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A Reliable Technique for Solving Gas Dynamic Equation using Natural Homotopy Perturbation Method

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Abstract- In this paper, a new analytical technique called Natural transform homotopy perturbation method has been successfully applied to obtain exact solution of nonlinear gas dynamic equation.

Application of the method to three test modelling problems from mathematical physics lead to sequence which tends to the exact solution of the problems.

The solution procedure shows the reliability of the method and is high accuracy evident.

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A Reliable Technique for Solving Gas Dynamic Equation using Natural Homotopy Perturbation Method

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Abstract- In this paper, a new analytical technique called Natural transform homotopy perturbation method has been successfully applied to obtain exact solution of nonlinear gas dynamic equation.

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I. Introduction

Gas dynamics is a science in the branch of fluid dynamics concerned with studying the motion of gases and its effects on physical systems, based on the principles of fluid mechanics and thermodynamics.

The science arises from the studies of gas flows, often around or within physical bodies. Examples of these studies include but not limited to choked flows in nozzles and valves, shock waves around jets, aerodynamic heating on atmospheric reentry vehicles and flow of gas fuel within a jet engine.

The equations of gas dynamics (Aminikah & Jamalian, 2013) are mathematical expressions based on the physical laws of conservation, namely, the laws of conservation of mass, conservation of momentum, conservation of energy and so forth.

The nonlinear equations of ideal gas dynamics are applicable for three types of nonlinear waves like shock fronts, rare factions and contact discontinuities.

Different types of gas dynamics equations in physics have been solved using Fourier transform A domian decomposition method (Ramezanpour, et al, 2013), homotopy analysis method (Jafari, et al, 2009), finite difference scheme (Rasulov & Karaguler, 2003), Reduced differential transform method (Keskin & Oturanc, 2010), Reconstruction of Variational Iteration method (Nikkar, 2012), Homotopy perturbation method (Jafari, et al, 2008), Variational iterative method (Jafari, et al, 2008), El-zaki transform homotopy perturbation method (Bhadane & Pradhan, 2013), Natural decomposition method (Maitama & Sabuwa, 2014), Variational homotopy perturbation method (Matinfar & Raeisi, 2011), Differential transform method, Modified homotopy perturbation method (Mohiuddin, 2015), Homotopy perturbation transform method (Singh, et al, 2012) and so on.

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In this study, we construct solution of gas dynamic equation by using a combination of Natural transform and homotopy perturbation methods (NHPM).

In one spatial dimension, the inviscid equations of gas dynamics can be written in the conservative form (Keskin & Oturanc, 2010) as:

$$v_t(x,t)-v(x,t)+\frac{1}{2}v_x{}^2(x,t)+v^2(x,t)=f(x,t)\,;\;\; 0\le x\le 1,\; t>0 \tag{1.1}$$

$$v(x,0)=g(x) \tag{1.2}$$

Notes

II. Analysis of the Method

For the purpose of illustration of the methodology of the proposed method, we write the gas dynamic equation in the standard operator form:

$$Dv(x,t)+Rv(x,t)+Nv(x,t)=f(x,t) \tag{2.1}$$

With the following initial conditions:

$$v(x,0)=g(x)\;\;;\;\; v_t(x,0)=h(x) \tag{2.2}$$

Where $D(v(x,t))=v_t(x,t)$ is a linear operator which has partial derivatives; $R(v(x,t))=v(x,t)$; $N(v(x,t))=\frac{1}{2}v_x{}^2(x,t)+v^2(x,t)$is a nonlinear term and $g(x)$ is an in homogenousterm.

Applying the Natural transform to equation (2.1) subject to the given initial condition, we have

$$N^+[Dv(x,t)]+N^+[Rv(x,t)]+N^+[Nv(x,t)]=N^+[f(x,t)] \tag{2.3}$$

using differentiation property of Natural transform and above initial conditions, we have

$$V(x,s,v)=\frac{1}{s}g(x)+\frac{v}{s^2}h(x)-\frac{v^2}{s^2}\mathrm{N}^+[RV(x,t)]-\frac{v^2}{s^2}\mathrm{N}^+[NV(x,t)]+\frac{v^2}{s^2}\mathrm{N}^+[f(x,t)] \tag{2.4}$$

Operating with the inverse Natural transform on both sides of equation (2.3), we have

$$v(x,t)=F(x,t)-\mathrm{N}^{-1}\left[\frac{v^2}{s^2}\mathrm{N}^+[Rv(x,t)+\mathrm{N}v(x,t)]\right] \tag{2.5}$$

where $F(x,t)$ represents the term arising from the source term and the prescribed initial condition.

Now, applying the homotopy perturbation method (HPM)

$$v(x,t)=\sum_{n=0}^{\infty}P^n v_n(x,t) \tag{2.6}$$

and the nonlinear term can be decomposed as

$$Nv(x,t)=\sum_{n=0}^{\infty}P^n H_n(v) \tag{2.7}$$

where $H_n(v)$ are He's polynomials and can be evaluated using the following formula:

$$H_n(v_1, v_2, \ldots, v_n) = \frac{1}{n!}\frac{\partial^n}{\partial P^n}\left[N\sum_{j=0}^{n} P^J V_J\right]_{p=0} \; ; \; n = 0, 1, 2, \ldots \tag{2.8}$$

Substituting equations (2.6) and (2.7) in (2.5); we have

Notes

$$\sum_{n=0}^{\infty} P^n v_n(x,t) = F(x,t) - P\left(\mathrm{N}^{-1}\left[\frac{v^2}{s^2}\mathrm{N}^+\left[R\sum_{n=0}^{\infty} P^n v_n(x,t) + \sum_{n=0}^{\infty} P^n H_n(v)\right]\right]\right) \tag{2.9}$$

which is the coupling of the Natural transform and the homotopy perturbation method using

He's polynomials.

Comparing the coefficient of same powers of P, we obtain the following approximations:

$$\begin{aligned}
P^0 &: v_0(x,t) = F(x,t) \\
P^1 &: v_1(x,t) = -\mathrm{N}^{-1}\left[\frac{v^2}{s^2}\mathrm{N}^+\left[Rv_0(x,t) + H_0(v)\right]\right] \\
P^2 &: v_2(x,t) = -\mathrm{N}^{-1}\left[\frac{v^2}{s^2}\mathrm{N}^+\left[Rv_1(x,t) + H_1(v)\right]\right] \\
P^3 &: v_3(x,t) = -\mathrm{N}^{-1}\left[\frac{v^2}{s^2}\mathrm{N}^+\left[Rv_2(x,t)\right] + H_2(v)\right]
\end{aligned} \tag{2.10}$$

and so on.

Thus, the series solution of equation (2.1) is

$$v(x,t) = \lim_{k \to \infty} \sum_{n=0}^{k} v_n(x,t) \tag{2.11}$$

III. Experimental Evaluation

In this section; we consider the following nonlinear homogenous and nonhomogenous gas dynamics equations:

Example 3.1: Consider the nonlinear homogenous gas dynamicequation [3-7]:

$$v_t(x,t) + \frac{1}{2}v^2{}_x(x,t) - v(x,t) + v^2(x,t) = 0 \tag{3.1}$$

with initial condition $$v(x,0) = g(x) = e^{-x} \tag{3.2}$$

applying the natural transform on both sides of equation (3.1) subject to the initial condition (3.2), we have

$$v(x,s) = \frac{e^{-x}}{s} + \frac{v}{s}\mathrm{N}^+\left[v(1-v) - \frac{1}{2}(v^2)_x\right] \tag{3.3}$$

The inverse of natural transform implies that

$$v(x\ t)=e^{-x}+\mathrm{N}^{-1}\left[\frac{v}{s}\mathrm{N}^{+}\left[v(1-v)-\frac{1}{2}\left(v^{2}\right)_{x}\right]\right] \tag{3.4}$$

Now, we apply the homotopy perturbation method to get

$$\sum_{n=0}^{\infty}P^{n}v_{n}(x,t)=e^{-x}+P\left(\mathrm{N}^{-1}\left[\frac{v}{s}\mathrm{N}^{+}\left[\sum_{n=0}^{\infty}P^{n}v_{n}(x,t)\left(1-\sum_{n=0}^{\infty}P^{n}v_{n}(x,t)\right)-\frac{1}{2}\left(\left(\sum_{n=0}^{\infty}P^{n}v_{n}(x,t)\right)^{2}\right)_{x}\right]\right]\right) \tag{3.5}$$

Comparing the coefficients of like powers of P in eqn (3.5); we obtain the following approximations.

$$P^{0}:\quad v_{0}(x,t)=e^{-x}$$

$$P^{1}:\quad v_{1}(x,t)=\mathrm{N}^{-1}\left[\frac{v}{s}\mathrm{N}^{+}\left[v_{0}(1-v_{0})-\frac{1}{2}\left(v_{0}{}^{2}\right)_{x}\right]\right]=e^{-x}t$$

$$P^{2}:\quad v_{2}(x,t)=\mathrm{N}^{-1}\left[\frac{v}{s}\mathrm{N}^{+}\left[v_{1}(1-v_{1})-\frac{1}{2}\left(v_{1}{}^{2}\right)_{x}\right]\right]=e^{-x}\frac{t^{2}}{2!}$$

$$P^{3}:\quad v_{3}(x,t)=\mathrm{N}^{-1}\left[\frac{v}{s}\mathrm{N}^{+}\left[v_{2}(1-v_{2})-\frac{1}{2}\left(v_{2}{}^{2}\right)_{x}\right]\right]=e^{-x}\frac{t^{3}}{3!}$$

and so on.

Therefore, the solution $v(x,t)$ is given by

$$v(x,t)=e^{-x}\left(1+t+\frac{t^{2}}{2!}+\frac{t^{3}}{3!}+\frac{t^{4}}{4!}+\quad...\right)=e^{t-x} \tag{3.6}$$

Obtained upon using the Taylor expansion for e^{t}. Higher number of iterations lead to convergence to the exact solution e^{t-x}. This result is similar to what obtained using HPTM [16], RDTM [7], VIM [5], RVIM [7], ADM [13], ETHPM [2], HPM [4], VHPM [10], MHPM [11] and NDM[9].

Example 3.2: Consider the nonlinear, nonhomogenous gas dynamic equation [2,12,13]

$$v_{t}(x,t)+\frac{1}{2}v_{x}{}^{2}(x,t)-v(x,t)+v^{2}(x,t)=-e^{t-x} \tag{3.7}$$

with initial condition $$v(x,0)=g(x)=1-e^{-x} \tag{3.8}$$

Applying the natural transform on both sides of equation (3.7) subject to the initial condition (3.8), we have

$$v(x,s)=\frac{1-e^{-x}}{s}-\frac{ve^{-x}}{s(s-v)}+\frac{v}{s}\left[\mathrm{N}^{+}\left[v(1-v)-\frac{1}{2}\left(v^{2}\right)_{x}\right]\right] \tag{3.9}$$

The inverse of Natural transform implies that:

Ref

4. Jafari, H.,Zabihi, M., Saidy, M., 2008.Application of homotopy perturbation method for solving gas dynamic equation. Applied Mathematical Sciences. 2, 2393-2396.

$$v(x,t)=\left(1-e^{t-x}\right)+\mathrm{N}^{-1}\left[\frac{v}{s}\mathrm{N}^{+}\left[v(1-v)-\frac{1}{2}\left(v^{2}\right)_{x}\right]\right] \tag{3.10}$$

Since $$\mathrm{N}^{-1}\left[\frac{ve^{-x}}{s(s-v)}\right]=e^{-x}\,\mathrm{N}^{-1}\left[\frac{v}{s(s-v)}\right]=e^{-x}\left(e^{t}-1\right)$$

Now, we apply the homotopy perturbation method to get:

$$\sum_{n=0}^{\infty}P^{n}v_{n}(x,t)=\left(1-e^{t-x}\right)+P\left(\mathrm{N}^{-1}\left[\frac{v}{s}\mathrm{N}^{+}\left[\sum_{n=0}^{\infty}P^{n}v_{n}(x,t)\left(1-\sum_{n=0}^{\infty}P^{n}v_{n}(x,t)\right)-\frac{1}{2}\left(\left(\sum_{n=0}^{\infty}P^{n}v_{n}(x,t)\right)^{2}\right)_{x}\right]\right]\right) \tag{3.11}$$

Comparing the coefficients of like powers of P in eqn (3.11), we obtain the following approximations:

$$P^{0}: v_{0}(x,t)=1-e^{t-x}$$

$$P^{1}: v_{1}(x,t)=\mathrm{N}^{-1}\left[\frac{v}{s}\mathrm{N}^{+}\left[v_{0}(1-v_{0})-\frac{1}{2}\left(v_{0}{}^{2}\right)_{x}\right]\right]=0$$

$$P^{2}: v_{2}(x,t)=N^{-1}\left[\frac{v}{s}\mathrm{N}^{+}\left[v_{1}(1-v_{1})-\frac{1}{2}\left(v_{1}{}^{2}\right)_{x}\right]\right]=0$$

$$P^{3}: v_{3}(x,t)=N^{-1}\left[\frac{v}{s}\mathrm{N}^{+}\left[v_{2}(1-v_{2})-\frac{1}{2}\left(v_{2}{}^{2}\right)_{x}\right]\right]=0$$

and so on.
Therefore, the solution $v(x,t)$ is given by

$$v(x,t)=1-e^{t-x} \tag{3.12}$$

Similar to results obtained using VIM [5], RVIM [12], ADM [13], ETHPM [2], VHPM [10]and NDM [9].

Example 3.3: Consider the non-homogenous, nonlinear gas dynamic equation [1,3]

$$v_{t}(x,t)+v(x,t)v_{x}(x,t)-v(x,t)+v^{2}(x,t)=-e^{t-x} \tag{3.13}$$

with initial condition $$v(x,0)=1-e^{-x} \tag{3.14}$$

Applying the natural transform on both sides of eqn (3.13) subject to the initial condition (3.14)we have

$$v(x,s)=\frac{1-e^{-x}}{s}-\frac{ve^{-x}}{s(s-v)}+\frac{v}{s}\mathrm{N}^{+}\left[v(1-v)-vv_{x}\right] \tag{3.15}$$

The inverse of Natural transform implies that

$$v(x,t)=1-e^{t-x}+\mathrm{N}^{-1}\left[\frac{v}{s}\mathrm{N}^{+}\left[v(1-v)-vv_{x}\right]\right] \tag{3.16}$$

Ref

12. Nikkar, A., 2012. A new approach for solving gas dynamic equation. Acta Technica Corviniensis Bulletin of Engineering. 4, 113-116.

Now, we apply the homotopy perturbation method to get:

$$\sum_{n=0}^{\infty} P^n v_n(x,t) = 1 - e^{t-x} + P\left(N^{-1}\left[\frac{v}{s}N^{+}\left[\sum_{n=0}^{\infty} P^n v_n(x,t)\left(1-\sum_{n=0}^{\infty} P^n v_n(x,t)\right) - \sum_{n=0}^{\infty} P^n v_n(x,t)\left(\sum_{n=0}^{\infty} P^n v_n(x,t)\right)_x\right]\right]\right) \tag{3.17}$$

Comparing the coefficients of like powers of P in eqn (3.17), we obtain the following approximations

$$P^0 : v_0(x,t) = 1 - e^{t-x}$$

$$P^1 : v_1(x,t) = \mathrm{N}^{-1}\left[\frac{v}{s}\mathrm{N}^{+}\left[v_0(1-v_0) - v_0 v_{0x}\right]\right] = 0$$

$$P^2 : v_2(x,t) = \mathrm{N}^{-1}\left[\frac{v}{s}\mathrm{N}^{+}\left[v_1(1-v_1) - v_1 v_{1x}\right]\right] = 0$$

$$P^3 : v_3(x,t) = \mathrm{N}^{-1}\left[\frac{v}{s}\mathrm{N}^{+}\left[v_2(1-v_2) - v_2 v_{2x}\right]\right] = 0$$

and so on.

Therefore, the solution $v(x,t)$ is given by

$$v(x,t) = 1 - e^{t-x} \tag{3.18}$$

Which is the exact solution of the problem.

IV. Conclusion

In the present work, we proposed a combination of Natural transform and Homotopy perturbation methods and successfully applied it to study the homogenous and non-homogenous cases of nonlinear gas dynamics equation.

The method gave closed form solution of the equations, with high accuracy, using the initial conditions and can be considered as a reliable refinement of existing techniques.

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Notes

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