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The American Physical Society and Errors in Gravitation

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Abstract- In spite of accurate predictions and an impressive explanation on perihelion of Mercury, Einstein's general relativity accepted by the APS is currently infested with unsolved problems due to his own earlier errors together with additional errors from his followers. Although subsequently his linearized equation has provided more impressive results, Einstein was puzzled because his equation has no wave solutions, but his linearized equation has. The root is due to his accumulated inadequate proposals in physics, and inadequacy in pure mathematics of many physicists such as Pauli. Moreover, the contributions from mathematicians such as Penrose and Yau made the situation worse because they use invalid assumptions due to not understanding physics. These errors are responsible for the lack of progress in gravitation of the APS.

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Abstract- In spite of accurate predictions and an impressive explanation on perihelion of Mercury, Einstein's general relativity accepted by the APS is currently infested with unsolved problems due to his own earlier errors together with additional errors from his followers. Although subsequently his linearized equation has provided more impressive results, Einstein was puzzled because his equation has no wave solutions, but his linearized equation has. The root is due to his accumulated inadequate proposals in physics, and inadequacy in pure mathematics of many physicists such as Pauli. Moreover, the contributions from mathematicians such as Penrose and Yau made the situation worse because they use invalid assumptions due to not understanding physics. These errors are responsible for the lack of progress in gravitation of the APS. Currently, there are ten major errors: 1) misinterpretation of the equivalence principle and the invalid covariance principle; 2) the conflict between the Einstein equation and $E = mc^2$; 3) inadequacy of Einstein's notion of photons and invalidity of $E = mc^2$; 4) the invalid Space time Singularity Theorems of Hawking & Penrose; 5) the non-existence of a dynamic solution and the Principle of Causality; 6) the rejection of the anti-gravitational coupling; 7) the absence of the reaction force for gravitational radiation; 8) the existence of repulsive gravitation; 9) the weight reduction of a charged capacitor and the current-mass interaction; 10) the necessary extension of general relativity to a five-dimensional space. A main problem is that physicists are incompetent in pure mathematics. This results in awarding the 2016 APS Medal for Exceptional Achievement in Research to Witten even without the support of experiments. There are three types of experiments that show formula $E = mc^2$ is invalid, but the APS still did not recognize them because of the incompetence of Dr. Eric J. Weinberg, Editor of the Physical Review D, in mathematics and physics. Moreover, many had made the same errors as Eric J. Weinberg because of the inadequacy in mathematics and physics. Thus, for the progress of physics, the APS must improve the level of pure mathematics and rectify the past errors in gravitation.

Keywords: anti-gravity coupling; gravitational radiation; repulsive gravitation; principle of causality; unification.

I. Introduction

Since Einstein's prediction on the bending of light rays was verified, general relativity is dominating astrophysics [1]. Subsequently, predictions based on the linearization of the Einstein equation were verified [1, 2]. Then, Einstein is established as a genius.

However, Einstein has the puzzle that the non-linear Einstein equation does not support the existence of a gravitational wave solution [3, 4] although the linearized equation assured the existence of a gravitational wave solution. In fact, Einstein concluded his talk on gravitational waves at Princeton University by saying [5] "If you ask me whether there are gravitational waves or not, I must answer that I do not know. But it is a highly interesting problem." Thus, Einstein's last words on this subject was that "I do not know." However, the Physical Review insisted that the gravitational wave solution exists because they believed that an unbounded solution is valid in physics so long as there is no singularity [6]. This started the conflict between Einstein and the American Physical Society (APS).

First, according to the principle of causality [7], a valid solution in physics must be bounded. Second, the existence of an unbounded gravitational solution is useless in physics because the calculation of a gravitational radiation requires a bounded solution [8, 9]. Thus, in 1937 this marked the beginning of a process that the APS added errors to general relativity, in addition to those errors already in Einstein's theory. Another example of such a conflict is the unbounded solution of Penrose [10] for an electromagnetic wave published in the Review of Modern Physics in 1964. His unbounded solution is in disagreement with the implicit assumption used in the calculation for the bending of light in which the gravity of the electromagnetic wave is assumed to be negligible.

Moreover, Einstein's equivalence principle was misinterpreted by the Wheeler School [2)] to become mathematically impossible [1, 11] (see Appendix A). Nevertheless, due to the general inadequacy of the APS in pure mathematics, their invalid interpretation was prevailingly accepted. Moreover, the APS was not aware that Einstein's covariance principle is inconsistent with Einstein's equivalence principle.

In this paper, we shall explain the errors in general relativity that have been accepted by the APS as valid because of inadequacy in mathematics and/or physics. In particular, the over-looked inconsistence in Einstein's theory must be identified and rectified.

II. The Invalid Covariance Principle and Measurements in General Relativity

Einstein's general relativity consists of three parts: 1) the equivalence principle; 2) the covariance principle; and 3) the Einstein field equation. Here, we

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shall discuss the equivalence principle and the covariance principle, but later the problems in the Einstein equation,

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = - \kappa T_{\mu\nu} , \quad (1)$$

where $g_{\mu\nu}$ is the space-time metric, $R_{\mu\nu}$ is the Ricci curvature tensor, $R = R_{\alpha\beta} g^{\alpha\beta}$ is the Ricci curvature, $T_{\mu\nu}$ is the sum of energy-stress tensors and κ is the coupling constant. Einstein's equivalence principle is a starting point of general relativity, and thus everybody paid lip service to it, but currently no text book interprets Einstein's equivalence principle correctly [11] except in Einstein's own book [12, 13].

As I have mentioned, the Wheeler School, Pauli [14] and the British Encyclopedia have misinterpreted this principle to become mathematically impossible [11]. In fact, Wald [2] has abandoned this principle and the 1993 Nobel Prize Committee for Physics incorrectly follows Wald [15]. Nevertheless, the consequences of this principle, i.e., the space contractions and the time dilation are supported by experiments [1, 12, 13]. Currently the covariance principle is most popular because of the mathematical freedom it provides.

However, as pointed out by Zhou [16], Einstein's covariance principle implicitly assigns different physical meaning to coordinates for different gauges. Thus, it is actually inconsistent with his equivalence principle that gives definite measurements to the time dilation and space contractions [17]. Note that the gauge independence of light bending [1] is only a coincidental, but not a proof for the gauge invariance since the same result can be obtained from unphysical gauges [18].

Einstein's "principle of covariance" has no theoretical basis beyond the principle of general relativity [17]. Philip Morrison of MIT [3] pointed out that the "covariance principle" is invalid since it disrupts the necessary continuity from special relativity to general relativity [17]. The covariance principle [12] was proposed to justify Einstein's adaptation of the distance in a Riemannian space. This has been pointed out by Whitehead [19] as invalid in physics. Nevertheless, Misner et al. [1, p. 430] claimed that the covariance principle can be verified, but they provided the opposite evidence. For instance, Will [1; p. 1067] claimed Whitehead's theory is invalid; but the solution of Whitehead is actually diffeomorphic to Einstein's [20].

Nevertheless, gauge invariance has a long history starting from electrodynamics. Then, gauge invariance has been developed according to non-Abelian gauge theories such as the Yang-Mills-Shaw theory [21, 22]. [4] They naively extended the invariance of the Abelian gauge to the cases of the Non-Abelian gauges in terms of mathematics. However, as shown by Aharonov & Bohm [23] the electromagnetic potentials actually are physically effective; and, as shown by Weinberg [24], all the physical non-Abelian gauge theories are not gauge invariant such that different masses can be generated. These facts support the view that gauge invariance of the whole theory would be a manifestation that there are some deficiencies [25, 26].

A counter example was provided by Bodenner & Will [27] and Gérard & Piereaux [28]. Although they showed that the deflection angle is gauge invariant to the second order, but the impact parameter b of the light ray and the shortest distance r_0 from the light ray to the center of the sun showed that these physical quantities cannot be both gauge invariant. From the harmonic gauge and the Schwarzschild gauge, one has respectively

$$b \approx 2\kappa M + r_0 , \quad (2a)$$

but

$$b \approx \kappa M + r_0 . \quad (2b)$$

Thus, Einstein's covariance principle is invalid since two physical quantities should not have gauge dependent relations.

Another counter example is the formulas for the de Sitter precession [29]. From the Maxwell-Newton Approximation [8], one would obtain a formula that is different from the formula obtained from the Kerr metric [2]. Unfortunately, the difference cannot be distinguished by the Stanford experiment, gravity Probe-B because this experiment detects only the time average, of which the difference is essentially zero [17]. Thus, this experiment of Stanford has a design failure. Moreover, the experiment on local light speeds [30] would show the break down of gauge invariance.

Now, we should find out what went wrong in Einstein's consideration on gauge invariance. To argue for the covariance principle, Einstein [12] claimed that for a circle in a uniformly rotating disk K' relative to an inertial system K, the quotient of the circumference and the diameter would be greater than π. This is due to that a measuring rod at rest relatively to K', when measured from K, the measuring-rod applied to the periphery undergoes a Lorentzian contraction, while the one applied along the radius does not. Nevertheless, many (included this author) had failed to see that his arguments are actually invalid. To see Einstein's errors, one must go into the details instead of glossing over his claims.

Consider a particle P resting at K'(r', ϕ', z', ct'). The local space of P is L*(dR, dX, dz', cdT) with a Minkowski metric. In K, P has a position (r, ϕ, z,) and its local space (dr, rdϕ, dz, cdt) has the Minkowski metric. These two local spaces have a relative velocity rΩ in the ϕ–direction. Moreover, let X be in the same direction of rdϕ.

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Then, according to special relativity, one has dz = dz' and dr = dR, and the Lorentz transformation as follows:

$$r d\phi = [1 - (r\Omega/c)^2]^{-1/2} [dX + r\Omega dT], \quad (3a)$$

and

$$c dt = [1 - (r\Omega/c)^2]^{-1/2} [c dT + (r\Omega/c) dX]; . \quad (3b)$$

or

$$dX = [1 - (r\Omega/c)^2]^{-1/2} [r d\phi - r\Omega dt], \quad (4a)$$

and

$$dT = [1 - (r\Omega/c)^2]^{-1/2} [dt - (r\Omega/c^2) r d\phi]. \quad (4b)$$

It follows that if dX is measured simultaneously (i.e. dt = 0) from K, from (4a) one has

$$dX = [1 - (r\Omega/c)^2]^{-1/2} [r d\phi]. \quad (5a)$$

This is a space contraction for L* (dX > r dϕ). For a clock fixed at L* (i.e., dX = 0), from (3b) we would have

$$c dT = [1 - (r\Omega/c)]^{1/2} c dt \quad (5b)$$

if measured from K. This is a time dilation for L* (dt > dT).

From (5a), Einstein concluded that U/D > π, where D is the diameter of a circle and U is its circumference. Since all the measurements in (5a) are done in K, Einstein mistakenly considered the integration as

$$U = (1/2)[1 - (D\Omega/2c)^2]^{-1/2} \oint D d\phi = \pi D [1 - (D\Omega/2c)^2]^{-1/2} \quad (6)$$

to be valid. However, the error is that the distance dX in (5a) is not in K', but in a local space L*, which depends on t and ϕ. Although all L*s at different ϕ are at rest relative to K', they are *under different accelerations*. Thus, integration (6) would not make sense as distance in K'. Moreover, the space K is in a relative motion with respect to K'. *This is in a different situation for space contractions and the time dilation since the space S and such a local space L are at rest with each other.*

Therefore, (5a) and (5b) actually have nothing to do with Einstein's equivalence principle. Einstein's claims for space contractions and the time dilation are supported with invalid arguments due to an over-sight on special relativity. Thus, Einstein had not provided a theoretical basis for his theory of measurement. Thus, it should be examined carefully.

Therefore, to clarify the issue of measurements, one should derive a space-time metric and show that such a metric satisfies Einstein's equivalence principle. We shall derive a metric for the case of a rotating disk. According to Landau and Lifshitz [31], the transformation to a uniformly rotating reference frame K'(x', y', z') [2)] with angular velocity Ω has the form,

$$x = x' \cos \Omega t - y' \sin \Omega t, \qquad y = x' \sin \Omega t + y' \cos \Omega t, \qquad \text{and} \qquad z = z', \quad (7a)$$

or

$$r = r', \qquad z = z', \qquad \text{and} \qquad \phi = \phi' + \Omega t \quad (7b)$$

Then a metric in terms of the coordinates in K'(x', y', z') can be obtained from(7b); and

$$dr = dr', \; dz = dz', \text{ and } d\phi = d\phi' + \Omega dt. \quad (7c)$$

The transformed metric in system K'*(x' y', z', ct) would have the following form,

$$ds^2 = (c^2 - \Omega^2 r^2) dt^2 - 2\Omega r^2 d\phi' dt - dr^2 - r^2 d\phi'^2 - dz'^2 \quad (8)$$

and

$$g^{ct\,ct} = 1, \quad g^{r'r'} = g^{z'z'} = -1, \; g^{\phi'\phi'} = -(1 - \Omega^2 r'^2/c^2)/r'^2, \; g^{\phi' ct} = g^{ct\,\phi'} = -\Omega/c \quad (9)$$

are the non-zero elements of the inverse metric, Then, one obtains that the force on a resting particle P with mass m is mv^2/r', which is also the force due to rotation. Note that (7a) implies also

$$r' = r, \quad x' = r \cos \phi', \quad \text{and} \quad y' = r \sin \phi', \quad (10)$$

Thus, (10) means K'(x', y' z') also has a Euclidean-like structure. Thus Einstein's claim of measurement would be incorrect.

The system K'* (x', y', z', ct) with metric (8) could have led to the "light speed" rdϕ'/dt larger than c because t is related to the local clocks resting at (x, y, z). Moreover, one could have misinterpreted metric (8) as having no space contraction. To rectify these problems, one must have a metric in terms of the local time t' of K'(x', y' z').

Now, consider the local space L*, from (4a), (4b) and (7c) we have,

$$dX = [1 - (r\Omega/c)^2]^{-1/2} r d\phi', \quad (11a)$$

and

$$dT = [1 - (r\Omega/c)^2]^{1/2} \{dt - [1 - (r\Omega/c)^2]^{-1} (r\Omega/c^2) r d\phi'\} \quad (11b)$$

Then we have

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$$ds^2 = (c^2 - \Omega^2 r^2)\,\{dt - [1 - (r\Omega/c)^2]^{-1}(r\Omega/c^2)\, r d\phi'\}^2 - dr^2 - [1 - (r\Omega/c)^2]^{-1} r^2\, d\phi'^2 - dz'^2. \quad (11c)$$

We note that local space L* is the local space of the Einstein-Minkowski condition. Consequently, we should have

$$ds^2 = g_{t't'}\, c^2 dt'^2 - dr'^2 - (1 - \Omega^2 r'^2/c^2)^{-1} r'^2\, d\phi'^2 - dz'^2. \quad (12)$$

Now, (11a) shows that the metric has space contractions. According to Landau & Lifshitz [31], we should have

$$ds^2 = (c^2 - \Omega^2 r'^2) dt'^2 - dr'^2 - (1 - \Omega^2 r'^2/c^2)^{-1} r'^2\, d\phi'^2 - dz'^2. \quad (13)$$

where

$$dT = [1 - (r\Omega/c)^2]^{1/2}\, dt' \quad \text{and} \quad cdt' = cdt - (r\Omega/c) r d\phi' [1 - (r\Omega/c)^2]^{-1}. \quad (14)$$

Eq. (14), which is different from (5b), implies that for a local clock fixed at K' an observer at K would have

$$dt' = dt. \quad (15a)$$

Thus, as Kündig [32] has shown, time dilation (14) can be verified. Moreover, since r = r', (11a) and (5a) imply

$$r d\phi\, [1 - (r\Omega/c)^2]^{-1/2} = dX = r' d\phi'\, [1 - (r\Omega/c)^2]^{-1/2}, \quad \text{and} \quad r d\phi' = r' d\phi \quad (15b)$$

Thus, *Einstein's claim of U/D > π is not valid*. 5) Note that $[1 - (r\Omega/c)^2]^{-1/2}\, r d\phi'$ is a distance measured in different L*. Therefore, the integration of (11a) is not a distance in K'. However, relations (5a) all are measured in K.

Moreover, Einstein's theory of measurement is invalid because Einstein incorrectly regarded space contractions, which are obtained from a local space at free fall, as measured in the frame of reference.

Note that metrics (8) and canonical metric (13) relate to each other with the relations (7c) and

$$cdt' = cdt - (r\Omega/c)\, r d\phi' [1 - (r\Omega/c)^2]^{-1}. \quad (16)$$

If one believed that dt' was a global variable, then one integrated them, and one would get (7b), and

$$ct' = ct - (r\Omega/c)\, r\phi' [1 - (r\Omega/c)^2]^{-1} \quad (17)$$

if (16) were integrable. A problem is that $\phi' \pm 2\pi$ is the same position, but t and t' would not be the same. As shown in (14), dt' is related to dT of the local inertial systems L*(dR, dX, dz', cdT) at different t, r, and ϕ', and thus (16) is not integrable.

Now, the Einstein-Minkowski condition [12] is satisfied, and Einstein's notion of local time and local clocks are supported. Thus, the Euclidean-like structure (10) is physically realizable in terms of measurements. In turn, this implies directly that Einstein's claim on larger circumference/diameter ratio of a circle is invalid.

Einstein stated that the light speed is measured "in the sense of Euclidean geometry [12]". In fact, *all Einstein's predictions are in terms of the Euclidean-like structure.* A ray of light, traveling at a shortest distance Δ from the center of sun with mass M, will be deflected by an amount [12, 13] $M\kappa/2\pi\Delta$. The secular rotation of the elliptic orbit of the planet in the same sense as the revolution of the planet, amounting in radius per revolution to $24\pi^3 a^2/(1 - e^2)c^2T^2$. In addition to Δ, *e* the numerical eccentricity and *a* the semi-major axis of the planetary orbit in centimeters are defined in terms of the Euclidean-like structure, and T the period of revolution in seconds is defined in terms of the time of a "quasi-Minkowskian space" [33]. Thus, it is clear that Einstein's theory of measurement has not been used in his calculations.

In other words, his invalid theory of measurement is actually independent of the theory of general relativity. Moreover, calculation for the bending of light has actually proved that his theory of measurement is experimentally invalid since it would imply only half of the observed value of light bending [34]. The correct theory is, however, just what Einstein practiced in his calculation of the bending of light [30].6) However, the covariance principle remains a useful mathematical tool [33]. Nevertheless, the approach of Wald [2], though accepted by the Nobel Prize Committee, is incorrect [35].

Now, it is clear that the covariance principle is invalid in physics. This principle is due to a confusion between physics and mathematics. Nevertheless, the linearized harmonic gauge is valid for the Maxwell-Newton Approximation [7-9] although the gauge for the Einstein equation is still not clear. Invalidity of the covariance principle leads to the important question of what the physical gauge is? Currently, we still do not yet have an answer. Zhou [16] chose the harmonic gauge. This seems appropriate since it is consistent with the linearized gauge for the case of Maxwell-Newton Approximation [7-9].

III. The Conflict between the Einstein Equation and E = MC²

We usually cite the achievements of Einstein on his field equation and the formula $E = mc^2$. However, few recognized that they are actually not always consistent [35]. Since the Einstein equation is the basis of the numerous predictions the only choice seems to

be just what can be done on the unverified E = mc^2 [36] if we do not want to abandon general relativity.
According to the Einstein equation (1), we have

$$R = R_{\alpha\beta}g^{\alpha\beta} = \kappa T_{\alpha\beta}g^{\alpha\beta}, \qquad (18)$$

for the space-time metric $g_{\alpha\beta}$. Since the electromagnetic energy-stress tensor is traceless, the electromagnetic energy cannot affect the Ricci curvature R, but the mass can. Thus, it is clear that the electromagnetic energy cannot be equivalent to mass.

One might argue that experiments show that a π_0 meson can decay into two photons. This would support that the electromagnetic energy is equivalent to mass, according to the proposal of Einstein [37] that photons consist of only electromagnetic energy. However, experimentally, Einstein's proposal is only partially verified since nobody has verified that the photons consist of only pure electromagnetic energy. In view of that all the charged particles are massive, it is entirely possible that the photons would include gravitational energy. Thus, we must investigate the gravity of an electromagnetic wave.

IV. Inadequacy of Einstein's Notion of Photons and Invalidity of E = MC^2

It is the bending of light ray that makes Einstein famous. It will be shown that problems related to the bending of light also exposed the shortcomings of Einstein. Moreover, the electromagnetic energy must generate different gravitation [35].

In Einstein's calculation, it is implicitly assumed that the gravity created by an electromagnetic wave is negligible. Since Einstein also claimed that any energy-momentum tensor can be a source of his equation. Since such gravity is very weak in physics, Journals such as the Chinese Physics B and Physical Review D believed that such gravity can be calculated with the perturbation approach, but did not do it. Mathematically, for a perturbation approach to be valid, a necessary condition is, however, that this problem has a bounded solution. This compatibility between mathematics and physics is crucial for the validity of a theory in physics. Thus, it is natural for Einstein [38] to believe that his equation could be used for such a case.

For a plane electromagnetic wave, the metric obtained by Penrose [10] is the following:

$$ds^2 = du\,dv + Hdu^2 - dx_i dx_i\,, \quad \text{where } u = ct - z,\; v = ct + z. \qquad (19)$$

$H = h_{ij}(u)x_i x_j$, $h_{ij}(u)$ is the energy-momentun tensor.

An obvious problem is that the solution (19) is not bounded. This violates the principle of causality and also common sense.

Moreover, since there are non-physical parameters (the choice of origin) that are unrelated to any physical causes, [7)] this obviously further violate the principle of causality. However, according to the standard of the Physical Review, the solution of Penrose was perfectly valid [6]. Penrose is essentially a mathematician, and thus causality was probably absent from his training, but this lack of awareness in causality from the editors for the Review of Modern Physics, should be a surprise.

Moreover, explicit calculation based in causality shows that it is impossible to have a bounded solution for the gravity of an electromagnetic wave [39, 40]. In order for Einstein's theory to make sense, the related Einstein equation with an electromagnetic wave as the source, must include a photonic energy-stress tensor with the anti-gravity coupling [39, 40].

For this case the related modified Einstein equation is the following:

$$G_{ab} \equiv R_{ab} - \frac{1}{2} g_{ab}R = - K[T(E)_{ab} - T(p)_{ab}], \qquad (20)$$

and

$$T_{ab} = - T(g)_{ab} = T(E)_{ab} - T(P)_{ab}\,,$$

where $T(E)_{ab}$ and $T(P)_{ab}$ are respectively the energy-stress tensors for the electromagnetic wave and the related photons. *Thus the photonic energy must include also the energy of its gravitational wave component.*

Therefore, Einstein also failed to see the need of an anti-gravity coupling in this case as in the case of a massive dynamic Einstein equation [41]. Note that it is this failure to see the need of the anti-gravity coupling that provides the erroneous theoretical basis for the space-time singularity theorems of Hawking and Penrose which actually are irrelevant to physics [42].

If the photons consist of only electromagnetic energy, there is a conflict since the photonic energy can be equivalent to mass and the electromagnetic energy is not. Now, this conflict is resolved since the photonic energy is the sum of electro-magnetic energy and gravitational energy, and thus it is established that E = mc^2 can be invalid.

Note that both quantum theory and relativity are based on the phenomena of light. It is gravity that makes the notion of photons compatible with the electromagnetic wave. Since Einstein proposed the photons before he conceived general relativity; understandably he failed to include gravitational wave energy. Now, since a charged particle is always massive, it would be natural to include the gravitational wave energy in a photon. Ohanian [43] incorrectly credited von Laue for complete proof of the equivalence of mass and photonic energy. However, the fact is that both von Laue and Einstein failed [12].

V. The Invalidity of the Spacetime Singularity Theorems in Physics

In physics, the existence of singularities suggests problematic assumptions. Nevertheless, in current general relativity, the existence of space-time singularities plays a central role on the notion of black holes and the expanding universe. [8)]

The existence of space-time singularities is due to the spacetime singularity theorems of Hawking and Penrose [2]. The mathematical validity of these theorems is highly reliable because Penrose has won his arguments in mathematics against the theoretical physicist, E. M. Lifshitz [44] in a long dispute. Moreover, the static Einstein equation has passed various tests.

A common implicit assumption on these singularity theorems is that all the coupling constants have the same sign. [9)] Such an assumption would be necessarily valid if the formula $E = mc^2$ is unconditional. However, theoretically it has been found that the equivalence between mass and the electromagnetic energy is in conflict with the static Einstein equation. Moreover, it has been found that the anti-gravitational coupling is necessary for the case of an electromagnetic wave.

Here, it will be shown that the assumption of unique coupling sign is, indeed, necessary for the singularity theorems (see Appendix: B). Therefore, the space-time singularity theorems of Hawking and Penrose are actually irrelevant to physics.

According to Misner et al. [1], the Einstein equation takes the following form.

$$G_{ab} \equiv R_{ab} - (1/2)\, g_{ab} R = 8\pi T_{ab}, \qquad (21)$$

This is different from eq. (1) since the R_{ab} and g_{ab} in (21) is - R_{ab} and - g_{ab} in equation (1). Thus, we would have

$$R_{ab} = 8\pi\, [T_{ab} - (1/2) g_{ab} T] \quad \text{where } T = g^{ab} T_{ab} \qquad (22)$$

Then,

$$R_{ab}\,\xi^a\xi^b = 8\pi\, [T_{ab} - (1/2) g_{ab} T]\, \xi^a\xi^b = 8\pi\, [T_{ab}\xi^a\xi^b + (1/2)T], \quad \text{for a unit timelike } \xi^a \qquad (23)$$

It is believed that for all physically reasonable classical matter the energy condition is non-negative, i.e.,

$$T_{ab}\,\xi^a\xi^b \geq 0 \qquad (24)$$

for all timelike ξ^a. This assumption is the weak energy condition. However, it also seems physically reasonable that the stress of matter will not become so large and negative as to make the right-hand side of eq. (23) negative. This assumption,

$$T_{ab}\,\xi^a\xi^b \geq -(1/2)T \qquad (25)$$

for all unit timelike unit vector ξ^a, is known as the strong energy condition. These energy-conditions (23)-(25) imply that

$$R_{ab}\,\xi^a\xi^b \geq 0. \qquad (26)$$

if all the coupling constants have the same sign. Thus, the discovery of the anti-gravity coupling implies the assumption of the space-time singularity is invalid in physics (see Appendix B), and thus the singularity theorems are irrelevant to physics.

VI. The Non-Existence of a Dynamic Solution for the Einstein Equation and the Principle of Causality

Because of inadequacy in non-linear mathematics, Einstein and his followers fail to see the non-existence of a dynamic solution for the Einstein equation [35]. [10)] This error led to accepting the misleading positive mass theorem of Yau and Schoen [45, 46] as generally valid, [11)] and thus delayed the development of general relativity for at least 13 years [47].

The issue on the existence of a dynamic solution started with the perihelion of Mercury. In 1915 Einstein obtained the expected value of the remaining perihelion with his equation, and thus was confident on his theory. His confidence was subsequently boosted by the confirmation of the bending of light [12]. However, unexpectedly his equation was questioned by Gullstrand [48], the Chairman (1922-1929) of the Nobel Prize for Physics. The perihelion of Mercury is actually a many-body problem [35], but Einstein had not shown that his calculation could be derived from such a step. Thus, Mathematician D. Hilbert, who approved Einstein's calculation, did not come to its defense [41].

Nevertheless, Einstein was awarded a Nobel Prize by virtue of his photoelectric effects [12)] instead of general relativity as expected [41]. The fact is, however, Gullstrand is right. In 1995, it was proven that Einstein's equation is incompatible with gravitational radiation and does not have a dynamic solution [7-9].

For space-time metric $g_{\mu\nu}$, for massive sources, the Einstein equation of 1915 [1, 2] is

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\kappa T(m)_{\mu\nu} \qquad (27)$$

where $G_{\mu\nu}$ is the Einstein tensor, $T(m)_{\mu\nu}$ is the energy-stress tensor for massive matter.·Eq. (27) implies no gravitational waves to carry away energy-momentum in vacuum and thus the principle of causality [13)] is violated. This is the physical reason that there is no dynamic solution for the Einstein equation. Thus, Einstein's theory is at least incomplete.

There are serious consequences from the mistaken existence of dynamic solutions for the Einstein equation. A well-known result is the existence of the so-called space-time singularities due to implicitly

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assuming the unique sign for all the coupling constants [2]. Another result is that the misleading [47] positive mass theorem of Yau and Schoen [45, 46]. Moreover, the unification of gravitation and electromagnetism was not recognized as necessary by others [35]. Thus, the criticism of Gullstrand turns out to be very crucially constructive and beneficial.

Nevertheless, there were erroneous claims for the existence of a dynamic solution because of inadequacy in non-linear mathematics among physicists. Mathematicians such as Yau [45] and Witten [46] supported the existence of a bounded dynamic solution with their misleadingly positive mass theorem [47]. 14) Moreover, Christodoulou and Klainerman [49] invalidly claimed they have constructed dynamic solutions [50]. Not only were their errors accepted by the 1993 Nobel Prize Committee for Physics [15] but also Christodoulou was awarded the 2011 Shaw Prize 15) in mathematics [51] in honor of his errors against the honorable Gullstrand. Subsequently, Christodoulou was accepted as a member of U.S. National Academy of Sciences. Thus, generations of physicists are misled into serious errors because of the over confidence. This is highlighted by a big error, the award of the APS Medal for Exceptional Achievement in Research without experimental support [52]. 16)

Moreover, Gullstrand was not the only theorist who questioned the existence of the bounded dynamic solution for the Einstein equation. As shown by Fock [53], any attempt to extend the Maxwell-Newton approximation to higher approximations leads to divergent terms. An independent supplementary evidence for the absence of a bounded dynamic solution is, as shown by Hu, Zhang & Ting [54], that a calculated gravitational radiation would depend on the approach used.

Nevertheless, many believed that the non-linear Einstein equation must have a bounded dynamic solution since the linearized Einstein equation has bounded dynamic solutions. Unfortunately, Einstein and his peers failed to see that for a dynamic case, these equations are actually independent equations [55]. The recent awards and honors to Christodoulou manifested an unpleasant fact that most of the theorists do not understand non-linear mathematics adequately.

To prepare those who would read the proof for the non-existence of dynamic solutions [7-9], we shall give examples in the literature to illustrate what has been wrong in their claims of the existence of dynamic solutions.

a) The Unbounded Wave Solutions

Now, consider a well-known metric obtained by Bondi, Pirani, & Robinson [56] as follows:

$$ds^2 = e^{2\varphi}\left(d\tau^2 - d\xi^2\right) - u^2\begin{bmatrix}\cosh 2\beta\left(d\eta^2 + d\varsigma^2\right) \\ +\sinh 2\beta\cos 2\theta\left(d\eta^2 - d\varsigma^2\right) \\ -2\sinh 2\beta\sin 2\theta\, d\eta\, d\varsigma\end{bmatrix} \tag{28a}$$

where ϕ, β and θ are functions of u $(=\tau-\xi)$. It satisfies the differential equation (i.e., their Eq. [2.8]),

$$2\phi' = u\left(\beta'^2 + \theta'^2\sinh^2 2\beta\right) \tag{28b}$$

which is a special case of $G_{\mu\nu} = 0$. They claimed this is a wave from a distant source. However, the metric is irreducibly unbounded because of the factor u^2, and linearization of (28b) does not make sense since u is not bounded.

Moreover, when gravity is absent, it is necessary to have $\phi = \sinh 2\beta = \sin 2\theta = 0$. These would reduce (28a) to

$$ds^2 = \left(d\tau^2 - d\xi^2\right) - u^2\left(d\eta^2 - d\zeta^2\right). \tag{28c}$$

However, this metric is not equivalent to the flat metric. Thus, metric (28c) violates the principle of causality.

Consider again the gravitational solution obtained by Penrose [10] for an electromagnetic plane wave as follows:

$$ds^2 = du\, dv + H du^2 - dx_i dx_i\,, \quad \text{where} \quad H = h_{ij}(u)x_i x_j \tag{29}$$

where $u = ct - z$, $v = ct + z$. However, there are non-physical parameters (the choice of origin) that are unrelated to any physical causes. Penrose [10] over-looked the principle of causality. Linearization of (29) also does not make sense.

Another example is the plane-wave solution of Liu & Zhou [57], which satisfies the harmonic gauge, is as follows:

$$ds^2 = dt^2 - dx^2 + 2F(dt - dx)^2 - \cosh 2\psi(e^{2\phi}dy^2 + e^{-2\phi}dz^2) - 2\sinh 2\psi\, dy\, dz. \tag{30}$$

where $\phi = \phi(u)$ and $\psi = \psi(u)$. Moreover, $F = F_P + H$, where

$$F_P = \frac{1}{2}(\dot{\psi}^2 + \dot{\phi}^2\cosh^2 2\psi)\,[\cosh 2\psi\,(e^{2\phi}y^2 + e^{-2\phi}z^2) + 2\sinh 2\phi\, yz], \tag{31}$$

and H satisfies the equation,

$$\cosh 2\psi (e^{-2\phi}H,_{22} + e^{2\phi} H,_{33}) - 2\sinh 2\psi H,_{23} = 0. \quad (32)$$

For the weak fields one has 1 >> |ϕ|, 1 >> |ψ|, but there is no weak approximation as claimed to be

$$ds^2 = dt^2 - dx^2 - (1 + 2\phi)\, dy^2 - (1 - 2\phi)dz^2 - 4\psi dydz \quad (33)$$

because F_p is not bounded unless $\dot{\phi}$ and $\dot{\psi}$ are zero (i.e., no wave).

The linearized equation for a dynamic case has been illustrated as incompatible with the Einstein equation. Thus, eq. (28b), eq, (29), and eq. (33) serve as examples. It will be shown that claimed bounded dynamic solutions actually do not exist.

b) *Errors of Misner, Thorne and Wheeler*

A "wave" form considered by Misner, Thorne, & Wheeler [1] is as follows:

$$ds^2 = c^2dt^2 - dx^2 - L^2\left(e^{2\beta}dy^2 + e^{-2\beta}dz^2\right) \quad (34)$$

where $L = L(u)$, $\beta = \beta(u)$, $u = ct - x$, and c is the light speed. Then, the Einstein equation $G_{\mu\nu} = 0$ becomes

$$\frac{d^2L}{du^2} + L\left(\frac{d\beta}{du}\right)^2 = 0 \quad (35)$$

Misner et al. [1] claimed that Eq. (35) has a bounded solution, compatible with a linearization of metric (34).

However, it has been shown with calculus [58] that Misner et al. are incorrect and eq. (35) does not have a physical solution that satisfies Einstein's requirement on weak gravity. In fact, $L(u)$ is unbounded even for a very small $\beta(u)$.

On the other hand, from the Maxwell-Newton approximation in vacuum, Einstein [59] obtained a solution as follows:

$$ds^2 = c^2dt^2 - dx^2 - (1+2\phi)dy^2 - (1-2\phi)dz^2 \quad (36)$$

where ϕ is a bounded function of u $(= ct - x)$. Note that metric (36) is the linearization of metric (34) if $\phi = \beta(u)$. Thus, the waves illustrate that the linearization is not valid for the dynamic case since eq. (35) does not have a weak wave solution. Since this crucial calculation can be proven with mathematics at the undergraduate level, it should not be surprising that Misner et al. [1] make other serious errors in mathematics and physics [11].

Note that Wheeler's student, Christodoulou [49] also falsely claimed the existence of bounded dynamic solutions [50].

c) *Errors of Wald*

Wald [2] also fails to see the non-existence of dynamic solutions. The linearized vacuum Einstein equation means

$$G^{(1)}_{\mu\nu}[\gamma^{(1)}_{\alpha\beta}] = 0 \text{ where } G^{(1)}_{\mu\nu} = \frac{1}{2}\partial^\alpha\partial_\alpha\bar{\gamma}_{\mu\nu} - \partial^\alpha\partial_\mu\bar{\gamma}_{\nu\alpha} - \partial^\alpha\partial_\nu\bar{\gamma}_{\mu\alpha} + \frac{1}{2}\eta_{\mu\nu}\partial^\alpha\partial^\beta\bar{\gamma}_{\alpha\beta} \quad (37)$$

and

$$G_{\mu\nu} = G_{\mu\nu}^{(1)} + G_{\mu\nu}^{(2)}, \quad \bar{\gamma}_{\mu\nu} = \gamma_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\gamma, \quad \text{and} \quad \gamma = \eta^{\alpha\beta}\gamma_{\alpha\beta}. \quad (38)$$

Thus, as Wald pointed out that we must correct $\gamma^{(1)}_{\mu\nu}$ by adding to it the term $\gamma^{(2)}_{\mu\nu}$ that satisfies

$$G^{(1)}_{\mu\nu}[\gamma^{(2)}{}_{\alpha\beta}] + G^{(2)}_{\mu\nu}\left[\gamma_{\alpha\beta}\right] = 0, \quad \text{where} \quad \gamma_{\mu\nu} = \gamma^{(1)}{}_{\mu\nu} + \gamma^{(2)}{}_{\mu\nu} \quad (39)$$

which is the correct form of eq. (4.4.52) in [2] (Wald did not distinguish $\gamma_{\mu\nu}$ from $\gamma^{(1)}{}_{\mu\nu}$). However, detailed calculation shows that this equation does not have a solution for the dynamic case [7-9] although it does have a solution for the static case. Wald believed that eq. (39) had a solution for the dynamic case, due to his false confidence on Yau's theorem [45].

VII. The Rejection of the Anti-Gravitational Coupling

Einstein was unaware of the need for an anti-gravitational coupling although Pauli [14] mentioned this possibility. It has been shown that anti-gravity coupling is needed in the calculation of the gravitational waves generated by an electromagnetic wave [39, 40]. This also leads to the conclusion that the anti-gravity coupling must be also present for the massive cases.

For the case with a massive source, there is a conflict between the Einstein equation, which has no dynamic solution and its linearized equation, which has a dynamic solution. The conflict is because the second order terms $G^{(2)}{}_{\mu\nu}$ cannot be eliminated in the Einstein equation. Thus, a simple solution is the 1995 update of the Einstein equation [8] as follows:

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -K\,[T(m)_{\mu\nu} - t(g)_{\mu\nu}], \quad (40)$$

where $t(g)_{\mu\nu}$ is the energy-stress tensor for gravity.[17] From the Lorentz-Levi-Einstein equation (40), in vacuum it is

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = K\, t(g)_{\mu\nu}. \quad (41)$$

Note that $t(g)_{\mu\nu}$ is equivalent to $G^{(2)}{}_{\mu\nu}$ (and Einstein's gravitational pseudotensor) in terms of his radiation formula.

Historically, equation (20) was first proposed by Lorentz [60] and later by Levi-Civita [61] in the following form,

$$\kappa t(g)_{ab} = G_{ab} + \kappa T_{ab} \quad (42)$$

such that the gravitational energy-stress tensor $t(g)_{ab}$ takes a covariant form although they have not proved its necessity. However, Einstein [62] objected to this form on the grounds that his field equation implies $t(g)_{ab} = 0$. A problem of Einstein is that he has never calculated a dynamic solution explicitly and thus failed to see its nonexistence.

In vacuum, the gravitational energy-stress tensor $t(g)_{\mu\nu}$ carries the energy-momentum as physics requires. Note that the linearized equation provides a valid approximation of the modified Einstein equation (40), but not the Einstein equation [12, 13]. This also solves Einstein's dilemma that his equation does not have an gravitational wave solution. Note also that the radiation of the binary pulsar can be calculated without detailed knowledge of $t(g)_{\mu\nu}$ [7-9].

It should be noted that the necessary existence of an anti-gravity coupling for a dynamic situation means the energy conditions in the singularity theorems [2] are not valid. Thus, the existence of a singularity is incorrect. Moreover, the claim of inevitably breaking of general relativity for microscopic phenomena is baseless since the notion of photons is based on gravitation. In other words, the contributions of Hawking and Penrose to physics are essentially zero if not negative.

VIII. The Absence of the Reaction Force for Gravitational Radiation

It has been shown that the existence of photons implies the existence of gravitational waves. Thus, the non-existence of a gravitational wave solution discovered by Einstein [3] means only that the Einstein equation must be rectified. Hence, another problem is that, just as in Maxwell's classical electromagnetism, there is also no radiation reaction force for gravitational waves in general relativity. Although an accelerated massive particle would create radiation, the metric elements in the geodesic equation are created by particles other than the test particle. In other words, not only the field equation, but also the equation of motion must be modified. Thus gravity cannot be just Riemannian geometry. *Therefore, general relativity is not yet complete, independent of the need of unification due to the existence of the charge-mass interaction (see next section).*

IX. The Existence of Repulsive Gravitation

Because the electromagnetic energy is not equivalent to mass, such energy must generate a different kind of gravitation.

On the other hand, due to believing in unconditional $E = mc^2$, nobody seriously studied repulsive gravitation generated by the electromagnetic energy before Lo [63] in 1997.

For a particle with charge q and mass M, the metric known as the Riessner-Nordstrom metric [1] is as follows:

$$ds^2 = \left(1 - \frac{2M}{r} + \frac{q^2}{r^2}\right) dt^2 - \left(1 - \frac{2M}{r} + \frac{q^2}{r^2}\right)^{-1} dr^2 - r^2 d\Omega^2, \quad (43)$$

(with c = 1) where r is the radial distance (in terms of the Euclidean-like structure [30]) from the particle center. In this metric (43), the gravitational components generated by electricity have not only a very different radial coordinate dependence but also a different sign that makes it a new repulsive gravity in general relativity [64].

Nevertheless, some still argued that the effective mass could be considered as $(M - q^2/2r)$ because the total electric energy outside a sphere of radius r is $q^2/2r$ [65], and thus (43) could be interpreted as supporting $m = E/c^2$. However, the strength of a gravitational force decreases everywhere after an increase of the electric energy.

Moreover, the gravitational forces would be different from the force created by the "effective mass" $M - q^2/2r$ because

$$-\frac{1}{2}\frac{\partial}{\partial r}\left(1 - \frac{2M}{r} + \frac{q^2}{r^2}\right) = -\left(\frac{M}{r^2} - \frac{q^2}{r^3}\right) > -\frac{1}{r^2}\left(M - \frac{q^2}{2r}\right). \quad (44)$$

Thus Will was defeated because he could not defend his interpretation of $m = E/c^2$ [63].

Nevertheless, theorists such as Herrera, Santos, & Skea [66] argued that M in (43) involves the electric energy. Then their metrics for a charged ball would increase its weight as the charge q increased [64]. (Their approach is essentially the same as that of Pekeris [67] in 1982. Apparently, theorists have run out of means that can be used against the repulsive force.) However, experiments support the opposite [68]. Nevertheless, Herrera et al. [66] are not alone in such an error. Nobel Laureate G. 't Hooft even claimed that the electric energy of an electron contributed to the inertial mass of an electron [69].[18] In fact, many Nobel Laureats incorrectly believed the validity of $E = mc^2$.[19] Nobody questioned this simple but incorrect result.

On the other hand, if the mass M is just the inertial mass of the particle, the weight of a charged metal ball can be reduced [64]. Thus, as Lo expected [63], experiments on two metal balls [68] confirm the charged ball has reduced weight. This is an experimental direct proof that the electric energy is not equivalent to mass. We recommend that the details of such experiments [70] should be continued such that this static case of general relativity is fully verified.

The existence of repulsive gravitation is important since it would solve a puzzle as to why we have never seen a black hole. Assuming gravity is always attractive to mass, simulation convinces Wheeler that a black hole must be formed [44]. More important, this new repulsive force is crucial for establishing the unification of gravitation and electromagnetism [65].

X. The Weight Reduction of a Charged Capacitor and the Current-Mass Interaction

Moreover, the weight of a charged capacitor is also reduced [71] instead of increased as Einstein claimed [72]. In fact, for a capacitor under the influence of a high electric voltage, the repulsive force would be able to lift its own weight plus a payload, but without a continuous supply of electric energy [73]. It is found that the lifting force is proportional to the square of the voltage difference V, and is diminishing faster than the attractive gravitational force. Moreover, this repulsive force is actually independent of the direction of the electric field since the force exists beyond a charged rolled-up capacitor, whose electric field has no definite direction [65]. Considering the charge Q = VC, C is the capacity of the capacitor, the force on a charged capacitor is essentially the same kind of force generated between a charge and a mass in general relativity.

However, a puzzle is why a charged capacitor exhibits the charge-mass repulsive force since a charged capacitor has no additional charges after charging. The difference of the situations is only some moving charges have become static. Thus, before the capacitor is charged, the repulsive force from the charges is canceled by the force generated by the moving charges.

If the static electric energy leads to a repulsive force toward a mass according to general relativity, the magnetic energy would lead to an attractive force from a current toward a mass [44]. Due to the fact that a charged capacitor has reduced weight, it is necessary to have the current-mass interaction be cancelled out by the effect of the charge-mass interaction. Thus, the existence of the current-mass attractive force would solve the puzzle.

The existence of such a current-mass attractive force has been discovered by Martin Tajmar and Clovis de Matos [74] from the European Space Agency. Martin et al found that a spinning ring of superconducting material increases its weight more than expected. Thus, they believed that general relativity was wrong. However, according to quantum theory, spinning super-conductors should produce a weak magnetic field. Thus, they also measured the current-mass interaction to the earth! From their findings, the current-mass interaction would generate a force which is perpendicular to the current.

One may ask what the formula for the current-mass force is. Unlike the charge-mass repulsive force, which can be derived from general relativity; this general force would be beyond general relativity since a current-mass interaction would involve the acceleration of a charge, this force would be time-dependent and generates electromagnetic radiation. Moreover, when the radiation is involved, the electromagnetic radiation reaction force and the variable of the fifth dimension must be considered [75]. Thus, we are not ready to derive the current-mass interaction yet.

Nevertheless, we may assume that, for a charged capacitor, the resulting force is the interaction of net macroscopic charges with the mass [71]. This also explains a predicted phenomenon, as reported by Liu [65] that it takes time for a capacitor to recover its weight after being discharged. Therefore, the electromagnetic energy in matter can reduce or increase its weight. Recently, the temperature dependence of gravitation has been demonstrated by the torsion balance [76].

XI. The Necessary Extension of General Relativity to a Five-Dimensional Space

According to the Reissner-Nordstrom metric for a particle Q with charge q and mass M, the static force that acts on a test particle P with mass m, for the first order approximation (since $g^{rr} \cong -1$) is

$$-m\frac{M}{r^2} + m\frac{q^2}{r^3} \qquad (45)$$

Note that the second term is a repulsive force due to the static charge-mass interaction. For a ball with a charge Q and mass M, the repulsive force is mQ^2/R^3, where R is the distance from the ball center [70].

Due to the reaction force being equal to, but in the opposite direction of, the acting force, the test particle P must create a field m/r^3 that couples to q^2. It thus follows that the force to particle Q is beyond current theoretical framework of gravitation + electromagnetism. As predicted by Lo, Goldstein and Napier [75], general relativity leads to a realization of the inadequacy of general relativity, just as electricity and magnetism lead to the exposition of their shortcomings.

If we consider the coupling with q^2, this naturally leads to a five-dimensional space of Lo et al [75]. Einstein & Pauli [77] did not recognize as Maxwell did, that unification necessarily leads to new interactions. Thus, they overlooked that the charge-mass interaction

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would be naturally generated in a five-dimensional theory. Unfortunately, their followers repeat their errors. Therefore, they failed to see the necessity, and even claimed Einstein's Unification is not valid.

However, in addition to the repulsive force acting from the test particle P to the charged ball, there is a repulsive force acting on a charged capacitor and the reduction of weight due to the increment of temperature [78, 79]. These forces can be understood only in terms of a five-dimensional theory. Thus, in gravitation, there are three types of forces, namely: 1) the mass-mass attractive interaction, 2) the charge-mass repulsive interaction, 3) the current-mass attractive interaction.

Some theorists have mistakenly assumed that the reduction of weight is due to a reduction of mass [78,79]. However, one can measure the inertial mass directly from acceleration of the subject and/or the period of such a pendulum [80]

XII. Conclusions and Discussions

A common mistake among those who work on gravitational waves is that they incorrectly believed that the linearized equation would give an approximate solution of the non-linear Einstein equation. However, the linearized equation and the non-linear Einstein equation actually are independent dynamic equations [55]. The linearized equation is a valid linerization for the Lorentz-Levi-Einstein equation (40), but not the Einstein equation which has no bounded dynamic solution.

Moreover, since the repulsive gravity can also lead to waves, the problem of gravitational waves is far more complicated. A useful feature of the gravitational wave based on repulsive gravitation is that it can be easily generated on earth. Thus this can be a new tool for communication because gravity can penetrate any medium [65].

Einstein is considered as the most prominent scientist of the 20 century since he has successfully challenged the authorities of sciences and forever changed the way we see the world. Unfortunately, his over-confidence on his theory also led to the invalid covariance principle and misinterpretations of experiments. Moreover, short-comings in physics together with his inadequacy in mathematics led to errors without knowing them. Those "so-called" experts such as Wheeler [1] and Wald [2] give more misinterpretations due to inadequacy in mathematics and physics, and thus add difficulties for understanding. 20)

This analysis shows that there are at least two areas that theoretical physicists should improve. One area is the non-linear mathematics related physics [35]. Another area is the application of the principle of causality. For instance, the failure to obtain the gravitational wave solution generated from an electromagnetic wave and the failure to recognize the non-existence of a dynamic solution are due to inadequate understanding on the principle of causality.

It has been shown that general relativity is still incomplete and its errors remain to be rectified. Moreover, the main obstacles for the development of general relativity are the errors of Einstein's earlier defective work and inadequate considerations, There are at least ten major errors of Einstein that prevent the development of general relativity.

An important interesting discovery is the existence of repulsive gravitation due to the charge-mass interaction [65, 71, 78-81]. Moreover, the electromagnetic energy can also generate attractive gravitation through the current-mass interaction. Thus, the physical picture provided by Galileo, Newton and Einstein is just too simple for gravitation which involves almost everything. Also, the notion of black holes is clearly questionable since gravity is no longer always attractive.

However, it is surprising that after the shortcomings are identified, Einstein's conjecture of unification between gravitation and electromagnetism are proven to be correct [35]. Moreover, since the charge-mass interaction is absent in quantum mechanics, it is not a final theory as Einstein claimed. Thus, Einstein remains the number one theorist in physics.

A lesson to be learned from Einstein is that humans are fallible, and thus over-confidence is often the starting point of a down fall. A problem of the physicists is that they have neglected the pure mathematics for too long, and a catch up program is now necessary. In contrast to the belief of Eric J, Weinberg, 21) pure mathematics is just as indispensible a tool as applied mathematics to physics. We should never forget that experiments and observations always provide us faithful guidance.

Philosopher Hu Shih once remarked that in science, one can have daring assumptions, but one must also be careful in his proof. Unfortunately, a problem of Einstein and many physicists are that they often have only the first half.

XIII. Acknowledgments

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Appendix A: Mathematical Foundation of Einstein's Equivalence Principle

Pauli's invalid version [14] has been mistaken as equivalent to Einstein's equivalence principle although Einstein has made clear it is a misinterpretation [82]. In "Gravitation" [1] of Misner et al, there is no reference to Einstein's equivalence principle. Instead, they misleadingly refer to Einstein's invalid 1911 assumption [83] [22)] and Pauli's version [14]. In addition, in their Eq. (40.14) Misner et al. [1] even failed to understand the local time of a particle at free fall.

The mathematical theorems [84] related to Einstein's equivalence principle are as follows:

Theorem 1. Given any point *P* in any Lorentz manifold (whose metric signature is the same as a Minkowski space) there always exist coordinate systems (x^{μ}) in which $\partial g_{\mu\nu}/\partial x^{\lambda} = 0$ at *P*.

Theorem 2. Given any time-like geodesic curve Γ there always exists a coordinate system (the so-called Fermi coordinates) (x^{μ}) in which $\partial g_{\mu\nu}/\partial x^{\lambda} = 0$ along Γ.

In these theorems, the local space of a particle is locally constant, but not necessarily Minkowski.

However, after some algebra, a local Minkowski metric exists at any given point and along any time-like geodesic curve Γ. What Einstein added is that such a locally constant metric must be Minkowski. This is the basis of the Einstein-Minkowski condition that Einstein uses to derive the gravitational redshifts [12].

Note that, Pauli's version [14] is a simplified but corrupted version of these theorems as follows:

> "For every infinitely small world region (i.e. a world region which is so small that the space- and time-variation of gravity can be neglected in it) there always exists a coordinate system K_0 (X_1, X_2, X_3, X_4) in which gravitation has no influence either in the motion of particles or any physical process."

He regards a local Minkowski space at a point is the same as the existence of local Minkowski spaces at a small world region.

Thus, an error is that Pauli extended the removal of uniform gravity to the removal of gravity in a small region. This becomes simply incorrect and even impossible in mathematics, but he does not see the difference because of inadequacy in mathematical analysis. He did not recognize that the removal of gravity in a small region, no matter how small, would be very different from a removal of gravity at one point. Apparently, Pauli [14], Eric J. Weinberg, [21)] Witten [46], [11)] the Wheeler School [1], and the British Encyclopedia did not understand the mathematics of the above theorems [84].

Appendix B: The Space-Time Singularity Theorems of Hawking and Penrose

For the convenience of the readers, the singularity theorems of Hawking and Penrose are listed below:

Theorem 1. Let (M, g_{ab}) be a globally hyperbolic spacetime with $R_{ab}\xi^a\xi^b \geq 0$ for all timelike ξ^a, which will be the case if Einstein equation is satisfied with the strong energy condition holding for matter. Suppose there exists a smooth (or at least C^2) spacelike Cauchy surface Σ for which the trace of the extrinsic curvature (for the past directed normal geodesic congruence) satisfies $0 > C \geq K$ everywhere C is a constant. Then no past directed timelike curve from Σ can have length greater than $3/|C|$. In particular, all past directed timelike geodesic are incomplete.

Theorem 2. Let (M, g_{ab}) be a strongly causal spacetime with $R_{ab}\xi^a\xi^b \geq 0$ for all timelike ξ^a, as will be the case if Einstein's equation is satisfied with the strong energy condition holding for matter. Suppose there exists a compact, edgeless, achronal smooth spacelike hypersurface S such that for the past directed normal geodesic congruence form S we have $0 > K$ everywhere on S. Let C denote the maximum value for K, so $0 > C \geq K$ everywhere on S. Then at least one inextendible past directed timelike geodesic from S has length no greater that $3/|C|$.

Theorem 3. Let (M, g_{ab}) be a connected, globally hyperbolic spacetime with a noncompact Cauchy surface Σ. Suppose $R_{ab}k^ak^b \geq 0$ for all null k^a, as will be the case if (M, g_{ab}) is a solution of Einstein's equation with matter satisfying the weak or strong energy condition. Suppose, further, that M contains a trapped surface T. Let $0 > \theta_0$ denote the maximum value of θ for both sets of orthogonal geodesic on T. Then at least one inextendible future directed orthogonal null geodesic from T has affine length no greater than $2/|\theta_0|$.

Theorem 4. Suppose a spacetime (M, g_{ab}) satisfies the following four conditions. (1) $R_{ab}v^av^b \geq 0$ for all timelike and null v^a, as will be the case if Einstein's equation is satisfied with the strong energy condition holding for matter. (2) The timelike and null generic conditions are satisfied. (3) No closed timelike curve exists. (4) At least one of the three properties holds: (a) (M, g_{ab}) posses a compact achronal set without edge [i.e., (M, g_{ab}) is a closed universe], (b) (M, g_{ab}) possesses a trapped surface, or (c) there exists a point $p \in M$ such that the expansion of the future (or past) directed null geodesics emanating from p becomes negative along each geodesic in this congruence. Then (M, g_{ab}) must contain at least one incomplete timelike or null geodesic.

Endnotes

1) Prof. S. Weinberg taught us that general relativity must be understood in terms of physics. This MIT tradition has a long history, starting from Rosen and

Einstein's paper of 1937, followed by Yilmaz, advocated by Weisskopf and Morrison, and so on. It is a pleasure to be able to contribute to such an outstanding tradition. However, repairs to the tradition are urgently needed since it has recently been broken by the Wheeler School [11] after Prof. Morrison passed away. It is hoped that MIT will soon be able to recover the tradition based on physics instead of opinions of the famous.

2) This is a popular mistake of physicists because of inadequate background in pure mathematics [14].
3) P. Morrison is a student of J. Robert Oppenheimer and thus has a very keen sense of physics.
4) C. N. Yang believed that Einstein's covariance principle is valid because he misinterpreted the gauge invariance [24].
5) Now, it is clear that Einstein's understanding of general relativity needs improvement.
6) Now, many theorists seem to have noticed this problem of inconsistency because Einstein's method is no longer used [1], but they choose the wrong choice.
7) This shows that Penrose is only a mathematician, but not a physicist.
8) The 2011 Nobel Prize has been awarded to Saul Perlmutter, Brian P. Schmidt and Adam G. Riess "*for the discovery of the accelerating expansion of the Universe through observations of distant supernovae*".
9) This incorrect implicit assumption was generally accepted because it was believed that $E= mc^2$ was unconditional.
10) Einstein failed to see the non-existence of a dynamic solution. Due to his inadequacy in mathematics, he does not know that for the dynamic case, the nonlinear Einstein equation and the linearized equation are independent [55].
11) From the theorem of Witten [46], it is clear that he also does not understand general relativity.
12) This is a good example that a Nobel Prize does not guarantee the validity of a theory, but only partial validity of a theory.
13) The time-tested assumption that phenomena can be explained in terms of identifiable causes is called the principle of causality. This is the basis of relevance for all scientific investigations. This principle implies that any parameter in a physical solution must be related to some physical causes. In general relativity, the normal metric is the flat metric.
14) In 1993, Yau and I met in the Chinese University of Hong Kong and discussed the non-existence of a dynamic solution [85]. Subsequently, Yau claimed that he has lost his earlier interest on the work of Christodoulou and Klainerman [49].
15) Some mathematicians did not take a responsible attitude toward a Prize. Members of the awarding committee often act on their faith of previous awards, but may not know the subject at all. For instance, Y.-T. Siu of Harvard agreed to award Christodoulou a 2011 Shaw Prize, although he does not understand general relativity nor non-linear mathematics [51].
16) The inappropriate award to Witten shows that the awarding committee of APS is incompetent in mathematics [52].
17) The exact form of the gravitational energy-stress tensor is still not known.
18) This manifests that G. 't Hooft does not understand special relativity and Newtonian mechanics adequately.
19) Another Nobel Laureate Wilczek [86] also applied $m = E/c^2$ without providing a justification. Since this relation in his theory is crucial for his Nobel Prize, Wilczek should work out a justification for its application.
20) The so-called experts often agree with Einstein's errors, but disagree with Einstein when he is correct [6, 11, 47, 52].
21) Eric J. Weinberg, editor of the Physical Review D, invalidly insists [7-9] that there is no difference in physics between Einstein's equivalence principle and Pauli's version [14] although Einstein pointed out that it is a misinterpretation [82]. He also accepts the unbounded solution because of inadequate understanding the principle of causality [7]. The root is that Eric J. Weinberg does not understand pure mathematics and thus failed to understand Einstein's equivalence principle as Pauli did [14]. Moreover, he does not understand not only non-linear mathematics, but also his calculus is defective. In addition, he simply does not have the wisdom to see that admitting past mistakes is the best choice for the APS as a scientific institute. Apparently, he is unaware of the 2003 work of Dmitriev et al. [78] on the weight reduction of the brass due to heat. As a result, erroneous papers on gravitation are still often published in the Physical Review D. Similarly, established journals such the Proceedings of the Royal Society, Annalen der Physik, etc. also just have too much blind faith on Einstein, and over-confidence on their out-dated knowledge in gravitation and experiments. Thus, they failed to adjust themselves to deal with new situations such as the non-linear mathematics.
22) Since the 1911 assumption of Einstein has been proven incorrect by experiments [12], it is beyond my comprehension that the Wheeler School [1] used it as the reference for Einstein's equivalence principle.

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