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Analytical Solutions for a Horizontally Oscillated Semi-Submerged Cylinder

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GJSFR-F Classification: MSC 2010: 33C10

ANALYTICALSOLUTIONSFORAHORIZONTALLYOSCILLATEDSEMISUBMERGEDCYLINDER

Strictly as per the compliance and regulations of:

Ref

1. Krylov, N. N. and Bogoliubov N. N., Introduction to Nonlinear Mechanics, Princeton University Press, New Jersey, 1947.

Analytical Solutions for a Horizontally Oscillated Semi-Submerged Cylinder

Shamima Aktar [α] & M. Abul Kawser [σ]

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I. Introduction

Krylov and Bogoliubov [1] initiated a perturbation method to obtain approximate solution (oscillatory type) of the second order nonlinear differential system with a small nonlinearity

$$\ddot{x}+\omega_0^2 x=-\varepsilon f(x,\dot{x}) \qquad (1)$$

where the over dots denote the differentiation with concerning t, $\omega_0>0$ and ε is a small parameter. This method has amplified and justified by Bogoliubov and Mitropolskii [2, 3]. Today the method is a well-known method as Krylov-Bogoliubov-Mitropolskii (KBM) [1, 2] in the literature of nonlinear oscillations. Popov [4] extended it to the following damped oscillatory system

$$\ddot{x}+c\dot{x}+\omega^2 x=-\varepsilon f(x,\dot{x}) \qquad (2)$$

where $c>0$, $\omega>0$ and $c<2\omega$. It is to be noted that if $c\geq 2\omega$, the system (2) becomes non-oscillatory. First, Murty *et al.* [5, 6] used this method to obtain an approximate solution of (2) characterized by non-oscillatory processes. In the case of over-damped systems, we know the characteristic roots of the unperturbed equation of (2) become real, unequal and negative inequality. The roots of the unperturbed equation of (1) are purely imaginary. On the contrary, these are complex conjugate with the negative real part, when $c<2\omega$ (considered by Popov[4]). Sattar [7] found an approximate solution of (2) characterized by critical damping. An asymptotic solution proposed by Kawser and Akbar [8] for the third order critically damped nonlinear system. Kawser

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and Sattar [9] suggested an asymptotic solution of a fourth order critically damped nonlinear system with pair-wise equal eigenvalues. Later, Kawser *et al.* [10] has developed a method for fourth order critically damped oscillatory nonlinear systems when the eigenvalues are complex and pair-wise equal. It has further extended by Kawser *et al.* [11] to fourth order critically undamped oscillatory nonlinear systems with pair-wise equal imaginary eigenvalues. Recently, Kawser *et al.* [12] presented a technique to obtain perturbation solutions of fifth order critically undamped nonlinear oscillatory systems with pair-wise equal imaginary eigenvalues.

In this paper, we have investigated the solutions of a horizontally semi-submerged cylinder in a liquid under oscillations due to the gravitational force for both oscillatory and damped oscillatory motions. So in these cases, the eigenvalues are imaginary and complex conjugate for undamped and damped motions respectively. For different sets of initial conditions the solutions show excellent coincidence with the numerical solutions obtained by the *Mathematica* program.

II. Formulation of the Problem

Suppose a half-submerged horizontal cylinder of radius (R) and length (l) is floating in a liquid. If we consider x is instantaneous displacement of a diametric plane from the equilibrium position.
Volume of cylinder from the bottom of height h is,

$$V_h = l\left[R^2\cos^{-1}\left(\frac{R-h}{R}\right)-(R-h)\sqrt{2Rh-h^2}\right] \tag{3}$$

Now using equation (3), we obtain the volume of cylinder from the bottom of height R *i.e.* volume of half cylinder is, $V_R = \frac{1}{2}\pi R^2 l$

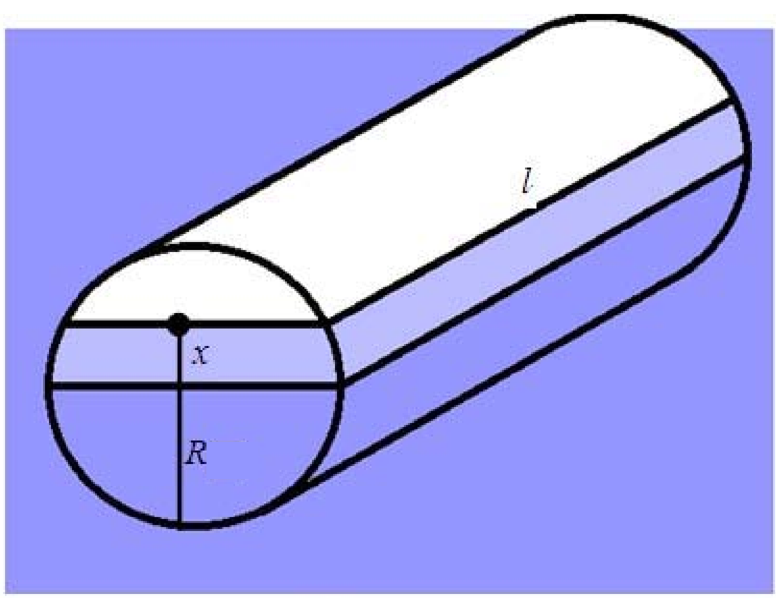

Figure 1: The oscillating semi-submerged cylinder in a liquid

And the volume of cylinder from the bottom of height $R+x$ is,

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12. Abul Kawser, M., Mahafujur Rahaman, Md. and Nurul Islam, Md., Perturbation Solutions of Fifth Order Critically Undamped Nonlinear Oscillatory Systems with Pairwise Equal Eigenvalues, International Journal of Mathematics and Computation, Vol. 28, pp. 22-39, 2017.

$$V_{R+x} = l\left[R^2 \cos^{-1}\left(\frac{R-R-x}{R}\right) - (R-R-x)\sqrt{2R(R+x)-(R+x)^2}\right]$$

Thus the volume of the partial part from the diametric plane of height x is

Notes

$$V_x = V_{R+x} - V_R$$

$$= l\left[R^2 \cos^{-1}\left(-\frac{x}{R}\right) + x(R^2-x^2)^{\frac{1}{2}} - \frac{\pi}{2}R^2\right]$$

$$= l\left[R^2\left\{\frac{\pi}{2}+\frac{x}{R}+\frac{1}{6}\frac{x^3}{R^3}+\frac{31}{40}\frac{x^5}{R^5}+\cdots\cdots\right\}\right.$$

$$\left. + xR\left\{1-\frac{x^2}{2R^2}-\frac{x^4}{8R^4}-\cdots\cdots\right\} - \frac{\pi}{2}R^2\right]$$

As x is very small and R is large, so neglecting the terms higher than $\left(\frac{x}{R}\right)^3$, we get

$$V_x = l\left[R^2\left\{\frac{\pi}{2}+\frac{x}{R}+\frac{x^3}{6R^3}\right\} + xR\left(1-\frac{x^2}{2R^2}\right) - \frac{\pi}{2}R^2\right]$$

$$= 2l\left[xR - \frac{x^3}{6R}\right]$$

Suppose m is the mass of the cylinder, m_1 is mass of the liquid occupied by the volume V_x, ρ is the density of the cylinder, 2ρ is the density of the liquid and g is the gravitational force. Suppose the semi-submerged cylinder is oscillating in the liquid without damping, then Newton's second law of motion gives

$$m\frac{d^2x}{dt^2} = -m_1 g$$

$$i.e., \frac{d^2x}{dt^2} + \frac{8g}{\pi R}x = \frac{4g}{3\pi R^3}x^3 \qquad (4)$$

Also if the half-submerged cylinder is floating in the liquid under damping, then the equation of the system is given by

$$\frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + \frac{8g}{\pi R}x = \frac{4g}{3\pi R^3}x^3 \qquad (5)$$

where $2k$ is the damping constant.

III. The Method

Consider a second order weakly nonlinear ordinary differential system

$$\ddot{x}+2k\dot{x}+\omega^2 x=-\varepsilon\, f(x,\dot{x}) \tag{6}$$

where over dots are used for the first and second derivatives of x concerning t; k is a non-negative constant, ε is a small parameter and $f(x,\dot{x})$ is the nonlinear function. Since the equation is second order, so, we shall get two eigenvalues for the damped oscillatory system and the eigenvalues are complex conjugate, *i.e.* $-k\pm i\lambda$ (say), where $\lambda=\sqrt{\omega^2-k^2}$ and $\omega>k$, and for oscillatory systems *i.e.* $k=0$, then the eigenvalues of system (6) are $\pm i\omega$.

When $\varepsilon=0$, then the equation (6) becomes linear, and the solution of the corresponding linear equation (6) is

$$x(t,0)=e^{-kt}\left(a_0\cos\lambda t+b_0\sin\lambda t\right) \tag{7}$$

where a_0 and b_0 are arbitrary constants.

Now we seek a solution of (6) that reduces to (7) as the limit $\varepsilon\to 0$. We look for an asymptotic solution of (6) is

$$x(t,\varepsilon)=e^{-kt}\left(a\cos\lambda t+b\sin\lambda t\right)+\varepsilon u_1\left(a,b,t\right)+O(\varepsilon^2) \tag{8}$$

where *a, b* are slowly varying functions of time, t and satisfy the following first order differential equations:

$$\frac{da}{dt}=\varepsilon A_1\left(a,b,t\right)+\ldots\ldots$$

$$\frac{db}{dt}=\varepsilon B_1\left(a,b,t\right)+\ldots\ldots \tag{9}$$

Now differentiating (8) twice times, substituting for the derivatives $\dot{x}$, $\ddot{x}$ and x in (6). Now utilizing relations (9) and comparing the coefficients of various powers of ε, we get

$$e^{-kt}\left\{\left(\frac{\partial A_1}{\partial t}+2\lambda B_1\right)\cos\lambda t+\left(-2\lambda A_1+\frac{\partial B_1}{\partial t}\right)\sin\lambda t\right\}$$

$$+\frac{\partial^2 u_1}{\partial t^2}+2k\frac{\partial u_1}{\partial t}+\omega^2 u_1=-f^{(0)}\left(a,b,t\right) \tag{10}$$

where $f^{(0)}=f\left(x_0,\dot{x}_0\right)$ and $x_0=e^{-kt}\left(a\cos\lambda t+b\sin\lambda t\right)$.

Notes

For the oscillatory system to obtain the solution, we have to put $k=0$ and replacing λ by ω in equation (10).
Thus for oscillatory system, we get

$$\left(\frac{\partial A_1}{\partial t}+2\omega B_1\right)\cos\omega t+\left(-2\omega A_1+\frac{\partial B_1}{\partial t}\right)\sin\omega t+\frac{\partial^2 u_1}{\partial t^2}+\omega^2 u_1=-f^{(0)}(a,b,t) \tag{11}$$

Usually, equation (10) or (11) is solved for the unknown functions A_1 and B_1 under the assumption that u_1 does not contain first harmonic terms. We shall follow this assumption (early imposed by KBM [1, 3]) partially to obtain approximate solutions of nonlinear systems with large damping.

IV. Oscillatory Motion

For the oscillatory motion from equation (4), we have

$$\ddot{x}+\frac{8g}{\pi R}x=\varepsilon x^3 \tag{12}$$

where $\varepsilon=\dfrac{4g}{3\pi R^3}$

Thus the solution of equation (12) is given by putting $k=0$ and replacing λ by ω in equation (8), we get

$$x(t,\varepsilon)=a\cos\omega t+b\sin\omega t+\varepsilon u_1(a,b,t) \tag{13}$$

where $\omega=\sqrt{\dfrac{8g}{\pi R}}$.
Comparing equation (12) with the equation (6), we obtain

$$f(x,\dot{x})=x^3$$

Therefore, $f^{(0)}=[a\cos\omega t+b\sin\omega t]^3$

$$=\frac{3}{4}\left(a^3+ab^2\right)\cos\omega t+\frac{3}{4}\left(a^2b+b^3\right)\sin\omega t$$

$$+\left(\frac{1}{4}a^3-\frac{3}{4}ab^2\right)\cos 3\omega t+\left(\frac{3}{4}a^2b-\frac{1}{4}b^3\right)\sin 3\omega t \tag{14}$$

Substituting $f^{(0)}$ from equation (14) into equation (11), we obtain

$$-2A_1\omega\sin\omega t+\frac{\partial A_1}{\partial t}\cos\omega t+2B_1\omega\cos\omega t+\frac{\partial B_1}{\partial t}\sin\omega t+\left(D^2+\omega^2\right)u_1$$

$$=\frac{3}{4}\left(a^3+ab^2\right)\cos\omega t+\frac{3}{4}\left(a^2b+b^3\right)\sin\omega t \tag{15}$$

Ref

1. Krylov, N. N. and Bogoliubov N. N., Introduction to Nonlinear Mechanics, Princeton University Press, New Jersey, 1947.

$$+\left(\frac{1}{4}a^3-\frac{3}{4}ab^2\right)\cos 3\omega t+\left(\frac{3}{4}a^2b-\frac{1}{4}b^3\right)\sin 3\omega t$$

According to our assumption, u_1 does not contain first harmonic terms of $f^{(0)}$. The following equations can be obtained by comparing the coefficients of $\sin\omega t$ and $\cos\omega t$ are the higher argument terms of $\sin\omega t$ and $\cos\omega t$ as

$$\left(D^2+4\omega^2\right)A_1=-\frac{3\omega}{2}b^3-\frac{3\omega}{2}a^2b \quad (16)$$

$$\left(D^2+4\omega^2\right)B_1=\frac{3\omega}{2}a^3+\frac{3\omega}{2}ab^2 \quad (17)$$

$$\left(D^2+\omega^2\right)u_1=\left(\frac{1}{4}a^3-\frac{3}{4}ab^2\right)\cos 3\omega t+\left(\frac{3}{4}a^2b-\frac{1}{4}b^3\right)\sin 3\omega t \quad (18)$$

The solutions of the equations (16) to (18) are respectively

$$A_1=-\frac{3(b^3+a^2b)}{8\omega} \quad (19)$$

$$B_1=\frac{3(a^3+ab^2)}{8\omega} \quad (20)$$

$$u_1=\left(3ab^2-a^3\right)\cos 3\omega t+\left(b^3-3a^2b\right)\sin 3\omega t \quad (21)$$

Substituting the values of A_1, B_1 from equations (19) and (20) into equation (9), we obtain

$$\frac{da}{dt}=-\varepsilon\frac{3(b^3+a^2b)}{8\omega} \quad (22)$$

$$\frac{db}{dt}=\varepsilon\frac{3(a^3+ab^2)}{8\omega} \quad (23)$$

Thus for the transformation $a=c\cos\phi$ and $b=-c\sin\phi$, the equations (21) to (23) respectively become

$$u_1=-\frac{c^3}{32\omega^2}\cos(3\omega t+3\phi) \quad (24)$$

And

$$\dot{c}=0$$

$$\dot{\phi}=-\frac{3\varepsilon c^2}{8\omega}$$

Notes

Or,
$$c = c_0 \tag{25}$$

$$\phi = \phi_0 - \frac{3\varepsilon c^2 t}{8\omega} \tag{26}$$

Thus by substituting $a = c\cos\phi$ and $b = -c\sin\phi$ into equation (13) and after simplification it becomes

Notes

$$x(t,\varepsilon) = c\cos(\omega t + \phi) + \varepsilon u_1 \tag{27}$$

Therefore, equation (27) represents the first order oscillatory solution of equation (12), where c, ϕ, u_1 are given by (25), (26) and (24).

V. Damped Oscillatory Motion

For the damped oscillatory motion, we have from equation (5)

$$\ddot{x} + 2k\dot{x} + \frac{8g}{\pi R}x = \varepsilon x^3 \tag{28}$$

where $\varepsilon = \dfrac{4g}{3\pi R^3}$

Comparing equation (28) with the equation (6), we obtain

$$f(x,\dot{x}) = x^3 \tag{29}$$

Therefore, $f^{(0)} = [e^{-kt}(a\cos\lambda t + b\sin\lambda t)]^3$

$$= e^{-3kt}\left(a^3\cos^3\lambda t + 3a^2 b\sin\lambda t\cos^2\lambda t + 3ab^2\sin^2\lambda t\cos\lambda t + b^3\sin^3\lambda t\right)$$

$$= e^{-3kt}\left[\frac{3}{4}\left(a^3 + ab^2\right)\cos\lambda t + \frac{3}{4}\left(a^2 b + b^3\right)\sin\lambda t\right. \tag{30}$$

$$\left. + \left(\frac{1}{4}a^3 - \frac{3}{4}ab^2\right)\cos 3\lambda t + \left(\frac{3}{4}a^2 b - \frac{1}{4}b^3\right)\sin 3\lambda t\right]$$

where $\lambda = \sqrt{\dfrac{8g}{\pi R} - k^2}$.

Substituting $f^{(0)}$ from equation (30) into equation (10), we obtain

$$e^{-kt}\left(-2A_1\lambda\sin\lambda t + \frac{\partial A_1}{\partial t}\cos\lambda t + 2B_1\lambda\cos\lambda t + \frac{\partial B_1}{\partial t}\sin\lambda t\right)$$

$$\left(D^2 + 2kD + \frac{8g}{\pi R}\right)u_1 = e^{-3kt}\left[\frac{3}{4}\left(a^3 + ab^2\right)\cos\lambda t + \frac{3}{4}\left(a^2 b + b^3\right)\sin\lambda t\right. \tag{31}$$

$$\left. + \left(\frac{1}{4}a^3 - \frac{3}{4}ab^2\right)\cos 3\lambda t + \left(\frac{3}{4}a^2 b - \frac{1}{4}b^3\right)\sin 3\lambda t\right]$$

Since, u_1 does not contain first harmonic terms, the following equations obtained by comparing the coefficients of $\sin\lambda t$ and $\cos\lambda t$ are the higher argument terms of $\sin\lambda t$ and $\cos\lambda t$ as

$$\left(D^2+4\lambda^2\right)A_1=-\frac{3}{2}e^{-2kt}\left\{k\left(a^3+ab^2\right)+\lambda\left(a^2b+b^3\right)\right\} \tag{32}$$

$$\left(D^2+4\lambda^2\right)B_1=\frac{3}{2}e^{-2kt}\left\{\lambda\left(a^3+ab^2\right)-k\left(a^2b+b^3\right)\right\} \tag{33}$$

$$\left(D^2+2kD+\frac{8g}{\pi R}\right)u_1=e^{-3kt}\cos 3\lambda t\left(\frac{1}{4}a^3-\frac{3}{4}ab^2\right)+e^{-3kt}\sin 3\lambda t\left(\frac{3}{4}a^2b-\frac{1}{4}b^3\right) \tag{34}$$

The solutions of the equations (32) to (34) are respectively

$$A_1=-\frac{3e^{-2kt}\left\{k\left(a^3+ab^2\right)+\lambda\left(a^2b+b^3\right)\right\}}{8\left(k^2+\lambda^2\right)} \tag{35}$$

$$B_1=\frac{3e^{-2kt}\left\{-k\left(a^2b+b^3\right)+\lambda\left(a^3+ab^2\right)\right\}}{8\left(k^2+\lambda^2\right)} \tag{36}$$

$$u_1=\frac{e^{-3kt}}{16(k^4+5k^2\lambda^2+4\lambda^4)}\Big[(a^3-3ab^2)\{(k^2-2\lambda^2)\cos 3\lambda t-3k\lambda\sin 3\lambda t\}$$
$$+(3a^2b-b^3)\{3k\lambda\cos 3\lambda t+(k^2-2\lambda^2)\sin 3\lambda t\}\Big] \tag{37}$$

Substituting the values of A_1, B_1 from equations (35) and (36) into equation (9), we obtain

$$\frac{da}{dt}=-\varepsilon\frac{3e^{-2kt}\left\{k\left(a^3+ab^2\right)+\lambda\left(a^2b+b^3\right)\right\}}{8\left(k^2+\lambda^2\right)} \tag{38}$$

$$\frac{db}{dt}=\varepsilon\frac{3e^{-2kt}\left\{-k\left(a^2b+b^3\right)+\lambda\left(a^3+ab^2\right)\right\}}{8\left(k^2+\lambda^2\right)} \tag{39}$$

Therefore, under the transformation, $a=c\sin\phi$ and $b=-c\sin\phi$ equations (37) to (39) respectively become

$$u_1=\frac{c^3e^{-3kt}}{16(k^4+5k^2\lambda^2+5\lambda^4)}[(k^2-2\lambda^2)\cos\left(3\lambda t+3\phi\right)-3k\lambda\sin\left(3\lambda t+3\phi\right)] \tag{40}$$

$$\dot{\phi}=-\frac{3\lambda\varepsilon c^2e^{-2kt}}{8(k^2+\lambda^2)}$$

Notes

Notes

$$\dot{c} = -\frac{3k\varepsilon c^3 e^{-2kt}}{8(k^2+\lambda^2)}$$

Or,
$$\phi = \phi_0 + \frac{3\lambda\varepsilon c^2}{16k\left(k^2+\lambda^2\right)}\left(e^{-2kt}-1\right) \tag{41}$$

$$c = c_0 + \varepsilon\frac{3c_0^{\ 3}}{16\left(k^2+\lambda^2\right)}\left(e^{-2kt}-1\right) \tag{42}$$

Thus by substituting $a_1 = c\cos\phi$ and $b_1 = -c\sin\phi$ into equation (8) and after simplification it becomes

$$x(t,\varepsilon) = ce^{-kt}\cos\left(\lambda t+\phi\right)+\varepsilon u_1 \tag{43}$$

Therefore, equation (43) represents a first order damped oscillatory solution of equation (28), where $c,\ \phi,\ u_1$ are given by (42), (41) and (40).

VI. Results and Discussion

To test the accuracy of our results, we match our results with the numerical results obtained by the *Mathematica* program for the different sets of initial conditions. Firstly, $x(t,\ \varepsilon)$ has been computed by analytic solution (27) for undamped motion in which $c,\ \phi$ are calculated by (25), (26) and u_1 is obtained from (24) together with three sets of initial conditions, which are obtained for different radius of the cylinder and gravitational force, $g = 9.8\ ms^{-2}$. The corresponding numerical solutions that computed by the *Mathematica* program for various values of time, t and all the results are showed in the *Figure 2 to Figure 4.* The numerical results for damped oscillatory motion obtained by the *Mathematica* program for same initial conditions are assimilating with the perturbation results. Here, $x\left(t,\varepsilon\right)$ has been computed by asymptotic solution (43), where $c,\ \phi$ are calculated by (42), (41) and u_1 is obtained from (40) with the same initial conditions, when $g = 9.8\ ms^{-2}$ and different values of damping constant $2k$. The comparative results of numerical and perturbation for various values of t are showed graphically in the *figure 5 to figure 7.*

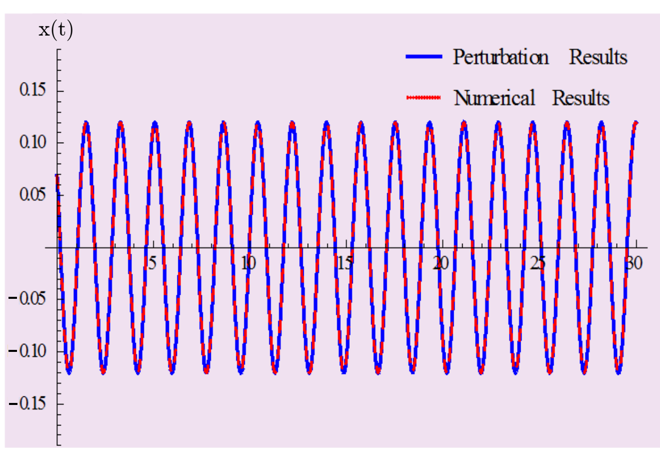

Figure 2: Comparison between perturbation and numerical results for $R = 2\ m,\ g = 9.8\ ms^{-2}$ with the initial conditions $c_0 = 0.12\ m,\ \phi_0 = 55°$

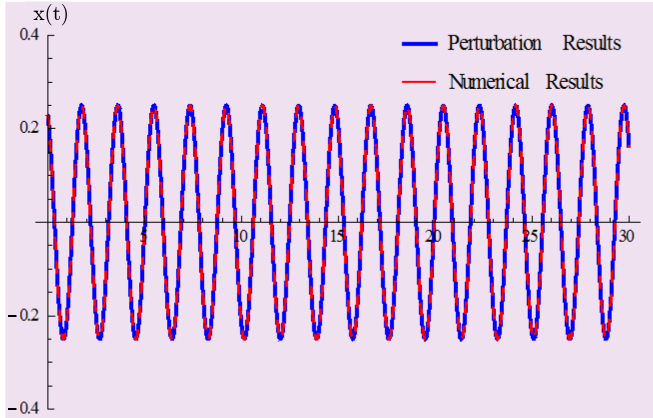

Figure 3: Comparison between perturbation and numerical results for $R = 2.2\ m,\ g = 9.8\ ms^{-2}$ with the initial conditions $c_0 = 0.25\ m,\ \phi_0 = 25°$

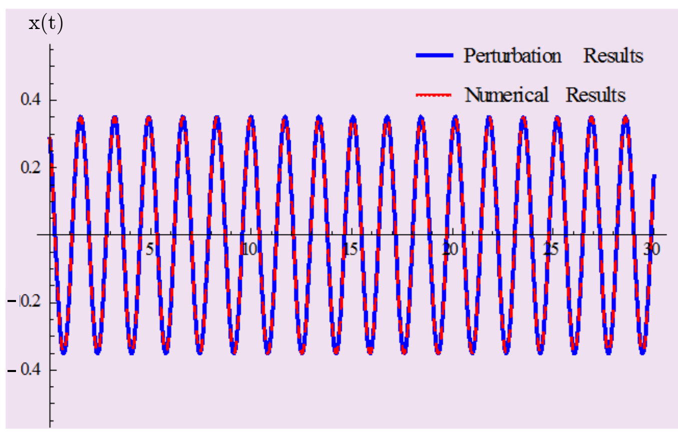

Figure 4: Comparison between perturbation and numerical results for $R = 1.8\ m,\ g = 9.8\ ms^{-2}$ with the initial conditions $c_0 = 0.35\ m,\ \phi_0 = 35°$

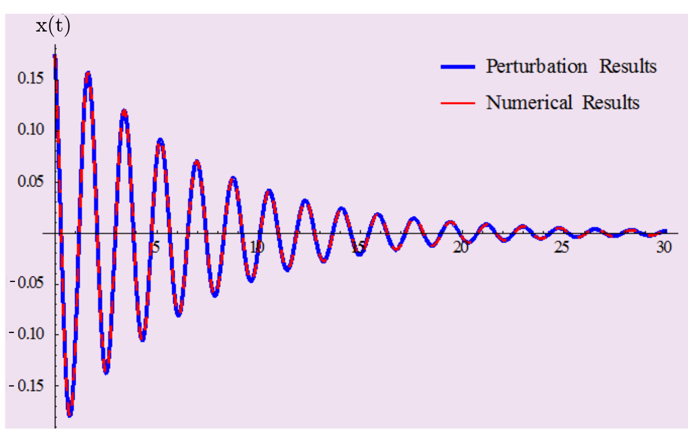

Figure 5: Comparison between perturbation and numerical results for $R = 2.0\ m,\ k = 0.15\ s^{-1},\ g = 9.8\ ms^{-2}$ with the initial conditions $c_0 = 0.20\ m,\ \phi_0 = 30°$

Notes

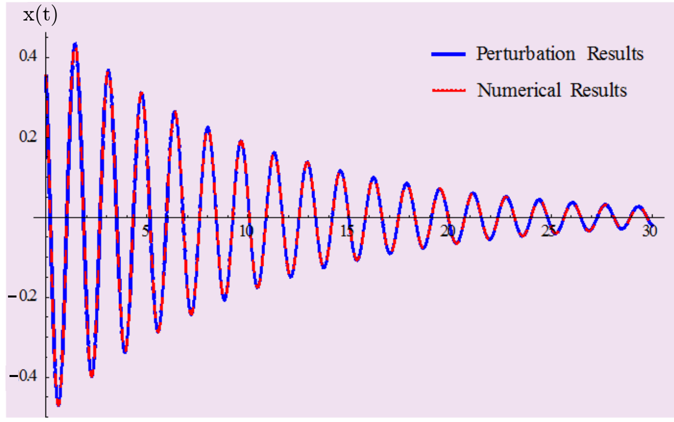

Figure 6: Comparison between perturbation and numerical results for $R=1.7\ m,\ k=0.10\ s^{-1},\ g=9.8\ ms^{-2}$ with the initial conditions $c_0=0.50\ m,\ \phi_0=45°$

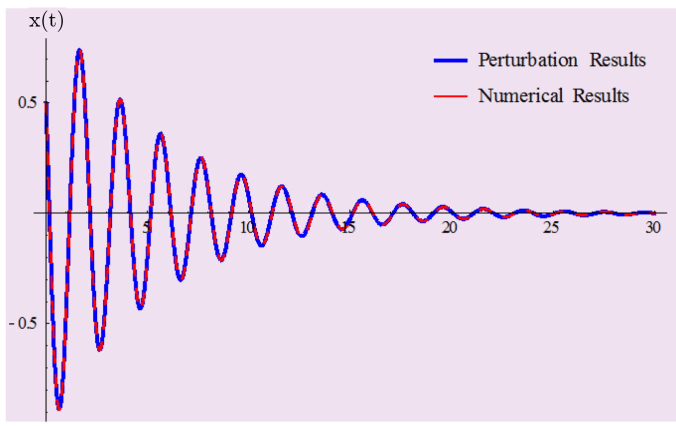

Figure 7: Comparison between perturbation and numerical results for $R=2.5\ m,\ k=0.18\ s^{-1},\ g=9.8\ ms^{-2}$ with the initial conditions $c_0=1.00\ m,\ \phi_0=60°$

VII. Conclusions

In this article, we have successfully applied the modified method to the half-submerged cylinder for oscillatory and damped oscillatory nonlinear systems. The system is oscillating in a liquid due to the gravitational force and upward pressure. A second order nonlinear equation has been derived from a horizontally half submerged cylinder, which is floating in a liquid for both oscillatory and damped oscillatory motion. Based on the modified KBM method transient responses of nonlinear differential systems have been investigated. For different sets of initial conditions, the modified KBM method provides solutions which show well agreement with the numerical solutions. In the KBM method, much error occurs in the case of rapid changes of x with respect to time, t. But it is noteworthy to observe from all figures, x changes rapidly in the time period $t=0$ to $t=30$, the results obtained via the modified KBM method show good coincidence with those obtained by the numerical method.

Notes

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