Global Journal of Science Frontier Research: F
Mathematics and Decision Sciences
Volume 17 Issue 7 Version 1.0 Year 2017
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Soliton-Like Solutions for Some Nonlinear Evolution Equations through the Generalized Kudryashov Method

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GJSFR-F Classification: MSC 2010: 34A34

SOLITONLIKESOLUTIONSFORSOMENONLINEAREVOLUTIONEQUATIONSTHROUGHTHEGENERALIZEDKUDRYASHOVMETHOD

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Md. Shafiqul Islam [α], Md. Babul Hossain [σ] & Md. Abdus Salam [ρ]

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I. Introduction

At the present time, investigating exact solutions of nonlinear evolution equations (NLEEs) are largely used as models to characterize physical phenomena in several fields of science and engineering, especially in biology, solid state physics, plasma, physics and fluid mechanics. Ultimately all the fundamental equations of physics are nonlinear and in general it's very complicate to solve explicitly these types of NLEEs. To solve the inherent nonlinear problems advance nonlinear techniques are very momentous; for the most part of those are involving dynamical system and related areas. Nonetheless, in the last few decades important development has been made and many influential methods for attaining exact solutions of NLEEs have been recommended in the works. Most of the methods found in the literature include, the tanh-sech method [1], simplest equation method [2],the homotopy perturbation method [3,4], Modified method of simplest equation [5,6], Bäcklund Transformations method [7],the (G'/G)-expansion method [8-13], the generalized Kudryashov Method [14,15], the Exp-function method [16,17], the $\exp(-\Phi(\xi))$-expansion method [18], the modified simple equation method [19], Improved F-expansion method [20-23] and so on.

In this article, we would like to discuss further (2+1)-dimensional Breaking Soliton equation, (2+1)-dimensional Burgers equation and (2+1)-dimensional Boussinesq equation by the generalized Kudryashov method. Consequently, more new exact traveling wave solutions have found through these three NLEEs. The (2+1)-dimensional Boussinesq describe the propagation of long waves in shallow water under gravity propagating in both directions. It also arises in other physical applications Such as nonlinear lattice waves, iron sound waves in plasma, and in vibrations in a nonlinear string. The Burgers equation is one of the fundamental model equations in fluid

Ref

1. Eslami, M. and Mirzazadeh, M. (2014)Exact solutions for power-law regularized long-wave and $R(m, n)$ equations with time-dependent coefficients. Reports on Mathematical Physics, 73(1), 77–90. (doi:10.1016/S0034-4877(14)60033-1).

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mechanics. It is also used to describe the structure of shock waves, traffic flow, and acoustic transmission. Burgers equation is completely integrable. The wave solutions of Burgers equation are single and multiple-front solutions.

The plan of this paper is as follows. In Sec. 2, we designate momentarily the generalized Kudryashov method. In Sec. 3, we apply the method to (2+1) -dimensional breaking soliton equation, (2+1)-dimensional Burgers equation and (2+1)-dimensional Boussinesq equation. In sec. 4, graphical representation of particular attained solutions and in sec. 5 Conclusions will be presented finally.

Notes

II. Algorithm of the Generalized Kudryashov Method

In this segment, we elect the generalized Kudryashovmethod looking for the exact traveling wave solutions of some NLEEs.
We consider the NLEEs of the form

$$\Psi(u,\frac{\delta u}{\delta t},\frac{\delta u}{\delta x},\frac{\delta u}{\delta y},\frac{\delta u}{\delta z},\frac{\delta^2 u}{\delta x^2},\frac{\delta^2 u}{\delta y^2},\frac{\delta^2 u}{\delta z^2},\cdots)=0\,, x\in\Psi\,, t>0\,, \quad (1)$$

where $u=u(x,y,z,t)$ is an unfamiliar function, Ψ is a polynomial in u and its innumerable partial derivatives, in which the highest order derivatives and nonlinear terms are engaged. The generalized Kudryashov method carries the following steps [24].

Step 1: The traveling wave transformation $u(x,y,t)=u(\eta),\eta=x+y-ct$ transform Eq. (1) into an ordinary differential equation

$$\mathrm{T}(u,\frac{du}{d\eta},\frac{d^2u}{d\eta^2},\cdots)=0\,, \quad (2)$$

Step 2: Assume that the solution of Eq. (3) has the following form

$$u(\eta)=\frac{\sum_{i=0}^{N} a_i Q^i(\eta)}{\sum_{j=0}^{M} b_j Q^j(\eta)}\,, \quad (3)$$

where $a_i\,(i=0,1,2,\ldots,N)$ and $b_j\,(j=0,1,2,\ldots,M)$ are constants to be determined later such $a_N\neq 0$ and $b_M\neq 0$, and $Q=Q(\eta)$ satisfies the ordinary differential equation

$$\frac{dQ(\eta)}{d\eta}=Q^2(\eta)-Q(\eta)\,. \quad (4)$$

The solutions of Eq. (4) are as follows:

$$Q(\eta)=\frac{1}{1\pm A\exp(\eta)}\,. \quad (5)$$

Step 3: Using the homogeneous balance method between the highest order derivatives and the nonlinear terms in Eq. (2),determine the positive integer numbers N and M in Eq. (3).

Step 4: Substituting Eqs. (3) and (4) into Eq. (2), we find a polynomial in Q^{i-j}, $(i,j=0,1,2,\cdots)$. In this polynomial equating all terms of same power and equating them to zero, we get a system of algebraic equations which can be solved by the Maple or Mathematica to get the unknown parameters $a_i (i = 0,1,2,...,N)$ and $b_j (j = 0,1,2,...,M)$, ω. Consequently, we obtain the exact solutions of Eq. (1).

III. Applications

a) The (2+1)-dimensional Breaking Soliton (BS) equation

Notes

In this subsection, we will implement the generalized Kudryashov method look for the exact solutions of the BS equation.

Let us consider the (2+1)-dimensional BS equation

$$u_{xxxy} - 2u_y u_{xx} - 4u_x u_{xy} + u_{xt} = 0. \tag{6}$$

We apply the traveling wave transformation of the form

$$u(\eta) = u(x, y, t), \qquad \eta = x + y - ct, \tag{7}$$

The wave transformation (7) reduces Eq. (6) into the following ordinary differential equation

$$u^{iv} - 6u'u'' - cu'' = 0, \tag{8}$$

Integrating Eq. (8) with respect to η and neglecting the constant of integration, we obtain

$$u''' - 3(u')^2 - cu' = 0, \tag{9}$$

Balancing homogeneously between the highest order nonlinear term $(u')^2$ and the derivative term u''' in Eq. (9), we attain

$$N = M + 1$$

If we choose $M = 1$ then $N = 2$

Hence for $M = 1$ and $N = 2$ Eq. (3) reduces to

$$u(\eta) = \frac{a_0 + a_1 Q + a_2 Q^2}{b_0 + b_1 Q}, \tag{10}$$

Where a_0, a_1, a_2, b_0 and b_1 are constants to be determined.

Now substituting Eq. (10) into Eq. (9), we get a polynomial in $Q(\eta)$, equating the coefficient of same power of $Q(\eta)$, we attain the following system of algebraic equations:

$$-6a_2 b_1^3 + 3a_2^2 b_1^2 = 0,$$

$$-6a_2^2 b_1^2 - 24a_0 b_0 b_1^2 + 12a_2 b_1^3 + 12a_2^2 b_0 b_1 = 0,$$

$$3a_2^2 b_1^2 - 6a_0 a_2 b_1^2 + 48a_2 b_0 b_1^2 + 6a_1 a_2 b_0 b_1 - 7a_2 b_1^3 - 36a_2 b_0^2 b_1 + 12a_2^2 b_0^2$$
$$+ ca_2 b_1^3 - 24a_2^2 b_0 b_1 = 0,$$

$$12a_2^2 b_0 b_1 + a_2 b_1^3 - ca_2 b_1^3 + 12a_1 a_2 b_0^2 - 12a_1 a_2 b_0 b_1 + 12a_0 a_2 b_1^2 - 12a_0 a_2 b_0 b_1 - 24a_2 b_0^3$$
$$+ 72a_2 b_0^2 b_1 + 4ca_2 b_0 b_1^2 - 24a_2^2 b_0^2 - 28a_2 b_0 b_1^2 = 0,$$

$$6a_0b_0^2b_1-6a_1b_0^2b_1+54a_2b_0^3-6a_0a_1b_0b_1+24a_0a_2b_0b_1+6a_1a_2b_0b_1+12a_2^2b_0^2+6a_0b_0b_1^2$$
$$+4a_2b_0b_1^2-41a_2b_0^2b_1-4ca_2b_0b_1^2+a_0b_1^3-ca_0b_1^3-24a_1a_2b_0^2-6a_0a_2b_1^2$$
$$+3a_0^2b_1^2-a_1b_0b_1^2+3a_1^2b_0^2+ca_1b_0b_1^2+5ca_2b_0^2b_1-6a_1b_0^3=0,$$

$$10a_1b_0^2b_1-6a_1^2b_0^2+12a_0a_1b_0b_1+2ca_1b_0^2b_1-ca_1b_0b_1^2-12a_0a_2b_0b_1+a_1b_0b_1^2-2ca_0b_0b_1^2$$
$$-38a_2b_0^3+12a_1a_2b_0^2-a_0b_1^3+5a_2b_0^2b_1-6a_0^2b_1^2-10a_0b_0b_1^2-5ca_2b_0^2b_1-12a_0b_0^2b_1$$
$$+ca_0b_1^3+12a_1b_0^3+2ca_2b_0^3=0,$$

$$-2ca_2b_0^3+ca_1b_0^3+3a_0^2b_1^2-7a_1b_0^3-2ca_1b_0^2b_1-6a_0a_1b_0b_1+8a_2b_0^3+3a_1^2b_0^2+7a_0b_0^2b_1$$
$$+4a_0b_0b_1^2-4a_1b_0^2b_1-ca_0b_0^2b_1+2ca_0b_0b_1^2=0,$$

$$-a_0b_0^2b_1-ca_1b_0^3+a_1b_0^3+ca_0b_0^2b_1=0.$$

Solving the above system of equations for a_0, a_1, a_2, b_0, b_1 and c, we attain the following values:

Set 1: $c=1, a_1=2b_0, a_2=0, b_1=0.$

Set 2: $c=1, a_0=\dfrac{b_0(-2b_1-2b_0+a_1)}{b_1}, a_2=0.$

Set 3: $c=1, a_0=\dfrac{b_0(a_1-2b_0)}{b_1}, a_2=2b_1.$

Set 4: $c=4, a_0=-0.50a_1, a_2=-4b_0, b_1=-2b_0.$

Set 1 Corresponds the following solutions for Breaking Soliton (BK) equation

$$u_1(\mu)=\frac{a_0+a_0A\exp(\eta)+2b_0}{(1+A\exp(\eta))b_0},$$

where $\eta=x+y-t$.

Set 2 Corresponds the following solutions for Breaking Soliton (BK) equation

$$u_2(\eta)=\frac{-2b_0b_1-2b_0b_1A\exp(\eta)-2b_0^2-2b_0^2A\exp(\eta)+a_1b_0+a_1b_0A\exp(\eta)+a_1b_1}{(b_0+b_0A\exp(\eta)+b_1)b_1}$$

where $\eta=x+y-t$.

Set 3 Corresponds the following solutions for Breaking Soliton (BK) equation

$$u_3(\eta)=\frac{a_1A\exp(\eta)-2b_0A\exp(\eta)-2b_0+a_1+2b_1}{(1+A\exp(\eta))b_1},$$

where $\eta=x+y-t$.

Set 4 Corresponds the following solutions for Breaking Soliton (BK) equation

$$u_4(\eta)=\frac{1}{2}\frac{a_1-a_1A^2\exp(2\eta)-8b_0}{(A^2\exp(2\eta)-1)b_0},$$

Notes

where $\eta = x + y - 4t$.

Remark: All of these solutions have been verified with Maple by substituting them into the original solutions.

b) The (2+1)-dimensional Burgers equation

In this subsection, we will construct the generalized Kudryashov method to find the exact traveling wave solutions of the Burgers equation. Let us consider the (2+1)-dimensional Burgers equation [25]

$$u_t - uu_x - u_{xx} - u_{yy} = 0, \tag{11}$$

Burgers equation arises in various areas of applied mathematics, such as modeling of gas dynamics and various vehicle densities in high way traffic [26].The wave transformation (7) reduces Eq. (11) into the following ordinary differential equations

$$cu' + uu' + 2u'' = 0, \tag{12}$$

Integrating Eq. (12) with respect to ξ and neglecting the constant of integration, we obtain

$$cu + \frac{u^2}{2} + 2u' = 0, \tag{13}$$

Considering the homogeneous balance between the highest order nonlinear term u^2 and the derivative term u' in Eq. (13), we attain

$$N = M + 1.$$

If we choose $M = 1$ then $N = 2$

Hence for $M = 1$ and $N = 2$ Eq. (3) reduces to

$$u(\eta) = \frac{a_0 + a_1 Q + a_2 Q^2}{b_0 + b_1 Q}, \tag{14}$$

Where a_0, a_1, a_2, b_0 and b_1 are constants to be determined.

Now substituting Eq. (14) into Eq. (13), we get a polynomial in $Q(\eta)$, equating the coefficient of same power of $Q(\eta)$, we attain the following system of algebraic equations:

$$4a_2 b_1 + a_2^2 = 0,$$

$$2ca_2 b_1 + 2a_1 a_2 + 8a_2 b_0 - 4a_2 b_1 = 0,$$

$$-4a_0 b_1 + 4a_1 b_0 + a_1^2 + 2ca_1 b_1 + 2ca_2 b_0 + 2a_0 a_2 - 8a_2 b_0 = 0,$$

$$2ca_0 b_1 + 4a_0 b_1 - 4a_1 b_0 + 2ca_1 b_0 + 2a_0 a_1 = 0,$$

$$a_0^2 + 2ca_0 b_0 = 0.$$

Ref

26. Polyanin, A. D. and Zaitsev, V. F. (2004) Handbook of Nonlinear Partial Differential Equations, Chapman & Hall/CRC, Boca Raton.

Solving the above system of equations for a_0, a_1, a_2, b_0, b_1 and c, we attain the following values:

Set 1: $c = 4, a_0 = 0, a_1 = 0, a_2 = -4b_1, b_0 = -0.50b_1$.

Set 2: $c = 2, a_0 = 0, a_1 = -4b_0 - 4b_1, a_2 = 0$.

Set 3: $c = 2, a_0 = 0, a_1 = -4b_0, a_2 = -4b_1$.

Set 4: $c = -2, a_0 = 4b_0, a_1 = -4b_0, a_2 = 0$.

Set 5: $c = -2, a_0 = -a_1 + 4b_1, a_1 = -4b_1, b_0 = b_1 - 0.25a_1$.

Set 6: $c = -4, a_0 = -4b_1, a_1 = 8b_1, a_2 = -4b_1, b_0 = -0.50b_1$.

Set 1 Corresponds the following solutions for Burgers equations

$$u_1(\eta) = \frac{8}{A^2 \exp(2\eta) - 1},$$

Where $\eta = x + y - 4t$.

Set 2 Corresponds the following solutions for Burgers equations

$$u_2(\eta) = -\frac{4(b_0 + b_1)}{b_0 + b_0 A \exp(\eta) + b_1},$$

Where $\xi = x + y - 2t$.

Set 3 Corresponds the following solutions for Burgers equations

$$u_3(\eta) = -\frac{4}{1 + A\exp(\eta)},$$

Where $\eta = x + y - 2t$.

Set 4 Corresponds the following solutions for Burgers equations

$$u_4(\eta) = \frac{4b_0 A\exp(\eta)}{b_0 + b_0 A\exp(\eta) + b_1},$$

Where $\mu = x + y + 2t$.

Set 5 Corresponds the following solutions for Burgers equations

$$u_5(\eta) = \frac{4A\exp(\eta)}{1 + A\exp(\eta)},$$

Where $\eta = x + y + 2t$.

Set 6 Corresponds the following solutions for Burgers equations

$$u_6(\eta) = \frac{8A^2 \exp(2\eta)}{A^2 \exp(2\eta) - 1},$$

Where $\eta = x + y + 4t$.

Notes

Remark: All of these solutions have been verified with Maple by substituting them into the original solutions.

c) The (2+1)-dimensional Boussinesq equation

In this subsection, we will use the generalized Kudryashov method to find the exact traveling wave solutions of the Boussinesq equation. Let us consider the (2+1)-dimensional Boussinesq equation [27] is in the form

$$u_{tt}-u_{xx}-u_{yy}-\left(u^2\right)_{xx}-u_{xxxx}=0, \tag{15}$$

which describes the propagation of gravity waves on the surface of water. The wave transformation (7) reduces Eq. (15) into the following ordinary differential equations

$$\left(c^2-2\right)u''-\left(u^2\right)''-u^{iv}=0, \tag{16}$$

Integrating Eq. (16) with respect to η and neglecting the constant of integration, we obtain

$$\left(c^2-2\right)u-u^2-u''=0, \tag{17}$$

Considering the homogeneous balance between the highest order nonlinear term u^2 and the derivative term u'' in Eq. (13), we attain $N=M+2$.

If we choose $M=1$ then $N=3$

Hence for $M=1$ and $N=3$ Eq. (3) reduces to

$$u(\eta)=\frac{a_0+a_1Q+a_2Q^2+a_3Q^3}{b_0+b_1Q}, \tag{18}$$

where a_0, a_1, a_2, a_3, b_0 and b_1 are constants to be determined.

Now substituting Eq. (18) into Eq. (17), we get a polynomial in $Q(\eta)$, equating the coefficient of same power of $Q(\eta)$, we attain the following system of algebraic equations:

$$a_3^2b_1+6a_3b_1^2=0,$$

$$a_3^2b_0+2a_2a_3b_1+16a_3b_0b_1+2a_2b_1^2-10a_3b_1^2=0,$$

$$12a_3b_0^2-3a_2b_1^2+6a_3b_1^2+2a_2a_3b_0+a_2^2b_1-27a_3b_0b_1+6a_2b_0b_1+2a_1a_3b_1-c^2a_3b_1^2=0,$$

$$a_2^2b_0+3a_2b_1^2+15a_3b_0b_1-2c^2a_3b_0b_1+2a_0a_3b_1+6a_2b_0^2+2a_1a_3b_0-21a_3b_0^2-c^2a_2b_1^2$$
$$-9a_2b_0b_1+2a_1a_2b_1=0,$$

$$7a_2b_0b_1+11a_3b_0^2-a_0b_1^2-c^2a_3b_0^2-c^2a_1b_1^2+a_1^2b_1+2a_1a_2b_0+2a_1b_0^2-2c^2a_2b_0b_1+2a_1b_1^2$$
$$+2a_0a_3b_0-10a_2b_0^2-2a_0b_0b_1+2a_0a_2b_1+a_1b_0b_1=0,$$

$$2a_0a_1b_1+2a_0a_2b_0+3a_0b_0b_1-c^2a_2b_0^2-2c^2a_1b_0b_1-c^2a_0b_1^2+a_1^2b_0+3a_0b_1^2-3a_1b_0^2$$
$$+3a_1b_0b_1+6a_2b_0^2=0,$$

$$3a_0b_0b_1+a_0^2b_1+3a_1b_0^2-2c^2a_0b_0b_1-c^2a_1b_0^2+2a_0a_1b_0=0,$$

$$2a_0b_0^2+a_0^2b_0-c^2a_0b_0^2=0.$$

Ref

27. Moleleki, L. D. and Khalique, C. M. (2013) Solutions and conservation laws of a (2+1) dimensional Boussinesq equation, Abstract and Applied Analysis, 2013, 8.

Solving the above system of equations for $a_0, a_1, a_2\, a_3, b_0, b_1$ and c, we attain the following values:

Set 1: $c = \pm\sqrt{3}, a_0 = 0, a_1 = 6b_0, a_2 = -6b_0 + 6b_1$.

Set 2: $c = \pm 1, a_0 = -b_0, a_1 = -b_1 + 6b_0, a_2 = -6b_0 + 6b_1$.

Set 1 Corresponds the following solutions for Boussinesq equation

$$u_1(\eta) = \frac{6A\exp(\eta)}{(1 + A\exp(\eta))^2},$$

where $\eta = x + y + \sqrt{3}t$.

Set 2 Corresponds the following solutions for Boussinesq equation

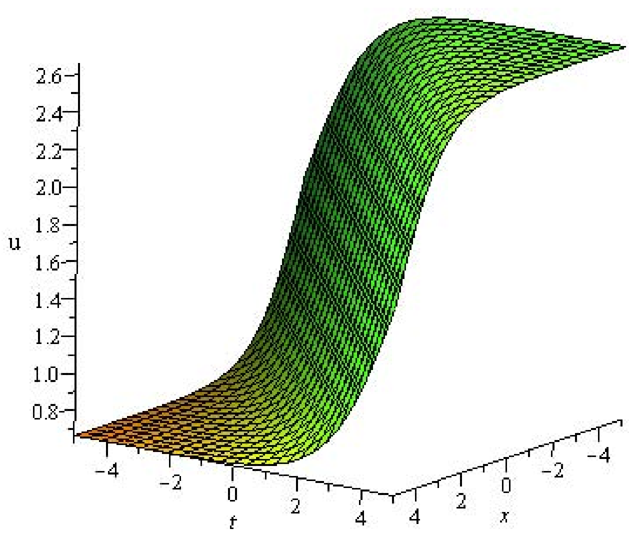
$$u_2(\eta) = -\frac{1 - 4A\exp(\eta) + A^2\exp(2\eta)}{(1 + A\exp(\eta))^2}.$$

where $\eta = x + y + t$.

Remark: All of these solutions have been verified with Maple by substituting them into the original solutions.

IV. Graphical Representation of Some Obtained Solutions

The graphical presentations of some obtained solutions are depicted in Figures 1–7 with the aid ofcommercial software Maple 13.



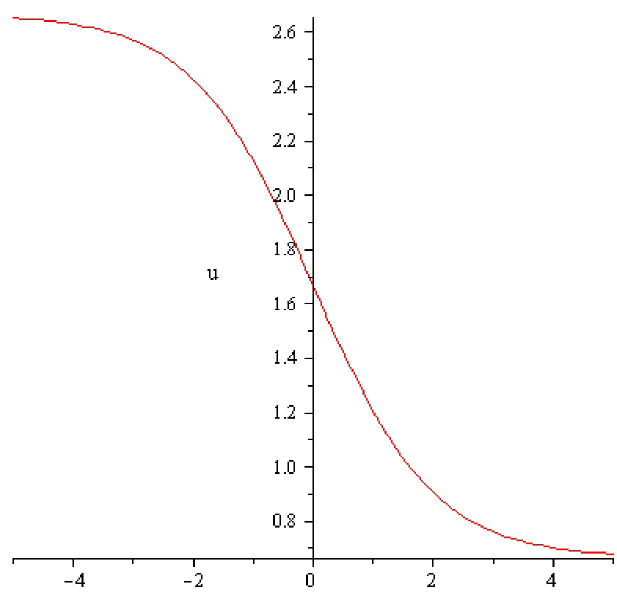


Fig. 1: Kink shaped soliton of BS equation for $A = 1, a_0 = 2, b_0 = 2, y = 0$ within the interval $-5 \le x, t \le 5$. (Only shows the shape of $u_1(\eta)$), the left figure shows the 3D plot and the right figure shows the 2D plot for $t = 0$

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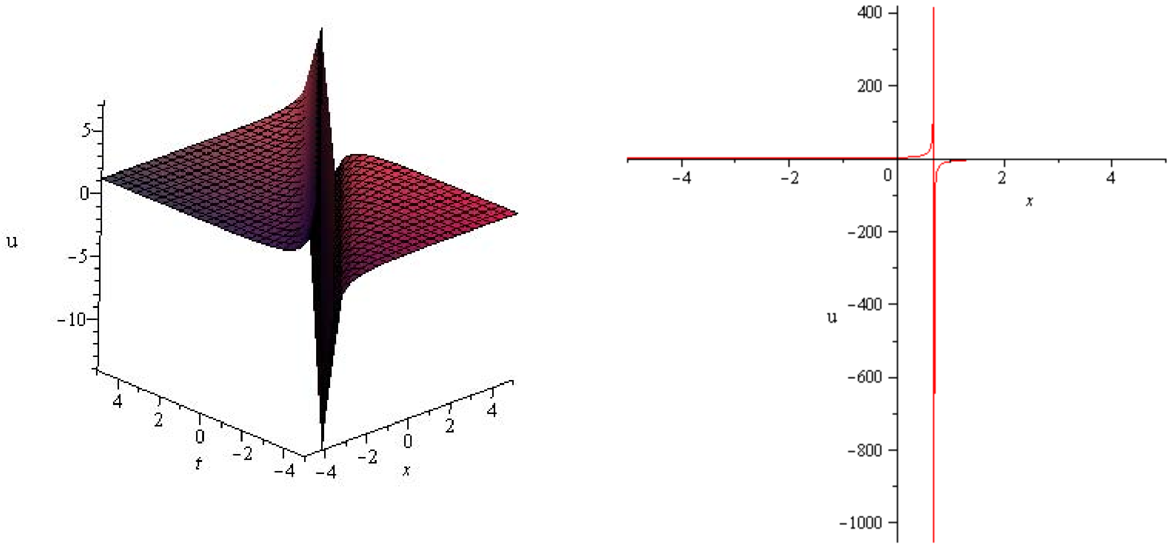

Fig. 2: Singular kink soliton of BK equation for $A=-0.50, a_1=2, b_0=3, b_1=5, y=0$ within the interval $-5\le x,t\le 5$. (Only shows the shape of $u_3(\eta)$), the left figure shows the 3D plot and the right figure shows the 2D plot for $t=0$

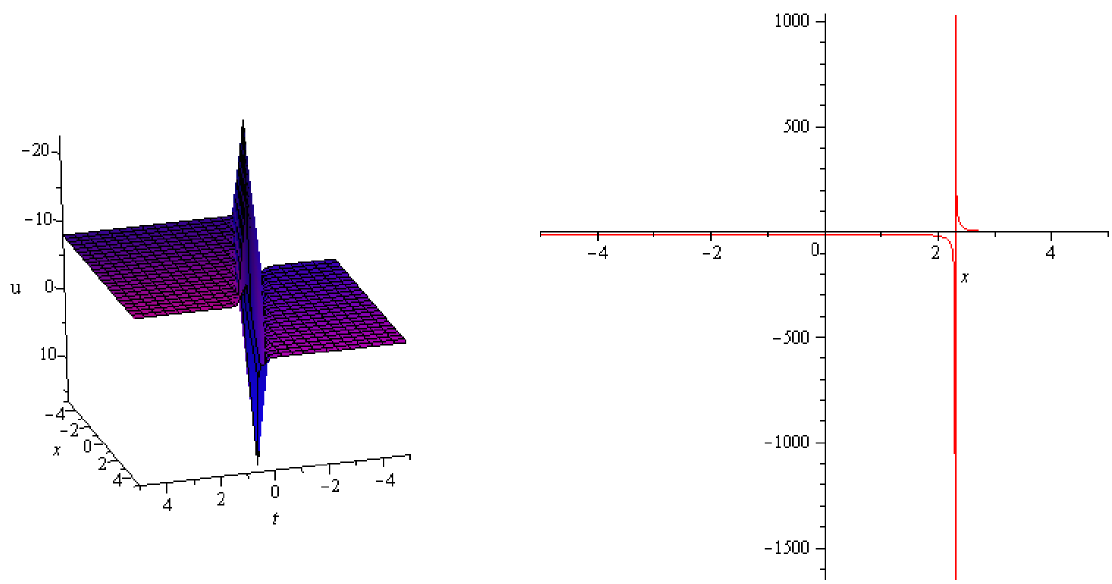

Fig. 3: Singular kink soliton of Burgers equation for $A=-0.10$, $y=0$ within the interval $-5\le x,t\le 5$. (Only shows the shape of $u_1(\eta)$), the left figure shows the 3D plot and the right figure shows the 2D plot for $t=0$

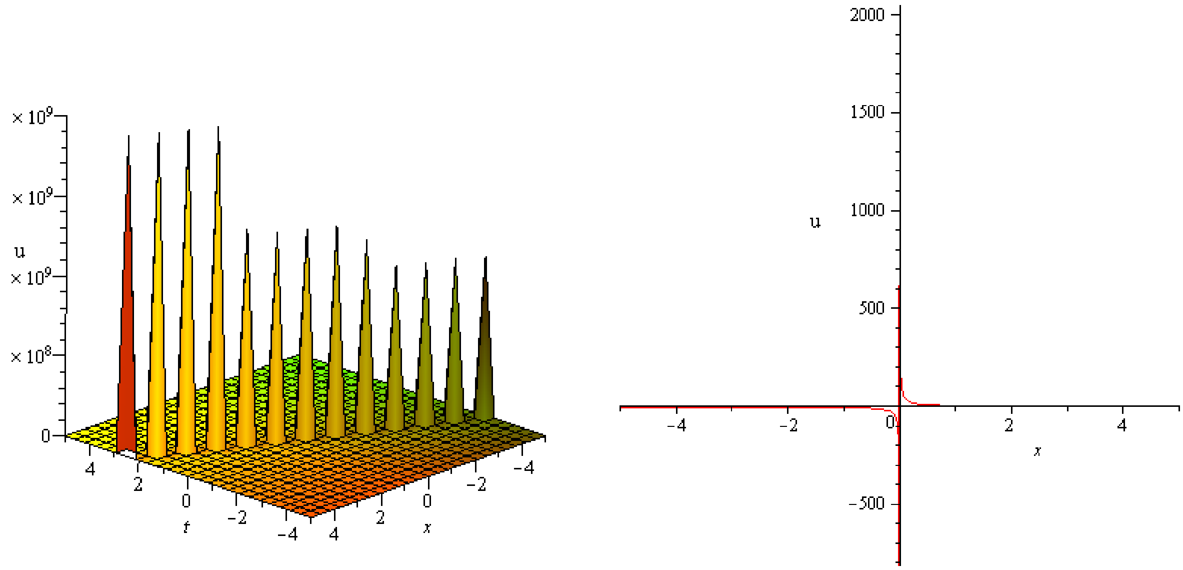

Fig. 4: Singular soliton of Burgers equation for $A=-1$, $y=0$ within the interval $-5 \le x,t \le 5$. (Only shows the shape of $u_3(\eta)$), the left figure shows the 3D plot and the right figure shows the 2D plot for $t=0$

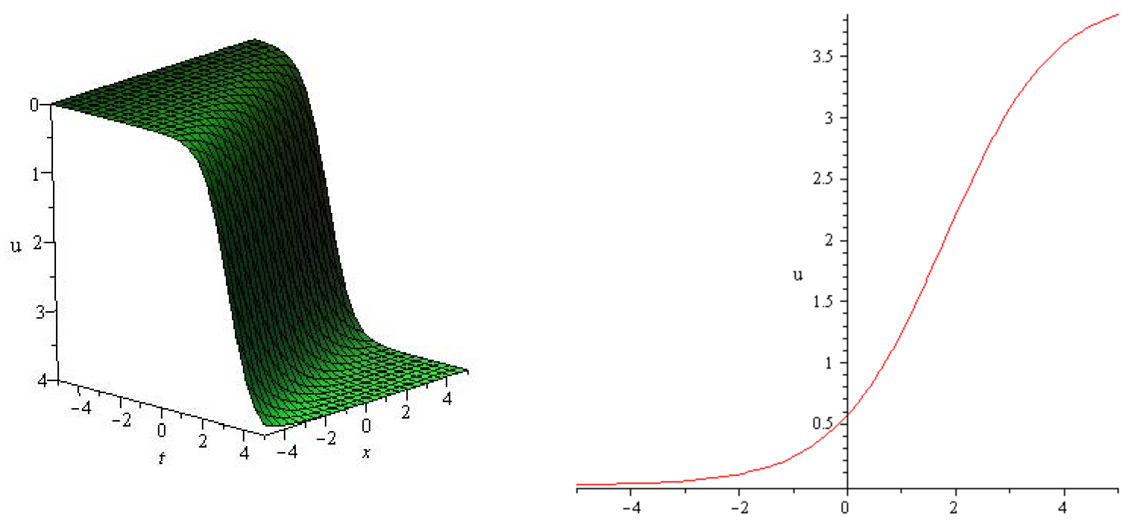

Fig. 5: Kink shaped soliton of Burgers equation for $A=0.50$, $b_0=1, b_1=2$, $y=0$ within the interval $-5 \le x,t \le 5$. (Only shows the shape of $u_4(\eta)$), the left figure shows the 3D plot and the right figure shows the 2D plot for $t=0$

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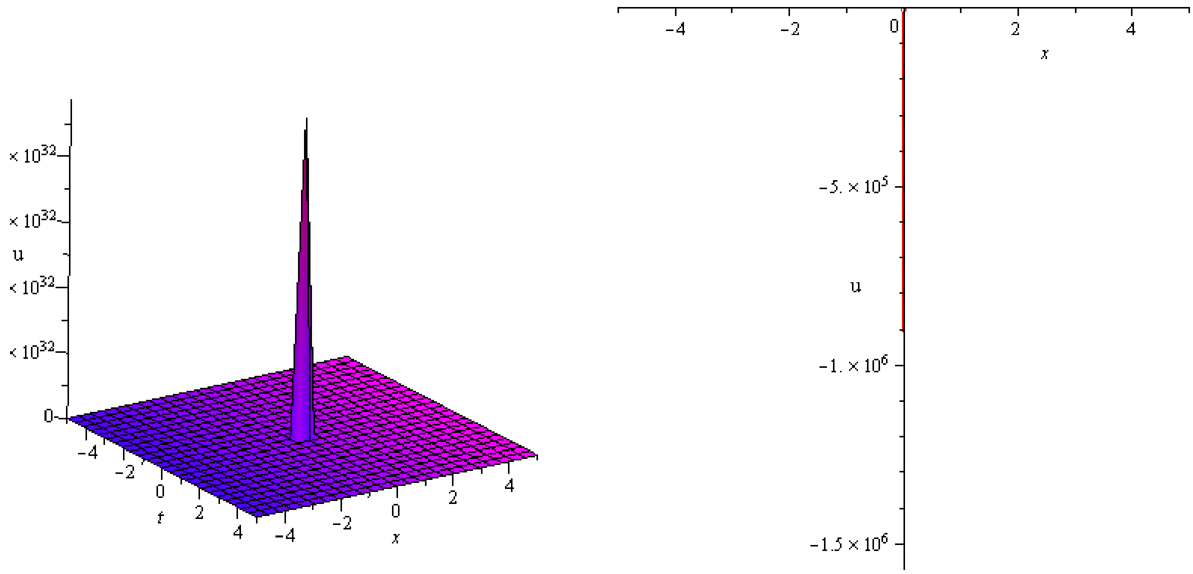

Fig. 6: Single soliton of Boussines q equation for $A=-1,\ y=0$ within the interval $-5\le x,t\le 5$. (Only shows the shape of $u_1(\eta)$), the left figure shows the 3D plot and the right figure shows the 2D plot for $t=0$

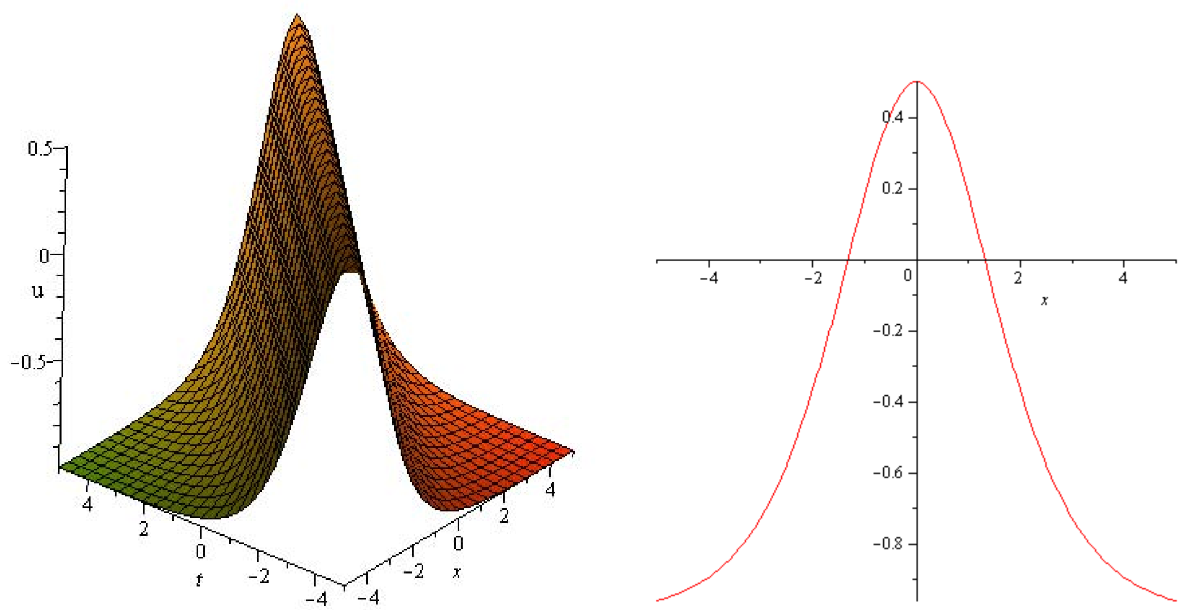

Fig. 7: Bell shaped solitonof Boussinesq equation for $A=1,\ y=0$ within the interval $-5\le x,t\le 5$. (Only shows the shape of $u_2(\eta)$), the left figure shows the 3D plot and the right figure shows the 2D plot for $t=0$

V. Conclusions

In this article, using the MAPLE 13 software the generalized Kudryashov method is executed to investigate the nonlinear evolution equations, namely (2+1)-dimensional Breaking soliton (BS) equation, (2+1)-dimensional Burgers equation, (2+1)-dimensional Boussinesq equation. All the attained solutions in this study verified

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these three NLEEs; we checked this using the MAPLE 13 software. Moreover, the obtained results in this work clearly demonstrate the reliability of the generalized Kudryashov method. This method can be more successfully applied to study nonlinear evolution equations, which frequently arise in nonlinear sciences.

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