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# General Coordinate Formula of Strain (Skew Reflection) of $\mathbb{R}^2$ onto $\mathbb{R}^2$

By Gedefa Negassa Feyissa & M.P. Chaudhary

*Madda Walabu University*

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*Keywords:* *coordinate formula, skew reflection.*

*GJSFR-F Classification:* *MSC 2010: 12E15*

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# General Coordinate Formula of Strain (Skew Reflection) of $\mathbb{R}^2$ onto $\mathbb{R}^2$

Gedefa Negassa Feyissa [α] & M.P. Chaudhary [σ]

*Abstract-* The authors establish a set of presumably new results, which provide general coordinate formula of strain(skew reflection) of $\mathbb{R}^2$ onto $\mathbb{R}^2$. Several closely related results are also considered.

*Keywords and Phrases: coordinate formula, skew reflection.*

## I. Introduction

**1.1.** *Definition:* Let $0 \neq k \neq 1$, and $l$ be a line. A strain (skew reflection) with the axis $l$ and coefficient $k$ denoted by $T_{l,k}$, keep $l$ pointwise invariant and maps every other point $P$ to a point $P'$ so that the line $\overline{PP'}$ is perpendicular to $l$.

Equation of strain with X-axis and coefficient $k$ is $T_{y=0,k}((p_1, p_2)) = (p_1, kp_2)$.
Equation of strain with Y-axis and coefficient $k$ is $T_{x=0,k}((p_1, p_2)) = (kp_1, p_2)$.

**1.2.** *Definition:* Rotation about a fixed point $C$ through a directed anggle $\theta$ is a transformation $\rho_{c,\theta}$, which fixes $C$ and sends $P$ to a point $P'$ such that $CP = CP'$, and $\theta$ is the measure of directed andgle from $\overline{CP}$ to $\overline{CP'}$.

## II. Rotation about Origin $O = (0, 0)$ and its Coordinate Equations

Let us consider in $XY$-plane, rotation from positive $X$-axis towards anticlockwise direction, after rotation of angle $\beta$ it reaches at any point $P = (x, y)$ which has distance $r$ from origin $O$. Further, the point $P$ reached to another poinr $P' = (x', y')$in the similar direction by rotation of angle $\theta$. Then we have following elementary results;

$$cos\beta = \frac{x}{r} \implies x = rcos\beta \tag{2.1}$$

$$sin\beta = \frac{y}{r} \implies y = rsin\beta \tag{2.2}$$

similarly, we have

$$cos(\theta + \beta) = \frac{x'}{r} \implies x' = rcos(\theta + \beta)$$

*Author α: Department of Mathematics, Madda Walabu University, Bale Robe, Ethiopia.*
*Author σ: Department of Mathematics, International Sciientific Research and Welfare Organization, New Delhi, India.*
*e-mail: dr.m.p.chaudhary@gmail.com*

$$\implies x' = rcos\theta \cos\beta - rsin\theta sin\beta = xcos\theta - ysin\theta \tag{2.3}$$

$$sin(\theta+\beta) = \frac{y'}{r} \implies y' = rsin(\theta+\beta)$$

$$\implies y' = rsin\theta \cos\beta + rcos\theta sin\beta = xsin\theta + ycos\theta \tag{2.4}$$

Therefore, we have

$$\rho_{O,\theta}((x,y)) = (xcos\theta - ysin\theta, xsin\theta + ycos\theta) \tag{2.5}$$

Notes

## III. Rotation about Arbitrary Centre $C = (c_1, c_2)$ and its Coordinate Equations

Let us consider an arbitrary centre $C = (c_1, c_2)$ in $XY$-plane, rotation from positive $X$-axis towards anticlockwise direction, after rotation of angle $\theta$ it reaches at any point $P = (x, y)$, which has distance $r$ from origin $O = (0, 0)$. Then we have following elementary results;

$$\rho_{C,\theta} = \tau_{O,C} \times \rho_{O,\theta} \times \tau_{C,O} \tag{3.1}$$

$$\implies \rho_{(c_1,c_2),\theta}((x,y)) = \tau_{(O,O),(c_1,c_2)}\big(\rho_{(O,O),\theta}\big(\tau_{(c_1,c_2),(O,O)}((x,y))\big)\big) \tag{3.2}$$

$$= \tau_{(O,O),(c_1,c_2)}\big(\rho_{(O,O),\theta}((x-c_1, y-c_2))\big) \tag{3.3}$$

$$= \tau_{(O,O),(c_1,c_2)}\big(((x-c_1)cos\theta - (y-c_2)sin\theta, (x-c_1)sin\theta + (y-c_2)cos\theta)\big) \tag{3.4}$$

$$= \big((x-c_1)cos\theta - (y-c_2)sin\theta + c_1, (x-c_1)sin\theta + (y-c_2)cos\theta + c_2\big) \tag{3.5}$$

## IV. Derivation of the General Coordinate Formula of Strain (Skew Reflection) of $\mathbb{R}^2$ onto $\mathbb{R}^2$

Let us assume

$$l : y = mx + b, m \neq 0 \quad \text{and} \quad \beta = arctan(m) \tag{4.1}$$

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$$T_{l,k}((p_1,p_2)) = \rho_{(\frac{-b}{m},0),\beta}(T_{y=0,k}(\rho_{(\frac{-b}{m},0),-\beta}((p_1,p_2)))) \tag{4.2}$$

$$= \rho_{(\frac{-b}{m},0),\beta}\Bigg(T_{y=0,k}\Bigg(\Bigg(\Bigg(p_1 + \frac{b}{m}\Bigg)cos(-\beta) - p_2 sin(-\beta)$$

$$-\frac{b}{m} + c_1, \Bigg(p_1 + \frac{b}{m}\Bigg)sin(-\beta) + p_2 cos(-\beta)\Bigg)\Bigg)\Bigg) \tag{4.3}$$

$$= \rho_{(\frac{-b}{m},0),\beta}\Bigg(\Bigg(\Bigg(p_1 + \frac{b}{m}\Bigg)cos(-\beta) - p_2 sin(-\beta)$$

$$-\frac{b}{m} + c_1, k\Bigg\{\Bigg(p_1 + \frac{b}{m}\Bigg)sin(-\beta) + p_2 cos(-\beta)\Bigg\}\Bigg)\Bigg) \tag{4.4}$$

$$= \Bigg(\Bigg(\Bigg(\left(p_1+\frac{b}{m}\right)cos(-\beta)-p_2sin(-\beta)\Bigg)cos\beta$$

$$-\left(k\left\{\left(p_1+\frac{b}{m}\right)sin(-\beta)+p_2cos(-\beta)\right\}\right)sin\beta-\frac{b}{m},$$

$$\left(\left(p_1+\frac{b}{m}\right)cos(-\beta)-p_2sin(-\beta)\right)sin\beta$$

$$+\left(k\left\{\left(p_1+\frac{b}{m}\right)sin(-\beta)+p_2cos(-\beta)\right\}\right)cos\beta\Bigg) \quad (4.5)$$

$$= \Bigg(\left(p_1+\frac{b}{m}\right)cos^2\beta+p_2sin\beta cos\beta+k\left(p_1+\frac{b}{m}\right)sin^2\beta-kp_2sin\beta cos\beta-\frac{b}{m},$$

$$\left(p_1+\frac{b}{m}\right)sin\beta cos\beta+p_2sin^2\beta-k\left(p_1+\frac{b}{m}\right)sin\beta cos\beta+kp_2cos^2\beta\Bigg) \quad (4.6)$$

$$= \Bigg(\left(p_1+\frac{b}{m}\right)cos^2\beta+mp_2cos^2\beta+k\left(p_1+\frac{b}{m}\right)sin^2\beta-mkp_2cos^2\beta-\frac{b}{m},$$

$$m\left(p_1+\frac{b}{m}\right)cos^2\beta+p_2sin^2\beta-mk\left(p_1+\frac{b}{m}\right)cos^2\beta+kp_2cos^2\beta\Bigg) \quad (4.7)$$

## V. Justification of General Formula

Let us consider $(p_1, p_2) \in l$, then we have

$$T_{l.k}((p_1,p_2)) = \Bigg(\left(p_1+\frac{b}{m}\right)cos^2\beta+mp_2cos^2\beta+k\left(p_1+\frac{b}{m}\right)sin^2\beta-mkp_2cos^2\beta$$

$$-\frac{b}{m}, m\left(p_1+\frac{b}{m}\right)cos^2\beta+p_2sin^2\beta-mk\left(p_1+\frac{b}{m}\right)cos^2\beta+kp_2cos^2\beta\Bigg)$$

$$= \Bigg(\left(\frac{p_2}{m}\right)cos^2\beta+mp_2cos^2\beta+k\left(\frac{p_2}{m}\right)sin^2\beta-mkp_2cos^2\beta$$

$$-\frac{b}{m}, m\left(\frac{p_2}{m}\right)cos^2\beta+p_2sin^2\beta-mk\left(\frac{p_2}{m}\right)cos^2\beta+kp_2cos^2\beta\Bigg)$$

$$= \Bigg(\left(\frac{p_2}{m}\right)cos^2\beta+mp_2cos^2\beta+k\left(\frac{p_2}{m}\right)sin^2\beta-mkp_2cos^2\beta$$

$$-\frac{b}{m}, p_2cos^2\beta+p_2sin^2\beta-kp_2cos^2\beta+kp_2cos^2\beta\Bigg)$$

$$= \Bigg(\left(\frac{p_2}{m}\right)cos^2\beta+mp_2cos^2\beta+k\left(\frac{p_2}{m}\right)sin^2\beta-mkp_2cos^2\beta-\frac{b}{m}, p_2(cos^2\beta+sin^2\beta)\Bigg)$$

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$$= \left(\left(\frac{p_2}{m}\right)cos^2\beta + mp_2cos^2\beta + k\left(\frac{p_2}{m}\right)sin^2\beta - mkp_2cos^2\beta - \frac{b}{m}, p_2\right)$$

$$= \left(\left(\frac{p_2}{m}\right)(1 - sin^2\beta) + mp_2cos^2\beta + k\left(\frac{p_2}{m}\right)sin^2\beta - mkp_2cos^2\beta - \frac{b}{m}, p_2\right)$$

$$= \left(\left(\frac{p_2}{m}\right) - \left(\frac{p_2}{m}\right)sin^2\beta + mp_2cos^2\beta + k\left(\frac{p_2}{m}\right)sin^2\beta - mkp_2cos^2\beta - \frac{b}{m}, p_2\right)$$

$$= \left(\left(\frac{p_2}{m}\right) - \left(\frac{p_2}{m}\right)sin^2\beta + m^2\frac{p_2}{m}cos^2\beta + k\left(\frac{p_2}{m}\right)sin^2\beta - mkp_2cos^2\beta - \frac{b}{m}, p_2\right)$$

$$= \left(\left(\frac{p_2}{m}\right) - \left(\frac{p_2}{m}\right)sin^2\beta + \frac{p_2}{m}tan^2\beta cos^2\beta + k\left(\frac{p_2}{m}\right)sin^2\beta - mkp_2cos^2\beta - \frac{b}{m}, p_2\right)$$

$$= \left(\left(\frac{p_2}{m}\right) - \left(\frac{p_2}{m}\right)sin^2\beta + \frac{p_2}{m}sin^2\beta + k\left(\frac{p_2}{m}\right)sin^2\beta - mkp_2cos^2\beta - \frac{b}{m}, p_2\right)$$

$$= \left(p_1 + \frac{b}{m} + k\left(\frac{p_2}{m}\right)sin^2\beta - mkp_2cos^2\beta - \frac{b}{m}, p_2\right)$$

$$= \left(p_1 + \frac{b}{m} + k\left(\frac{p_2}{m}\right)sin^2\beta - m^2k\left(\frac{p_2}{m}\right)cos^2\beta - \frac{b}{m}, p_2\right)$$

$$= \left(p_1 + \frac{b}{m} + k\left(\frac{p_2}{m}\right)sin^2\beta - k\left(\frac{p_2}{m}\right)tan^2\beta cos^2\beta - \frac{b}{m}, p_2\right)$$

$$= \left(p_1 + \frac{b}{m} + k\left(\frac{p_2}{m}\right)sin^2\beta - k\left(\frac{p_2}{m}\right)sin^2\beta - \frac{b}{m}, p_2\right)$$

$$= (p_1, p_2)$$

$$\implies T_{l.k}((p_1, p_2)) = (p_1, p_2)$$

Hence, it keeps $l$ pointwise invariant.

Further, let $T_{l.k}((p_1, p_2)) = (p_1', p_2')$, then the slop of the line between two points $P = (p_1, p_2)$ and $P' = (p_1', p_2')$ is given as;

$$\frac{(p_2' - p_2)}{(p_1' - p_1)} = \frac{m\left(p_1 + \frac{b}{m}\right)cos^2\beta + p_2sin^2\beta - mk\left(p_1 + \frac{b}{m}\right)cos^2\beta + kp_2cos^2\beta - p_2}{\left(p_1 + \frac{b}{m}\right)cos^2\beta + mp_2cos^2\beta + k\left(p_2 + \frac{b}{m}\right)sin^2\beta - mkp_2cos^2\beta - \frac{b}{m} - p_1}$$

$$= \frac{m(1-k)\left(p_1 + \frac{b}{m}\right)\cos^2\beta + (k-1)p_2cos^2\beta}{(k-1)\left(p_1 + \frac{b}{m}\right)\sin^2\beta + m(1-k)p_2cos^2\beta} = \frac{\left\{\frac{m(1-k)\left(p_1+\frac{b}{m}\right)\cos^2\beta+(k-1)p_2cos^2\beta}{cos^2\beta}\right\}}{\left\{\frac{(k-1)\left(p_1+\frac{b}{m}\right)\sin^2\beta+m(1-k)p_2cos^2\beta}{cos^2\beta}\right\}}$$

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$$= \frac{m(1-k)\left(p_1+\frac{b}{m}\right)+(k-1)p_2}{m^2(k-1)\left(p_1+\frac{b}{m}\right)+m(1-k)p_2} = \frac{m(1-k)\left(p_1+\frac{b}{m}\right)+(k-1)p_2}{m\left\{m(k-1)\left(p_1+\frac{b}{m}\right)+(1-k)p_2\right\}}$$

$$= \frac{m\{-(k-1)\}\left(p_1+\frac{b}{m}\right)+(p_2\{-(1-k)\}}{m\left\{m(k-1)\left(p_1+\frac{b}{m}\right)+(1-k)p_2\right\}} = \frac{-\left\{m(k-1)\left(p_1+\frac{b}{m}\right)+(1-k)p_2\right\}}{m\left\{m(k-1)\left(p_1+\frac{b}{m}\right)+(1-k)p_2\right\}}$$

$$= \frac{-1}{m}$$

$$\Longrightarrow \frac{(p_2'-p_2)}{(p_1'-p_1)} = \frac{-1}{m}$$

Hence, line passing through two point $P=(p_1,p_2)$ and $P'=(p_1',p_2')$ i.e line $\overline{PP'}$ is perpendicular to $l$.

## VI. Conclusion

The results on general coordinate formula of strain (skew reflection) of $\mathbb{R}^2$ onto $\mathbb{R}^2$ are summarized as follows;

1. Equation of strain with X-axis and coefficient $k$ is

$$T_{y=0,k}((p_1,p_2)) = (p_1,kp_2)$$

2. Equation of strain with Y-axis and coefficient $k$ is

$$T_{x=0,k}((p_1,p_2)) = (kp_1,p_2)$$

3. If, $l: y=mx+b, m\neq 0$ and $\beta = arctan(m)$; then

$$T_{l.k}((p_1,p_2)) = \Bigg(\left(p_1+\frac{b}{m}\right)cos^2\beta + mp_2cos^2\beta + k\left(p_1+\frac{b}{m}\right)sin^2\beta$$

$$-mkp_2cos^2\beta - \frac{b}{m}, m\left(p_1+\frac{b}{m}\right)cos^2\beta + p_2sin^2\beta - mk\left(p_1+\frac{b}{m}\right)cos^2\beta + kp_2cos^2\beta\Bigg).$$

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