Global Journal of Science Frontier Research: F
Mathematics and Decision Sciences
Volume 17 Issue 5 Version 1.0 Year 2017
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

The Study of Effects of Gravity Modulation on Double Diffusive Convection in Oldroyd-B Liquids

By R. K. Vanishree & K. Anjana
Maharani Science College for Women

Abstract- The effect of gravity modulation is analysed in Oldroyd-B liquids subjected to double diffusive convection. Both linear and non-linear analysis has been done. A regular perturbation technique has been employed to arrive at the thermal Rayleigh number. The results show that stress relaxation destabilises the system whereas strain retardation parameter and Lewis number stabilises the system. Truncated Fourier series expansion gives a system of Lorentz equations that represent the non-linear analysis. Nusselt and Sherwood numbers are used to quantify the heat and mass transfer respectively. It is observed that Lewis number and strain retardation parameter decreases heat and mass transfer and stress relaxation parameter increases them. It is seen that modulation gives rise to sub-critical motion.

Keywords: double diffusive convection, gravity modulation, oldroyd-B liquids, rayleigh-bénard convection.

GJSFR-F Classification: *MSC 2010: 83C27*

THESTUDYOFEFFECTSOFGRAVITYMODULATIONONDOUBLEDIFFUSIVECONVECTIONINOLDROYDBLIQUIDS

The Study of Effects of Gravity Modulation on Double Diffusive Convection in Oldroyd-B Liquids

R. K. Vanishree [α] & K. Anjana [σ]

Abstract- The effect of gravity modulation is analysed in Oldroyd-B liquids subjected to double diffusive convection. Both linear and non-linear analysis has been done. A regular perturbation technique has been employed to arrive at the thermal Rayleigh number. The results show that stress relaxation destabilises the system whereas strain retardation parameter and Lewis number stabilises the system. Truncated Fourier series expansion gives a system of Lorentz equations that represent the non-linear analysis. Nusselt and Sherwood numbers are used to quantify the heat and mass transfer respectively. It is observed that Lewis number and strain retardation parameter decreases heat and mass transfer and stress relaxation parameter increases them. It is seen that modulation gives rise to sub-critical motion.

Keywords: double diffusive convection, gravity modulation, oldroyd-B liquids, rayleigh-bénard convection.

I. Introduction

The study of convection in non-Newtonian liquids has been a topic of interest due to its usage as a working media in many engineering and industrial applications. Viscoelastic fluids exhibits both solid and liquid properties and find application in diverse fields such as geothermal energy modeling, crystal growth, solar receivers etc. Some other applications include chemical industry, bioengineering, petroleum industry and so on. These liquids are defined by constitutive equations which include complex differential operators. They also include relaxation and retardation times. As they possess both elastic (property of solids) and viscosity (property of liquids) leading to a unique instability patterns such as overstability which is not observed in Newtonian fluids. This is the main reason why many researchers have studied Rayleigh-Benard convection in a rectangular layer of viscoelastic fluid heated from below (Vest and Apaci [1], Sokolov and Tanner[2, Green[3], Siddheshwar *et. al.* [4]).

Oldroyd-B liquid is a type of viscoelastic fluid. The study of stationary and oscillatory convection in viscoelastic fluids gave information about the formation of pattern in these fluids (Li and Khayat [5,6]). It was also found that a thin layer of fluid when heated from below sets up oscillatory convection. Siddheshwar and Krishna [7] investigated Rayleigh– Bénard convection in a viscoelastic fluid and found that the ratio of strain retardation parameter to the stress relaxation parameter should be less than one for convection to set in.

The nonlinear stability analysis under the influence of gravity modulation in viscoelastic fluids was studied by Siddheshwar [8] who found that modulation helps in controlling the onset of convection. The destabilizing and stabilizing effects of rotation

Ref

1. Charles M. Vest, Vedat S. Arpaci, 1969, *"Stability of natural convection in a vertical slot"*, J. Fluid Mech., 36, pp. 1-15.

Author α: Dept. of Mathematics, Maharani Science College for Women, Bengaluru, Karnataka, India. e-mail: vanirkmscw@gmail.com
Author σ: Dept. of Mathematics, Mount Carmel College, Bengaluru, Karnataka, India. e-mail: anjana.kenath86@gmail.com

on Oldroyd-B liquids were found by Sharma [9]. In spite of these studies not many literature exists on non-linear convection in Oldroyd-B liquids.

The classical Rayleigh– Bénard convection arises due to temperature gradient alone. In most practical cases convection may be caused not just due to one gradient, but multiple gradients. Double diffusion occurs when there are two components whose rates of diffusion are different. These varying diffusivities give rise to unpredictable movement of fluid particles, thus making these problems interesting. In majority of cases the two varying components are temperature and solute (Mojtabi and Charrier-Mojtabi [10]). In such fluids density variations depend on both thermal and solutal gradients which diffuse at different rates. This leads to the formation of salt fingers or oscillations in the fluid layer. Malashetty and Swamy [11] found that there is a competition between the processes of thermal diffusion, solute diffusion and viscoelasticity that causes the convection to set in through oscillatory mode rather than stationary. A common example of double diffusive convection is found in the ocean (Stommel *et al.* [12]). Stommel noticed that with the decrease in solute quantity there was a large amount of potential energy available. Further study on this was done by Stern [13, 14] who made the observation that if there are two diffusing components in a system, then the behaviour of the system depend on whether the solute component is stabilizing or destabilizing. Siddheshwar and Pranesh [15] studied the effects of temperature modulation and g-gitter on magneto-convection in a weak electrically conducting fluid with internal angular momentum. The effects of temperature modulation on double diffusion were found by Bhadauria [16]. Double diffusive magneto convection in viscoelastic fluids was investigated by Narayana *et. al.* [17]. A stability analysis of chaotic and oscillatory magneto-convection in binary viscoelastic fluids with gravity modulation was done by Bhadauria and Kiran [18]. A Ginzburg–Landau model was adopted to find the effects of the parameters. It was found that gravity modulation can be used to either advance or delay convection by varying its frequency. Siddheshwar *et. al.* [19] analyzed the heat transport by stationary magneto-convection in Newtonian liquids under g-gitter and temperature modulation and obtained similar. Kiran [20] used the Darcy model the porous medium to study the nonlinear thermal convection in a porous medium saturated with viscoelastic nanofluids and found that frequency of modulation can be varied to get the desired results with respect to onset of convection.

In this paper we use linear and non-linear stability analysis to investigate the effects of gravity modulation on double diffusive convection in Oldroyd-B liquid.

II. Mathematical Formulation

Consider a layer of Oldroyd-B liquid held between two parallel plates at z = 0 and z = d. The two plates are maintained at two different temperatures with the difference in temperatures and solute concentrations ΔT and ΔS respectively. This causes variable heating of the fluid particles and hence, variable movements. That is, a temperature gradient arises and in turn gives rise to convection. The fluid density is assumed to be a linear function of temperature, T, and solute concentration, S. A Cartesian co-ordinate system is taken with origin in the lower boundary and z-axis vertically upwards (fig 1).

Ref

9. R.C. Sharma, 2006, *"Effect of rotation on thermal instability of a viscoelastic fluid."* Acta Physiol. Hung., 40, pp. 11–17.

Notes

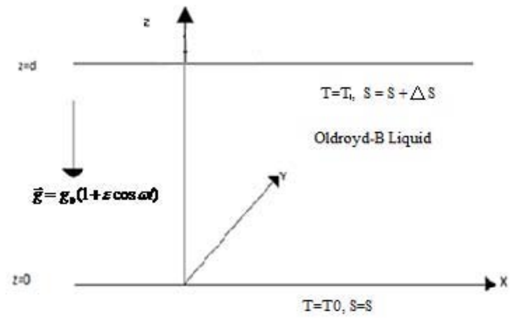

Fig.: Physical Configuration

Nomenclature	
d	thickness of the liquid
k	dimensionless wave number
pr	Prandtl number
q	velocity
Ra	thermal Rayleigh number
Rs	solutal Rayleigh number
t	time
T	temperature
T_o	constant temperature of the upper boundary
T_R	reference temperature
Le	Lewis number
Greek symbols	
α	thermal expansion coefficient
ε	amplitude of modulation
κ	thermal diffusivity
κ_s	solutal diffusivity
λ_1	stress relaxation coefficient
λ_2	strain retardation coefficient
Λ	elastic ratio(λ_2 / λ_1)
μ	viscosity
ω	frequency of modulation
ρ	density

Thus the governing equations for Rayleigh-Benard situation of an Oldroyd-B liquid are:

Continuity Equation:

$$\nabla.\vec{q} = 0 \tag{1}$$

Conservation of momentum:

$$\rho_0\left(\frac{\partial \vec{q}}{\partial t}\right) = -\nabla p + \rho \vec{g}(t) + \nabla.\tau' \tag{2}$$

Rheological Equation:

$$\left(1+\lambda_1 \frac{\partial}{\partial t}\right)\tau' = \left(1+\lambda_2 \frac{\partial}{\partial t}\right)\left(\nabla \vec{q} + \nabla \vec{q}^{\,tr}\right) \tag{3}$$

Conservation of Energy:

$$\frac{\partial T}{\partial t} + (\vec{q}.\nabla)T = \kappa \nabla^2 T \tag{4}$$

Conservation of Species:

$$\frac{\partial S}{\partial t} + (\vec{q}.\nabla)S = \kappa_s \nabla^2 S \tag{5}$$

Energy Equation:

$$\rho = \rho_0\left(1-\alpha(T_b - T_0) + \alpha_s(S_b - S_0)\right) \tag{6}$$

The variation of gravity with time is given by

$$\vec{g}(t) = g_0(1+\delta\varepsilon \cos \omega t)\vec{k} \tag{7}$$

Where δ is the amplitude of gravity modulation and ε is a small quantity indicative of weak variation.

III. Basic State

In the basic state the fluid is at rest. Therefore, the velocity is zero and the other parameters are function of z alone.

$$\vec{q} = \vec{q}(b) = 0,\quad p = p_b(z),\quad \rho = \rho_b(z),$$
$$S = S_b(z),\quad T = T_b(z) \tag{8}$$

The temperature T_b, pressure p_b and density ρ_b satisfy

$$\frac{dp}{dz} + \rho g(1+\varepsilon \cos \omega t) = 0 \tag{9}$$

$$\frac{\partial T_b}{\partial t} = \kappa \frac{\partial^2 T_b}{\partial z^2} \tag{10}$$

Using the boundary condition, the above equation yields

$$T_b = -\frac{\Delta T}{d} + T_0 \tag{11}$$

$$\rho = \rho_0\left(1-\alpha(T_b - T_0) + \alpha_s(S_b - S_0)\right) \tag{12}$$

The rheological equation takes the form

$$\left(1+\lambda_1 \frac{\partial}{\partial t}\right)\left[\rho_0 \frac{\partial \vec{q}}{\partial t} + \nabla p + \rho g(1+\varepsilon \cos \omega t)\right]$$

Notes

$$= \mu\left(1+\lambda_2 \frac{\partial}{\partial t}\right)\nabla^2 \vec{q} \tag{13}$$

IV. Stability Analysis

The infinitesimal perturbations on the basic state are superimposed to study the stability of the system. The basic state is slightly perturbed by an infinitesimal perturbation as given in eq. (14). The primes denote the perturbations.

$$\vec{q} = \vec{q}', p = p_b + p', \rho = \rho_b + \rho', T = T_b + T', S = S_b + S' \tag{14}$$

Substituting eq. (14) in the governing equations and with the help of the basic state solutions, we get eq. (15) – (17) for the perturbations

$$\frac{\partial T'}{\partial t} + w'\frac{\partial T'}{\partial z} + w'\frac{\partial T_b}{\partial z} = \kappa\nabla^2 T' \tag{15}$$

$$\frac{\partial S'}{\partial t} + w'\frac{\partial S'}{\partial z} + w'\frac{\partial S_b}{\partial z} = \kappa_s\nabla^2 S' \tag{16}$$

$$\left(1+\lambda_1 \frac{\partial}{\partial t}\right)\begin{bmatrix}\rho_0 \dfrac{\partial\left(\nabla^2 w'\right)}{\partial t} - \alpha\rho_0 g(1+\varepsilon\cos\omega t)\nabla_1^2 T' \\ +\alpha_s\rho_0 g(1+\varepsilon\cos\omega t)\nabla_1^2 S'\end{bmatrix}$$

$$= \mu\left(1+\lambda_2 \frac{\partial}{\partial t}\right)\nabla^4 w' \tag{17}$$

Eq. (20) is used to arrive at the non-dimensional form of the above equations.

$$w^* = \frac{w'}{\kappa/d},\ t^* = \frac{t}{d^2/\kappa},\ \theta = \frac{T'}{\Delta T},\ \phi = \frac{S'}{\Delta S},$$

$$\nabla^* = d\nabla,\ \left(x^*, y^*, z^*\right) = \left(\frac{x}{d}, \frac{y}{d}, \frac{z}{d}\right) \tag{18}$$

Since we consider only two-dimensional disturbances, we introduce the stream function ψ such that

$$u = \frac{\partial\psi}{\partial z},\ w = -\frac{\partial\psi}{\partial x} \tag{19}$$

and all the terms are independent of y. The resulting non-dimensional equations are:

$$\left[-\left(1+\Lambda_1\frac{\partial}{\partial t}\right)\frac{1}{\Pr}\nabla^2\frac{\partial}{\partial t} + \left(1+\Lambda_2\frac{\partial}{\partial t}\right)\nabla^4\right]\frac{\partial\psi}{\partial x} =$$

$$\left(1+\Lambda_1\frac{\partial}{\partial t}\right)\left[Ra(1+\varepsilon\cos\omega t)\nabla_1^2\theta + Rs(1+\varepsilon\cos\omega t)\nabla_1^2\phi\right], \tag{20}$$

$$\left(\frac{\partial}{\partial t} - \nabla^2\right)\theta + (\vec{q}.\nabla)\theta = \frac{\partial\psi}{\partial x}, \tag{21}$$

$$\left(\frac{\partial}{\partial t}-\frac{1}{Le}\nabla^2\right)\phi+(\vec{q}.\nabla)\phi=-\frac{\partial\psi}{\partial x}. \quad (22)$$

The non-dimensional parameters appearing in eq. (20) – (22) are the Prandtl number, Thermal Rayleigh number, Solutal Rayleigh number, stress relation parameter and strain retardation parameter, which are given in equation (23).

$$\Lambda_1=\frac{\lambda_1\kappa}{d^2},\ \Lambda_2=\frac{\lambda_2\kappa}{d^2},\ Le=\frac{\kappa}{\kappa_s},\quad \Pr=\frac{\mu}{\rho_0\kappa},$$

$$Ra=\frac{\alpha\rho_0 g\Delta T d^3}{\mu\kappa},\quad Rs=\frac{\alpha_s\rho_0 g\Delta S d^3}{\mu\kappa} \quad (23)$$

V. Linear Stability Analysis

In this section, we discuss the linear stability analysis by considering marginal and over-stable states. The solution of this analysis is of great utility in the local non-linear stability analysis discussed in the later sections. The linearlized equations after neglecting the non linear terms are:

$$\left[-\left(1+\Lambda_1\frac{\partial}{\partial t}\right)\frac{1}{\Pr}\nabla^2\frac{\partial}{\partial t}+\left(1+\Lambda_2\frac{\partial}{\partial t}\right)\nabla^4\right]\frac{\partial\psi}{\partial x}=$$

$$\left(1+\Lambda_1\frac{\partial}{\partial t}\right)\left[Ra(1+\varepsilon\cos\omega t)\nabla_1^2\theta+Rs(1+\varepsilon\cos\omega t)\nabla_1^2\phi\right], \quad (24)$$

$$\left(\frac{\partial}{\partial t}-\nabla^2\right)\theta=\frac{\partial\psi}{\partial x}, \quad (25)$$

$$\left(\frac{\partial}{\partial t}-\frac{1}{Le}\nabla^2\right)\phi=-\frac{\partial\psi}{\partial x}. \quad (26)$$

Eq. (24) – (26) are reduced to a single equation by eliminating θ and φ to get an equation in terms of the stream function, ψ.

$$\left(\frac{\partial}{\partial t}-\nabla^2\right)\left(\frac{\partial}{\partial t}-\frac{1}{Le}\nabla^2\right)\left[\begin{array}{l}\left(1+\Lambda_1\frac{\partial}{\partial t}\right)\frac{1}{pr}\frac{\partial}{\partial t}\\ \qquad -\left(1+\Lambda_2\frac{\partial}{\partial t}\right)\nabla^2\end{array}\right]\nabla^2\psi$$

$$=\left(1+\Lambda_1\frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial t}-\nabla^2\right)\left(Ra(1+\varepsilon\cos\omega t)\frac{\partial^2\psi}{\partial x^2}\right)$$

$$-\left(1+\Lambda_1\frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial t}-\frac{1}{Le}\nabla^2\right)Rs(1+\varepsilon\cos\omega t)\frac{\partial^2\psi}{\partial x^2} \quad (27)$$

VI. Perturbation Procedure

We seek the eigenfunction, ψ, and eigenvalue Ra of eq. (27) for the basic temperature distribution that departs from the linear profile by using quantities of order ε. Thus, the eigenvalues of the present problem differ from those of double diffusive

Notes

convection in Oldroyd-B liquids by quantities of ε. The solution of eq. (27) is sought in the form

$$\psi = \psi_0 + \varepsilon\psi_1 + \varepsilon^2\psi_2 + ...$$

$$Ra = Ra_0 + \varepsilon Ra_1 + \varepsilon^2 Ra_2 + ... \tag{28}$$

Malkus and Veronis [21] first used this type of expansion in connection with the study of finite amplitude convection. Here ψ_0 and Ra_0 are the eigenfunction and eigenvalue respectively of the unmodulated system and (ψ_i, Ra_i), i>1 are the corrections due to modulation of ψ_0 and Ra_0.

These expansions are used in eq. (28) and the coefficients of various powers of ε are equated to obtain the following system of equations

$$L\psi_0 = 0 \tag{29}$$

$$L\psi_1 = \left(\frac{\partial}{\partial t} - \frac{1}{Le}\nabla^2\right)\left(1+\Lambda_1\frac{\partial}{\partial t}\right)\left(Ra_1\frac{\partial^2\psi_0}{\partial x^2} + Ra_0\cos\Omega t\frac{\partial^2\psi_0}{\partial x^2}\right)$$

$$+\left(\frac{\partial}{\partial t} - \nabla^2\right)\left(1+\Lambda_1\frac{\partial}{\partial t}\right)Rs\cos\Omega t\frac{\partial^2\psi_0}{\partial x^2} \tag{30}$$

$$L\psi_2 = \left(\frac{\partial}{\partial t} - \frac{1}{Le}\nabla^2\right)\left(1+\Lambda_1\frac{\partial}{\partial t}\right)$$

$$\left(Ra_0\cos\Omega t\frac{\partial^2\psi_1}{\partial x^2} + Ra_1\frac{\partial^2\psi_1}{\partial x^2} + Ra_1\cos\Omega t\frac{\partial^2\psi_1}{\partial x^2} + Ra_2\frac{\partial^2\psi_0}{\partial x^2}\right) \tag{31}$$

$$-\left(\frac{\partial}{\partial t} - \nabla^2\right)\left(1+\Lambda_1\frac{\partial}{\partial t}\right)Rs\cos\Omega t\frac{\partial^2\psi_1}{\partial x^2}$$

Where,

$$L = \left(\frac{\partial}{\partial t} - \nabla^2\right)\left(\frac{\partial}{\partial t} - \frac{1}{Le}\nabla^2\right)$$

$$\left[\left(1+\Lambda_1\frac{\partial}{\partial t}\right)\frac{1}{pr}\frac{\partial}{\partial t} - \left(1+\Lambda_2\frac{\partial}{\partial t}\right)\nabla^2\right]\nabla^2$$

$$-\left(1+\Lambda_1\frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial t} - \nabla^2\right)Ra_0\frac{\partial^2}{\partial x^2}$$

$$+\left(1+\Lambda_1\frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial t} - \frac{1}{Le}\nabla^2\right)Rs\frac{\partial^2}{\partial x^2} \tag{32}$$

Each ψ_n is required to satisfy the boundary condition

$$\psi = \nabla^2\psi = \nabla^4\psi = 0 \text{ at } z = 0,\ 1. \tag{33}$$

a) Solution to the zero^th order problem

The absence of gravity modulation is equivalent to the zeroth order problem. The general solution of eq. (29), obtained at $o(\varepsilon^0)$, is the marginally stable solution of the zeroth order problem. The marginally stable solutions are

Ref

21. W.V.R. Malkus, G. Veronis, 1958, *"Finite amplitude cellular convection"*, J. Fluid Mech., 4, pp. 225-260.

$$\psi_0 = \sin(\pi\alpha x)\sin(\pi z) \tag{34}$$

with the corresponding eigenvalue R_{a0} given by

$$Ra_0 = \frac{\delta^6}{\pi^2 a^2} + LeR_s + \varepsilon^2 Ra_2 \tag{35}$$

b) Solution to the first order problem

Substituting Eq. (34) in Eq. (30), we get

$$\begin{aligned} L\psi_1 &= \frac{-\pi^2\alpha^2 k^2}{Le} Ra_0 \cos\Omega t\psi_0 + Ra_0\pi^2\alpha^2\Omega\sin\Omega t\psi_0 \\ &+ Ra_0\pi^2\alpha^2\Lambda_1\Omega^2\cos\Omega t\psi_0 + Rs\pi^2\alpha^2k^2\cos\Omega t\psi_0 \\ &- Rs\pi^2\alpha^2\Omega\sin\Omega\psi_0 - Rs\pi^2\alpha^2\Lambda_1\Omega^2\cos\Omega t\psi_0 \end{aligned} \tag{36}$$

Where,

$$L(\omega, n) = Y_1 + iY_2 \tag{37}$$

$$\begin{aligned} Y_1 &= \frac{-k^8}{Le} + \frac{k^6\Lambda_1\Omega^2}{\Pr} + k^4\Omega^2 - \frac{k^2\Lambda_1\Omega^4}{\Pr} + \frac{k^4\Omega^2}{\Pr} \\ &+ k^6\Lambda_2\Omega^2 + \frac{k^4\Omega^2}{Le\Pr} + \frac{k^6\Lambda_2\Omega^2}{Le} + \pi^2\alpha^2 Ra\left(\frac{k^2}{Le} - \Lambda_1\Omega^2\right) \\ &- \pi^2\alpha^2 Rs\left(k^2 - \Lambda_1\Omega^2\right) \end{aligned} \tag{38}$$

$$\begin{aligned} Y_2 &= \frac{k^6\Omega}{Le\Pr} + \frac{k^8\Lambda_2\Omega}{Le} - \frac{k^2\Omega^3}{\Pr} - k^4\Lambda_2\Omega^3 + k^6\Omega \\ &- \frac{k^4\Lambda_1\Omega^3}{\Pr} + \frac{k^6\Omega}{Le} - \frac{k^4\Lambda_1\Omega^3}{Le\Pr} - \pi^2\alpha^2 Ra\left(\Omega + \frac{k^2\Lambda_1\Omega}{Le}\right) \\ &+ \pi^2\alpha^2 Rs\left(\Omega + k^2\Omega\right) \end{aligned} \tag{39}$$

Eq. (36) is inhomogeneous and its solution poses a problem since it contains resonance terms The solvability condition requires that the first non-zero correction to R_0. The steady part of eq. (36) is orthogonal to sin πz. We take the time average and get the following expression for the correction Rayleigh number.

$$Ra_{2C} = \frac{Le}{2\pi^2\alpha^2k^2\,|\,L(\Omega,n)\,|^2}\left[\begin{aligned}&\left(X_1X_3\Omega + X_2X_4 + X_3^2\right)Y_1 \\ &+ \left(X_1X_4\Omega + X_2X_3 - X_3X_4\right)Y_2\end{aligned}\right] \tag{40}$$

Where,

$$X_1 = -Ra_0\pi^2\alpha^2 - \frac{Ra_0\pi^2\alpha^2k^2\Lambda_1}{Le} + Rs\pi^2\alpha^2 + Rs\pi^2\alpha^2k^2\Lambda_1 \tag{41}$$

$$\begin{aligned} X_2 &= -\frac{Ra_0\pi^2\alpha^2k^2}{Le} + Ra_0\pi^2\alpha^2\Omega^2\Lambda_1 + Rs\pi^2\alpha^2k^2 \\ &- Rs\pi^2\alpha^2\Omega^2\Lambda_1 \end{aligned} \tag{42}$$

$$X_3 = Ra_0\pi^2\alpha^2\Omega - Rs\pi^2\alpha^2\Omega \tag{43}$$

Notes

$$X_4 = \frac{-Ra_0\pi^2\alpha^2k^2}{Le} + Rs\pi^2\alpha^2k^2 + Ra_0\pi^2\alpha^2\Omega^2\Lambda_1 \tag{44}$$

$$-Rs\pi^2\alpha^2\Omega^2\Lambda_1$$

Notes

The linear theory predicts only the condition for the onset of convection and is silent about the heat and mass transport. We now embark on a non-linear analysis by means of truncated representation of Fourier series to find the effects of various parameters on finite amplitude convection and to know the amount of heat and mass transport.

VII. Non Linear Theory

A non-linear analysis is done to study the amount of heat and mass transfer due to the various parameters. Using the stream functions given by eq. (19)

$$\left(1+\Lambda_1\frac{\partial}{\partial t}\right)\left[\frac{1}{PR}\frac{\partial}{\partial t}\left(\nabla^2\psi\right)+Ra\frac{\partial\theta}{\partial x}-Rs\frac{\partial\varphi}{\partial x}\right]$$

$$=\left(1+\Lambda_1\frac{\partial}{\partial t}\right)\nabla^4\psi \tag{45}$$

$$\frac{\partial\theta}{\partial t} - J(\psi,\theta) + (1-\varepsilon f)\frac{\partial\psi}{\partial x} = \frac{1}{Le}\nabla^2\theta \tag{46}$$

$$\frac{\partial\varphi}{\partial t} - J(\psi,\phi) + \frac{\partial\psi}{\partial x} = \frac{1}{Le}\nabla^2\phi \tag{47}$$

An infinite series representation is used to find the solutions to eq. (45)– (47). The amplitudes depend only on time. Only one term in the Fourier representation for the stream function may be retained with two terms in the temperature expressions to retain some nonlinearity.
Eq. (45) is decomposed into two first order equations since it is a second order equation.

$$\frac{1}{\Pr}\frac{\partial}{\partial t}(\nabla^2\psi) = -Ra\frac{\partial\theta}{\partial x} + Rs\frac{\partial\phi}{\partial x} + \Lambda\nabla^4\psi + M \tag{48}$$

where
$$\Lambda = \frac{\Lambda_2}{\Lambda_1} \tag{49}$$

and
$$\Lambda_1\frac{\partial M}{\partial t} = -M + (1-\Lambda)\nabla^4\psi \tag{50}$$

The stream function, ψ, the temperature distribution, θ, concentration distribution, φ and M are represented as follows:

$$\psi = A(t)\sin(\pi\alpha x)\sin(\pi z) \tag{51}$$

$$\theta = B(t)\cos(\pi\alpha x)\sin(\pi z) + C(t)\sin(2\pi z) \tag{52}$$

$$\phi = E(t)\cos(\pi\alpha x)\sin(\pi z) + F(t)\sin(2\pi z) \tag{53}$$

$$M = G(t)\sin(\pi\alpha x)\sin(\pi z) \tag{54}$$

where A(t), B(t), C(t), E(t), F(t) and, G(t) are the amplitudes to be determined from the dynamics of the system.

Projecting eq. (46), (47), (48) and (50) onto the modes (51) - (54) and following the standard orthogonalization procedure, we obtain the following non-linear autonomous system of differential equations (generalized Lorenz model [22]):

$$\dot{A}(t) = \frac{-Ra\Pr\pi\alpha}{k^2}B(t) + \frac{Rs\Pr\pi\alpha}{k^2}E(t) - \Lambda\Pr k^2 A(t) - \frac{\Pr}{k^2}G(t) \tag{55}$$

$$\dot{B}(t) = (\varepsilon f - 1)\pi\alpha A(t) - k^2 B(t) \tag{56}$$

$$\dot{C}(t) = \frac{\pi^2\alpha}{2}A(t)B(t) - 4\pi^2 C(t) \tag{57}$$

$$\dot{E}(t) = -\pi\alpha A(t) - \frac{k^2}{Le}E(t) \tag{58}$$

$$\dot{F}(t) = \frac{\pi^2\alpha}{2}A(t)E(t) - \frac{4\pi^2}{Le}F(t) \tag{59}$$

$$\dot{G}(t) = \frac{-1}{\Lambda_1}G(t) + \frac{(1-\Lambda)}{\Lambda_1}k^4 A(t) \tag{60}$$

Where the over dot denotes the time derivative with respect to t. More modes other than the minimal ones have not been considered in the study in view of the observation by Siddheshwar and Titus [23] that additional modes do not significantly alter the results on the onset of convection as well as transport.

VIII. Heat Transport

In non-linear study of convection, the heat transport across the layers of fluid is important. The onset of convection can be easily determined by analyzing the increase and decrease in heat transport. In the basic state, transfer of heat takes place only due to convection.

If H_T is the rate of heat transfer / unit area, then

$$H_T = -\chi\left\langle\frac{\partial T_{total}}{\partial z}\right\rangle_{z=0}, \tag{61}$$

where the bracket corresponds to a horizontal average and

$$T_{total} = \left[T_0 - \frac{\Delta T}{d}z\right] + T(x,z)$$

The first term is the temperature distribution due to conduction state prevalent before convection sets in. The second term represents the convective heat transport.

The Nusselt number Nu is defined by

$$Nu = \frac{H_T}{\kappa\Delta T/d}. \tag{62}$$

Alternately, Nu may be directly defined in terms of the non-dimensional quantities as follows:

Ref

23. P.G. Siddheshwar, P.S. Titus, 2013, *"Nonlinear Rayleigh–Bénard convection with variable heat source"*, J. Heat transfer, 135, pp. 122502.

$$Nu = \frac{\left[\frac{k_c}{2\pi}\int_0^{2\pi/k_c}(1-z+T)_z\,dx\right]}{\left[\frac{k_c}{2\pi}\int_0^{2\pi/k_c}(1-z)_z\,dx\right]} = 1+\frac{4k_c^2A^2}{4\delta^2}. \tag{63}$$

Notes

The solutal gradient, arising due to double diffusive convection, causes a transfer of mass in the fluid system. This is quantified using Sherwood number given by

$$Sh = 1+\frac{4k_c^2Le^2A^2}{4\delta^2} \tag{64}$$

We use these expressions to determine the effects of various parameters of the problem on heat and mass transfer.

III. Results and Discussions

In this paper an attempt is made to study the effects of gravity modulation on double diffusive convection in Oldroyd-B liquids. The following effects on the classical Rayleigh-Benard problem are considered

i) Stress relaxation parameter.
ii) Strain retardation parameter.
iii) Lewis number.
iv) Frequency of modulation.

These are represented by $\Lambda_1, \Lambda_2,$ Le and ω. The effects of these parameters on heat and mass transfer are also analyzed. In the case of thermal modulation the amplitude, ε, is small compared with the imposed steady temperature difference. The validity of the results obtained here depends on the value of modulating frequency, ω. When ω < 1, the period of modulation is large and hence, the disturbance grows to such an extent as to make finite amplitude effects important. When $\omega \to \infty$, $Ra_{2c} \to 0$. Thus, modulation becomes small. Therefore, we choose moderate values of ω

Graphs of Ra_{2c} versus ω are plotted for varying values of the parameters which represent the linear part of the problem (figs (2) – (5)). The effects of gravity modulation on non-linear stability analysis are also discussed using graphs of Nusselt number versus time and Sherwood number versus time. The non – autonomous Lorenz model obtained is solved numerically. The parameters of the system are Lewis number, Le, Stress relaxation parameter, Λ_1, Strain retardation parameter, Λ_2, Prandtl number, Pr, Solutal Rayleigh number, Rs and frequency of modulation, ω, which influence the heat and mass transfer.

The linear stability analysis is discussed through graphs of correction Rayleigh number, Ra_{2c}, as a function of frequency of modulation, ω. Figures (2) – (6) are the corresponding graphs.

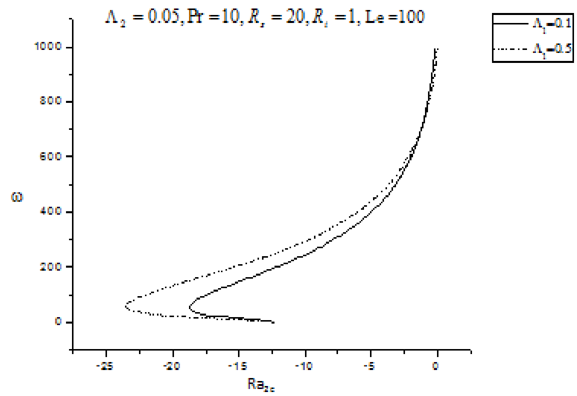

Fig. 2: Graph of Ra_{2c} vs t for different values of Λ_1

Fig (2) is the graphs for different values of the stress relaxation parameter, Λ_1, for fixed values of other parameters. It is evident from the graph that increase in Λ_1, causes a decrease in the value of Ra_{2c}. This, in turn, causes acceleration in the onset of convection, thus, destabilizing the system

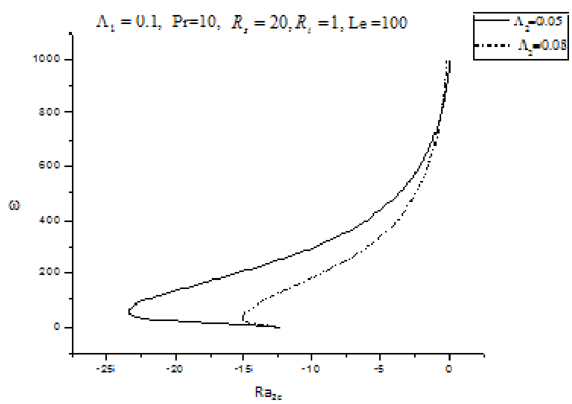

Fig. 3: Graph of Ra_{2c} vs t for different values of Λ_2

Fig (3) is the graphs of Ra_{2c} versus ω for varying values of Λ_2. The other parameters remain fixed. It can be seen that Λ_2 causes an effect opposite to that of Λ_1. With increase in Λ_2, Ra_{2c} also increases, thus delaying the convective process

Notes

Notes

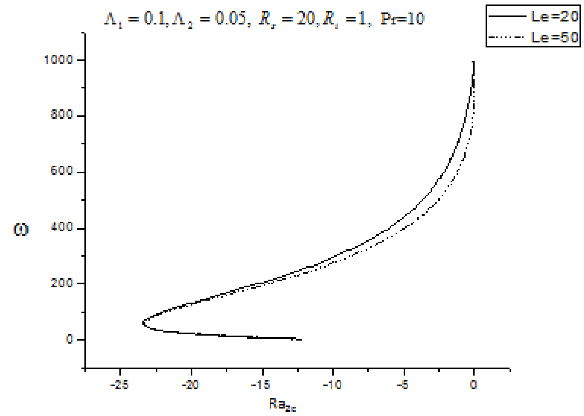

Fig. 4: Graph of Ra_{2c} vs t for different values of Le

Fig (4) is a graph of Ra_{2c} versus ω for different values of Le. As can be seen from the graph the increasing values of Le results in the increase in Ra_{2c}. This delays the onset of convection. Le is the ratio of thermal to solutal diffusivities. As Le increases the Solutal diffusivity decreases and the thermal diffusivity increases. This results in more heat transfer

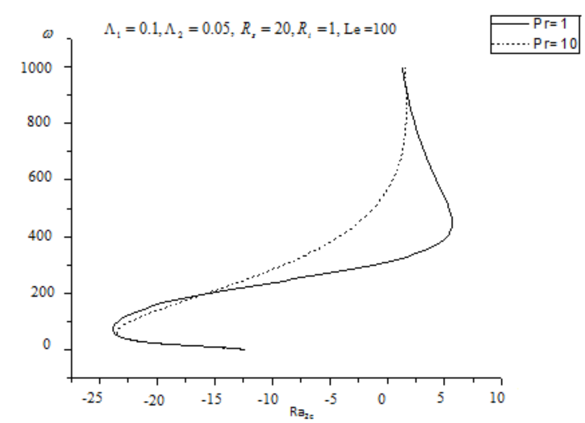

Fig. 5: Graph of Ra_{2c} vs t for different values of Pr

Figure (5) is the graph varying the values of Pr. It is appropriate here to note that Pr does not significantly affect the values of Ra_{2c}. The graphs in fig (2)-(5) depict sub critical motion. There is a steady line as well as a parabolic profile. This parabolic part is subject to finite amplitude instabilities

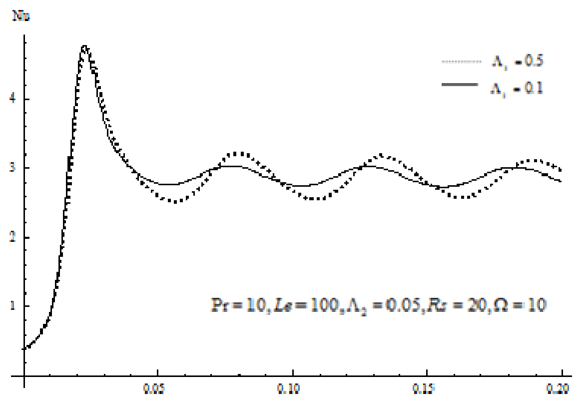

Fig. 6: Graph of Nu vs t for different values of Λ_1

Figures (6) – (15) are the graphs of heat and mass transfer. They represent the non-linear theory. Nusselt number, Nu, and Sherwood number, Sh, are used to plot these graphs as functions of time. Figures (6) – (10) show the effects of the different parameters on the Nusselt number. Fig (6) shows that Λ_1 causes an increase in the Nusselt number and in turn, the heat transport. This is obvious as Λ_1 causes a decrease in Ra_{2c}

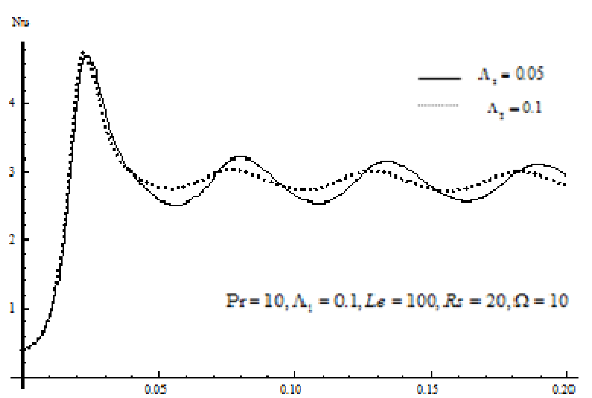

Fig. 7: Graph of Nu vs t for different values of Λ_2

The opposite result is seen for Λ_2 (Fig (7)). That is, as Λ_2 increases Nusselt number decreases, thus reducing the heat transfer. This is again an expected result as Λ_2 was found to cause an increase in the value of Ra_{2c}. Therefore, its stabilizing effects are affirmed here

Notes

Notes

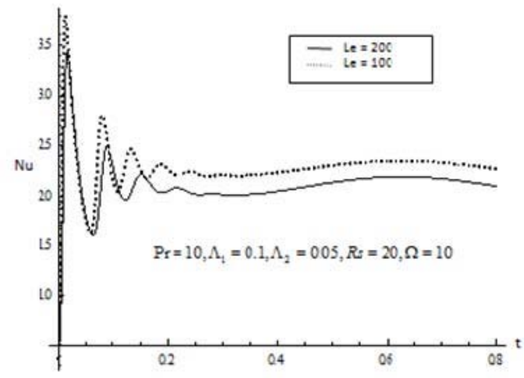

Fig. 8: Graph of Nu vs t for different values of Le

Fig (8) Shows the graphs of Nusselt number versus time for varying values of Le. It can be seen that the increasing values of Le decreases Nu, thus reducing heat transfer.

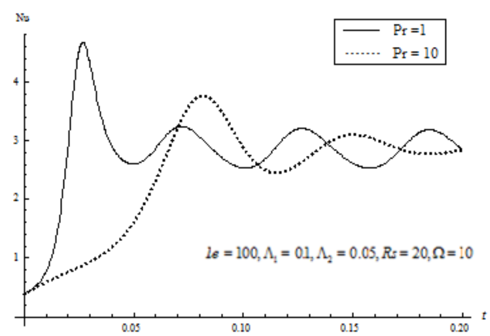

Fig. 9: Graph of Nu vs t for different values of Pr

Fig (9) Shows the pattern in Nu when Prandtl number is varied. The variation in the values of Nu can be observed for smaller values of Pr. As the value of Pr increases largely the values of Nu becomes more or less similar. Pr is the property of the type of fluid and hence smaller values of it differentiates the fluids

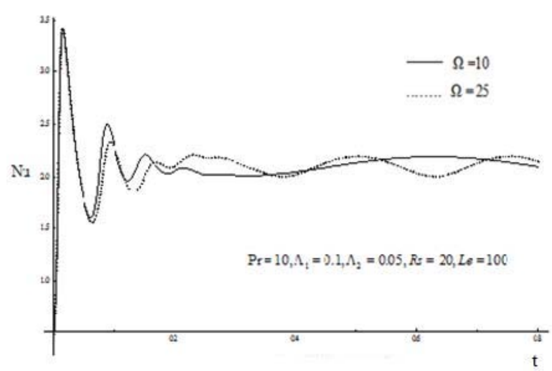

Fig. 10: Graph of Nu vs t for different values of Ω

Fig (10) is the graphs of Nusselt number versus time for varying values of the frequency of modulation, ω. It is evident that the increase in ω results in the decrease of Nu, Therefore, higher the frequency of modulation, lesser is the transport of heat. Thus, the frequency can be controlled to get desirable results in the system

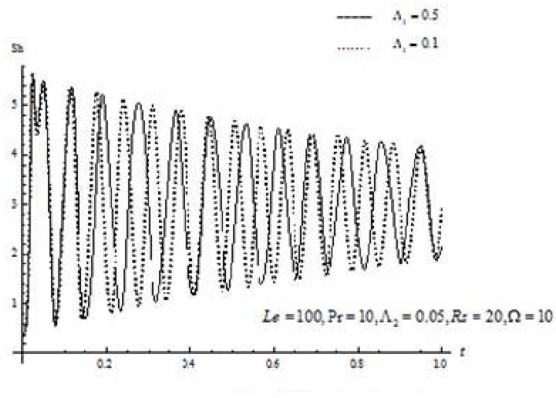

Fig. 11: Graph of Sh vs t for different values of Λ_1

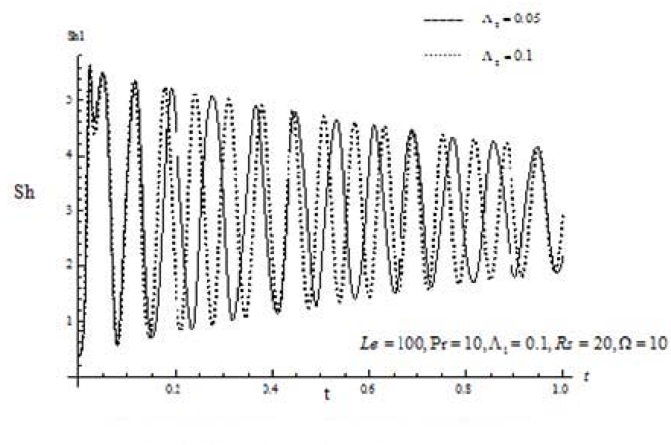

Fig. 12: Graph of Sh vs t for different values of Λ_2

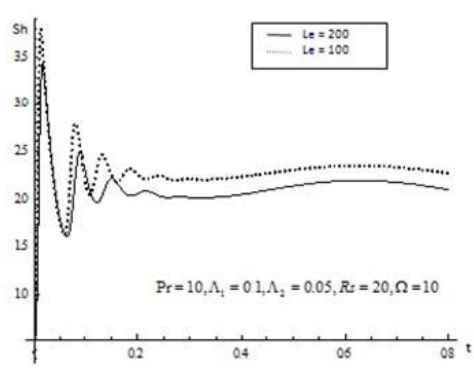

Fig. 13: Graph of Sh vs t for different values of Le

Notes

Notes

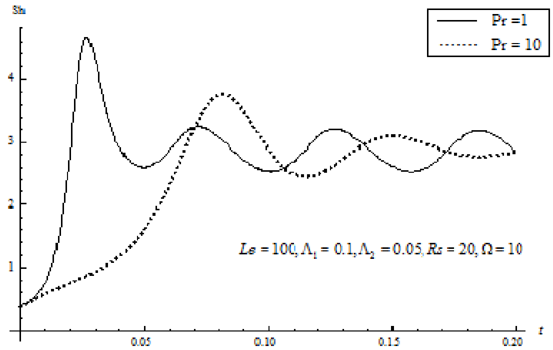

Fig. 14: Graph of Sh vs t for different values of Pr

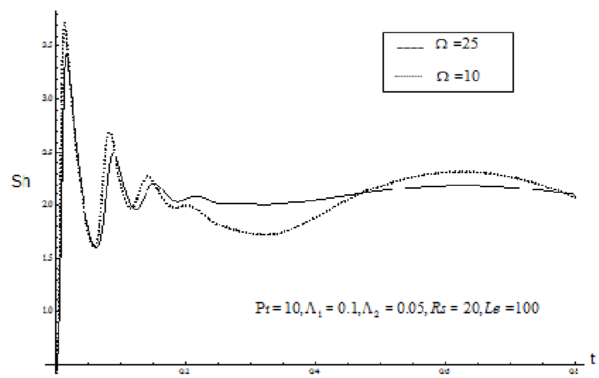

Fig. 15: Graph of Sh vs t for different values of Ω

Figures (11) – (15) are graphs of Sherwood number versus time for the same parameters mentioned above. These graphs show a pattern similar to that of Nusselt number. Therefore, heat and mass transfer show same type of variations for all the parameters.

IV. Conclusions

1. The stress relaxation parameter, Λ_1, and strain retardation parameter, Λ_2, have opposing effects on the stability with Λ_1 destabilizing the system and thereby increasing the heat transfer.
2. Lewis number, Le, stabilizes the system thereby decreasing the heat transfer.
3. Effect of the frequency of modulation, ω, is to decrease the heat transfer.
4. Sherwood number behaves in a way similar to Nusselt number.
5. Modulation can be used as effective means of controlling convection.

References Références Referencias

1. Charles M. Vest, Vedat S. Arpaci, 1969, "*Stability of natural convection in a vertical slot*", J. Fluid Mech., 36, pp. 1-15.
2. M. Sokolov, R.I. Tanner, 1972, "*Convective Stability of a General Viscoelastic Fluid Heated from Below*", Phys. Fluids, 15, pp. 534.
3. T Green III, 1968, "*Oscillating Convection in an Elasticoviscous Liquid*", Phys. Fluids, 11, pp. 1410.

4. P. G. Siddeshwar, G.N. Sekhar, and G. Jayalatha, 2010, "*Effect of time-periodic vertical oscillations of the Rayleigh–Bénard system on nonlinear convection in viscoelastic liquids*" Non-Newtonian Fluid Mech., 165, pp. 1412–1418.
5. Zhenyu Li and Roger E. Khayat, 2005, "*Three-dimensional thermal convection of viscoelastic fluids*" Phys. Rev. E, 71,pp. 221-251.
6. Z. Li, R.E. Khayat, 2005, "*Finite-amplitude Rayleigh–Benard convection and pattern selection for viscoelastic fluids*", J. Fluid Mech. 529. 2217
7. Pradeep G. Siddheshwar and C. V. Sri Krishna, 2001, "*Rayleigh-Benard Convection in a viscoelastic fluid-filled high-porosity medium with non-uniform basic temperature gradient*" Int. J. Math. Math. Sci., 25, pp. 609-619.
8. M. S. Malashetty, P. G. Siddheshwar, and Mahantesh Swamy, 2006, "*Effect of Thermal Modulation on the Onset of Convection in a Viscoelastic Fluid Saturated Porous Layer*" Transport Porous Med., 62, pp 55-79.
9. R.C. Sharma, 2006, "*Effect of rotation on thermal instability of a viscoelastic fluid.*" Acta Physiol. Hung., 40, pp. 11–17.
10. Mojtabi A., Charrier–Mojtabi M.C., 2000, Hand Book of Porous Media, Marcel Dekkes, New York, pp. 559–603.
11. M.S. Malashetty, M.Swamy, 2010, "*Onset of double diffusive convection in a viscoelastic fluid layer*", J. Non-Newtonian Fluid Mech., 165, pp. 1129 – 1138.
12. H. M. Stommel, A. B. Arons, Blanchard, 1956, "*An oceanography curiosity: the perpetual salt fountain*" Deep-Sea Res, 3. pp. 152-153.
13. M. Stern, 1960, "*The Salt Fountain and Thermohaline Convection*" Tellus, 12, pp.172-175.
14. M. Stern, 1969, "*Convective instability of salt fingers*", J. Fluid Mech., 35, pp. 209-218.
15. P. G. Siddheshwar and S. Pranesh, 1999, "*Effect of temperature/gravity modulation on the onset of magneto-convection in weak electrically conducting fluids with internal angular momentum.*" Int. J. Magn. Magn. Mater., 192, pp. 159-176.
16. B. S. Bhadauria, 2006, "*Temperature Modulation of Double Diffusive Convection in a Horizontal Fluid Layer,*" Z. Naturforsch, 61a, pp. 335 – 344.
17. M. Narayana, S.N. Gaikwad, P. Sibanda, R.B. Malge, 2013, "*Double Diffusive Magneto Convection in Viscoelastic fluids,*" Int. J. of Heat and Mass Transfer, 6, pp. 194-20.
18. B.S. Bhadauria and Palle Kiran, 2015, "*Chaotic and oscillatory magneto-convection in a binary viscoelastic fluid under g-jitter*", Int .J. Heat Mass Tran., 84, pp. 610-624.
19. P.G. Siddheshwar, B.S. Bhadauria, Pankaj Mishra, Atul K. Srivastava, 2012, "*Study of heat transport by stationary magneto-convection in a Newtonian liquid under temperature or gravity modulation using Ginzburg–Landau model*", Int. J. Nonlinear Mech., 47, pp. 418-425.
20. Palle Kiran, 2016, "*Nonlinear thermal convection in a viscoelastic nanofluid saturated porous medium under gravity modulation*", Ain Shams Engineering Journal, 7, pp. 639-651.

Notes

21. W.V.R. Malkus, G. Veronis, 1958, "*Finite amplitude cellular convection*", J. Fluid Mech., 4, pp. 225-260.
22. E. N. Lorenz, 1963, "*Deterministic non-periodic flow*", J. Atoms Sci., 20, pp. 130-141.
23. P.G. Siddheshwar, P.S. Titus, 2013, "*Nonlinear Rayleigh–Bénard convection with variable heat source*", J. Heat transfer, 135, pp. 122502.

Notes

Notes

This page is intentionally left blank