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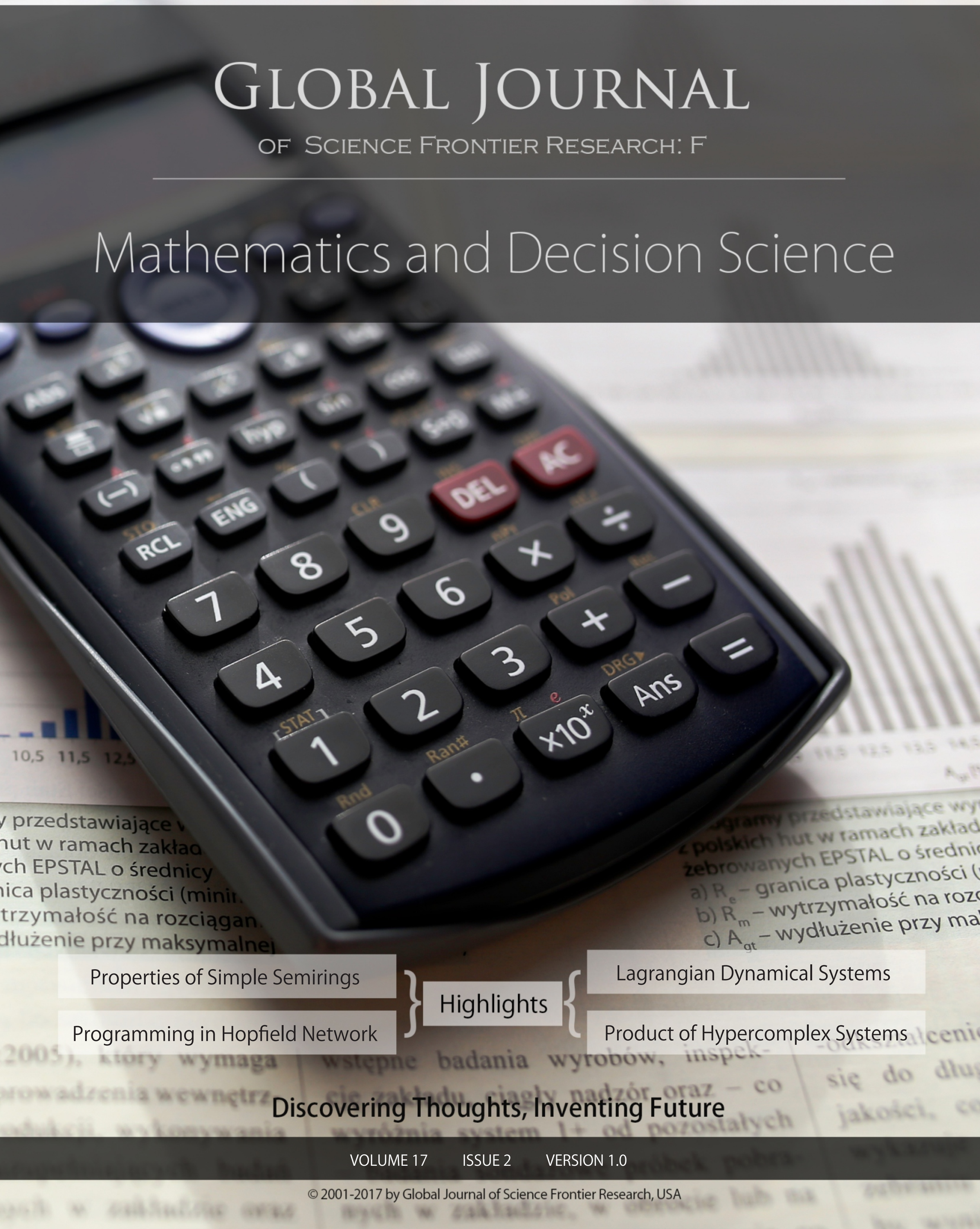

Highlights

- Properties of Simple Semirings
- Programming in Hopfield Network
- Lagrangian Dynamical Systems
- Product of Hypercomplex Systems

Discovering Thoughts, Inventing Future

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Properties of Simple Semirings

By P. Sreenivasulu Reddy & Abduselam Mahamed Derdar
Samara University

Abstract- Authors determine different properties of simple semiring which was introduced by Golan [3]. We also proved some results based on the papers of Fitore Abdullahu [1]. P. Sreenivasulu Reddy and Guesh Yfter tela [4].

GJSFR-F Classification: *MSC 2010: 16Y60*

PROPERTIESOFSIMPLESEMIRINGS

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Properties of Simple Semirings

P. Sreenivasulu Reddy [α] & Abduselam Mahamed Derdar [σ]

Abstract- Authors determine different properties of simple semiring which was introduced by Golan [3]. We also proved some results based on the papers of Fitore Abdullahu [1]. P. Sreenivasulu Reddy and Guesh Yfter tela [4].

I. Introduction

This paper reveals the properties of simple semirings by considering that the multiplicative semigroup is singular.

1.1. Definition: A semigroup (S, .) is said to be left(right) singular if it satisfies the identity $ab = a$ ($ab = b$) for all a, b in S

1.2. Definition: A semigroup (S, .) is rectangular if it satisfies the identity $aba = a$ for all a, b in S.

1.3. Definition: A semigroup S is called medial if $xyzu = xzyu$, for every x,y,z,u in S.

1.4. Definition: A semigroup S is called left (right) semimedial if it satisfies the identity $x^2yz = xyxz$ ($zyx^2 = zxyx$), where $x,y,z \in S$ and x, y are idempotent elements.

1.5. Definition: A semigroup S is called a semimedial if it is both left and right semimedial.

Example: The semigroup S is given in the table is I-semimedial

*	a	b	c
a	b	b	b
b	b	b	b
c	c	c	c

1.6. Definition: A semigroup S is called I- left(right) commutative if it satisfies the identity $xyz = yxz$ ($zxy = zyx$), where x, y are idempotent elements.

1.7. Definition: A semigroup S is called I-commutative if it satisfies the identity $xy = yx$, where $x,y \in S$ and x, y are idempotent elements.

Example: The semigroup S is given in the table is I-commutative.

*	a	b	c
a	b	b	a
b	b	b	b
c	c	b	c

Author α σ: Department of mathematics, Samara University, Semera, Afar Regional State, Ethiopia. e-mail: skgm.org@gmail.com

1.8. Definition: A semigroup S is called I-left(right) distributive if it satisfies the identity xyz = xyxz (zyx = zxyx), where x,y,z ∈ S and x, y are idempotent elements.

1.9. Definition: A semigroup S is called I-distributive if it is both left and right distributive

1.10. Definition: A semigroup S is said to be cancellative for any a, b, ∈S, then ac = bc ⇒ a = b and ca = cb ⇒ a = b

1.11. Definition: A semigroup S is called diagonal if it satisfies the identities $x^2 = x$ and xyz = xz.

1.12. Definition: [3] A semiring S is called simple if a + 1 = 1 + a = 1 for any a∈S.

1.13. Definition: A semiring (S, +, .) with additive identity zero is said to be zero sum free semiring if x + x = 0 for all x in S.

1.14. Definition: . A semiring (S, +, .) is said to be zero square semiring if $x^2 = 0$ for all x in S, where 0 is multiplicative zero.

1.15. Theorem: A simple semiring is additive idempotent semiring.

Proof: Let (S, +, .) be a simple semiring. Since (S, +, .) is simple, for any a∈S, a + 1 = 1. (Where 1 is the multiplicative identity element of S. S^1 = SU {1}.)

Now a = a.1 = a(1 + 1) = a + a ⇒ a = a + a ⇒ S is additive idempotent semiring.

1.16. Theorem: Let (S, +, .) be a simple semiring. Then the following statements are holds:
(i) a+b+1 = 1 (ii) ab +1 = 1 (iii) $a^n + 1 = 1$ (iv) $(ab)^n + 1 = 1$ (v) $(ab)^n + (ba)^n = a + b$ For all a,b ∈S.

Proof: Proof for (i) and (ii) are trivial. Proof for (iii), (iv) and (v) are by mathematical induction.

1.17. Theorem: Let (S, +, .) be a simple semiring in which (S, .) is singular then (S, .) is rectangular band.

Proof: Let (S,+,.) be a simple semiring and (S, .) be a singular i.e, for any a,b∈S ab = a. ⇒ aba = aa ⇒ aba = a (since (S, .) is singular) ⇒ (S, .) is rectangular band.

1.18. Theorem: Let (S, +, .) be a simple semiring in which (S, .) is singular then (S, +) is one of the following:
(i) I-medial (ii) I-semimedial (iii) I-distributive (iv) L-commutative
(v) R-commutative (vi) I-commutative (vii) external commutative
(viii) conditional commutative. (ix) digonal

Proof: Let (S, + ,.) be a semiring in which (S, .) is a singular. Assume that S satisfies the identity 1+a = 1 for any a ∈S. Now for any a, b, c, d ∈S.

(i) Consider a + b + c + d = a + (b + c) + b = a + c + b + d ⇒ (S, +) is I- medial.

(ii) Consider a + a + b + c = a +(a + b) + c = a + (b + a) + c = a + b + a + c ⇒ a + a + b + c = a + b + a + c ⇒ (S, +) is I- left semi medial.

Again b + c + a + a = b + (c + a) + a = b + (a + c) + a = b + a + c + a ⇒ b + c + a + a = b + a + c + a ⇒ (S, +) is I-right semi medial.
Therefore, (S, +) is I-semi-medial.
(iii) consider a + b + c = (a) + b + c = a + a + b + c = a + (a + b) + c = a + (b + a) + c = a + b + a + c ⇒ (S, +) is I-left distributive.

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Consider $b + c + a = b + c + (a) = b + c + a + a = b + (c + a) + a = b + (a + c) + a = b + a + c + a \Rightarrow b + c + a = b + a + c + a \Rightarrow (S, +)$ is I right-distributive. Hence $(S, +)$ is I-distributive.

Similarly we can prove the remaining.

1.19. Theorem: Let $(S, +, .)$ be a simple semiring and $(S, .)$ is singular then $(S, +)$ is (i) quasi-seprative (ii) weakly-seperative (iii) seperative.

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Proof: Let $(S,+, .)$ be a simple semiring and $(S, .)$ is a singular i.e, for any $a,b\in S$, $ab = a$. Since S is simple, $1+a = a+1 = 1$, for all $a\in S$. Let $a + a = a + b \Rightarrow a + a +1 = a + b + 1 \Rightarrow a + 1 = b + 1 \Rightarrow a = b$. Again, $a + b = b + b \Rightarrow a + b + 1 = b + b + 1 \Rightarrow a + 1 = b + 1 \Rightarrow a = b$. Hence $a + a = a + b = b + b \Rightarrow a = b \Rightarrow (S, +)$ is quasi-seperative.

(ii) Let $a + b = (a) + b = ba + b = b + ab = b + a \Rightarrow a + b = b + a \rightarrow (1)$

From (i) and (ii) $a + a = a + b = b + a = b + b \Rightarrow a = b \Rightarrow (S, +)$ is weakly seperative

(iii) Let $a + a = a + b$
$b + b = b + a$

From (1) $a + b = b + a$ and from theorem 1.15 $(S, +)$ is a band

Therefore, $a = a + a = a + b = b + b = b \Rightarrow a = b$.

Hence $(S, +)$ is seperative.

1.20. Theorem: Let $(S, +, .)$ be a simple semiring in which $(S, .)$ is singular then $(S, +)$ is cancellative in which case —S— = 1.

Proof: Let $(S, +, .)$ be a simple semiring in which $(S, .)$ is singular. Since S is simple then for any $a\in S$, $1 + a = a + 1 = 1$.

Let a,b,c, $\in$S. To prove that $(S, +)$ is cancellative, for any a, b, c $\in$S, consider $a + c = b + c$. Then $a + c.1 = b + c.1 \Rightarrow a + c(a + 1)$
$= b + c(b + 1) \Rightarrow a + ca + c = b + cb + c \Rightarrow a + ca + cac = b + cb + cbc$ (since $(S, .)$ is rectangular) $\Rightarrow a + ca(1 + c) = b + cb(1 + c)$
$\Rightarrow a + ca.1 = b + cb.1 \Rightarrow a + ca = b + cb.1 \Rightarrow a + ca = b + cb$
$\Rightarrow (1 + c)a = (1 + c)b \Rightarrow 1.a = 1.b \Rightarrow a = b \Rightarrow a + c = b + c$
$\Rightarrow a = b. \Rightarrow (S. +)$ is right cancellative

Again $c + a = c + b \Rightarrow c.1 + a = c.1 + b \Rightarrow c(1 + a) + a = c(1 + b) + b \Rightarrow c + ca + a = c + cb + b \Rightarrow cac + c + a = cbc + cb + b \Rightarrow cac + ca + a = abc +cb + b \Rightarrow ca(c + 1)+ a = cb(c + 1) + b \Rightarrow ca.1 + a = cb.1 + b$
$\Rightarrow a + a = cb + b \Rightarrow (c + 1)a = (c + 1)b \Rightarrow 1.a = 1.b \Rightarrow a = b \Rightarrow c + a = c + b \Rightarrow a = b \Rightarrow (S, +)$ is left cancellative.

Therefore, $(S,+)$ is cancellative semigroup. Since S is simple semiring we have $1 + a = 1 \Rightarrow 1 + a = 1 + 1$. But $(S, +)$ is cancellative $\Rightarrow a = 1$ for all $a\in S$. Therefore —S— = 1.

1.21. Theorem: Let $(S, +, .)$ be a simple semiring in which $(S, .)$ is singular then $(S, +)$ is one of the following: i) singular ii) rectangular band iii) left(right) semi-normal iv) regular v) normal vi) left(right) quasi-normal vii) left(right) semi-regular.

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Proof: Let (S, +, .) be a simple semiring in which (S, .) is singular. Since S is simple then for any a, b, c$\in$S, i) $1 + a = 1 \Rightarrow b + ba = b \Rightarrow b + a = b \Rightarrow$ (S, +) is left singular. Again $1 + b = 1 \Rightarrow a + ba = a \Rightarrow a + b = a \Rightarrow$ (S, +) is right singular.
Therefore, (S, +) is singular.

Since (S, +) is singular, it is easy to prove (S, +) is ii) rectangular band, iii) left(right) semi-normal, iv) regular, v) normal and vi) left(right) quasi-normal.

vii) Let $a + b + c + a = a + (b) + (c) + (a) = a + b + a + c + a + c + a \Rightarrow a + b + c + a = a + b + a + c + a + (c) + a \Rightarrow a + b + a + c + a + b + c + a \Rightarrow$ (S, +) left semi-regular.
Similarly, we can prove (S, +) is right semi-regular.

1.22. Theorem: Let (S, +, .) be a simple semiring in which (S, .) is singular then (S, .) is one of the following: i) left semi-normal ii) left semi-regular iii) right semi-normal iv) right semi-regular v) regular vi) normal vii) left quasi-normal viii) right quasi-normal ix) I-medial (x) I-semimedial (xi) I-distributive (xii) L-commutative (xiii) R-commutative (xiv) I-commutative (xv) external commutative (xvi) conditional commutative (xvii) digonal (xviii) quasi-seprative (xix) weakly-seperative (xx) seperative.

Proof: Proof of the theorem is similar to 1.21.Theorem, 1.18.theorem and 1.19.theorem.

1.23. Theorem: Let (S, +, .) be a simple semiring with additive identity zero in which (S, .) is singular then (S, +, .) zero sum free semiring if and only if (S, +, .) is zero square semiring.

Proof: Let (S, +, .) be a simple semiring with additive identity zero in which (S, .) is singular. Since S is simple then for any a$\in$S, $1 = a + 1 \Rightarrow a = aa + a \Rightarrow a^2 = a + a \Rightarrow a^2 = 0$ (Since (S, +, .) is zero sum free semiring) $\Rightarrow$ (S, +, .) is zero square semiring.

Conversely, let (S, +, .) is zero square semiring then $a^2 = 0$ $1 = a + 1 \Rightarrow a = aa + a \Rightarrow a^2 = a + a \Rightarrow 0 = a + a$ (Since (S, +, .) is zero square semiring) $\Rightarrow$ (S, +, .) is zero sum free semiring.

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Lagrangian Dynamical Systems on Clifford *Kähler* Manifolds

By Gebreel Mohammed Khur Baba Gebreel & Mohammed Ali Bashir
West Kordufan University

Abstract- In his paper we obtained a canonical local basis $\{J_i\}, i = \overline{1,5}$ of vector bundle V on Clifford $Kähler$ manifold (M,V). The paths of semispray on Clifford $Kähler$ manifold are infact the solutions of Euler-Lagrange equations.

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Lagrangian Dynamical Systems on Clifford *Kähler* Manifolds

Gebreel Mohammed Khur Baba Gebreel [α] & Mohammed Ali Bashir [σ]

Abstract- In his paper we obtained a canonical local basis $\{\boldsymbol{J_i}\}, \boldsymbol{i} = \overline{\mathbf{1,5}}$ of vector bundle $\boldsymbol{V}$ on Clifford ***Kähler*** manifold $(\boldsymbol{M}, \boldsymbol{V})$.the paths of semispray on Clifford ***Kähler*** manifold are infact the solutions of Euler-Lagrange equations.

I. Introduction

It is well-known that modern differential geometry express explicitly the dynamics of Lagrangians.

Therefore we explain that if M is an m-dimensional configuration manifold and $L : TM \rightarrow R$ is a regular lagrangian function, then there is a unique vector field ξ on TM such that dynamics equations is determined by:

$$i_{\xi}\Phi_l = d\mathrm{E}_l \quad \rightarrow \quad (1)$$

Where Φ_l indicates the symplectic form and E_l is the energy associated to L [1,2].

The Triple (TM, Φ_l, ξ) in named lagrangian system on the tangent bundle TM.

It is known, there are many studies about Lagrangian mechanics, formalisms, systems and equations such as real, complex, paracomplex and other analogues[1,3] and there in. so, it may be possible to produced different analogues in different spaces.

The goal of finding new dynamics equations is both a new expansion and contribution to science to explain physical events.

Sir William Rowan Hamilton invented quaternions as an extension to the complex numbers.

Hamilton's defining relation is most succinctly written as:

$$i^2 = j^2 = k^2 = ijk = -1 \quad \rightarrow \quad (2)$$

If it is compared to the calculus of vectors , quaternion's have slipped into the realm of obscurity. They do however still find use in the in the computation of rotations. A lot of physical laws in classical, relativistic, and quantum mechanics can be written pleasantly by means of quaternions. Some physicists hope they will find deeper understanding of the universe by restating basic principles in terms of quaternion algebra. It is well-known that quaternions are useful for representing rotations in both quantum and classical mechanics[4]. Clifford manifolds are also quaternion manifolds[5].

II. Preliminaries

In this paper, all mathematical objects and mappings are assumed to be smooth, i.e. infinitely differentiable and Einstein convention of summarizing is adopted.

Author α σ: West Kordufan University. e-mail: gebreel6600@gmail.com

$\mathcal{F}(M), \mathcal{X}(M) and\, \Lambda^1(M)$ define the set of functions on M, the set of vector fields on M and the set of 1-forms on M,respectively.

a) Theorem

Let f be differentiable $\phi,\ \psi$ are 1-form, then [6]:

(i) $d(f\phi) = df\wedge\phi + fd\phi$

(ii) $d(\phi\wedge\psi)$=d$\phi\wedge\psi - \phi\wedge d\psi$

b) Definition (Kronecker's delta)

Kronecker's delta denote by δ and defined as follows [7,8] :

$$\delta_i^j = \begin{cases} 1 \ ; & if \quad i = j \\ 0 \ ; & if \quad i \neq j \end{cases}$$

c) Clifford Kähler Manifolds

Now, here we extend and rewrite the main concepts and structures given in [5,9,10]. Let M be a real smooth manifold of dimension m. Suppose that there is a 6-dimensional vector bundle V consisting of $F_i(i = 1,2,\dots,6)$ tensors of type (1,1) over M. Such a local basis $\{F_1, F_2, \dots, F_6\}$ is named a canonical local basis of the bundle V in a neighborhood U of M. Then V is called an almost Clifford structure in M. The pair $(M,V)$is named an almost Clifford manifold with V. Thus, an almost Clifford manifold M is of $dimension\, m = 8n$. If there exists on (M,V) a global basis $\{F_1, F_2, \dots, F_6\}$, then (M,V) is called an almost Clifford manifold; the basis $\{F_1, F_2, \dots, F_6\}$ is said to be a global basis for V.

An almost Clifford connection on the almost Clifford manifold (M,V) is a linear connection $\nabla\, on\, M$ which preserves by parallel transport the vector bundle V. This means that if Φ is a cross-section (local-global) of the bundle V. Then $\nabla_X\Phi$ is also a cross-section (local-global, respectively) of V ,X being an arbitrary vector field of M.

If for any canonical basis $\{J_i\}, i = \overline{1,6}\, of\, V$ in a coordinate neighborhood U, the identities

$$g(J_iX, J_iY) = g(X,Y)\,,\ \forall X,Y \in \chi(M), i = 1,2,\dots,6 \quad \rightarrow \tag{3}$$

Hold, the triple (M, g, V) is called an almost Clifford Hermitian manifold or metric Clifford manifold denoting by V an almost Clifford structure V and by g a Riemannian metric and by (g,V) an almost Clifford metric structure.

Since each $J_i(i = 1,2,\dots,6)$ is almost Hermitian structure with respect to g, setting

$$\Phi_i(X,Y) = g(J_iX,Y), \quad i = 1,2,\dots,6 \qquad \rightarrow \tag{4}$$

For any vector fields $X\, and\, Y$, we see that Φ_i are 6-local 2-forms.

If the Levi-Civita connection $\nabla = \nabla^g\, on\, (M, g, V)$ preserves the vector bundle V by parallel transport, then (M, g, V) is named a Clifford $Kähler$ manifold, and an almost Clifford structure Φ_i of M is said to be a Clifford $Kähler$ structure. Suppose that let

$$\{x_i\,, x_{n+i}\,, x_{2n+i}\,, x_{3n+i}\,, x_{4n+i}\,, x_{5n+i}\,, x_{6n+i}\,, x_{7n+i}\}\,, i = \overline{1,n}$$

Ref

9. K. Yano, M. Kon, Structures on Manifolds, Series in Pure Mathematics –Volume 3, World Scientific Publishing co. Pte. Ltd., Singore, 1984.

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be a real coordinate system on (M,V). Then we denote by

$$\left\{\frac{\partial}{\partial x_i},\frac{\partial}{\partial x_{n+i}},\frac{\partial}{\partial x_{2n+i}},\frac{\partial}{\partial x_{3n+i}},\frac{\partial}{\partial x_{4n+i}},\frac{\partial}{\partial x_{5n+i}},\quad\frac{\partial}{\partial x_{6n+i}},\frac{\partial}{\partial x_{7n+i}}\right\},$$

$$\{dx_i,dx_{n+i},dx_{2n+i},dx_{3n+i},dx_{4n+i},dx_{5n+i},dx_{6n+i},dx_{7n+i}\}$$

The natural bases over R of the tangent space $T(M)$ and the cotangent space $T^*(M)\ of\ M$, respectively.

By structure $\{J_1,J_2,J_3,J_4,J_5,J_6\}$ the following expressions are given

$$J_1\left(\frac{\partial}{\partial x_i}\right)=\frac{\partial}{\partial x_{n+i}}\qquad J_2\left(\frac{\partial}{\partial x_i}\right)=\frac{\partial}{\partial x_{2n+i}}\qquad J_3\left(\frac{\partial}{\partial x_i}\right)=\frac{\partial}{\partial x_{3n+i}}$$

$$J_1\left(\frac{\partial}{\partial x_{n+i}}\right)=-\frac{\partial}{\partial x_i}\quad J_2\left(\frac{\partial}{\partial x_{n+i}}\right)=-\frac{\partial}{\partial x_{4n+i}}\quad J_3\left(\frac{\partial}{\partial x_{n+i}}\right)=-\frac{\partial}{\partial x_{5n+i}}$$

$$J_1\left(\frac{\partial}{\partial x_{2n+i}}\right)=\frac{\partial}{\partial x_{4n+i}}\quad J_2\left(\frac{\partial}{\partial x_{2n+i}}\right)=-\frac{\partial}{\partial x_i}\quad J_3\left(\frac{\partial}{\partial x_{2n+i}}\right)=-\frac{\partial}{\partial x_{6n+i}}$$

$$J_1\left(\frac{\partial}{\partial x_{3n+i}}\right)=\frac{\partial}{\partial x_{5n+i}}\quad J_2\left(\frac{\partial}{\partial x_{3n+i}}\right)=\frac{\partial}{\partial x_{6n+i}}\quad J_3\left(\frac{\partial}{\partial x_{3n+i}}\right)=-\frac{\partial}{\partial x_i}$$

$$J_1\left(\frac{\partial}{\partial x_{4n+i}}\right)=-\frac{\partial}{\partial x_{2n+i}}\quad J_2\left(\frac{\partial}{\partial x_{4n+i}}\right)=\frac{\partial}{\partial x_{n+i}}\quad J_3\left(\frac{\partial}{\partial x_{4n+i}}\right)=\frac{\partial}{\partial x_{7n+i}}$$

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$$J_4\left(\frac{\partial}{\partial x_{3n+i}}\right)=-\frac{\partial}{\partial x_{7n+i}}\quad J_5\left(\frac{\partial}{\partial x_{3n+i}}\right)=\frac{\partial}{\partial x_{n+i}}\quad J_6\left(\frac{\partial}{\partial x_{3n+i}}\right)=\frac{\partial}{\partial x_{2n+i}}$$

$$J_4\left(\frac{\partial}{\partial x_{4n+i}}\right)=-\frac{\partial}{\partial x_i}\quad J_5\left(\frac{\partial}{\partial x_{4n+i}}\right)=\frac{\partial}{\partial x_{6n+i}}\quad J_6\left(\frac{\partial}{\partial x_{4n+i}}\right)=\frac{\partial}{\partial x_{5n+i}}$$

$$J_4\left(\frac{\partial}{\partial x_{5n+i}}\right)=\frac{\partial}{\partial x_{6n+i}}\quad J_5\left(\frac{\partial}{\partial x_{5n+i}}\right)=-\frac{\partial}{\partial x_i}\quad J_6\left(\frac{\partial}{\partial x_{5n+i}}\right)=-\frac{\partial}{\partial x_{4n+i}}$$

$$J_4\left(\frac{\partial}{\partial x_{6n+i}}\right)=-\frac{\partial}{\partial x_{5n+i}}\quad J_5\left(\frac{\partial}{\partial x_{6n+i}}\right)=-\frac{\partial}{\partial x_{4n+i}}\quad J_6\left(\frac{\partial}{\partial x_{6n+i}}\right)=-\frac{\partial}{\partial x_i}$$

$$J_4\left(\frac{\partial}{\partial x_{7n+i}}\right)=\frac{\partial}{\partial x_{3n+i}}\quad J_5\left(\frac{\partial}{\partial x_{7n+i}}\right)=\frac{\partial}{\partial x_{2n+i}}\quad J_6\left(\frac{\partial}{\partial x_{7n+i}}\right)=\frac{\partial}{\partial x_{n+i}}$$

III. Lagrangian Mechanics

In this section, we introduce Euler-Lagrange equations for quantum and classical mechanics by means of canonical local basis $\{J_i\}, i=\overline{1,6}$ *of* Von Clifford *Kähler* manifold (M,V). We say that the Euler-Lagrange equations using basis $\{J_1,J_2,J_3\}$ of V on (R^{8n},V) are introduce in[5]. In this study, we obtain that they are the same as the obtained by operators J_1, J_2, J_3 *of* V on Clifford *Kähler* manifold (M,V).
If we express them, they are respectively:
First:

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_i}\right)+\frac{\partial L}{\partial x_{n+i}}=0\,,\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{n+i}}\right)-\frac{\partial L}{\partial x_i}=0\,,\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{2n+i}}\right)+\frac{\partial L}{\partial x_{4n+i}}=0,$$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{3n+i}}\right)+\frac{\partial L}{\partial x_{5n+i}}=0\,,\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{4n+i}}\right)-\frac{\partial L}{\partial x_{2n+i}}=0,\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{5n+i}}\right)-\frac{\partial L}{\partial x_{3n+i}}=0,$$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{6n+i}}\right)+\frac{\partial L}{\partial x_{7n+i}}=0\,,\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{7n+i}}\right)-\frac{\partial L}{\partial x_{6n+i}}=0.$$

Second:

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_i}\right)+\frac{\partial L}{\partial x_{2n+i}}=0,\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{n+i}}\right)-\frac{\partial L}{\partial x_{4n+i}}=0,\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{2n+i}}\right)-\frac{\partial L}{\partial x_i}=0,$$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{3n+i}}\right)+\frac{\partial L}{\partial x_{6n+i}}=0,\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{4n+i}}\right)+\frac{\partial L}{\partial x_{n+i}}=0,\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{5n+i}}\right)-\frac{\partial L}{\partial x_{7n+i}}=0,$$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{6n+i}}\right)-\frac{\partial L}{\partial x_{3n+i}}=0,\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{7n+i}}\right)+\frac{\partial L}{\partial x_{5n+i}}=0.$$

Third:

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_i}\right)+\frac{\partial L}{\partial x_{3n+i}}=0,\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{n+i}}\right)-\frac{\partial L}{\partial x_{5n+i}}=0,\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{2n+i}}\right)-\frac{\partial L}{\partial x_{6n+i}}=0,$$

Ref

5. M. Tekkoyun, Lagrangian Mechanics on the standard Clifford *Kähler* Manifolds, arXiv:0902.3724v1.

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{3n+i}}\right)-\frac{\partial L}{\partial x_i}=0,\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{4n+i}}\right)+\frac{\partial L}{\partial x_{7n+i}}=0,\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{5n+i}}\right)+\frac{\partial L}{\partial x_{n+i}}=0,$$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{6n+i}}\right)+\frac{\partial L}{\partial x_{2n+i}}=0\,,\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{7n+i}}\right)-\frac{\partial L}{\partial x_{4n+i}}=0.$$

Notes

Here, only we derive Euler-Lagrange equations using operators $J_4\,,J_5,J_6$ of V on Clifford *Kähler* manifold(M,V).
Fourth:

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_i}\right)+\frac{\partial L}{\partial x_{4n+i}}=0\,,\qquad \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{n+i}}\right)-\frac{\partial L}{\partial x_{2n+i}}=0\,,\qquad \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{2n+i}}\right)+\frac{\partial L}{\partial x_{n+i}}=0$$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{3n+i}}\right)-\frac{\partial L}{\partial x_{7n+i}}=0,\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{4n+i}}\right)-\frac{\partial L}{\partial x_i}=0,\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{5n+i}}\right)+\frac{\partial L}{\partial x_{6n+i}}=0\,,$$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{6n+i}}\right)-\frac{\partial L}{\partial x_{5n+i}}=0\,,\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{7n+i}}\right)+\frac{\partial L}{\partial x_{3n+i}}=0.$$

Such that the equations are named Euler-Lagrange equations structured on Clifford *Kähler* manifold (M,V) by means of $\Phi_L^{J_4}$ and in the case, the triple $(M,\Phi_L^{J_4},\xi)$ is said to be a mechanical system on Clifford *Kähler* manifold (M,V).

Fifth, we obtain Euler-Lagrange equations for quantum and classical mechanics by means of $\Phi_L^{J_5}$ on Clifford *Kähler* manifold(M,V).

Let J_5 be another local basis component on the Clifford *Kähler* manifold(M,V), and $\{x_i\,,x_{n+i}\,,x_{2n+i}\,,x_{3n+i}\,,x_{4n+i}\,,x_{5n+i}\,,x_{6n+i}\,,x_{7n+i}\}\,,i=\overline{1\,,n}$ be its coordinate functions.
Let semisparay be the vector field ξ defined by:

$$\xi=X^i\frac{\partial}{\partial x_i}+X^{n+i}\frac{\partial}{\partial x_{n+i}}+X^{2n+i}\frac{\partial}{\partial x_{2n+i}}+X^{3n+i}\frac{\partial}{\partial x_{3n+i}}+X^{4n+i}\frac{\partial}{\partial x_{4n+i}}$$

$$+X^{5n+i}\frac{\partial}{\partial x_{5n+i}}+X^{6n+i}\frac{\partial}{\partial x_{6n+i}}+X^{7n+i}\frac{\partial}{\partial x_{7n+i}}\qquad\rightarrow\qquad(5)$$

Where

$$X^i=\dot{x}_i\,,X^{n+i}=\dot{x}_{n+i}\,,X^{2n+i}=\dot{x}_{2n+i}\,,X^{3n+i}=\dot{x}_{3n+i}\,,X^{4n+i}=\dot{x}_{4n+i}$$

$$X^{5n+i}=\dot{x}_{5n+i}\,,X^{6n+i}=\dot{x}_{6n+i}\,,X^{7n+i}=\dot{x}_{7n+i}.$$

This equation (5) can be written concise manner

$$\xi=\sum_{a=0}^{7}X^{an+i}\frac{\partial}{\partial x_{an+i}}\qquad\rightarrow\qquad(6)$$

And the dot indicates the derivative with respect to time t. The vector field defined by

$$V_{J_5} = J_5(\xi) = X^i \frac{\partial}{\partial x_{5n+i}} - X^{n+i} \frac{\partial}{\partial x_{3n+i}} - X^{2n+i} \frac{\partial}{\partial x_{7n+i}} + X^{3n+i} \frac{\partial}{\partial x_{n+i}} + X^{4n+i} \frac{\partial}{\partial x_{6n+i}}$$

$$- X^{5n+i} \frac{\partial}{\partial x_i} - X^{6n+i} \frac{\partial}{\partial x_{4n+i}} + X^{7n+i} \frac{\partial}{\partial x_{2n+i}} \quad \rightarrow \quad (7)$$

Is named Liouville vector field on Clifford *Kähler* manifold (M, V).

The maps explained by $T, P: M \rightarrow R$ such that:

$$T = \frac{1}{2} m_i (\dot{x}_i^2 + \dot{x}_{n+i}^2 + \dot{x}_{2n+i}^2 + \dot{x}_{3n+i}^2 + \dot{x}_{4n+i}^2 + \dot{x}_{5n+i}^2 + \dot{x}_{6n+i}^2 + \dot{x}_{7n+i}^2)$$

$$\therefore \ T = \frac{1}{2} m_i \sum_{a=0}^{7} \dot{x}_{an+i}^2 \qquad , \quad P = m_i g h$$

Are said to be the kinetic energy and the potential energy of the system, respectively. Here $m_i, g \ and \ h$ stand for mass of a mechanical system having m particles, the gravity acceleration and distance to the origin of a mechanical system on Clifford *Kähler* manifold (M, V), respectively.

Then $L: M \rightarrow R$ is a map that satisfies the conditions:

i) $L = T - P$ is a Lagrangian function.

ii) the function given by $E_L^{J_5} = V_{J_5}(L) - L$, is energy function.

The operator i_{J_5} induced by J_5 and defined by:

$$i_{J_5} \omega(X_1, X_2, \dots, X_r) = \sum_{i=1}^{r} \omega(X_1, \dots, J_5 X_i, \dots, X_r) \quad \rightarrow \quad (8)$$

Is called vertical derivation, where $\omega \in \Lambda^r M, X_i \in \mathcal{X}(M)$. The vertical differentiation d_{J_5} is determined by:

$$d_{J_5} = [i_{J_5}, d] = i_{J_5} d - d i_{J_5} \quad \rightarrow \quad (9)$$

Where d is the usual exterior derivation. We saw that the closed Clifford *Kähler* form is the closed 2-form given by $\Phi_L^{J_5} = -d d_{J_5} L$ such that

$$d_{J_5} = \frac{\partial}{\partial x_{5n+i}} dx_i - \frac{\partial}{\partial x_{3n+i}} dx_{n+i} - \frac{\partial}{\partial x_{7n+i}} dx_{2n+i} + \frac{\partial}{\partial x_{n+i}} dx_{3n+i} + \frac{\partial}{\partial x_{6n+i}} dx_{4n+i}$$

$$- \frac{\partial}{\partial x_i} dx_{5n+i} - \frac{\partial}{\partial x_{4n+i}} dx_{6n+i} + \frac{\partial}{\partial x_{2n+i}} dx_{7n+i}$$

And given by operator

$$d_{J_5} : \mathcal{F}(M) \rightarrow \Lambda^1 M \quad \rightarrow \quad (10)$$

Then

$$\Phi_L^{J_5} = -\frac{\partial^2 L}{\partial x_j \partial x_{5n+i}} dx_j \wedge dx_i + \frac{\partial^2 L}{\partial x_j \partial x_{3n+i}} dx_j \wedge dx_{n+i} + \frac{\partial^2 L}{\partial x_j \partial x_{7n+i}} dx_j \wedge dx_{2n+i}$$

Notes

Notes

$$- \frac{\partial^2 L}{\partial x_j \partial x_{n+i}} dx_j \wedge dx_{3n+i} - \frac{\partial^2 L}{\partial x_j \partial x_{6n+i}} dx_j \wedge dx_{4n+i} + \frac{\partial^2 L}{\partial x_j \partial x_i} dx_j \wedge dx_{5n+i}$$

$$+ \frac{\partial^2 L}{\partial x_j \partial x_{4n+i}} dx_j \wedge dx_{6n+i} - \frac{\partial^2 L}{\partial x_j \partial x_{2n+i}} dx_j \wedge dx_{7n+i} - \frac{\partial^2 L}{\partial x_{n+j} \partial x_{5n+i}} dx_{n+j} \wedge dx_i$$

$$+ \frac{\partial^2 L}{\partial x_{n+j} \partial x_{3n+i}} dx_{n+j} \wedge dx_{n+i} + \frac{\partial^2 L}{\partial x_{n+j} \partial x_{7n+i}} dx_{n+j} \wedge dx_{2n+i} - \frac{\partial^2 L}{\partial x_{n+j} \partial x_{n+i}} dx_{n+j} \wedge dx_{3n+i}$$

$$- \frac{\partial^2 L}{\partial x_{n+j} \partial x_{6n+i}} dx_{n+j} \wedge dx_{4n+i} + \frac{\partial^2 L}{\partial x_{n+j} \partial x_i} dx_{n+j} \wedge dx_{5n+i} + \frac{\partial^2 L}{\partial x_{n+j} \partial x_{4n+i}} dx_{n+j} \wedge dx_{6n+i}$$

$$- \frac{\partial^2 L}{\partial x_{n+j} \partial x_{2n+i}} dx_{n+j} \wedge dx_{7n+i} - \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{5n+i}} dx_{2n+j} \wedge dx_i + \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{3n+i}} dx_{2n+j} \wedge dx_{n+i} +$$

$$\frac{\partial^2 L}{\partial x_{2n+j} \partial x_{7n+i}} dx_{2n+j} \wedge dx_{2n+i} - \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{n+i}} dx_{2n+j} \wedge dx_{3n+i} - \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{6n+i}} dx_{2n+j} \wedge dx_{4n+i} +$$

$$\frac{\partial^2 L}{\partial x_{2n+j} \partial x_i} dx_{2n+j} \wedge dx_{5n+i} + \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{4n+i}} dx_{2n+j} \wedge dx_{6n+i} - \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{2n+i}} dx_{2n+j} \wedge dx_{7n+i}$$

$$- \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{5n+i}} dx_{3n+j} \wedge dx_i + \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{3n+i}} dx_{3n+j} \wedge dx_{n+i} + \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{7n+i}} dx_{3n+j} \wedge dx_{2n+i}$$

$$- \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{n+i}} dx_{3n+j} \wedge dx_{3n+i} - \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{6n+i}} dx_{3n+j} \wedge dx_{4n+i} + \frac{\partial^2 L}{\partial x_{3n+j} \partial x_i} dx_{3n+j} \wedge dx_{5n+i}$$

$$+ \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{4n+i}} dx_{3n+j} \wedge dx_{6n+i} - \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{2n+i}} dx_{3n+j} \wedge dx_{7n+i} - \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{5n+i}} dx_{4n+j} \wedge dx_i$$

$$+ \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{3n+i}} dx_{4n+j} \wedge dx_{n+i} + \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{7n+i}} dx_{4n+j} \wedge dx_{2n+i} - \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{n+i}} dx_{4n+j} \wedge dx_{3n+i}$$

$$- \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{6n+i}} dx_{4n+j} \wedge dx_{4n+i} + \frac{\partial^2 L}{\partial x_{4n+j} \partial x_i} dx_{4n+j} \wedge dx_{5n+i} + \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{4n+i}} dx_{4n+j} \wedge dx_{6n+i}$$

$$- \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{2n+i}} dx_{4n+j} \wedge dx_{7n+i} - \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{5n+i}} dx_{5n+j} \wedge dx_i + \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{3n+i}} dx_{5n+j} \wedge dx_{n+i}$$

$$+ \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{7n+i}} dx_{5n+j} \wedge dx_{2n+i} - \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{n+i}} dx_{5n+j} \wedge dx_{3n+i} - \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{6n+i}} dx_{5n+j} \wedge dx_{4n+i}$$

$$+ \frac{\partial^2 L}{\partial x_{5n+j} \partial x_i} dx_{5n+j} \wedge dx_{5n+i} + \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{4n+i}} dx_{5n+j} \wedge dx_{6n+i} - \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{2n+i}} dx_{5n+j} \wedge dx_{7n+i} -$$

$$\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{5n+i}} dx_{6n+j}\wedge dx_i + \frac{\partial^2 L}{\partial x_{6n+j}\partial x_{3n+i}} dx_{6n+j}\wedge dx_{n+i} + \frac{\partial^2 L}{\partial x_{6n+j}\partial x_{7n+i}} dx_{6n+j}\wedge dx_{2n+i}$$

$$-\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{n+i}} dx_{6n+j}\wedge dx_{3n+i} - \frac{\partial^2 L}{\partial x_{6n+j}\partial x_{6n+i}} dx_{6n+j}\wedge dx_{4n+i} + \frac{\partial^2 L}{\partial x_{6n+j}\partial x_{i}} dx_{6n+j}\wedge dx_{5n+i}$$

$$+\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{4n+i}} dx_{6n+j}\wedge dx_{6n+i} - \frac{\partial^2 L}{\partial x_{6n+j}\partial x_{2n+i}} dx_{6n+j}\wedge dx_{7n+i} - \frac{\partial^2 L}{\partial x_{7n+j}\partial x_{5n+i}} dx_{7n+j}\wedge dx_{i} +$$

$$\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{3n+i}} dx_{7n+j}\wedge dx_{n+i} + \frac{\partial^2 L}{\partial x_{7n+j}\partial x_{7n+i}} dx_{7n+j}\wedge dx_{2n+i} - \frac{\partial^2 L}{\partial x_{7n+j}\partial x_{n+i}} dx_{7n+j}\wedge dx_{3n+i}$$

$$-\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{6n+i}} dx_{7n+j}\wedge dx_{4n+i} + \frac{\partial^2 L}{\partial x_{7n+j}\partial x_{i}} dx_{7n+j}\wedge dx_{5n+i} + \frac{\partial^2 L}{\partial x_{7n+j}\partial x_{4n+i}} dx_{7n+j}\wedge dx_{6n+i}$$

$$-\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{2n+i}} dx_{7n+j}\wedge dx_{7n+i}.$$

Let ξ be the second order differential equation by determined Eq(1) and given by Eq(5) and

$$i_\xi \Phi_L^{J_5} = -X^i \frac{\partial^2 L}{\partial x_j \partial x_{5n+i}} \delta_i^j\, dx_i + X^i \frac{\partial^2 L}{\partial x_j \partial x_{5n+i}} dx_j + X^i \frac{\partial^2 L}{\partial x_j \partial x_{3n+i}} \delta_i^j\, dx_{n+i} - X^{n+i} \frac{\partial^2 L}{\partial x_j \partial x_{3n+i}} dx_j$$

$$+X^i \frac{\partial^2 L}{\partial x_j \partial x_{7n+i}} \delta_i^j\, dx_{2n+i} - X^{2n+i} \frac{\partial^2 L}{\partial x_j \partial x_{7n+i}} dx_j - X^i \frac{\partial^2 L}{\partial x_j \partial x_{n+i}} \delta_i^j\, dx_{3n+i} + X^{3n+i} \frac{\partial^2 L}{\partial x_j \partial x_{n+i}} dx_j$$

$$-X^i \frac{\partial^2 L}{\partial x_j \partial x_{6n+i}} \delta_i^j\, dx_{4n+i} + X^{4n+i} \frac{\partial^2 L}{\partial x_j \partial x_{6n+i}} dx_j + X^i \frac{\partial^2 L}{\partial x_j \partial x_{i}} \delta_i^j\, dx_{5n+i} - X^{5n+i} \frac{\partial^2 L}{\partial x_j \partial x_{i}} dx_j +$$

$$X^i \frac{\partial^2 L}{\partial x_j \partial x_{4n+i}} \delta_i^j\, dx_{6n+i} - X^{6n+i} \frac{\partial^2 L}{\partial x_j \partial x_{4n+i}} dx_j - X^i \frac{\partial^2 L}{\partial x_j \partial x_{2n+i}} \delta_i^j\, dx_{7n+i} + X^{7n+i} \frac{\partial^2 L}{\partial x_j \partial x_{2n+i}} dx_j$$

$$-X^{n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{5n+i}} \delta_{n+i}^{n+j}\, dx_i + X^i \frac{\partial^2 L}{\partial x_{n+j} \partial x_{5n+i}} dx_{n+j} + X^{n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{3n+i}} \delta_{n+i}^{n+j}\, dx_{n+i}$$

$$-\; X^{n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{3n+i}} dx_{n+j} \;+\; X^{n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{7n+i}} \delta_{n+i}^{n+j}\, dx_{2n+i} - \; X^{2n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{7n+i}} dx_{n+j} -$$

$$X^{n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{n+i}} \delta_{n+i}^{n+j}\, dx_{3n+i} \;+\; X^{3n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{n+i}} dx_{n+j} - X^{n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{6n+i}} \delta_{n+i}^{n+j}\, dx_{4n+i}$$

$$+\, X^{4n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{6n+i}} dx_{n+j} + X^{n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{i}} \delta_{n+i}^{n+j}\, dx_{5n+i} - X^{5n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{i}} dx_{n+j}$$

$$+X^{n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{4n+i}} \delta_{n+i}^{n+j}\, dx_{6n+i} - \; X^{6n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{4n+i}} dx_{n+j} - X^{n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{2n+i}} \delta_{n+i}^{n+j}\, dx_{7n+i}$$

$$X^{7n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{2n+i}} dx_{n+j} - X^{2n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{5n+i}} \delta_{2n+i}^{2n+j}\, dx_i + X^i \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{5n+i}} dx_{2n+j}$$

$$+X^{2n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{3n+i}} \delta_{2n+i}^{2n+j}\, dx_{n+i} - X^{n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{3n+i}} dx_{2n+j} + X^{2n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{7n+i}} \delta_{2n+i}^{2n+j}\, dx_{2n+i} -$$

Notes

$$X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{7n+i}}dx_{2n+j}-X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{n+i}}\delta_{2n+i}^{2n+j}dx_{3n+i}+X^{3n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{n+i}}dx_{2n+j}$$

$$-X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{6n+i}}\delta_{2n+i}^{2n+j}dx_{4n+i}+X^{4n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{6n+i}}dx_{2n+j}+X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{i}}\delta_{2n+i}^{2n+j}dx_{5n+i}$$

$$-X^{5n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{i}}dx_{2n+j}+X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{4n+i}}\delta_{2n+i}^{2n+j}dx_{6n+i}-X^{6n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{4n+i}}dx_{2n+j}$$

$$-X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{2n+i}}\delta_{2n+i}^{2n+j}dx_{7n+i}+X^{7n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{2n+i}}dx_{2n+j}-X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{5n+i}}\delta_{3n+i}^{3n+j}dx_{i}$$

$$+X^{i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{5n+i}}dx_{3n+j}+X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{3n+i}}\delta_{3n+i}^{3n+j}dx_{n+i}-X^{n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{3n+i}}dx_{3n+j}+$$

$$X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{7n+i}}\delta_{3n+i}^{3n+j}dx_{2n+i}-X^{2n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{7n+i}}dx_{3n+j}-X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{n+i}}\delta_{3n+i}^{3n+j}dx_{3n+i}$$

$$+X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{n+i}}dx_{3n+j}-X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{6n+i}}\delta_{3n+i}^{3n+j}dx_{4n+i}+X^{4n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{6n+i}}dx_{3n+j}$$

$$+X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{i}}\delta_{3n+i}^{3n+j}dx_{5n+i}-X^{5n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{i}}dx_{3n+j}+X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{4n+i}}\delta_{3n+i}^{3n+j}dx_{6n+i}$$

$$-X^{6n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{4n+i}}dx_{3n+j}-X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{2n+i}}\delta_{3n+i}^{3n+j}dx_{7n+i}+X^{7n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{2n+i}}dx_{3n+j}-$$

$$X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{5n+i}}\delta_{4n+i}^{4n+j}dx_{i}+X^{i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{5n+i}}dx_{4n+j}+X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{3n+i}}\delta_{4n+i}^{4n+j}dx_{n+i}$$

$$-X^{n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{3n+i}}dx_{4n+j}+X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{7n+i}}\delta_{4n+i}^{4n+j}dx_{2n+i}-X^{2n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{7n+i}}dx_{4n+j}$$

$$-X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{n+i}}\delta_{4n+i}^{4n+j}dx_{3n+i}+X^{3n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{n+i}}dx_{4n+j}-X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{6n+i}}\delta_{4n+i}^{4n+j}dx_{4n+i}$$

$$+X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{6n+i}}dx_{4n+j}+X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{i}}\delta_{4n+i}^{4n+j}dx_{5n+i}-X^{5n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{i}}dx_{4n+j}+$$

$$X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{4n+i}}\delta_{4n+i}^{4n+j}dx_{6n+i}-X^{6n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{4n+i}}dx_{4n+j}-X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{2n+i}}\delta_{4n+i}^{4n+j}dx_{7n+i}$$

$$+X^{7n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{2n+i}}dx_{4n+j}-X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{5n+i}}\delta_{5n+i}^{5n+j}dx_{i}+X^{i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{5n+i}}dx_{5n+j}+$$

$$X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{3n+i}}\delta_{5n+i}^{5n+j}dx_{n+i}-X^{n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{3n+i}}dx_{5n+j}+X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{7n+i}}\delta_{5n+i}^{5n+j}dx_{2n+i}$$

$$-X^{2n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{7n+i}}dx_{5n+j}-X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{n+i}}\delta_{5n+i}^{5n+j}dx_{3n+i}+X^{3n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{n+i}}dx_{5n+j}$$

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$$-X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{6n+i}}\delta_{5n+i}^{5n+j}dx_{4n+i}+X^{4n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{6n+i}}dx_{5n+j}+X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{i}}\delta_{5n+i}^{5n+j}dx_{5n+i}$$

$$-X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{i}}dx_{5n+j}+X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{4n+i}}\delta_{5n+i}^{5n+j}dx_{6n+i}-X^{6n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{4n+i}}dx_{5n+j}$$

$$-X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{2n+i}}\delta_{5n+i}^{5n+j}dx_{7n+i}+X^{7n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{2n+i}}dx_{5n+j}-X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{5n+i}}\delta_{6n+i}^{6n+j}dx_{i}$$

$$+X^{i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{5n+i}}dx_{6n+j}+X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{3n+i}}\delta_{6n+i}^{6n+j}dx_{n+i}-X^{n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{3n+i}}dx_{6n+j}$$

$$+X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{7n+i}}\delta_{6n+i}^{6n+j}dx_{2n+i}-X^{2n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{7n+i}}dx_{6n+j}-X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{n+i}}\delta_{6n+i}^{6n+j}dx_{3n+i}$$

$$+X^{3n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{n+i}}dx_{6n+j}-X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{6n+i}}\delta_{6n+i}^{6n+j}dx_{4n+i}+X^{4n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{6n+i}}dx_{6n+j}$$

$$+X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{i}}\delta_{6n+i}^{6n+j}dx_{5n+i}-X^{5n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{i}}dx_{6n+j}+X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{4n+i}}\delta_{6n+i}^{6n+j}dx_{6n+i}$$

$$-X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{4n+i}}dx_{6n+j}-X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{2n+i}}\delta_{6n+i}^{6n+j}dx_{7n+i}+X^{7n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{2n+i}}dx_{6n+j}$$

$$-X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{5n+i}}\delta_{7n+i}^{7n+j}dx_{i}+X^{i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{5n+i}}dx_{7n+j}+X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{3n+i}}\delta_{7n+i}^{7n+j}dx_{n+i}$$

$$-X^{n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{3n+i}}dx_{7n+j}+X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{7n+i}}\delta_{7n+i}^{7n+j}dx_{2n+i}-X^{2n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{7n+i}}dx_{7n+j}$$

$$-X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{n+i}}\delta_{7n+i}^{7n+j}dx_{3n+i}+X^{3n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{n+i}}dx_{7n+j}-X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{6n+i}}\delta_{7n+i}^{7n+j}dx_{4n+i}$$

$$+X^{4n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{6n+i}}dx_{7n+j}+X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{i}}\delta_{7n+i}^{7n+j}dx_{5n+i}-X^{5n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{i}}dx_{7n+j}$$

$$+X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{4n+i}}\delta_{7n+i}^{7n+j}dx_{6n+i}-X^{6n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{4n+i}}dx_{7n+j}-X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{2n+i}}\delta_{7n+i}^{7n+j}dx_{7n+i}$$

$$+X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{2n+i}}dx_{7n+j}$$

Since the closed Clifford *Kähler* form $\Phi_L^{J_5}$ *on M* is the symplectic structure.

$$E_L^{J_5}=V_{J_5}(L)-L=X^{i}\frac{\partial L}{\partial x_{5n+i}}-X^{n+i}\frac{\partial L}{\partial x_{3n+i}}-X^{2n+i}\frac{\partial L}{\partial x_{7n+i}}+X^{3n+i}\frac{\partial L}{\partial x_{n+i}}+$$

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$$X^{4n+i}\frac{\partial L}{\partial x_{6n+i}}-X^{5n+i}\frac{\partial L}{\partial x_i}-X^{6n+i}\frac{\partial L}{\partial x_{4n+i}}+X^{7n+i}\frac{\partial L}{\partial x_{2n+i}}-L \quad \to (11)$$

And thus

$$dE_L^{J_5}=X^i\frac{\partial^2 L}{\partial x_j\partial x_{5n+i}}dx_j-X^{n+i}\frac{\partial^2 L}{\partial x_j\partial x_{3n+i}}dx_j-X^{2n+i}\frac{\partial^2 L}{\partial x_j\partial x_{7n+i}}dx_j+X^{3n+i}\frac{\partial^2 L}{\partial x_j\partial x_{n+i}}dx_j$$

$$+X^{4n+i}\frac{\partial^2 L}{\partial x_j\partial x_{6n+i}}dx_j-X^{5n+i}\frac{\partial^2 L}{\partial x_j\partial x_i}dx_j-X^{6n+i}\frac{\partial^2 L}{\partial x_j\partial x_{4n+i}}dx_j+X^{7n+i}\frac{\partial^2 L}{\partial x_j\partial x_{2n+i}}dx_j$$

$$+X^i\frac{\partial^2 L}{\partial x_{n+j}\partial x_{5n+i}}dx_{n+j}-X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{3n+i}}dx_{n+j}-X^{2n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{7n+i}}dx_{n+j}+$$

$$X^{3n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{n+i}}dx_{n+j}+X^{4n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{6n+i}}dx_{n+j}-X^{5n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_i}dx_{n+j}-$$

$$X^{6n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{4n+i}}dx_{n+j}+X^{7n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{2n+i}}dx_{n+j}+X^i\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{5n+i}}dx_{2n+j}$$

$$-X^{n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{3n+i}}dx_{2n+j}-X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{7n+i}}dx_{2n+j}+X^{3n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{n+i}}dx_{2n+j}$$

$$+X^{4n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{6n+i}}dx_{2n+j}-X^{5n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_i}dx_{2n+j}-X^{6n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{4n+i}}dx_{2n+j}$$

$$+X^{7n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{2n+i}}dx_{2n+j}+X^i\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{5n+i}}dx_{3n+j}-X^{n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{3n+i}}dx_{3n+j}$$

$$-X^{2n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{7n+i}}dx_{3n+j}+X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{n+i}}dx_{3n+j}+X^{4n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{6n+i}}dx_{3n+j}$$

$$-X^{5n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_i}dx_{3n+j}-X^{6n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{4n+i}}dx_{3n+j}+X^{7n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{2n+i}}dx_{3n+j}$$

$$+X^i\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{5n+i}}dx_{4n+j}-X^{n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{3n+i}}dx_{4n+j}-X^{2n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{7n+i}}dx_{4n+j}$$

$$+X^{3n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{n+i}}dx_{4n+j}+X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{6n+i}}dx_{4n+j}-X^{5n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_i}dx_{4n+j}$$

$$-X^{6n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{4n+i}}dx_{4n+j}+X^{7n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{2n+i}}dx_{4n+j}+X^i\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{5n+i}}dx_{5n+j}$$

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$$-X^{n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{3n+i}}dx_{5n+j} - X^{2n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{7n+i}}dx_{5n+j} + X^{3n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{n+i}}dx_{5n+j}$$

$$+X^{4n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{6n+i}}dx_{5n+j} - X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{i}}dx_{5n+j} - X^{6n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{4n+i}}dx_{5n+j}$$

$$+X^{7n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{2n+i}}dx_{5n+j} + X^{i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{5n+i}}dx_{6n+j} - X^{n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{3n+i}}dx_{6n+j}$$

$$-X^{2n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{7n+i}}dx_{6n+j} + X^{3n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{n+i}}dx_{6n+j} + X^{4n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{6n+i}}dx_{6n+j}$$

$$-X^{5n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{i}}dx_{6n+j} - X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{4n+i}}dx_{6n+j} + X^{7n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{2n+i}}dx_{6n+j}$$

$$+X^{i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{5n+i}}dx_{7n+j} - X^{n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{3n+i}}dx_{7n+j} - X^{2n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{7n+i}}dx_{7n+j}$$

$$+X^{3n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{n+i}}dx_{7n+j} + X^{4n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{6n+i}}dx_{7n+j} - X^{5n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{i}}dx_{7n+j}$$

$$-X^{6n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{4n+i}}dx_{7n+j} + X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{2n+i}}dx_{7n+j} - \frac{\partial L}{\partial x_j}dx_j - \frac{\partial L}{\partial x_{n+j}}dx_{n+j}$$

$$-\frac{\partial L}{\partial x_{2n+j}}dx_{2n+j} - \frac{\partial L}{\partial x_{3n+j}}dx_{3n+j} - \frac{\partial L}{\partial x_{4n+j}}dx_{4n+j} - \frac{\partial L}{\partial x_{5n+j}}dx_{5n+j} - \frac{\partial L}{\partial x_{6n+j}}dx_{6n+j}$$

$$-\frac{\partial L}{\partial x_{7n+j}}dx_{7n+j}$$

By means of Eq(1), we calculate the following expressions

$$-X^{i}\frac{\partial^2 L}{\partial x_j\partial x_{5n+i}}\delta_i^j\,dx_i + X^{i}\frac{\partial^2 L}{\partial x_j\partial x_{3n+i}}\delta_i^j\,dx_{n+i} + X^{i}\frac{\partial^2 L}{\partial x_j\partial x_{7n+i}}\delta_i^j\,dx_{2n+i} - X^{i}\frac{\partial^2 L}{\partial x_j\partial x_{n+i}}\delta_i^j\,dx_{3n+i}$$

$$-X^{i}\frac{\partial^2 L}{\partial x_j\partial x_{6n+i}}\delta_i^j\,dx_{4n+i} + X^{i}\frac{\partial^2 L}{\partial x_j\partial x_{i}}\delta_i^j\,dx_{5n+i} + X^{i}\frac{\partial^2 L}{\partial x_j\partial x_{4n+i}}\delta_i^j\,dx_{6n+i} - X^{i}\frac{\partial^2 L}{\partial x_j\partial x_{2n+i}}\delta_i^j\,dx_{7n+i}$$

$$-X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{5n+i}}\delta_{n+i}^{n+j}\,dx_i + X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{3n+i}}\delta_{n+i}^{n+j}\,dx_{n+i} + X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{7n+i}}\delta_{n+i}^{n+j}\,dx_{2n+i}$$

$$-X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{n+i}}\delta_{n+i}^{n+j}\,dx_{3n+i} - X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{6n+i}}\delta_{n+i}^{n+j}\,dx_{4n+i} + X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{i}}\delta_{n+i}^{n+j}\,dx_{5n+i} +$$

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$$X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{4n+i}}\delta_{n+i}^{n+j}dx_{6n+i}-X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{2n+i}}\delta_{n+i}^{n+j}dx_{7n+i}-X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{5n+i}}\delta_{2n+i}^{2n+j}dx_{i}$$

$$+X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{3n+i}}\delta_{2n+i}^{2n+j}dx_{n+i}+X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{7n+i}}\delta_{2n+i}^{2n+j}dx_{2n+i}-$$

$$X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{n+i}}\delta_{2n+i}^{2n+j}dx_{3n+i}-X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{6n+i}}\delta_{2n+i}^{2n+j}dx_{4n+i}+$$

$$X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{i}}\delta_{2n+i}^{2n+j}dx_{5n+i}+X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{4n+i}}\delta_{2n+i}^{2n+j}dx_{6n+i}-$$

$$X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{2n+i}}\delta_{2n+i}^{2n+j}dx_{7n+i}-X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{5n+i}}\delta_{3n+i}^{3n+j}dx_{i}+X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{3n+i}}\delta_{3n+i}^{3n+j}dx_{n+i}$$

$$+X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{7n+i}}\delta_{3n+i}^{3n+j}dx_{2n+i}-X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{n+i}}\delta_{3n+i}^{3n+j}dx_{3n+i}-$$

$$X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{6n+i}}\delta_{3n+i}^{3n+j}dx_{4n+i}+X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{i}}\delta_{3n+i}^{3n+j}dx_{5n+i}+$$

$$X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{4n+i}}\delta_{3n+i}^{3n+j}dx_{6n+i}-X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{2n+i}}\delta_{3n+i}^{3n+j}dx_{7n+i}$$

$$-X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{5n+i}}\delta_{4n+i}^{4n+j}dx_{i}+X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{3n+i}}\delta_{4n+i}^{4n+j}dx_{n+i}$$

$$X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{7n+i}}\delta_{4n+i}^{4n+j}dx_{2n+i}-X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{n+i}}\delta_{4n+i}^{4n+j}dx_{3n+i}-$$

$$X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{6n+i}}\delta_{4n+i}^{4n+j}dx_{4n+i}+X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{i}}\delta_{4n+i}^{4n+j}dx_{5n+i}+$$

$$X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{4n+i}}\delta_{4n+i}^{4n+j}dx_{6n+i}-X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{2n+i}}\delta_{4n+i}^{4n+j}dx_{7n+i}$$

$$-X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{5n+i}}\delta_{5n+i}^{5n+j}dx_{i}+X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{3n+i}}\delta_{5n+i}^{5n+j}dx_{n+i}+$$

$$X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{7n+i}}\delta_{5n+i}^{5n+j}dx_{2n+i}-X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{n+i}}\delta_{5n+i}^{5n+j}dx_{3n+i}-$$

$$X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{6n+i}}\delta_{5n+i}^{5n+j}dx_{4n+i} + X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_i}\delta_{5n+i}^{5n+j}dx_{5n+i} +$$

$$X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{4n+i}}\delta_{5n+i}^{5n+j}dx_{6n+i} - X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{2n+i}}\delta_{5n+i}^{5n+j}dx_{7n+i}$$

$$- X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{5n+i}}\delta_{6n+i}^{6n+j}dx_i + X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{3n+i}}\delta_{6n+i}^{6n+j}dx_{n+i} +$$

$$X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{7n+i}}\delta_{6n+i}^{6n+j}dx_{2n+i} - X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{n+i}}\delta_{6n+i}^{6n+j}dx_{3n+i} -$$

$$X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{6n+i}}\delta_{6n+i}^{6n+j}dx_{4n+i} + X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_i}\delta_{6n+i}^{6n+j}dx_{5n+i} +$$

$$X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{4n+i}}\delta_{6n+i}^{6n+j}dx_{6n+i} - X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{2n+i}}\delta_{6n+i}^{6n+j}dx_{7n+i}$$

$$- X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{5n+i}}\delta_{7n+i}^{7n+j}dx_i + X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{3n+i}}\delta_{7n+i}^{7n+j}dx_{n+i} +$$

$$X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{7n+i}}\delta_{7n+i}^{7n+j}dx_{2n+i} - X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{n+i}}\delta_{7n+i}^{7n+j}dx_{3n+i} -$$

$$X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{6n+i}}\delta_{7n+i}^{7n+j}dx_{4n+i} + X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_i}\delta_{7n+i}^{7n+j}dx_{5n+i} +$$

$$X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{4n+i}}\delta_{7n+i}^{7n+j}dx_{6n+i} - X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{2n+i}}\delta_{7n+i}^{7n+j}dx_{7n+i} +$$

$$\frac{\partial L}{\partial x_j}dx_j + \frac{\partial L}{\partial x_{n+j}}dx_{n+j} + \frac{\partial L}{\partial x_{2n+j}}dx_{2n+j} + \frac{\partial L}{\partial x_{3n+j}}dx_{3n+j} + \frac{\partial L}{\partial x_{4n+j}}dx_{4n+j} +$$

$$\frac{\partial L}{\partial x_{5n+j}}dx_{5n+j} + \frac{\partial L}{\partial x_{6n+j}}dx_{6n+j} + \frac{\partial L}{\partial x_{7n+j}}dx_{7n+j} = 0$$

If a curve determined by $\alpha: R \to M$ is taken to be an integral curve of ξ, then we found equation as follows:

$$-X^{i}\frac{\partial^2 L}{\partial x_j\partial x_{5n+i}}dx_j - X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{5n+i}}dx_j - X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{5n+i}}dx_j - X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{5n+i}}dx_j$$

$$-X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{5n+i}}dx_j - X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{5n+i}}dx_j - X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{5n+i}}dx_j -$$

Notes

$$X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{5n+i}}dx_j+X^{i}\frac{\partial^2 L}{\partial x_j\partial x_{3n+i}}dx_{n+j}+X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{3n+i}}dx_{n+j}+X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{3n+i}}dx_{n+j}$$

$$+X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{3n+i}}dx_{n+j}+X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{3n+i}}dx_{n+j}+X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{3n+i}}dx_{n+j}+$$

Notes

$$X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{3n+i}}dx_{n+j}+X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{3n+i}}dx_{n+j}+X^{i}\frac{\partial^2 L}{\partial x_j\partial x_{7n+i}}dx_{2n+j}+$$

$$X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{7n+i}}dx_{2n+j}+X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{7n+i}}dx_{2n+j}+X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{7n+i}}dx_{2n+j}+$$

$$X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{7n+i}}dx_{2n+j}+X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{7n+i}}dx_{2n+j}+X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{7n+i}}dx_{2n+j}+$$

$$X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{7n+i}}dx_{2n+j}+-X^{i}\frac{\partial^2 L}{\partial x_j\partial x_{n+i}}dx_{3n+j}-X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{n+i}}dx_{3n+j}-$$

$$X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{n+i}}dx_{3n+j}-X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{n+i}}dx_{3n+j}-X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{n+i}}dx_{3n+j}-$$

$$X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{n+i}}dx_{3n+j}-X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{n+i}}dx_{3n+j}-X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{n+i}}dx_{3n+j}-$$

$$X^{i}\frac{\partial^2 L}{\partial x_j\partial x_{6n+i}}dx_{4n+j}-X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{6n+i}}dx_{4n+j}-X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{6n+i}}dx_{4n+j}-$$

$$X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{6n+i}}dx_{4n+j}-X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{6n+i}}dx_{4n+j}-X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{6n+i}}dx_{4n+j}-$$

$$X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{6n+i}}dx_{4n+j}-X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{6n+i}}dx_{4n+j}+X^{i}\frac{\partial^2 L}{\partial x_j\partial x_i}dx_{5n+j}+X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_i}dx_{5n+j}$$

$$+X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_i}dx_{5n+j}+X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_i}dx_{5n+j}+X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_i}dx_{5n+j}+$$

$$X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_i}dx_{5n+j}+X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_i}dx_{5n+j}+X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_i}dx_{5n+j}+$$

$$X^{i}\frac{\partial^2 L}{\partial x_j\partial x_{4n+i}}dx_{6n+j}+X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{4n+i}}dx_{6n+j}+X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{4n+i}}dx_{6n+j}+$$

$$X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{4n+i}}dx_{6n+j}+X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{4n+i}}dx_{6n+j}+X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{4n+i}}dx_{6n+j}+$$

$$X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{4n+i}}dx_{6n+j}+X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{4n+i}}dx_{6n+j}-X^{i}\frac{\partial^2 L}{\partial x_{j}\partial x_{2n+i}}dx_{7n+j}-$$

$$X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{2n+i}}dx_{7n+j}-X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{2n+i}}dx_{7n+j}-X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{2n+i}}dx_{7n+j}-$$

$$X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{2n+i}}dx_{7n+j}-X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{2n+i}}dx_{7n+j}-X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{2n+i}}dx_{7n+j}-$$

$$X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{2n+i}}dx_{7n+j}+\frac{\partial L}{\partial x_j}dx_j+\frac{\partial L}{\partial x_{n+j}}dx_{n+j}+\frac{\partial L}{\partial x_{2n+j}}dx_{2n+j}+\frac{\partial L}{\partial x_{3n+j}}dx_{3n+j}+$$

$$\frac{\partial L}{\partial x_{4n+j}}dx_{4n+j}+\frac{\partial L}{\partial x_{5n+j}}dx_{5n+j}+\frac{\partial L}{\partial x_{6n+j}}dx_{6n+j}+\frac{\partial L}{\partial x_{7n+j}}dx_{7n+j}=0 \quad \rightarrow \quad (12)$$

Or

$$-[X^{i}\frac{\partial^2 L}{\partial x_{j}\partial x_{5n+i}}+X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{5n+i}}X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{5n+i}}+X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{5n+i}}+X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{5n+i}}$$

$$+X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{5n+i}}+X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{5n+i}}+X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{5n+i}}]\,dx_j+\frac{\partial L}{\partial x_j}dx_j+$$

$$[X^{i}\frac{\partial^2 L}{\partial x_{j}\partial x_{3n+i}}+X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{3n+i}}+X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{3n+i}}+X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{3n+i}}+X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{3n+i}}$$

$$+X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{3n+i}}+X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{3n+i}}+X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{3n+i}}]\,dx_{n+j}+\frac{\partial L}{\partial x_{n+j}}dx_{n+j}+$$

$$[X^{i}\frac{\partial^2 L}{\partial x_{j}\partial x_{7n+i}}+X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{7n+i}}+X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{7n+i}}+X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{7n+i}}+X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{7n+i}}$$

$$+X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{7n+i}}+X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{7n+i}}+X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{7n+i}}]\,dx_{2n+j}+\frac{\partial L}{\partial x_{2n+j}}dx_{2n+j}$$

$$-[X^{i}\frac{\partial^2 L}{\partial x_{j}\partial x_{n+i}}+X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{n+i}}+X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{n+i}}+X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{n+i}}+X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{n+i}}$$

$$+X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{n+i}}+X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{n+i}}+X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{n+i}}]\,dx_{3n+j}+\frac{\partial L}{\partial x_{3n+j}}dx_{3n+j}$$

$$-[X^{i}\frac{\partial^2 L}{\partial x_{j}\partial x_{6n+i}}+X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{6n+i}}+X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{6n+i}}+X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{6n+i}}+X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{6n+i}}$$

$$+X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{6n+i}}+X^{6n+i}\frac{\partial^2 L}{\partial x_{j}\partial x_{6n+i}}+X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{6n+i}}]\,dx_{4n+j}+\frac{\partial L}{\partial x_{4n+j}}dx_{4n+j}$$

Notes

$$+[X^i\frac{\partial^2L}{\partial x_j\partial x_i}+X^{n+i}\frac{\partial^2L}{\partial x_{n+j}\partial x_i}+X^{2n+i}\frac{\partial^2L}{\partial x_{2n+j}\partial x_i}+X^{3n+i}\frac{\partial^2L}{\partial x_{3n+j}\partial x_i}+X^{4n+i}\frac{\partial^2L}{\partial x_{4n+j}\partial x_i}$$

$$+X^{5n+i}\frac{\partial^2L}{\partial x_{5n+j}\partial x_i}+X^{6n+i}\frac{\partial^2L}{\partial x_{6n+j}\partial x_i}+X^{7n+i}\frac{\partial^2L}{\partial x_{7n+j}\partial x_i}]dx_{5n+j}+\frac{\partial L}{\partial x_{5n+j}}dx_{5n+j}$$

Notes

$$+[X^i\frac{\partial^2L}{\partial x_j\partial x_{4n+i}}+X^{n+i}\frac{\partial^2L}{\partial x_{n+j}\partial x_{4n+i}}+X^{2n+i}\frac{\partial^2L}{\partial x_{2n+j}\partial x_{4n+i}}+X^{3n+i}\frac{\partial^2L}{\partial x_{3n+j}\partial x_{4n+i}}+X^{4n+i}\frac{\partial^2L}{\partial x_{4n+j}\partial x_{4n+i}}$$

$$+X^{5n+i}\frac{\partial^2L}{\partial x_{5n+j}\partial x_{4n+i}}+X^{6n+i}\frac{\partial^2L}{\partial x_{6n+j}\partial x_{4n+i}}+X^{7n+i}\frac{\partial^2L}{\partial x_{7n+j}\partial x_{4n+i}}]\,dx_{6n+j}+\frac{\partial L}{\partial x_{6n+j}}dx_{6n+j}$$

$$-[X^i\frac{\partial^2L}{\partial x_j\partial x_{2n+i}}+X^{n+i}\frac{\partial^2L}{\partial x_{n+j}\partial x_{2n+i}}+X^{2n+i}\frac{\partial^2L}{\partial x_{2n+j}\partial x_{2n+i}}+X^{3n+i}\frac{\partial^2L}{\partial x_{3n+j}\partial x_{2n+i}}+X^{4n+i}\frac{\partial^2L}{\partial x_{4n+j}\partial x_{2n+i}}$$

$$+X^{5n+i}\frac{\partial^2L}{\partial x_{5n+j}\partial x_{2n+i}}+X^{6n+i}\frac{\partial^2L}{\partial x_{6n+j}\partial x_{2n+i}}+X^{7n+i}\frac{\partial^2L}{\partial x_{7n+j}\partial x_{2n+i}}]\,dx_{7n+j}+\frac{\partial L}{\partial x_{7n+j}}dx_{7n+j}=0$$

In this equation can be concise manner

$$-\sum_{a=0}^{7}X^{an+i}\frac{\partial^2L}{\partial x_{an+j}\partial x_{5n+i}}dx_j+\frac{\partial L}{\partial x_j}dx_j+\sum_{a=0}^{7}X^{an+i}\frac{\partial^2L}{\partial x_{an+j}\partial x_{3n+i}}dx_{n+j}+\frac{\partial L}{\partial x_{n+j}}dx_{n+j}$$

$$+\sum_{a=0}^{7}X^{an+i}\frac{\partial^2L}{\partial x_{an+j}\partial x_{7n+i}}dx_{2n+j}+\frac{\partial L}{\partial x_{2n+j}}dx_{2n+j}-\sum_{a=0}^{7}X^{an+i}\frac{\partial^2L}{\partial x_{an+j}\partial x_{n+i}}dx_{3n+j}+$$

$$\frac{\partial L}{\partial x_{3n+j}}dx_{3n+j}-\sum_{a=0}^{7}X^{an+i}\frac{\partial^2L}{\partial x_{an+j}\partial x_{6n+i}}dx_{4n+j}+\frac{\partial L}{\partial x_{4n+j}}dx_{4n+j}+\sum_{a=0}^{7}X^{an+i}\frac{\partial^2L}{\partial x_{an+j}\partial x_i}dx_{5n+j}$$

$$+\frac{\partial L}{\partial x_{5n+j}}dx_{5n+j}+\sum_{a=0}^{7}X^{an+i}\frac{\partial^2L}{\partial x_{an+j}\partial x_{4n+i}}dx_{6n+j}+\frac{\partial L}{\partial x_{6n+j}}dx_{6n+j}-$$

$$\sum_{a=0}^{7}X^{an+i}\frac{\partial^2L}{\partial x_{an+j}\partial x_{2n+i}}dx_{7n+j}+\frac{\partial L}{\partial x_{7n+j}}dx_{7n+j}=0 \qquad \rightarrow \qquad (13)$$

Then we find the equations

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_i}\right)+\frac{\partial L}{\partial x_{5n+i}}=0\,,\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{n+i}}\right)-\frac{\partial L}{\partial x_{3n+i}}=0\,,\quad \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{2n+i}}\right)-\frac{\partial L}{\partial x_{7n+i}}=0\,,$$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{3n+i}}\right)+\frac{\partial L}{\partial x_{n+i}}=0\,,\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{4n+i}}\right)+\frac{\partial L}{\partial x_{6n+i}}=0\,,\quad \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{5n+i}}\right)-\frac{\partial L}{\partial x_i}=0\,,$$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{6n+i}}\right)-\frac{\partial L}{\partial x_{4n+i}}=0\,,\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{7n+i}}\right)+\frac{\partial L}{\partial x_{2n+i}}=0 \qquad \rightarrow \qquad (14)$$

Such that the equations expressed in Eq(14) are named Euler-Lagrange equations structured on Clifford *Kähler* manifold (M,V) by means of $\Phi_L^{J_5}$ in the case, the triple $(M,\Phi_L^{J_5},\xi)$ is called a mechanical system on Clifford *Kähler* manifold (M,V).

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Notes

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Study on Prey-Predator Model with Predator in Disease and Harvesting

By Ahmed Buseri Ashine
Madda Walabu University

Abstract- In this article, predator prey interactions where the predator is exposed to the risk of disease and harvesting is proposed. Equilibrium points, boundedness, and non-periodic solutions of the model are obtained. Local stability and global stability were discussed. The equilibrium was stable locally, but not globally.

Keywords: prey-predator, stability, harvesting, dulac's criterion.

GJSFR-F Classification: MSC 2010: 92B05

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Strictly as per the compliance and regulations of :

Ref

2. Chevé, M., R. Congar, et al. (2010). Resilience and stability of harvested predator-prey systems to infectious diseases in the predator.

Study on Prey-Predator Model with Predator in Disease and Harvesting

Ahmed Buseri Ashine

Abstract- In this article, predator prey interactions where the predator is exposed to the risk of disease and harvesting is proposed. Equilibrium points, boundedness, and non-periodic solutions of the model are obtained. Local stability and global stability were discussed. The equilibrium was stable locally, but not globally.

Keywords: prey-predator, stability, harvesting, dulac's criterion.

AMS (MOS) subject classification: 34, 37, and 65.

I. Introduction

Prey-predator models are of great interest to researchers in mathematics and ecology because they deal with environmental problems such as community's morbidity and how to control it, optimal harvest policy to sustain a community, and others. In the physical sciences, generic models can be constructed to explain a variety of phenomena. However, in the life sciences a model only describes a particular situation. Simple models such as the Lotka-Volterra are not able to tell us what is going on in the majority of cases. One of the reasons is due to the complexity of the biological ecosystem. Hence, we still seek for a variety of models to describe nature.

Theoretical and numerical studies of these models are able to give us an understanding of the interactions that is taking place. A particular class of models considers the existence of a disease in the predator or prey. Several models were constructed to study particular cases. To ensure the existence of the species involved, one of the steps taken is to harvest the infected species. In this paper, we consider the case where the infected predator is harvested. Several related theoretical studies have been conducted.

Amongst them are studies on the disease spread among the prey and the epidemic among predators with action incidence [6], the role of transmissible disease in the Holling Tanner predator prey model [4], the analysis of prey predator model with disease in the prey [7], another's study the disease in Lotka Volterra, [3]study the dynamics of a fisher resource system in an aquatic environment in two zones harvest in reserve area,[5] study the harvesting of infected prey,[1]show the stability analysis of harvesting,[2] Study the stability of harvested when the disease affects the predator by using the reproduction number.

The model is introduced after this section, followed by analysis on boundedness and properties of the solutions.

Author: Department of Mathematics, Madda Walabu University, Bale Robe, Ethiopia, East Africa. e-mail: amebuseri@gmail.com

II. The Mathematical Model

Consider the following dynamical system:

$$\begin{cases} \dfrac{dx}{dt} = a(1-x)x - b(y+z)x \\ \dfrac{dy}{dt} = cxy + \alpha yz - h_1 y \\ \dfrac{dz}{dt} = cxz - \alpha yz - h_2 z \end{cases} \quad (1)$$

Notes

where *x, y, z* are the prey, infected predator and susceptible predator respectively; a is the growth rate of prey; b, c the capture rate $(b > c)$, α is the contact rate between the susceptible and infected predator; h_1, and h_2 are the harvest rates of the infected and susceptible predator respectively, and assume that the less effective predator shall be easier to harvest $h_1 > h_2$, so; it is better also to assume infected predator not become susceptible again and finally the disease does not affect the ability of the infected predator attacking prey.

a) *Bounded of solutions*

Theorem 1. The solution of system (1) is bounded.

Proof

Let the function $w(x,y,z) = x(t) + y(t) + z(t)$ and let μ be a positive number such that $0 < \eta < h_2$.

Then, $w'(t) + uw = ax(1-x) + \eta x - (b-c)(y+z)x - (h_1 - \eta)y - (h_2 - \eta)z$

$$w'(t) + uw < -a\left(x^2 - \left(\frac{a+\eta}{a}\right)x + \left(\frac{a+\eta}{2a}\right)^2\right) + \frac{1}{a}\left(\frac{a+\eta}{2}\right)^2$$

$$\text{Let } \frac{1}{a}\left(\frac{a+\eta}{2}\right)^2 = v$$

$$w'(t) + uw(t) \le v$$

$$0 < w(x,y,z) \le \frac{v}{u}\left(1 - e^{-ut}\right) + e^{-ut}(x,y,z)\big|_{t\to 0}$$

Theorem 2. The system (1) has no periodic solution.

Let $F(x,y) = ax - ax^2 - bxy$, $G(x,y) = cxy - h_1 y$,

$M(x,z) = ax - ax^2 - bxz$, $N(x,z) = cxz - h_2 z$.

Define afunction $H(x,y) = \dfrac{1}{xz}$.

Then $Q(x,y) = \dfrac{\partial(HF)}{\partial x} + \dfrac{\partial(HG)}{\partial y} = -\dfrac{a}{y}$

It's clear that is no change in change sign; therefore, this system cannot have any periodic solution in the xy-plane.

Again, $Q(x,z) = \dfrac{\partial(HM)}{\partial x} + \dfrac{\partial(HN)}{\partial z} = -\dfrac{a}{z}$

There is no change in sign; therefore, there is no periodic solution in xz- plane. Hence, the system has no periodic solution.

b) Equilibrium

The dynamical system (1) has the following five fixed points: the origin(E_1), a predator fee fixed point (E_2), a disease free fixed point (E_3), a fixed point when all predator infected (E_4), and a fixed for which both population survive(E_5):

Notes

$$E_1 : (x, y, z) = (0,0,0)$$

$$E_2 : (x, y, z) = (1,0,0)$$

$$E_3 : (x, y, z) = (x_2, 0, z_2)\text{; where } x_2 = \frac{h_2}{c}, z_2 = \frac{a(1-x)}{b}$$

$$E_4 : (x, y, z) = (x_3, y_3, 0)\text{; where } x_3 = \frac{h_1}{c}, y_3 = \frac{a(1-x)}{b}$$

$$E_5 : (x^*, y^*, z^*) = \left(1 - \frac{b}{a\alpha}(h_1 - h_2), \frac{cx^* - h_2}{\alpha}, \frac{h_1 - bx^*}{\alpha}\right)$$

III. Stability

The Jacobian matrix of system (1) is given by:

$$J(x, y, z) = \begin{pmatrix} a - 2ax - b(y+z) & -bx & -bx \\ cy & cx + \alpha z - h_1 & \alpha y \\ cz & -\alpha z & cx - \alpha y - h_2 \end{pmatrix}$$

Case 1. The system without Disease

When infected predators eradicate, the system (1) becomes:

$$\begin{cases} \dfrac{dx}{dt} = a(1-x)x - bxz \\ \dfrac{dz}{dt} = cxz - h_2 z \end{cases} \tag{1a}$$

The equilibrium (nontrivial) are $E_c'(1,0)$, $E_c'(x_2, z_2)$ where, $x_2 = \frac{h_2}{c}, z_2 = \frac{a}{b}(1 - x_2)$

Proposition 1. $E_c'(1,0)$ is stable when $h_2 > c$ and unstable otherwise.

Proof: The eigenvalues near the first equilibrium are $-a$ and $c - h_2$. This completes the proof.

Theorem 3. If the equilibrium $E_c'(x_2, z_2)$ is locally stable, then the basin of attraction of this equilibrium is denoted by $B(E_c'(x_2, z_2))$,

where $B(E_2') = \{(x, z) \in \Re_+^{\,2} : x > \frac{h_2}{c}, z > \frac{a}{b}(1-x) with \frac{h_2}{c} z < \frac{a}{b}(1-x)x\}$

Proof: Let $V(x, z)$ be a function where

$V(x,z) = \left(x - x_2 - x_2 \log^{\frac{x}{x_2}} \right) + \left(z - z_2 - z_2 \log^{\frac{z}{z_2}} \right)$, the

$$\frac{dV}{dt} = -a(x-x_2)^2 - (b-c)(x-x_2)(z-z_2) < 0$$

Remark: The eigenvalues near $E_2'(x_2, z_2)$ are $\frac{-ax_2}{2} \pm \frac{\sqrt{a^2 x_2^2 - 4ah_2(1-x_2)}}{2}$ and $h_2 + \alpha z_2 - h_1$, and stable when $1 - \frac{a\alpha(1-x_2)}{b(h_1 - h_2)} > 0$

Case 2. When all predators become infected

When all predators become infected the subsystem of system (1) becomes:

$$\begin{cases} \frac{dx}{dt} = a(1-x)x - bxy \\ \frac{dy}{dt} = cxy - h_1 y \end{cases} \tag{1b}$$

The equilibrium (nontrivial) are $E_c'(1,0)$, $E_c'(x_3, y_3)$ where, $x_3 = \frac{h_1}{c}$, $y_3 = \frac{a}{b}(1-x_3)$

Proposition 2. $E_c'(1,0)$ is stable when $h_1 > c$ and unstable otherwise.

Proof: The eigenvalues near $E_c'(1,0)$ are $-a$ and $c - h_1$. This completes the proof.

Theorem 4. Assume the equilibrium $E_c'(x_3, y_3)$ is locally stable, the basin of attraction of this equilibrium is denoted by $B(E_c'(x_3, y_3))$ where $B(E_c') = \{(x,z) \in \Re_+^{\ 2} : x > x_3, y > y_3\}$

Proof. The proof is the same as in theorem (3).

Proposition 3. The equilibrium $E_c'(x_3, y_3, 0)$ is stable with condition $0 > 1 - \frac{a\alpha(1-x_3)}{b(h_1 - h_2)}$

Proposition 4. The stability near the equilibrium $E_c^*(x^*, y^*, z^*)$ is given by equation

$\lambda^3 + A\lambda^2 + B\lambda + C = 0$ where

$A = ax^* > 0$, $B = bx^* c(y^* + z^*) + \alpha^2 y^* z^*$, $C = a\alpha^2 x^* y^* z^* > 0$

$AB - C = ax^*(bx^* c(y^* + z^*)) > 0$

From Routh-Hurwitz stability criterion it is stable.

IV. Conclusion

In this paper, the discussion and analysis model prey predator interaction with harvesting of predator is presented. Boundedness of solution, and equilibriums points with their conditions were discussed. The basin attraction of some of equilibrium points was also calculated. Finally, the result shows us the infected predator increases while the susceptible predator decreasing.

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Smoothness for Some Selected Test Functions Relative to Shape Parameter via IMQ

By Issa, K. & Sanni B. O.

Kwara State University

Abstract- Radial basis function (RBF) approximation has the potential to provide accurate function approximations for large data site given at scattered node locations which yields smooth solutions for a given number of node points especially when the basis functions are scaled to be nearly at and when the shape parameter is choose wisely. In this paper, we concentrate on the choice of shape parameter, which must be choose wisely and the simplest strategy we adopt is to perform a series of interpolation experiments by varying the interval of shape parameter, and then pick the \best" one. The \best" was pick by checking the errors for different data sites and the smoothness of the error graphs. The results shows that the choice of interval for the shape parameter give better accuracy and smoothness of the graphs.

Keywords: *radial basis functions, inverse multiquadric (IMQ), test function, shape parameter (ϵ).*

GJSFR-F Classification: *MSC 2010: 65L99*

SMOOTHNESSFORSOMESELECTEDTESTFUNCTIONSRELATIVETOSHAPEPARAMETERVIAIMQ

Strictly as per the compliance and regulations of :

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Smoothness for Some Selected Test Functions Relative to Shape Parameter via IMQ

Issa, K. [α] & Sanni B. O. [σ]

Abstract- Radial basis function (RBF) approximation has the potential to provide accurate func-tion approximations for large data site given at scattered node locations which yields smooth solutions for a given number of node points especially when the basis functions are scaled to be nearly at and when the shape parameter is choose wisely. In this paper, we concentrate on the choice of shape parameter, which must be choose wisely and the simplest strategy we adopt is to perform a series of interpolation experiments by varying the interval of shape parameter, and then pick the \best" one. The \best" was pick by checking the errors for different data sites and the smoothness of the error graphs. The results shows that the choice of interval for the shape parameter give better accuracy and smoothness of the graphs.

Keywords: radial basis functions, inverse multiquadric (IMQ), test function, shape parameter (ϵ).

I. Introduction

Radial basis function (RBF) approximation [8,10] is emerging as an important method for approximation, and interpolation of test functions for data given at scattered node locations, with computational domains in higher dimensions. And the application of radial functions to the solution of the scattered data interpolation problem benefits from the fact that the interpolation problem becomes insensitive to the dimension (d) of the space in which the data sites lie. In recent years, various RBF-based methods have gained fast growing attention from a broad range of scientific computing and engineering applications, such as multivariate scattered data processing [8,11], interpolation of functions [1,4,6,7,18] numerical solutions of partial differential equations (PDEs) [3,12–15], just to mention a few. The main advantages are spectral convergence rates that can be achieved using infinitely smooth basis functions, geometrical flexibility, and ease of implementation with shape parameter play important role in determine the accuracy of the interpolant [1,19].

a) Basic idea of RBF

Define the RBF interpolant of a function $f(x)$ as

$$\Gamma_f(x) = \sum_{k=1}^{N} \lambda_k \varphi(r, \epsilon), \ r = \parallel x - x_k \parallel, \ x \in \mathbb{R}^d \tag{1.1}$$

where the value of the interpolant at any location x is obtained as weighted sum of r and the coefficients $\lambda_k, k = 1, \cdots, N$ depend on the right hand side $f(x_j)$ and ϵ is called the

*Author α σ: Department of Statistics and Mathematical Sciences, Kwara State University, Malete, P.M.B 1530, Ilorin, Kwara State, Nigeria.
e-mails: issakazeem@yahoo.com, bashirsanni4@gmail.com*

shape parameter which must be choosing wisely, for better accuracy. The coefficients λ_k are determined by enforcing the interpolation condition

$$\Gamma_f(x_j) = f(x_j), \quad j = 1, \ldots, K \tag{1.2}$$

and when $N = K$, (1.2) becomes $N \times N$ linear system of the form

$$\theta_{NN}\sigma = F \tag{1.3}$$

where $F = \{f(x_j)\}_{j=1}^{N}$, $\sigma = \{\lambda_j\}_{j=1}^{N}$ and θ_{NN} is an $N \times N$ matrix whose entries are $\theta_{NN_{(j,k)}} = \varphi(\| x_j - x_k \|, \epsilon)$.

Therefore, RBF interpolant at M evaluation points on set $\Pi := \{x_i\}_{i=1}^{M}$ read

$$\Gamma_f(x_i) = \theta_{MN}\theta_{NN}^{-1}F \tag{1.4}$$

and some of the commonly used RBF are shown in the Table 1

Table 1: Some commonly used RBF

Type of RBF	$\varphi(r, \epsilon)$
Gaussian	$\exp\left(-\epsilon^2 r^2\right)$
Inverse Multiquadric (IMQ)	$\left(1 + \epsilon^2 r^2\right)^{-\frac{1}{2}}$
Multiquadric	$\left(1 + \epsilon^2 r^2\right)^{\frac{1}{2}}$
Thin Plate Spline (TPS)	$r^3 \log(r)$
Wendland	$(1 - \epsilon r)_+^4 \, (4r + 1)$

II. Role of Shape Parameter (ϵ)

The shape parameter has important consequences on the stability and accuracy of RBF interpolants, as the different choices of ϵ lead to different results in terms of computational error and the smaller the shape parameter, the flatter or wider the basis function (see figure 1).

In this work, we are generally interested in the range of the shape parameter that will incurs numerical stability, the higher accuracy of RBF interpolant and the smoothness of the error graphs. Hence, basically, we would like to choose the best shape parameter ϵ, to achieve best shape of the basis functions with respect to RBF interpolation error.

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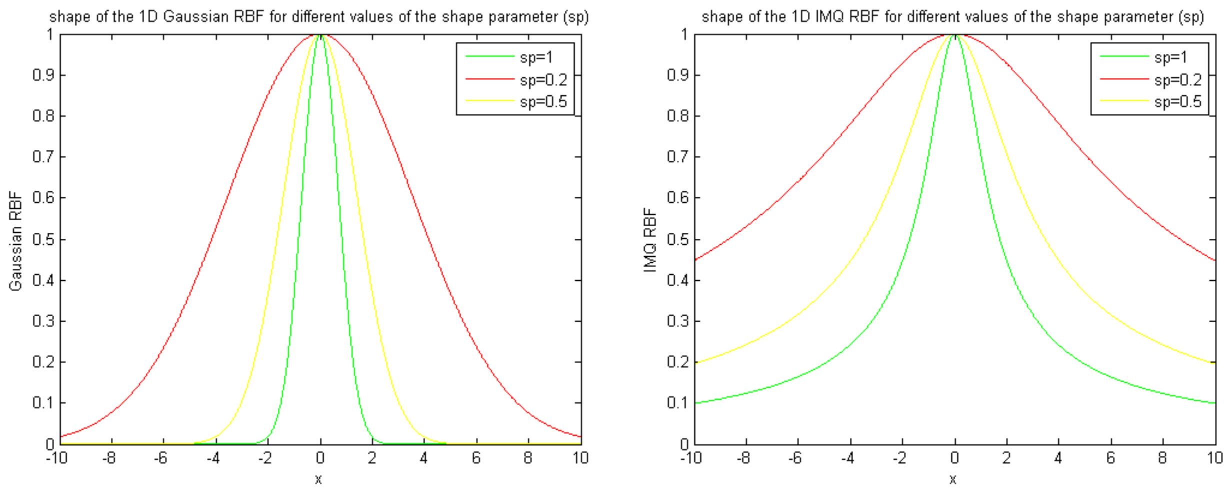

Figure 1: Examples of the shape of the 1D Gaussian and IMQ RBFs for di˙erent values of the shape parameter (sp)

III. Selection of Range of Shape Parameter (ϵ) for IMQ

To determine the best shape parameter ϵ^*, different strategies have been used by different authors, Xiang and Wang [22] applied trigonometric variable shape parameter (ϵ) to generalized Multiquadric RBF, Sarra and Sturgill [23] showed that the random variable shape parameter produces the most accurate results, if the centers are uniformly spaced, Biazar and Hosami [1] developed an algorithm for determining an interval for MQ shape parameter, In this work, we are focusing our attention on *Trial and Error* approach. The *Trial and Error* algorithm used in this work helps us to clarify quite well how the shape parameter acts on the basis functions (as shown in Figure 1) we choose for the interpolation, and so give us insight about the choice of a small range in which we will set ϵ. The algorithm is given below:

Trial and Error Algorithm

Data: f: function to be interpolated, Θ: set of data, B base of **S** interpolation subspace, Π: set of evaluation points

Result: ϵ^* optimal shape parameter

Define the data sites X from Θ

Define a large range for the choice of ϵ, in an interval $[a,\ b]$;

Define how many shape parameters are going to be tested, namely k;

for $i = 1, \ldots, k$

for **do**

$\epsilon_i = a + (i-1)\,\frac{b-a}{k}$;

Solve $\theta\,(X, B(X,\epsilon))\,\sigma = F$;

Evaluate $\Gamma_f(X, \epsilon; x)$;

Evaluate the error $\varepsilon_i = \mathrm{RMS} = \sqrt{\frac{\sum_{x\in\Pi} |f(x)-\Gamma_f(x)|^2}{\Pi}}$

end

Ref

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Plot the (RMS) ϵ_i versus ε_i.
Find the index i^*=minimum ε_k
Set $\epsilon^* = \epsilon_{i^*}$
Compute the interpolant with ϵ^*

IV. Numerical Experiments

In this section, we implement the algorithm discussed in section 3 on three test functions in $\mathbb{R}^2$ (since RBF methods are dimension blind) using inverse multiquadrics (IMQ) RBF with different knots N and M evaluation points by setting $\Theta = [0,\ 1]^2$, $\mathbf{x} = (x, y)$, $\mathbf{x} \in \Theta \subseteq \mathbb{R}^2$, $f(\mathbf{x}) \in \mathbb{R}$. Firstly, the algorithm is applied to find the range of the shape parameter for each test function, then the interpolant is recomputed by using the range of the shape parameter strategy in the proposed interval (see Figure 2). The accuracy are computed by finding the Root Mean Square (RMS) error approach and then pick the best (see Figure 3). The Root Mean Square (RMS) errors were calculated by the formula

$$\varepsilon_i = \text{RMS} = \sqrt{\frac{\sum_{x\in\Pi} |f(\mathbf{x}) - \Gamma_f(\mathbf{x})|^2}{\Pi}}$$

Example 4.1: We consider 2D Franke function

$$f(\mathbf{x}) = \frac{3}{4}e^{-\left((9x-2)^2+\frac{(9y-2)^2}{4}\right)} + \frac{3}{4}e^{-\left(\frac{(9x+1)^2}{49}+\frac{(9y+1)^2}{10}\right)} + \frac{1}{2}e^{-\left((9x-7)^2+\frac{(9y-3)^2}{4}\right)} - \frac{1}{5}e^{-\left((9x-4)^2+(9y-7)^2\right)} \tag{4.1}$$

The RMS errors are tabulated in Table 2 with different ϵ, the corresponding graph is presented in Figure 2 while Figure 4 shows the interpolant at $N = 4225$ and its corresponding maximum error and the ϵ (with respect to the smoothness of the error graph) used for the computation of the interpolant is presented in Figure 3

Example 4.2: We consider 2D Ackley function

$$f(\mathbf{x}) = -20e^{(-\frac{1}{5}\sqrt{\frac{1}{2}(x^2+y^2)}} - e^{\frac{1}{2}(\cos(2\pi x)+\cos(2\pi y))} + e^1 + 20 \tag{4.2}$$

The RMS errors are tabulated in Table 3 with different ϵ and the corresponding graph is presented in Figure 2 while Figure 5 shows the interpolant at $N = 4225$ and its corresponding maximum error and the ϵ (with respect to the smoothness of the error graph) used for the computation of the interpolant is presented in Figure 3

Example 4.3: We implement Trial and Error on 2D Beale function

$$f(\mathbf{x}) = \left(\frac{3}{2} - x + xy\right)^2 + \left(\frac{9}{4} - xy^2\right)^2 + \left(\frac{21}{8} - x + xy^3\right)^2 \tag{4.3}$$

Figure 6 display the interpolant that correspond to Figure 3 reference to the smoothness of the error graph while Table 4 is the RMS errors and its shape parameters.

V. Conclusion

In this paper, Trial and Error algorithm is proposed to determine the range of shape parameter for IMQ interpolation that produces the small error for different data sites (N) respect

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to the smooth error graph. The implementation of the scheme are illustrated on 3 selected test functions. Numerical results demonstrate that our scheme is an effective and reliable numerical technique for interpolating functions.

Table 2: Shape parameters (ϵ) and its corresponding RMS errors for Franke function (Example 4.1)

data sites (N)	$\epsilon = 6.0$	$\epsilon = 6.2$	$\epsilon = 6.4$	$\epsilon = 6.6$	$\epsilon = 6.8$	$\epsilon = 7.0$
9	1.921311e-001	1.940839e-001	1.960197e-001	1.979357e-001	1.998295e-001	2.016994e-001
25	3.080634e-002	3.158354e-002	3.240742e-002	3.327625e-002	3.418810e-002	3.514088e-002
36	1.928667e-002	1.972654e-002	2.019069e-002	2.067860e-002	2.118985e-002	2.172405e-002
289	2.500556e-004	2.751982e-004	3.009742e-004	3.272891e-004	3.540543e-004	3.811861e-004
1089	3.543262e-006	4.311747e-006	5.184915e-006	6.167679e-006	7.264324e-006	8.478486e-006
4225	6.912828e-009	7.890668e-009	9.963406e-009	1.362776e-008	1.814020e-008	2.402932e-008

Table 3: Shape parameters (ϵ) and its corresponding RMS errors for Ackley function (Example 4.2)

data sites (N)	$\epsilon = 7.0$	$\epsilon = 7.2$	$\epsilon = 7.4$	$\epsilon = 7.6$	$\epsilon = 7.8$	$\epsilon = 8.0$
9	4.372483e-001	4.638774e-001	4.903400e-001	5.166114e-001	5.426708e-001	5.685013e-001
25	1.191346e-001	1.188800e-001	1.187683e-001	1.188320e-001	1.191021e-001	1.196076e-001
36	8.979158e-002	8.958214e-002	8.934477e-002	8.909150e-002	8.883506e-002	8.858881e-002
289	4.959255e-003	5.231761e-003	5.504290e-003	5.776340e-003	6.047447e-003	6.317180e-003
1089	1.357913e-004	1.505501e-004	1.665895e-004	1.838989e-004	2.024630e-004	2.222622e-004
4225	6.598121e-006	6.093603e-006	6.372372e-006	6.208202e-006	6.156283e-006	6.123180e-006

Table 4: Shape parameters (ϵ) and its corresponding RMS errors for Beale function (Example 4.3)

data sites (N)	$\epsilon = 6.0$	$\epsilon = 6.2$	$\epsilon = 6.4$	$\epsilon = 6.6$	$\epsilon = 6.8$	$\epsilon = 7.0$
9	4.687441e-001	5.164511e-001	5.695731e-001	6.267950e-001	6.870994e-001	7.497095e-001
25	2.558036e-001	2.492158e-001	2.418695e-001	2.338618e-001	2.252999e-001	2.163039e-001
36	2.088475e-001	2.086083e-001	2.077394e-001	2.062678e-001	2.042220e-001	2.016324e-001
289	1.056356e-002	1.152670e-002	1.251113e-002	1.351372e-002	1.453141e-002	1.556133e-002
1089	1.656199e-004	1.978607e-004	2.340463e-004	2.743377e-004	3.188737e-004	3.677704e-004
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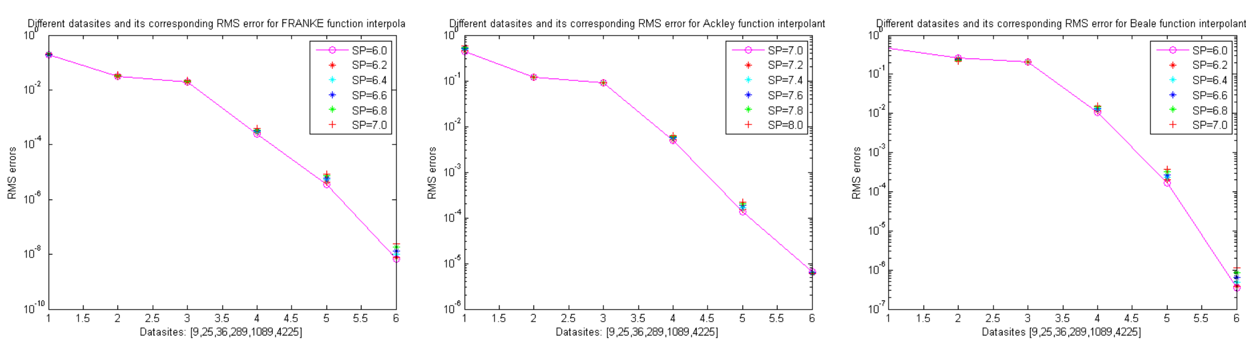

Figure 2: Test functions with different shape parameters (sp) and their corresponding RMS errors

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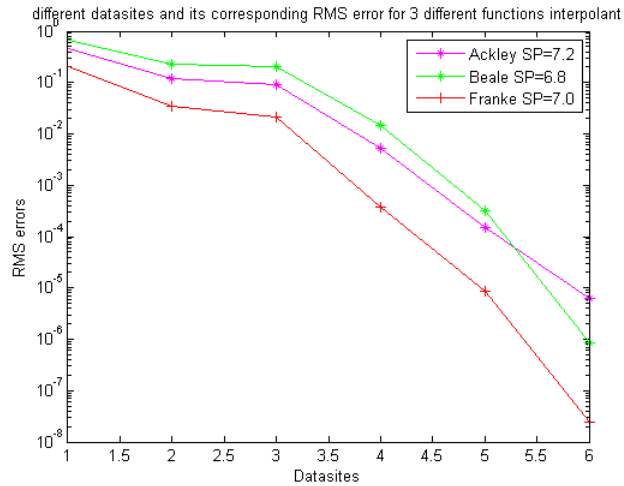

Figure 3: RMS error for the 3 Test functions with best shape parameter (sp)

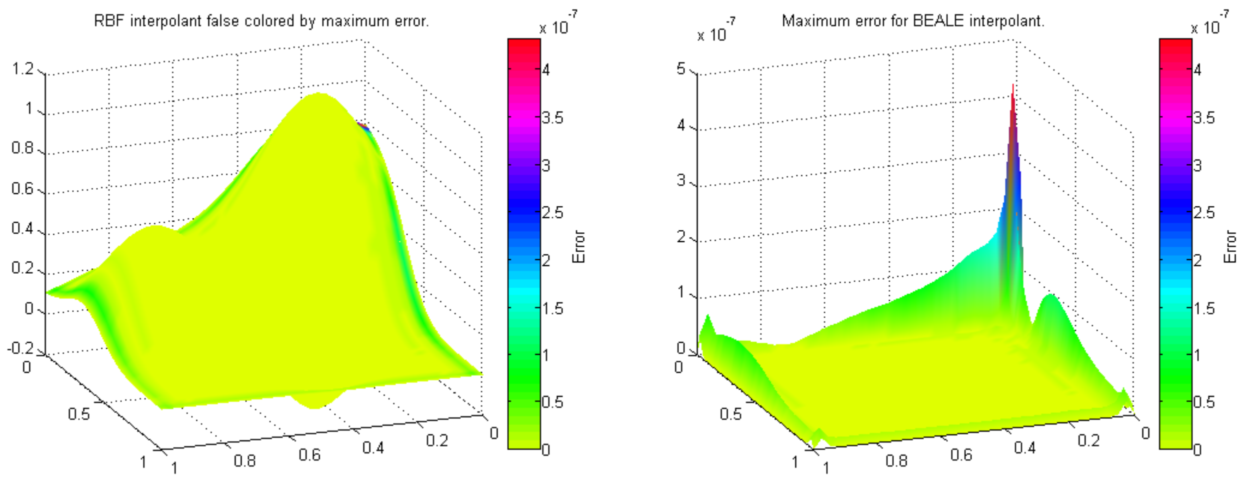

Figure 4: Franke function interpolant and its maximum error

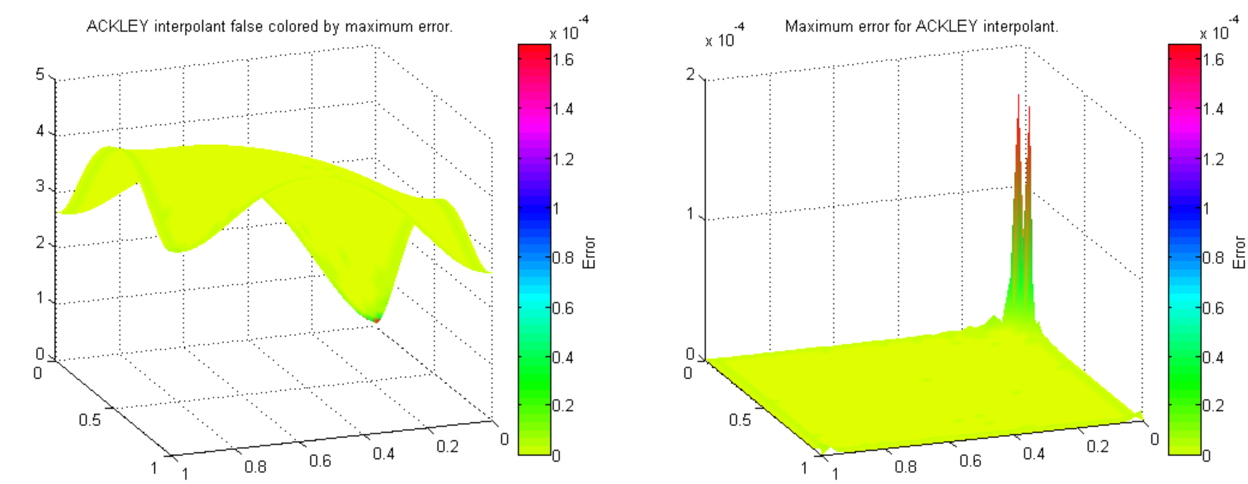

Figure 5: Ackley function interpolant and its maximum error

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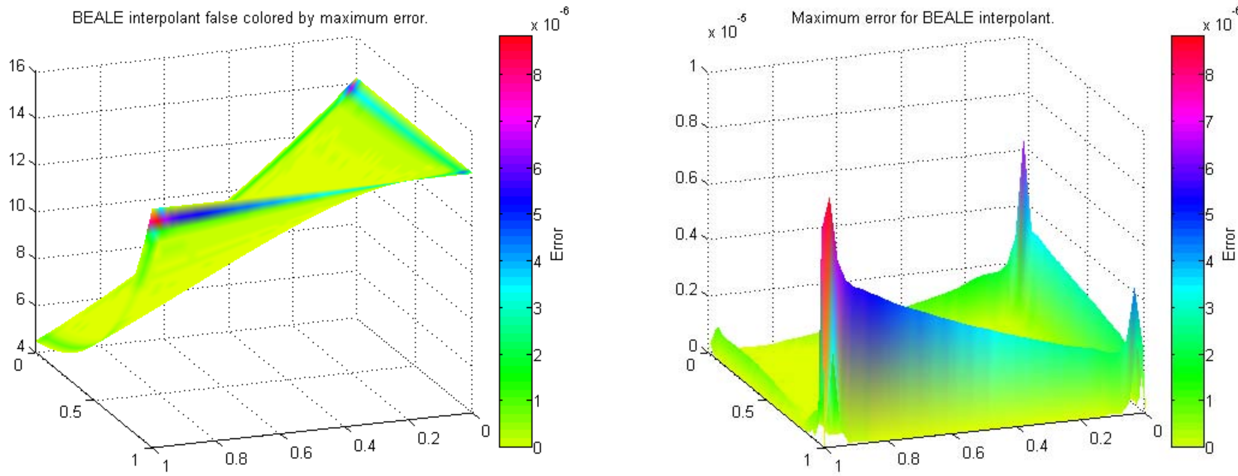

Figure 6: Beale function interpolant and its maximum error

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Notes

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Fish Harvesting Experienced by Depensation Growth Function

By Getachew Abiye Salilew
Madda Walabu Universty

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Keywords: mathematical bio-economics, overfishing.

GJSFR-F Classification: *MSC 2010: 91B84*

FISHHARVESTINGEXPERIENCEDBYDEPENSATIONGROWTHFUNCTION

Strictly as per the compliance and regulations of :

Ref

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Fish Harvesting Experienced by Depensation Growth Function

Getachew Abiye Salilew

Abstract- Getachew [1] introduced there is overfishing in Lake Tana, Amhara, Ethiopia using critical depensation model. In this paper, we applied this technique for depensation model. We investigated that if we consider depensation model we found that there is overfishing.

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I. Introduction

The economics of renewable resource use is essentially a multi-disciplinary undertaking, integrating both biological and economic aspects. When modeling the dynamics of the resource, one has to choose a level of analysis. An intuitive entity is the organism itself (a fish, a tree, or a cow) that experiences growth and mortality. While growth and mortality determine the dynamics of the existing number of individuals, it is the potential to reproduce which characterizes renewable resources. Resources whose reproduction is completely outside of the control of the resource users can perhaps best be analyzed in the framework of "eating a cake of unknown size". We will meet resources whose reproduction can be completely controlled when discussing forestry issues, but aquaculture could be another example. For most resource management problems however, it will be useful to model a reproduction function which depends in some (possibly highly nonlinear, possibly very stochastic) way on the existing number of individuals, which in turn are influenced by the current exploitation regime. In the absence of regulation control over harvesting behavior, the resource stocks are subject to open access [1]. In addition to the viewpoint of an organism, one could also focus on the dynamics of the underlying processes. Most models will take an aggregated view means analyzing a fishery, a forest, an ecosystem as a whole.

In different renewable resource management, it is important to balance ecological and economic needs. For example if we consider one of the renewable resource (fish), the fishery management is the consideration of the ecological effects of harvesting. Fisherman work to provide fish for a growing human population but because of this some fish populations have been dangerously declining. A major focus in fishery management is how best to ensure harvesting sustainability [2, 4, 5, 6]. The object of the management is to devise harvesting strategies that will not drive species to extinction. Therefore, the notion of persistence, extinction times of the populations and precautionary harvesting policy, is always critical. A control variable of every fishery

Author: Department of Mathematics, College of Natural and Computational Science, Madda Walabu Universty, Bale Robe, Ethiopia.
e-mail: getachewabye@gmail.com

management is the fishing effort [3, 8], which is defined as a measure of the intensity of fishing operations. As fishery management is the balance between harvesting and its ecological implications, it is important to fish in such a way that a species is sustainable and not in danger of becoming extinct. Mathematical bio economics is the study of the management of renewable resources. It takes into consideration not only economic questions like revenue, cost, price, effort etc., but also the impact of this demand on the resource. The aim of fish harvesting management is to gain a sustainable development of activity so that, future generation can also benefit from the resources. In this paper we consider depensation (weak Alee effect) deterministic model with a constant harvest rate as well as time dependent. Optimization and numerical calculations were used to determine the harvest rate that produces maximum yield under different population density scenarios. The dynamic mathematical models set on the background of biology and economics knowledge. The integration of these seemingly different subjects namely mathematics, biology and economics creates the source of interesting results and give valuable applications for the peoples living with fishing activities and those policy makers who involved control of overfishing.

II. Mathematical Biology of Depensation Model

Deterministic models of fishery populations can be classified into three types namely compensation, depensation and critical depensation. Compensation model is a growth type where population declination is compensated by increased growth rate. Depensation model is the opposite case to composition growth model. The critical depensation modelis the generalized logistic model which is extremely in opposite of the depensation model. A population's dynamics are depensatory or depensation is said to occur if the per- capita rate of growth decreases as the density decreases to low levels. Component of the life-history such as fecundity or survival during a particular stage or the mechanisms that affect these components (such as group defense or mate-finding difficulty) are called depensatory if they decrease the per-capita growth rate as abundance declines to low levels. Depensation model is the label most often used in fisheries. The strong depensation model is called critical depensation model. By the work done [9] some populations experience reduced rates of survival and reproduction when reduced to very low densities. Mathematical biology expression of depensation is given by growth model as:

$$(dx/dt) = rx^a[1 - (x/k)] \tag{1}$$

Here in the growth model (1), $x(t)$ represents fish biomass, r represents intrinsic growth rate of fish, k is ecological carrying capacity, $a \geq 0$ and $a \neq 1$, and (dx/dt) growth rate of fish without harvest. Model (1) has the property that for $a > 1$ there is, at low stock levels, depensation, which is a situation where the proportionate growth rate is an increasing function of the stock size, as opposed to being a decreasing function (compensation) in the simple logistic case where $a = 1$. The biological growth model (1) exhibiting depensation at the stock level below, y_0, and compensation thereafter as shown the figure below.

Ref

3. F. Brauer and C. Castillo-Chavez, *Mathematical Models in Population Biology and Epidemiology*, Springer-Verlag, 2001.

Notes

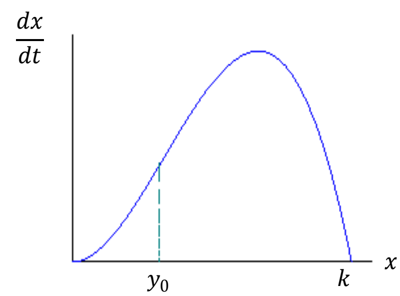

Figure 1: Growth curve of depensation model for $r = 1.4,\ a = 2k = 3.2,\ y_0 = 1$

In the figure 1 we have shown the rate change curve of the depensation model for some positive particular values of the parameters as shown. The curve is plotted for the population size function $x(t)$ versus the population rate change function $x' = (dx/dt)$. The maximum rate change of population, $Max(x')$, occurs when the population size be $x(t) = (ak/(a+1))$ for $a > 0$ and the corresponding maximum rate change of population is given by $Max(x') = \{r(ak/(a+1))^a(1-(a/(a+1)))\}$.

III. Solution of the Depensation Model

The solution of depensation model (1) with initial condition $x(0) = x_0$ is obtained as follows. Using techniques of separable variables the model (1) can be rewritten as $[\,dx/x^a(k-x)] = (r/k)dt$ and integrating both sides we get $\int[\,1/x^a(k-x)]dx = \int(r/k)dt$. To integrate the left hand side, we have to consider the following two cases by assuming that the initial value $x(0) = x_0$; a is an integer and $a > 1$ and then applying integration by partial fraction.

Case1: Let a is an even integer so that $a = 2n,\ n \in \mathrm{Z}^+$.

$\int[1/x^a(k-x)]\,dx = \int[1/x^{2n}(k-x)]\,dx = \int[1/(x^2)^n\,(k-x)]\,dx = (r/k)\,t + \ell$, where $\ell \in \mathcal{R}$

If $n = 1$ then $a = 2$ and thus we do have $\int[1/x^a(k-x)]dx = \int[1/x^2(k-x)]dx$. The solution is obtained using integration by partial fraction and is $ln\left(\frac{x}{k-x}\right) - \frac{k}{x} = rkt + \ln\left(\frac{x_0}{k-x_0}\right) - \frac{k}{x_0}$. If $n = 2$ then $a = 4$ and thus we do have $\int[1/x^4(k-x)]dx = (r/k)t$. The solution is obtained using integration by partial fraction and is:

$$\ln\left(\frac{x}{k-x}\right) - \frac{k}{x} - \frac{k^2}{2x^2} - \frac{k^3}{3x^3} = rk^3t + \ln\left(\frac{x_0}{k-x_0}\right) - \frac{k}{x_0} - \frac{k^2}{2(x_0)^2} - \frac{k^3}{3(x_0)^3}$$

In general by mathematical induction for $a = 2n$ using partial fraction we get the solution:

$$\int \frac{dx}{(x^2)^n(k-x)} = \int (\frac{A_1x+B_1}{x^2} + \frac{A_2x+B_2}{(x^2)^2} + \cdots + \frac{A_nx+B_n}{(x^2)^n} + \frac{c}{k-x})dx = (r/k)t + \ell$$

This is the general solution. Here $A_1 = \frac{1}{k^{2n}},\ A_2 = \frac{1}{k^{2n-2}},\ \cdots,\ A_{n-1} = \frac{1}{k^4},\ A_n = \frac{1}{k^2}$; $B_1 = \frac{1}{k^{2n-1}},\ B_2 = \frac{1}{k^{2n-3}}, \ldots,\ B_{n-1} = \frac{1}{k^3},\ B_n = \frac{1}{k}$; $c = A_n = \frac{1}{k^2}$ and $\ell \in \mathcal{R}$.

Case 2: Let a is an odd integer so that $a = 2n + 1,\ n \in Z^+$.

$\int[1/x^a(k-x)]dx = \int\left[1/x^{(2n+1)}(k-x)\right]dx = (r/k)t + \ell$, where $\ell \in \mathcal{R}$

If $n = 1$ then $a = 3$ and thus we have, $\int[1/x^3(k-x)]dx = (r/k)t$. The solution is obtained using integration by partial fraction and is:

$$\ln\left(\frac{x}{k-x}\right) - \frac{k}{x} - \frac{k^2}{2x^2} = rk^2t + \ln\left(\frac{x_0}{k-x_0}\right) - \frac{k}{x_0} - \frac{k^2}{2(x_0)^2}$$

If $n = 2$ then $a = 5$ and thus we do have $\int[1/x^5(k-x)]dx = (r/k)t$. The solution is obtained using integration by partial fraction and is:

$$\ln\left(\frac{x}{k-x}\right) - \frac{k}{x} - \frac{k^2}{2x^2} - \frac{k^3}{3x^3} - \frac{k^4}{4x^4} = rk^4t + \ln\left(\frac{x_0}{k-x_0}\right) - \frac{k}{x_0} - \frac{k^2}{2(x_0)^2} - \frac{k^3}{3(x_0)^3} - \frac{k^4}{4(x_0)^4}$$

In general by mathematical induction for $a = 2n + 1$ and using partial fraction we get the solution:

$$\int \frac{dx}{x^{2n+1}(k-x)} = \int \left(\frac{A_1}{x} + \frac{A_2}{x^2} + \frac{A_3}{x^3} + \cdots + \frac{A_{2n}}{x^{2n}} + \frac{A_{2n+1}}{x^{2n+1}} + \frac{B}{k-x}\right)dx = (r/k)t + \ell$$

This is the general solution. Where $A_1 = \frac{1}{k^{2n+1}}$, $A_2 = \frac{1}{k^{2n}}$, $A_3 = \frac{1}{k^{2n-1}}$, $A_4 = \frac{1}{k^{2n-2}}$, $\cdots$, $A_{2n} = \frac{1}{k^2}$, $A_{2n+1} = \frac{1}{k}$, $B = \frac{1}{k^{2n+1}}$ and $\ell \in \mathcal{R}$.

When we combine the above two cases we found that $\forall a \in Zl^+$ and $a \geq 2$ the general implicit solution of the depensation model is:

$$\int \frac{dx}{x^a(k-x)} = ln\left(\frac{x}{k-x}\right) - \sum_{n=1}^{a-1} \frac{1}{n}\left(\frac{k}{x}\right)^n = r(k^{a-1})t + \ell \quad (2)$$

Result (3) is the required particular implicit solution of the depensation model (1).

$$ln\left(\frac{x}{k-x}\right) - \sum_{n=1}^{a-1} \frac{1}{n}\left(\frac{k}{x}\right)^n = r(k^{a-1})t + ln\left(\frac{x_0}{k-x_0}\right) - \sum_{n=1}^{a-1} \frac{1}{n}\left(\frac{k}{x_0}\right)^n \quad (3)$$

The following graph represents the stock level of the depensation model.

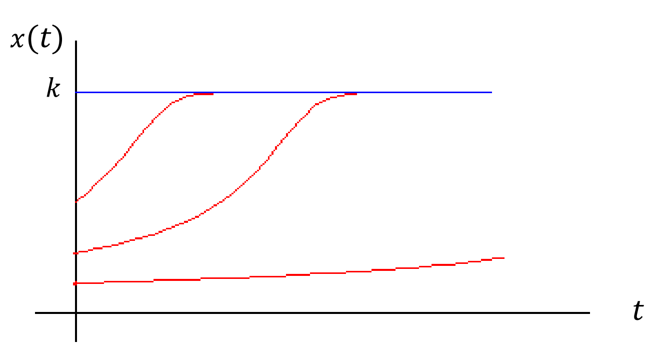

Figure 2: Typical solution curve for depensation model for $r = 1.6$, $k = 2.0$, $a = 3.0$

Notes

Notes

In figure 2 we have time series plot for depensation model which verifying local stability of the three equilibrium point in model (1). The depensation model has two equilibrium points namely the trivial and non-trivial equilibrium points $x = 0$ or $x = k$ respectively which are obtained by making $\frac{dx}{dt} = rx^a\left(1 - \frac{x}{k}\right) = 0$. The equilibrium point $x = 0$ is semi stable since $f'(0) = 0$. The non-trivial equilibrium point $x = k$ is stable for $r > 0$ and unstable for $r < 0$[7, 11].

IV. Mathematical Bio-Economics of Depensation Model

A fishery is an area with an associated fish or aquatic population which is harvested for its commercial or recreational value. Population dynamics describes the ways in which a given population grows and shrinks over time, as controlled by birth, death, and emigration or immigration. It is the basis for understanding changing fishery patterns and issues such as habitat destruction, predation and optimal harvesting rates. With the natural positive population growth the population size can be brought down whenever harvesting is introduced. Schaefer catch equation is a bilinear short-term harvest function and it assumes that effort always removes a constant proportion of the stock. Depensation Mathematical Bio-Economics model is given by $\frac{dx}{dt} = f(x) - h(E, x)$ where $f(x) = rx^a[1 - (x/k)]$ is the growth function of fish and $h(E, x) = qEx$ is the harvest function of fish. And thus we do have

$$(dx/dt) = rx^a[1 - (x/k)] - qEx \tag{4}$$

Here in the model (4), $x(t)$ represents fish biomass, r represents intrinsic growth rate of fish, k is ecological carrying capacity, $a \geq 0$ and $a \neq 1$, t is time, (dx/dt) is growth rate of the fish with harvest function $h(E, x)$.

a) *Equilibrium Points of Bio-Economics of Depensation Model*

The equilibrium points of model (4) are obtained by making $(dx/dt) = 0 \Leftrightarrow rx^a[1 - (x/k)] - qEx = 0$. This implies that the trivial equilibrium point $x = 0$ or the non-trivial equilibrium point: $rx^{a-1}(1 - (x/k)) - qE = 0$ for $a > 1$ are the equilibrium points. If we take $a = 2$, we get, $rx^2 - rkx + qkE = 0$. So that the non-trivial equilibrium points are

$$x_1 = \frac{rk + \sqrt{(rk)^2 - 4rqkE}}{2r} \quad or \quad x_2 = \frac{rk - \sqrt{(rk)^2 - 4rqkE}}{2r} \tag{5}$$

Provided that $rk > 4qE$ and since $rk > \sqrt{(rk)^2 - 4rkqE}$ both equilibrium points are positive for positive parameters r, q, E and k.

The stability analysis of the equilibrium points is obtained by identifying the algebraic sign of the first derivative of the function at each equilibrium points. That is, $(dx/dt) = g(x) = rx^a[1 - (x/k)] - qEx$. So, its first derivative for $a = 2$ is, $g'(x) = 2rx - \frac{3r}{k}x^2 - qE$. Since $g'(0) = -qE < 0$ implies equilibrium point $x = 0$ is stable. Next we have, $g'(x_1) = -(rk/2) + 2qE - \frac{1}{2}\sqrt{(rk)^2 - 4rkqE} < 0$ this implies that x_1 in (5) is stable. Further we have, $g'(x_2) = -\frac{rk}{2} + 2qE + \frac{1}{2}\sqrt{(rk)^2 - 4rkqE}$ this implies that, $x_2 = \frac{rk - \sqrt{(rk)^2 - 4rqkE}}{2r}$ is stable if $\sqrt{(rk)^2 - 4rkqE} < rk - 4qE$ and unstable otherwise.

b) Maximum Sustainable Yield (MSY) Of The Depensation Model

Schaefer catch equation is a bilinear short-term harvest function and it assumes that effort always removes a constant proportion of the stock.

$$H(E,x) = qEx \tag{6}$$

Where H =catch measured in terms of biomass; E fishing effort and q is a constant catchability of coefficient. And substituting the non-trivial bio-economic equilibrium points of (5) in (6) gives the harvesting function as a function of effort E. Let $H(x_1,E) = H_1(E)$ and $H(x_2,E) = H_2(E)$ then we got

$$H_1(E) = (qE/2r)\left[kr + \sqrt{(kr)^2 - 4krqE}\right] \tag{7}$$

$$H_2(E) = (qE/2r)\left[kr - \sqrt{(kr)^2 - 4krqE}\right] \tag{8}$$

The effort at the maximum sustainable yield denoted by E_{MSY} is obtained by making the first derivative of H with respect to the effort E equal to zero. That is $\frac{d(H_1)}{dE} = 0$, gives $E = 0$ or $E = \frac{2kr}{9q}$. Thus, $E_{MSY} = \frac{2kr}{9q}$. We have the same result for, $\frac{d(H_2)}{dE} = 0$.

And thus the corresponding Maximum Sustainable Yield in (7) to be

$$MSY_1 = H_1(E_{MSY}) = \frac{4k^2r}{27}$$

Again the corresponding Maximum Sustainable Yield in (8) to be

$$MSY_2 = H_2(E_{MSY}) = \frac{2k^2r}{27}$$

From this we conclude that, $MSY_1 = 2MSY_2$.

c) The Open Access Yield (OAY) For The Depensation Model

A work done in [10] shows that economic models of fishery are underlined by biological models and it is impossible to formulate any useful economic model of fishery without specifying the underlining biological dynamics of the fishery. Based on constant price and unit cost of effort the total revenue denoted by TR will be calculated using the formula $TR(E) = p.H(E)$, where p is the average price per kilogram of fish. The relationship between cost and effort is assumed to be linear and then the total cost of fishing effort denoted by TC is defined as $TC(E) = c.E$, where c is the unit cost of effort that includes cost of labor and capital and E is the unit of effort and thus the total economic rent of fishery denoted by TER defined as

$$TER(E) = TR(E) - TC(E) \tag{9}$$

At the open access point, total fishing costs are equal to total revenues from the fishery. Then the open access effort is obtained by equating $TR(E) = TC(E)$. Where $TR(E) = pqxE$ and $TC(E) = cE$ which yields $pqxE = cE$. To calculate the effort for the Open Access Yield we used two non-trivial equilibria in (5). And thus we have two

Ref

10. G. Munro, *Fisheries, Extended Jurisdiction and the Economics of Common Property Resources*. The Canadian Journal of Economics 15(1982).

equations namely $pqx_1E = cE$ and $pqx_2E = cE$. And substituting their corresponding values respectively gives

$$\frac{pq}{2r}E\left(kr + \sqrt{(kr)^2 - 4krqE}\right) = cE \tag{10}$$

$$\frac{pq}{2r}E\left(kr - \sqrt{(kr)^2 - 4krqE}\right) = cE \tag{11}$$

Notes

From (10), we have $E = 0$ $E = \frac{cr}{pq^2}\left(1 - \frac{c}{pqk}\right)$ that is $E_{OAY} = \frac{cr}{pq^2}\left(1 - \frac{c}{pqk}\right)$ provided, $c < pqk$.

And thus the corresponding Open Access Yield to be

$$OAY_1 = H_1(E_{OAY}) = \frac{q}{2r}.E_{OAY}\left(kr + \sqrt{(kr)^2 - 4krqE_{OAY}}\right)$$

$$OAY_1 = \frac{c}{2pq}(1 - \frac{c}{pqk})\left(kr + \sqrt{(kr)^2 - \frac{4kr^2c}{pq}(1 - \frac{c}{pqk})}\right)$$

Equation (11) gives $E = 0$ or $E = \frac{cr}{pq^2}\left(1 - \frac{c}{pqk}\right)$ that is $E_{OAY} = \frac{cr}{pq^2}\left(1 - \frac{c}{pqk}\right)$ provided, $c < pqk$.

And thus the corresponding Open Access Yield to be

$$OAY_2 = H_2(E_{OAY}) = \frac{q}{2r}E_{OAY}\left(kr - \sqrt{(kr)^2 - 4krqE_{OAY}}\right)$$

$$OAY_2 = \frac{c}{2pq}\left(1 - \frac{c}{pqk}\right)\left(kr - \sqrt{(kr)^2 - \frac{4kr^2c}{pq}(1 - \frac{c}{pqk})}\right)$$

d) The Maximum Economic Yield (MEY) of Depensation Model

The maximum economic yield is attained at the profit maximizing level of effort which is obtained using equation (9). So, $[d(TER(E))/dE] = 0$ implies $[d(TR(E))/dE] = [d(TC(E))/dE]$. To calculate the effort for the Maximum Economic Yield we used two non-trivial equilibria x_1 and x_2 in (5). And thus we have two equations namely $[d(pqx_1E)/dE] = [\,d\,(cE)/dE\,]$ and $[\,d(pqx_2E)/dE] = [\,d(cE)/dE]$. And substituting the corresponding values of x_1 and x_2 in these equations give respectively

$$\frac{d}{dE}\left(\frac{pq}{2r}E\left(kr + \sqrt{(kr)^2 - 4krqE}\right)\right) = \frac{d(cE)}{dE} \tag{12}$$

$$\frac{d}{dE}\left(\frac{pq}{2r}E\left(kr - \sqrt{(kr)^2 - 4krqE}\right)\right) = \frac{d(cE)}{dE} \tag{13}$$

From equation (12), we do have the following

$$E^2 - \frac{2r}{9kq^2}\left[k\left(kq + \frac{2c}{p}\right) - \frac{2c^2}{p^2q}\right]E + \frac{r^2ck}{9pq^3}\left(1 - \frac{c}{pqk}\right) = 0$$

Setting $A = k\left(kq + \frac{2c}{p}\right) - \frac{2c^2}{p^2q}$ and $B = 1 - \frac{c}{pqk}$, we obtain

$$E=\frac{r}{3q}\left[\frac{A}{3kq}\pm\sqrt{\left(\frac{A}{3kq}\right)^2-\frac{ckB}{pq}}\right],$$ provided that $(A/3kq)^2\geq(ckB/pq)$ and, $A>0$.

Thus efforts at maximum economic yield are:

$$E_{MEY_1}=\frac{r}{3q}\left[\frac{\left[k\left(kq+\frac{2c}{p}\right)-\frac{2c^2}{p^2q}\right]}{3kq}+\sqrt{\left(\frac{1}{3kq}\right)^2\left[k\left(kq+\frac{2c}{p}\right)-\frac{2c^2}{p^2q}\right]^2-\frac{ck}{pq}\left[1-\frac{c}{pqk}\right]}\right]$$

$$E_{MEY_2}=\frac{r}{3q}\left[\frac{\left[k\left(kq+\frac{2c}{p}\right)-\frac{2c^2}{p^2q}\right]}{3kq}-\sqrt{\left(\frac{1}{3kq}\right)^2\left[k\left(kq+\frac{2c}{p}\right)-\frac{2c^2}{p^2q}\right]^2-\frac{ck}{pq}\left[1-\frac{c}{pqk}\right]}\right]$$

And thus the corresponding Maximum Economic Yields in (7) to be

$$MEY_1=H_1\left(E_{MEY_1}\right)$$

$$MEY_1=\frac{1}{6}\left[\frac{\left[k\left(kq+\frac{2c}{p}\right)-\frac{2c^2}{p^2q}\right]}{3kq}+\sqrt{\left(\frac{1}{3kq}\right)^2\left[k\left(kq+\frac{2c}{p}\right)-\frac{2c^2}{p^2q}\right]^2-\frac{ck}{pq}\left[1-\frac{c}{pqk}\right]}\right]\times$$

$$\times\left(kr+\sqrt{(kr)^2-\frac{4kr^2}{3}\left[\frac{\left[k\left(kq+\frac{2c}{p}\right)-\frac{2c^2}{p^2q}\right]}{3kq}+\sqrt{\left(\frac{1}{3kq}\right)^2\left[k\left(kq+\frac{2c}{p}\right)-\frac{2c^2}{p^2q}\right]^2-\frac{ck}{pq}\left[1-\frac{c}{pqk}\right]}\right]}\right)$$

$$MEY_2=H_1\left(E_{MEY_2}\right)$$

$$MEY_2=\frac{1}{6}\left[\frac{\left[k\left(kq+\frac{2c}{p}\right)-\frac{2c^2}{p^2q}\right]}{3kq}-\sqrt{\left(\frac{1}{3kq}\right)^2\left[k\left(kq+\frac{2c}{p}\right)-\frac{2c^2}{p^2q}\right]^2-\frac{ck}{pq}\left[1-\frac{c}{pqk}\right]}\right]\times$$

$$\times\left(kr+\sqrt{(kr)^2-\frac{4kr^2}{3}\left[\frac{\left[k\left(kq+\frac{2c}{p}\right)-\frac{2c^2}{p^2q}\right]}{3kq}-\sqrt{\left(\frac{1}{3kq}\right)^2\left[k\left(kq+\frac{2c}{p}\right)-\frac{2c^2}{p^2q}\right]^2-\frac{ck}{pq}\left[1-\frac{c}{pqk}\right]}\right]}\right)$$

Similarly from equation (13), we have the same effort as the above. And thus the corresponding Maximum Economic Yields in (8) to be

$$MEY_3=\frac{1}{6}\left[\frac{\left[k\left(kq+\frac{2c}{p}\right)-\frac{2c^2}{p^2q}\right]}{3kq}+\sqrt{\left(\frac{1}{3kq}\right)^2\left[k\left(kq+\frac{2c}{p}\right)-\frac{2c^2}{p^2q}\right]^2-\frac{ck}{pq}\left[1-\frac{c}{pqk}\right]}\right]\times$$

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$$\times\left(kr-\sqrt{(kr)^2-\frac{4kr^2}{3}\left[\frac{\left[k\left(kq+\frac{2c}{p}\right)-\frac{2c^2}{p^2q}\right]}{3kq}+\sqrt{\left(\frac{1}{3kq}\right)^2\left[k\left(kq+\frac{2c}{p}\right)-\frac{2c^2}{p^2q}\right]^2-\frac{ck}{pq}\left[1-\frac{c}{pqk}\right]}\right]}\right)$$

$$MEY_4=\frac{1}{6}\left[\frac{\left[k\left(kq+\frac{2c}{p}\right)-\frac{2c^2}{p^2q}\right]}{3kq}-\sqrt{\left(\frac{1}{3kq}\right)^2\left[k\left(kq+\frac{2c}{p}\right)-\frac{2c^2}{p^2q}\right]^2-\frac{ck}{pq}\left[1-\frac{c}{pqk}\right]}\right]\times$$

$$\times\left(kr-\sqrt{(kr)^2-\frac{4kr^2}{3}\left[\frac{\left[k\left(kq+\frac{2c}{p}\right)-\frac{2c^2}{p^2q}\right]}{3kq}-\sqrt{\left(\frac{1}{3kq}\right)^2\left[k\left(kq+\frac{2c}{p}\right)-\frac{2c^2}{p^2q}\right]^2-\frac{ck}{pq}\left[1-\frac{c}{pqk}\right]}\right]}\right)$$

V. Parameter Estimation

a) Basic Parameters Estimation

Using the time series data [1], we have the following parameters estimation.

Table 1

Parameters	Symbol	Value
Carrying capacity	k	$2.57\times10^{13}\,kg\ of\ fish$
Catch ability constant	q	$2.197\times10^{-11}\,perday$
Cost of effort	c	$182.50\ birr/kg$
Price of effort	p	$11 birr/kg$
Intrinsic growth rate	r	0.5

Table 2: Parameter estimation for depensation model

Description	Formula	Value [kg per day]
E_{MSY}	$2rk/9q$	1.3×10^{23}
E_{OAY}	$cr(1-(c/pqk))/pq^2$	1.7×10^{22}
E_{MEY}	$r(A+\sqrt{A^2-B})/3q$	1.29×10^{23}
MSY	$4k^2r/27$	4.9×10^{25}
OAY	$c(1-(c/pqk))\left(kr+\sqrt{(kr)^2-4r^2B}\right)/2pq$	91.4×10^{23}
MEY	$MEY_1=\varphi\left(kr+\sqrt{(kr)^2-(4kr^2\varphi/3)}\right)/6$	48.9×10^{24}

Where; $A=\left[\frac{k\left(kq+\frac{2c}{p}\right)-\frac{2c^2}{p^2q}}{3kq}\right]$, $B=\frac{ck}{pq}\left[1-\frac{c}{pqk}\right]$, and $\varphi=A+\sqrt{A^2-B}$

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Table 3: Parameter estimation to depensation model for different values of r.

r=0.5	R=1	R=2
$E_{MSY} = 1.3 \times 10^{23}$	$E_{MSY} = 2.6 \times 10^{23}$	$E_{MSY} = 5.20 \times 10^{23}$
$MSY_1 = 4.89 \times 10^{25}$	$MSY_1 = 9.795 \times 10^{25}$	$MSY_1 = 19.59 \times 10^{25}$
$MSY_2 = 2.45 \times 10^{25}$	$MSY_2 = 4.898 \times 10^{25}$	$MSY_2 = 9.795 \times 10^{25}$
$E_{OAY} = 1.67 \times 10^{22}$	$E_{OAY} = 3.336 \times 10^{22}$	$E_{OAY} = 6.67 \times 10^{22}$
$OAY_1 = 91.47 \times 10^{23}$	$OAY_1 = 182.94 \times 10^{23}$	$OAY_1 = 365.885 \times 10^{23}$
$OAY_2 = 2.77 \times 10^{23}$	$OAY_2 = 5.53 \times 10^{23}$	$OAY_2 = 11.07 \times 10^{23}$
$E_{MEY_1} = 1.29 \times 10^{23}$	$E_{MEY_1} = 2.58 \times 10^{23}$	$E_{MEY_1} = 5.162 \times 10^{23}$
$E_{MEY_2} = 8.4 \times 10^{21}$	$E_{MEY_2} = 16.81 \times 10^{21}$	$E_{MEY_2} = 33.62 \times 10^{21}$
$MEY_1 = 48.97 \times 10^{24}$	$MEY_1 = 97.94 \times 10^{24}$	$MEY_1 = 195.885 \times 10^{24}$
$MEY_2 = 46.79 \times 10^{23}$	$MEY_2 = 93.58 \times 10^{23}$	$MEY_2 = 187.156 \times 10^{23}$
$MEY_3 = 23.94 \times 10^{24}$	$MEY_3 = 47.88 \times 10^{24}$	$MEY_3 = 95.75 \times 10^{24}$
$MEY_4 = 6.92 \times 10^{22}$	$MEY_4 = 13.84 \times 10^{22}$	$MEY_4 = 27.68 \times 10^{22}$

b) The Economic Model Estimation

In this section we calculated the profit for depensation models by using real data [1]. In a commercial fishery, the appropriate measure of gross benefits is the total revenue that accrues to firms. Assuming that fish are sold in a competitive market, each firm takes the market price p as given and so the revenue obtained from a harvest H is given by $TR(E) = pH(E)$. And finally the economic rent or profit denoted by P is defined in terms of total cost TC and total revenue TR by $P = TR - TC$.

c) The depensation economic model parameter estimations

In this case we do have the following parameter estimation with the harvest at the given type of Effort is obtained by: $H(E) = \frac{qE}{2r}\left(kr + \sqrt{(kr)^2 - 4krqE}\right)$.

Table 4: The depensation economic model parameter estimations

r	c	p	q	k	E_{MSY}	E_{OAY}	E_{MEY}	$H(E_{MSY})$	$H(E_{OAY})$	$H(E_{MEY})$
0.5	182.5	11	2.1×10^{-11}	2.5×10^{13}	1.3×10^{23}	1.6×10^{22}	1.2×10^{23}	4.8×10^{25}	91.4×10^{23}	48.9×10^{24}

And thus the profit with different type of harvest function is given by:
The depensation economic model for Maximum Sustainable Yield (MSY)

$$P(E_{MSY}) = TR(E_{MSY}) - TC(E_{MSY}) = p.H(E_{MSY}) - c.E_{MSY}$$

$$P(E_{MSY}) = 53.877549 \times 10^{25} - 237.330848 \times 10^{23} = 51.50424 \times 10^{25}\ birr$$

The depensation economic model for Open Access Yield (OAY)

$$P(E_{OAY}) = TR(E_{OAY}) - TC(E_{OAY}) = p.H(E_{OAY}) - c.E_{OAY}$$

$$P(E_{OAY}) = 1006.1903 \times 10^{23} - 30.44 \times 10^{23} = 975.7503 \times 10^{23}\ birr$$

The depensation economic model for Maximum Economic Yield (MEY)

$$P(E_{MEY}) = TR(E_{MEY}) - TC(E_{MEY}) = p.H(E_{MEY}) - c.E_{MEY}$$

$$P(E_{MEY}) = 538.684 \times 10^{24} - 235.5227436 \times 10^{23} = 515.13173 \times 10^{24}\ birr$$

VI. Results and Conclusions

Biologically overfishing occurs when fish species are caught at a rate faster than they can reproduce. A continuous increase of effort might result in an increase catch

Ref
1. Getachew Abiye Salilew, *Mathematical Bio-Economics of Fish Harvesting with Critical Depensation in Lake Tana*, TJPRC: JMCAR 3(2016) 1-14.

but at a decreasing rate or more effort may result in proportionality a smaller harvest, which means the additional effort will have less return.

Using data [1] there is over fishing for different cases of the natural growth rates $r = 0.5$, $r = 1$ and $r = 2$ of fish as in table 3. Without loss of any generality we prefer to analyze the tabular approximate value for $r = 0.5$ as our choice of the parameter is similar to that of $r = 1$ and $r = 2$. When the natural growth rate $r = 0.5$, carrying capacity $k = 2.57 \times 10^{13} kg\ of\ fish$, effort for maximum sustainable yield, $E_{MSY} = 1.300443 \times 10^{23}\ kg\ of\ fish$, effort for open access yield $E_{OAY} = 0.1668069 \times 10^{23}\ kg\ of\ fish$, effort for maximum economic yield $E_{MEY} = 1.2905355813 \times 10^{23}\ kg\ of\ fish$. And thus we observed from these values that all efforts are greater than the carrying capacity therefore there is overfishing if we consider the depensation model.

In the economic point of view we have the approximate price of total population of fish in [1] is, $28.2 \times 10^{13}\ birr$. In the depensation economic model parameter estimations we have: $P(E_{MSY}) = 51.50424 \times 10^{25} birr$, $P(E_{OAY}) = 975.7503 \times 10^{23}\ birr$ and $P(E_{MEY}) = 515.13173 \times 10^{24}\ birr$. And thus in the depensation model the economic rent or the profit obtained by all kinds of effort are greater than the price of the total population of fish and therefore there is overfishing. To keep the sustainability of fish we must reduce the effort levels.

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Positive Definite and Related Functions in the Product of Hypercomplex Systems

By A. S. Okb El Bab, A. M. Zabel, Hossam A. Ghany & M. Zakarya

Helwan University

Abstract- The main aim of this paper is to explore harmonic properties of functions defined in the product of hypercomplex systems. By means of the generalized translation operators, the precise definition of the product of commutative hypercomplex systems is given and full description for its properties are shown. The integral representations of positive definite function defined in the product of commutative normal hypercomplex systems are given. Furthermore, we present the necessary and sufficient conditions guarantees the property of positive definite function in the product of hypercomplex systems.

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GJSFR-F Classification: *MSC 2010: 43A62, 43A22, 43A10.*

POSITIVEDEFINITEANDRELATEDFUNCTIONSINTHEPRODUCTOFHYPERCOMPLEXSYSTEMS

Positive Definite and Related Functions in the Product of Hypercomplex Systems

A. S. Okb El Bab [α], A. M. Zabel [σ], Hossam A. Ghany [ρ] & M. Zakarya [ω]

Abstract- The main aim of this paper is to explore harmonic properties of functions defined in the product of hypercomplex systems. By means of the generalized translation operators, the precise definition of the product of commutative hypercomplex systems is given and full description for its properties are shown. The integral representations of positive definite function defined in the product of commutative normal hypercomplex systems are given. Furthermore, we present the necessary and sufficient conditions guarantees the property of positive definite function in the product of hypercomplex systems.

Keywords: hypercomplex systems; generalized translation operators; direct product; positive definite.

I. Introduction

Harmonic analysis in hypercomplex systems (HCSs) dates back to Delsartes and Levitans work during the 1930s and 1940s, but the substantial development had to wait till the 1990s when Berezansky and Kondratiev [1] put HCSs in the right setting for harmonic analysis. Recently, some authors as Zabel and Bin Dehaish [2,3], Bin Dehaish [4], Okb El Bab, Zabel and Ghany [5] and Okb El Bab, Ghany and Boshnaa [6], studied some important subjects related to harmonic analysis in HCSs. Furthermore, Okb El Bab, Ghany, Hyder and Zakarya [7,8], studied some important subjects related to a construction of non-Gaussian white noise analysis using the theory of HCSs.

Generalized translation operators (GTOs) were first introduced by Delsarte [9] as an object that generalizes the idea of translation on a group. Later, they were systematically studied by Levitan [10–14], for some classes of GTOs, he obtained generalizations of harmonic analysis, the Lie theory, the theory of almost periodic functions, the theory of group representations, etc. The fact that GTOs arise in various problems of analysis is explained by Vainerman and Litvinov in [15]. Transformations of Fourier type for which the Plancherel theorem and the inversion formula hold, as a rule, are closely related to families of GTOs. According to Section 1 in [1], each hypercomplex system (HCS) can be associated with a family of GTOs. So, we begin with recalling some necessary facts deal with theory of GTOs and reviewing the conditions that distinguish the class of HCSs from the class of GTOs [2,14].

Let $L_1(Q,m)$ be a HCS with basis Q and let Φ be a space of complex valued functions on Q. Assume that an operator valued function $Q \ni p \mapsto L_p : \Phi \to \Phi$ is given such that the function $g(p) = (L_p f)(q)$ belongs to Φ for any $f \in \Phi$ and any fixed $q \in Q$.

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Author ρ: Department of Mathematics, Helwan University, Cairo, Egypt. e-mail: h.abdelghany@yahoo.com
Author α σ: Department of Mathematics, Faculty of Science, Al-Azhar University, Cairo, Egypt.
Author ω: Department of Mathematics, Faculty of Science, Al-Azhar University, Egypt. e-mail: mohammed zakarya@rocketmail.com

Definition 1.1. The operators $L_p,\ p \in Q$ are called GTOs, provided that the following axioms are satisfied.

I. Associativity axiom. The equality

$$\big(L_p^q(L_q f)\big)(r) = \big(L_q^r(L_p f)\big)(r) \tag{1.1}$$

holds for any elements $p, q \in Q$.

II. There exists an element $e \in Q$ such that $L_e = I$, where I is the identity operator in Φ.

Definition 1.2. The GTOs are called commutative if for any $p, q \in Q$, we have $\big(L_p^r(L_q f)\big)(r)$ $= \big(L_q^r(L_p f)\big)(r)$. For commutative GTOs L_p the following equality is satisfied.

$$\big(L_p f\big)(q) = \big(L_q f\big)(p), \quad p, q \in Q. \tag{1.2}$$

In this paper we are interest in the case where Q is locally compact space with regular Borel measure m positive on open sets and bounded GTOs L_p act in the space of functions $\Phi = L_2(Q, m)$.

Definition 1.3. Given an involutive homeomorphism $Q \ni p \mapsto p^* \in Q$. The GTOs L_p are involutive if the equalities

$$\big(L_p f\big)(q) = \overline{\big(L_{q^*} f^*\big)(p^*)}, \quad \big(f \in L_2(Q, m), f^*(p) = \overline{f(p^*)}\,\big), \tag{1.3}$$

and $e^* = e$ hold for almost all $p, q \in Q$.

Definition 1.4. The GTOs L_p preserve positivity if $\big(L_p f\big)(q) \geq 0$ almost everywhere in m whenever $f(q) \geq 0$.

Definition 1.5. The family of operators L_p is called weakly continuous if the operator-valued function $Q \ni p \mapsto L_p$ is weakly continuous.

Definition 1.6. Let L_p^* be the operator adjoint to L_p. The measure m is called strongly invariant if $L_p^* = L_{p^*}$ for all $p \in Q$. We say that the measure m unimodular if $m(A) = m(A^*)$ for all $A \in \mathcal{B}(Q)$.

Assume that the GTOs L_p satisfy the finiteness condition:

(F) For any $A, B \in \mathcal{B}_0(Q)$, there is a compact set F so large that $\big(L_p f\big)(q) = 0$ for almost all $p \in A$ and $q \in B$ provided that supp $f \cap F = \emptyset$.

Lemma 1.1. *[1] If weakly continuous GTOs L_p are commutative, then relation (1.2) holds for almost all p and q.*

Lemma 1.2. *[1] Let m be a measure strongly invariant with respect to the GTOs L_p $(p \in Q)$ which preserve the unit element and satisfy the finiteness condition **(F)**, Then*

$$\int_Q \big(L_p f\big)(q)dq = \int_Q f(q)dq, \qquad (p \in Q \text{ and } f \in L_{2,0}(Q, m)), \tag{1.4}$$

where $L_{2,0}(Q, m)$ is the subspace of finite functions from $L_2(Q, m)$.

Theorem 1.3. *[1] There exists a one-to-one correspondence between normal HCSs with basis unity e and weakly continuous families of bounded involutive GTOs L_p satisfying the finiteness condition, preserving positivity in the space $L_2(Q, m)$ with unimodular strongly invariant measure m,*

Notes

and preserving the unit element. Convolution in the HCS $L_1(Q,m)$ and the corresponding family of GTOs L_p satisfy the relation

$$(f*g)(p)=\int_Q (L_pf)(q)\,g(q^*)dq=(L_pf,g^*),\ \ (f,g\in L_2(Q,m)). \tag{1.5}$$

Moreover, the HCS $L_1(Q,m)$ is commutative if and only if the GTOs L_p, $p\in Q$ are commutative.

Notes

In this paper we can generalized the concept of HCS to the direct product of HCSs. This work can be immediately generalized to a direct product of any finite number of HCSs. While, the case of infinite number of HCSs is still open. Moreover, it is fairly easy to observe that all our results for direct product of HCSs can be easily investigated for direct products of semigroups and hypergroups (See [16, 17]).

This paper is organized as follows: In section 2, we give the basic definition of the direct product of HCSs and discuss its objects like convolution, characters, normality and commutativity preserving. In section 3, we give an example to improve the concept of direct product of HCSs. In section 4, we introduce and analyze the concept of positive definite functions on the direct product of commutative normal HCSs, and we present the integral representation of positive definite functions. Section 5 is employed for conclusion.

II. Direct Product of Hypercomplex Systems

Suppose that $L_{p_i}(i=1,2)$ be GTOs associated with normal HCSs $L_1(Q_1,m_1)$ and $L_1(Q_2,m_2)$ with basis unity e_1 and e_{2x} respectively. We denote, $\mathbf{H}_1=L_1(Q_1,m_1)$ and $\mathbf{H}_2=L_1(Q_2,m_2)$. The direct product of the GTOs L_{p_1} and L_{p_2} $(p_1\in Q_1,\ p_2\in Q_2)$ is defined as the operator-valued function

$$Q_1\times Q_2\ni(p_1,p_2)\mapsto L_{(p_1,p_2)}=L_{p_1}\otimes L_{p_2}:\mathbf{H}_1\otimes\mathbf{H}_2\to\mathbf{H}_1\otimes\mathbf{H}_2. \tag{2.1}$$

It is clear that the operators $L_{(p_1,p_2)}$ $((p_1,p_2)\in Q_1\times Q_2)$ form in $\mathbf{H}_1\otimes\mathbf{H}_2$ a family of GTOs satisfying the conditions of Theorem 1.3. The HCS $\mathbf{H}_1\otimes\mathbf{H}_2$ constructed from the GTOs $L_{(p_1,p_2)}$ is called the direct product of the HCSs $L_1(Q_1,m_1)$ and $L_1(Q_2,m_2)$. To denote the operation of taking the direct product, we write

$$\mathbf{H}_1\otimes\mathbf{H}_2=L_1(Q_1\times Q_2,m_1\otimes m_2)=L_1(Q_1,m_1)\otimes L_1(Q_2,m_2).$$

The following Lemma shows that the operation of taking the direct product preserves commutativity

Lemma 2.1. The direct product of two commutative HCSs is commutative.

Proof. Let $\mathbf{H}_1,\mathbf{H}_2$ be two commutative HCSs and L_p,L_q be the corresponding GTOs, respectively. According to the above Theorem, it is sufficient to prove that the GTOs $L_{(p_1,p_2)}=L_{p_1}\otimes L_{p_2}$, $L_{(q_1,q_2)}=L_{q_1}\otimes L_{q_2}$ $\big((p_1,p_2),(q_1,q_2)\in Q_1\times Q_2\big)$ are commutative. So from definition 1.2 and Eq.1.2 we have,

$$\big(L_{(p_1,p_2)}\big(L_{(q_1,q_2)}f\big)\big)(r_1,r_2)=\big(L_{(q_1,q_2)}\big(L_{(p_1,p_2)}f\big)\big)(r_1,r_2), \tag{2.2}$$

and hence, the Lemma is proved. ■

There are important concepts related to any HCS like structure measure, multiplicative measure, characters and normality. Now, we transfer these concepts to the direct product of two HCSs.

Let Q_1 and Q_2 be complete separable locally compact metric spaces. $\mathcal{B}(Q_1 \times Q_2)$ is the σ-algebra of Borel subsets from $Q_1 \times Q_2$, and $\mathcal{B}_0(Q_1 \times Q_2)$ be the subring of $\mathcal{B}(Q_1 \times Q_2)$ which consists of sets with compact closure. We will consider the Borel measures, that is, positive regular measures on $\mathcal{B}(Q_1 \times Q_2)$, finite on compact sets. The spaces of continuous functions, of finite continuous functions, of continuous functions vanishing at infinity and of bounded functions are denoted by $C(Q_1 \times Q_2)$, $C_0(Q_1 \times Q_2)$, $C_\infty(Q_1 \times Q_2)$ and $C_b(Q_1 \times Q_2)$, respectively.

Let $A_1 \times A_2, B_1 \times B_2 \in \mathcal{B}_0(Q_1 \times Q_2)$ and let $\mathcal{K}_{(A_1\times A_2)}$ and $\mathcal{K}_{(B_1\times B_2)}$ be the characteristic functions of $A_1 \times A_2, B_1 \times B_2$, respectively. By using Eq.(1.5), we can set up the structure measure of the HCS $\mathbf{H}_1 \otimes \mathbf{H}_2$ as follows

$$\begin{aligned} c\big(A_1 \times A_2, B_1 \times B_2, (r_1, r_2)\big) &= \mathcal{K}_{(A_1\times A_2)} * \mathcal{K}_{(B_1\times B_2)}(r_1, r_2) \\ &= \int\limits_{Q_1\times Q_2} \big(L_{(r_1,r_2)}\mathcal{K}_{(A_1\times A_2)}\big)(q_1, q_2)\mathcal{K}_{(B_1\times B_2)}(q_1^*, q_2^*)d(q_1, q_2), \quad (2.3) \end{aligned}$$

where $(r_1, r_2) \in Q_1 \times Q_2$. This structure measure is said to be commutative whenever

$$c\Big(A_1 \times A_2, B_1 \times B_2, (r_1, r_2)\Big) = c\Big(B_1 \times B_2, A_1 \times A_2, (r_1, r_2)\Big). \quad (2.4)$$

A regular Borel measure $m := m_1 \otimes m_2$ on $\mathcal{B}_0(Q_1 \times Q_2)$ is called multiplicative if

$$\int\limits_{Q_1\times Q_2} c\Big(A_1 \times A_2, B_1 \times B_2, (r_1, r_2)\Big)dm(r_1, r_2) = m(A_1 \times A_2)m(B_1 \times B_2). \quad (2.5)$$

By using Eq.(1.5), we can define the convolution in $\mathbf{H}_1 \otimes \mathbf{H}_2$ as follows

$$\begin{aligned} \big(f * g\big)(p_1, p_2) &= \int\limits_{Q_1\times Q_2} \big(L_{(p_1,p_2)}f\big)(q_1, q_2)g(q_1^*, q_2^*)d(q_1, q_2) \\ &= \big(L_{(p_1,p_2)}f, g^*\big)_{(\mathbf{H}_1\otimes\mathbf{H}_2)_2}, \quad (2.6) \end{aligned}$$

where $f, g \in L_2\big(Q_1 \times Q_2, m_1 \otimes m_2\big) := (\mathbf{H}_1 \otimes \mathbf{H}_2)_2$.

A non zero measurable and bounded almost everywhere function $Q_1 \times Q_2 \ni (r_1, r_2) \mapsto \chi(r_1, r_2) \in \mathbb{C}$ is said to be a character of HCS $\mathbf{H}_1 \otimes \mathbf{H}_2$, if the equality

$$\int\limits_{Q_1\times Q_2} c\big(A_1 \times A_2, B_1 \times B_2, (r_1, r_2)\big)\chi(r_1, r_2)dm(r_1, r_2) = \chi(A_1 \times A_2)\chi(B_1 \times B_2) \quad (2.7)$$

holds for any $A_1 \times A_2, B_1 \times B_2 \in \mathcal{B}_0(Q_1 \times Q_2)$. Every direct product of HCSs has at least one character, namely, the function $\chi = 1$. A non zero measurable complex-valued function $\chi(r_1, r_2)$, $(r_1, r_2) \in Q_1 \times Q_2$ is called a generalized character of $\mathbf{H}_1 \otimes \mathbf{H}_2$, if the equality (2.7) holds.

The HCS $\mathbf{H}_1 \otimes \mathbf{H}_2$ is said to be normal, if there exists an involution homomorphism $Q_1 \times Q_2 \ni (r_1, r_2) \mapsto (r_1^*, r_2^*) \in Q_1 \times Q_2$, such that $m(E_1 \times E_2) = m(E_1^* \times E_2^*)$ $\big(E_1 \times E_2 \in \mathcal{B}(Q_1 \times Q_2)\big)$ and for all $A_1 \times A_2, B_1 \times B_2, C_1 \times C_2 \in \mathcal{B}_0(Q_1 \times Q_2)$, we have

$$\begin{aligned} c\big(A_1 \times A_2, B_1 \times B_2, C_1 \times C_2\big) &= c\big(C_1 \times C_2, B_1^* \times B_2^*, A_1 \times A_2\big), \\ &= c\big(A_1^* \times A_2^*, C_1 \times C_2, B_1 \times B_2\big), \quad (2.8) \end{aligned}$$

Notes

where

$$c(A_1 \times A_2, B_1 \times B_2, C_1 \times C_2) = \int_{C_1 \times C_2} c(A_1 \times A_2, B_1 \times B_2, (r_1, r_2))dm(r_1, r_2). \tag{2.9}$$

A normal HCS $\mathbf{H}_1 \otimes \mathbf{H}_2$ possesses a basis unity if there exists a point $(e_1, e_2) \in Q_1 \times Q_2$ such that $(e_1, e_2) = (e_1^*, e_2^*)$ and

Notes

$$c(A_1 \times A_2, B_1 \times B_2, (e_1 \times e_2)) = m((A_1^* \times A_2^*) \cap (B_1 \times B_2)), \tag{2.10}$$

where $A_1 \times A_2, B_1 \times B_2 \in \mathcal{B}(Q_1 \times Q_2)$.

A normal HCS $\mathbf{H}_1 \otimes \mathbf{H}_2$ is called Hermitian if $(r_1^*, r_2^*) = (r_1, r_2)$ fore all $(r_1, r_2) \in Q_1 \times Q_2$. Every Hermitian direct product of HCSs is commutative. We should remark that, for a normal HCS $\mathbf{H}_1 \otimes \mathbf{H}_2$, the mapping

$$\mathbf{H}_1 \otimes \mathbf{H}_2 \ni f(r_1, r_2) \mapsto f^*(r_1, r_2) \in \mathbf{H}_1 \otimes \mathbf{H}_2 \tag{2.11}$$

is an involution in the Banach algebra $\mathbf{H}_1 \otimes \mathbf{H}_2$. A character χ of a normal HCS $\mathbf{H}_1 \otimes \mathbf{H}_2$ is said to be Hermitian if

$$\chi(r_1^*, r_2^*) = \overline{\chi(r_1, r_2)}, \quad (r_1, r_2) \in Q_1 \times Q_2. \tag{2.12}$$

Denote the families of characters, of generalized characters and of bounded Hermitian characters by $\mathbf{X}$, $\mathbf{X}_g$ and $\mathbf{X}_h$, respectively.

The following result gives us the criterium of the generalized characters of a normal commutative direct product of HCSs.

Lemma 2.2. *In order that a function $\chi(r_1, r_2) \in C(Q_1 \times Q_2)$ be a generalized character of the normal commutative direct product of HCS $\boldsymbol{H}_1 \otimes \boldsymbol{H}_2$ with basis unity (e_1, e_1) it is necessary and sufficient that the equality*

$$(L_{(p_1,p_2)}\chi)(q_1, q_2) = \chi(p_1, p_2)\chi(q_1, q_2), \tag{2.13}$$

hold for almost all $(p_1, p_2), (q_1, q_2) \in (Q_1 \times Q_2)$.

Proof. Assume that a function $\chi \in X_g$. Then, we have

$$\begin{aligned}
\chi(A_1 \times A_2)\chi(B_1 \times B_2) &= \int_{Q_1 \times Q_2} c(A_1 \times A_2, B_1 \times B_2, (r_1, r_2))\chi(r_1, r_2)d(r_1, r_2) \\
&= \int_{Q_1 \times Q_2} \int_{B_1^* \times B_2^*} (L_{(r_1,r_2)}\mathcal{K}_{(A_1 \times A_2)})(s_1, s_2)d(s_1, s_2)\chi(r_1, r_2)d(r_1, r_2) \\
&= \int_{B_1 \times B_2} \int_{Q_1 \times Q_2} (L_{(s_1^*,s_2^*)}\mathcal{K}_{(A_1 \times A_2)})(r_1, r_2)\chi(r_1, r_2)d(r_1, r_2)d(s_1, s_2) \\
&= \int_{B_1 \times B_2} \int_{A_1 \times A_2} (L_{(s_1,s_2)}\chi)(r_1, r_2)d(r_1, r_2)d(s_1, s_2) \qquad (2.14)
\end{aligned}$$

for any $A_1 \times A_2, B_1 \times B_2 \in \mathcal{B}_0(Q_1 \times Q_2)$, which yields 2.13. The converse statement can be proved by analogy. ■

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Practically, to illustrate the concept of direct product of HCSs, we give an example as follows:
Example 2.1. Let $Q_1 = G_1, Q_2 = G_2$ be commutative locally compact groups. It is easy to see that $Q_1 \times Q_2 = G_1 \times G_2$ is commutative locally compact group with unity (e_1, e_2), where e_1 and e_2 are the unities of G_1 and G_2, respectively. Consider its group algebra, i.e., a set $L_1(G_1 \times G_2, m)$ of functions defined on the group $G_1 \times G_2$ and summable with respect to the Haar measure $m := m_1 \otimes m_2$. So, we can define the involution

$$G_1 \times G_2 \ni (p_1, p_2) \longmapsto (p_1^*, p_2^*) \in G_1 \times G_2. \tag{2.15}$$

In this case, where

$$\big(L_{(p_1,p_2)} f\big)(q_1, q_2) = f\big((q_1, q_2)(p_1, p_2)\big), \quad (p_1, p_2), (q_1, q_2) \in G_1 \times G_2, \tag{2.16}$$

we have the convolution

$$\big(f * g\big)(p_1, p_2) = \int\limits_{Q_1 \times Q_2} f\big((q_1, q_2)(p_1, p_2)\big) g(q_1^*, q_2^*) d(q_1, q_1) \tag{2.17}$$

Also, the structure measure has the form,

$$c\big(A_1 \times A_2, B_1 \times B_2, (r_1, r_2)\big) = m\big((A_1^{-1} \times A_2^{-1})(r_1, r_2) \cap (B_1 \times B_2)\big), \tag{2.18}$$

where $A_1 \times A_2, B_1 \times B_2 \in \mathcal{B}(G_1 \times G_2), (r_1, r_2) \in G_1 \times G_2$. Thus, we obtain the direct product of the commutative HCSs $\mathbf{H}_1 \otimes \mathbf{H}_2$. This direct product is also commutative and with basis unity (e_1, e_2). In particular, if $G_1 \times G_2 = \mathbb{R} \times \mathbb{R}$ is an additive groups of all real numbers. For such HCSs it is possible to introduce generalized translation $L_{(p_1,p_2)}$:

$$\mathbb{R} \times \mathbb{R} \ni (p_1, p_2) \longmapsto \big(L_{(p_1,p_2)} f\big)(q_1, q_2) \in \mathbb{C}, \quad f \in C(\mathbb{R} \times \mathbb{R}),$$

where $\big(L_{(p_1,p_2)} f\big)(q_1, q_2) = f\big((q_1, q_2) + (p_1, p_2)\big)$. By using the operators $L_{(p_1,p_2)}$, one can rewrite the involution and convolution as follows respectively:

$$\mathbb{R} \times \mathbb{R} \ni (p_1, p_2) \longmapsto (p_1^*, p_2^*) := (p_1^{-1}, p_2^{-1}) \in \mathbb{R} \times \mathbb{R}, \tag{2.19}$$

$$\begin{aligned} \big(f * g\big)(p_1, p_2) &= \int\limits_{\mathbb{R} \times \mathbb{R}} f(q_1, q_2)\big(L_{(q_1^*, q_2^*)} g\big)(p_1, p_2) d(q_1, q_1) \\ &= \int\limits_{\mathbb{R} \times \mathbb{R}} f(q_1, q_2) g\big((p_1, p_2) - (q_1, q_2)\big) d(q_1, q_1), \end{aligned} \tag{2.20}$$

where $(q_1^*, q_2^*) = (-q_1, -q_2)$ in additive groups $\mathbb{R} \times \mathbb{R}$, $f, g \in \mathbf{H}_1 \otimes \mathbf{H}_2$ and the functions $\chi(t_1, t_2) = e^{i(t_1,t_2)(s_1,s_2)}$, $\big((s_1, s_2) \in \mathbb{R} \times \mathbb{R}\big)$ are characters.

Actually, there are many examples can be modified to the case of direct product of HCSs. For more details see [1, 19].

III. Positive Definite Functions on Direct Product of HCSs

In this section, we present a concept of positive definite functions on a commutative normal direct product of HCSs with basis unity. So, we give the following definitions and the important concepts of positive definite functions.

Notes

Definition 3.1. An essentially bounded function $\Theta(p_1,p_2)$ $\big((p_1,p_2)\in Q_1\times Q_2\big)$ is called positive definite if

$$\int\limits_{Q_1\times Q_2}\Theta(p_1,p_2)\Big(x^* * x\Big)(p_1,p_2)d(p_1,p_2)\geq 0 \tag{3.1}$$

for all $x\in\mathbf{H}_1\otimes\mathbf{H}_2$. We also, present another definition of positive definiteness as the following.

Notes

Definition 3.2. A continuous bounded function $\Theta(p_1,p_2)$ $\big((p_1,p_2)\in Q_1\times Q_2\big)$ is called positive definite if the inequality

$$\sum_{i,j=1}^{n}\lambda_i\overline{\lambda_j}\big(L_{\left((p_1)^*,(p_2)^*\right)_i}\Theta\big)(p_1,p_2)_j\geq 0 \tag{3.2}$$

holds for all $(p_i,p_j),...,(p_n,p_n)\in Q_1\times Q_2$, $(p_1,p_2)_i^*:=\big((p_1)_i^*,(p_2)_i^*\big),(p_1,p_2)_j:=\big((p_1)_j,(p_2)_j\big)$, $\big(i,j=1,...,n\ (n\in\mathbb{N})\big)$ and $\lambda_1,...,\lambda_n\in\mathbb{C}$.

Lemma 3.1. If the GTOs $L_{(t_1,t_2)}$ extended to $L_\infty: C_b(Q_1\times Q_2)\to C_b\Big((Q_1\times Q_2)\times(Q_1\times Q_2)\Big)$. Then the definitions 3.1 and 3.2 are equivalent for the functions $\phi\in C_b(Q_1\times Q_2)$.

Proof. From definition 3.1, we have

$$\begin{aligned}
&\int\limits_{Q_1\times Q_2}\phi(r_1,r_2)\Big(x^* * x\Big)(t_1,t_2)d(t_1,t_2)\\
&=\int\limits_{Q_1\times Q_2}\phi(t_1,t_2)\int\limits_{Q_1\times Q_2}\big(L_{(s_1,s_2)}x\big)(t_1,t_2)\overline{x(s_1,s_2)}d(s_1,s_2)d(t_1,t_2)\\
&=\int\limits_{Q_1\times Q_2}\int\limits_{Q_1\times Q_2}\big(L_{(s_1^*,s_2^*)}\phi\big)(t_1,t_2)\overline{x(s_1,s_2)}d(s_1,s_2)x(t_1,t_2)d(t_1,t_2)\\
&=\int\limits_{Q_1\times Q_2}\int\limits_{Q_1\times Q_2}\big(L_{(t_1,t_2)}\phi\big)(s_1^*,s_2^*)x(t_1,t_2)\overline{x(s_1,s_2)}d(t_1,t_2)d(s_1,s_2)\\
&\geq\ 0,
\end{aligned}\tag{3.3}$$

where $x\in\mathbf{H}_1\otimes\mathbf{H}_2$. By the condition, we have $\big(L_{(t_1,t_2)}\phi\big)(s_1^*,s_2^*)\in C_b\big((Q_1\times Q_2)\times(Q_1\times Q_2)\big)$, then the last inequality clearly implies (3.2). Let us prove the converse assertion. Let $Q_n\times Q_n$ be an increasing sequence of compact sets covering the entire $Q_1\times Q_2$. We consider a function $\Omega(r_1,r_2)\in C_0(Q_1\times Q_2)$ and set $\lambda_i=\Omega(r_1,r_2)_i$ in (3.2) This yields

$$\sum_{i,j=1}^{n}\Big(L_{\left(r_1^*,r_2^*\right)_i}\phi\Big)(r_1,r_2)_j\,\Omega(r_1,r_2)_i\,\overline{\Omega(r_1,r_2)_j}\geq 0. \tag{3.4}$$

By integrating this inequality with respect to each $(r_i,r_j),...,(r_n,r_n)$, over the set $Q_k\times Q_k$ $(k\in\mathbb{N})$ and collecting similar terms, we conclude that

$$\begin{aligned}
&nm(Q_k\times Q_k)\int\limits_{Q_k\times Q_k}\big(L_{(r_1^*,r_2^*)}\phi\big)(r_1,r_2)\,|\Omega(r_1,r_2)|^2d(r_1,r_2)\\
&+\ n(n-1)\int\limits_{Q_k\times Q_k}\int\limits_{Q_k\times Q_k}\big(L_{(r_1^*,r_2^*)}\phi\big)(s_1,s_2)\,\Omega(r_1,r_2)\,\overline{\Omega(s_1,s_2)}d(r_1,r_2)d(s_1,s_2)\\
&\geq\ 0
\end{aligned}\tag{3.5}$$

Further, we divide this inequality by n^2 and pass to the limit as $n \to \infty$. We get

$$\int\limits_{Q_k \times Q_k} \int\limits_{Q_k \times Q_k} \left(L_{(r_1^*, r_2^*)}\phi\right)(s_1, s_2)\, \Omega(r_1, r_2)\, \overline{\Omega(s_1, s_2)} d(r_1, r_2) d(s_1, s_2) \geq 0 \tag{3.6}$$

for each $k \in \mathbb{N}$. By passing to the limit as $k \to \infty$. and applying Lebesgue theorem, we see that (3.1) holds for all functions from $C_0(Q_1 \times Q_2)$. Approximating an arbitrary function from $\mathbf{H}_1 \otimes \mathbf{H}_2$ by finite continuous functions, we arrive at (3.1) ■

By $\mathcal{P}(Q_1 \times Q_2)$ we denote the set of all positive definite functions.

Lemma 3.2. *If x belongs to $(\boldsymbol{H}_1 \otimes \boldsymbol{H}_2)_2$, then* $\left(x^* * x\right) \in \mathcal{P}(Q_1 \times Q_2)$.

Proof. The proof is an immediately consequence of Lemma 3.3 in [1]. ■

Definition 3.3. For any function $x \in \mathbf{H}_1 \otimes \mathbf{H}_2$ and any character $\chi \in \mathbf{X}$, we set

$$\widehat{x}(\chi) = \int\limits_{Q_1 \times Q_2} x(r_1, r_2) \overline{\chi(r_1, r_2)} d(r_1, r_2). \tag{3.7}$$

This integral exists and $\widehat{x}$ is a continuous function on $\mathbf{X}$. It is called a Fourier transform of the function $x \in \mathbf{H}_1 \otimes \mathbf{H}_2$.

It well known that, every positive definite function in a HCS has a unique integral representation with respect to a nonnegative finite regular measure defined on the family of Hermitian characters (see Theorem 3.1 in [1]). Theorem 3.3 below gives a similar representation, but for positive definite functions in $\mathbf{H}_1 \otimes \mathbf{H}_2$.

Theorem 3.3. *Every function $\Theta \in \mathcal{P}(Q_1 \times Q_2)$ admits a unique representation in the form of an integral*

$$\Theta(r_1, r_2) = \int\limits_{\boldsymbol{X}_h} \chi(r_1, r_2) d\mu(\chi), \qquad (r_1, r_2) \in Q_1 \times Q_2, \tag{3.8}$$

where μ is a nonnegative finite regular measure on the space $\boldsymbol{X}_h$. Conversely, each function of the form (3.8) belongs to $\mathcal{P}(Q_1 \times Q_2)$.

Proof. Let $\Theta \in \mathcal{P}(Q_1 \times Q_2)$. Consider a continuous functional Φ in $\mathbf{H}_1 \otimes \mathbf{H}_2$ defined as follows

$$\Phi(x) = \int\limits_{Q_1 \times Q_2} \Theta(r_1, r_2) x(r_1, r_2) d(r_1, r_2), \quad (x \in \mathbf{H}_1 \otimes \mathbf{H}_2). \tag{3.9}$$

It is clear that this functional is positive. The functional Φ can be extended to a positive functional $\widetilde{\Phi}$ in a commutative normal direct product of HCSs with basis unity $\mathbf{H}_1 \otimes \mathbf{H}_2$. To do this, it suffices to show that

- The functional Φ is real $\left(\text{i.e., } \Phi(x^*) = \overline{\Phi(x)} \text{ for all } x \in \mathbf{H}_1 \otimes \mathbf{H}_2\right)$,
- The inequality $\left|\Phi(x)\right|^2 \leq C\Phi(x^*)$ holds, where C is a constant.

Let $e_n \in \mathbf{H}_1 \otimes \mathbf{H}_2$ be an approximative unit, that is, $e_n(r_1, r_2) \geq 0$, $e_n(r_1, r_2) = e_n(r_1^*, r_2^*)$ $(r_1, r_2) \in Q_1 \times Q_2$, $\|e_n\|_{\mathbf{H}_1 \otimes \mathbf{H}_2} = 1$ and for all $x \in \mathbf{H}_1 \otimes \mathbf{H}_2$, $\lim\limits_{n \to \infty} e_n * x = x$ weakly in $\mathbf{H}_1 \otimes \mathbf{H}_2$. Since Φ is positive, we have

$$\Phi(x^*) = \lim_{n \to \infty} \Phi(e_n^*(r_1, r_2) * x^*) = \lim_{n \to \infty} \overline{\Phi(x * e_n(r_1, r_2))} = \overline{\Phi(x)} \tag{3.10}$$

Ref

1. Yu. M. Berezansky and A. A. Kalyuzhnyi, Harmonic analysis in hypercomplex systems, Naukova Dumka, Kyiv: Ukrania, 1992.

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for all $x \in \mathbf{H}_1 \otimes \mathbf{H}_2$. Further, by using Lemma 1.3 in [1], we obtain

$$\begin{aligned} \left|\Phi(x)\right|^2 &= \lim_{n\to\infty} \left|\Phi\big(e_n(r_1,r_2)*x\big)\right|^2 \\ &\leq \lim_{n\to\infty} \Phi\big(e_n^*(r_1,r_2)*e_n(r_1,r_2)\big)\Phi\big(x^* * x\big) \\ &\leq \|\Phi\|\Phi\big(x^* * x\big). \end{aligned} \tag{3.11}$$

Consequently, it is possible to extend Φ to a positive functional $\widetilde{\Phi}$ on $\mathbf{H}_1 \otimes \mathbf{H}_2$. By virtue of the theorem on representations of positive functionals on commutative Banach $*$-algebras with identity element, the functional $\widetilde{\Phi}$ (and, hence, Φ) can be uniquely represented in the form

$$\Phi(x) = \int\limits_{\mathbf{X}_h}\int\limits_{Q_1\times Q_2} x(r_1,r_2)\chi(r_1,r_2)d(r_1,r_2)d\mu(\chi), \tag{3.12}$$

where μ is a finite regular Borel measure on $\mathcal{B}_0(\mathbf{X}_h)$. From Eqs.(3.9) and (3.12), we obtain the following relation

$$\Theta(r_1,r_2) = \int\limits_{\mathbf{X}_h} \chi(r_1,r_2)d\mu(\chi),$$

almost everywhere on $Q_1 \times Q_2$. Since the characters of $\mathbf{H}_1 \otimes \mathbf{H}_2$ are continuous, both functions in this equality are also continuous. This yields Eq.(3.8). The second part of the theorem follows from the relation

$$\begin{aligned} &\int\limits_{Q_1\times Q_2}\int\limits_{\mathbf{X}_h} \chi(r_1,r_2)d\mu(\chi)\big(x^* * x\big)(r_1,r_2)d(r_1,r_2) \\ &= \int\limits_{\mathbf{X}_h}\int\limits_{Q_1\times Q_2} \big(x^* * x\big)(r_1,r_2)\chi(r_1,r_2)d(r_1,r_2)d\mu(\chi) \\ &= \int\limits_{\mathbf{X}_h} \left|\widehat{x}(\chi)\right|^2 d\mu(\chi) \geq 0, \end{aligned} \tag{3.13}$$

where $\widehat{x}(\chi)$ is the Fourier transform of the functions $x \in \mathbf{H}_1 \otimes \mathbf{H}_2$. For all $\chi \in \mathbf{X}_h$, we have $\widehat{(x^*)}(\chi) = \overline{(\widehat{x})(\chi)}$, In particular, $\big(\widehat{x^* * x}\big)(\chi) = \left|\widehat{x}(\chi)\right|^2$. See [1] and the Lebesgue theorem on the limit transition. ■

Corollary 3.4. If the product of any two Hermitian characters is positive definite in $\mathbf{H}_1 \otimes \mathbf{H}_2$*, then the product of any two continuous positive definite functions in* $\mathbf{H}_1 \otimes \mathbf{H}_2$ *is also positive definite.*

Proof. Let χ and are two Hermitian characters and positive definite in $\mathbf{H}_1 \otimes \mathbf{H}_2$, by virtue of Theorem 3.3, we have

$$\begin{aligned} &\int\limits_{Q_1\times Q_2} f(r_1,r_2)g(r_1,r_2)\big(x^* * x\big)(r_1,r_2)d(r_1,r_2) \\ &= \int\limits_{Q_1\times Q_2}\int\limits_{\mathbf{X}_h} \chi(r_1,r_2)d\mu(\chi)\int\limits_{\mathbf{X}_h} \quad (r_1,r_2)d\nu(\quad)\big(x^* * x\big)(r_1,r_2)d(r_1,r_2) \\ &= \int\limits_{\mathbf{X}_h}\int\limits_{\mathbf{X}_h}\int\limits_{Q_1\times Q_2} \chi(r_1,r_2)\quad (r_1,r_2)\big(x^* * x\big)(r_1,r_2)d(r_1,r_2)d\mu(\chi)d\nu(\quad) \geq 0 \end{aligned} \tag{3.14}$$

for all $f, g \in \mathcal{P}(Q_1 \times Q_2)$, $x \in \mathbf{H}_1 \otimes \mathbf{H}_2$. ■

Ref

1. Yu. M. Berezansky and A. A. Kalyuzhnyi, Harmonic analysis in hypercomplex systems, Naukova Dumka, Kyiv: Ukrania, 1992.

Corollary 3.5. Assume that $\mathbf{H}_1 \otimes \mathbf{H}_2$ is a commutative direct product of HCSs with basis unity, then a continuous bounded function $\varphi(r_1, r_1)$ is positive definite in the sense of (3.1) if and only if it is positive definite in the sense of (3.2). Moreover, it has the following properties.

(i) $\varphi(e_1, e_2) \geq 0$,

(ii) $\varphi(r_1^*, r_2^*) = \overline{\varphi(r_1, r_2)}$,

(iii) $|\varphi(r_1, r_2)| \leq \varphi(e_1, e_2)$,

(iv) $|(L_{(s_1,s_2)}\varphi)(t_1, t_2)|^2 \leq (L_{(s_1^*,s_2^*)}\varphi)(s_1, s_2)(L_{(t_1^*,t_2^*)}\varphi)(t_1, t_2)$,

(v) $|\varphi(s_1, s_2) - \varphi(t_1, t_2)|^2 \leq 2\varphi(e_1, e_2)\big[\varphi(e_1, e_2) - Re\big(L_{(s_1,s_2)}\varphi\big)(t_1^*, t_2^*)\big]$.

Proof. The first part of this Corollary we can found it from Lemma 3.1, from Theorem 3.3, we can proof the second part from **(i)** to **(v)** as following

$$\varphi(e_1, e_2) = \int_{\mathbf{X}_h} \chi(e_1, e_2) d\mu(\chi) = \mu(\mathbf{X}_h) \geq 0, \tag{3.15}$$

$$\varphi(r_1^*, r_2^*) = \int_{\mathbf{X}_h} \chi(r_1^*, r_2^*) d\mu(\chi) = \int_{\mathbf{X}_h} \overline{\chi(r_1, r_2)} d\mu(\chi) = \overline{\varphi(r_1, r_2)}, \tag{3.16}$$

$$\Big|\varphi(r_1, r_2)\Big| \leq \int_{\mathbf{X}_h} \Big|\chi(r_1, r_2)\Big| d\mu(\chi) \leq \mu(\mathbf{X}) = \varphi(e_1, e_2), \tag{3.17}$$

$$\begin{aligned}
\big|(L_{(s_1,s_2)}\varphi)(t_1, t_2)\big|^2 &= \Big|\int_{\mathbf{X}_h} \chi(s_1, s_2)\chi(t_1, t_2) d\mu(\chi)\Big|^2 \\
&\leq \int_{\mathbf{X}_h} \Big|\chi(s_1, s_2)\Big|^2 d\mu(\chi) \int_{\mathbf{X}_h} \Big|\chi(t_1, t_2)\Big|^2 d\mu(\chi) \\
&= \int_{\mathbf{X}_h} \chi(s_1, s_2)\chi(s_1^*, s_2^*) d\mu(\chi) \int_{\mathbf{X}_h} \chi(t_1, t_2)\chi(t_1^*, t_2^*) d\mu(\chi) \\
&= \big(L_{(s_1^*,s_2^*)}\varphi\big)(s_1, s_2)\big(L_{(t_1^*,t_2^*)}\varphi\big)(t_1, t_2),
\end{aligned} \tag{3.18}$$

Finally,

$$\begin{aligned}
\big|\varphi(s_1, s_2) - \varphi(t_1, t_2)\big|^2 &= \Big|\int_{\mathbf{X}_h} \chi(s_1, s_2) d\mu(\chi) - \int_{\mathbf{X}_h} \chi(t_1, t_2) d\mu(\chi)\Big|^2 \\
&\leq \Big|\int_{\mathbf{X}_h} \big(\chi(s_1, s_2) - \chi(t_1, t_2) d\mu(\chi)\Big|^2 \\
&\leq \mu(\mathbf{X}_h) \int_{\mathbf{X}_h} \Big|\big(\chi(s_1, s_2) - \chi(t_1, t_2)\Big|^2 d\mu(\chi) \\
&= \varphi(e_1, e_2) \int_{\mathbf{X}_h} \Big(\big|\chi(s_1, s_2)\big|^2 + \big|\chi(t_1, t_2)\big|^2 - 2Re\, \chi(s_1, s_2)\overline{\chi(t_1, t_2)}\Big) d\mu(\chi) \\
&\leq \varphi(e_1, e_2) \int_{\mathbf{X}_h} 2\Big(1 - Re\, \big(L_{(s_1,s_2)}\chi\big)(t_1^*, t_2^*)\Big) d\mu(\chi) \\
&= 2\varphi(e_1, e_2)\Big[\varphi(e_1, e_2) - Re\, \big(L_{(s_1,s_2)}\varphi\big)(t_1^*, t_2^*)\Big].
\end{aligned} \tag{3.19}$$

Hence, the Corollary is proved. ∎

In the remaining part of this section, we present the necessary and sufficient conditions guarantees that the property of positive definiteness on the direct product of HCSs is preserved under the usual function product.

Let $\mathbf{H}_1 \otimes \mathbf{H}_2$ be a commutative direct product of HCSs. The following two lemmas are in fact, an adaption of whatever done for semigroups in Berg et al. [18]. We will not repeat the proof, wherever the proof for semigroups can be applied to the HCSs [5]. In our work, we can applay it to the direct product of HCSs $\mathbf{H}_1 \otimes \mathbf{H}_2$ with necessary modification.

Lemma 3.6. (*i*) *The sum and the point-wise limit of positive definite functions in $\mathbf{H}_1 \otimes \mathbf{H}_2$ are also positive definite.*

(*ii*) *Let ϕ be a continuous positive definite function on $Q \times Q$ and define $\Phi : \mathbf{H}_1 \otimes \mathbf{H}_2 \longrightarrow \mathbb{C}$ by $\Phi(x) := \int_{Q_1 \times Q_2} \phi(s_1, s_2) dm(s_1, s_2)$, $x \in \mathbf{H}_1 \otimes \mathbf{H}_2$. Then Φ is positive definite in $\mathbf{H}_1 \otimes \mathbf{H}_2$.*

Proof. The proof is as the case of semigroups and HCSs [5, 18]. ∎

Lemma 3.7. *A bounded measurable function $\phi \in C_c(Q_1 \times Q_2)$ is positive definite if and only if there exists a in $(\mathbf{H}_1 \otimes \mathbf{H}_2)_2$ such that $\phi = \bullet\,\widetilde{\ }$, where*

$$f \bullet \widetilde{g}(r_1, r_2) = \int_{Q_1 \times Q_2} f\big((r_1, r_2) * (s_1, s_2)\big)\overline{g(s_1, s_2)} dm(s_1, s_2), \tag{3.20}$$

for all $f,\ g \in (\mathbf{H}_1 \otimes \mathbf{H}_2)_2$.

Proof. The proof is as Lemma 7.2.4 in Pederson [20]. ∎

Theorem 3.8. *Let ϕ_1 and ϕ_2 belongs to $C_c(Q_1 \times Q_2)$, then the product $\phi_1.\phi_2$ is positive definite on $Q_1 \times Q_2$ if and only if ϕ_1 and ϕ_2 are positive definite on $Q_1 \times Q_2$.*

Proof. From Lemma 3.7, there exist $f, g \in (\mathbf{H}_1 \otimes \mathbf{H}_2)_2$ such that $\phi_1 = f \bullet \widetilde{f}$, $\phi_2 = g \bullet \widetilde{g}$. So, we have

$$\begin{aligned}
\phi_1 \,.\, \phi_2(r_1, r_2) &= \Big(f \bullet \widetilde{f}(r_1, r_2)\Big).\Big(g \bullet \widetilde{g}(r_1, r_2)\Big) \\
&= \int_{Q_1 \times Q_2} f\Big((r_1, r_2) * (s_1, s_2)\Big)\overline{f(s_1, s_2)} dm(s_1, s_2) \\
&\times \int_{Q_1 \times Q_2} g\Big((r_1, r_2) * (t_1, t_2)\Big)\overline{g(t_1, t_2)} dm(t_1, t_2) \\
&= \int_{Q_1 \times Q_2} \int_{Q_1 \times Q_2} f\Big((r_1, r_2) * (s_1, s_2)\Big) g\Big((r_1, r_2) * (t_1, t_2)\Big) \\
&\times \overline{f(s_1, s_2) g(t_1, t_2)}\, dm(s_1, s_2)\, dm(t_1, t_2) \\
&= \int_{Q_1 \times Q_2} \int_{Q_1 \times Q_2} f \,.\, g\Big((r_1, r_2) * (s_1, s_2), (r_1, r_2) * (t_1, t_2)\Big) \\
&\times \overline{f.g\big((s_1, s_2), (t_1, t_2)\big)}\, dm(s_1, s_2)\, dm(t_1, t_2) \\
&= \int \int f \,.\, g\Big((r_1, r_2) * \big((s_1, s_2), (t_1, t_2)\big)\Big)
\end{aligned}$$

Ref

18. C. Berg, J. P. R. Christensen, P. Ressel, Harmonic Analysis on Semigroups: Theory of Positive Definite and Related Functions, Springer-Verlag: Berlin, Heidelberg, New York, 1984.

$$\times \quad \overline{f\,.\,g\big((s_1,s_2),(t_1,t_2)\big)}^{\,Q_1\times Q_2\, Q_1\times Q_2}\, dm(s_1,s_2)\, dm(t_1,t_2). \tag{3.21}$$

Applying Fubini's theorem to the right hand side, we get

$$\phi_1.\phi_2(r_1,r_2) \;= \int\limits_{(Q_1\times Q_2)\times(Q_1\times Q_2)} f\,.\,g\Big((r_1,r_2) * \big((s_1,s_2),(t_1,t_2)\big)\Big)$$

$$\times \quad \overline{f\,.\,g\big((s_1,s_2),(t_1,t_2)\big)}\, dn\big((s_1,s_2),(t_1,t_2)\big). \tag{3.22}$$

This implies that $\phi_1.\phi_2(r_1,r_2) = f\,.\,g \bullet \widetilde{f\,.\,g}(r_1,r_2)$. ■

IV. Conclusion

A direct product of two HCSs is precisely defined via the theory of GTOs. We showed that, under some conditions, the properties of commutativity, normality are preserved under the operation of taking the direct product. Some examples were given to improve the concept of direct product of HCSs. Also, we transferred the objects of harmonic analysis, namely, the criteria of positive definite, the integral representation of positive definite functions, the positive definiteness of the product of two HCSs.

This work can be immediately generalized to a direct product of any finite number of HCSs. While, the case of infinite number of HCSs is still open. Moreover, it is fairly easy to observe that all our results for direct product of HCSs can be easily investigated for direct products of semigroups and hypergroups (See [16–18]).

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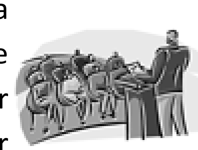

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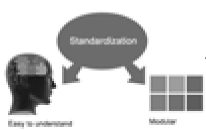

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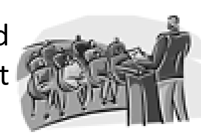

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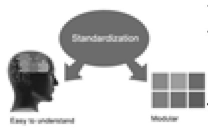

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- Never confuse figures with tables - there is a difference.

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- As forever, use past tense when you submit to your results, and put the whole thing in a reasonable order.
- Put figures and tables, appropriately numbered, in order at the end of the report
- If you desire, you may place your figures and tables properly within the text of your results part.

Figures and tables

- If you put figures and tables at the end of the details, make certain that they are visibly distinguished from any attach appendix materials, such as raw facts
- Despite of position, each figure must be numbered one after the other and complete with subtitle
- In spite of position, each table must be titled, numbered one after the other and complete with heading
- All figure and table must be adequately complete that it could situate on its own, divide from text

Discussion:

The Discussion is expected the trickiest segment to write and describe. A lot of papers submitted for journal are discarded based on problems with the Discussion. There is no head of state for how long a argument should be. Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implication of the study. The purpose here is to offer an understanding of your results and hold up for all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of result should be visibly described. Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved with prospect, and let it drop at that.

- Make a decision if each premise is supported, discarded, or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."
- Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work
- You may propose future guidelines, such as how the experiment might be personalized to accomplish a new idea.
- Give details all of your remarks as much as possible, focus on mechanisms.
- Make a decision if the tentative design sufficiently addressed the theory, and whether or not it was correctly restricted.
- Try to present substitute explanations if sensible alternatives be present.
- One research will not counter an overall question, so maintain the large picture in mind, where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

- When you refer to information, differentiate data generated by your own studies from available information
- Submit to work done by specific persons (including you) in past tense.
- Submit to generally acknowledged facts and main beliefs in present tense.

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	A-B	**C-D**	**E-F**
Abstract	Clear and concise with appropriate content, Correct format. 200 words or below	Unclear summary and no specific data, Incorrect form Above 200 words	No specific data with ambiguous information Above 250 words
Introduction	Containing all background details with clear goal and appropriate details, flow specification, no grammar and spelling mistake, well organized sentence and paragraph, reference cited	Unclear and confusing data, appropriate format, grammar and spelling errors with unorganized matter	Out of place depth and content, hazy format
Methods and Procedures	Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads	Difficult to comprehend with embarrassed text, too much explanation but completed	Incorrect and unorganized structure with hazy meaning
Result	Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake	Complete and embarrassed text, difficult to comprehend	Irregular format with wrong facts and figures
Discussion	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

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