Global Journal of Science Frontier Research: F
Mathematics and Decision Sciences
Volume 17 Issue 8 Version 1.0 Year 2017
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Functional Product of Graphs and Multiagent Systems

By A. R. G. Lozano, A. S. Siqueira & S. R. P. Mattos
Department of Mathematics of ECELAH / UNIGRANRIO

Abstract- In this work, the concepts of functional product of graphs and equitable total coloring were used to propose a model of connection among the multiagent systems. We show how to generate a family of regular graphs that admits a range vertex coloringof order Δ with $\Delta + 1$ colors, denominated harmonic graphs. We prove that the harmonic graphs do not have cut vertices. We also show that the concept of equitable total coloring can be used to elaborate parallel algorithms that are independent of the network topology. Finally, we show a model of connection among multiagent systems (MAS) based on the use of harmonic graphs as a support for the construction of P2P overlay network topologies used for the communication among these systems.

Keywords: functional product of graphs, harmonics graphs, multiagent system.

GJSFR-F Classification: MSC 2010: 05C50 eJournal Print Journal epaper open standard 3d virtual journal

Functional Product of Graphs and Multiagent Systems

A. R. G. Lozano [α], A. S. Siqueira [σ] & S. R. P. Mattos [ρ]

Abstract- In this work, the concepts of functional product of graphs and equitable total coloring were used to propose a model of connection among the multiagent systems. We show how to generate a family of regular graphs that admits a range vertex coloringof order Δ with Δ + 1 colors, denominated harmonic graphs. We prove that the harmonic graphs do not have cut vertices. We also show that the concept of equitable total coloring can be used to elaborate parallel algorithms that are independent of the network topology. Finally, we show a model of connection among multiagent systems (MAS) based on the use of harmonic graphs as a support for the construction of P2P overlay network topologies used for the communication among these systems.

Keywords: functional product of graphs, harmonics graphs, multiagent system.

I. Introduction

Historically, product graphs, more specifically the cartesian product graphs, have been widely used as the topology of interconnection networks. Classical topologies such as mesh, hyper-star, star-cube, hypercube, and torus are obtained through the cartesian product of graphs. Currently, the concept of interconnection networks (physical structures) does not have the same relevance as before, However, the concept of multi agent systems (MAS), in which two or more agents work together to perform certain tasks, has been increasingly gaining space and applicability [9, 16]. It is on this tripod (functional product of graphs, harmonic graphs, and multiagent systems) that this work is supported.

In this article, we prove that the functional product of graphs allows building harmonic graphs from any regular graph and that the harmonic graphs do not have cut vertices. We show that a family of harmonic graphs disposes of a scalable and recursive structure since, from an initial basic instance, it can expand dynamically its form maintaining properties, such as connectedness and regularity. We also show that the concept of equitable total coloring can be used to elaborate parallel algorithms that are independent of the network topology. Finally, we present a model of connection among MAS through the use of harmonic graphs as a support for the construction of these topologies. Therefore, the main contributions of this work are the theorems 3.3, 3.4, 3.5, and the application of harmonic graphs as P2P overlay network topologies for the communication among multi agent systems.

Ref

9. Lesser, V. R. Cooperative multiagent systems: A personal view of the state of the art. *Knowledge and Data Engineering, IEEE Transactions11*, 01 (1999), 133–142.

Author α: Department of Mathematics, FFP/UERJ. Postgraduate Program in Teaching of Sciences of ECELAH/ UNIGRANRIO. Research Group in Discrete and Computational Mathematics. e-mail: arglozano@terra.com.br

Author σ: Postgraduate Program in Humanities, Cultures and Arts of ECELAH/UNIGRANRIO. Research Group in Discrete and Computational Mathematics. e-mail: asiqueira@unigranrio.edu.br

Author ρ: Department of Mathematics of ECELAH/UNIGRANRIO. Research Group in Discrete and Computational Mathematics. e-mail: rickdemattos@unigranrio.edu.br

This text is organized as follows: in section 2, we present the concept of the functional product of graphs, idea that generalizes the cartesian product of graphs. Section 3 approaches the construction of harmonic graphs. In section 4, we enter in the computational aspect, and we present the concepts of the agents and multi agent systems. We also highlight the advantages of implementing a peer-to-peer communication system in the communication of the agents of a MAS, and we present a model of connection among MAS using harmonic graphs. In section 6, we make the final considerations.

II. Functional Product of Graphs

We can find introductory concepts about graphs and coloring in [3] and [20]. More specific concepts about coloring, such as equitable total coloring and range coloring of order k, can be viewed in [6] and [5] respectively. After that, we present the concept of the functional product of graphs, which also appears in [10] and [12]. To provide a better understanding of this section, some definitions and primary notations are necessary.

a) *Definitions and primary notations*

- $\{u, v\}$ or uv denotes an edge of graph G, in which u and v are adjacent;
- $\Delta(G)$ or Δ, if there is no ambiguity, denotes the maximum degree of graph G;
- $F(X)$ denotes the set of all bijections of X in X;
- $D(G)$ denotes the digraph obtained by replacing each edge uv of graph G by arcs (u, v) and (v, u) while maintaining the same set of vertices;
- D denotes the set of the digraphs that satisfy the following conditions:
  1. (u, v) is an arc of the digraph if and only if (v, u) is also an arc of the digraph;
  2. No two arcs are alike.
- If $\vec{G} \in D$, $G(\vec{G})$ denotes the graph obtained by replacing each pair of arcs (u, v) and (v, u) of $\vec{G}$ by edge uv while maintaining the same set of vertices;
- If A is a set, $|A|$ denotes the cardinality of A;
- C_n denotes the cycle of n vertices;
- K_n denotes the complete graph of n vertices.

Definition 2.1. The digraphs $\vec{G}_1(V_1, E_1)$ and $\vec{G}_2(V_2, E_2)$ are said to be functionally connected by the applications f_1: $E_1 \rightarrow F(V_2)$ and f_2: $E_2 \rightarrow F(V_1)$ if f_1 and f_2 are such that:

1. For every arc $(u,v) \in E_1$, if $(v,u) \in E_1$, then $f_1((u,v)) = (f_1((v,u)))^{-1}$;
2. For every $(x,y) \in E_2$, if $(y,x) \in E_2$, then $f_2((x,y)) = (f_2((y,x)))^{-1}$;
3. For every pair of arcs $(u,v) \in E_1$ and $(x,y) \in E_2$, it has that $f_2((x,y))(u) \neq v$ or $f_1((u,v))(x) \neq y$.

The applications f_1 and f_2 are called linking applications.

Definition 2.2. Let $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ be graphs, if $D(G_1)$ and $D(G_2)$ are functionally connected by applications f_1: $E(D(G_1)) \rightarrow F(V_2)$ and f_2: $E(D(G_2)) \rightarrow F(V_1)$, then the graphs $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ are said to be *functionally connected* by the same applications.

Definition 2.3. Let $\vec{G}_1(V_1, E_1)$ and $\vec{G}_2(V_2, E_2)$ be *digraphs* functionally connected by applications f_1: $E_1 \rightarrow F(V_2)$ and f_2: $E_2 \rightarrow F(V_1)$, the *functional product* of digraph $\vec{G}_1$ by digraph $\vec{G}_2$ according to f_1 and f_2, denoted by $(\vec{G}_1, f_1) \times (\vec{G}_2, f_2)$, is digraph $\vec{G}^*(V^*, E^*)$ defined by:

Ref

6. Fu, H. Some results on equalized total coloring. *Congressus Numerantium*(1994), 111–120.

- $V^* = V_1 \times V_2$.
- $((u,x),(v,y)) \in E^*$ *if and only if one of following conditions is true:*
  1. $(u, v) \in E_1$ *and* $f_1((u, v))(x) = y$;
  2. $(x, y) \in E_2$ *and* $f_2((x, y))(u) = v$.

Notes

Definition 2.4. Let $G_1(V_1, E_1)$ *and* $G_2(V_2, E_2)$ *be graphs functionally connected by applications* $f_1: E(D(G_1)) \rightarrow F(V_2)$ *and* $f_2: E(D(G_2)) \rightarrow F(V_1)$, *the functional product of graph* G_1 *by graph* G_2, *denoted by* $(G_1, f_1) \times (G_2, f_2)$, *is graph* G $(\vec{G}^* (V^*, E^*))$, *such that* $\vec{G}^* (V^*, E^*) = (D(G_1), f_1) \times (D(G_2), f_2)$.

Figures 1, 2 and 3 present the functional product between two paths P_3. The linking applications f_1 *and* f_2 are defined by $f_1(x) = r_2$ for every edge $x \in E_1$ and $f_2(y) = r_1$ for every edge $y \in E_2$, in which $r_1(v_i) = v_{i+1(mod3)}$ and $r_2(v_i) = v_{i+2(\text{mod}3)}$, with $i \in \{1, 2, 3\}$. Figure 1 makes reference to the definitions 2.1 and 2.2 while figures 2 and 3 illustrate the definitions 2.3 and 2.4 respectively.

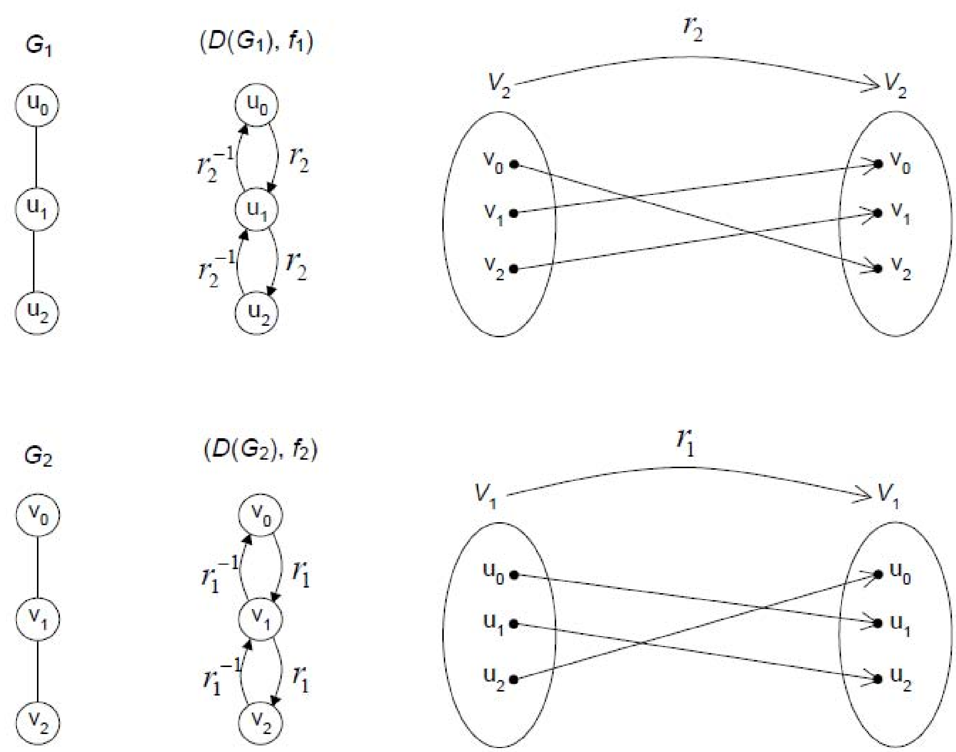


Figure 1: Graphs G_1 and G_2, the respective digraphs, and associated bijections r_1 and r_2

Global Journal of Science Frontier Research (F) Volume XVII Issue VIII Version I Year 2017

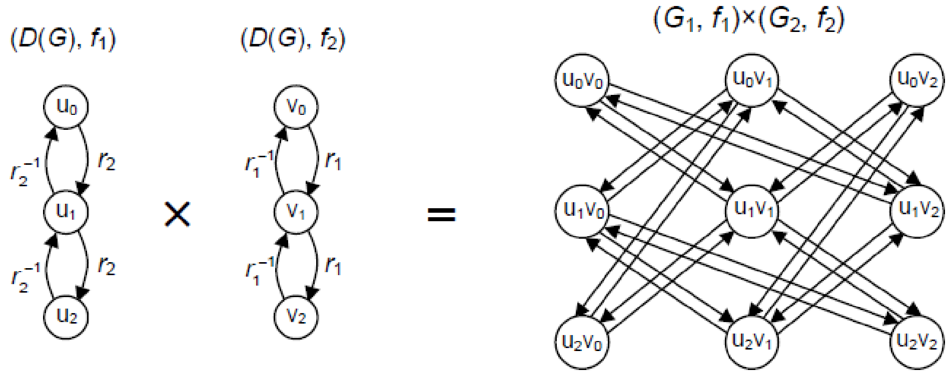


Figure 2: Functional product among digraphs $D(G_1)$ and $D(G_2)$ according to f_1 and f_2

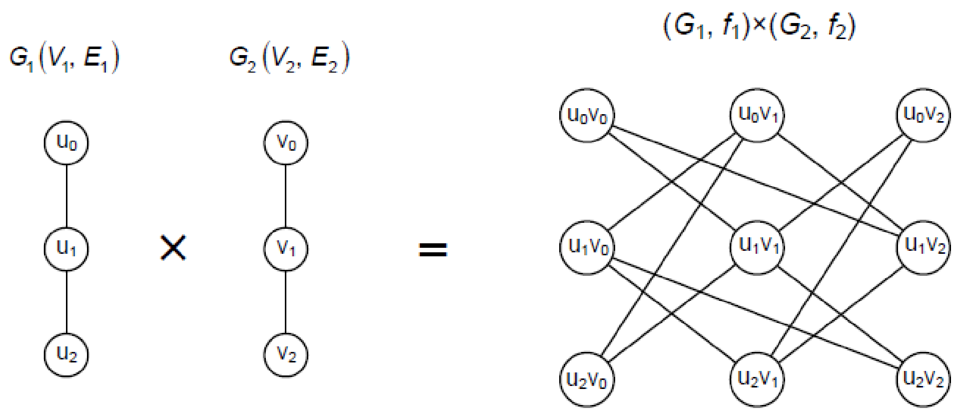


Figure 3: Functional product among graphs G_1 and G_2 according to f_1 and f_2

III. Harmonic Graphs

In this section, we presented the main contributions of this paper, the Theorems 3.3 and 3.4, which show how to build harmonic graphs from the functional product of graphs, and the Theorem 3.5 that proves that harmonic graphs do not have *articulation points*. To provide a better comprehension of this results, we enunciate some important concepts, the Petersen theorem, described in [20], and the result that guarantees the extension of a range coloring of order Δ to an equitable total coloring, which also appears in [11].

Definition 3.5. Let $G(V, E)$ be a graph, $C=\{c_1, c_2, c_3 \ldots c_p\}$ be a set of colors, with $p \in \mathbb{N}$ and a natural number k, such that $k \leq \Delta(G)$, an application $f: V \rightarrow C$ is a range vertices coloring of order k of G if for every $v \in V$, it has that $d(v) < k$, then $|c(N(v))| = d(v)$, otherwise $|c(N(v))| \geq k$, such that $|c(N(v))|$ is the cardinality of the set of colors used in the neighborhood of v [5].

Definition 3.6. A regular graph $G(V, E)$ is said to be harmonic if it admits a range vertices coloring of order Δ with $\Delta + 1$ colors [11].

Definition 3.7. A vertex in a connected graph is an articulation point or a cut vertex if by removing it, the graph becomes disconnected [4].

Theorem 3.1. If $G(V, E)$ is a $2k$-regular graph, then G is 2-factorized [20].

Ref
20. Yap, H. *Total colorings of graphs.* Springer, Berlin, 1996.

Theorem 3.2. Let G (V, E) a regular graph and $c{:}V \rightarrow C = \{1, 2, 3, \ldots \Delta + 1\}$ a range coloring of order Δ of vertices of G, then the natural extension from c to G is an equitable total coloring [11].

The following theorem shows how to generate a harmonic graph from any regular graph and its complement.

Theorem 3.3. For every regular graph G and its complement G^*, there are linking applications f_1 and f_2, such that (G, f_1) x (G^*, f_2) is a harmonic graph.

Proof. Initially, note that for every regular graph G, if $n = |V(G)|$ is odd, then $\Delta(G)$ and $\Delta(G')$ are even. If $n = |V(G)|$ is even, then $\Delta(K_n)$ is odd and, as $\Delta(K_n) = \Delta(G) + \Delta(G')$, it has that $\Delta(G)$ or $\Delta(G')$ is even. Suppose that $\Delta(G')$ is even, by theorem 3.1, there is a decomposition in 2-factors of G'. Let $F_1, F_2, F_3, \ldots, F_t$ be the 2-factors of decomposition of G', each 2-factors F_i is replaced by anoriented cycle, and we define the application $b{:}\ V(F) \rightarrow V(F)$ such that if $(u,v) \in E(F)$, then $b(u) = v$.

Note that b is a bijection, and each 2-factors have associated a bijection of vertices of G. The application f_1 associates the identity to every pair of arcs associated to the edges of G. The application f_2 associates the bijection b to every arc of the cycle. In the cycle, in the opposite direction, we associate the reverse bijection. Now, if $V(G) = \{v_0, v_1, v_2, \ldots, v_p\}$, we give the color p to each vertex in the form (x, vp). By construction, the coloring obtained in (G,f_1) x (G',f_2) is a range coloring of order Δ with $\Delta+1$ colors. If $\Delta(G')$ is odd, then $\Delta(G)$ is even. Therefore, just change the positions of G *and* G', in the previous reasoning, to obtain the desired result. Then, (G,f_1) x (G',f_2) is a harmonic graph.

Theorem 3.4. Let G and G^* be a regular graph and its complement, if $\Delta(G^*)$ is even, then for any graph G', such that $\Delta(G^*) = \Delta(G')$, there are linking applications f_1 and f_2, such that (G, f_1) x (G', f_2) is a harmonic graph.

Proof. Just note that both G' and H decompose themselves in the same amount of 2-factors. Let $F_1, F_2, F_3, \ldots, F_t$ be 2-factors of decomposition of G', $r_1, r_2, \ldots, r_t$ be the associated bijections, and $K_1, K_2, \ldots, K_t$ be the 2-factors of decomposition of H, which will be replaced by oriented cycles $O_1, O_2, \ldots, O_t$., the application f_1 makes the identity correspond to all the edges of G. The application f_2 makes the bijection r_i correspond to each oriented arc O_i, and the bijection r^{-1} corresponds to the arc of opposite direction, for every $i \in 1, 2, \ldots, t$. Now, if $V(G) = \{v_1, v_2, \ldots, v_p\}$, we give the color p to each vertex in the form (x, v_p). Again, by construction, the coloring obtained in (G, f_1) x (H, f_2) is arange coloring of order Δ with $\Delta+1$ colors. So, (G, f_1) x (H, f_2) is a harmonic graph.

Theorem 3.5. Harmonic graphs do not have cut vertices.

Proof. Let $G(V, E)$ be a harmonic graph and c: $V \rightarrow C = \{0,1,2,\ldots,\Delta\}$ be a range coloring of order Δ of the vertices of G, suppose by absurdity that G has a cut vertex $u \in V$ and, without losing generality, suppose that the vertex u was colored with the color 0. Let G' (V', E') be one of the connected components obtained by removing u of graph G, observe that the colors of $C - \{0\} = \{1, 2, \ldots, \Delta\}$ are used the same number of times in G' because, in a range coloring of order Δ, all of the adjacent vertices are colored with distinct colors, so given two arbitrary colors $i \in C - \{0\}$ and $j \in C - \{0\}$, every vertex of V', with the color i, has one and only one neighbor with the color j. Denote by V_i' the set of vertices

Ref

11. Lozano, A. R. G., Siqueira, A. S., Friedmann, C., and Jurkiewicz, S. Relationship between equitable total coloring and range coloring in some regular graphs. *Pesquisa Operacional 36*, 1(2016), 101–111.

of G' colored with the color $i \in C$, let $q = |V_i|$, $i \in C - \{0\}$ if $|V_0'| = q$, then all of the vertices of V' colored with colors different from 0 have a neighbor in V_0', so none of them can be neighbor of u, which is an absurdity. If $|V_0'| < q$, then it exists at least Δ vertices of G' with color other than 0 that do not have neighbors in V_0'. But, the number of neighbors of u in G' is less than Δ, so it exists vertices of V' with color other than 0 that do not have neighbor with color 0, which is an absurdity.

From the previous Theorem, it is obtained, immediately, the following corollary.

Corollary 3.1 Let u, v∈V be any two vertices of G(V, E), if G is a harmonic graph, then it exists a cycle in G that contains u and v.

IV. Multiagent Systems and Peer-to-Peer Communication System

According to Russel and Norvig[16], "an agent is anything that can be viewed as perceiving its environment through sensors and acting upon that environment through actuators". According to Lesser [9]" Multi agent systems are computational systems in which two or more agents interact or work together to perform some set of tasks or to satisfy some set of goals". The investigation of multi agent systems is focused on the development of computational principles and models to construct, analyze, and implement forms of interaction and coordination of agents in small or large-scale societies [9].

A peer-to-peer system implements an abstract overlay network on top of the network topology. The overlay network is a "virtual" network and the peers are connected to each other through logical connections, in which all of them should cooperate among themselves providing part of its resources on behalf of the accomplishment of a certain service [2].The objective of a peer-to-peer (P2P) system is to share computational resources through direct communication among its components therefore any device can access directly the resources of other devices of the system without any centralized control [2].

The combination between peer-to-peer network and multi agent systems has presented great solutions for the realization of applications that expand themselves on the internet. In [7], these two technologies were used to create an intelligent peer-to-peer infrastructure, which allows a dynamic network of intelligent agents while it manages several ways of discovering, cooperating, and executing efficiently computational resources. RETSINA [8,19] is a MAS infrastructure that uses the P2P Gnutella[4] network and some protocols based on DHT to extend the discovery services. ZHANG [21] proposes a peer-to-peer multi agent system that supports the execution of tasks of electronic commerce facilitating a dynamic selection of partners and allowing the use of heterogeneous agents.

The structured P2P overlay networks are characterized by a well-defined topology. Peers are positioned in a controlled way and the resources are distributed in a deterministic way making their location in the overlay network more efficient. Currently, we find several topologies implementing overlays networks on P2P systems. For example, Pastry [15] and Tapestry [22] are mesh-based, Chord [18] implements ring topology, and CAN [13] the d-dimensional torus.

V. Model of Connection among Multiagent Systems

In this paper, *multiagent systems connection* is a linking between two or more multiagent systems, in such way that agents of one MAS can communicate with agents of another MAS. It enables access to services, resource sharing, and guarantees the joint work.

[4] It is a network of file-sharing used mainly for the exchange of songs, movies and software [1].

Ref

16. Russel, S., and Norvig, P. *Artificial Intelligence: A Modern Approach.* Prentice Hall Series in Artificial Intelligence, New Jersey, 2004.

The reasons to amplify a computational structure range from opening a new sector of a company to the necessity of sharing servers interconnected to the internet to supply the demand of online sales during the launch of a product or on special dates, such as Christmas, for example.

In the proposed model, multiagent systems are overlaid by a P2P infrastructure that guarantees the interaction/communication among MAS agents. Therefore, each MAS agent corresponds to a peer of the P2P overlay network. The P2P overlay network architecture is represented by a graph $G(V, E)$, in which the vertices are the peers and the edges are the bonds(links) between the peers. Consequently, a vertex of the graph corresponds to a MAS agent, and graph $G(V, E)$ represents a multiagent system. Figure 4 illustrates an example in which three MAS are intended to be connected.

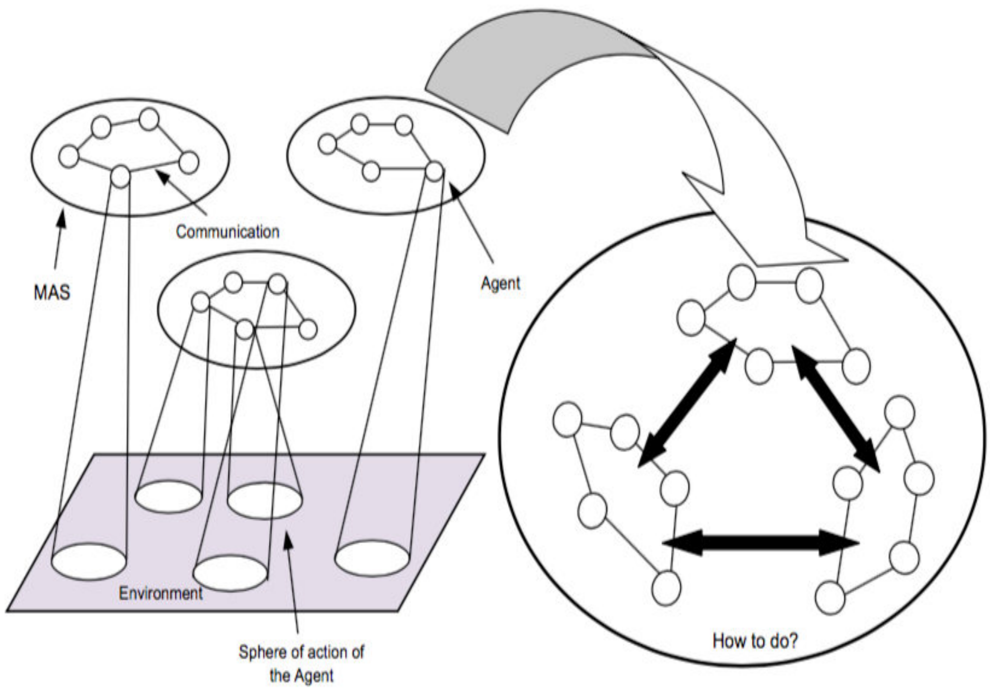


Figure 4: Structure of three MAS (left); Connection among three MAS (right). Adapted from Reis [14]

Initially, we are going to show how the theorem 3.4 allows making the connection among three multiagent systems. A graph C_5 will be used to represent the topology of each MAS to be connected, and a graph C_3 will be used to describe the type of connection, in other words, the way of making the connection among the three multiagent systems. Note that the graphs C_5 and C_3 satisfy the conditions of the theorem 3.4, so the harmonic graph illustrated in figure 5 can be constructed.

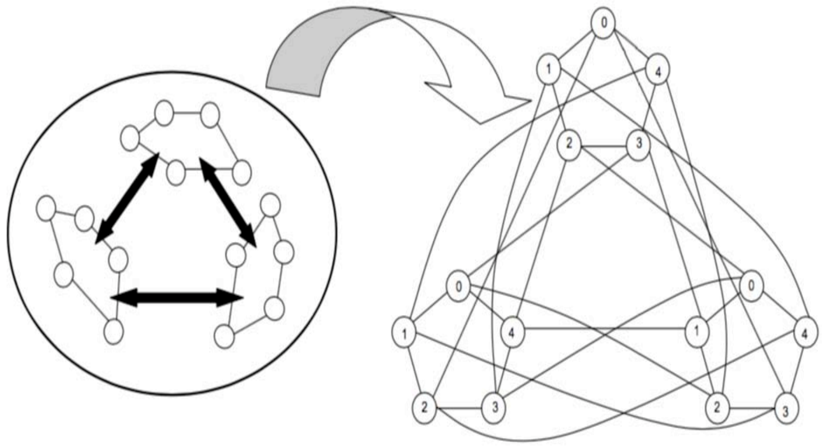


Figure 5: Process of construction of a range colored harmonic graph of order 4 with 5 colors

Now, we are going to show how to expand the structure of a MAS from the application of the theorem 3.3. Figures 6 and 7 illustrate this construction. Again, we used a graph C_5 as an example.

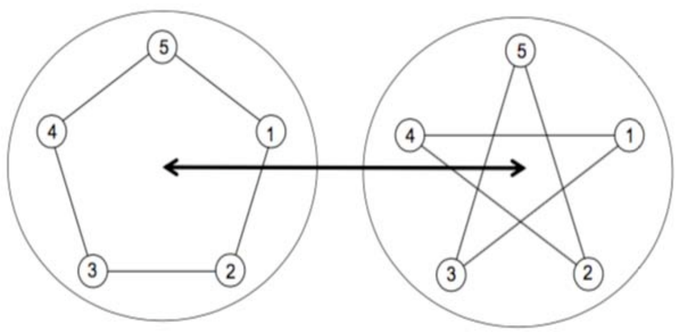


Figure 6: Graph G and its complement G^*(topology of the MAS)

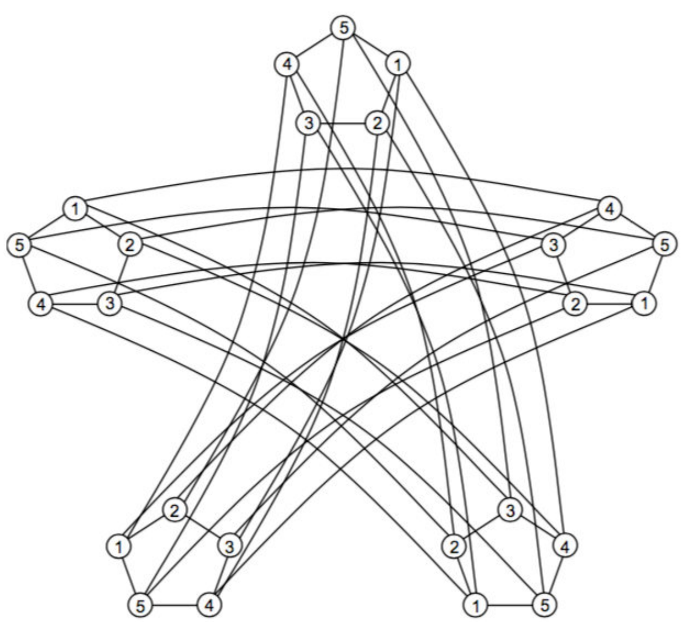


Figure 7: Resulting range colored harmonic graph of order 4 with 5 colors

Notes

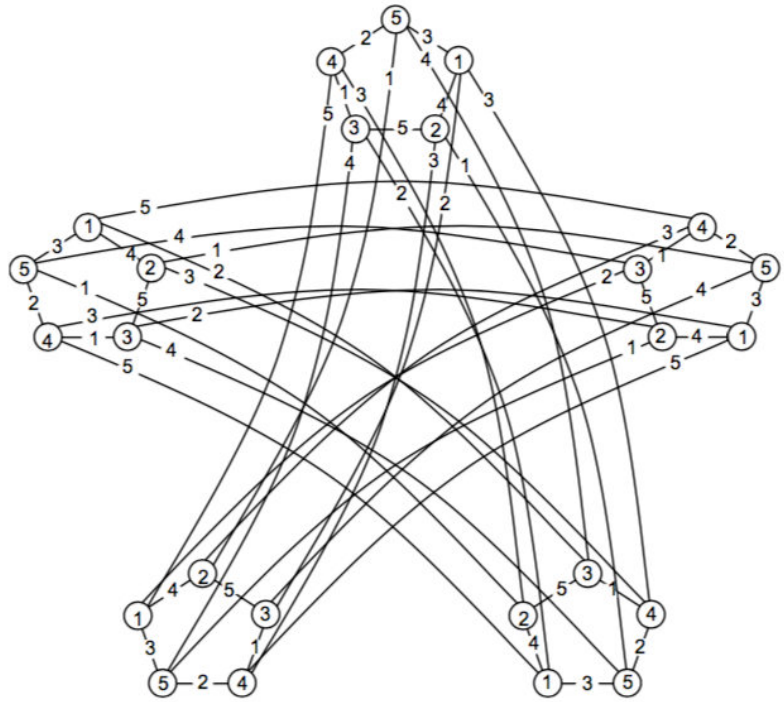


Figure 8: Harmonic graph total and equitably colored with 5 colors (resulting model of the multiagent system)

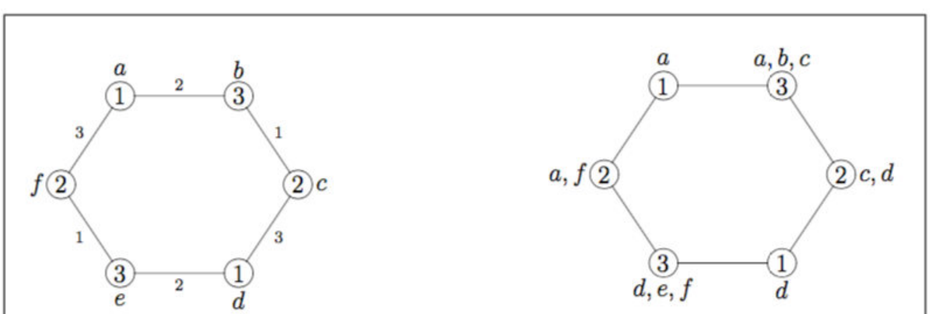
a) *Parallel Algorithm of Complete Exchange*

In this section, we present an algorithm of complete exchange that aims to guarantee the interaction /communication among every present agent of the P2P overlay network. In an algorithm that involves a complete exchange of information, each processor has information, and it is necessary that every processor knows all of the information. The algorithm bellow does not intend to be optimum, its objective is to show that from the total coloring it is possible to build algorithms that are independent of the topology.

Algorithm1: Complete Exchange of Information

```
Entrance: total and equitably colored graph G
  Variables: k: entire (total number of necessary colors)
             x[k]: array of color
             aux: color
start
 | Step 1:
 | For i= 1 up to k make
 |  | x[i] = i
 | end
end
start
 | Step 2:
 | for i= 1 a k make
 |  | Transmit the information through whichever edge with the color i, in
 |  | the direction of the vertex in which the color has the highest rate
 |  | in the array x[k]
 | end
end
start
 | Step 3:
 | aux:
 | x[k]
 | For i= k going down to 1 make
 |  | x[i] = x[i− 1]
 |  | x[1] = aux
 | end
end
start
 | Step 4:
 | repeat
 |  | the steps 2 and 3
 | until all of the vertices receive all the information;
end
```

Figures 9, 10, 11, 12, and 13 detail the functioning of the algorithm. Figure 9a shows a graph C_6 (MAS topology) in which the numbering of the vertices and edges represents an equitable total coloring, and the letters above the vertices symbolize the information contained in these vertices (agents). As a way to facilitate the comprehension of the algorithm, in the following figures, the numbering of the vertices indicates the exchanges of indexes of the *array* of color proposed in step 3.



(a) Step 1 (b) Step 2

Figure 9: Step-by-step of the Algorithm (original position)

Notes

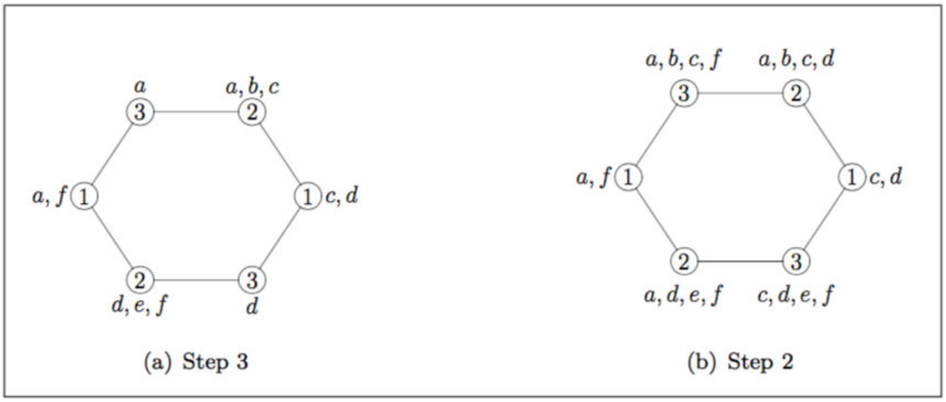


Figure 10: Step-by-step of the Algorithm (1st exchange)

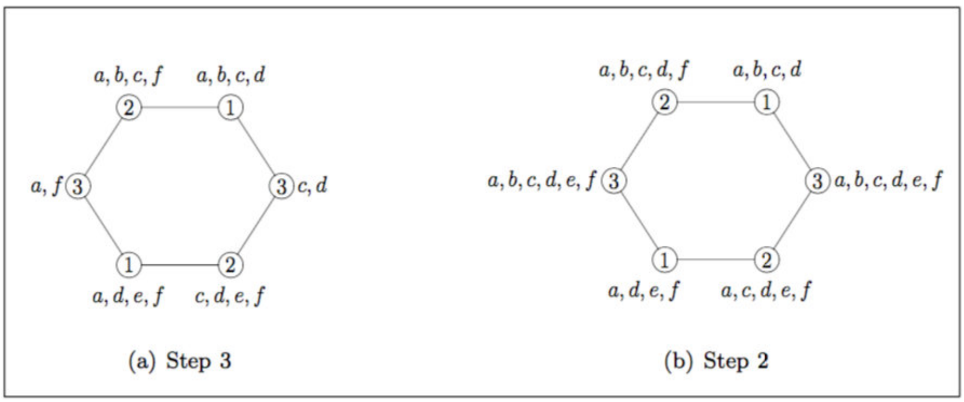


Figure 11: Step-by-step of the Algorithm (2nd exchange)

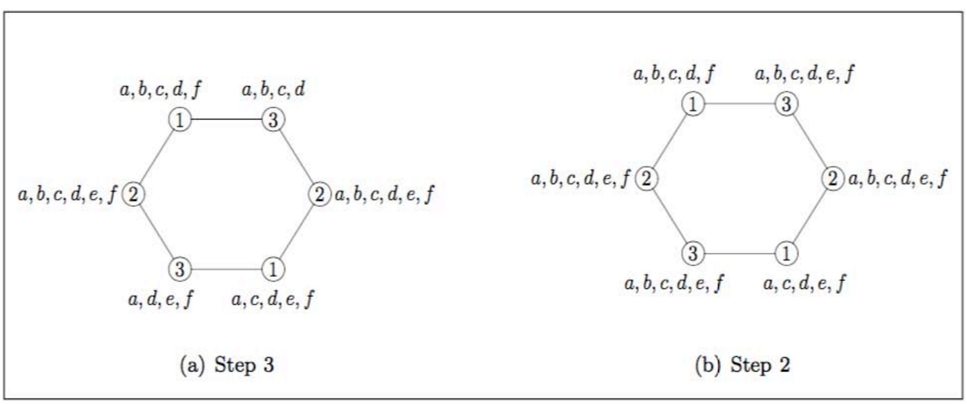


Figure 12: Step-by-step of the Algorithm (3rd exchange)

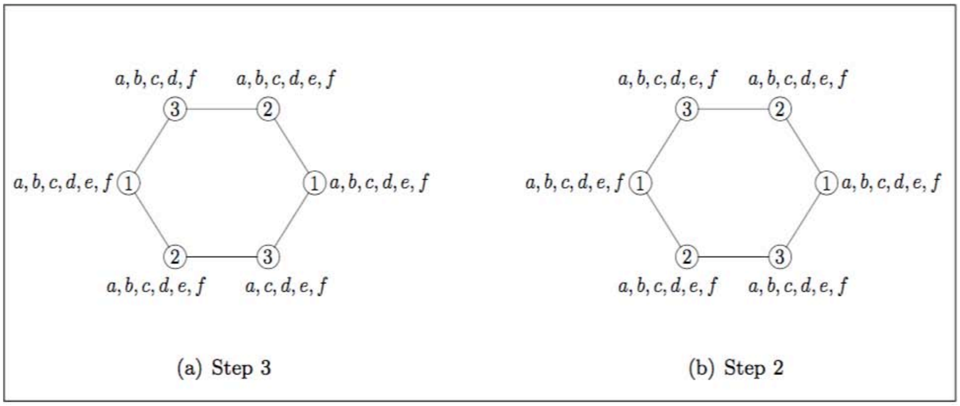


Figure 13: Step-by-step of the Algorithm (4th exchange)

b) *Algorithm analysis*

Theorem 5.6. Let *x* and *y* be adjacent vertices of graph *G(V, E)* associated to a network, vertex *y* will receive the information of *x* in a maximum of *k* repetitions of steps 2 and 3 of the algorithm.

Proof. If two vertices are adjacent, then they cannot have the same color, Consequently, they have distinct numbers. Step 2 guarantees that the exchange of information will always be in the direction of the vertex from the smallest to the biggest index in the array of color. With the exchange of the index of the array of color proposed by step 3, after at least *k* exchanges, every vertex will have at least once a bigger index than its neighbor. Therefore, in a maximum of *k* repetitions of steps 2 and 3, two adjacent vertices exchange information.

Theorem 5.7. If d is the length of the longest path between any two vertices of graph G, then in the $d\,.k^2$ *steps all of the vertices will receive all the information.*

Proof. Note that the algorithm executes *k* steps in each passage through step 2 since *k* colors were used in the total coloring of the graph. At each exchange in the indices of the array of color predicted by step 3, we return to step 2 therefore, after *k* exchanges, we have $k.k = k^2$ steps performed by the algorithm. As demonstrated previously, after *k* repetitions of steps 2 and 3, in other words, $k.\ k = k^2$ steps of the algorithm, any two adjacent vertices exchange information. In practice, it means that, after k^2 steps of the algorithm, a given vertex has the information of all of its neighbors. Therefore, the following steps of the algorithm guarantee that non-adjacent vertices exchange information. If *d* is the length of the longest way between any two vertices of graph G, then, after $d.k^2$ steps of the algorithm, all of the vertices will have received all of the system information.

c) *Advantages of the Proposed Model*

The implementation of topologies that admit equitable total coloring can make the processing more efficient because it allows a natural division of the network resources, in which at least $\frac{t}{\Delta+2}$ processors or connections can be used simultaneously, let t be the number of elements of the graph associated to the network, i.e., number of vertices plus number of edges and Δ the maximum degree of the graph.

An equitable total coloring obtained by the natural extension of the range coloring of order Δ with $\Delta + 1$ colors guarantees that neighbor vertices always have distinct colors. In the case of the graph having an even maximum degree, the equitable total coloring is obtained with $\Delta + 1$ colors [17], fact that provides to the topology a processing optimization because in every given moment or the processor (vertex of color i) is executing

Ref

17. Siqueira, A. S. *Coloração Total Equilibrada em Subfamílias de Grafos Regulares.* Tese de D.Sc., COPPE/UFRJ, Rio de Janeiro, Brasil, 2011.

a task or it is receiving information from one of its neighbors through one of its links (edge of color j), in other words, in every moment, every processor of the system is being activated. In this sense, the use of algorithms based on equitable total coloring does not allow the existence of idle processors in any step of the computation.

Moreover, it is not difficult to verify that topologies such as mesh, hypercube, and torus do not always admit a range vertex coloring of order Δ with $\Delta + 1$ colors. In contrast, among the topologies obtained by the cartesian product, the hypercube is the most scalable and the only one that allows recursive increase while preserves its original structure. Under this perspective, harmonic graphs, besides being scalable and having a recursive structure, they also present as an advantage the fact that they admit an equitable total coloring obtained by the natural extension of a range coloring of order Δ with $\Delta + 1$ colors.

VI. Conclusions

In this article, we used the concept of the functional product of graphs to build a family of regular graphs that admits a range vertex coloring Δ with $\Delta + 1$ colors, denominated harmonic graphs. We also proved that the harmonic graphs do not have cut vertices. We showed that the family of harmonic graphs offers advantages in its implementation, as P2P overlay network topology for the communication among MAS because it disposes of a scalable and recursive structure since, from an initial basic instance, it can expand its form dynamically maintaining properties, such as connectedness and regularity. Moreover, a topology based on harmonic graphs offers security against failures, since it does not have cut vertices. We also showed that, from the concept of equitable total coloring, the confection of parallel algorithms could be done generically, which guarantees a natural division of the resources of a network of connections. Finally, we presented a model of connection among multiagent systems based on the use of harmonic graphs as a support for the construction of P2P overlay network topologies used for the communication among the MAS.

References Références Referencias

1. The Gnutella protocol specification, 2000. http://dss.clip2.com/GnutellaProtocolo4.pdf
2. Androutsellis-Theotokis, S., and Spinellis, D. A survey of peer-to-peer content distribution technologies. *ACM Computing Surveys(CSUR)36*, 4 (2004),335–371.
3. Bondy, J., and Murty, U. *Graph Theory with Applications*. North-Holland, New York, 1976.
4. Diestel, R. *Graph Theory*. Springer-Verlag, New York, 1997.
5. Friedmann, C. V. P., Lozano, A. R. G., Markenzon, L., and Waga, C. F. E. M. Total coloring of block-cactus graphs. *The journal of combinatorial mathematics and combinatorial computing 78* (2011), 273–283.
6. Fu, H. Some results on equalized total coloring. *Congressus Numerantium* (1994), 111–120.
7. Helin, H., Klusch, M., Lopes, A. L., Fernández, A., Shumacher, M., Schuldt, H., Bergenti, F., and Kinnunem, A. Cascom: Context-aware service coordination in mobile 2 penvironments. In *MATES* (2005), Springer, pp.242–243.
8. Langley, B. K., Paolucci, M., and Sycara, K. Discovery of infrastructure in multi-agent systems. In *Proceedings of the second international joint conference on Autonomous agents and multiagent systems* (2003), ACM, pp. 1046–1047.

9. Lesser, V. R. Cooperative multiagent systems: A personal view of the state of the art. *Knowledge and Data Engineering, IEEE Transactionson11*, 01 (1999), 133–142.
10. Lozano, A. R. G., Siqueira, A. S., Mattos, S. R. P., and Jurkiewicz, S. Functional product of graphs: basic ideas and some properties. In *10th Andalusian Meeting on Discrete Mathematics* (2017), pp. 155–158.
11. Lozano, A. R. G., Siqueira, A. S., Friedmann, C., and Jurkiewicz, S. Relationship between equitable total coloring and range coloring in some regular graphs. *Pesquisa Operacional 36*, 1 (2016), 101–111.
12. Lozano, A. R. G., Siqueira, A. S., Jurkiewicz, S., and Friedmann, C. *Produto funcional de grafos. Tema-Tend. Mat. Apl.Comput. 2* (2013), 221–232.
13. Ratnasamy, S., Francis, P., Handley, M., Karp, R., and Shenker, S. *Ascalable content-address able network*, vol.31. ACM, NewYork, USA, 2001.
14. Reis, L. *Coordination in Multi-Agent Systems: Applications in University Management and Robotic Soccer*. Ph.D. thesis, FEUP, Porto, Portugal, 2003.
15. Rowstron, A., and Druschel, P. Pastry: Scalable, decentralized object location, and routing for large-scalepeer-to-peer systems. In *Middleware 2001* (2001), Springer, pp.329–350.
16. Russel, S., and Norvig, P. *Artificial Intelligence: A Modern Approach*. Prentice Hall Series in Artificial Intelligence, New Jersey, 2004.
17. Siqueira, A. S. *Coloração Total Equilibrada em Subfamilias de Grafos Regulares*. Tese de D.Sc., COPPE/UFRJ, Rio de Janeiro, Brasil, 2011.
18. Stoica, I., Morris, R., Karger, D., Kaashoek, M.F., and Balakrishnan, H. Chord: A scalable peer-to-peer lookup service for internet applications. *ACM SIGCOMM Computer Communication Review 31*, 4 (2001), 149–160.
19. Sycara, K., Paolucci, M., Van Velsen, M., and Giampapa, J. The retsinamas infrastructure. *Autonomous agents and multi-agent systems 7*, 1-2 (2003), 29–48.
20. Yap, H. *Total colorings of graphs*. Springer, Berlin, 1996.
21. Zhang, Z. E-commerce based agents over p2p network. In *Management of e-Commerce and e-Government, 2008.I CMECG'08. International Conference on* (2008), IEEE, pp.77–81.
22. Zhao, B. Y., Huang, L., Stribling, J., Rhea, S. C., Joseph, A. D., and Kubiatowicz, J. D. Tapestry: A resilient global-scale over lay for service deployment. *Selected Areasin Communications, IEEE Journal on 22*, 1 (2004), 41–53.

Notes