Global Journal of Science Frontier Research: F
Mathematics and Decision Sciences
Volume 17 Issue 1 Version 1.0 Year 2017
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Testing Restricted Mean Vector under Alternatives Hypothesis

By F. Mim & A. K. Majumder
Jahangirnagar University

Abstract- In most of the statistical models, the sign of the parameters are known is advance. In order to test the validity of the model, estimation and testing parameters to be done at the initial stage. In this case, usual unrestricted estimation and testing procedures may result in incorrect solutions. Usually, two-sided F or χ^2 testing are not suitable as well as unconstraint optimization solutions can give wrong estimate. In multivariate analysis, we usually apply two-sided Hotelling's -T^2 for testing mean vector. This test may not be appropriate for testing when an order restriction is imposed among several p-variate normal mean vector. The main objective of this paper is for a given a multivariate normal population with unknown covariance matrix to develop a new testing procedure when the mean vector slipped to the right or to the left or both. So, we proposed a new distance based one sided and partially one sided Hotelling's -T^2 to test restricted mean vectors. Monte Carlo simulations are conducted to compare power properties of the proposed DT^2 along with their respective conventional counterparts.

Keywords: distance–based test, weighted mixture distribution, simulation, one-sided and partially one-sided hotelling's-T^2, power.

GJSFR-F Classification: MSC 2010: 97G70

TESTINGRESTRICTEDMEANVECTORUNDERALTERNATIVESHYPOTHESIS

Strictly as per the compliance and regulations of :

© 2017. F. Mim & A. K. Majumder. This is a research/review paper, distributed under the terms of the Creative Commons Attribution-Noncommercial 3.0 Unported License http://creativecommons.org/licenses/by-nc/3.0/), permitting all non commercial use, distribution, and reproduction in any medium, provided the original work is properly cited.

Testing Restricted Mean Vector under Alternatives Hypothesis

F. Mim [α] & A. K. Majumder [σ]

Abstract- In most of the statistical models, the sign of the parameters are known is advance. In order to test the validity of the model, estimation and testing parameters to be done at the initial stage. In this case, usual unrestricted estimation and testing procedures may result in incorrect solutions. Usually, two-sided F or χ^2 testing are not suitable as well as unconstraint optimization solutions can give wrong estimate. In multivariate analysis, we usually apply two-sided Hotelling's -T^2 for testing mean vector. This test may not be appropriate for testing when an order restriction is imposed among several p-variate normal mean vector. The main objective of this paper is for a given a multivariate normal population with unknown covariance matrix to develop a new testing procedure when the mean vector slipped to the right or to the left or both. So, we proposed a new distance based one sided and partially one sided Hotelling's -T^2 to test restricted mean vectors. Monte Carlo simulations are conducted to compare power properties of the proposed DT^2 along with their respective conventional counterparts. It is found that our proposed DT^2 test shows substantially improved power than the usual two-sided test in all situations.

Keywords: distance–based test, weighted mixture distribution, simulation, one-sided and partially one-sided hotelling's-T^2, power.

I. Introduction

Multivariate analysis considers joint effect of a set of variables simultannously. It can be applied in different ares of scientific research viz, Medical, Zoological, Botanical etc. In medical science, diagnosis of diseases of patients are based on several physical or clinical conditions. Simultaneous physical or clinical conditions of patients with interaction can be very useful to correctly diagnose diseases of patients. Statistical multivariate analysis such as multivariate mean vector, cluster analysis, factor analysis etc. can be very useful to diagnosis or identify factors of diseases. In recent year there has been a significant amount of interest in developing tests that incorporate one sided information. For example, in the case of ordered treatment means or the testing in which a treatment is better than the control when the responses are ordinal. The application of restricted hypothesis can be found in clinical trials design to test superiority of a combination therapy (Laska and Meisner, 1989and Sarka et al.,1995). The extensive literature concerning this problem has been appeared by Barlow et al.(1972),Robertson et al. (1988) and Silvapulle and Sen (2005). Bartholomew (1959a,b)derived a likelihood ratio test for homogeneity of k univariate normal means against ordered alternatives. The problems with ordered parameters have been studied to some extents by Chacko (1963) and Shorack (1967). Kudo (1963) considered a p - dimensional normal distribution with unknown mean $\mu = (\mu_1, \mu_2, ..., \mu_p)$ and known covariance matrix Σ. The problem of testing was $H_o : \mu = 0$ against the restricted

Author α: Lecturer, Department of Statistics, Mawlana Bhasani Science and Technology University. e-mail: faijun2005@yahoo.com
Author σ: Professor, Department of Statistics, Jahangirnagar University.

© 2017 Global Journals Inc. (US)

alternative $H_a : \mu_i \geq 0 (i = 1,2,......., p)$, where the inequality is strict for at least one value of i. He obtained the test statistic based on the likelihood ratio criterion and discussed its existence and geometric nature and also gave a scheme for its computation. Perlman (1969) studied this problem assuming that Σ is completely unknown. Tang et al. (1989) proposed a new multivariate statistic for this problem of testing with known covariance matrix and investigated its null distribution. Robertson and Wegman (1978) obtained the likelihood ratio test statistic for testing the isotonicness of several univariate normal means against all alternative hypotheses. They calculated its exact critical values at different significance levels for some of the normal distributions and simulated the power by Monte Carlo experiment. Also they considered the test of trend for an exponential class of distributions.

Hotelling's T^2 is a very versatile test statistic to test the homogeneity of several multivariate normal means against the unrestricted alternative hypothesis. In this paper, our main objective is to develop one-sided and partially one-sided testing approach for testing restricted multivariate mean vector using Hotelling's T^2 type test along with Distance-Based estimation technique (See for example, Majumder and King (1999)) and make a comparative study between the usual two-sided Hotelling's T^2 and distance-based Hotelling's T^2 (DT^2) tests.

The paper is organized as follows. In Section 2, we represent the hypothesis used to develop the test. Our proposed DT^2 along with usual T^2 test are discussed in Section 3. Calculation of weights and Monte Carlo simulation are given in Section 4 and 5. In Section 6, we introduce the design matrix. A comparison is made between the powers of two-sided T^2 and DT^2 tests in Section 7. Finally, Section 8 contains concluding remarks.

II. Hypothesis

Multivariate one-sided hypothesis-testing problems are very common in real life. The likelihood ratio test (LRT) and union intersection test (UIT) are widely used for testing such problems. It is argued that, for many important multivariate one-sided testing problems, the LRT and UIT fail to adapt to the presence of sub-regions of varying dimensionalities on the boundary of the null parameter space and thus give undesirable results. Several improved tests are proposed that do adapt to the varying dimensionalities and hence reflect the evidence provided by the data more accurately than the LRT and UIT (See Perlman, Lang). Suppose that $X_{i1}, X_{i2},........., X_{ini}$ are random vectors from a p - dimensional normal distribution $Np(\mu_i, \Sigma)$ with unknown mean vector μ_i and nonsingular covariance matrix $\Sigma_i, i = 1,2,......,k$. We assume that Σ is unknown. In order to test the mean vector we consider the following hypotheses

$$\text{H1} \quad H_0^1 : \mu = 0 \quad \text{vs} \quad H_a^1 : \mu > 0,$$

$$\text{H2} \quad H_0^2 : \mu = 0 \quad \text{vs} \quad H_a^2 : \mu < 0,$$

$$\text{H3} \quad H_0^3 : \mu = 0 \quad \text{vs} \quad H_a^3 : \mu_k > 0, \mu_l \neq 0, \mu_m < 0.$$

where, $k \neq l \neq m$, $\mu = (\mu_1, \mu_2, \ldots, \mu_p)'$ *is a* $(p \times 1)$ matrix.

Notes

© 2017 Global Journals Inc. (US)

III. Distance-Based One-Sided Tests

Notes

Distance-based approach suggests that we have to determine whether the estimated parameters under test likely to be closer to null hypothesis or to alternative hypothesis. For testing two or more parameters, we can use the normal Euclidean measure of distance to determine closest point in the maintained hypothesis parameter space to the unconstrained estimate. But, in the case of a non-orthogonal design matrix, we use a transformation of the original model to understand the incorporation variance-covariance or information matrix to measure the distance in general situations. It is also worth noting that Shapiro (1988, p.50) used the variance-covariance matrix in a metric to determine the closest point in maintained hypothesis from the estimated value of the parameter. Also, Kodde and Palm (1986) used a distance function and metric spaces to develop a test based on their distance function. Majumder (1999) utilizes these ideas to develop distance-based approach for testing one-sided or partially one-sided hypothesis of any parametric model. In this paper, we apply Majumder and King's (1999) distance based approach. Majumder's (1999) approach is outlined below for general testing problem: suppose, we are interested in testing a hypothesis of a parametric model in which the parameter of interest, θ, is restricted under the alternative hypothesis. More specifically, we wish to test

$$H_0 : \theta = 0 \qquad \text{versus} \qquad H_a : \theta \in B$$

based on the $n \times 1$ random vector y whose distribution has probability density function $f(y, \theta)$ where $\theta \in R^p$ is a subvector of an unknown parameter $\Theta \in R^s$ and B is a subset of R^p. Let $\hat{\theta}$ be a suitable estimate of θ such that $\hat{\theta}$ is asymptotically distributed as normal with variance-covariance matrix $cI^{-1}(\theta)$ where c is a constant and $I(\theta)$ is the information matrix. As θ is an element of B, and B is a subset of R^p, elements of the parameter vector θ can be either positive, negative or both. Therefore, a test under this hypothesis is either one-sided when all values of θ are positive (or more accurately constrained to have a particular sign-positive or negative) or partially one-sided when some of the values are positive and some are unconstrained. Following Shapiro (1988, eq.21, p.50), Kodde and Palm's (1986) Majumder (1999) suggest that we should determine the closest point in the maintained hypothesis from the unconstrained point. This closest point is the solution of the following distance function or optimal function in the metric $cI^{-1}(\theta)$ of the parameter vector $\hat{\theta}$

$$\left\|\breve{\theta} - \hat{\theta}\right\|^2 = (\breve{\theta} - \hat{\theta})' I(\theta)(\breve{\theta} - \hat{\theta})$$

subject to $\breve{\theta} \in B$.

The closest point or optimized $\breve{\theta}$ can be used in any appropriate two-sided tests to obtain the corresponding distance-based one-sided and partially one-sided tests. The asymptotic null hypothesis distribution generally follows a mixture of the corresponding two-sided distributions.

a) *Distance-Based One-Sided LR test*

The Likelihood ratio (LR) test requires calculation of both restricted and unrestricted estimators. If both are simple to compute, this will be a convenient way to proceed.

The general form of two-sided LR statistic is,

© 2017 Global Journals Inc. (US)

$$LR = 2\left(l(\hat{\theta}_a) - l(\hat{\theta}_0)\right), \quad (3.1.1)$$

where, $l(\hat{\theta}_0)$ and $l(\hat{\theta}_a)$ are the unrestricted and restricted maximized log-likelihood functions, respectively. The asymptotic null hypothesis distribution of (4.1.1) follows a central chi-square distribution with k degrees of freedom. But the two-sided LR test is not appropriate when the alternative hypothesis becomes strictly one-sided. In our distance-based LR test we have to estimate the optimum values of $l(\hat{\theta}_0)$ and $l(\hat{\theta}_a)$ according to the general formulation of distance-based approach (see for example, Majumder (1999), Basak, T. (2004), Rois etl. (2008)), subject to the restrictions $\mathrm{H1}$ and $\mathrm{H2}$, discussed in section 2. Then the statistic becomes,

$$DLR = L\breve{R} = 2\left(l(\breve{\theta}_a) - l(\breve{\theta}_0)\right), \quad (3.1.2)$$

where, $l(\breve{\theta}_0)$ is the unrestricted optimized value and $l(\breve{\theta}_a)$ is the optimized value subject to the restrictions, $\mathrm{H1}$ and $\mathrm{H2}$.

Under the null hypothesis the distribution of the statistic (3.1.2) follows asymptotically weighted mixture of chi-square distribution with p degrees of freedom (see for example, Kodde and Palm (1986), Shaprio (1988), Majumder (1999)).

b) Distance-Based One-Sided Hotelling T^2 (DT^2) Test

In this testing procedure we wish to maximize

$$L(\theta,\Sigma) = \frac{1}{(2\pi)^{np/2}|\Sigma|^{n/2}} \exp\left[-\frac{1}{2}\sum_{i=1}^{n}(x_i-\mu)'\Sigma^{-1}(x_i-\mu)\right], \quad (3.1.3)$$

with respect to μ, Σ and subject to the restrictions : $\mu > 0 \; or \; \mu < 0 \; or \; \mu_k < 0, \mu_l \neq 0, \mu_m > 0$ where, $k \neq l \neq m$.

In our DT^2 we have to estimate the optimum values of μ under the above restrictions according to the general formulation of distance-based approach (See for example, Majumder (1999), Basak, T. (2004), Rois etl. (2008)). Replacing the usual maximum likelihood estimate of μ in two-sided T^2 by the optimal value we get the following one-sided or partially one-sided T^2 statistic,

$$T^2 = n\left(\breve{\mu} - \mu_0\right)' S^{-1}\left(\breve{\mu} - \mu_0\right) \quad (3.1.4)$$

where, $\breve{\mu}$ is the optimized value . Under the null hypothesis the distribution of the test statistic (3.4) follows weighted mixture of $\frac{(n-1)p}{n-p} \times F_{p,(n-p)}$ distribution.

IV. Determination of Weights

Weights of DT^2 can be calculated from likelihood ratio test. Monfort's (1980) one-sided LR test statistic is given below:

$$S_{LR} = 2(L(\tilde{\mu},\Sigma) - L(\hat{\mu},\Sigma) = L(\tilde{w})\text{-}L(\hat{w}_0) \quad (4.1)$$

where, $\tilde{w} = (\tilde{\mu}', \tilde{\Sigma}')'$, $\hat{w}_0 = (\hat{\mu}', \hat{\Sigma}_0')'$

Notes

© 2017 Global Journals Inc. (US)

The asymptotic null hypothesis distribution of (4.1) is a probability mixture of independent chi-squared distributions with different degrees of freedom and is given by,

$$\Pr(S_{LR} < c) = \sum_{i=1}^{n} w(p,i)\Pr(F < c), \text{ for } c \in R, \tag{4.2}$$

where, F denotes a random variable having F distribution with i degrees of freedom and R is the two sided LR statistic defined as,

Notes

$$R = \operatorname{sgn}(\hat{\mu} - \mu_0)\sqrt{2\{L(\hat{\mu}, \Sigma) - L(\tilde{\mu}_0, \Sigma)\}}. \tag{4.3}$$

The weight, $w(p, i)$, $i = 1, 2, \cdots, p$ denote the power function of times any i elements of $\tilde{\mu}$ are strictly positive and the remaining $p-i$ elements are zero under the null hypothesis.

V. Monte Carlo Simulation

Monte Carlo simulations are carried out to compare the powers of the usual T^2 and DT^2 for testing restricted multivariate mean vector. The powers were estimated by calculating the percentage of times the test statistics exceeded the appropriate 5% critical values. Here each entry is based on at least 5000 runs.

a) Experimental design

In order to compare the power properties of proposed DT^2 test with usual T^2, we use artificially generated data. Since Hotelling T^2 statistic is a generalization of Student's t statistic and Student's t statistic follows normal distribution, we generate data of size 100 from Multivariate Normal distribution by Spectral decomposotion with mean vector assumed in hypothesises and covarinaces Σ_1 and Σ_2. To make the comparison appropriate, we consider hypothetical covariance matrices

$$\Sigma_1 = \begin{bmatrix} 20 & -5 \\ -5 & 13 \end{bmatrix}, \ \Sigma_2 = \begin{bmatrix} 23.65 & 18.43 & 13.78 \\ 8.43 & 32.28 & 25.18 \\ 13.78 & 25.18 & 42.53 \end{bmatrix}.$$

We perform our experiment for different values of the parameters $\mu = (\mu_1, \mu_2, \mu_3)'$ ($\mu = 0, .3, .5, .7, 1$). On the basis of the above covariance matrix we estimate simulated powers of all the considerate tests for testing strictly one-sided and partially one-sided hypothesis.

For testing, H1, H2 and H3 we use, $\mu = (0, 0.3, 0.5, 0{\cdot}7, 1)'$, $\mu = (0, -0.3, -0.5, -0.7, -1)'$ and $\mu = (0, 0.3, -0.5, 0.7, 1)'$, respectively, for each design matrix.

VI. Results

This section compares the power of the existing Hotelling's T^2 and proposed DT^2 for testing H1, H2, H3 for different design matrices defined in Section 6.The estimated simulated powers of these tests are presented in Tables 1-2 with figures 1-2 for the defined design matrixes Σ_1 and Σ_2 .

© 2017 Global Journals Inc. (US)

For the design matrix Σ_1 and $\mu=(\mu_1,\mu_2)'$, simulated powers of DT^2 test and the usual T^2 test when the alternative hypothesis is of the form H_a^1, H_a^2 are presented in Table 1.

Table 1: Simulated power comparisons of the DT^2 and T^2 statistics for hypothesis H1 when covariance matrix $\sum_1$

μ_1	μ_2	DT^2	T^2
0	0	0.05	0.05
	0.3	0.209	0.1036
	0.7	0.934	0.5819
	1	0.998	0.7352
0	0	0.092	0.0854
	0.3	0.387	0.1881
	0.7	0.972	0.7433
	1	0.999	0.8173
1	0	0.850	0.2841
	0.3	0.954	0.4418
	0.7	0.999	0.763
	1	1	0.9251
1	0	0.994	0.5371
	0.3	0.999	0.6867
	.5	1	0.7914
	0.7	1	.8942

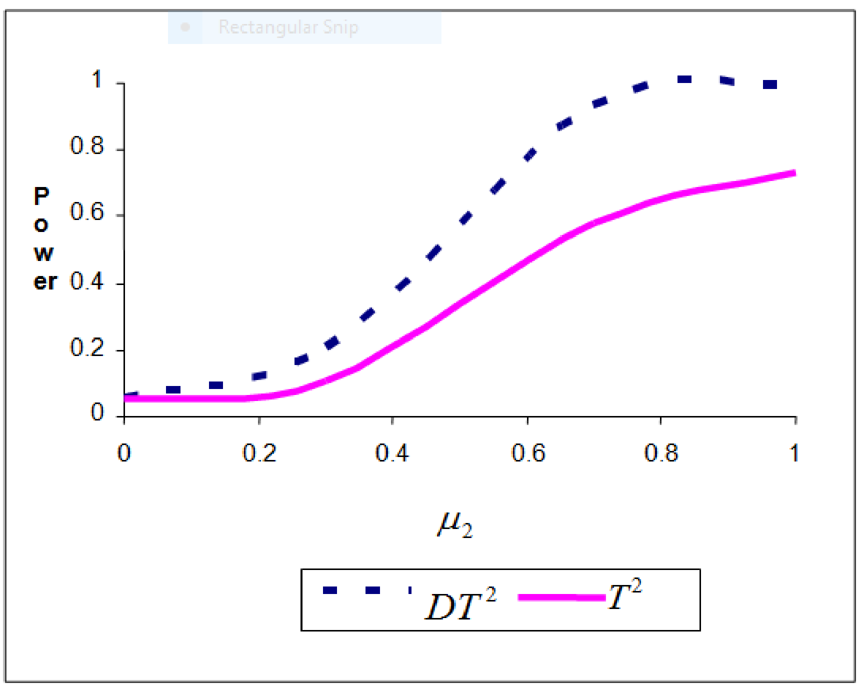

Figure 1: Power curves of DT^2 *and* T^2 *tests of hypothesis,* H1, *for* $p=2$ *and fixed* $\mu_1=0$

Table 1 reveals the estimated powers of Hotelling's T^2 and our proposed distance based Hotelling's T^2 for the design matrix Σ_1 and $\mu=(\mu_1,\mu_2)'$, when the alternative hypothesis is of the form H_a^1, H_a^2. We observe that the power of the DT^2 test is higher than the usual T^2 test in all cases. For example, the powers of the DT^2 and T^2 tests are 0.2099 and 0.1036, respectively, for $\mu_1=0,\mu_2=0.3$.

Notes

© 2017 Global Journals Inc. (US)

Table 2 shows, simulated powers for testing H_a^3 when the mean vector and design matrix is of the form $\mu = (\mu_1, \mu_2, \mu_3)'$ and Σ_2, respectively. Also, we observe that the powers of the DT^2 and T^2 test when $\mu_1 = 0, \mu_2 = 0.5, \mu_3 = 0.3$ are 0.912, 0.341, respectively, for the design matrix Σ_2. Figure 1 and 2 explore that powers of all tests are increases as increases.

Notes

Table 1: Simulated power comparisons of the DT^2 and T^2 statistic for hypothesis H3 when covariance matrix Σ_2

μ_1	μ_2	μ_3	DT^2	T^2
		0	0.05	0.05
		0.3	0.219	0.09
0	0	0.5	0.568	0.141
		0.7	0.911	0.281
		1	0.998	0.472
		0	0.773	0.23
		0.3	0.912	0.341
0	.5	0.5	0.901	0.253
		0.7	0.996	0.601
		1	1	0.485
		0	0.5	0.159
		0.3	0.795	0.239
	0	0.5	0.854	0.219
.3		0.7	0.988	0.468
		1	1	0.49
		0	0.979	0.454
		0.3	0.993	0.576
0.5	. 5	0.5	0.997	0.401
		0.7	0.999	0.787
		1	1	0.597

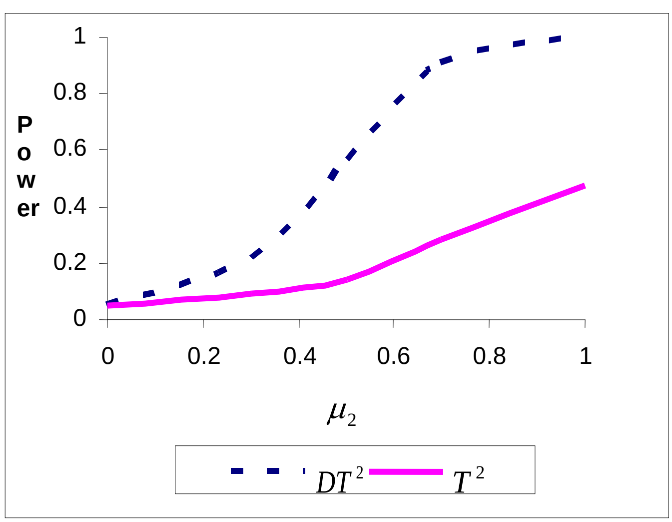

Figure 2: Power curves of DT^2 and T^2 tests of hypothesis H1, for $p = 3$, Σ_2 and fixed $\mu_1 = 0, \mu_2 = 0$.

© 2017 Global Journals Inc. (US)

We observe from Monte Carlo simulation study that in all cases our proposed DT^2 test gives higher power than two-sided T^2 test. All the figures and tables represent that the simulated power of all one-sided T^2 test is always superior to its usual two-sided counterpart.

VII. Conclusions

This paper develops distance based one-sided and partially one-sided testing approach for testing multivariate mean vector under considered restricted alternative hypothesis. We observe from Monte Carlo simulation studies in different cases. All the figures and tables represent that the simulated power of one-sided and partially one-sided test is always superior to the usual two-sided test. Monte Carlo results show that the power of the proposed DT^2 test is better than that of the usual test based on the adjusted F distribution. Therefore, we advocate the use of D T^2 test to test the multivariate mean vector when the alternative hypothesis is strictly one-sided or partially one-sided.

Notes

References Références Referencias

1. Barlow, R. E., Bartholomew, D. J., Bremner, J. M. and Bd, H. D. (1972). Statistical Inference under Order Restrictions: *The Theory and Application of Isotonic Regression*. Wiely, New York.
2. Bartholomew, D. J. (1959a). A test of homogeneity for ordered alternatives. *Biometrika*, 46, 36-48.
3. Bartholomew, D. J. (1959b). A test of homogeneity for ordered alternatives II. *Biometrika*, 46, 328-33 5.
4. Basak, T. (2004), Modified Breusch-Godfrey Test for Testing Higher Order Autocorrelation When Alternative Hypothesis is Either One-sided or Partially One-sided - A Distance-Based Approach, An Unpublished Thesis, Jahangirnagar University, Bangladesh.
5. Chacko, V. J. (1963). Testing homogeneity against ordered alternatives. *The Annals of Mathematical Statistics*, 34,945-956.
6. Kodde, D. A. and Palm, F. C., (1986), Wald Criteria for Jointly Testing Equality and Inequality Restrictions, *Econometrica*, 54, 1243-1248.
7. Kudo, A. (1963). A multivariate analogue of the one-sided test. *Biometrika*, 50,403-41 8.
8. Laska, E. M. and Meisner, M. J. (1989). Testing whether identified treatment is best. *Biometries*, 45, 1 139-1 151.
9. Majumder, A. K. (1999), One-sided and Partially One-sided Hypothesis Testing in Econometrics, An Unpublished Thesis, Monash University, Australia. No. (Dec., 1963), pp.403-418
10. Majumder, A. K., & King, M. L. (1999). One-sided test of multiple inequality constraints-a distance-based approach, Paper presented at the Econometric Society Australasian Meeting 7-10 July. Canberra.
11. Perlman, M. D. (1969). One-sided testing problems in multivariate analysis. The Annals of Mathematical Statistics, 40(2), 549-567.
12. Robertson, T. and Wegman, E.T. (1978). Likelihood ratio tests for order restrictions in exponential families. *The Annals of Statistics*, 6(3), 485-505.
13. Robertson, T., Wright, F. T. and Dykstra, R. L. (1988). Order Restricted Statistical Inference. Wiely, New York.
14. Rois, R., Basak, T., & Majumder, A. K. (2008). On the Test of Restricted Higher Order Autocorrelation: A Wald-Type Distance-Based Approach. *Jahangirnagar University Journal of Science*, 31(1), 127-142.

© 2017 Global Journals Inc. (US)

Notes

15. Sarka, S. K., Snapinn, S., and Wang, W. (1995). On improving the min test for the analysis of combination drug trails (Corr: 1998V60 p180-181). *Journal of Statistical Computation and Simulation*, **51**, 197-2 13.
16. Shapiro, A. (1988), Towards a Unified Theory of Inequality Constrained Testing in Multivariate Analysis, *International Statistical Review*, 56, 49-62.
17. Shorack, G. R. (1967). Testing against ordered alternative in model I analysis of variance: normal theory and nonparametric. *The Annals of Mathematical Statistics*, 38, 1740- 1752.
18. Silvapulle, M. J. and Sen, P. K. (2005). *Constrained Statistical Inference: Inequality, Order, and Shape Restrictions*, Wiely, New York.
19. Tang, D.I., Gnecco, C. and Geller, N. (1989). An approximate likelihood ratio test for a normal mean vector with nonnegative components with application to clinical trials. *Biometrika*, 76, 577-583.

© 2017 Global Journals Inc. (US)

Global Journals Inc. (US) Guidelines Handbook 2017

www.GlobalJournals.org