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Properties of Simple Semirings

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Properties of Simple Semirings

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Abstract- Authors determine different properties of simple semiring which was introduced by Golan [3]. We also proved some results based on the papers of Fitore Abdullahu [1]. P. Sreenivasulu Reddy and Guesh Yfter tela [4].

I. Introduction

This paper reveals the properties of simple semirings by considering that the multiplicative semigroup is singular.

1.1. Definition: A semigroup (S, .) is said to be left(right) singular if it satisfies the identity ab = a (ab = b) for all a, b in S

1.2. Definition: A semigroup (S, .) is rectangular if it satisfies the identity aba = a for all a, b in S.

1.3. Definition: A semigroup S is called medial if xyzu = xzyu, for every x,y,z,u in S.

1.4. Definition: A semigroup S is called left (right) semimedial if it satisfies the identity $x^2yz = xyxz$ ($zyx^2 = zxyx$), where $x,y,z \in S$ and x, y are idempotent elements.

1.5. Definition: A semigroup S is called a semimedial if it is both left and right semimedial.

Example: The semigroup S is given in the table is I-semimedial

*	a	b	c
a	b	b	b
b	b	b	b
c	c	c	c

1.6. Definition: A semigroup S is called I- left(right) commutative if it satisfies the identity xyz = yxz (zxy = zyx), where x, y are idempotent elements.

1.7. Definition: A semigroup S is called I-commutative if it satisfies the identity xy = yx, where $x,y \in S$ and x, y are idempotent elements.

Example: The semigroup S is given in the table is I-commutative.

*	a	b	c
a	b	b	a
b	b	b	b
c	c	b	c

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1.8. Definition: A semigroup S is called I-left(right) distributive if it satisfies the identity xyz = xyxz (zyx = zxyx), where x,y,z ∈ S and x, y are idempotent elements.

1.9. Definition: A semigroup S is called I-distributive if it is both left and right distributive

1.10. Definition: A semigroup S is said to be cancellative for any a, b, ∈S, then ac = bc ⇒ a = b and ca = cb ⇒ a = b

1.11. Definition: A semigroup S is called diagonal if it satisfies the identities $x^2 = x$ and xyz = xz.

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1.12. Definition: [3] A semiring S is called simple if a + 1 = 1 + a = 1 for any a∈S.

1.13. Definition: A semiring (S, +, .) with additive identity zero is said to be zero sum free semiring if x + x = 0 for all x in S.

1.14. Definition: . A semiring (S, +, .) is said to be zero square semiring if $x^2 = 0$ for all x in S, where 0 is multiplicative zero.

1.15. Theorem: A simple semiring is additive idempotent semiring.

Proof: Let (S, +, .) be a simple semiring. Since (S, +, .) is simple, for any a∈S, a + 1 = 1. (Where 1 is the multiplicative identity element of S. S^1 = SU {1}.)

Now a = a.1 = a(1 + 1) = a + a ⇒ a = a + a ⇒ S is additive idempotent semiring.

1.16. Theorem: Let (S, +, .) be a simple semiring. Then the following statements are holds:

(i) a+b+1 = 1 (ii) ab +1 = 1 (iii) $a^n + 1 = 1$ (iv) $(ab)^n + 1 = 1$ (v) $(ab)^n + (ba)^n = a + b$ For all a,b ∈S.

Proof: Proof for (i) and (ii) are trivial. Proof for (iii), (iv) and (v) are by mathematical induction.

1.17. Theorem: Let (S, +, .) be a simple semiring in which (S, .) is singular then (S, .) is rectangular band.

Proof: Let (S,+,.) be a simple semiring and (S, .) be a singular i.e, for any a,b∈S ab = a. ⇒ aba = aa ⇒ aba = a (since (S, .) is singular) ⇒ (S, .) is rectangular band.

1.18. Theorem: Let (S, +, .) be a simple semiring in which (S, .) is singular then (S, +) is one of the following:

(i) I-medial (ii) I-semimedial (iii) I-distributive (iv) L-commutative
(v) R-commutative (vi) I-commutative (vii) external commutative
(viii) conditional commutative. (ix) digonal

Proof: Let (S, + ,.) be a semiring in which (S, .) is a singular. Assume that S satisfies the identity 1+a = 1 for any a ∈S. Now for any a, b, c, d ∈S.

(i) Consider a + b + c + d = a + (b + c) + b = a + c + b + d ⇒ (S, +) is I- medial.

(ii) Consider a + a + b + c = a +(a + b) + c = a + (b + a) + c = a + b + a + c ⇒ a + a + b + c = a + b + a + c ⇒ (S, +) is I- left semi medial.

Again b + c + a + a = b + (c + a) + a = b + (a + c) + a = b + a + c + a ⇒ b + c + a + a = b + a + c + a ⇒ (S, +) is I-right semi medial.

Therefore, (S, +) is I-semi-medial.

(iii) consider a + b + c = (a) + b + c = a + a + b + c = a + (a + b) + c = a + (b + a) + c = a + b + a + c ⇒ (S, +) is I-left distributive.

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Consider b + c + a = b + c + (a) = b + c + a + a = b + (c + a) + a = b + (a + c) + a = b + a + c + a ⇒ b + c + a = b + a + c + a ⇒ (S, +) is I right-distributive. Hence (S, +) is I-distributive.

Similarly we can prove the remaining.

1.19. Theorem: Let (S, +, .) be a simple semiring and (S, .) is singular then (S, +) is (i) quasi-seprative (ii) weakly-seperative (iii) seperative.

Notes

Proof: Let (S,+, .) be a simple semiring and (S, .) is a singular i.e, for any a,b∈S, ab = a. Since S is simple, 1+a = a+1 = 1, for all a∈S. Let a + a = a + b ⇒ a + a +1 = a + b + 1 ⇒ a + 1 = b + 1 ⇒ a = b. Again, a + b = b + b ⇒ a + b + 1 = b + b + 1 ⇒ a + 1 = b + 1 ⇒ a = b. Hence a + a = a + b = b + b ⇒ a = b ⇒ (S, +) is quasi-seperative.

(ii) Let a + b = (a) + b = ba + b = b + ab = b + a ⇒ a + b = b + a→ (1)

From (i) and (ii) a + a = a + b = b + a = b + b ⇒ a = b ⇒ (S, +) is weakly seperative

(iii) Let a + a = a + b
b + b = b + a

From (1) a + b = b + a and from theorem 1.15 (S, +) is a band

Therefore, a = a + a = a + b = b + b = b ⇒ a = b.

Hence (S, +) is seperative.

1.20. Theorem: Let (S, +, .) be a simple semiring in which (S, .) is singular then (S, +) is cancellative in which case —S— = 1.

Proof: Let (S, +, .) be a simple semiring in which (S, .) is singular. Since S is simple then for any a∈S, 1 + a = a + 1 = 1.

Let a,b,c, ∈S. To prove that (S, +) is cancellative, for any a, b, c ∈S, consider a + c = b + c. Then a + c.1 = b + c.1 ⇒ a + c(a + 1)
= b + c(b + 1) ⇒ a + ca + c = b + cb + c ⇒ a + ca + cac = b + cb + cbc (since (S, .) is rectangular) ⇒ a + ca(1 + c) = b + cb(1 + c)
⇒ a + ca.1 = b + cb.1 ⇒ a + ca = b + cb.1 ⇒ a + ca = b + cb
⇒ (1 + c)a = (1 + c)b ⇒ 1.a = 1.b ⇒ a = b ⇒ a + c = b + c
⇒ a = b. ⇒ (S. +) is right cancellative

Again c + a = c + b ⇒ c.1 + a = c.1 + b ⇒ c(1 + a) + a = c(1 + b) + b ⇒ c + ca + a = c + cb + b ⇒ cac + c + a = cbc + cb + b ⇒ cac + ca + a = abc +cb + b ⇒ ca(c + 1)+ a = cb(c + 1) + b ⇒ ca.1 + a = cb.1 + b
⇒ a + a = cb + b ⇒ (c + 1)a = (c + 1)b ⇒ 1.a = 1.b ⇒ a = b ⇒ c + a = c + b ⇒ a = b ⇒ (S, +) is left cancellative.

Therefore, (S,+) is cancellative semigroup. Since S is simple semiring we have 1 + a = 1 ⇒ 1 + a = 1 + 1. But (S, +) is cancellative ⇒ a = 1 for all a∈S. Therefore —S— = 1.

1.21. Theorem: Let (S, +, .) be a simple semiring in which (S, .) is singular then (S, +) is one of the following: i) singular ii) rectangular band iii) left(right) semi-normal iv) regular v) normal vi) left(right) quasi-normal vii) left(right) semi-regular.

Proof: Let (S, +, .) be a simple semiring in which (S, .) is singular. Since S is simple then for any a, b, c∈S, i) 1 + a = 1⇒ b + ba = b ⇒ b + a = b ⇒ (S, +) is left singular. Again 1 + b = 1 ⇒ a + ba = a ⇒ a + b = a ⇒ (S, +) is right singular.

Therefore, (S, +) is singular.

Since (S, +) is singular, it is easy to prove (S, +) is ii) rectangular band, iii) left(right) semi-normal, iv) regular, v) normal and vi) left(right) quasi-normal.

vii) Let a + b + c + a = a + (b) + (c) + (a) = a + b + a + c + a + c + a ⇒ a + b + c + a = a + b + a + c + a + (c) + a ⇒ a + b + a + c + a + b + c + a ⇒ (S, +) left semi-regular.

Similarly, we can prove (S, +) is right semi-regular.

1.22. Theorem: Let (S, +, .) be a simple semiring in which (S, .) is singular then (S, .) is one of the following: i) left semi-normal ii) left semi-regular iii) right semi-normal iv) right semi-regular v) regular vi) normal vii) left quasi-normal viii) right quasi-normal ix) I-medial (x) I-semimedial (xi) I-distributive (xii) L-commutative (xiii) R-commutative (xiv) I-commutative (xv) external commutative (xvi) conditional commutative (xvii) digonal (xviii) quasi-seprative (xix) weakly-seperative (xx) seperative.

Proof: Proof of the theorem is similar to 1.21.Theorem, 1.18.theorem and 1.19.theorem.

1.23. Theorem: Let (S, +, .) be a simple semiring with additive identity zero in which (S, .) is singular then (S, +, .) zero sum free semiring if and only if (S, +, .) is zero square semiring.

Proof: Let (S, +, .) be a simple semiring with additive identity zero in which (S, .) is singular. Since S is simple then for any a∈S, 1 = a + 1 ⇒ a = aa + a ⇒ a^2 = a + a ⇒ a^2 = 0 (Since (S, +, .) is zero sum free semiring) ⇒ (S, +, .) is zero square semiring.

Conversely, let (S, +, .) is zero square semiring then a^2 = 0 1 = a + 1 ⇒ a = aa + a ⇒ a^2 = a + a ⇒ 0 = a + a (Since (S, +, .) is zero square semiring) ⇒ (S, +, .) is zero sum free semiring.

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