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Bounce Behavior of Kantowski-Sach Cosmological Model in General Relativity

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Abstract- Kantowski-Sach cosmological model has obtained in the general theory of relativity. The source for energy-momentum tensor is assumed a perfect fluid. The field equations have solved by using special form of the average scale factor $R(t)=\left((t-t_0)^2+\frac{t_0}{1-\beta}\right)^{\frac{1}{1-\beta}}$ proposed by Cai et al. The physical properties and the bouncing behavior of the model are also discussed.

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BOUNCEBEHAVIOROFKANTOWSKISACHCOSMOLOGICALMODELINGENERALRELATIVITY

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Bounce Behavior of Kantowski-Sach Cosmological Model in General Relativity

H. R. Ghate [α] & Yogendra D. Patil [σ]

Abstract- Kantowski-Sach cosmological model has obtained in the general theory of relativity. The source for energy-momentum tensor is assumed a perfect fluid. The field equations have solved by using special form of the average scale factor $R(t)=\left((t-t_0)^2+\dfrac{t_0}{1-\beta}\right)^{\frac{1}{1-\beta}}$ proposed by Cai et al. The physical properties and the bouncing behavior of the model are also discussed.

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I. Introduction

Astronomical observational data obtained from high *red shift* surveys of Supernovae (SnIa) by Riess et al. [1], Perlmutter et al. [2] and Bennett et al. [3] indicated that our universe is expanding with acceleration. Also, observations such as Cosmic Microwave Background Radiations [4] and Large-scale structure [5] provide indirect evidence for the late time accelerated expansion of the universe. The accelerating expansion of the universe is driven by a mysterious component with high negative pressure known as dark energy (DE). In spite of all these attempts, DE is still the open question to the theoretical physicists because its nature is unknown. According to the astronomical observations, the DE currently accounts for about 73% of the total mass/energy of the universe and only 27% of a combination of dark matter and baryonic matter [6]. The DE universe may have very interesting implications for the future [7,8]. A different way of accounting for the DE without any extra components is the modification of gravity [9,10].

The idea that instead of originating from a Big Bang singularity, the universe has emerged from a cosmological bounce has a long history [11]. Novello et al. [12,13] realized that a bouncing cosmology with a matter-dominated phase of contraction during which scales which are probed today, a cosmological observations exit a Hubble radius can provide an alternative to the current inflationary universe paradigm of cosmological structure formation. According to Cai et al. [10], the solution of the singularity problem of the standard Big Bang cosmology is known as bouncing universe. A bouncing universe with an initial contraction to a non-vanishing minimal radius and then subsequent an expanding phase provides a possible solution to the singularity problem of the standard Big Bang cosmology. Moreover, for the universe entering into the hot Big Bang era after the bouncing, the equation of state (EoS) of the matter content ω in the universe must transit from $\omega<-1$ to $\omega>-1$. In the contracting phase, the scale factor $R(t)$ is decreasing, this means $\dot{R}(t)<0$ and in the expanding universe, scale factor $\dot{R}(t)>0$. Finally at the bouncing point, $\dot{R}(t)=0$ and near this point $\ddot{R}(t)>0$, for a period of time. It is also discussed with other view that in the bouncing cosmology, the Hubble parameter H passes across zero $(H=0)$ from $H<0$ to $H>0$. Cai et al. have investigated bouncing universe with quintom matter. He showed that a bouncing universe has an initial narrow state by a minimal radius and then develops to an expanding phase. This means for the universe arriving to the Big-Bang era after the bouncing, the EoS parameter should crossing from $\omega<-1$ to $\omega>-1$. Sadatian [14] have studied rip singularity scenario and bouncing universe in a Chaplygin gas dark energy model. Recently, Bamba et al. [15] have investigated bounce cosmology from $f(R)$ gravity and $f(R)$ bi-gravity. Astashenok [16] has studied effective energy models and dark energy models with bounce in frames of $f(T)$ gravity. Solomans et al. [17] have investigated bounce behavior in Kantowski-Sach and Bianchi cosmology. Silva et al. [18] have studied bouncing solutions in Rastall's theory with a barotropic fluid. Brevik and Timoshkin [19] have obtained inhomogeneous dark fluid matter leading to a bounce cosmology. Singh et al. [20] have studied k-essence cosmologies in Kantowski-Sach cosmological Sachs and Bianchi space-times.

In this paper, Bouncing behavior of Kantowski-Sachs cosmological model has been obtained in the general theory of relativity. This work is organized as follows: In section 2, the metric and field equations have presented. The field equations have solved in section 3 by using the physical condition that the expansion scalar θ is proportional to shear scalar σ and the special form of average scale factor $R(t)=\left[(t-t_0)^2+\dfrac{t_0}{1-\beta}\right]^{\frac{1}{1-\beta}}$ proposed by Cai *et al.* [10]. The physical and geometrical behavior of the model have been discussed

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in section 4. In the last section 5, concluding remarks have been expressed.

II. Metric and Field Equations

Kantowski-Sachs metric is considered in the form

$$ds^2 = dt^2 - a^2 dr^2 - b^2\left(d\theta^2 + \sin^2\theta d\psi^{2)}\right), \tag{1}$$

where $A(t)$ and $B(t)$ are functions of cosmic time t.

The energy-momentum tensor when the source for energy is assumed a perfect fluid given by

$$T_i^j = (\rho + p)u_i u^j - p g_i^j, \tag{2}$$

where u^i is the flow of vector satisfying $g_{ij}u^i u^j = 1$. Here ρ is the total energy density of perfect fluid and p is the corresponding pressure. For the perfect fluid, p and ρ are related by and equation of state

$$p = \omega\rho, \quad 0 \le \omega \le 1. \tag{3}$$

In co-moving system of co-ordinates, using equation (2), one can find

$$T_0^0 = \rho \text{ and } T_1^1 = T_2^2 = T_3^3 = -p. \tag{4}$$

The Einstein's field equations are given by

$$R_i^j - \frac{1}{2}g_i^j R = -T_i^j, \tag{5}$$

where R_i^j is a Ricci tensor, R is the Ricci scalar.

Using equation (2), for the metric (1), the field equations are given by

$$2\frac{\dot{a}\dot{b}}{ab} + \frac{\dot{b}^2}{b^2} + \frac{1}{b^2} = \rho \tag{6}$$

$$2\frac{\ddot{b}}{b} + \frac{\dot{b}^2}{b^2} + \frac{1}{b^2} = -\omega\rho \tag{7}$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\dot{a}\dot{b}}{ab} = -\omega\rho, \tag{8}$$

where an overhead dot $\left(\cdot\right)$ denotes differentiation with respect to t.

The average scale factor R and volume scalar V are given by

$$R^3 = V = ab^2. \tag{9}$$

The generalized mean Hubble's parameter H is defined by

$$H = \frac{\dot{R}}{R} = \frac{1}{3}\left(H_r + H_\theta + H_\psi\right), \tag{10}$$

where the directional Hubble parameters H_r, H_θ and H_ψ are given by

$$H_r = \frac{\dot{a}}{a}, \quad H_\psi = H_\theta = \frac{\dot{b}}{b}, \tag{11}$$

The expansion scalar θ and shear scalar σ are given by

$$\theta = 3H = \left(\frac{\dot{a}}{a} + 2\frac{\dot{b}}{b}\right), \tag{12}$$

$$\sigma^2 = \frac{1}{2}\left[\sum_{i=1}^{n} H_i^2 - 3H^2\right]. \tag{13}$$

The deceleration parameter (DP) q is defined by

$$q = -1 + \frac{d}{dt}\left(\frac{1}{H}\right).$$

III. The Solution of Field Equations

The field equations (6) to (8) are a system of three highly non-linear differential equations in four unknowns A, B, ρ and ω. The system is thus initially undetermined. We need one extra physical condition to solve the field equations completely.

We assume that the expansion scalar (θ) is proportional to the shear scalar (σ). This condition leads to

$$\frac{1}{\sqrt{3}}\left(\frac{\dot{a}}{a} + \frac{\dot{b}}{b}\right) = \alpha_0\left(\frac{\dot{a}}{a} + 2\frac{\dot{b}}{b}\right).$$

which yields

$$\frac{\dot{a}}{a} = m\frac{\dot{b}}{b}$$

where α_0 and m are constants.

Above equation, on integration, reduces to

$$a = \eta(\mathrm{b})^m,$$

where η is an integration constant.

Here, for simplicity and without loss of generality, we assume that $\eta = 1$.

Hence, we have

$$a = (\mathrm{b})^m, \quad (m \ne 1). \tag{15}$$

Collins et al. [21] have pointed out that for spatially homogeneous metric, the normal congruence

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to the homogeneous expansion satisfies that the condition $\frac{\sigma}{\theta}$ is constant.

In cosmology, the constant deceleration parameter is commonly used by several researchers [22-26], as it duly gives a power law for metric function or corresponding quantity.

The motivation to choose time-dependent deceleration parameter (DP) is behind the fact that the expansion of the universe was decelerating in the past and accelerating at present as observed by recent observations of Type Ia Supernova [1,2, 27-29] and CMB anisotropies [3,31]. Also, the transition redshift from deceleration expansion to accelerated expansion is about 0.5. Now for a Universe which was decelerating in past and accelerating at the present time, the DP must show signature flipping [31-33]. So, in general, the DP is not a constant but time variable. The motivation to choose the following scale factor is that it provides a time-dependent DP.

Under above motivations, we use a special form of deceleration parameter as

$$q=-\frac{R\ddot{R}}{R^2}=-1+\frac{d}{dt}\left(\frac{1}{H}\right)=-1+\frac{1}{2}\left[(1-\beta)-\frac{t_0}{(t-t_0)^2}\right],\beta<1 \quad (16)$$

where R is average scale factor of the universe.

This form is proposed by Cai *et al.* [10] and then modified by Sadatian [11].

After integration of (16), we obtain the Hubble parameter as

$$H=\frac{\dot{R}}{R}=\frac{2(t-t_0)}{(1-\beta)(t-t_0)^2+t_0}.$$

Again integrating, the average scale factor which is time dependent given by

$$R(t)=\left[(t-t_0)^2+\frac{t_0}{1-\beta}\right]^{\frac{1}{1-\beta}}, \quad (17)$$

where t_0 is initial time and $\beta<1$ is constant.

Solving the equations $a=b^m$ and $R(t)=(ab^2)^{\frac{1}{3}}$ and using (17), we get

$$a=\left[(t-t_0)^2+\frac{t_0}{1-\beta}\right]^{\frac{3m}{(1-\beta)(m+2)}} \quad (18)$$

With the help of equation (17), equation (15) takes the form

$$b=\left[(t-t_0)^2+\frac{t_0}{1-\beta}\right]^{\frac{3}{(1-\beta)(m+2)}}. \quad (19)$$

Using above two equations (18) and (19), the metric (1) takes the form

$$ds^2=dt^2-\left[(t-t_0)^2+\frac{t_0}{1-\beta}\right]^{\frac{6m}{(1-\beta)(m+2)}}dr^2-\left[(t-t_0)^2+\frac{t_0}{1-\beta}\right]^{\frac{6}{(1-\beta)(m+2)}}\left(d\theta^2+\sin^2\theta d\psi^2\right). \quad (20)$$

Equation (20) represents Kantowski-Sachs cosmological model with time-dependent scale factor.

IV. Physical Properties of the Model

The physical quantities such as special volume V, Hubble parameter H, expansion scalar θ, mean anisotropy A_m, shear scalar σ^2, energy density ρ, equation of state parameter ω are obtained as follows :

The average scale factor is

$$R(t)=\left[(t-t_0)^2+\frac{t_0}{1-\beta}\right]^{\frac{1}{1-\beta}}.$$

From fig. 4.1 (a), in the earlier stage, the scale factor is strictly decreasing $(\dot{R}(t)<0)$ and in the expanding phase the scale factor increases rapidly $(\dot{R}(t)>0)$. Hence our model is bouncing at $t=t_0$ $(\dot{R}(t)=0)$.

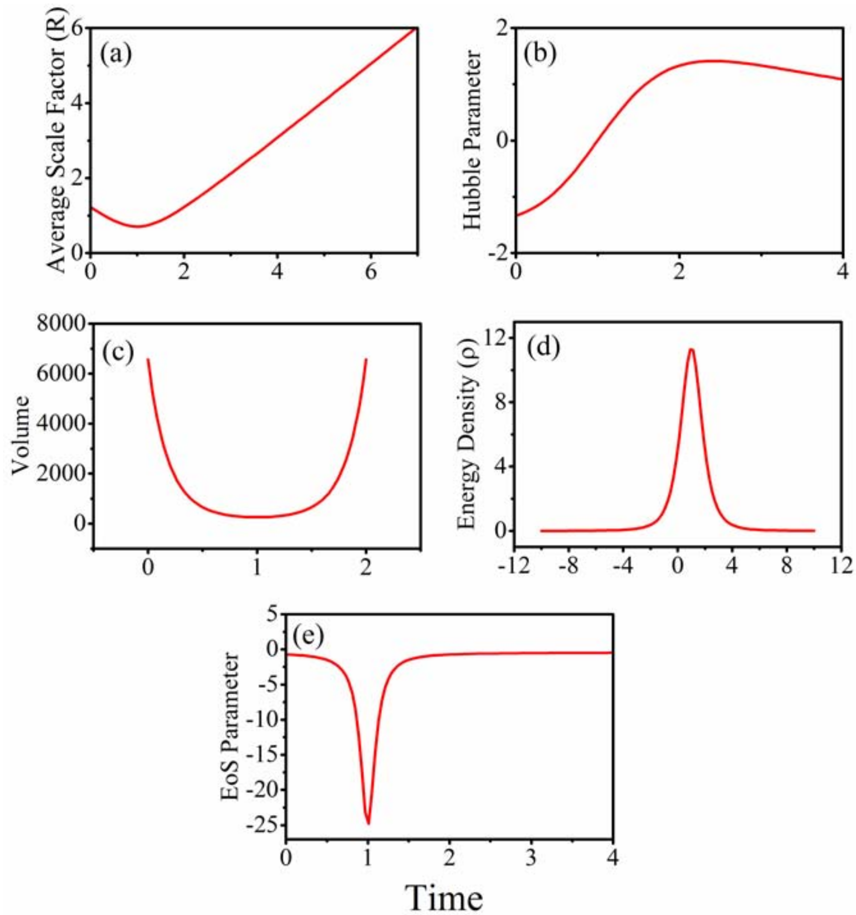

Fig. 4.1: Plots of time versus (a) Average scale factor (b) Hubble parameter (c) Spatial Volume (d) Energy density (e) EoS parameter for the values $\beta = 0.5$, $t_0 = 1$

The spatial volume is in the form

$$V = R^3 = \left[(t - t_0)^2 + \frac{t_0}{1-\beta}\right]^{\frac{3}{1-\beta}} . \quad (21)$$

The spatial volume is finite at time $t = 0$ and increases with increasing value of time hence the model starts expanding with finite volume.

The Hubble parameter is given by

$$H = \frac{2(t - t_0)}{(1-\beta)}\left[(t - t_0)^2 + \frac{t_0}{(1-\beta)}\right]^{-1} . \quad (22)$$

From fig. 4.1 (b) the Hubble parameter $H < 0$ for $t < 1$ and $H > 0$ for $t > 1$ indicating that H passes across zero $(H = 0)$ at $t = 1$, which represents that the universe is bouncing at $t = 1$.

The expansion scalar is

$$\theta = 3H = \frac{32(t - t_0)}{(1-\beta)(t - t_0)^2 + \frac{t_0}{1-\beta}} . \quad (23)$$

The mean anisotropy parameter is

$$Am = 2\frac{(m-1)^2}{(m+2)^2} = cons\tan t(\neq 0, form \neq 1) . \quad (24)$$

The shear scalar is

$$\sigma^2 = 12\frac{(m-1)^2 (t - t_0)^2}{(m+2)^2 (1-\beta)^2}\left[(t - t_0)^2 + \frac{t_0}{(1-\beta)}\right]^{-2} . \quad (25)$$

We observe that

$$\lim_{t \to \infty} \frac{\sigma^2}{\theta^2} = \frac{(m-1)^2}{3(m+2)^2} (\neq 0 \text{ , for } m \neq 1) . \quad (26)$$

The mean anisotropy parameter A_m is constant and $\lim_{t \to \infty} \frac{\sigma^2}{\theta^2} \neq 0$ is also constant. Hence the model is anisotropic throughout the evolution of the universe except at $m = 1$ i.e the model does not approach isotropy.

The matter energy density is given by

$$\rho = \frac{36(2m+1)(t-t_0)^2}{(1-\beta)^2(m+2)^2}\left[(t-t_0)^2+\frac{t_0}{1-\beta}\right]^{-2}+\left[(t-t_0)^2+\frac{t_0}{1-\beta}\right]^{\frac{-6}{(1-\beta)(m+2)}} \tag{27}$$

From fig. 4.1(d), the energy density decreases at the early stage of evolution when $t<1$ and goes into the hot Big Bang era. The model bounces at $t=1$ and after bouncing the energy density rapidly increases for $t>1$.

The equation of state (EoS) parameter ω is given by

$$\omega = -\left\{\frac{\frac{108(t-t_0)^2}{(1-\beta)(m+2)^2}\left[(t-t_0)^2+\frac{t_0}{1-\beta}\right]^{-2}-\frac{24(t-t_0)^2}{(1-\beta)(m+2)}\left[(t-t_0)^2+\frac{t_0}{1-\beta}\right]^{-2}+\frac{12}{(1-\beta)(m+2)}\left[(t-t_0)^2+\frac{t_0}{1-\beta}\right]^{-1}+\left[(t-t_0)^2+\frac{t_0}{1-\beta}\right]^{\frac{-6}{(1-\beta)(m+2)}}}{\frac{36(2m+1)(t-t_0)^2}{(1-\beta)^2(m+2)^2}\left[(t-t_0)^2+\frac{t_0}{1-\beta}\right]^{-2}+\left[(t-t_0)^2+\frac{t_0}{1-\beta}\right]^{\frac{-6}{(1-\beta)(m+2)}}}\right. \tag{28}$$

A bouncing universe model has an initial narrow state by a non-zero minimal radius and then develops to an expanding phase. For the universe going into the hot Big Bang era after the bouncing, the equation of state parameter of the universe crosses from $\omega<-1$ to $\omega>-1$, From fig. 4.1 (e), before bouncing point at $t=1$, we see that the EoS parameter $\omega<-1$ and after the bounce, the universe enter into the hot Big Bang ear and occurs the Big rip singularity. Further the EoS parameter $\omega>-1$ for $t>1$. Hence our model is bouncing at $t=1$.

V. Conclusion

Kantowski-Sachs cosmological model has been investigated in the general theory of relativity. The source for energy-momentum tensor is a perfect fluid. The field equations have been solved by using time-dependent deceleration parameter. The mean anisotropy parameter A_m is constant and $\lim_{t\to\infty}\frac{\sigma^2}{\theta^2}(\neq 0)$ is also constant, hence the model is anisotropic throughout the evolution of the universe except at $m=1$ *i.e.* the model does not approach isotropy. It is interesting to note that the behavior of the model is bouncing as the Hubble parameter H passes across zero $(H=0)$ from $H<0$ to $H>0$, for some finite time $t=t_0$. Also the energy density decreases at the early stage of evolution and rapidly increases showing big bounce at $t=t_0$. The Hubble parameter $H<0$, for $t<t_0$ and $H>0$, for $t>t_0$ indicating that H passes across zero $(H=0)$ at $t=t_0$. $(t\neq t_0)$ which represents the model is bouncing at $t=t_0$. The skew-ness parameter $\omega<-1$ before the bounce at $t=t_0$ and $\omega>-1$ after the bounce.

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