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On a General Class of Multiple Eulerian Integrals with Multivariable A-Functions

By Frederic Ayant

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Ref

6. I.S. Gradsteyn and I.M. Ryxhik, Table of integrals, series and products: Academic press, New York 1980.

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I. Introduction and Prerequisites

The well-known Eulerian Beta integral [6]

$$\int_a^b (z-a)^{\alpha-1}(b-t)^{\beta-1}\mathrm{d}t = (b-a)^{\alpha+\beta-1}B(\alpha,\beta)(Re(\alpha)>0, Re(\beta)>0, b>a)$$

Is a basic result of evaluation of numerous other potentially useful integrals involving various special functions and polynomials. The mathematicians Raina and Srivastava [7], Saigo and Saxena [8], Srivastava and Hussain [11], Srivastava and Garg [10] et cetera have established some Eulerian integrals involving the various general class of polynomials, Meijer's G-function and Fox's H-function of one and more variables with general arguments. Recently, several Authors study some multiple Eulerian integrals, see Bhargava [2], Goyal and Mathur [5], Ayant [1] and others. The aim of this paper, we obtain general multiple Eulerian integrals of the product of two multivariable A-functions defined by Gautam et al [4], a class of multivariable polynomials [10] and the extension of the Hurwitz-Lerch Zeta function.

The last function noted $\phi(z,\mathfrak{s},a)$ is introduced by Srivastava et al ([15], eq.(6.2), page 503) as follows:

$$\phi^{(\rho_1,\cdots,\rho_{\mathbf{p}},\sigma_1,\cdots,\sigma_{\mathbf{q}})}_{(\lambda_1,\cdots,\lambda_{\mathbf{p}},\mu_1,\cdots,\mu_{\mathbf{q}})}(z;\mathfrak{s},a) = \sum_{R=0}^{\infty}\frac{\prod_{j=1}^{\mathbf{p}}(\lambda_j)_{R\rho_j}}{(a+R)^{\mathfrak{s}}\prod_{j=1}^{\mathbf{q}}(\mu_j)_{R\sigma_j}}\times\frac{z^R}{R!}$$

with : $\mathbf{p},\mathbf{q}\in\mathbb{N}_0, \lambda_j\in\mathbb{C}(j=1,\cdots,\mathbf{p}), a,\mu_j\in\mathbb{C}\setminus\mathbb{Z}_0^* \quad (j=1,\cdots,\mathbf{q}), \rho_j,\sigma_k\in\mathbb{R}^+$

$(j=1,\cdots,\mathbf{p}; k=1,\cdots,\mathbf{q})$

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where $\Delta > -1$ when $\mathfrak{s}, z \in \mathbb{C}$; $\Delta = -1$ and $\mathfrak{s} \in \mathbb{C}, when |z| < \bigtriangledown^*$, $\Delta = -1$ and $Re(\chi) > \frac{1}{2}$ when $|z| = \bigtriangledown^*$

$$\bigtriangledown^* = \prod_{j=1}^{\mathbf{p}} \rho_j^{\rho_j} \prod_{j=1}^{\mathbf{q}} \sigma_j^{\sigma_j} \, ; \Delta = \sum_{j=1}^{\mathbf{q}} \sigma_j - \sum_{j=1}^{\mathbf{p}} \rho_j ; \chi = \mathfrak{s} + \sum_{j=1}^{\mathbf{q}} \mu_j - \sum_{j=1}^{\mathbf{p}} \lambda_j + \frac{\mathbf{p}-\mathbf{q}}{2}$$

We shall call these conditions the conditions (f) and $\bar{A}_R = \dfrac{\prod_{j=1}^{\mathbf{p}} (\lambda_j)_{R\rho_j}}{(a+R)^{\mathfrak{s}} \prod_{j=1}^{\mathbf{q}} (\mu_j)_{R\sigma_j}}$

The multivariable A-function is a generalization of the multivariable H-function studied by Srivastava and Panda [13,14]. The A-function of r-variables is defined and represented in the following manner.

$$A(z_1, \cdots, z_r) = A_{p,q:p_1,q_1;\cdots;p_r,q_r}^{m,n:m_1,n_1;\cdots;m_r,n_r} \left(\begin{array}{c|c} \mathrm{z}_1 & (\mathrm{a}_j; A_j^{(1)}, \cdots, A_j^{(r)})_{1,p} : (\mathrm{c}_j^{(1)}, C_j^{(1)})_{1,p_1}; \cdots ; (c_j^{(r)}, C_j^{(r)})_{1,p_r} \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_r & (\mathrm{b}_j; B_j^{(1)}, \cdots, B_j^{(r)})_{1,q} : (\mathrm{d}_j^{(1)}, D_j^{(1)})_{1,q_1}; \cdots ; (d_j^{(r)}, D_j^{(r)})_{1,q_r} \end{array} \right)$$

$$= \frac{1}{(2\pi\omega)^r} \int_{L_1} \cdots \int_{L_r} \psi(s_1, \cdots, s_r) \prod_{k=1}^{r} \theta_k(s_k) z_k^{s_k} \, \mathrm{d}s_1 \cdots \mathrm{d}s_r \tag{1.3}$$

where $\phi(s_1, \cdots, s_r), \theta_i(s_i), i = 1, \cdots, r$ are given by:

$$\psi(s_1, \cdots, s_r) = \frac{\prod_{j=1}^{m} \Gamma(b_j - \sum_{i=1}^{r} B_j^{(i)} s_i) \prod_{j=1}^{n} \Gamma(1 - a_j + \sum_{i=1}^{r} A_j^{(i)} s_j)}{\prod_{j=n+1}^{p} \Gamma(a_j - \sum_{i=1}^{r} A_j^{(i)} s_j) \prod_{j=m+1}^{q} \Gamma(1 - b_j + \sum_{i=1}^{r} B_j^{(i)} s_j)}$$

$$\theta_i(s_i) = \frac{\prod_{j=1}^{n_i} \Gamma(1 - c_j^{(i)} + C_j^{(i)} s_i) \prod_{j=1}^{m_i} \Gamma(d_j^{(i)} - D_j^{(i)} s_i)}{\prod_{j=n_i+1}^{p_i} \Gamma(c_j^{(i)} - C_j^{(i)} s_i) \prod_{j=m_i+1}^{q_i} \Gamma(1 - d_j^{(i)} + D_j^{(i)} s_i)}$$

Here $m, n, p, m_i, n_i, p_i, c_i \in \mathbb{N}^*; i = 1, \cdots, r \, ; a_j, b_j, c_j^{(i)}, d_j^{(i)}, A_j^{(i)}, B_j^{(i)}, C_j^{(i)}, D_j^{(i)} \in \mathbb{C}$

The multiple integral defining the A-function of r variables converges absolutely if:

$$|arg(\Omega_i) z_k| < \frac{1}{2} \eta_k \pi, \xi^* = 0, \eta_i > 0 \tag{1.4}$$

$$\Omega_i = \prod_{j=1}^{p} \{A_j^{(i)}\}^{A_j^{(i)}} \prod_{j=1}^{q} \{B_j^{(i)}\}^{-B_j^{(i)}} \prod_{j=1}^{q_i} \{D_j^{(i)}\}^{D_j^{(i)}} \prod_{j=1}^{p_i} \{C_j^{(i)}\}^{-C_j^{(i)}} ; i = 1, \cdots, r \tag{1.5}$$

$$\xi_i^* = Im\Big(\sum_{j=1}^{p} A_j^{(i)} - \sum_{j=1}^{q} B_j^{(i)} + \sum_{j=1}^{q_i} D_j^{(i)} - \sum_{j=1}^{p_i} C_j^{(i)}\Big); i = 1, \cdots, r \tag{1.6}$$

$$\eta_i = Re \left(\sum_{j=1}^{n} A_j^{(i)} - \sum_{j=n+1}^{p} A_j^{(i)} + \sum_{j=1}^{m} B_j^{(i)} - \sum_{j=m+1}^{q} B_j^{(i)} + \sum_{j=1}^{m_i} D_j^{(i)} - \sum_{j=m_i+1}^{q_i} D_j^{(i)} + \sum_{j=1}^{n_i} C_j^{(i)} - \sum_{j=n_i+1}^{p_i} C_j^{(i)} \right) \tag{1.7}$$

for $i = 1, \cdots, r$

Ref

14. H.M. Srivastava and R.Panda, Some bilateral generating functions for a class of generalized hypergeometric polynomials, J. Reine Angew. Math. (1976), 265-274.

Notes

If all the poles of (1.6) are simple, then the integral (1.4) can be evaluated with the help of the residue theorem to give

$$A(z_1,\cdots,z_r)=\sum_{G_k=1}^{m_k}\sum_{g_k=0}^{\infty}\phi\frac{\prod_{k=1}^{r}\phi_k z_k^{\eta_{G_k,g_k}}(-)^{\sum_{k=1}^{r}g_k}}{\prod_{k=1}^{r}\delta_{G^{(k)}}^{(k)}\prod_{k=1}^{r}g_k!} \tag{1.8}$$

where ϕ and ϕ_i are defined by

$$\phi=\frac{\prod_{j=1}^{n}\Gamma^{A_j}\left(1-a_j+\sum_{i=1}^{r}\alpha_j^{(i)}S_k\right)}{\prod_{j=n+1}^{p}\Gamma^{A_j}\left(a_j-\sum_{i=1}^{r}\alpha_j^{(i)}S_k\right)\prod_{j=1}^{q}\Gamma^{B_j}\left(1-bj+\sum_{i=1}^{r}\beta_j^{(i)}S_k\right)}$$

and

$$\phi_i=\frac{\prod_{j=1}^{n_i}\Gamma^{C_j^{(i)}}\left(1-c_j^{(i)}+\gamma_j^{(i)}S_k\right)\prod_{j=1}^{m_i}\Gamma\left(d_j^{(i)}-\delta_j^{(i)}S_k\right)}{\prod_{j=n_i+1}^{p_i}\Gamma^{C_j^{(i)}}\left(c_j^{(i)}-\gamma_j^{(i)}S_k\right)\prod_{j=m_i+1}^{q_i}\Gamma^{D_j^{(i)}}\left(1-d_j^{(i)}+\delta_j^{(i)}S_k\right)},i=1,\cdots,r$$

where

$$S_k=\eta_{G_k,g_k}=\frac{d_{g_k}^{(k)}+G_k}{\delta_{g_k}^{(k)}}\text{ for }k=1,\cdots,r$$

which is valid under the following conditions: $\epsilon_{M_k}^{(k)}[p_j^{(k)}+p_k']\neq\epsilon_j^{(k)}[p_{M_k}+g_k]$

We shall note $A(z_1,\cdots,z_r)=A_1(z_1,\cdots,z_r)$ and

$$A(z_1',\cdots,z_s')=A_{p',q':p_1',q_1';\cdots;p_s',q_s'}^{m',n':m_1',n_1';\cdots;m_s',n_s'}\left(\begin{array}{c|c} z_1' & (\mathrm{a}'_j;A_j'^{(1)},\cdots,A_j'^{(s)})_{1,p'}:(\mathrm{c}'^{(1)}_j,C_j'^{(1)})_{1,p_1'};\cdots;(c_j'^{(s)},C_j'^{(s)})_{1,p_s'} \\ \cdot & \\ \cdot & \\ \cdot & \\ z_s' & (\mathrm{b}'_j;B_j'^{(1)},\cdots,B_j'^{(s)})_{1,q'}:(\mathrm{d}'^{(1)}_j,D_j'^{(1)})_{1,q_1'};\cdots;(d_j'^{(s)},D_j'^{(s)})_{1,q_s'} \end{array}\right)$$

$$=\frac{1}{(2\pi\omega)^s}\int_{L_1'}\cdots\int_{L_s'}\zeta(t_1,\cdots,t_s)\prod_{k=1}^{s}\phi_k(t_k)z_k'^{t_k}\mathrm{d}t_1\cdots\mathrm{d}t_s \tag{1.9}$$

where $\zeta(t_1,\cdots,t_s),\phi_i(t_i),i=1,\cdots,s$ are given by :

$$\phi'(t_1,\cdots,t_s)=\frac{\prod_{j=1}^{m'}\Gamma(b_j'-\sum_{i=1}^{s}B_j'^{(i)}t_i)\prod_{j=1}^{n'}\Gamma(1-a_j'+\sum_{i=1}^{s}A_j'^{(i)}t_j)}{\prod_{j=n'+1}^{p'}\Gamma(a_j'-\sum_{i=1}^{s}A_j'^{(i)}t_j)\prod_{j=m'+1}^{q'}\Gamma\left(1-b_j'+\sum_{i=1}^{s}B_j'^{(i)}t_j\right)}$$

$$\theta_i'(t_i)=\frac{\prod_{j=1}^{n_i'}\Gamma(1-c_j'^{(i)}+C_j'^{(i)}t_i)\prod_{j=1}^{m_i'}\Gamma(d_j'^{(i)}-D_j'^{(i)}t_i)}{\prod_{j=n_i'+1}^{p_i'}\Gamma(c_j'^{(i)}-C_j'^{(i)}t_i)\prod_{j=m_i'+1}^{q_i'}\Gamma(1-d_j'^{(i)}+D_j'^{(i)}t_i)}$$

Here $m',n',p',m_i',n_i',p_i',c_i'\in\mathbb{N}^*;i=1,\cdots,r;a_j',b_j',c_j'^{(i)},d_j'^{(i)},A_j'^{(i)},B_j'^{(i)},C_j'^{(i)},D_j'^{(i)}\in\mathbb{C}$

The multiple integral defining the A-function of r variables converges absolutely if:

$$|arg(\Omega_i')z_k'|<\frac{1}{2}\eta_k'\pi,\xi'^*=0,\eta_i'>0 \tag{1.10}$$

$$\Omega_i' = \prod_{j=1}^{p'}\{A_j'^{(i)}\}^{A_j'^{(i)}}\prod_{j=1}^{q'}\{B_j'^{(i)}\}^{-B_j'^{(i)}}\prod_{j=1}^{q_i'}\{D_j'^{(i)}\}^{D_j'^{(i)}}\prod_{j=1}^{p_i'}\{C_j'^{(i)}\}^{-C_j'^{(i)}}; i=1,\cdots,s \tag{1.11}$$

$$\xi_i'^{*} = Im(\sum_{j=1}^{p'}A_j'^{(i)} - \sum_{j=1}^{q'}B_j'^{(i)} + \sum_{j=1}^{q_i'}D_j'^{(i)} - \sum_{j=1}^{p_i'}C_j'^{(i)}); i=1,\cdots,s \tag{1.12}$$

$$\eta_i' = Re\left(\sum_{j=1}^{n'}A_j'^{(i)} - \sum_{j=n'+1}^{p'}A_j'^{(i)} + \sum_{j=1}^{m'}B_j'^{(i)} - \sum_{j=m'+1}^{q'}B_j'^{(i)} + \sum_{j=1}^{m_i'}D_j'^{(i)} - \sum_{j=m_i'+1}^{q_i'}D_j'^{(i)} + \sum_{j=1}^{n_i'}C_j'^{(i)} - \sum_{j=n_i'+1}^{p_i'}C_j'^{(i)}\right)$$

$$i=1,\cdots,s \tag{1.13}$$

Srivastava and Garg [10] introduced a class of multivariable polynomials as follows

$$S_L^{h_1,\cdots,h_u}[z_1,\cdots,z_u] = \sum_{R_1,\cdots,R_u=0}^{h_1R_1+\cdots h_uR_u\leqslant L}(-L)_{h_1R_1+\cdots+h_uR_u}B(L;R_1,\cdots,R_u)\frac{z_1^{R_1}\cdots z_u^{R_u}}{R_1!\cdots R_u!} \tag{1.14}$$

The coefficients $B(L;R_1,\cdots,R_u)$ are arbitrary real or complex constants.

We shall note

$$B_u = \frac{(-L)_{h_1R_1+\cdots+h_uR_u}B(L;R_1,\cdots,R_u)}{R_1!\cdots R_u!}$$

II. Integral Representation of Generalized Hypergeometric Function

The following generalized hypergeometric function regarding multiple integrals contour is also required [12, page 39 eq.30]

$$\frac{\prod_{j=1}^{P}\Gamma(A_j)}{\prod_{j=1}^{Q}\Gamma(B_j)}\,{}_PF_Q\left[(A_P);(B_Q);-(x_1+\cdots+x_r)\right]$$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\frac{\prod_{j=1}^{P}\Gamma(A_j+s_1+\cdots+s_r)}{\prod_{j=1}^{Q}\Gamma(B_j+s_1+\cdots+s_r)}\Gamma(-s_1)\cdots\Gamma(-s_r)x_1^{s_1}\cdots x_r^{s_r}\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{2.1}$$

where the contours are of Barnes type with indentations, if necessary, to ensure that the poles $\Gamma(A_j+s_1+\cdots+s_r)$ of are separated from those of $\Gamma(-s_j), j=1,\cdots,r$. The above result (2.1) is easily established by an appeal to the calculus of residues by calculating the residues at the poles of $\Gamma(-s_j), j=1,\cdots,r$
The equivalent form of Eulerian beta integral is given by (1.1):

III. Main Integral

We shall note:

$X = m_1', n_1'; \cdots; m_s', n_s'; 1,0; \cdots; 1,0; 1,0; \cdots; 1,0$

$Y = p_1', q_1'; \cdots; p_s', q_s'; 0,1; \cdots; 0,1; 0,1; \cdots; 0,1$

$$\mathbb{A} = \left[1+\sigma_i^{(1)} - \sum_{k'=1}^{u}R_{k'}\rho_i''^{(1,k')} - \sum_{k=1}^{r}\eta_{G_k,g_k}\rho_i^{(1,k)} - \theta_i^{(1)}R;\rho_i'^{(1,1)},\cdots,\rho_i'^{(1,s)},\tau_i^{(1,1)},\cdots,\tau_i^{(1,l)},1,0,\cdots,0\right]_{1,s},\cdots,$$

Ref

10. H.M. Srivastava and M. Garg, Some integrals involving general class of polynomials and the multivariable H function. Rev. Roumaine. Phys. 32 (1987) 685-692.

Notes

$$[1+\sigma_i^{(T)}-\sum_{k'=1}^{u}R_{k'}\rho_i''^{(T,k')}-\sum_{k=1}^{r}\eta_{G_k,g_k}\rho_i^{(T,k)}-\theta_i^{(T)}R;\rho_i'^{(T,1)},\cdots,\rho_i'^{(T,s)},\tau_i^{(T,1)},\cdots,\tau_i^{(T,l)},1,0,\cdots,0]_{1,s},$$

$$[1-A_j;0,\cdots,0,1,\cdots,1,0,\cdots,0]_{1,P},$$

$$[1-\beta_i-\sum_{k'=1}^{u}R_{k'}\eta_i''^{(k')}-\sum_{k=1}^{r}\eta_{G_k,g_k}\eta_i^{(k)}-R\lambda_i;\eta_i'^{(1)},\cdots,\eta_i'^{(s)},\theta_i^{(1)},\cdots,\theta_i^{(l)},1,\cdots,1,0,\cdots,0]_{1,s}$$

$$\mathbf{A}=(a_j';A_j'^{(1)},\cdots,A_j'^{(s)},0,\cdots,0,0,\cdots,0)_{1,p'}:(c_j'^{(1)},C_j'^{(1)})_{1,p_1'};\cdots;(c_j'^{(r)},C_j'^{(s)})_{1,p_s'}$$

$$(1,0),\cdots,(1,0);(1,0),\cdots,(1,0)$$

$$\mathbb{B}=[1+\sigma_i^{(1)}-\sum_{k'=1}^{u}R_{k'}\rho_i''^{(1,k')}-\sum_{k=1}^{r}\eta_{G_k,g_k}\rho_i^{(1,k)}-\theta_i^{(1)}R;\rho_i'^{(1,1)},\cdots,\rho_i'^{(1,s)},\tau_i^{(1,1)},\cdots,\tau_i^{(1,l)},0,\cdots,0]_{1,s},\cdots,$$

$$[1+\sigma_i^{(T)}-\sum_{k'=1}^{u}R_{k'}\rho_i''^{(T,k')}-\sum_{k=1}^{r}\eta_{G_k,g_k}\rho_i^{(T,k)}-\theta_i^{(T)}R;\rho_i'^{(T,1)},\cdots,\rho_i'^{(T,s)},\tau_i^{(T,1)},\cdots,\tau_i^{(T,l)},0,\cdots,0]_{1,s},$$

$$[1-B_j;0,\cdots,0,1,\cdots,1,0,\cdots,0]_{1,Q},$$

$$[1-\alpha_i-\beta_i-\sum_{k'=1}^{u}(\delta_i''^{(k')}+\eta_i''^{(k')})R_{k'}-\sum_{k=1}^{r}(\delta_i^{(k)}+\eta_i^{(k)})\eta_{G_k,g_k}-(\zeta_i+\lambda_i)R;$$

$$(\delta_i'^{(1)}+\eta_i'^{(1)}),\cdots,(\delta_i'^{(s)}+\eta_i'^{(s)}),(\mu_i^{(1)}+\theta_i^{(1)}),\cdots,(\mu_i^{(l)}+\theta_i^{(l)}),1,\cdots,1]_{1,s}$$

$$\mathbf{B}=(b_j';B_j'^{(1)},\cdots,B_j'^{(s)},0,\cdots,0,0,\cdots,0)_{1,q'}:\ (\mathrm{d}_j'^{(1)},D_j'^{(1)})_{1,q_1'};\cdots;(d_j'^{(s)},D_j'^{(s)})_{1,q_s'};$$

$$(0,1),\cdots,(0,1);(0,1),\cdots,(0,1)$$

We have the following multiple Eulerian integrals, we obtain the A-function of $(r+l+T)-$variables.

Theorem

$$\int_{u_1}^{v_1}\cdots\int_{u_t}^{v_t}\prod_{i=1}^{t}\left[(x_i-u_i)^{\alpha_i-1}(v_i-x_i)^{\beta_i-1}\prod_{j=1}^{T}(U_i^{(j)}x_i+V_i^{(j)})^{\sigma_i^{(j)}}\right]$$

$$A_1\left(\begin{array}{c}\mathrm{z}_1\prod_{i=1}^{t}\left[\frac{(x_i-u_i)^{\delta_i^{(1)}}(v_i-x_i)^{\eta_i^{(1)}}}{\prod_{j=1}^{T}\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{\rho_i^{(j,1)}}}\right]\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_r\prod_{i=1}^{t}\left[\frac{(x_i-u_i)^{\delta_i^{(r)}}(v_i-x_i)^{\eta_i^{(r)}}}{\prod_{j=1}^{T}\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{\rho_i^{(j,r)}}}\right]\end{array}\right)A_2\left(\begin{array}{c}\mathrm{z}_1'\prod_{i=1}^{t}\left[\frac{(x_i-u_i)^{\delta_i'^{(1)}}(v_i-x_i)^{\eta_i'^{(1)}}}{\prod_{j=1}^{T}\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{\rho_i'^{(j,1)}}}\right]\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_s'\prod_{i=1}^{t}\left[\frac{(x_i-u_i)^{\delta_i'^{(s)}}(v_i-x_i)^{\eta_i'^{(s)}}}{\prod_{j=1}^{T}\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{\rho_i'^{(j,s)}}}\right]\end{array}\right)$$

$$S_L^{h_1,\cdots,h_u}\left(\begin{array}{c} z''_1 \prod_{i=1}^{t}\left[\frac{(x_i-u_i)^{\delta_i''^{(1)}}(v_i-x_i)^{\eta_i''^{(1)}}}{\prod_{j=1}^{T}\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{\rho_i''^{(j,1)}}}\right] \\ \cdot \\ \cdot \\ \cdot \\ z''_u \prod_{i=1}^{t}\left[\frac{(x_i-u_i)^{\delta_i''^{(u)}}(v_i-x_i)^{\eta_i''^{(u)}}}{\prod_{j=1}^{T}\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{\rho_i''^{(j,u)}}}\right] \end{array}\right)$$

$$\phi_{(\lambda_1,\cdots,\lambda_{\mathbf{p}},\mu_1,\cdots,\mu_{\mathbf{q}})}^{(\rho_1,\cdots,\rho_{\mathbf{p}},\sigma_1,\cdots,\sigma_{\mathbf{q}})}\left[\prod_{j=1}^{t}\left[\frac{(x_i-u_i)^{\zeta_i}(v_i-x_i)^{\lambda_i}}{\prod_{j=1}^{T}\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{\theta_i^{(j)}}}\right];\mathfrak{s},a\right]$$

$${}_PF_Q\left[(A_P);(B_Q);-\sum_{k=1}^{l}g_k\prod_{i=1}^{t}\left[\frac{(x_i-u_i)^{u_i^{(k)}}(v_i-x_i)^{\theta_i^{(r)}}}{\prod_{j=1}^{T}\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{\tau_i^{(j,k)}}}\right]\right]\mathrm{d}x_1\cdots\mathrm{d}x_t$$

$$=\frac{\prod_{j=1}^{Q}\Gamma(B_j)}{\prod_{j=1}^{P}\Gamma(A_j)}\prod_{j=1}^{t}\left[(v_i-u_i)^{\alpha_i+\beta_i-1}\prod_{j=1}^{W}\left(u_iU_i^{(j)}+V_i^{(j)}\right)^{\sigma_i^{(j)}}\prod_{j=W+1}^{T}\left(u_iU_i^{(j)}+V_i^{(j)}\right)^{\sigma_i^{(j)}}\right]$$

$$\sum_{R=0}^{\infty}\sum_{R_1,\cdots,R_u=0}^{h_1R_1+\cdots h_uR_u\leqslant L}\sum_{G_k=1}^{m_k}\sum_{g_k=0}^{\infty}\phi\frac{\prod_{k=1}^{r}\phi_k z_k^{\eta_{G_k,g_k}}(-)^{\sum_{k=1}^{r}g_k}}{\prod_{k=1}^{r}\delta_{G^{(k)}}^{(k)}\prod_{k=1}^{r}g_k!}\bar{A}_R b_u z_1''^{R_1}\cdots z_u''^{R_u}E_{ij}$$

$$A_{sT+P+2s+p',sT+Q+s+q':Y}^{m',sT+P+2s+n':X}\left(\begin{array}{c|c} \mathrm{z}'_1 w_1 & \mathbb{A},\ \mathbf{A} \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{z}'_1 w_s & \cdot \\ \mathrm{g}_1 W_1 & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{g}_l W_l & \cdot \\ \mathrm{G}_1 & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{G}_T & \mathbb{B},\ \mathbf{B} \end{array}\right) \qquad (3.1)$$

where

$$E_{ij}=\frac{1}{\prod_{i=1}^{t}\prod_{j=1}^{W}(u_iU_i^{(j)}+V_i^{(j)})^{\sum_{k'=1}^{u}\rho_i''^{(j,k')}R_{k'}+\sum_{k=1}^{r}\rho_i^{(j,k)}\eta_{G_k,g_k}+\theta_i^{(j)}R}}$$

$$\times\frac{\prod_{i=1}^{t}(v_i-u_i)^{\sum_{k'=1}^{u}(\delta_i''^{(k')}+\eta_i''^{(k')})R_{k'}+\sum_{k=1}^{r}(\delta_i^{(k)}+\eta_i^{(k)})\eta_{G_k,g_k}+(\zeta_i+\lambda_i)R}}{\prod_{i=1}^{t}\prod_{j=W+1}^{T}(u_iU_i^{(j)}+V_i^{(j)})^{\sum_{k'=1}^{u}\rho_i''^{(j,k')}R_{k'}+\sum_{k=1}^{r}\rho_i^{(j,k)}\eta_{G_k,g_k}+\theta_i^{(j)}R}}$$

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$$w_m = \prod_{i=1}^{t}\left[(v_i-u_i)^{\delta_i'^{(m)}+\eta_i'^{(m)}}\prod_{j=1}^{W}\left(u_iU_i^{(j)}+V_i^{(j)}\right)^{-\rho_i'^{(j,m)}}\prod_{j=W+1}^{T}\left(u_iU_i^{(j)}+V_i^{(j)}\right)^{\rho_i'^{(j,m)}}\right], m=1,\cdots,s$$

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$$W_k = \prod_{i=1}^{t}\left[(v_i-u_i)^{\mu_i^{(k)}+\theta_i^{(k)}}\prod_{j=1}^{W}\left(u_iU_i^{(j)}+V_i^{(j)}\right)^{-\tau_i^{(j,k)}}\prod_{j=W+1}^{T}\left(u_iU_i^{(j)}+V_i^{(j)}\right)^{-\tau_i^{(j,k)}}\right], k=1,\cdots,l$$

$$G_j = \prod_{i=1}^{t}\left[\frac{(v_i-u_i)U_i^{(j)}}{u_iU_i^{(j)}+V_i^{(j)}}\right], j=1,\cdots,W$$

$$G_j = -\prod_{i=1}^{t}\left[\frac{(v_i-u_i)U_i^{(j)}}{u_iU_i^{(j)}+V_i^{(j)}}\right], j=W+1,\cdots,T$$

$$\sum_{G_k=1}^{m_k}\sum_{g_k=0}^{\infty}=\sum_{G_1,\cdots,G_r=1}^{m_1,\cdots,m_r}\sum_{g_1,\cdots,g_r=0}^{\infty}$$

Provided that:

(A) $W\in[0,T]; u_i,v_i\in\mathbb{R}; i=1,\cdots,t$

(B)$min\{\delta_i^{(g)},\eta_i^{(g)},\delta_i'^{(h)},\eta_i'^{(h)},\delta_i''^{(k)},\eta_i''^{(k)},\zeta_i,\eta_i\}\geqslant 0; g=1,\cdots,r; i=1,\cdots,t; h=1,\cdots,s; k=1,\cdots,u$

$min\{\rho_i^{(j,g)},\rho_i'^{(j,h)},\rho_i''^{(j,k')},\theta_i^{(j)},\tau_i^{(j,k)}\}\geqslant 0; j=1,\cdots,T; i=1,\cdots,t; g=1,\cdots,r; h=1,\cdots,s; k'=1,\cdots,u; k=1,\cdots,l$

(C) $\sigma_i^{(j)}\in\mathbb{R}, U_i^{(j)}, V_i^{(j)}\in\mathbb{C}, z_{i'}, z_{j'}, z''_{k'}, g_k, G_j\in\mathbb{C}; i=1,\cdots,t; j=1,\cdots,T; i'=1,\cdots,r;$

$j'=1,\cdots,s; k'=1,\cdots,u; k=1,\cdots,l$

(D)$max\left[\left|\frac{(v_i-u_i)U_i^{(j)}}{u_iU_i^{(j)}+V_i^{(j)}}\right|\right]<1, i=1,\cdots,s; j=1,\cdots,W$ and

$$max\left[\left|\frac{(v_i-u_i)U_i^{(j)}}{u_iU_i^{(j)}+V_i^{(j)}}\right|\right]<1, i=1,\cdots,s; j=W+1,\cdots,T$$

(E) $\left|arg\left(z_i\prod_{j=1}^{T}(U_i^{(j)}x_i+V_i^{(j)})^{\rho_i^{(j,k)}}\right)\right|<\frac{1}{2}\eta_k\pi, k=1,\cdots,r$, where

$$\eta_i = Re\left(\sum_{j=1}^{n}A_j^{(i)}-\sum_{j=n+1}^{p}A_j^{(i)}+\sum_{j=1}^{m}B_j^{(i)}-\sum_{j=m+1}^{q}B_j^{(i)}+\sum_{j=1}^{m_i}D_j^{(i)}-\sum_{j=m_i+1}^{q_i}D_j^{(i)}+\sum_{j=1}^{n_i}C_j^{(i)}-\sum_{j=n_i+1}^{p_i}C_j^{(i)}\right)$$

$$-\delta_i^{(k)} - \eta_i^{(k)} - \sum_{j=1}^{T} \rho_i^{(j,k)} > 0$$

$$\left| arg \left(z'_i \prod_{j=1}^{T} (U_i^{(j)} x_i + V_i^{(j)})^{\rho_i'^{(j,k)}} \right) \right| < \frac{1}{2} \eta'_k \pi, k = 1, \cdots, s \text{ , where}$$

$$\eta'_i = Re \left(\sum_{j=1}^{n'} A_j'^{(i)} - \sum_{j=n'+1}^{p'} A_j'^{(i)} + \sum_{j=1}^{m'} B_j'^{(i)} - \sum_{j=m'+1}^{q'} B_j'^{(i)} + \sum_{j=1}^{m'_i} D_j'^{(i)} - \sum_{j=m'_i+1}^{q'_i} D_j'^{(i)} + \sum_{j=1}^{n'_i} C_j'^{(i)} - \sum_{j=n'_i+1}^{p'_i} C_j'^{(i)} \right)$$

$$-\delta_i'^{(k)} - \eta_i'^{(k)} - \sum_{j=1}^{T} \rho_i'^{(j,k)} > 0$$

(F) $Re \left(\alpha_i + \zeta_i R + \sum_{j=1}^{r} \delta_i^{(j)} \eta_{G_j,g_j} \right) + \sum_{k=1}^{s} \delta_i'^{(k)} \min_{1 \leqslant j \leqslant m'_k} Re \left(\frac{d_j'^{(k)}}{D_j'^{(k)}} \right) > 0$ and

$$Re \left(\beta_i + \lambda_i R + \sum_{j=1}^{r} \eta_i^{(j)} \eta_{G_j,g_j} \right) + \sum_{k=1}^{s} \eta_i'^{(k)} \min_{1 \leqslant j \leqslant m'_k} Re \left(\frac{d_j'^{(k)}}{D_j'^{(k)}} \right) > 0 \text{ for } i = 1, \cdots, t$$

(G). $P \leqslant Q + 1$ The equality holds, when, also

either $P > Q$ and $\sum_{k=1}^{l} \left| g_k \left(\prod_{j=1}^{T} (U_i^{(j)} x_i + V_i^{(j)})^{\tau_i^{(j,k)}} \right) \right|^{\frac{1}{Q-P}} < 1 \qquad (u_i \leqslant x_i \leqslant v_i; i = 1, \cdots, t)$

or $P \leqslant Q$ and $\max_{1 \leqslant k \leqslant l} \left[\left| \left(g_k \prod_{j=1}^{T} (U_i^{(j)} x_i + V_i^{(j)})^{-\tau_i^{(j,k)}} \right) \right| \right] < 1 \qquad (u_i \leqslant x_i \leqslant v_i; i = 1, \cdots, t)$

(H) the conditions (f) are satisfied

Proof

To establish the formula (3.1), we first express the extension of the Hurwitz-Lerch Zeta function, the class of multivariable polynomials $S_L^{h_1, \cdots, h_u}[.]$ and the multivariable A-function $A_1(z_1, \cdots, z_r)$ in series with the help of (1,2), (1,14) and (1.18) respectively, use integral contour representation with the help of (1.9) for the multivariable A-function $A_2(z'_1, \cdots, z'_s)$ occurring on its left-hand side and use the integral contour representation with the help of (2.1) for the Generalized hypergeometric function ${}_PF_Q(.)$. Changing the order of integration and summation (which is easily seen to be justified due to the absolute convergence of the integral and the summations involved in the process). Now we write:

$$\prod_{j=1}^{T} (U_i^{(j)} x_i + V_i^{(j)})^{K_i^{(j)}} = \prod_{j=1}^{W} (U_i^{(j)} x_i + V_i^{(j)})^{K_i^{(j)}} \prod_{j=W+1}^{T} (U_i^{(j)} x_i + V_i^{(j)})^{K_i^{(j)}} \tag{3.2}$$

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where $K_i^{(j)}=\upsilon_i^{(j)}-\theta_i^{(j)}R-\sum_{l=1}^{r}\rho_i^{(j,l)}\eta_{G_l,g_l}-\sum_{l=1}^{s}\rho_i'^{(j,l)}\psi_l-\sum_{l=1}^{v}\rho_i''^{(j,v)}K_l$ where $i=1,\cdots,t;j=1,\cdots,T$

and express the factors occurring in R.H.S. Of (3.1) in terms of following Mellin-Barnes integral contour, we obtain:

$$\prod_{j=1}^{W}(U_i^{(j)}x_i+V_i^{(j)})^{K_i^{(j)}}=\prod_{j=1}^{W}\left[\frac{(U_i^{(j)}u_i+V_i^{(j)})^{K_i^{(j)}}}{\Gamma(-K_i^{(j)})}\right]\frac{1}{(2\pi\omega)^W}\int_{L_1'}\cdots\int_{L_W'}\prod_{j=1}^{W}\left[\Gamma(-\zeta_j')\Gamma(-K_i^{(j)}+\zeta_j')\right.$$

$$\left.\prod_{j=1}^{W}\left[\frac{(U_i^{(j)}(x_i-u_i)}{(u_iU_i^{(j)}+V_i^{(j)})}\right]^{\zeta_j'}\right]\mathrm{d}\zeta_1'\cdots\mathrm{d}\zeta_W' \tag{3.3}$$

and

$$\prod_{j=W+1}^{T}(U_i^{(j)}x_i+V_i^{(j)})^{K_i^{(j)}}=\prod_{j=W+1}^{T}\left[\frac{(U_i^{(j)}v_i+V_i^{(j)})^{K_i^{(j)}}}{\Gamma(-K_i^{(j)})}\right]\frac{1}{(2\pi\omega)^{T-W}}\int_{L_{W+1}'}\cdots\int_{L_T'}\prod_{j=W+1}^{T}\left[\Gamma(-\zeta_j')\Gamma(-K_i^{(j)}+\zeta_j')\right.$$

$$\left.\prod_{j=W+1}^{T}\left[-\frac{(U_i^{(j)}(v_i-x_i)}{(v_iU_i^{(j)}+V_i^{(j)})}\right]^{\zeta_j'}\right]\mathrm{d}\zeta_{W+1}'\cdots\mathrm{d}\zeta_T' \tag{3.16}$$

We apply the Fubini's theorem for multiple integral. Finally evaluating the innermost **x**-integral with the help of (1.1) and reinterpreting the multiple Mellin-Barnes integrals contour regarding the multivariable A-function of $(r+l+T)-$ variables, we obtain the formula (3.7).

IV. Particular Cases

a) Srivastava-Daoust polynomial [9]

$$If \quad B(L;R_1,\cdots,R_u)=\frac{\prod_{j=1}^{\bar{A}}(a_j)_{R_1\theta_j'+\cdots+R_u\theta_j^{(u)}}\prod_{j=1}^{B'}(b_j')_{R_1\phi_j'}\cdots\prod_{j=1}^{B^{(u)}}(b_j^{(u)})_{R_u\phi_j^{(u)}}}{\prod_{j=1}^{\bar{C}}(c_j)_{R_1\psi_j'+\cdots+R_u\psi_j^{(u)}}\prod_{j=1}^{D'}(d_j')_{R_1\delta_j'}\cdots\prod_{j=1}^{D^{(u)}}(d_j^{(u)})_{R_u\delta_j^{(u)}}}$$

we have

Corollary 1

$$\int_{u_1}^{v_1}\cdots\int_{u_t}^{v_t}\prod_{i=1}^{t}\left[(x_i-u_i)^{\alpha_i-1}(v_i-x_i)^{\beta_i-1}\prod_{j=1}^{T}(U_i^{(j)}x_i+V_i^{(j)})^{\sigma_i^{(j)}}\right]$$

$$F_{\bar{C}:D';\cdots;D^{(u)}}^{1+\bar{A}:B';\cdots;B^{(u)}}\left(\begin{array}{c} z_1''\prod_{i=1}^{t}\left[\frac{(x_i-u_i)^{\delta_i''^{(1)}}(v_i-x_i)^{\eta_i''^{(1)}}}{\prod_{j=1}^{T}\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{\rho_i''^{(j,1)}}}\right]\\ \vdots \\ z_u''\prod_{i=1}^{t}\left[\frac{(x_i-u_i)^{\delta_i''^{(u)}}(v_i-x_i)^{\eta_i''^{(u)}}}{\prod_{j=1}^{T}\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{\rho_i''^{(j,u)}}}\right]\end{array}\right)$$

Ref

9. H.M. Srivastava H.M. and M.C. Daoust, Certain generalized Neumann expansions associated with Kampé de Fériet function. Nederl. Akad. Wetensch. Proc. Ser A72 = Indag Math 31(1969), 449-457.

$$A_1\begin{pmatrix} \mathrm{z}_1\prod_{i=1}^t\left[\frac{(x_i-u_i)^{\delta_i^{(1)}}(v_i-x_i)^{\eta_i^{(1)}}}{\prod_{j=1}^T\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{\rho_i^{(j,1)}}}\right]\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_r\prod_{i=1}^t\left[\frac{(x_i-u_i)^{\delta_i^{(r)}}(v_i-x_i)^{\eta_i^{(r)}}}{\prod_{j=1}^T\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{\rho_i^{(j,r)}}}\right]\end{pmatrix} A_2\begin{pmatrix} \mathrm{z}_1'\prod_{i=1}^t\left[\frac{(x_i-u_i)^{\delta_i'^{(1)}}(v_i-x_i)^{\eta_i'^{(1)}}}{\prod_{j=1}^T\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{\rho_i'^{(j,1)}}}\right]\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_s'\prod_{i=1}^t\left[\frac{(x_i-u_i)^{\delta_i'^{(s)}}(v_i-x_i)^{\eta_i'^{(s)}}}{\prod_{j=1}^T\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{\rho_i'^{(j,s)}}}\right]\end{pmatrix}$$

$$\phi^{(\rho_1,\cdots,\rho_{\mathbf{p}},\sigma_1,\cdots,\sigma_{\mathbf{q}})}_{(\lambda_1,\cdots,\lambda_{\mathbf{p}},\mu_1,\cdots,\mu_{\mathbf{q}})}\left[\prod_{j=1}^t\left[\frac{(x_i-u_i)^{\zeta_i}(v_i-x_i)^{\lambda_i}}{\prod_{j=1}^T\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{\theta_i^{(j)}}}\right];\mathfrak{s},a\right]$$

$${}_PF_Q\left[(A_P);(B_Q);-\sum_{k=1}^l g_k\prod_{i=1}^t\left[\frac{(x_i-u_i)^{u_i^{(k)}}(v_i-x_i)^{\theta_i^{(r)}}}{\prod_{j=1}^T\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{\tau_i^{(j,k)}}}\right]\right]\mathrm{d}x_1\cdots\mathrm{d}x_t$$

$$=\frac{\prod_{j=1}^Q\Gamma(B_j)}{\prod_{j=1}^P\Gamma(A_j)}\prod_{j=1}^t\left[(v_i-u_i)^{\alpha_i+\beta_i-1}\prod_{j=1}^W\left(u_iU_i^{(j)}+V_i^{(j)}\right)^{\sigma_i^{(j)}}\prod_{j=W+1}^T\left(u_iU_i^{(j)}+V_i^{(j)}\right)^{\sigma_i^{(j)}}\right]$$

$$\sum_{R=0}^{\infty}\sum_{R_1,\cdots,R_u=0}^{h_1R_1+\cdots h_uR_u\leqslant L}\sum_{G_k=1}^{m_k}\sum_{g_k=0}^{\infty}\phi\frac{\prod_{k=1}^r\phi_k z_k^{\eta_{G_k,g_k}}(-)^{\sum_{k=1}^r g_k}}{\prod_{k=1}^r\delta_{G^{(k)}}^{(k)}\prod_{k=1}^r g_k!}\bar{A}_R b_u' z_1''^{R_1}\cdots z_u''^{R_u}E_{ij}$$

$$A^{m',sT+P+2s+n':X}_{sT+P+2s+p',sT+Q+s+q':Y}\left(\begin{array}{c|c} \mathrm{z}_1'w_1 & \mathbb{A},\ \mathbf{A}\\ \cdot & \cdot\\ \cdot & \cdot\\ \mathrm{z}_1'w_s & \cdot\\ \mathrm{g}_1W_1 & \cdot\\ \cdot & \cdot\\ \cdot & \cdot\\ \mathrm{g}_lW_l & \cdot\\ \mathrm{G}_1 & \cdot\\ \cdot & \cdot\\ \cdot & \cdot\\ \mathrm{G}_T & \mathbb{B},\ \mathbf{B}\end{array}\right) \tag{4.1}$$

where $b_u'=\dfrac{(-L)_{h_1R_1+\cdots+h_uR_u}B(L;R_1,\cdots,R_u)}{R_1!\cdots R_u!}$, $B[L;R_1,\ldots,R_u]$ *is defined by (4.1)*

Notes

The validity conditions are the same that (3.1).

b) A-function of one variable

If $r=s=1$, the multivariable A-functions reduce to A-functions of one variable defined by Gautam et Asgar [3]. We have.

$$\int_{u_1}^{v_1}\cdots\int_{u_t}^{v_t}\prod_{i=1}^{t}\left[(x_i-u_i)^{\alpha_i-1}(v_i-x_i)^{\beta_i-1}\prod_{j=1}^{T}(U_i^{(j)}x_i+V_i^{(j)})^{\sigma_i^{(j)}}\right]$$

$$A_1\left(\mathrm{z}\prod_{i=1}^{t}\left[\frac{(x_i-u_i)^{\delta_i^{(1)}}(v_i-x_i)^{\eta_i^{(1)}}}{\prod_{j=1}^{T}\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{\rho_i^{(j,1)}}}\right]\right)A_2\left(\mathrm{z'}\prod_{i=1}^{t}\left[\frac{(x_i-u_i)^{\delta_i'^{(1)}}(v_i-x_i)^{\eta_i'^{(1)}}}{\prod_{j=1}^{T}\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{\rho_i'^{(j,1)}}}\right]\right)$$

$$S_L^{h_1,\cdots,h_u}\left(\begin{array}{c}\mathrm{z}_1''\prod_{i=1}^{t}\left[\frac{(x_i-u_i)^{\delta_i''^{(1)}}(v_i-x_i)^{\eta_i''^{(1)}}}{\prod_{j=1}^{T}\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{\rho_i''^{(j,1)}}}\right]\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_u''\prod_{i=1}^{t}\left[\frac{(x_i-u_i)^{\delta_i''^{(u)}}(v_i-x_i)^{\eta_i''^{(u)}}}{\prod_{j=1}^{T}\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{\rho_i''^{(j,u)}}}\right]\end{array}\right)$$

$$\phi_{(\lambda_1,\cdots,\lambda_{\mathbf{p}},\mu_1,\cdots,\mu_{\mathbf{q}})}^{(\rho_1,\cdots,\rho_{\mathbf{p}},\sigma_1,\cdots,\sigma_{\mathbf{q}})}\left[\prod_{j=1}^{t}\left[\frac{(x_i-u_i)^{\zeta_i}(v_i-x_i)^{\lambda_i}}{\prod_{j=1}^{T}\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{\theta_i^{(j)}}}\right];\mathfrak{s},a\right]$$

$${}_PF_Q\left[(A_P);(B_Q);-\sum_{k=1}^{l}g_k\prod_{i=1}^{t}\left[\frac{(x_i-u_i)^{u_i^{(k)}}(v_i-x_i)^{\theta_i^{(r)}}}{\prod_{j=1}^{T}\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{\tau_i^{(j,k)}}}\right]\right]\mathrm{d}x_1\cdots\mathrm{d}x_t$$

$$=\frac{\prod_{j=1}^{Q}\Gamma(B_j)}{\prod_{j=1}^{P}\Gamma(A_j)}\prod_{j=1}^{t}\left[(v_i-u_i)^{\alpha_i+\beta_i-1}\prod_{j=1}^{W}\left(u_iU_i^{(j)}+V_i^{(j)}\right)^{\sigma_i^{(j)}}\prod_{j=W+1}^{T}\left(u_iU_i^{(j)}+V_i^{(j)}\right)^{\sigma_i^{(j)}}\right]$$

$$\sum_{k''=0,or1}^{\infty_*}\sum_{R=0}^{\infty}\sum_{K_1=0}^{[N_1/\mathfrak{M}_1]}\cdots\sum_{K_v=0}^{[N_v/\mathfrak{M}_v]}\sum_{G_1=1}^{m_1}\sum_{g_1=0}^{\infty}\phi_1\frac{z_k^{\eta_{G_1,g_1}}(-)^{g_1}}{\delta_{G^{(1)}}^{(1)}g_1!}\bar{A}_Rb_uz_1''^{R1}\cdots z_u''^{R_u}E_{ij}$$

$$A_{sT+P+2s+p',sT+Q+s+q':Y_1}^{m,sT+P+2s+n':X_1}\left(\begin{array}{c|c}\mathrm{z'w}_1 & \mathbb{A}_1,\ \mathbf{A}_1\\ \mathrm{g}_1W_1 & \cdot\\ \cdot & \cdot\\ \cdot & \cdot\\ \mathrm{g}_lW_l & \cdot\\ \mathrm{G}_1 & \cdot\\ \cdot & \cdot\\ \cdot & \\ \mathrm{G}_T & \mathbb{B}_1,\ \mathbf{B}_1\end{array}\right) \qquad (4.2)$$

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3. B.P. Gautam, A.S. Asgar and A.N.Goyal. The A-function. Revista Mathematica. Tucuman (1980).

The validity conditions are the same that (3.1) with $r=s=1$. The quantities $\phi_1, V_1, W_1, \mathbb{A}_1, \mathbb{B}_1, \mathbf{A}_1, \mathbf{B}_1$ are equal to $\phi_k, V, W, \mathbb{A}, \mathbb{B}, \mathbf{A}, \mathbf{B}$ respectively for $r=s=1$.

Remark: By the similar procedure, the results of this document can be extended to the product of any finite number of multivariable A-functions and a class of multivariable polynomials defined by Srivastava and Garg [10].

V. Conclusion

Our main integral formula is unified in nature and possesses manifold generality. It acts a capital formula and using various particular cases of the multivariable A-function, a general class of multivariable polynomials and the generalization of the Hurwitz-Lerch Zeta function, we can obtain a large number of other integrals involving simpler special functions and polynomials of one and several variables.

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Notes

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