Global Journal of Science Frontier Research: F
Mathematics and Decision Sciences
Volume 17 Issue 7 Version 1.0 Year 2017
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Twin Prime Number Theorem

By Yin Yue Sha
Zhejiang University

Abstract- Let Pt(N) be the number of twin primes less than or equal to N, Pi (3≤Pi≤Pm) be taken over the odd primes less than or equal to √N, then exists the formula as follows:

$Pt(N) \geq INT\{ N\times(1-1/2)\times\prod(1-2/Pi) \} - 2$

$\geq INT\{ Ct\times 2N/(Ln(N))^2 \} - 2$

$Pt(N) \geq INT\{ 0.660\times 2N/(Ln(N))^2 \} - 2 \geq 0.660\times 2N/(Ln(N))^2 - 3$

$\prod(Pi(Pi-2)/(Pi-1)^2) \geq Ct = 0.6601618158\ldots$

Where the INT { } expresses the taking integer operation of formula spread out type in { }.

Keywords: *twin prime, bilateral sieve method.*

GJSFR-F Classification: *MSC 2010: 70A05, 70E55, 70G10*

TWINPRIMENUMBERTHEOREM

Strictly as per the compliance and regulations of:

Global Journal of Science Frontier Research (F) Volume XVII Issue VII Version I Year 2017

Twin Prime Number Theorem

Yin Yue Sha

Abstract- Let Pt(N) be the number of twin primes less than or equal to N, Pi ($3 \leqslant Pi \leqslant Pm$) be taken over the odd primes less than or equal to √N, then exists the formula as follows:

$Pt(N) \geqslant INT\{ N \times (1-1/2) \times \Pi\ (1-2/Pi) \} - 2$

$\geqslant INT\{ Ct \times 2N/(Ln\ (N))^2 \} - 2$

$Pt(N) \geqslant INT\{ 0.660 \times 2N/(Ln\ (N))^2 \} - 2 \geqslant 0.660 \times 2N/(Ln\ (N))^2 - 3$

$\Pi\ (Pi(Pi-2)/(Pi-1)^2) \geqslant Ct = 0.6601618158\cdots$

Where the INT { } expresses the taking integer operation of formula spread out type in { }.

Keywords: twin prime, bilateral sieve method.

I. The Twin Prime Number

There exists a prime P for which the Twin number Q=2+P is also prime. The Twin Primes shall be denoted by the representation 2=Q−P=(2+P)−P, where P and Q are primes and prime P{P<Q} is a Twin prime of even integer 2. Looking at the Twin partition a different way, we can look at the number of distinct representations (or Twin primes) that exist for 2.

For example, as noted at the beginning of this discussion:

$$2 = 05 - 03 = (2+03) - 03;\ 2 = 07 - 05 = (2+05) - 05;$$
$$2 = 13 - 11 = (2+11) - 11;\ 2 = 19 - 17 = (2+17) - 17;$$

where 3, 5, 11, and 17 are Twin primes of even integer 2.

II. The Sieve Method about the Twin Primes

The 2 is an even integer, Ti is a positive integer less than or equal to N, then exists the formula as follows:

$$2 = (2+Ti) - Ti \qquad (1)$$

where Ti and 2+Ti are two positive integers less than or equal to N+2.

If Ti and 2+Ti any one can be divided by the prime anyone more not large than √(N+2), then sieves out the positive integer Ti; If both Pt and 2+Pt can not be

Author: Dongling Engineering Center, Ningbo Institute of Technology, Zhejiang University, Tao Yuan Xin Cun, Heng Xi Town, Ning Bo, Z.J.315131, China. e-mail: shayinyue@qq.com

divided by all primes more not large than √(N+2), then both the Pt and 2+Pt are primes at the same time, where the prime Pt is a Twin prime of even integer 2.

III. The Total of Representations

The 2 is an even integer, then exists the formula as follows:

$$2 = (2+Ti)-Ti$$

where Ti is a positive integer less than or equal to N.

In terms of the above formula we can obtain the array as follows:

$$(2+1,\ 1),\ (2+2,\ 2),\ (2+3,\ 3),\ (2+4,\ 4),\ (2+5,\ 5), \cdots ,\ (2+N,\ N).$$

From the above arrangement we can obtain the formula about the total of Twin numbers of even integer 2 as follows:

$$Ti(N) = N = \text{Total of integers Ti more not large than N} \qquad (2)$$

IV. The Bilateral Sieve Method of Even Prime 2

It is known that the number 2 is an even prime, and above arrangement from (2+1, 1) to (2+N, N) can be arranged to the form as follows:

$$(2+1,\ 1),\ (2+3,\ 3),\ (2+5,\ 5),\ \ldots,\ (2+N-X{:}X< 2,\ N-X{:}X< 2).$$

$$(2+2,\ 2),\ (2+4,\ 4),\ (2+6,\ 6),\ \ldots,\ (2+N-X{:}X< 2,\ N-X{:}X< 2),$$

From the above arrangement we can known that: Because the even integer 2 can be divided by the even prime 2, therefore, both Ti and 2+Ti can be or can not be divided by the even prime 2 at the same time.

The number of integers Ti that Ti and 2+Ti anyone can be divided by the even prime 2 is:

$$INT\ (\ N\times(1/2)\ ).$$

The number of integers Ti that both Ti and 2+Ti can not be divided by the even prime 2 is:

$$N-INT(N\times(1/2))=INT\{\ N-N\times(1/2)\}=INT\{\ N\times(1-1/2)\} \qquad (3)$$

Where the INT { } expresses the taking integer operation of formula spread out type in { }.

V. The Bilateral Sieve Method of Odd Prime 3

It is known that the number 3 is an odd prime, and above arrangement from (2+1, 1) to (2+N, N) can be arranged to the form as follows:

$$(2+1,\ 1),\ (2+4,\ 4),\ (2+7,\ 7),\ \ldots,\ (2+N-X{:}X< 3,\ N-X{:}X< 3),$$

$$(2+2,\ 2),\ (2+5,\ 5),\ (2+8,\ 8),\ \ldots,\ (2+N-X{:}X< 3,\ N-X{:}X< 3),$$

$$(2+3,\ 3),\ (2+6,\ 6),\ (2+9,\ 9),\ \ldots,\ (2+N-X{:}X< 3,\ N-X{:}X< 3).$$

Notes

From the above arrangement we can known that:

The even integer 2 can not be divided by the odd prime 3, then both Ti and 2+Ti can not be divided by the odd prime 3 at the same time, that is the Ti and 2+Ti only one can be divided or both the Ti and 2+Ti can not be divided by the odd prime 3.

The number of integers Ti that the Ti and 2+Ti anyone can be divided by the odd prime 3 is: INT(N×(2/3)).

The number of integers Ti that both the Ti and 2+Ti can not be divided by the odd prime 3 is:

$$N-INT(N\times(2/3))=INT\{N-N\times(2/3)\}=INT\{N\times(1-2/3)\} \quad (4)$$

Where the INT { } expresses the taking integer operation of formula spread out type in { }.

VI. The Sieve Function of Bilateral Sieve Method

The 2 is an even integer, then exists the formula as follows:

$$2 = (2 + Ti) - Ti$$

where Ti is the natural integer less than or equal to N.
In terms of the above formula we can obtain the array as follows:

$$(2+1,\ 1),\ (2+2,\ 2),\ (2+3,\ 3),\ (2+4,\ 4),\ (2+5,\ 5),\ \ldots,\ (2+N,\ N).$$

Let Pi be an odd prime less than or equal to √(N+2), then the above arrangement can be arranged to the form as follows:

$$(2+1,\ 1),\ (2+Pi+1,\ Pi+1),\ \ldots,\ (2+N-X{:}X< Pi,\ N-X{:}X< Pi),$$

$$(2+2,\ 2),\ (2+Pi+2,\ Pi+2),\ \ldots,\ (2+N-X{:}X< Pi,\ N-X{:}X< Pi),$$

$$(2+3,\ 3),\ (2+Pi+3,\ Pi+3),\ \ldots,\ (2+N-X{:}X< Pi,\ N-X{:}X< Pi),$$

$$\ldots\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots$$

$$(2+Pi,\ Pi),\ (2+2Pi,\ 2Pi),\ \ldots,\ (2+N-X{:}X< Pi,\ N-X{:}X< Pi).$$

The even integer 2 can not be divided by the odd prime Pi, then both the Ti and 2+Ti can not be divided by the odd prime Pi at the same time, that is the Ti and 2+Ti only one can be divided or both the Ti and 2+Ti can not be divided by the odd prime Pi.

The number of integers Ti that the Ti and 2+Ti anyone can be divided by the odd prime Pi is INT(N×(2/Pi)).

The number of integers Ti that both the Ti and 2+Ti can not be divided by the odd prime Pi is

$$N-INT\ (N\times(2/Pi)\ )\ =\ INT\ \{\ N-N\times(2/Pi)\ \}\ =\ INT\{\ N\times(1-2/Pi)\ \} \quad (5)$$

Let Pt(N) be the number of Twin Primes less than or equal to (N+2), Pi be taken over the odd primes less than or equal to √(N+2), then exists the formulas as follows:

$$Pt(N) \geqslant INT\{ N\times(1-1/2)\times\prod(1-2/Pi) \} - 2 \quad (6)$$

Where the INT { } expresses the taking integer operation of formula spread out type in { }.

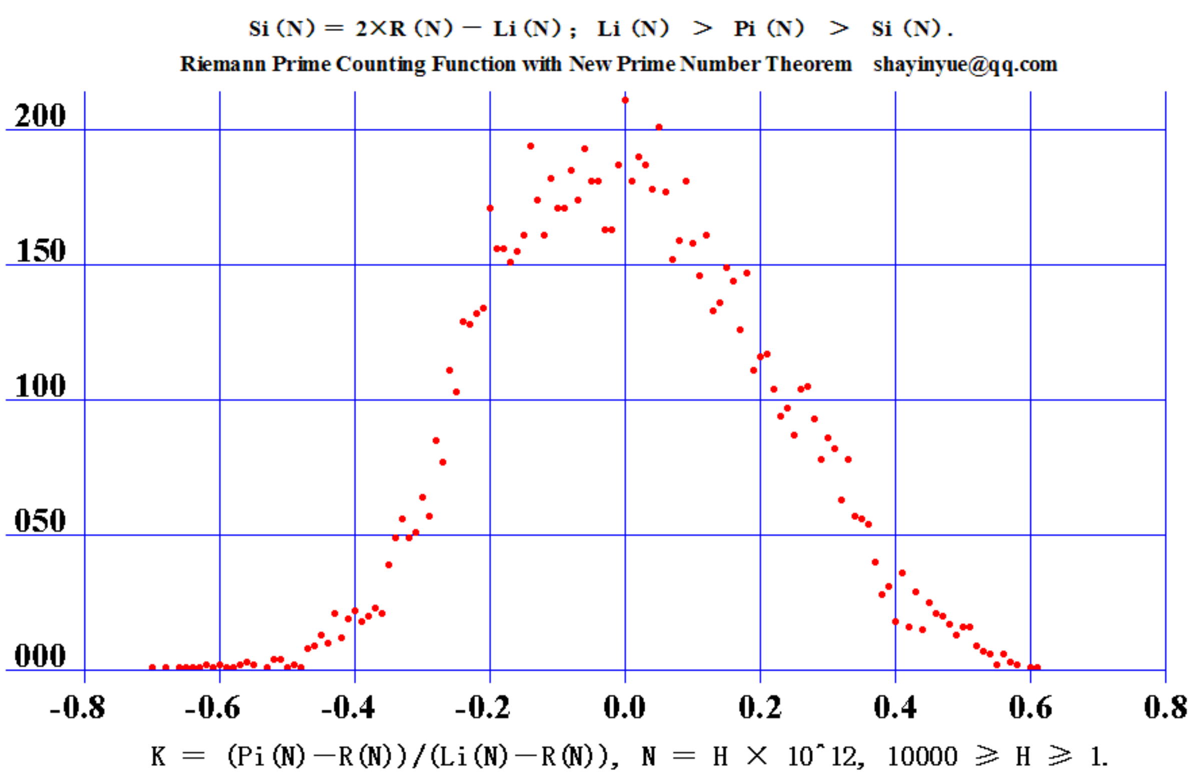
VII. New Prime Number Theorem

Let Pi(N) be the number of primes less than or equal to N, Pi (3⩽Pi⩽Pm) be taken over the odd primes less than or equal to √N, then exists the formulas as follows:

$$Pi(N \mid N\geqslant 10\hat{}4) = INT\{ N\times(1-1/2)\times\prod(1-1/Pi) + m + 1 \} - 1 \quad (7)$$

$$\geqslant INT\{N\times(1-1/2)\times\prod(1-1/Pi)\} \geqslant INT\{N/Ln(N)\} \quad (8)$$

$$Pi(N) = R(N) + K\times(Li(N) - R(N)),\ 1 > K > -1. \quad (9)$$



Figure

VIII. The Twin Prime Theorem

From above we can obtain that:

Let Pt(N) be the number of Twin Primes less than or equal to (N+2), Pi(3⩽Pi⩽Pm) be taken over the odd primes less than or equal to √(N+2), then exists the formulas as follows:

Notes

$$Pt(N) \geqslant INT\ \{\ N\times(1-1/2)\times\prod(1-2/Pi)\ \}-2 \tag{10}$$

Apply the Prime Number Theorem, from above formula we can obtain the formula as follows:

$$Pt(N \mid N\geqslant 10\hat{}4) \geq INT\ \{\ N\times(1-1/2)\times\prod(1-2/Pi)\ \}-2$$

$$\geqslant INT\ \{Ct\times 2N/(Ln\ (N))\hat{}2\}-2 \tag{11}$$

$$\geqslant Ct\times 2N/(Ln(N))\hat{}2-3 \tag{12}$$

$$\prod(Pi(Pi-2)/(Pi-1)\hat{}2) \geqslant Ct = 0.6601618158\ldots \tag{13}$$

When the number $N \rightarrow \infty$, we can obtain the formula as follows:

$$Pt(N \mid N\rightarrow\infty) \geqslant 0.660\times 2N/(Ln\ (N))\hat{}2-3 \rightarrow \infty \tag{14}$$

IX. Conclusion

There are infinitely many pairs of Twin primes which difference by 2.

Notes

This page is intentionally left blank