form.

Let M be a n-dimensional Lorentzian manifold admitting a unit timelike concircular vector field ξ, called the characteristic vector field of the manifold. Then we have

$$g(\xi,\xi) = -1. \tag{2.1}$$

Since ξ is a unit concircular vector field, there exists a non-zero 1-form η such that

$$g(X,\xi) = \eta(X), \tag{2.2}$$

the equation of the following form holds

$$(\nabla_X \eta)(Y) = \alpha[g(X,Y) + \eta(X)\eta(Y)], \quad (\alpha \neq 0) \tag{2.3}$$

for all vector fields X, Y, where ∇ denotes the operator of covariant differentiation with respect to Lorentzian metric g and α is a non-zero scalar function satisfying

$$\nabla_X \alpha = (X\alpha) = d\alpha(X) = \rho\eta(X), \tag{2.4}$$

ρ being a certain scalar function given by $\rho = -(\xi\alpha)$. If we put

$$\phi X = \frac{1}{\alpha}\nabla_X \xi, \tag{2.5}$$

then from (2.3) and (2.4), we have

$$\phi X = X + \eta(X)\xi, \tag{2.6}$$

from which it follows that ϕ is a symmetric $(1,1)$ tensor. Thus the Lorentzian manifold M together with the unit timelike concircular vector field ξ, its associated 1-form η and $(1,1)$ tensor field ϕ is said to be a Lorentzian concircular structure manifold (briefly $(LCS)_n$-manifold). Especially, if we take $\alpha = 1$, then we can obtain the Lorentzian para-Sasakian structure of Matsumoto [16]. In a $(LCS)_n$-manifold, the following relations hold ([20], [21]):

Ref

20. A.A. Shaikh, *Lorentzian almost paracontact manifolds with a structure of concircular type*, Kyungpook Math. J., 43 (2) (2003), 305-314.

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Notes

$$\eta(\xi) = -1, \quad \phi\xi = 0, \quad \eta(\phi X) = 0, \tag{2.7}$$

$$g(\phi X, \phi Y) = g(X,Y) + \eta(X)\eta(Y), \tag{2.8}$$

$$\eta(R(X,Y)Z) = (\alpha^2 - \rho)[g(Y,Z)\eta(X) - g(X,Z)\eta(Y)], \tag{2.9}$$

$$(\nabla_X \phi)(Y) = \alpha[g(X,Y)\xi + 2\eta(X)\eta(Y)\xi + \eta(Y)X], \tag{2.10}$$

$$S(X,\xi) = (n-1)(\alpha^2 - \rho)\eta(X), \tag{2.11}$$

$$S(\phi X, \phi Y) = S(X,Y) + (n-1)(\alpha^2 - \rho)\eta(X)\eta(Y), \tag{2.12}$$

for any vector fields X, Y, Z, where R, S denote respectively the curvature tensor and the Ricci tensor of the manifold.

III. Generalized Pseudo-Ricci Symmetric $(LCS)n$-Manifold

Let M be a $(2n+1)$-dimensional generalized pseudo-Ricci symmetric $(LCS)_n$-manifold. Then by definition, we have

$$(D_X S)(Y,Z) = 2A(X)S(Y,Z) + B(Y)S(X,Z) + C(Z)S(X,Y), \tag{3.1}$$

where A, B and C are three non-zero 1-forms.
Putting $Z = \xi$ in (3.1) and using (2.11), we get

$$\begin{aligned}(D_X S)(Y,\xi) &= 2(n-1)(\alpha^2 - \rho)A(X)\eta(Y) + (n-1)(\alpha^2 - \rho)B(Y)\eta(X) \\ &+ C(\xi)S(X,Y).\end{aligned} \tag{3.2}$$

Also we have

$$\begin{aligned}(D_X S)(Y,\xi) &= (n-1)(2\alpha\rho - \beta)\eta(X)\eta(Y) + (n-1)(\alpha^2 - \rho)\alpha[g(X,Y) + \eta(X)\eta(Y)] \\ &- \alpha S(X,Y).\end{aligned} \tag{3.3}$$

By using (3.3) in (3.2), we get

$$\begin{aligned}&(n-1)(2\alpha\rho - \beta)\eta(X)\eta(Y) + (n-1)(\alpha^2 - \rho)\alpha[g(X,Y) + \eta(X)\eta(Y)] - \alpha S(X,Y) \\ &= 2(n-1)(\alpha^2 - \rho)A(X)\eta(Y) + (n-1)(\alpha^2 - \rho)B(Y)\eta(X) + C(\xi)S(X,Y).\end{aligned} \tag{3.4}$$

Again putting $X = Y = \xi$ in (3.2), yields

$$2A(\xi) + B(\xi) + C(\xi) = -\frac{2\alpha\rho - \beta}{\alpha^2 - \rho}. \tag{3.5}$$

Taking $X = \xi$ in (3.2) and by virtue of (3.5), we have

$$B(Y) = -\eta(Y)B(\xi). \tag{3.6}$$

Similarly by taking $Y = \xi$ in (3.2) and using (3.5), we get

$$A(X) = -\eta(X)A(\xi). \tag{3.7}$$

Substituting $X = Y = \xi$ in (3.1) and by virtue of (3.5), we obtain

$$C(Z) = -C(\xi)\eta(Z). \tag{3.8}$$

In view of (3.6), (3.7) and (3.8), we have

$$2A(X) + B(X) + C(X) = \frac{2\alpha\rho - \beta}{\alpha^2 - \rho}\eta(X) \;\; for \;\; all \;\; X.$$

Hence we can state the following theorem:

Theorem 3.1. *In a* $(2n+1)$*-dimensional generalized pseudo Ricci-symmetric* $(LCS)n$*-manifold the sum of* $2A$, B *and* C *is always nonzero.*

IV. Almost Pseudo Ricci-Symmetric $(LCS)n$-Manifold

In 2007, Chaki and Kawaguchi [8] introduced a type of non-flat Riemannian manifold whose Ricci tensor S of type $(0,2)$ satisfies the condition

$$(D_X S)(Y,Z) = [A(X) + B(X)]S(Y,Z) + A(Y)S(X,Z) + A(Z)S(X,Y), \tag{4.1}$$

where A, B are non-zero 1-forms called the associated 1-forms and D denotes the operator of covariant differentiation with respect to the metric g. Such a manifold was called an almost pseudo Ricci symmetric manifold. If in particular $A = B$ then the manifold becomes a pseudo Ricci symmetric manifold introduced by Chaki [7].

Let us consider M be an almost pseudo Ricci-symmetric $N(k)$-contact metric manifold. Now putting $Z = \xi$ in (4.1), we get

$$(D_X S)(Y,\xi) = [A(X) + B(X)]S(Y,\xi) + A(Y)S(X,\xi) + A(\xi)S(X,Y). \tag{4.2}$$

By using (2.3), (2.4), (2.5) and (2.11) in (4.2), we have

$$\begin{aligned}(n-1)(2\alpha\rho - \beta)\eta(X)\eta(Y) &= (n-1)(\alpha^2 - \rho)[A(X) + B(X)]\eta(Y) \\ &+ (n-1)(\alpha^2 - \rho)A(Y)\eta(X) + A(\xi)S(X,Y).\end{aligned} \tag{4.3}$$

Putting $X = \xi$ in (4.3), we get

$$\begin{aligned}-(2\alpha\rho - \beta)\eta(Y) &= (\alpha^2 - \rho)[A(\xi) + B(\xi)]\eta(Y) \\ &- (\alpha^2 - \rho)A(Y) + A(\xi)(\alpha^2 - \rho)\eta(Y).\end{aligned} \tag{4.4}$$

Again putting $Y = \xi$ in (4.4), gives

$$3A(\xi) + B(\xi) = -\frac{(2\alpha\rho - \beta)}{(\alpha^2 - \rho)}. \tag{4.5}$$

Ref

7. M.C. Chaki, *On pseudo Ricci symmetric manifolds*, Bulg. J. Phys., 15 (1988), 526-531.

Next putting $Y = \xi$ in (4.3), we get

$$-(2\alpha\rho - \beta)\eta(X) = -(\alpha^2 - \rho)[A(X) + B(X)] + 2(\alpha^2 - \rho)A(\xi)\eta(X). \tag{4.6}$$

Now replacing Y by X in (4.4) and adding with (4.6), and by virtue of (4.5), we obtain

$$3A(X) + B(X) = \frac{(2\alpha\rho - \beta)}{(\alpha^2 - \rho)}\eta(X). \tag{4.7}$$

Hence we can state the following:

Theorem 4.2. *An $(2n+1)$-dimensional $(LCS)_n$-manifold is almost pseudo Ricci-symmetric, if the sum $3A + B$ is nowhere zero.*

V. Semi Pseudo Ricci-Symmetric $(LCS)n$-Manifold

The notion of semi pseudo Ricci Symmetric Manifolds introduced by Tarafdar and Jawarneh in 1993 [24], a non flat Riemannian manifold whose Ricci tensor S satisfies:

$$(D_X S)(Y, Z) = A(Y)S(X, Z) + A(Z)S(X, Y), \tag{5.1}$$

where A is a non-zero 1-form.
Put $Z = \xi$ in (5.1), we get

$$(n-1)(2\alpha\rho - \beta)\eta(X)\eta(Y) + (n-1)(\alpha^2 - \rho)\alpha[g(X, Y) + \eta(X)\eta(Y)] - \alpha S(X, Y) \tag{5.2}$$

$$= 2(n-1)(\alpha^2 - \rho)A(X)\eta(Y) + (n-1)(\alpha^2 - \rho)B(Y)\eta(X) + C(\xi)S(X, Y).$$

Putting $X = \xi$ in (5.2), gives

$$(\alpha^2 - \rho)A(Y) = [(2\alpha\rho - \beta) + (\alpha^2 - \rho)A(\xi)]\eta(Y). \tag{5.3}$$

Again putting $Y = \xi$ in (5.3), we have

$$A(\xi) = -\frac{(2\alpha\rho - \beta)}{(\alpha^2 - \rho)}. \tag{5.4}$$

Using (5.4) in (5.3), gives

$$A(Y) = \frac{(2\alpha\rho - \beta)}{(\alpha^2 - \rho)}\eta(Y). \tag{5.5}$$

Hence we can state the following theorem:

Theorem 5.3. *In a semi pseudo Ricci-symmetric $(LCS)_n$-manifold, the 1-form A is always non zero and is given by (5.5).*

Also from above theorem we can state the following corollary:
Corollary 5.1. *There exists a semi pseudo Ricci-symmetric $(LCS)_n$-manifold.*

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Distribution Natural Waves in an Infinite Viscoelastic Cylinder with Radial Cracks and Wedges

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Abstract- This paper deals with the distribution of natural waves of an infinite cylinder with radial crack and wedge. The task is put in cylindrical coordinates. Viscoelastic cylinder with radial crack is a limiting case of the wedge with an angle 360^0. With the help of the Navier equations and physical system received six differential equations. After not complicated conversion obtained spectral boundary value problem for systems of ordinary and partial differential equations complex the coefficient, which is then solved by the direct and orthogonal shooting with a combination of the method of Mueller on a complex arithmetic. A dispersion relation for a cylinder with radial crack and the wedge was got.

Keywords: *crack, viscoelastic cylinder, freezing procedure, the navier equation, orthogonal shooting, ordinary differential equations.*

GJSFR-F Classification: *MSC 2010: 00A69, 00A79*

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Safarov Ismail Ibrahimovich [α], Akhmedov Maqsud Sharipovich [σ] & Boltayev Zafar Ixtiyarovich [ρ]

Annotation- This paper deals with the distribution of natural waves of an infinite cylinder with radial crack and wedge. The task is put in cylindrical coordinates. Viscoelastic cylinder with radial crack is a limiting case of the wedge with an angle 360^0. With the help of the Navier equations and physical system received six differential equations. After not complicated conversion obtained spectral boundary value problem for systems of ordinary and partial differential equations complex the coefficient, which is then solved by the direct and orthogonal shooting with a combination of the method of Mueller on a complex arithmetic. A dispersion relation for a cylinder with radial crack and the wedge was got.

Keywords: crack, viscoelastic cylinder, freezing procedure, the navier equation, orthogonal shooting, ordinary differential equations.

I. Introductions

One of the central tasks of dynamic elasticity theory is the study of the spread of a perturbation of the stress-strain state in deformed bodies with geometric structures that combine the concept of a mechanical waveguide [1,2,3]. The main features are the length of the waveguide in one direction, as well as restrictions and localization of the wave beam in other directions. Accounting for the damping capacity of the waveguide material plays an important role in the dynamic behavior of the structure [4,5]. It leads to a marked weakening of the natural oscillations, a significant decrease in amplitudes of forced vibrations and smoothing of the stresses in the concentration zone of the oscillations. The complexity of their solutions for many reasons, for example, rheological properties of real waveguides, not the classic geometric shapes and so on. N., Resulting in a wide variety of schematized models to describe in some approximation of real phenomena and makes it difficult to create a unified mathematical model of the mechanical system [6] . In the viscoelastic cylinder with radial crack is a limiting case of the wedge with an angle 360^0. A method and a solution algorithm for the study of wave propagation in viscoelastic cylinder with radial crack and wedge with an arbitrary angle vertex.

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6. Safarov I.I., Teshaev M.KH, Boltaev Z.I.. Mathematical modeling of wave process in a mechanical waveguide taking into account the internal friction. Germany. LAP. 2013. 243p.

II. Waves in an Infinite Cylinder with Radial Crack

The propagation of harmonic waves in an infinite elastic cylinder with radial crack is dialed. The task is put in cylindrical coordinates. The elastic cylinder with radial crack is a limiting case of the wedge with an angle 360^0. The basic equations of motion of an elastic medium, which occupies a region B are defined by three groups of relations [6]:

$$\frac{\partial \sigma_{ik}}{\partial x_k} = \rho \frac{\partial^2 u_i}{\partial t^2};$$

$$\varepsilon_{ik} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i}\right), \quad \sigma_{ik} = \tilde{\lambda}\,\theta\delta_{ik} + 2\tilde{\mu}\varepsilon_{ik}. \qquad (1)$$

Here σ_{ik} - stress tensor, ε_{ik} - strain tensor, θ - volumetric deformation, $\tilde{\lambda}$ and $\tilde{\mu}$ - operator elastic moduli [7,8,9]:

$$\tilde{\lambda}\varphi(t) = \lambda_{01}\left[\varphi(t) - \int_0^t R_\lambda(t-\tau)\varphi(t)d\tau\right];$$

$$\tilde{\mu}\varphi(t) = \mu_{01}\left[\varphi(t) - \int_0^t R_\mu(t-\tau)\varphi(t)d\tau\right] \qquad (2)$$

$\varphi(t)$ – Arbitrary function of time; $R_\lambda(t-\tau)$ and $R_\mu(t-\tau)$ – core and relaxation λ_{01}, μ_{01} – Instant elastic modules. We accept the integral terms in (2) small, then the function $\varphi(t) = \psi(t)e^{-i\omega_R t}$, where $\psi(t)$- a slowly varying function of time, ω_R - real constant. Next, using the freezing procedure [9], we note the relation (3) approximate species

$$\bar{\lambda}\varphi = \lambda_{01}\left[1 - \Gamma_\lambda^C(\omega_R) - i\Gamma_\lambda^S(\omega_R)\right];$$

$$\bar{\mu}\varphi = \mu_{01}\left[1 - \Gamma_\mu^C(\omega_R) - i\Gamma_\mu^S(\omega_R)\right]\varphi, \qquad (3)$$

Where

$$\Gamma_\lambda{}^C(\omega_R) = \int_0^\infty R_\lambda(\tau)\cos\omega_R\tau\,d\tau;\ \Gamma_\lambda{}^S(\omega_R) = \int_0^\infty R_\lambda(\tau)\sin\omega_R\tau\,d\tau,$$

$$\Gamma_\mu{}^C(\omega_R) = \int_0^\infty R_\mu(\tau)\cos\omega_R\tau\,d\tau, \Gamma_\mu{}^S(\omega_R) = \int_0^\infty R_\mu(\tau)\sin\omega_R\tau\,d\tau.$$

Respectively cosine and sine Fourier transform of the relaxation of the core material. As an example, the viscoelastic material take three parametric relaxation nucleus $R_\lambda(t) = R_\mu(t) = Ae^{-\beta t}/t^{1-\alpha}$. On the effect of the function $R(t-\tau)$ superimposed usual requirements integral ability, continuity (except $t=\tau$), sign certainty and monotony:

9. Koltunov M.A.. Creep and relaxation. - M.: Higher School, 1976. - 276p.

$$R \rangle\, 0,\ \ \frac{dR(t)}{dt} \le 0,\ 0\langle \int_0^\infty R(t)dt \langle 1.$$

In a cylindrical coordinate system, the equation (1), (2), (3) have the form

$$\frac{\partial\sigma_{rr}}{dr} + \frac{\sigma_{rr} - \sigma_{r\varphi}}{r} + \frac{1}{r}\frac{\partial\sigma_{r\varphi}}{\partial\varphi} + \frac{\partial\sigma_{rz}}{\partial z} = \rho\frac{\partial^2 u_r}{dt^2};$$

$$\frac{1}{r}\frac{\partial\sigma_{\varphi\varphi}}{\partial\varphi} + \frac{2\sigma_{r\varphi}}{r} + \frac{\partial\sigma_{r\varphi}}{\partial r} + \frac{\partial\sigma_{z\varphi}}{\partial z} = \rho\frac{\partial^2 u_\varphi}{\partial t^2}; \tag{4}$$

$$\frac{\partial\sigma_{zz}}{\partial z} + \frac{\partial\sigma_{rz}}{\partial r} + \frac{\sigma_{zz}}{r} + \frac{1}{r}\frac{\partial\sigma_{z\varphi}}{\partial\varphi} = \rho\frac{\partial^2 u_z}{\partial t^2};$$

$$\varepsilon_{rr} = \frac{\partial u_r}{\partial r};\ \ \varepsilon_{zz} = \frac{\partial u_z}{\partial z};\ \varepsilon_{\varphi\varphi} = \frac{1}{r}\frac{\partial u_r}{\partial\varphi} + \frac{u_r}{r};$$

$$\varepsilon_{r\varphi} = \frac{1}{2}\left(\frac{1}{r}\frac{\partial u_r}{\partial\varphi} + \frac{\partial u_\varphi}{\partial r} - \frac{u_\varphi}{r}\right); \varepsilon_{rz} = \frac{1}{2}\left(\frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z}\right); \tag{5}$$

$$\varepsilon_{\varphi z} = \frac{1}{2}\left(\frac{\partial u_\varphi}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial\varphi}\right);$$

$$\sigma_{rr} = \lambda\left(\frac{\partial u_r}{\partial r} + \frac{1}{r}\frac{\partial u_\varphi}{\partial\varphi} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z}\right) + 2\mu\frac{\partial u_r}{\partial r};$$

$$\sigma_{r\varphi} = 2\mu\varepsilon_{r\varphi} = \mu\left(\frac{1}{r}\frac{\partial u_r}{\partial\varphi} + \frac{\partial u_\varphi}{\partial r} - \frac{u_\varphi}{r}\right);$$

$$\sigma_{rz} = 2\mu\varepsilon_{rz} = \mu\left(\frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z}\right);$$

$$\sigma_{\varphi\varphi} = \lambda\left(\frac{\partial u_r}{\partial r} + \frac{1}{r}\frac{\partial u_\varphi}{\partial\varphi} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z}\right) + 2\mu\left(\frac{1}{r}\frac{\partial u_\varphi}{\partial\varphi} + \frac{u_r}{r}\right);$$

$$\sigma_{\varphi z} = \mu\left(\frac{\partial u_\varphi}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial\varphi}\right),$$

$$\sigma_{zz} = \lambda\left(\frac{\partial u_r}{\partial r} + \frac{1}{r}\frac{\partial u_\varphi}{\partial r} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z}\right) + 2\mu\frac{\partial u_z}{\partial z}. \tag{6}$$

Where $\sigma_{rr}, \sigma_{r\varphi}, \sigma_{rz}, \sigma_{\varphi\varphi}, \sigma_{\varphi z}, \sigma_{zz}$ - respectively components of the stress tensor; $\varepsilon_{rr}, \varepsilon_{r\varphi}, \varepsilon_{rz}, \varepsilon_{\varphi\varphi}, \varepsilon_{\varphi z}, \varepsilon_{zz}$ -respectively components of the strain tensor. The link between stress and strain is given in the third chapter. Equations (4), (5) and (6) after algebraic manipulations are identical to the system of six differential equations solved with respect to the first derivative of the radial coordinate [10,11]

Ref

10. Safarov I.I., Teshaev M.KH. Boltaev Z.I.. Distribution of linear waves in extended lamellar bodies. LAP, Lambert Academic Publishing (Germany). 2016. 315 p.

$$\begin{cases} \dfrac{\partial u_r}{\partial r} = \dfrac{1}{K}\sigma_{rr} - \dfrac{\lambda}{K}\left(\dfrac{1}{r}\dfrac{\partial u_\varphi}{\partial \varphi} + \dfrac{u_r}{r} + \dfrac{\partial u_z}{\partial z}\right); \\ \dfrac{\partial u_\varphi}{\partial r} = \dfrac{1}{\mu}\sigma_{r\varphi} - \dfrac{1}{r}\left(\dfrac{\partial u_r}{\partial \varphi} - u_\varphi\right); \\ \dfrac{\partial u_z}{\partial r} = \dfrac{1}{\mu}\sigma_{rz} - \dfrac{\partial u_r}{\partial z}; \\ \dfrac{\partial \sigma_{rr}}{\partial r} = \rho\dfrac{\partial^2 u_r}{\partial t^2} - \dfrac{\tilde{A}}{r} - \dfrac{1}{r}\dfrac{\partial \sigma_{r\varphi}}{\partial \varphi} - \dfrac{\partial \sigma_{rz}}{\partial z}; \\ \dfrac{\partial \sigma_{r\varphi}}{\partial r} = \rho\dfrac{\partial^2 u_\varphi}{\partial t^2} - \dfrac{1}{r}\dfrac{\partial}{\partial \varphi}\left[\sigma_{rr} - \tilde{A}\right] - \dfrac{2\sigma_{r\varphi}}{r} - \dfrac{\partial}{\partial z}\tilde{B}; \\ \dfrac{\partial \sigma_{rz}}{\partial r} = \rho\dfrac{\partial^2 u_z}{\partial t^2} - \dfrac{\partial}{\partial z}\left[\sigma_{rr} - 2\mu\left(\dfrac{\partial u_r}{\partial r} - \dfrac{\partial u_z}{\partial z}\right)\right] - \dfrac{\sigma_{rz}}{r} - \dfrac{1}{r}\dfrac{\partial}{\partial \varphi}\tilde{B}; \end{cases} \tag{7}$$

where we use the notation

$$\tilde{A} = 2\mu\left[\frac{\partial u_r}{\partial r} - \frac{1}{r}\left(\frac{\partial u_\varphi}{\partial \varphi} + u_r\right)\right];$$

$$\tilde{B} = \mu\left(\frac{\partial u_\varphi}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial \varphi}\right).$$

Boundary conditions are given in the form:

$$r = r_0 \to 0, R: \sigma_{rz} = \sigma_{rr} = \sigma_{r\varphi} = 0; \tag{8}$$

$$r = 0,\ \ 2\pi: u_\varphi = 0; \sigma_{\varphi z} = \sigma_{\varphi r} = 0. \tag{9}$$

Condition (8) $r = 0$ physically result can be interpreted as limiting the transition from the hollow cylinder with the inner free surface to the solid. Inner radius tends to zero. In the case of harmonic waves traveling along the axis z, solution of (7), (8), (9) admits separation of variables [12,13]

$$\begin{aligned} u_r &= w(r)\cos\frac{\varphi}{2}\cos(kz - \omega t); \\ u_\varphi &= v(r)\sin\frac{\varphi}{2}\cos(kz - \omega t); \\ u_z &= u(r)\cos\frac{\varphi}{2}\sin(kz - \omega t); \end{aligned} \tag{10,a}$$

$$\sigma_{rr} = \sigma(r)\cos\frac{\varphi}{2}\cos(kz - \omega t);$$

$$\sigma_{r\varphi} = \tau_\varphi(r)\ \ \sin\frac{\varphi}{2}\cos(kz - \omega t);$$

$$\sigma_{rz} = \tau_z(r)\cos\frac{\varphi}{2}\sin(kz - \omega t),$$

Ref
12. Safarov I.I, Boltaev Z. I., Akhmedov M. Sh. Properties of wave motion in a fluid-filled cylindrical shell/ LAP, Lambert Academic Publishing . 2016 -105 p.

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or

$$u_r = w(r)\cos\frac{\varphi}{2}e^{i(kz-\omega t)};$$

$$u_\varphi = v(r)\sin\frac{\varphi}{2}e^{i(kz-\omega t)};$$

$$u_z = u(r)\cos\frac{\varphi}{2}e^{i(kz-\omega t)};$$

$$\sigma_{rr} = \sigma(r)\cos\frac{\varphi}{2}e^{i(kz-\omega t)};$$

$$\sigma_{r\varphi} = \tau_\varphi(r)\sin\frac{\varphi}{2}e^{i(kz-\omega t)}; \qquad (10,b)$$

$$\sigma_{rz} = \tau_z(r)\cos\frac{\varphi}{2}e^{i(kz-\omega t)}.$$

Given (10), problem (7), (8), (9) is transformed into a spectral boundary value problem for a system of ordinary differential equations with complex the coefficient:

$$\begin{cases} w' = \dfrac{\sigma}{k} - \dfrac{\lambda}{k}\left(ku + \dfrac{v}{2r} + \dfrac{w}{r}\right); \\ v' = \dfrac{\tau_\varphi}{\mu} + \dfrac{\vartheta}{r} + \dfrac{w}{2r}; \\ u' = \dfrac{\tau_z}{\mu} + kw; \\ \sigma' = -\omega^2\rho w + \dfrac{\tilde{a}}{r} - \dfrac{\tau_\varphi}{2r} - k\tau_z; \\ \tau_\varphi' = -\omega^2\rho\vartheta - \dfrac{2\tau_\varphi}{r} + (\sigma + \tilde{a})\dfrac{1}{2r} - k\tilde{b}; \\ \tau_z{}' = -\omega^2\rho u - \dfrac{\tau_z}{r} - \dfrac{\tilde{b}}{2r} + k(\sigma + 2\mu(ku - w')), \end{cases} \qquad (11)$$

$$(\ldots)' = \frac{d}{dr}.$$

Here $\tilde{a} = 2\mu\left(\dfrac{\vartheta + w}{2r} - w'\right)$; $\tilde{b} = \mu\left(-\dfrac{u}{2r} - k\vartheta\right)$.

with boundary conditions

$$r = r_0 \to 0 : \sigma = \tau_\varphi = \tau_z = 0;$$

$$r = R : \qquad \sigma = \tau_\varphi = \tau_z = 0. \qquad (12)$$

Thus formulated spectral boundary value problem (11), (12) describing the propagation of harmonic waves in an infinite cylinder with radial crack. Note that the choice of boundary conditions at the edges of the slit (9) led primarily to separate variables to the coordinates r and φ, which greatly simplifies the solution of the original problem. Separation of variables is also possible in the case of the following boundary conditions:

$$\begin{aligned}&\varphi=0: \quad \sigma_{\varphi\varphi}=0; \quad u_r=u_z=0;\\&\varphi=2\pi: \sigma_{\varphi\varphi}=0; \quad u_r=u_z=0.\end{aligned} ; \qquad (13)$$

Indeed, performing in (7), (8) the change of variables so as to satisfy the conditions (13)

$$\begin{aligned}
&u_r=\tilde{w}(r)\sin\frac{\varphi}{2}\cos(kz-\omega t);\\
&u_\varphi=\tilde{\vartheta}(r)\cos\frac{\varphi}{2}\cos(kz-\omega t);\\
&u_z=\tilde{u}(r)\sin\frac{\varphi}{2}\sin(kz-\omega t);\\
&\sigma_{rr}=\tilde{\sigma}(r)\sin\frac{\varphi}{2}\cos(kz-\omega t);\\
&\sigma_{r\varphi}=\tau_\varphi(r)\cos\frac{\varphi}{2}\cos(kz-\omega t);\\
&\sigma_{rz}=\tau_z(r)\sin\frac{\varphi}{2}\sin(kz-\omega t);
\end{aligned} \qquad (14)$$

We obtain spectral boundary value problem with complex coefficients and roots

$$\begin{aligned}
&\tilde{w}'=\frac{\tilde{\sigma}}{k}-\frac{\lambda}{k}\left(k\tilde{u}-\frac{\tilde{v}}{2r}+\frac{\tilde{w}}{r}\right);\\
&\tilde{u}'=\frac{\tilde{\tau}_\varphi}{\mu}+\frac{\tilde{v}}{r}-\frac{\tilde{w}}{2r};\\
&\tilde{u}=\frac{\tilde{\tau}_z}{\mu}+k\tilde{w};\\
&\tilde{\sigma}'=-\rho\omega^2\tilde{w}+\frac{2\mu}{r}\left(-\frac{\tilde{v}}{2r}+\frac{\tilde{w}}{r}-\tilde{w}'\right)+\frac{\tilde{\tau}_\varphi}{2r}-k\tilde{\tau};\\
&\tilde{\tau}_\varphi'=-\rho\omega^2\tilde{v}-\frac{2\tilde{\tau}_\varphi}{r}-\frac{1}{2r}\left(\tilde{\sigma}+2\mu\left(-\frac{\tilde{v}}{2r}+\frac{\tilde{w}}{r}-\tilde{w}\right)-k\left(\frac{\tilde{u}}{2r}-k\tilde{v}\right)\right);\\
&\tilde{\tau}_z{}'=-\rho\omega^2\tilde{u}-\frac{\tilde{\tau}_z}{r}+\frac{\mu}{2r}\left(\frac{u}{2r}-k\tilde{v}\right)+k\left(\tilde{\sigma}+2\mu(k\tilde{u}-\tilde{w}')\right),
\end{aligned} \qquad (15)$$

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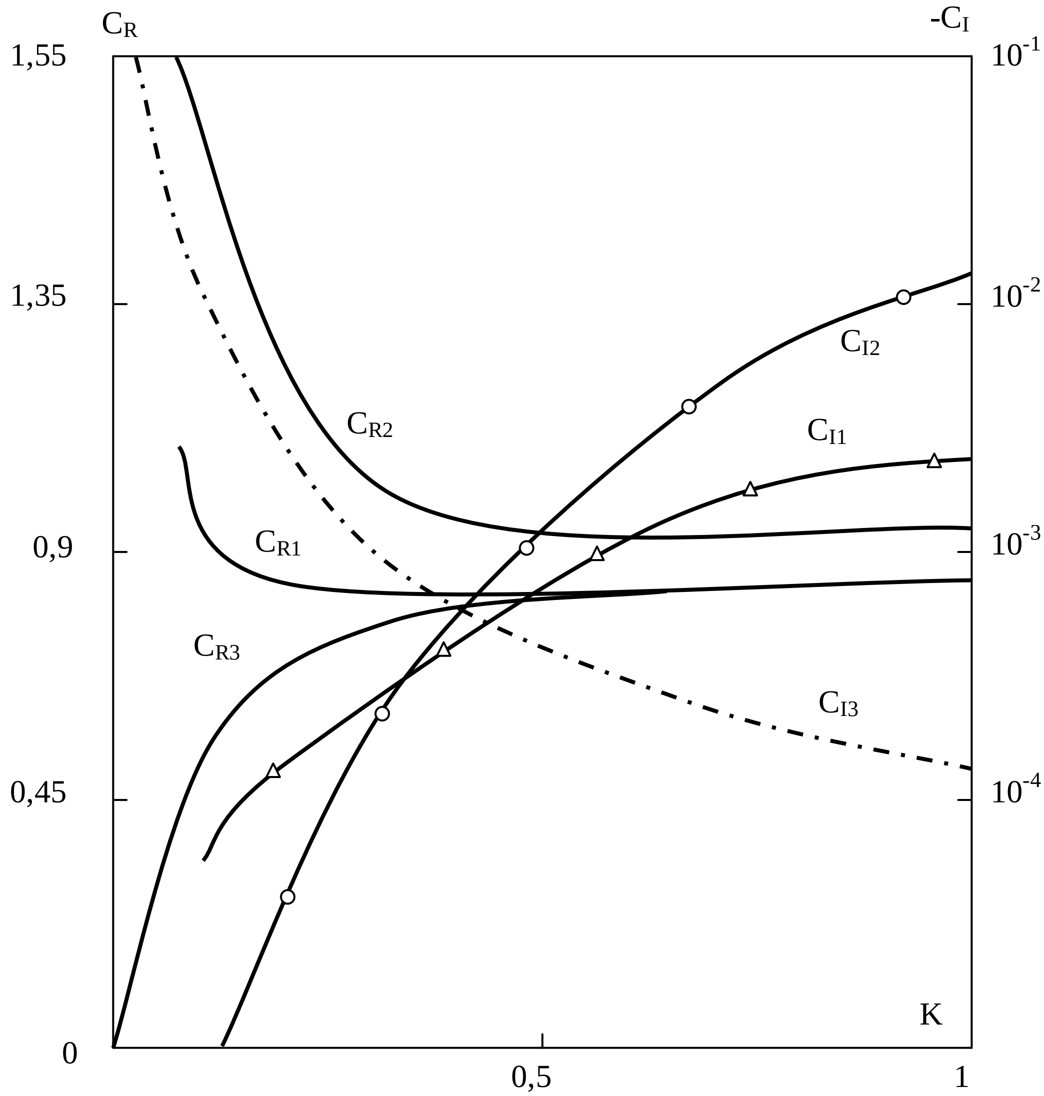

Figure 1: Changes in the real and imaginary parts of the frequency of oscillation on κ
With the boundary conditions

$$r = r_0 \to 0 : \tilde{\sigma} = \tilde{\tau}_{\varphi} = \tilde{\tau}_z = 0;$$
$$r = R : \tilde{\sigma} = \tilde{\tau}_{\varphi} = \tilde{\tau}_z = 0 \qquad (16)$$

It is easy to see that the problem (15), (16) reduces to the problem (11), (12) by replacing

$$\tilde{\tau}_z = \tau_z,\ \ \tilde{\tau}_{\varphi} = -\tau_{\varphi},\ \tilde{\sigma} = \sigma,\ \ \tilde{w} = w,\ \ \tilde{u}_{\varphi} = -u_{\varphi},\ \tilde{u}_z = u_z.$$

The solution of (11), (12) was carried out by the orthogonal shooting Godunov [14]. Dimensionless quantities in the formulation of the problem chosen in such a way that the shear rate C_s, density ρ and the outer radius R has the single value. Fig. I shows dispersion curves of the first two modes in infinite cylinder with viscoelastic radial thickness (curves 1 and 2). For comparison, the same figure shows the dependence of the phase velocity of the wave number of the first bending mode vibrations of a solid cylinder (curve 3) without gaps. final solution of the problem has been previously found Pohgomerom Cree and with the help of special functions (5) the solution was used for testing tasks.

Ref

14. S.K. Godunov. On the numerical solution of boundary value problems for systems of linear ordinary differential equations. - Successes of Mathematical Sciences, 1061, T.16, № 3, 171-174p.

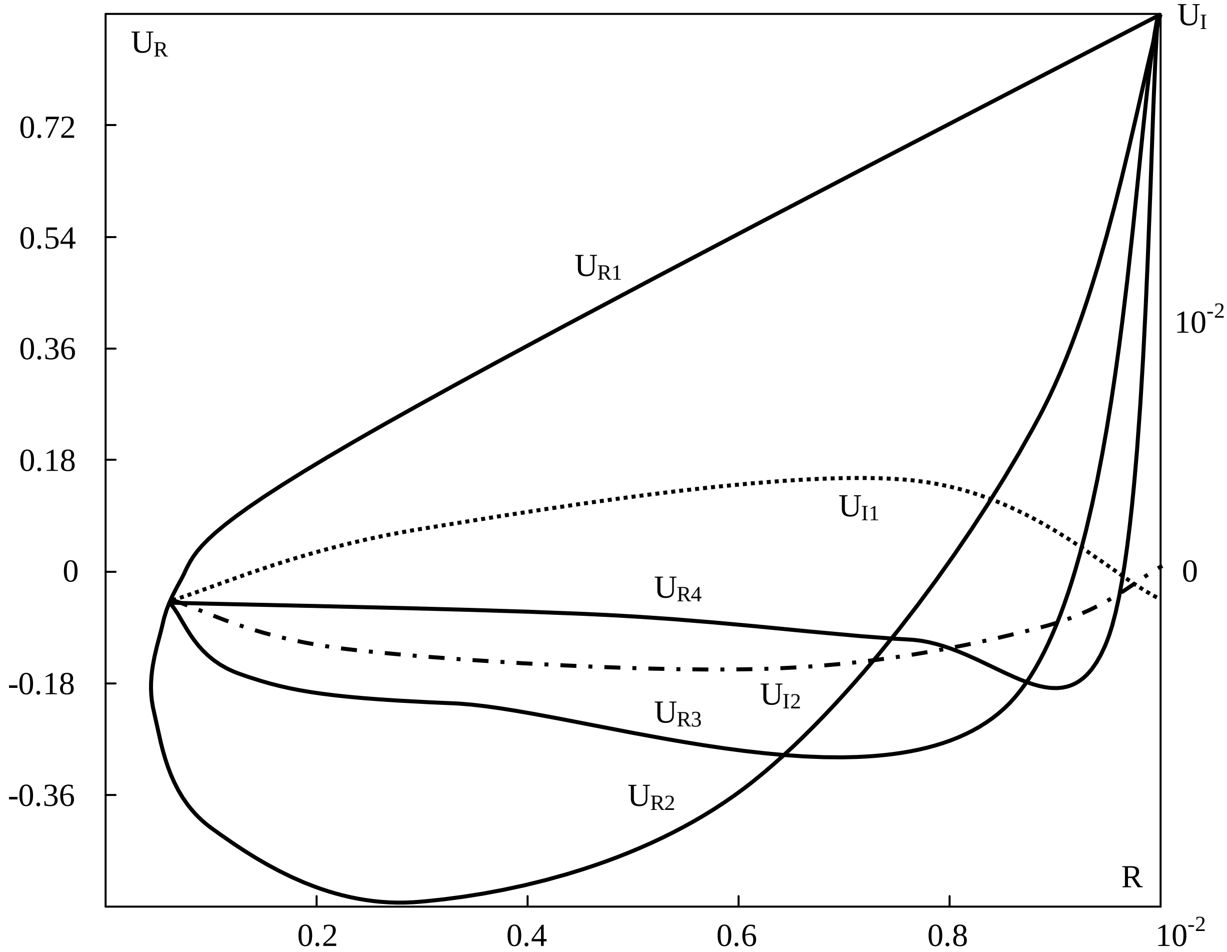

Figure 2: Changes in the real and imaginary parts of the waveform U_R and U_I on R

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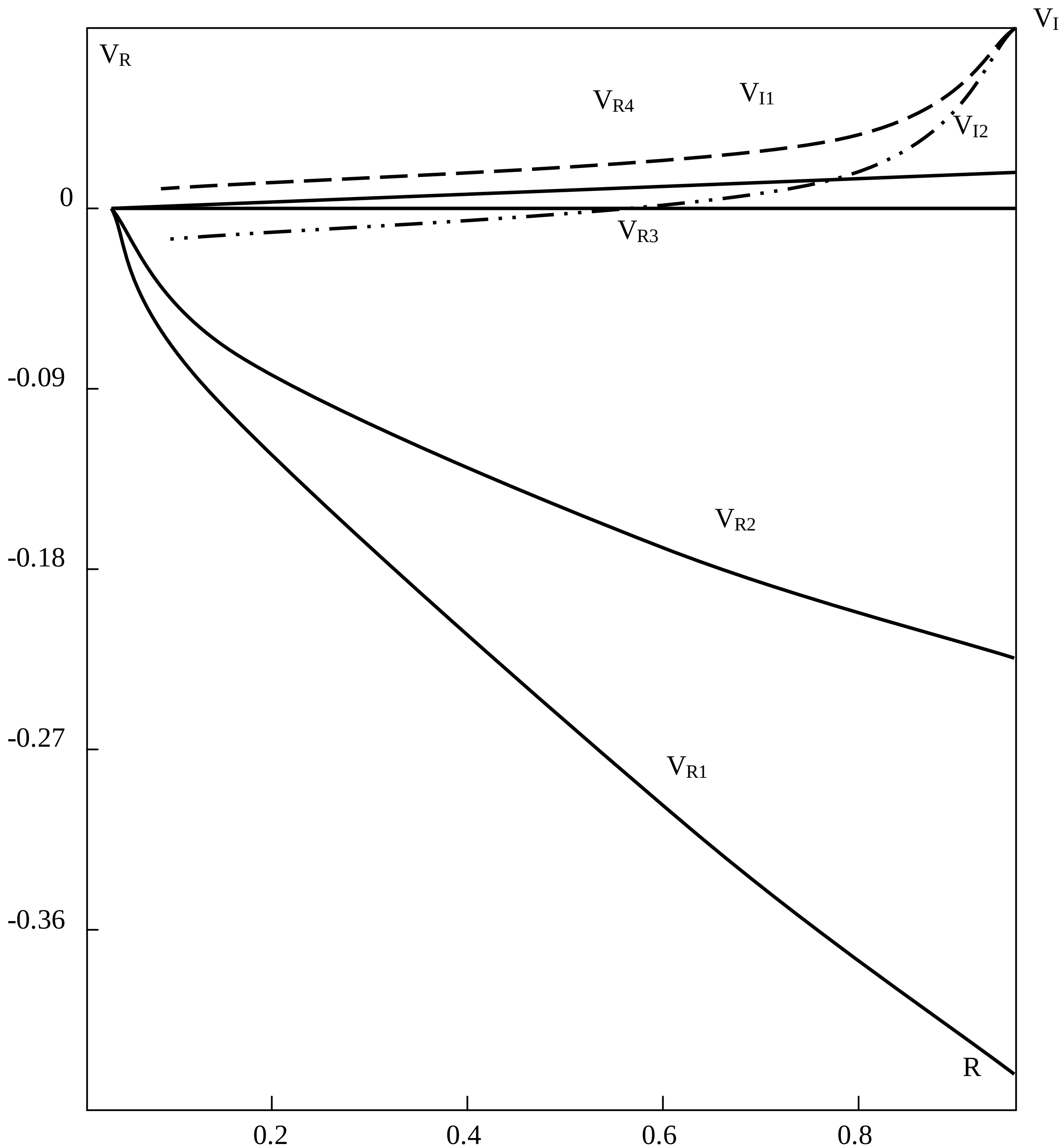

Figure 3: Changes in the real and imaginary parts of the waveform V_R and V_I on R

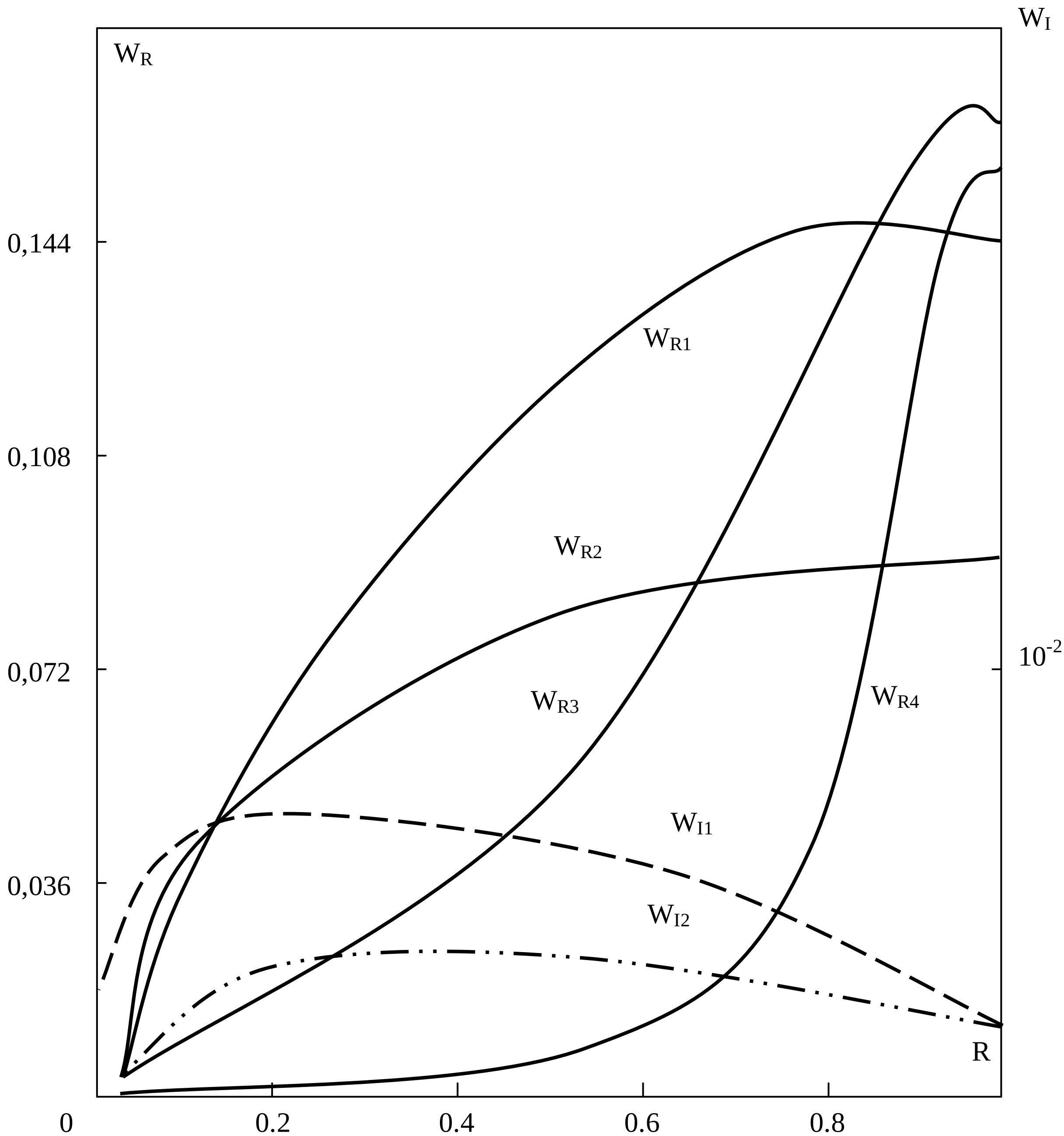

Figure 4: Changing the real and imaginary parts of the waveform on R

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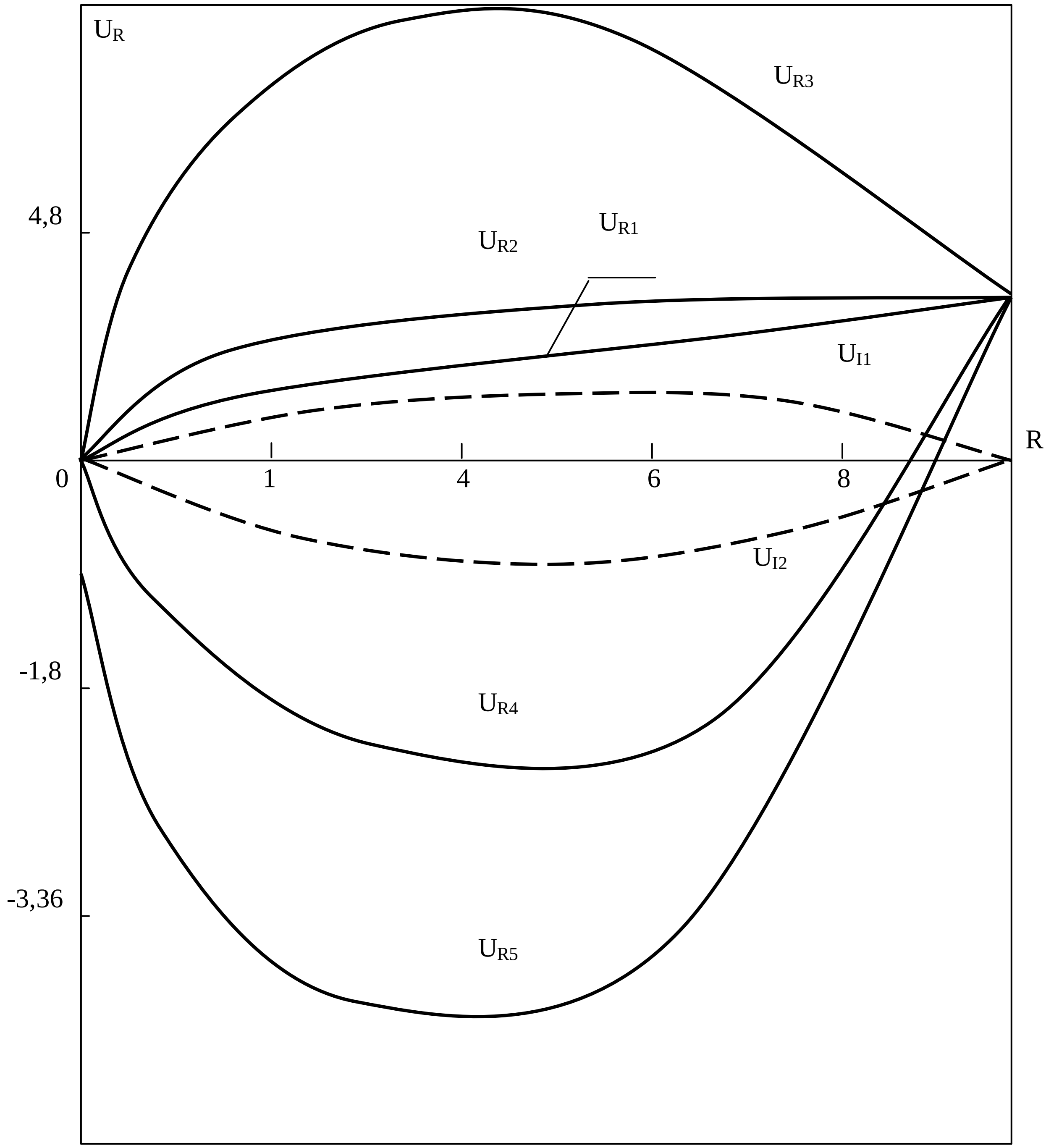

Figure 5: Changing the real and imaginary parts of the waveform on R

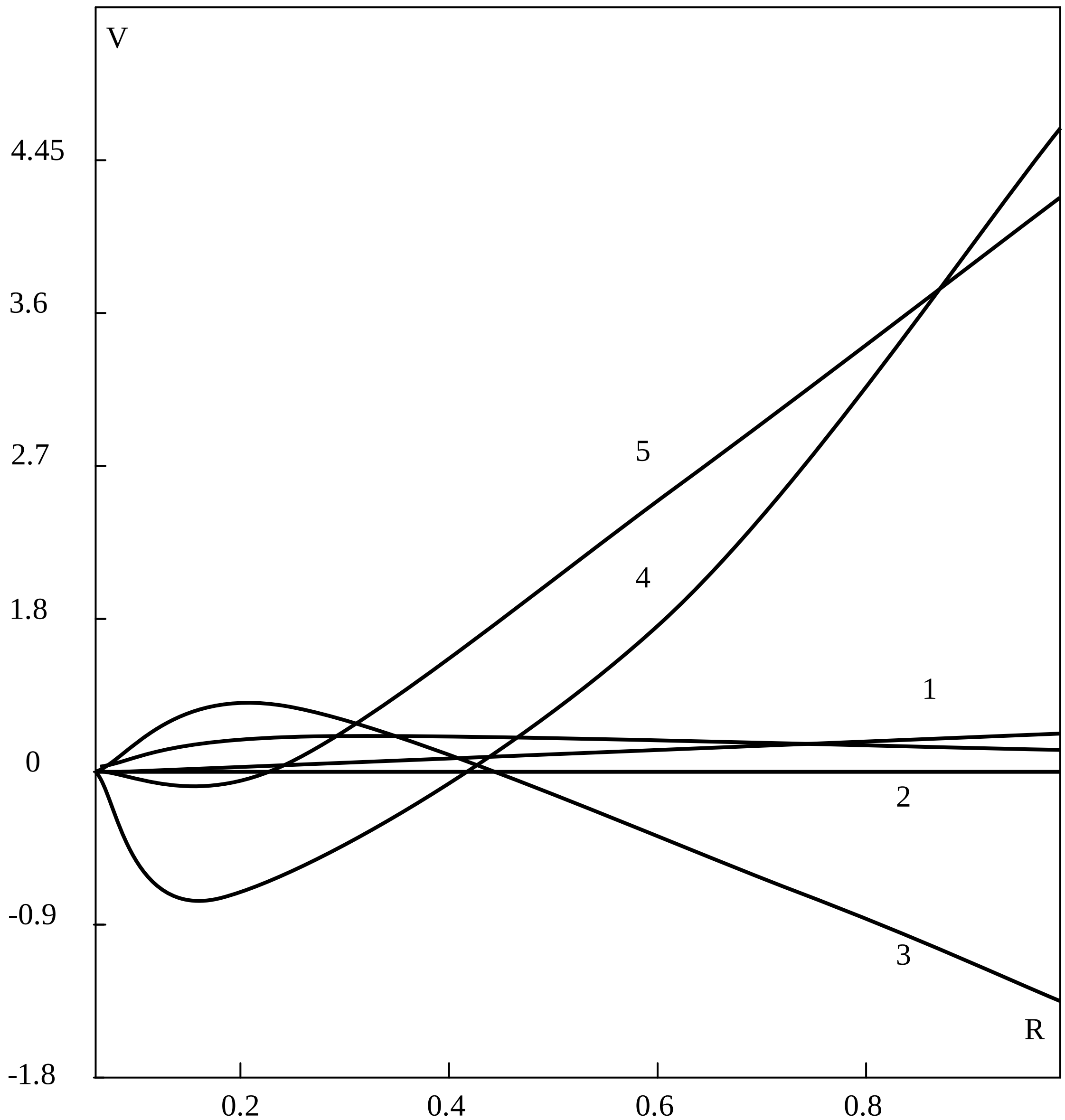

Figure 6: Change the absolute value of the waveform $V^2 = V_R^{\ 2} + V_I^{\ 2}$ on R

Notes

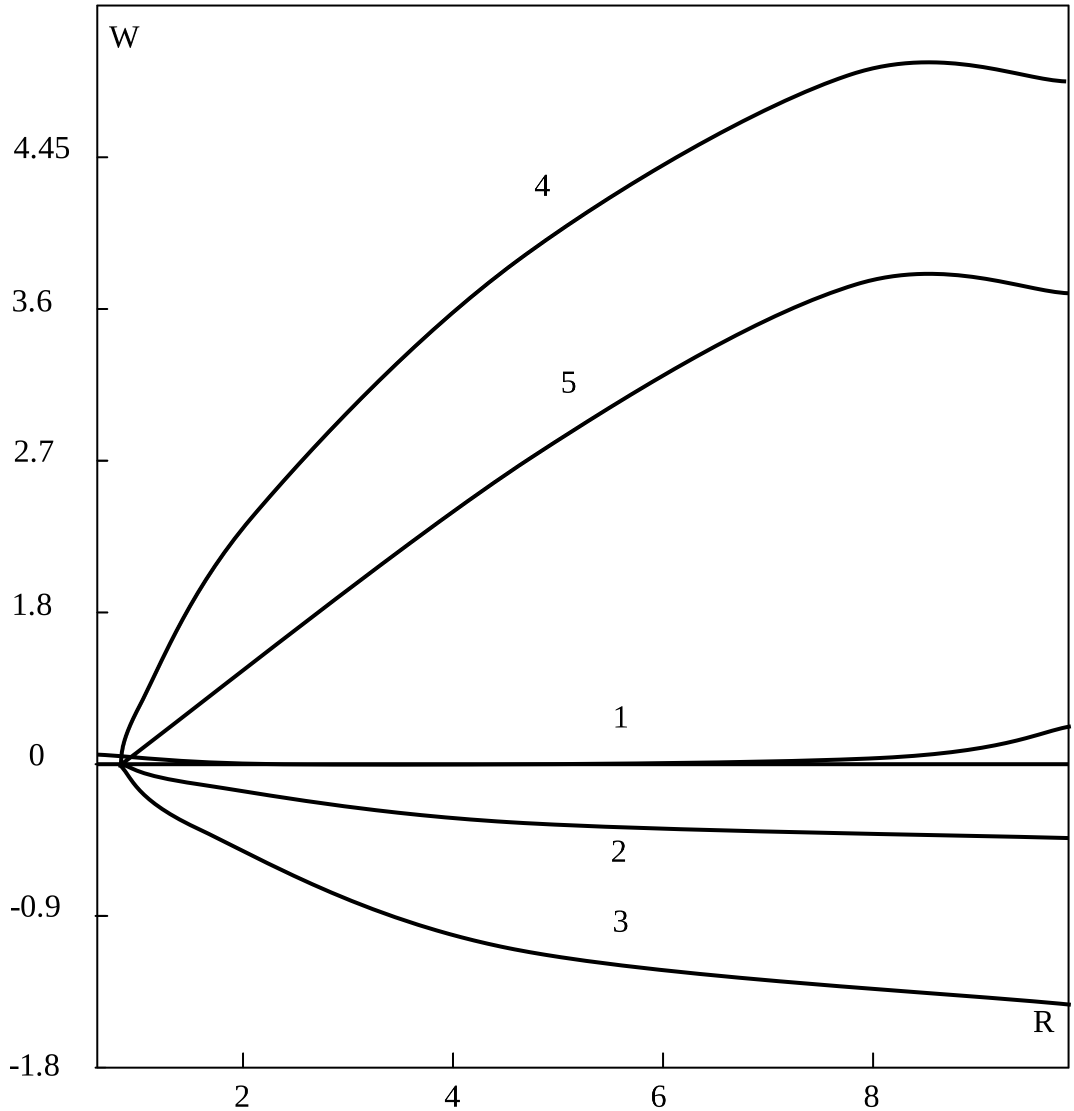

Figure 7: Change the absolute value of the waveform $W^2 = W_R^{\ 2} + W_I^{\ 2}$ on R

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Note the characteristics of the curve 3: at the origin of the phase velocity is equal to zero, but not infinite, is committed to the Rayleigh wave velocity for a half. In the case of a cylinder with radial crack the first mode has a cut-off frequency and phase velocity tends to infinity. At large wave numbers limit the phase velocity of this mode also coincides with the velocity of the Rayleigh wave. At the cutoff frequency axial displacement are zero and cylinder vibrations occur in the plane strain condition. In the second mode at a cut-off frequency are observed only real and opinions of the axial displacement, circumferential and radial displacement are zero. The evolution of the form of the solution of the complex movements of the first and second modes depending on the wave number is shown in Figures 2-4 and 5-7, respectively.

The curves are numbered in order of growth κ. Note the strong dependence on the wave number of forms. With the growth of the wave number in the first mode are localized oscillations near the outer surface of the cylinder. It is characteristic that the second mode, which is on the small wave numbers, is a form of predominantly axial vibration, with growth κ gradually turning into a form of predominantly radial oscillations.

III. Waves in a Deformable Wedge with an Arbitrary Angle Vertices

In this section we consider the propagation of harmonic waves in an infinite elastic wedge with an arbitrary angle peaks. For a description of the wave process, use the above relations in the preceding paragraph (1), (2), (3). Resolving equation system coincides with the system (7) is also saved without changing the boundary conditions on the surface (8). The boundary conditions for φ for any angle of the wedge when the free lateral surfaces should be written in the form:

$$\varphi = -\frac{\varphi_0}{2}, \frac{\varphi_0}{2}: \quad \sigma_{\varphi\varphi} = \sigma_{\varphi r} = \sigma_{\varphi z} = 0, \tag{17}$$

where φ_0 - angle at the apex of the wedge. Harmonic waves propagating along the z axis, the essence of the solution of the problem (7), (8), (9), (17) periodic in z and time. Terms periodicity allows to eliminate the dependence of the main unknowns on the time axis and the z coordinate with the following change of variables:

$$\begin{aligned}
u_r &= w(r,\varphi)e^{i(\kappa z-\omega t)};\\
u_\varphi &= v(r,\varphi)e^{i(\kappa z-\omega t)};\\
u_z &= \tilde{u}(r,\varphi)e^{i(\kappa z-\omega t)};\\
\sigma_{rr} &= \sigma(r,\varphi)e^{i(\kappa z-\omega t)};\\
\sigma_{rz} &= \tau_z(r,\varphi)e^{i(\kappa z-\omega t)};\\
\sigma_{r\varphi} &= \tau_\varphi(r,\varphi)e^{i(\kappa z-\omega t)};
\end{aligned} \tag{18}$$

Under the condition (17), separation of variables r and φ, as in the previous paragraph, it is already impossible. Taking into account (18), the system of equations (7) takes the form:

$$\begin{cases}
w' = \dfrac{\sigma}{k} - \dfrac{\bar{\lambda}}{k}\left(ku + \dfrac{1}{r}\left(w + \dfrac{\partial v}{\partial \varphi}\right)\right)\\
v' = \dfrac{\tau_\varphi}{\bar{\mu}} + \dfrac{1}{r}\left(v - \dfrac{\partial w}{\partial \varphi}\right)\\
u' = \dfrac{\tau_z}{\bar{\mu}} + kw\\
\sigma' = -\omega^2 \rho w + \dfrac{1}{r}\left(A - \dfrac{\partial \tau_\varphi}{\partial \varphi}\right) - k\tau_z\\
\tau'_\varphi = -\omega^2 \rho v - \dfrac{1}{r}\left(\dfrac{\partial (A+\sigma)}{\partial \varphi} + 2\tau_\varphi\right) - kB\\
\tau'_z = -\omega^2 \rho u - \dfrac{1}{r}\left(\dfrac{\partial B}{\partial \varphi} + \tau_z\right) + k(\sigma + 2\bar{\mu}(ku - w'))
\end{cases} \tag{19}$$

Notes

Notes

$$\begin{cases} w' = \dfrac{\sigma}{k} - \dfrac{\overline{\lambda}}{k}\left(ku + \dfrac{1}{r}\left(w + \dfrac{\partial v}{\partial \varphi}\right)\right) \\ v' = \dfrac{\tau_{\varphi}}{\overline{\mu}} + \dfrac{1}{r}\left(v - \dfrac{\partial w}{\partial \varphi}\right) \\ u' = \dfrac{\tau_z}{\overline{\mu}} + kw \\ \sigma' = -\omega^2 \rho w + \dfrac{1}{r}\left(A - \dfrac{\partial \tau_{\varphi}}{\partial \varphi}\right) - k\tau_z \\ \tau_{\varphi}' = -\omega^2 \rho v - \dfrac{1}{r}\left(\dfrac{\partial (A+\sigma)}{\partial \varphi} + 2\tau_{\varphi}\right) - kB \\ \tau_z' = -\omega^2 \rho u - \dfrac{1}{r}\left(\dfrac{\partial B}{\partial \varphi} + \tau_z\right) + k\left(\sigma + 2\overline{\mu}(ku - w')\right) \end{cases} \tag{19}$$

Where

$$A = 2\overline{\mu}\left(\frac{1}{2}\left(\frac{\partial v}{\partial \varphi} + w\right) - w'\right);\ B = \overline{\mu}\left(\frac{1}{r}\frac{\partial u}{\partial \varphi} - kv\right).$$

Similarly transformed boundary conditions (8)

$$r = 0, R : \sigma = \tau_{\varphi} = \tau_z = 0. \tag{20}$$

It is easy to see that the components of the stress tensor, $\sigma_{\varphi\varphi}, \sigma_{\varphi z}$ and σ_{zz} expressed in terms of the main unknowns on formulas:

$$\begin{gathered} \sigma_{\varphi\varphi} = \sigma_{rr} + 2\overline{\mu}\left(\frac{1}{r}\frac{\partial u_{\varphi}}{\partial \varphi} + \frac{u_r}{r} - \frac{\partial u_r}{\partial r}\right); \\ \sigma_{\varphi z} = \overline{\mu}\left(\frac{\partial u_z}{\partial \varphi} + \frac{\partial u_{\varphi}}{\partial z}\right); \\ \sigma_{zz} = \sigma_{rr} + 2\overline{\mu}\left(\frac{\partial u_z}{\partial z} - \frac{\partial u_r}{\partial r}\right). \end{gathered} \tag{21}$$

Then, taking into account the first equation (21), the boundary conditions (20) take the form:

$$\begin{gathered} \sigma_{\varphi} = A + \sigma_r = a\sigma_r + b\frac{1}{r}\left(\frac{\partial v}{\partial \varphi} + w\right) + cku = 0; \\ \varphi = -\frac{\varphi_0}{2}, \frac{\varphi_0}{2} : \quad \tau_{\varphi} = 0, B = \overline{\mu}\left(\frac{\partial u}{r\partial \varphi} - kr\right) = 0, \end{gathered} \tag{22}$$

where

$$a = 1 + \frac{2\overline{\mu}}{k};\ b = 2\overline{\mu}\left(1 + \frac{\overline{\lambda}}{k}\right), c = 2\overline{\mu}\frac{\overline{\lambda}}{k}.$$

A boundary value problem for the alignment of the system in the frequency of derivatives (19) (20) (22) can be reduced to a boundary value problem for a system of ordinary differential equations by the method of lines that will be used in solving a software unit orthogonal shooting method. According to the method of direct rectangular domain of the function key unknown is covered by lines parallel to the axis *r* and evenly spaced (Figure 8).

The solution is sought only on these lines, and the directional derivative φ, is replaced by the approximate finite differences. Used a second-order approximation formulas for the first and second derivatives are of the form [14,15]:

$$y_{i,\varphi} \cong \frac{y_{i+1} - y_{i-1}}{2\Delta} \cong \frac{-3y_i + 4y_{i+1} - y_{i+2}}{2\Delta} \cong \frac{3y_i - 4y_{i-1} + y_{i-2}}{2\Delta} \qquad (23)$$

$$y''_{i,\varphi} \cong \frac{y_{i+1} - 2y_i + y_{i-1}}{\Delta^2} \qquad (24)$$

where i it varies from 0 to $N+1\left(\overline{i=0\ N+1}\right)$, y_i - the projection of the unknown function on the line number *i*; Δ – move partition to the coordinate φ

As a result, the main vector of the sample of unknown total *6N* dimension can be written as:

$$Y = \left(\{w_i\}, \{v_i\}, \{u_i\}, \{\sigma_{ri}\}, \{\tau_{\varphi i}\}, \{\tau_{zi}\}\right)^T \quad i = \overline{1, N} \qquad (25)$$

The central difference (23), (25) are used for domestic direct *(1<i<N)*, the difference between the left and right (24), (25) make it possible to take into account the boundary conditions for φ. In the first case, the derivative with respect φ on the right sides of equations (19) is expressed by the formulas:

$$1 < i < N$$

$$\begin{aligned}
&w_{i,\varphi} = (w_{i+1} - w_{i-1})/2\Delta;\ u_{i,\varphi} = (u_{i+1} - u_{i-1})/2\Delta;\\
&v_{i,\varphi} = (v_{i+1} - v_{i-1})/2\Delta; \tau_{\varphi_i,\varphi} = \left(\tau_{\varphi(i+1)} - \tau_{\varphi(i-1)}\right)/2\Delta\\
&\tau_{\varphi_i,\varphi} = \left(\tau_{\varphi(i+1)} - \tau_{\varphi(i-1)}\right)/2\Delta;\\
&\sigma_{\varphi_i,\varphi} = a(\sigma_{i+1} - \sigma_{i-1})\Big/ 2\Delta + \frac{b}{r}\left[(v_{i+1} - 2v_i + v_{i-1})/\Delta^2 + w_{i,\varphi}\right] + cku_{i,\varphi};\\
&B_i = (u_{i+1} - 2u_i + u_{i-1})/\Delta^2/k - kv_{i,\varphi}.
\end{aligned} \qquad (26)$$

The boundary conditions at $\varphi = -\frac{\varphi_0}{2}$ accounted for in the equations corresponding straight $i = I$. For the main unknown outside the boundary conditions w_i, v_i, u_i Use the right difference (24):

Ref

15. B.I. Myachenkov, I.V. Grigorev. Calculation of composite shell structures on the computer: Spavochnik. -Moscow: Engineering, 1981-216 p.

$$w_{i,\varphi}=(-3w_1+4w_2-w_3)/2\Delta;$$
$$v_{i,\varphi}=(-3v_1+4v_2-v_3)/2\Delta; \qquad (27)$$
$$u_{i,\varphi}=(-3u_1+4u_2-u_3)/2\Delta.$$

For variable τ_φ Conditions (22) are recorded by means of the central difference

$$\tau_{\varphi_i,\varphi}\cong(\tau_{\varphi_2}-\tau_{\varphi_0})/2\Delta=-\tau_{\varphi_2}/2\Delta. \qquad (28)$$

The first and third of the conditions (22) is taken into account in the approximation of the derivatives of the function in the software φ σ_φ

$$\sigma_{\varphi_1,\varphi}\cong(\sigma_{\varphi_2}-\sigma_{\varphi_0})/2\Delta=\sigma_{\varphi_2}\Big/2\Delta=\left(a\sigma_{r_2}+\frac{b}{r}[(v_3-v_1)/2\Delta+w_2]-cku_2\right)\Big/2\Delta;$$
$$B_{1,\varphi}\cong(B_2-B_0)/2\Delta=B_2/2\Delta=[(u_3-u_1)/2\Delta/r-kv_2]/2\Delta. \qquad (29)$$

Similarly, derivatives are presented to the line with number i = N, taking into account the boundary conditions at $\varphi=\frac{\varphi_0}{2}$. The only difference is the replacement of the right finite difference Left:

$i{=}N:$

$$w_{i,\varphi}=(3w_N-4w_{N-1}+w_{N-2})/2\Delta;\ v_{i,\varphi}=(3v_N-\ldots)/2\Delta;$$
$$U_{i,\varphi}=(U_{i+1}-U_{i-1})/2\Delta;u_{i,\varphi}=(3u_N-\ldots)/2\Delta;$$
$$\tau_{\varphi,\varphi}=-\tau_{\varphi(N-1)}/2\Delta;\tau_{\varphi_i,\varphi}=(\tau_{\varphi(i+1)}-\tau_{\varphi(i-1)})/2\Delta; \qquad (30)$$
$$\sigma_{i,\varphi}=\left(a\sigma_{N-1}+\frac{b}{r}[(v_N-v_{N-2})/2\Delta+w_{N-1}]+cku_{N-1}\right)\Big/2\Delta=-\frac{\sigma_{N-1}}{2\Delta},$$
$$B_{i,\varphi}=-[(u_N-u_{N-2})/2\Delta/r-kv_{N-1}]\Big/2\Delta=-\frac{B_{N-1}}{2\Delta}.$$

The number of lines can be reduced by half if the conditions of use of anti symmetry transverse plate vibrations at $\varphi=0$

$$w=u=\sigma_\varphi=0 \qquad (31)$$

The corresponding difference ratio, taking into account the conditions (31) can be written as:

$i=N:$

$$w_{i,\varphi}=-w_{N-1}/2\Delta;\ u_{i,\varphi}=-u_{N-1}/2\Delta;$$
$$v_{i,\varphi}=(3v_N-\ldots)/2\Delta;\ \tau_{\varphi_i,\varphi}=(3\tau_{\varphi N}-4\tau_{\varphi(N-1)}+\tau_{\varphi(N-2)})/2\Delta;$$

Notes

$$\sigma_{i,\varphi} = -\left(a\sigma_{N-1} + \frac{b}{r}\left[(v_N - v_{N-2})/2\Delta + w_{N-1} \right] + cku_{N-1} \right) \Big/ 2\Delta = -\frac{\sigma_{N-1}}{2\Delta}; \qquad (32)$$

$$B_{i,\varphi} = -(-2u_N + u_{N-1})/\Delta^2/r - kv_{i,\varphi}.$$

The resolution of a system of ordinary differential equations according to (21) has the form:

$$w_i' = \sigma_i/k - a(ku_i + (w_i + v_{i,\varphi})/R); \;\; v_i' = \tau_{\varphi i} + (v_i - w_{i,\varphi})/R;$$

$$u_i' = \tau_{zi} + kw_i; \; \sigma_i' = -\omega^2 w_i + \left[2\left((w_i + v_{i,\varphi})/R - w_i' \right) - \tau_{\varphi_i,\varphi} \right] \Big/ r - k\tau_{zi};$$

$$\tau_{zi}' = -\omega^2 u_i - (B_{i,\varphi} + \tau_{zi})/r + k(\sigma + 2(ku_i - w_i')); \qquad (33)$$

$$\tau_{\varphi i}' = -\omega^2 v_i + (\sigma_{i,\varphi} + 2\tau_{\varphi i})/r - k(u_{i,\varphi}/R - kv_i).$$

Segment Ref:

In equations (33) the expression for the derivatives wiφ, $v_{i\varphi}$, $u_{i\varphi}$,$\sigma_{i,\varphi}$ $\beta_{i,\varphi}$ $\tau_{i,\varphi}$ are selected from (29) - (32) depending on the boundary conditions of the coordinate φ. free surface conditions equivalent (20) and forming together with the equations (33), the boundary value problem, is obtained in the form of

$$B_i = 0, \tau_{\varphi i} = 0, \sigma_\varphi = 0. \qquad (i=1,N) \qquad (34)$$

Thus, the initial spectral problem (19), (20), (22) by means of sampling coordinate φ by the method of direct reduced to the canonical problem (33), (34), for solutions which use the method of orthogonal sweep method previously used. The table shows the limit values of the phase velocity of the first edge of fashion, depending on the angle of the wedge. Found phase velocity for a material with a Poisson's ratio $\upsilon = 0{,}25$ Kirchhoff theory on plates - Love (column 3), Timoshenko - (column 4), contained within this section of the wedge method for calculating three-dimensional (column 5 - 6) and the formula $C_0 = C_ч \sin(m\varphi)$ /8/, $m = 1, 2, \ldots, m\varphi < 90^0$ (column 6). Column 5 corresponds to the embodiment of calculation with three internal lines (N = 3) and the boundary conditions (17), column 6 corresponds to the boundary conditions:

$$\varphi = -\frac{\varphi_0}{2}: \quad \sigma_{\varphi\varphi} = \sigma_{\varphi ч} = \sigma_{\varphi z} = 0; \quad \varphi = 0: \quad u_r = u_z = \sigma_{\varphi\varphi} = 0.$$

In accordance with the numerical results and shown in Table 1, embodiments of methods for calculating the Kirchhoff - Love, a three-dimensional theory of Timoshenko and agree with each other within 7 % for a thickness of the wedge angles of the base h_2, not exceeding 0.5 (the wedge angle $\varphi_0 = 28^0$). Note that for the angle $\varphi = 90^0$ limiting phase velocity was calculated as in [16,17,18], where the value is for her 0,90I $(\nu = 0{,}25)$. Thus, in contrast to the waveguides with a rectangular cross-section in the tapered waveguide with a sufficiently small wedge angle in the analysis of the dispersion relations of the first mode is permissible to use the theory of plates Kirchhoff - Love. Established fact is explained by the phenomenon of localization waveform near the acute angle of the wedge, as described in [8]. This phenomenon should be seen as a characteristic feature of the dynamic behavior of a plate of variable thickness.

Ref

16. H.B. Tolipov, S.Yu. Gurevich, AV Gerenshteyn. Propagation of elastic waves in the wedge: Monograph Chelyabinsk. 2003 – 122 p.

Notes

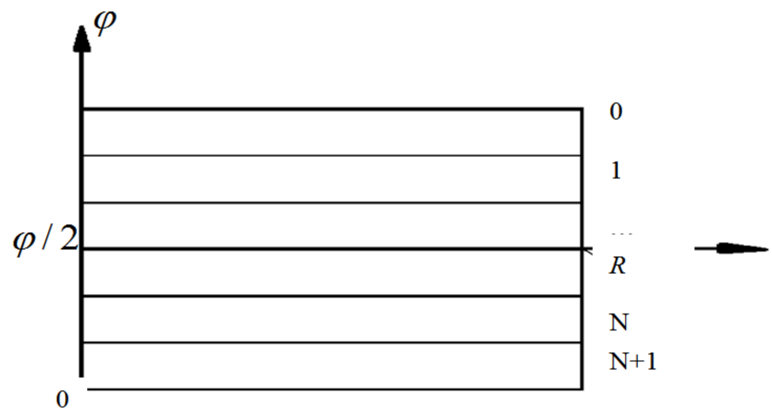

Figure 8: The settlement scheme

Table 1

R^*	φ_0	*К/Л*	*Т*	*З(1)*	*З(2)*	**По работе** [8]
0,2	11^0	0,2	0,196	-	-	0,182
0,3	17^0	0,3	0,286	0,308	0,298	0,276
0,5	28^0	0,5	0,442	0,475	0,462	0,433
0,7	38^0	0,7	0,563	0,605	0,592	0,574
1	53^0	1	0,691	0,741	0,729	0,736
2	90^0	2	0,864	0,908	-	0,92

IV. Conclusions

1. It was revealed that in an elastic cylinder with radial crack no waves having a real part of the phase velocity, localized near the axis of the cylinder.
2. The results of calculation of the maximum speed the spread of the first tapered waveguide modes in the theory of plates Kirchhoff - Love and dynamic elasticity does not differ by more than 6% to the top of the wedge angles not exceeding 28^0. *At* $28^0 < \varphi < 90^0$ characterized by certain surcharges to 20%. Thus, for small wedge angles permissible use of the simplified theory of Kirchhoff - Love and Timoshenko in the whole wavelength range. Thus, for small wedge angles permissible use of the simplified theory of Kirchhoff - Love and Timoshenko in the whole wavelength range.
3. Accounting for the viscoelastic properties of the wedge material reduces the real part of the wave propagation velocity is 10-15%, as well as to evaluate the ability of the system damping in general. The work was supported by the Foundation for Fundamental Research Ф-4-14 of the Republic of Uzbekistan.

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A Study on Sensitivity and Robustness of One Sample Test Statistics to Outliers

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Abstract- Outliers are observations that stand too different from others in a set of observations. When present in a data set, they affect both descriptive and inferential statistics. This work therefore, studies the sensitivity and robustness of one sample test statistics to outliers so as to know the appropriate one to test hypothesis about the population parameter when outliers are present. One sample test statistics considered are: parametrics test (Student t-test and z-test), non-parametric test (Wilcoxon Sign test (Distribution Sign test (DST), Asymptotic Sign test (AST)), Wilcoxon Signed rank test (Distribution Wilcoxon Signed rank test (DWST) and Asymptotic (AWST)), t-test for rank transformation (Rt-test) and Trimmed t-test statistics (Tt-test). Monte Carlo experiments, replicated five thousand (5000) times, were conducted at eight (8) sample sizes (10, 15, 20, 25, 30, 35, 40 and 50) by simulating data from normal distribution. At each of the sample sizes, 10% and 20% of the generated data were randomly selected and invoked with various magnitude of outliers (-10, -9, -8,… 8, 9, 10).

Keywords: *outliers, type 1 error rate, sensitivity, robustness, inferential test statistics, levels of significance.*

GJSFR-F Classification: *MSC 2010: 13P25*

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A Study on Sensitivity and Robustness of One Sample Test Statistics to Outliers

Kayode Ayinde α, Taiwo Joel Adejumo σ & Gbenga Sunday Solomon ρ

Abstract- Outliers are observations that stand too different from others in a set of observations. When present in a data set, they affect both descriptive and inferential statistics. This work therefore, studies the sensitivity and robustness of one sample test statistics to outliers so as to know the appropriate one to test hypothesis about the population parameter when outliers are present. One sample test statistics considered are: parametrics test (Student t-test and z-test), non-parametric test (Wilcoxon Sign test (Distribution Sign test (DST), Asymptotic Sign test (AST)), Wilcoxon Signed rank test (Distribution Wilcoxon Signed rank test (DWST) and Asymptotic (AWST)), t-test for rank transformation (Rt-test) and Trimmed t-test statistics (Tt-test). Monte Carlo experiments, replicated five thousand (5000) times, were conducted at eight (8) sample sizes (10, 15, 20, 25, 30, 35, 40 and 50) by simulating data from normal distribution. At each of the sample sizes, 10% and 20% of the generated data were randomly selected and invoked with various magnitude of outliers (-10, -9, -8,… 8, 9, 10). The test statistics were compared at three levels of significance, 0.1, 0.05 and 0.01. A test is considered robust if its estimated error rate approximates the true error rate and has the highest number of times it approximates the error rate when counted over the percentage (%) of outliers, magnitudes of outliers and levels of significance; and if the counts is minimum the test statistics is sensitive. At all the three (3) levels of significance, results revealed that Type 1 error rates of Student t-test, Rt-test and AWST statistics are good; and that z-test and Student t-test statistics are most sensitive to outliers. The statistics robustness is affected by the levels of significance in that the sign test (DST and AST) is robust at 0.1; Tt-test and Wilcoxon Sign Rank test (DWST and AWST) at 0.05; and DST, AWST, Tt-test and AST at 0.01 level of significance. Consequently, the Sign test and Tt-test statistics are recommended for hypothesis testing in the presence of outliers.

Keywords: outliers, type 1 error rate, sensitivity, robustness, inferential test statistics, levels of significance.

I. Introduction

Edgeworth (1887) defined outliers as discordant observations which present the appearance of differing in respect of their law of frequency from other observations with which they are combined. Hawkins (1980) defined outlier as an observation which deviates so much from the other observations as to arouse suspicious that it was generated by a different mechanism. Also, Barnett, Lewis and John (1994) indicated outlier as outlying observation which appears to deviate markedly from other members of the sample in which it occurs. According to Osborne and Amy (2004), some of the major causes of outliers in the data set include: variability in the measurement or measurement error, data collection errors, data entry errors, invalidity of theory, intentional or motivated mis-reporting, standardization failure and faulty distributional assumptions. If outliers are present in the data and deleted, Osborne and Amy (2004) claimed that their deleterious effect may: alter the odds of making both Type 1 and Type 11 errors, seriously bias or influence estimates that may be of substantive interest,

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generally serve to increase error variance and reduce the power of statistical tests. Some descriptive statistics of a data set such as arithmetic mean have been noted to be affected by outliers while the median is not being affected, other measures especially the arithmetic mean is affected. Most parametric test statistics including the Student t-test (Gosset, 1908), z-test (Gauss, 1809), etc. are directly meant for hypothesis testing about the mean while the non- parametric ones test hypothesis about the median. The non-parametric inferential test statistics for one sample problem include the Sign test (John, 1710) and Wilcoxon signed rank test (Wilcoxon, 1945). The t-test for rank transformation (Rt-test) by Conover and Ronald (1981) tried to bridge the gap between the parametric and the non-parametric while the trimmed t-test (Tt-test) by Yuen (1974) excluded outliers in its test procedure. The effect of outliers on these various test statistics needs to be investigated as this inevitably affects inference. Therefore, this research work examines the effect of outliers on these test statistics so as to determine the sensitive and robust ones to outliers.

Notes

II. Review on Some Inferential Test Statistics

Many inferential test statistics which are meant for hypothesis testing about the mean and median of one sample problem have been discussed in literatures. Some of these test statistics are discussed as follows:

a) *Parametric statistics*

Parametric tests are tests in which probability density function of the population in which sample is taken is known or assumed to be normal. The data used must at least be at interval scale and method does not require ranking of observations. However, it has long been established that moderate violations of parametric assumptions have little or no effect on substantive conclusions in most instances (Cohen, 1969).

i. *One sample z-test*

The z-test is one of the most popular techniques for statistical inference based on the assumption of normal distribution (Gauss, 1809; Agresti and Finlay, 1997). The test statistic requires the variance of the population. The test statistic for one sample z-test is defined as:

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} \sim N(0,1) \tag{1}$$

where: $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$ is the sample mean, μ_0 is the population mean or hypothesized mean, σ is the standard deviation and n is sample size.

ii. *One sample student t-test*

The student t-test was developed in the early 1900s by a statistician named (Gosset, 1900) who was working at the Guinness brewery and is called the student t-test because of proprietary rights. A single sample t-test is used to determine whether the mean of a sample is different from a known population mean. The t-test uses the mean, standard deviation and number of samples to calculate the test statistic. Plackett and Barnard, (1990).

One sample t-test has all the assumptions of z-test except that of small sample size. Its test statistic is as follows:

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \tag{2}$$

Notes

where $s = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$ (the standard deviation), $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$ is the sample mean, μ_0 is the hypothesized mean value and n is the sample size. This t-statistic is known to follow a t-distribution with n-1 degree of freedom.

b) Non-parametric statistics

Non-parametric test is an alternative to parametric tests when the exact distributions are not known or when the level of measurement is weaker than the interval level. Moreover, non-parametric tests are generally designed to test hypothesis that do not concern population parameters, but are based on the shape of the population frequency distributions. Generally, if the data do not meet the criteria for a parametric test (normally distributed, equal variance and continuous), it must be analyzed with a nonparametric test.

i. *The sign test*

The sign test which was discovered by John (1710) is used to compare a single sample with some hypothesized value, and it is therefore of use in the situation in which the one sample or paired t-test might traditionally be applied. The sign test is so called because it allocates a sign, either positive (+) or negative (-) to each observation according to whether it is greater or less than some hypothesized median value and considers whether this is substantially different from what we would expect by chance. The test statistics are T^+or T^-. UnderH_0, binomial distribution is used and H_0 is rejected if P (T≤ T^+) ¡α or P (T≤ T^-) <α. $\alpha/2$ is used instead of α when the test is two-tail. Asymptotically, the sign test has its distribution following binomial (n,$1/2$). The asymptotic test statistic for sign test is:

$$z = \frac{T^+ - \frac{n}{2}}{\sqrt{\frac{n}{4}}} \sim N(0,1) \tag{3}$$

ii. *Wilcoxon signed rank test*

Wilcoxon signed-rank test is named after Wilcoxon (1945) who in a single paper proposed both the test and rank-sum test for two independent samples. The test was further popularized by Siegel (1956) who used the symbol T for value related to, but not the same.

The asymptotic distribution of Wilcoxon signed rank test is:

$$T = \frac{T^+ - E_0(T^+)}{\sqrt{V_0(T^+)}} \sim N(0,1) \tag{4}$$

where $E_0(T^+) = \frac{(n+1)}{4}$ and $V_0(T^+) = \sqrt{\frac{n(n+1)(2n+1)}{24}}$

c) T-test for Rank Transformation in one and matched pairs sample

Conover and Ronald (1981) proposed t-test for rank transformation which bridged the gap between parametric and non-parametric test statistics. The test statistic is defined as follows: Let $D_1, D_2, \ldots, D_n$ represent independent random variables with a common mean where in the case of matched pairs (X_i, Y_i); $D_i = X_i - Y_i$. But, for one sample $D_i = X_i - \mu_0$ where μ_0 is the hypothesized mean value and R_i = (sign D_i) x (rank of / D_i/). The test statistic is defined as:

$$T = \frac{\sum_{l=1}^{n} R_i}{\sqrt{\sum_{l=1}^{n} R_i^{\,2}}} \tag{5}$$

The alternative t-test statistic is computed on the signed ranks as;

$$t_R = \frac{\sum_{l=1}^{n} R_i}{\sqrt{\dfrac{n\sum_{l=1}^{n} R_i^{\,2} - (\sum_{l=1}^{n} R_i)^2}{n-1}}} \tag{6}$$

which is compared with the t-distribution (n-1) degree of freedom.
Also, t_R can be expressed as:

$$t_R = \frac{T}{\sqrt{\dfrac{n}{n-1} - \dfrac{T^2}{n-1}}} \tag{7}$$

which is a monotonic function of T.

d) The Trimmed t-test

Yuen (1974) proposed the Trimmed t-test for the independent two-sample case, under unequal population variances (Keselman, Wilcox, Algina, and Fradette, 2008).

In each sample, the trimmed mean is computed by removing g-observations from each tail of the distribution. The trimmed mean is computed as follows:

$$\overline{X}_t = \frac{X_{g+1} + X_{g+2} + \ldots + X_{n-g}}{n-2g} \tag{8}$$

Where $x_1, \ldots, x_n$ are the ordered values in a sample.
g = observations trimmed from each tail of the sample distribution.
n – 2g = the number of observations in the trimmed sample.

In addition to the trimmed mean, the Winsorized mean is required to compute the appropriate variance estimate. Instead of "trimming" this method replaces the most extreme g- observations by the next-most-extreme value. The Winsorized mean is computed as:

$$\overline{X}_w = \frac{([g+1]X_{g+1} + X_{g+2} + \ldots + [g+1]X_{n-g})}{n} \tag{9}$$

Notes

The Winsorized sum-of-squared derivation is computed as:

$$\mathrm{SSD_w} = [\mathrm{g}+1][\mathrm{x_{g+1}} - \bar{\mathrm{X}}_\mathrm{w}]^2 + [\mathrm{x_{g+2}} - \bar{\mathrm{X}}_\mathrm{w}]^2 + \cdots + [\mathrm{g}+1][\mathrm{x_{n-g}} - \bar{\mathrm{X}}_\mathrm{w}]^2 \qquad (10)$$

The Winsorized variance is obtained as:

Notes

$$S_w^2 = \frac{SSD_w}{n-2g-1} \qquad (11)$$

Also, the obtained value of the trimmed t-test for one sample is obtained by dividing the difference between the trimmed mean and the hypothesized mean by the estimated standard error.
Hence, the test statistic is defined as:

$$t_w = \frac{\bar{x}_t - \mu_t}{S_w \big/ \sqrt{n-2g-1}} \quad \text{where } t_w \text{ follows a t-distribution} \qquad (12)$$

III. Methodology

a) *The Monte Carlo Experiments*

Data were generated from the univariate normal distribution using Monte Carlo simulation procedures with the aids of R-statistical programming codes.

X_i is generated with mean (\`) =10 and standard deviation (σ) = 5. The percentage (%) of outliers to be invoked into the generated data were 10% and 20% while the magnitude of the outliers (f) were taken as: -10, -9, -8, …, 8, 9, and 10. The experiments were conducted five thousand times (R=5000) at eight sample sizes namely: 10, 15, 20, 25, 30, 35, 40 and 50.

The procedures for invoking the outliers and estimation of the Type 1 errors are as follows:

(i) Choose a percentage of the data to be replaced with outliers.

(ii) Choose a particular magnitude of outlier to invoke into the generated data

(iii) Choose a sample size to work with, say n.

(iv) Generate random sample with size n from a normal distribution with $\mu = 10$ and $\sigma = 5$; $X_i \sim N(10, 25)$

(v) Randomly select those observations making up the percentage of the generated data to be replaced with outliers.

(vi) Invoke outliers as follows:

$$X(i)_{outlier} = f^*\mathrm{Max}\,(X_i) + X_i \qquad (13)$$

Where: X_i = selected generated observation i

$X(i)_{outlier}$ = Outlier to replace X_i

f = Magnitude of outliers

$\mathrm{Max}(X_i)$ = Maximum of the generated data

(vii) Replace the outliers in the data originally generated in (iv)

(viii) Apply the various test statistics and keep their p-values.

(ix) For each test statistics in (viii), define:

$$G_i = \begin{cases} 1, if \ p - value < \alpha \\ 0, otherwise \end{cases} \quad (14)$$

Where α is a preselected level of significance, say 0.1.

(x) Repeat steps (v) to (ix) five thousand (5000) times, R= 5000.

(xi)For each of the test statistics, sum the results obtained in step (x), i.e

$$G = \sum_{i=1}^{R} G_i \quad (16)$$

(xii) For each of the test statistics, divide the result in step (xi) by the number of replications to estimate the Type 1 error of each test statistics, i.e

$$Q_\alpha = \frac{\sum_{i=1}^{R} G_i}{R} = \frac{G}{R} \quad (17)$$

(xiii) Choose another magnitude of outlier to invoke into the generated data and repeat step (v) to (xii).

(xiv) Repeat step (v) to (xiii) until all the magnitudes of the outliers are exhausted.

(xv) Choose another sample size to work with and repeat step (v) to (ix)

(xvi) Repeat step (v) to (xv) until all the sample sizes are exhausted.

(xvii) Choose another percentage of the data to be replaced with outliers and repeat step (ii) to (xvi).

(xviii) Repeat step (ii) to (xvii) until all the levels of percentages are exhausted.

b) Sensitivity and Robustness of the Test Statistics

The test statistics were considered robust if their estimated Type 1 error rates at different % and magnitude of outliers are within the preferred interval of levels of significance suggested by Kuranga (2015) and used by Ayinde et.al (2016). This is presented in Table 1.

On the other hand, the test statistics were considered sensitive if as percentage of outliers and magnitude of outliers increase as the Type 1 error rates of the test statistics also increases. i.e many do not fall into the preferred intervals (the null hypothesis is rejected often).

Table 1: The True level of significance and preferred interval

Level of significance	Preferred interval
0.1	0.095 - 0.14
0.05	0.045 - 0.054
0.01	0.005 - 0.014

Source: Kuranga (2015) and Ayinde et.al (2016)

The number of times Type 1 error rates fell within the preferred interval was counted over the levels of percentage of outliers, magnitude of outliers and sample size. A test statistic that is robust is expected to have the highest number of counts, the mode; and that which is sensitive is to have the smallest.

Notes

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IV. Results and Discussion

The results of Type 1 error rates of one sample inferential test statistics as affected by outliers at 0.1, 0.05 and 0.01 levels of significance are respectively presented in Table 2, Table 3 and Table 4 and discussed.

a) *Results of Type 1 Error Rates on one sample test statistics at 0.1 level of significance*

From Table 2, it can be observed that all of the Type 1 error rates of Student t-test, Rt-test and AWST performed very well across all sample sizes. The z-test, DWST, AST and Tt-test, in this order also did well but not at all the sample sizes. However, the performance of DST is not good across all sample sizes.

Table 2: Results of Type 1 Error Rates on one sample test statistics at 0.1 level of significance

Sample size

Test statistics	10	15	20	25	30	35	40	50
Sd.t-test	0.0962	0.1026	0.1012	0.098	0.098	0.1	0.096	0.0974
z-test	0.0954	0.0978	0.1004	0.098	0.1	0.09	0.096	0.0948
Rt-test	0.105	0.0972	0.0952	0.099	0.101	0.1	0.097	0.0986
DWST	0.0828	0.0972	0.0952	0.094	0.095	0.09	0.093	0.0986
AWST	0.105	0.1098	0.0952	0.099	0.101	0.1	0.097	0.1004
DST	0.0208	0.0356	0.0366	0.043	0.091	0.09	0.075	0.062
AST	0.1082	0.119	0.1114	0.105	0.091	0.09	0.075	0.1152
Tt-test	0.0956	0.0978	0.1	0.099	0.094	0.09	0.094	0.0978

Source: Simulation results

b) *Results of Type 1 Error Rates on one sample test statistics at 0.05 level of significance*

From Table 3, it can be observed that all of the Type 1 error rates of the test statistics generally did well except that of the Sign test

Table 3: Results of Type 1 Error Rates on one sample test statistics at 0.05 level of significance

Sample size

Test statistics	10	15	20	25	30	35	40	50
Sd.t-test	0.0486	0.0504	0.046	0.05	0.05	0.045	0.05	0.0502
z-test	0.0492	0.0502	0.051	0.05	0.05	0.048	0.05	0.0482
Rt-test	0.0468	0.0552	0.05	0.05	0.05	0.048	0.05	0.0494
DWST	0.0468	0.048	0.046	0.05	0.05	0.046	0.05	0.0478
AWST	0.0468	0.048	0.046	0.05	0.05	0.046	0.05	0.0488
DST	0.0208	0.0356	0.037	0.04	0.04	0.041	0.04	0.033
AST	0.0208	0.0356	0.037	0.04	0.04	0.041	0.04	0.062
Tt-test	0.0488	0.0492	0.051	0.05	0.05	0.044	0.05	0.0546

Source: Simulation results

c) Results of Type 1 Error Rates on one sample test statistics at 0.01 level of significance

From Table 4, it can be seen that the Type 1 error rates of all test statistics did well across all sample sizes but the performance of DST and AST are only good in some instances.

Table 4: Results of Type 1 Error Rates on one sample test statistics at 0.01 level of significance

Sample size

Test statistics	10	15	20	25	30	35	40	50
Sd.t-test	0.0082	0.0104	0.009	0.01	0.01	0.009	0.01	0.011
z-test	0.0098	0.0116	0.01	0.01	0.01	0.007	0.01	0.0094
Rt-test	0.0116	0.0126	0.011	0.01	0.01	0.009	0.01	0.011
DWST	0.009	0.0076	0.008	0.01	0.01	0.008	0.01	0.0104
AWST	0.0058	0.0062	0.007	0.01	0.01	0.008	0.01	0.0104
DST	0.0012	0.0074	0.003	0	0.01	0.004	0.01	0.007
AST	0.0012	0.0074	0.009	0.01	0.01	0.004	0.01	0.007
Tt-test	0.0076	0.0102	0.008	0.01	0.01	0.009	0.01	0.01

Source: Simulation results

Note: Estimated Type 1 error rates in the preferred interval are in bold form.

d) Results of overall number of times Type 1 Error rates approximate true levels of significance when counted over levels of significance

Results of counting the number of times the Type 1 error rates of the inferential test statistics approximate the true level of significance when counted over the three (3) levels of significance is presented in Table 5 and graphically represented in Figure 1.

From Table 5 and Figure 1, it can be seen that the Type 1 error rate of student t-test and AWST, Rt-test and z-test are generally very good while that of the Sign test not good.

Table 5: Overall Total number of Times Type 1 Error Rates approximates True levels of significance when counted over levels of significance

Sample size

Test statistics	10	15	20	25	30	35	40	50	Total	Rank
Sd.t-test	3	3	3	3	3	3	3	3	24	1.5
z-test	3	3	3	3	3	2	3	2	22	4
Rt-test	3	2	3	3	3	3	3	3	23	3
DWST	2	3	3	2	3	2	2	3	20	5.5
AWST	3	3	3	3	3	3	3	3	24	1.5
DST	0	1	0	0	1	0	1	1	4	8
AST	1	2	2	2	1	0	1	2	11	7
Tt-test	3	2	3	3	2	2	2	3	20	5.5

Source: Counted from Table 2, 3 and 4

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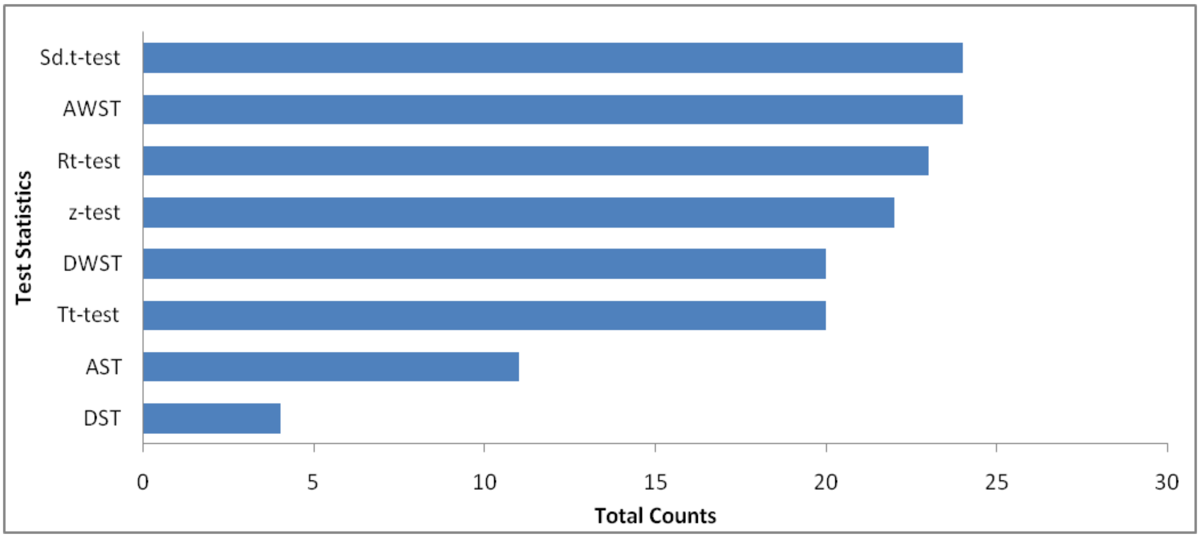


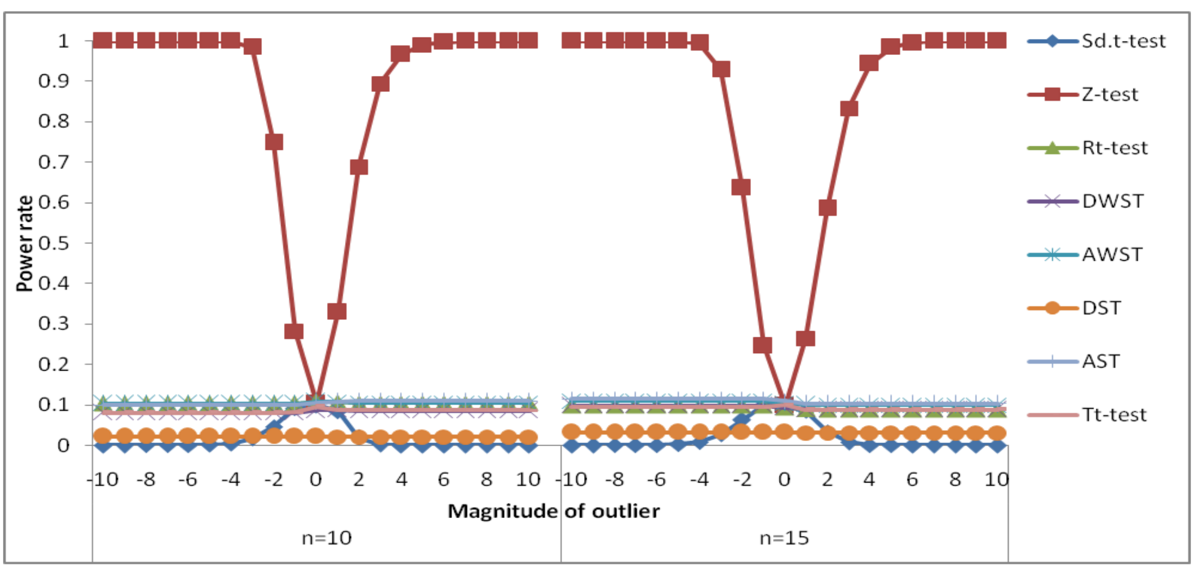
Figure 1: Bar chart showing overall total number of times Type 1 Error Rates approximate true levels of significance

e) Sensitivity and Robustness Investigation of One Sample test Statistics

Sample graphs of Type 1 error rates of the inferential test statistics at different levels of significance, percentage of outliers and magnitude of outliers are presented and discussed. Adejumo (2016) provided the details of the graphs and the simulation results. However, Figure 2, Figure 3 and Figure 4 are sample graphs of the results for 0.1, 0.05 and 0.01 level of significance at 10% and 20% outliers respectively and discussed.

The test statistics are affected by the magnitude, percentage of outlier and levels of significance. From the graphical representations, it can be observed that as the sample size, magnitude and percentage of outliers are increasing the more the sensitivity of z-test and student t-test to outliers at all levels of significance.

Summarily, the overall results of the further counts over the levels of significance are presented in Table 6 and Figure 5. From Table 6 and Figure 5, it can be concluded that the z-test and student t- statistics are very sensitive and AST and Tt-test are robust to outliers.



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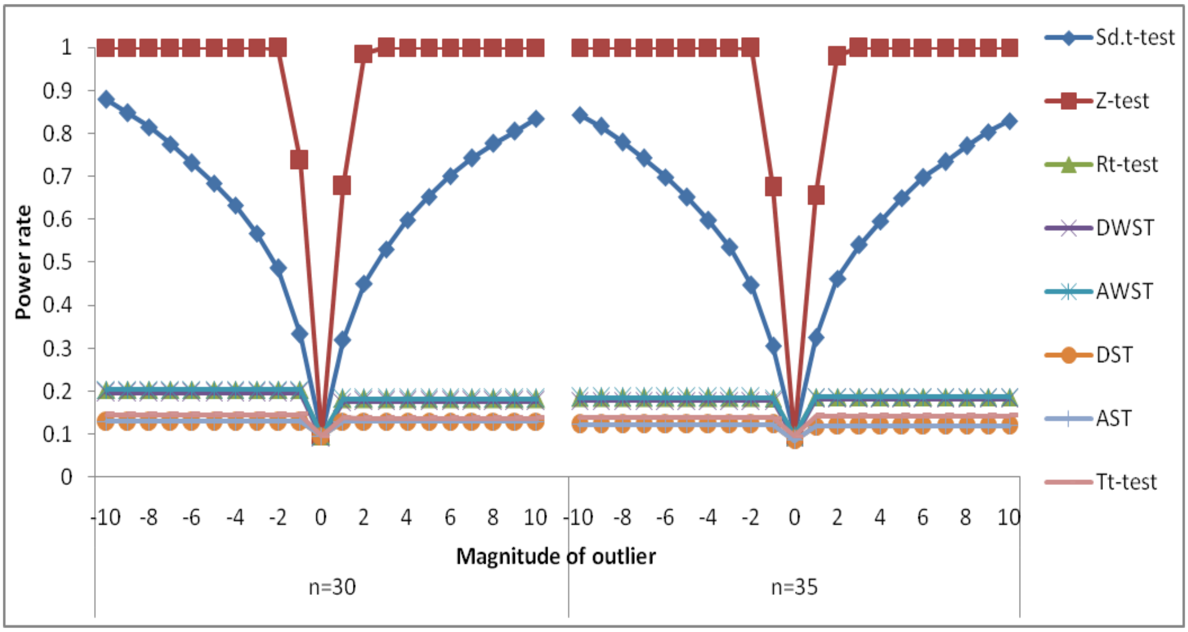

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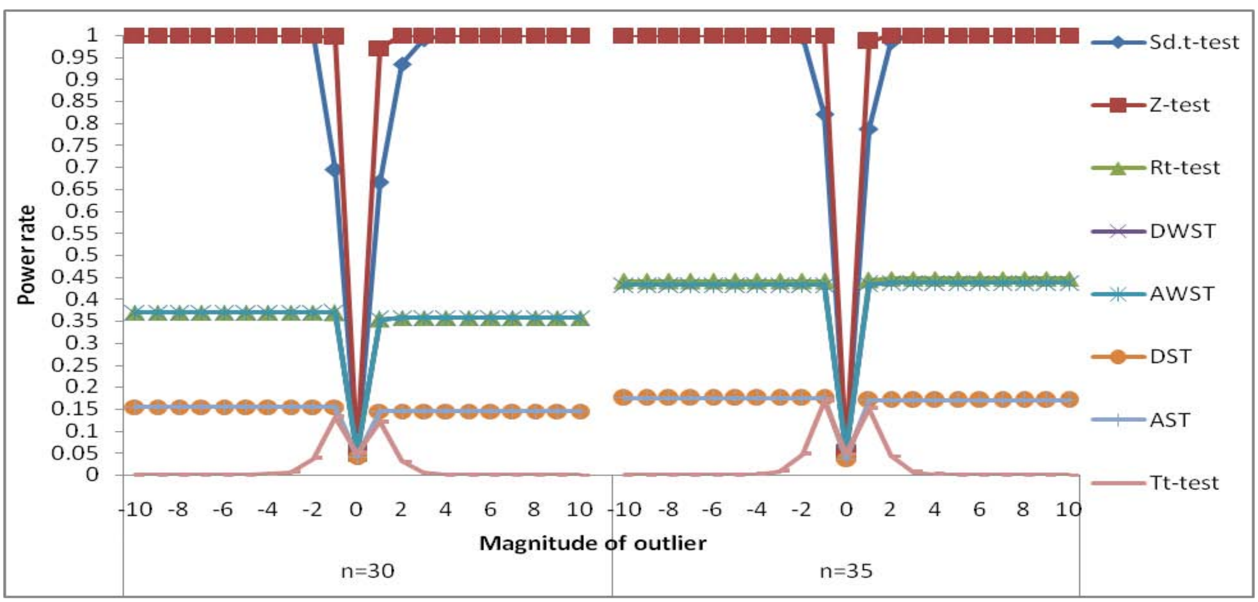
Figure 2: Graphical representation of power rate of one sample test statistics with 10% outlier at 0.1 level of significance



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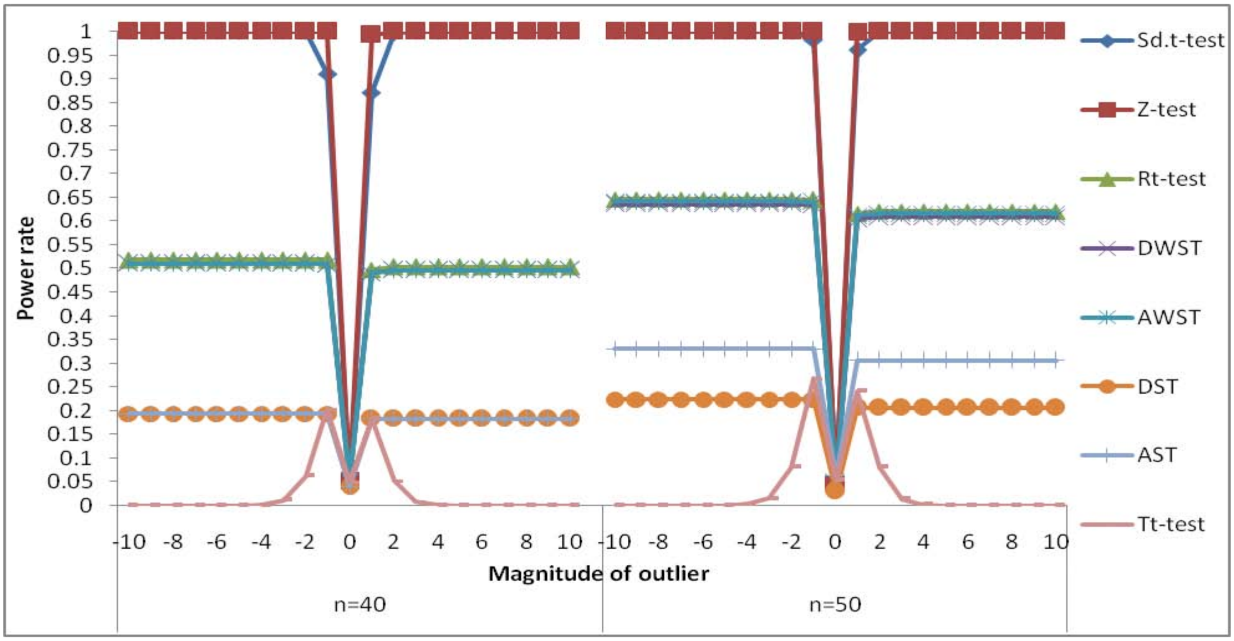

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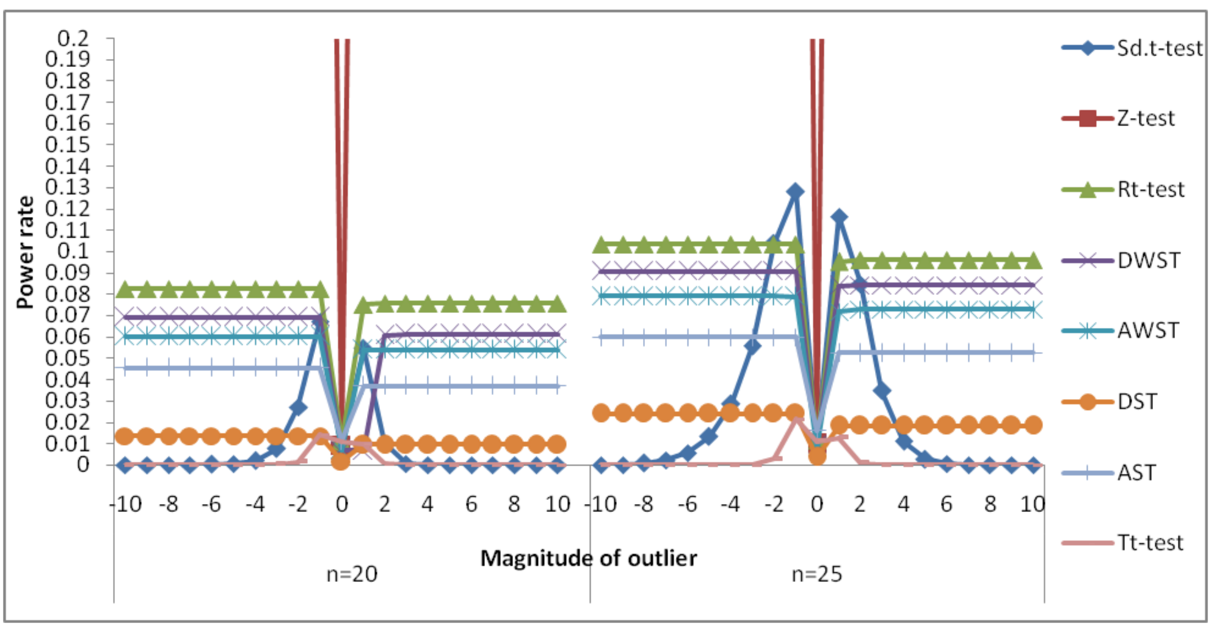
Figure 3: Graphical representation of power rate of one sample test statistics with 20% outlier at 0.05 level of significance



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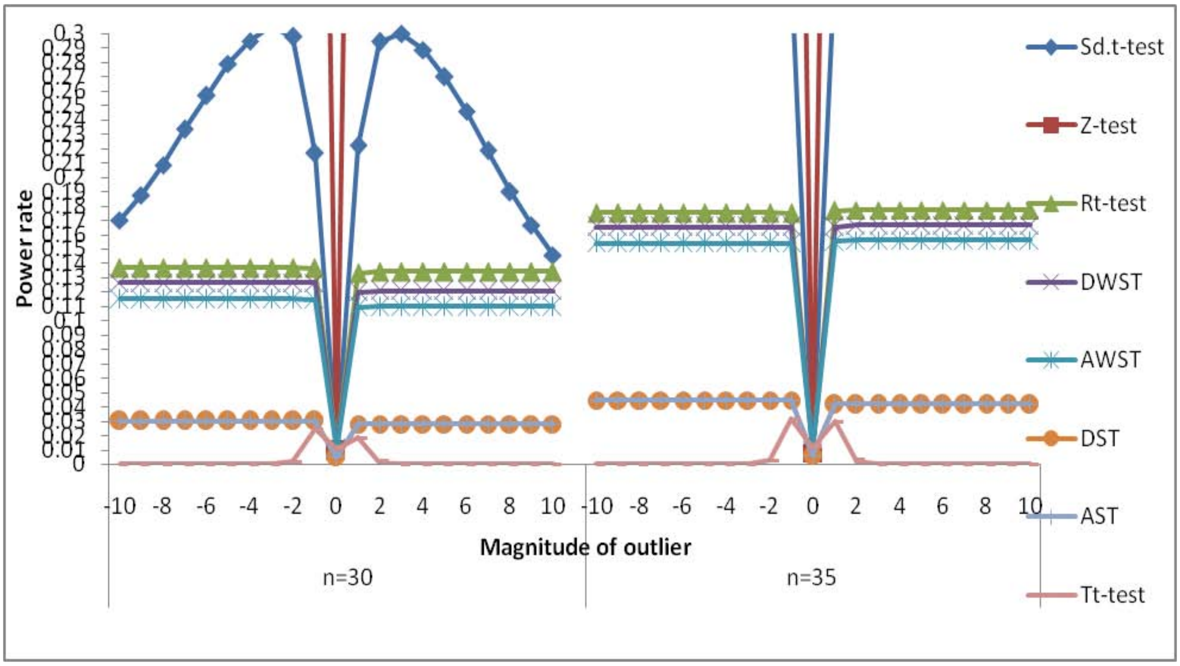

(b)

Figure 4: Graphical representation of power rate of one sample test statistics with 20% outlier at 0.01 level of significance

Table 6: Overall total number of times Power Rates approximates True levels of significance when counted over percentage of outliers and levels of significance for one sample problem

	Sample size									
Test Statistics	**10**	**15**	**20**	**25**	**30**	**35**	**40**	**50**	**Total**	**Rank**
Sd.t-test	10	9	12	11	8	8	7	8	73	7
z-test	6	5	6	4	6	6	6	6	45	8
Rt-test	66	41	3	14	6	6	6	6	148	5
DWST	44	55	8	44	4	5	6	6	172	4
AWST	86	66	15	33	6	5	6	6	223	3
DST	0	32	19	0	20	22	12	2	107	6
AST	22	54	45	34	41	42	32	4	274	1
Tt-test	26	40	56	64	18	19	7	8	238	2

Source: Counted from Simulation results

Notes

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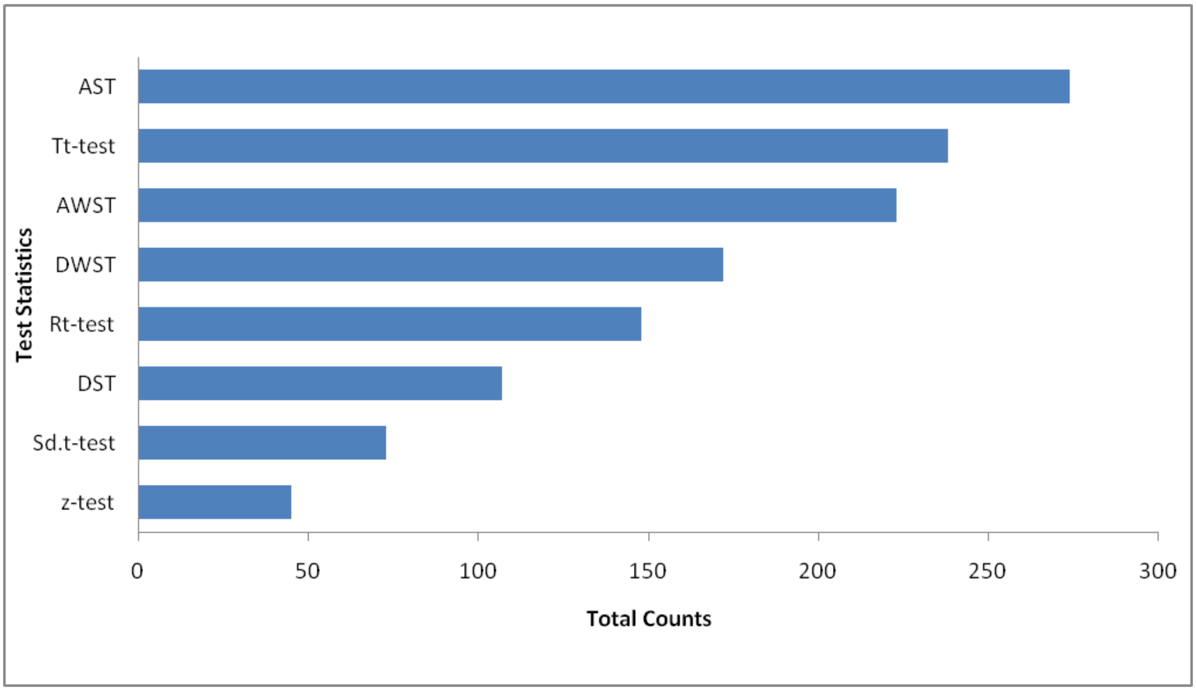

Figure 5: Bar chart showing overall total number of times Power Rates approximate true level of significance when counted over percentage of outlier and all levels of significance for one sample problem

V. Summary and Conclusion

Summary of findings and conclusions of the work is hereby presented in Table 6 as follows:

Table 6: Summary of findings on the One Sample Test Statistics

α	One Sample Investigation		
	Type 1 Error	**Robustness**	**Sensitivity**
0.1	Sd.t-test, Rt-test, AWST	AST, DST	z-test, Sd.t-test
0.05	z-test, Sd.t-test, DWST, AWST	Tt-test, DWST	z-test, Sd.t-test
0.01	z-test, Sd.t-test, DWST, AWST, Rt-test, Tt-test	DST, AWST, Tt-test, AST	z-test, Sd.t-test
Overall	Sd.t-test, AWST	AST, Tt-test	z-test, Sd.t-test

From Table 6, the following can be observed:

The z-test and Student t- statistics are the most test statistics sensitive to outliers at all levels of significance.

At 0.1 level of significance, Student t-test, Rt-test and AWST have better Type 1 error while, the Sign tests, AST and DST in this order, are robust.

At 0.05 level of significance, the Type 1 error rate of z-test, student t-test DWST and AWST are good while, Tt-test and Wilcoxon Sign Rank test (DWST and AWST) in this order, are robust.

At 0.01 level of significance, the Type 1 error rate of all the test statistics are good except sign test meanwhile, DST, AWST, Tt-test and AST, in this order, are robust to outliers.

Summarily, over all levels of significance it can be concluded that, student t-test and AWST are the test statistics that have better Type 1 error rate, AST and Tt-test are the most robust test statistics to outliers whereas, z-test and Student t- statistics were identified to be the most test statistics sensitive to outliers.

Notes

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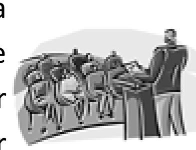

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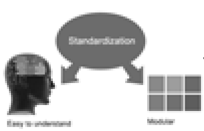

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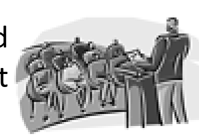
The "FARSC" is a dignified title which is accorded to a person's name viz. Dr. John E. Hall, Ph.D., FARSC or William Walldroff, M.S., FARSC.

The IFOARS institution is entitled to form a Board comprised of one Chairperson and three to five board members preferably from different streams. The Board will be recognized as "Institutional Board of Open Association of Research Society"-(IBOARS).

The Institute will be entitled to following benefits:

The IBOARS can initially review research papers of their institute and recommend them to publish with respective journal of Global Journals. It can also review the papers of other institutions after obtaining our consent. The second review will be done by peer reviewer of Global Journals Incorporation (USA) The Board is at liberty to appoint a peer reviewer with the approval of chairperson after consulting us.

The author fees of such paper may be waived off up to 40%.



The Global Journals Incorporation (USA) at its discretion can also refer double blind peer reviewed paper at their end to the board for the verification and to get recommendation for final stage of acceptance of publication.

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The Board can also play vital role by exploring and giving valuable suggestions regarding the Standards of "Open Association of Research Society, U.S.A (OARS)" so that proper amendment can take place for the benefit of entire research community. We shall provide details of particular standard only on receipt of request from the Board.

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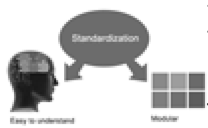

After nomination of your institution as "Institutional Fellow" and constantly functioning successfully for one year, we can consider giving recognition to your institute to function as Regional/Zonal office on our behalf.
The board can also take up the additional allied activities for betterment after our consultation.

The following entitlements are applicable to individual Fellows:

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Note :

"

- In future, if the board feels the necessity to change any board member, the same can be done with the consent of the chairperson along with anyone board member without our approval.
- In case, the chairperson needs to be replaced then consent of 2/3rd board members are required and they are also required to jointly pass the resolution copy of which should be sent to us. In such case, it will be compulsory to obtain our approval before replacement.
- In case of "Difference of Opinion [if any]" among the Board members, our decision will be final and binding to everyone.

"

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The paper should be in proper format. The format can be downloaded from first page of 'Author Guideline' Menu. The Author is expected to follow the general rules as mentioned in this menu. The paper should be written in MS-Word Format (*.DOC,*.DOCX).

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1. General,

2. Ethical Guidelines,

3. Submission of Manuscripts,

4. Manuscript's Category,

5. Structure and Format of Manuscript,

6. After Acceptance.

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Research articles: These are handled with small investigation and applications

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The recommended size of original research paper is less than seven thousand words, review papers fewer than seven thousands words also.Preparation of research paper or how to write research paper, are major hurdle, while writing manuscript. The research articles and research letters should be fewer than three thousand words, the structure original research paper; sometime review paper should be as follows:

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(e) Resources and techniques with sufficient complete experimental details (wherever possible by reference) to permit repetition; sources of information must be given and numerical methods must be specified by reference, unless non-standard.

(f) Results should be presented concisely, by well-designed tables and/or figures; the same data may not be used in both; suitable statistical data should be given. All data must be obtained with attention to numerical detail in the planning stage. As reproduced design has been recognized to be important to experiments for a considerable time, the Editor has decided that any paper that appears not to have adequate numerical treatments of the data will be returned un-refereed;

(g) Discussion should cover the implications and consequences, not just recapitulating the results; conclusions should be summarizing.

(h) Brief Acknowledgements.

(i) References in the proper form.

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It is vital, that authors take care in submitting a manuscript that is written in simple language and adheres to published guidelines.

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Metric SI units are supposed to generally be used excluding where they conflict with current practice or are confusing. For illustration, 1.4 l rather than 1.4 × 10-3 m3, or 4 mm somewhat than 4 × 10-3 m. Chemical formula and solutions must identify the form used, e.g. anhydrous or hydrated, and the concentration must be in clearly defined units. Common species names should be followed by underlines at the first mention. For following use the generic name should be constricted to a single letter, if it is clear.

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- One should avoid outdated words.

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Acknowledgements: Please make these as concise as possible.

References

References follow the Harvard scheme of referencing. References in the text should cite the authors' names followed by the time of their publication, unless there are three or more authors when simply the first author's name is quoted followed by et al. unpublished work has to only be cited where necessary, and only in the text. Copies of references in press in other journals have to be supplied with submitted typescripts. It is necessary that all citations and references be carefully checked before submission, as mistakes or omissions will cause delays.

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24. Never copy others' work: Never copy others' work and give it your name because if evaluator has seen it anywhere you will be in trouble.

25. Take proper rest and food: No matter how many hours you spend for your research activity, if you are not taking care of your health then all your efforts will be in vain. For a quality research, study is must, and this can be done by taking proper rest and food.

26. Go for seminars: Attend seminars if the topic is relevant to your research area. Utilize all your resources.

27. Refresh your mind after intervals: Try to give rest to your mind by listening to soft music or by sleeping in intervals. This will also improve your memory.

28. Make colleagues: Always try to make colleagues. No matter how sharper or intelligent you are, if you make colleagues you can have several ideas, which will be helpful for your research.

29. Think technically: Always think technically. If anything happens, then search its reasons, its benefits, and demerits.

30. Think and then print: When you will go to print your paper, notice that tables are not be split, headings are not detached from their descriptions, and page sequence is maintained.

31. Adding unnecessary information: Do not add unnecessary information, like, I have used MS Excel to draw graph. Do not add irrelevant and inappropriate material. These all will create superfluous. Foreign terminology and phrases are not apropos. One should NEVER take a broad view. Analogy in script is like feathers on a snake. Not at all use a large word when a very small one would be sufficient. Use words properly, regardless of how others use them. Remove quotations. Puns are for kids, not grunt readers. Amplification is a billion times of inferior quality than sarcasm.

32. Never oversimplify everything: To add material in your research paper, never go for oversimplification. This will definitely irritate the evaluator. Be more or less specific. Also too, by no means, ever use rhythmic redundancies. Contractions aren't essential and shouldn't be there used. Comparisons are as terrible as clichés. Give up ampersands and abbreviations, and so on. Remove commas, that are, not necessary. Parenthetical words however should be together with this in commas. Understatement is all the time the complete best way to put onward earth-shaking thoughts. Give a detailed literary review.

33. Report concluded results: Use concluded results. From raw data, filter the results and then conclude your studies based on measurements and observations taken. Significant figures and appropriate number of decimal places should be used. Parenthetical remarks are prohibitive. Proofread carefully at final stage. In the end give outline to your arguments. Spot out perspectives of further study of this subject. Justify your conclusion by at the bottom of them with sufficient justifications and examples.

34. After conclusion: Once you have concluded your research, the next most important step is to present your findings. Presentation is extremely important as it is the definite medium though which your research is going to be in print to the rest of the crowd. Care should be taken to categorize your thoughts well and present them in a logical and neat manner. A good quality research paper format is essential because it serves to highlight your research paper and bring to light all necessary aspects in your research.

Informal Guidelines of Research Paper Writing

Key points to remember:

- Submit all work in its final form.
- Write your paper in the form, which is presented in the guidelines using the template.
- Please note the criterion for grading the final paper by peer-reviewers.

Final Points:

A purpose of organizing a research paper is to let people to interpret your effort selectively. The journal requires the following sections, submitted in the order listed, each section to start on a new page.

The introduction will be compiled from reference matter and will reflect the design processes or outline of basis that direct you to make study. As you will carry out the process of study, the method and process section will be constructed as like that. The result segment will show related statistics in nearly sequential order and will direct the reviewers next to the similar intellectual paths throughout the data that you took to carry out your study. The discussion section will provide understanding of the data and projections as to the implication of the results. The use of good quality references all through the paper will give the effort trustworthiness by representing an alertness of prior workings.

Writing a research paper is not an easy job no matter how trouble-free the actual research or concept. Practice, excellent preparation, and controlled record keeping are the only means to make straightforward the progression.

General style:

Specific editorial column necessities for compliance of a manuscript will always take over from directions in these general guidelines.

To make a paper clear

· Adhere to recommended page limits

Mistakes to evade

- Insertion a title at the foot of a page with the subsequent text on the next page
- Separating a table/chart or figure - impound each figure/table to a single page
- Submitting a manuscript with pages out of sequence

In every sections of your document

· Use standard writing style including articles ("a", "the," etc.)

· Keep on paying attention on the research topic of the paper

· Use paragraphs to split each significant point (excluding for the abstract)

· Align the primary line of each section

· Present your points in sound order

· Use present tense to report well accepted

· Use past tense to describe specific results

· Shun familiar wording, don't address the reviewer directly, and don't use slang, slang language, or superlatives

· Shun use of extra pictures - include only those figures essential to presenting results

Title Page:

Choose a revealing title. It should be short. It should not have non-standard acronyms or abbreviations. It should not exceed two printed lines. It should include the name(s) and address (es) of all authors.

Abstract:

The summary should be two hundred words or less. It should briefly and clearly explain the key findings reported in the manuscript--must have precise statistics. It should not have abnormal acronyms or abbreviations. It should be logical in itself. Shun citing references at this point.

An abstract is a brief distinct paragraph summary of finished work or work in development. In a minute or less a reviewer can be taught the foundation behind the study, common approach to the problem, relevant results, and significant conclusions or new questions.

Write your summary when your paper is completed because how can you write the summary of anything which is not yet written? Wealth of terminology is very essential in abstract. Yet, use comprehensive sentences and do not let go readability for briefness. You can maintain it succinct by phrasing sentences so that they provide more than lone rationale. The author can at this moment go straight to shortening the outcome. Sum up the study, with the subsequent elements in any summary. Try to maintain the initial two items to no more than one ruling each.

- Reason of the study - theory, overall issue, purpose
- Fundamental goal
- To the point depiction of the research
- Consequences, including definite statistics - if the consequences are quantitative in nature, account quantitative data; results of any numerical analysis should be reported
- Significant conclusions or questions that track from the research(es)

Approach:

- Single section, and succinct
- As a outline of job done, it is always written in past tense
- A conceptual should situate on its own, and not submit to any other part of the paper such as a form or table
- Center on shortening results - bound background information to a verdict or two, if completely necessary
- What you account in an conceptual must be regular with what you reported in the manuscript
- Exact spelling, clearness of sentences and phrases, and appropriate reporting of quantities (proper units, important statistics) are just as significant in an abstract as they are anywhere else

Introduction:

The **Introduction** should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable to comprehend and calculate the purpose of your study without having to submit to other works. The basis for the study should be offered. Give most important references but shun difficult to make a comprehensive appraisal of the topic. In the introduction, describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will have no attention in your result. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here. Following approach can create a valuable beginning:

- Explain the value (significance) of the study
- Shield the model - why did you employ this particular system or method? What is its compensation? You strength remark on its appropriateness from a abstract point of vision as well as point out sensible reasons for using it.
- Present a justification. Status your particular theory (es) or aim(s), and describe the logic that led you to choose them.
- Very for a short time explain the tentative propose and how it skilled the declared objectives.

Approach:

- Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done.
- Sort out your thoughts; manufacture one key point with every section. If you make the four points listed above, you will need a least of four paragraphs.

- Present surroundings information only as desirable in order hold up a situation. The reviewer does not desire to read the whole thing you know about a topic.
- Shape the theory/purpose specifically - do not take a broad view.
- As always, give awareness to spelling, simplicity and correctness of sentences and phrases.

Procedures (Methods and Materials):

This part is supposed to be the easiest to carve if you have good skills. A sound written Procedures segment allows a capable scientist to replacement your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt for the least amount of information that would permit another capable scientist to spare your outcome but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section. When a technique is used that has been well described in another object, mention the specific item describing a way but draw the basic principle while stating the situation. The purpose is to text all particular resources and broad procedures, so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step by step report of the whole thing you did, nor is a methods section a set of orders.

Materials:

- Explain materials individually only if the study is so complex that it saves liberty this way.
- Embrace particular materials, and any tools or provisions that are not frequently found in laboratories.
- Do not take in frequently found.
- If use of a definite type of tools.
- Materials may be reported in a part section or else they may be recognized along with your measures.

Methods:

- Report the method (not particulars of each process that engaged the same methodology)
- Describe the method entirely
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures
- Simplify - details how procedures were completed not how they were exclusively performed on a particular day.
- If well known procedures were used, account the procedure by name, possibly with reference, and that's all.

Approach:

- It is embarrassed or not possible to use vigorous voice when documenting methods with no using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result when script up the methods most authors use third person passive voice.
- Use standard style in this and in every other part of the paper - avoid familiar lists, and use full sentences.

What to keep away from

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings - save it for the argument.
- Leave out information that is immaterial to a third party.

Results:

The principle of a results segment is to present and demonstrate your conclusion. Create this part a entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Carry on to be to the point, by means of statistics and tables, if suitable, to present consequences most efficiently.You must obviously differentiate material that would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matter should not be submitted at all except requested by the instructor.

Content

- Sum up your conclusion in text and demonstrate them, if suitable, with figures and tables.
- In manuscript, explain each of your consequences, point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation an exacting study.
- Explain results of control experiments and comprise remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or in manuscript form.

What to stay away from

- Do not discuss or infer your outcome, report surroundings information, or try to explain anything.
- Not at all, take in raw data or intermediate calculations in a research manuscript.
- Do not present the similar data more than once.
- Manuscript should complement any figures or tables, not duplicate the identical information.
- Never confuse figures with tables - there is a difference.

Approach

- As forever, use past tense when you submit to your results, and put the whole thing in a reasonable order.
- Put figures and tables, appropriately numbered, in order at the end of the report
- If you desire, you may place your figures and tables properly within the text of your results part.

Figures and tables

- If you put figures and tables at the end of the details, make certain that they are visibly distinguished from any attach appendix materials, such as raw facts
- Despite of position, each figure must be numbered one after the other and complete with subtitle
- In spite of position, each table must be titled, numbered one after the other and complete with heading
- All figure and table must be adequately complete that it could situate on its own, divide from text

Discussion:

The Discussion is expected the trickiest segment to write and describe. A lot of papers submitted for journal are discarded based on problems with the Discussion. There is no head of state for how long a argument should be. Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implication of the study. The purpose here is to offer an understanding of your results and hold up for all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of result should be visibly described. Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved with prospect, and let it drop at that.

- Make a decision if each premise is supported, discarded, or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."
- Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work
- You may propose future guidelines, such as how the experiment might be personalized to accomplish a new idea.
- Give details all of your remarks as much as possible, focus on mechanisms.
- Make a decision if the tentative design sufficiently addressed the theory, and whether or not it was correctly restricted.
- Try to present substitute explanations if sensible alternatives be present.
- One research will not counter an overall question, so maintain the large picture in mind, where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

- When you refer to information, differentiate data generated by your own studies from available information
- Submit to work done by specific persons (including you) in past tense.
- Submit to generally acknowledged facts and main beliefs in present tense.

Please carefully note down following rules and regulation before submitting your Research Paper to Global Journals Inc. (US):

Segment Draft and Final Research Paper: You have to strictly follow the template of research paper. If it is not done your paper may get rejected.

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- Methods to avoid Plagiarism is applied by us on every paper, if found guilty, you will be blacklisted by all of our collaborated research groups, your institution will be informed for this and strict legal actions will be taken immediately.)
- To guard yourself and others from possible illegal use please do not permit anyone right to use to your paper and files.

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Topics	Grades		
	A-B	**C-D**	**E-F**
Abstract	Clear and concise with appropriate content, Correct format. 200 words or below	Unclear summary and no specific data, Incorrect form Above 200 words	No specific data with ambiguous information Above 250 words
Introduction	Containing all background details with clear goal and appropriate details, flow specification, no grammar and spelling mistake, well organized sentence and paragraph, reference cited	Unclear and confusing data, appropriate format, grammar and spelling errors with unorganized matter	Out of place depth and content, hazy format
Methods and Procedures	Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads	Difficult to comprehend with embarrassed text, too much explanation but completed	Incorrect and unorganized structure with hazy meaning
Result	Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake	Complete and embarrassed text, difficult to comprehend	Irregular format with wrong facts and figures
Discussion	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

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