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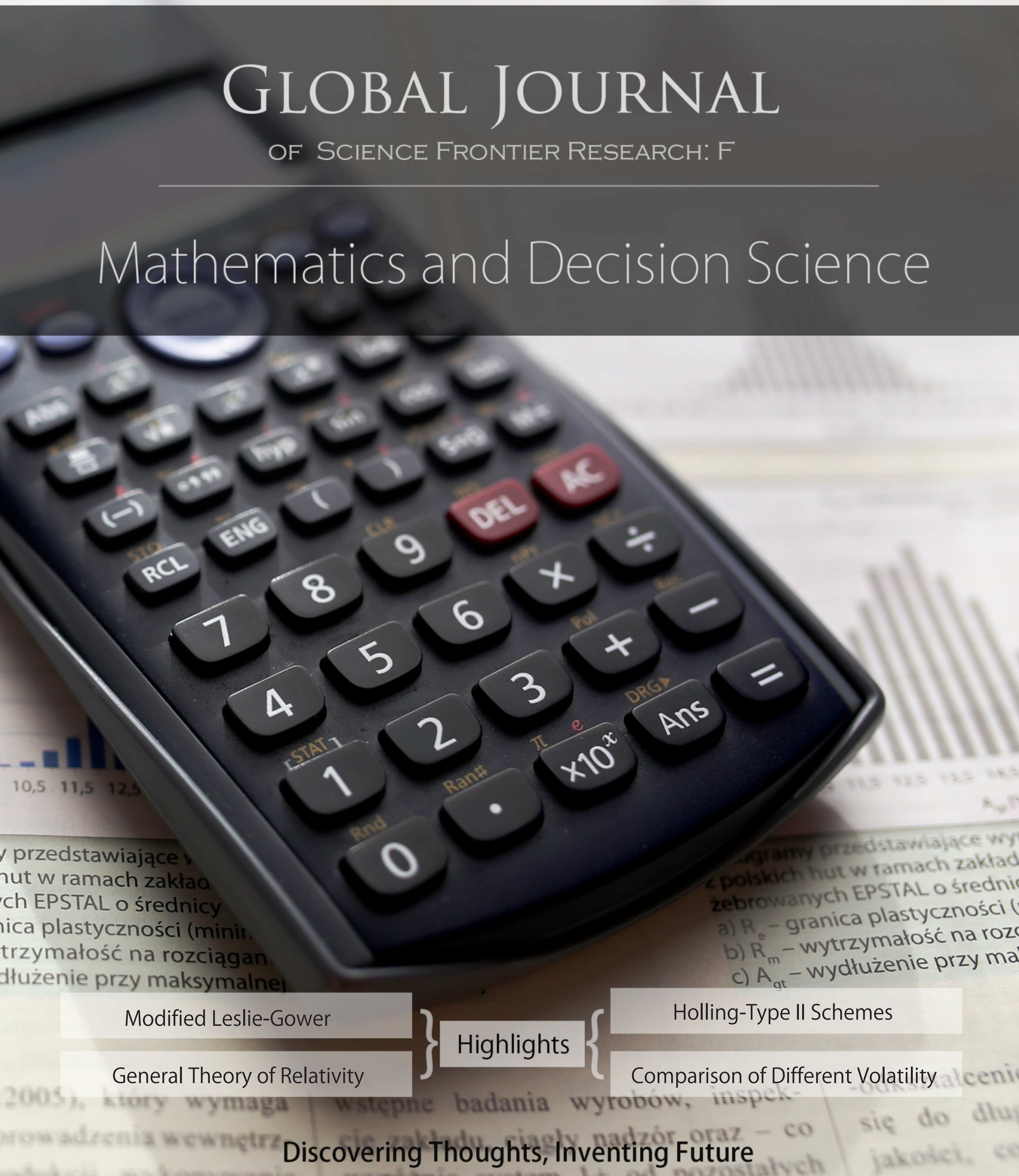

Highlights

- Modified Leslie-Gower
- General Theory of Relativity
- Holling-Type II Schemes
- Comparison of Different Volatility

Discovering Thoughts, Inventing Future

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Deriving Kalman Filter - An Easy Algorithm

By Amaresh Das & Faisal Alkhateeb
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Abstract- The Kalman filter may be easily understood by the econometricians, and forecasters if it is cast as a problem in Bayesian inference and if along the way some well-known results in multivariate statistics are employed. The aim is to motivate the readers by providing an exposition of the key notions of the predictive tool and by laying its derivation in a few easy steps. The paper does not deal with many other ad hoc techniques used in adaptive Kalman filtering.

Keywords: *bayes's theorem, state-space forecasting.*

GJSFR-F Classification: *MSC 2010: 11Y16*

DERIVINGKALMANFILTERANEASYALGORITHM

Notes

Deriving Kalman Filter - An Easy Algorithm

Amaresh Das [α] & Faisal Alkhateeb [σ]

Abstract- The Kalman filter may be easily understood by the econometricians, and forecasters if it is cast as a problem in Bayesian inference and if along the way some well-known results in multivariate statistics are employed. The aim is to motivate the readers by providing an exposition of the key notions of the predictive tool and by laying its derivation in a few easy steps. The paper does not deal with many other ad hoc techniques used in adaptive Kalman filtering.

Keywords: bayes's theorem, state-space forecasting.

I. Introduction

The Kalman filter wants to find at each iteration, the most likely cause of the measurement of Y_t given the approximation made by a flawed estimation the linear dynamics[1]. What is important here is not only that we have the measurement and the prediction, but knowledge of how each is flawed.[2] In the Kalman case, this knowledge is given by the covariance matrixes (essentially fully describing the distribution of the measurement and prediction for the Gaussian case). The main principle of forecasting is to find the model that will produce the best forecasts, not the best fit to the historical data. The model that explains the historical data best may not be best predictive model[3].The power of the Kalmancomes from its ability not only to perform this estimation once (a simple Bayesian task) but to use both estimates and knowledge of their distributions to a distribution for the updated estimate, thus iteratively calculating the best solution for state at each iteration[4].

Let Y_t , Y_{t+1}, ...Y_1, the data (which may be either scalar or vertical) denote the observed values of a variable of interest at times t, t-1, ... 1. We assume that Y_t

[1] The famous work by [1] was extension of Weiner's classical work. They focused attention upon aclass of linear minimum-error variance sequential error estimation algorithm. While the problem of linear minimum variance sequential filtering

[2] In the Kalman case, this knowledge is given by the covariance matrixes (essentially fully describing the distribution of the measurement and prediction for the Gaussian case.While many derivations of the Kalmanfilter are available, utilizing the orthogonalityprinciple orfnding iterative updates to the Best Linear Unbiased Estimator (BLUE), we will derive the Kalmanfilter here using a Bayesian approach, where 'best' is interpretedin the Maximum A-Posteriori (MAP) sense [2]instead of Gaussian innovations. This forecasting algorithm [5] is very flexible method that is particularly suitable in nonstationary time series. The Eq [7] used the method to forecast demand in the alcoholic drink industry over a period that included record demand followed by a drought and the imposition of a new excise duty.

[3] The future may not be described by the same probability as the past. Perhaps neither the past nor the future is a sample from any probability distribution. The time series could be nothing more than a non-recurrent historical record. •The model may involve too many parameters. Over fitted models could account for noise or other features in the data that are unlikely to extend into the future.The error involved in fitting a large number of parameters may be damaging to forecast accuracy, even when the model is correctly specified.

[4] It will be very convenient for the readers to remember the keywords used in the text:Filtering- When we estimate the *current* value given past and current observations, Smoothing: - When estimating *past* values given present and past measures, and Prediction - : When estimating a probable future value given the present and the past measures.

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depends on an unobservable be either a scalar or a vector whose dimension is independent of the dimensions of Y_t the relationship between Y_t and ϕ_t is linear and is specified by the observation equation

$$Y_t = \varpi_t \phi_t + \upsilon_t \tag{1}$$

where ϖ_t is a known quantity. The observation error υ_t is assumed to be normally distributed with mean zero and a known variance υ_t denoted as $\upsilon_t \rightarrow N(0, \upsilon_t)$

The essential difference between the Kalman filter and the conventional linear model representation is that in the former, the state of nature - analogous to the regression coefficients of the latter – is not assumed to be a constant but may change with time. This dynamic feature is incorporated via the system equation wherein

$$\phi_t = \Psi_t \phi_t + \zeta_t \tag{2}$$

Ψ_t being a known quantity and the system equation error $\zeta_t \rightarrow N(0, \zeta_t)$ with ζ_t known. Since there are physical systems for which the state of nature ϕ_t changes over time according to a relationship prescribed by engineering or scientific principles, the ability to include a knowledge of the system behavior in the statistical model is an apparent source of attractiveness of the Kalman filter. Note that the relationships (1) and (2) specified through ϖ_t and ψ_t may or may not change with time, as is also true of the variance υ_t and ζ_t we have subscripted these here for the sake of generality. In addition to the usual linear model assumptions regarding the error terms, we also postulate that υ_t is independent of time. The extension of the case of dependency is straightforward.

II. Extension of the Concept

To look at how the Kalman filter model might be employed in practice, we consider a situation in the context of statistical quality control. Here the observation Y_t is a simple (approximately normal) transform of the number of defectives observed in a sample obtained at time t, while ϕ_{1t} and ϕ_{2t}, represents, respectively, the true defective index of the process and the drift of the index. We have here as the observation equation

$$Y_1 = \phi_{1t} + \upsilon_{1t} \tag{3}$$

and as the system equations

$$\phi_{1,t} = \phi_{2,t} + \zeta_{2,t}$$

$$\phi_{2,t} = \phi_{2,t-1} + \zeta_{2,t}$$

In vector notation, the system of equation becomes $\phi_t = \psi\phi_{t-1} + \pi_t$ where

$$\phi_t = \begin{bmatrix} \phi_{1t} \\ \phi_{2t} \end{bmatrix} \text{and } \pi_t = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \zeta_{1t} \\ \zeta_{2t} \end{bmatrix}$$

$$\zeta = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$$

Notes

does not change with time.

If we examine Y_t -.Yt-1. for this model, we observe that, under the assumptions of constant variance, $\upsilon_t = \upsilon$ and $\zeta_t = \zeta$ the autocorrelation structure of the difference is identical to that of ARIMA (0, 1, 1) process in the sense of [1]. Although with a correspondence is sometimes easily discernible, we should in general, not because of the discrepancies in the philosophies and methodologies involved, consider the two approaches to be equivalent.

Ref

1. Box and Jenkins G M (1970) *Time Series Analysis, Forecasting and Control*, San Francisco, Holden- Day

III. Recursive Procedure of the Filter

The Kalman filter refers to a recursive procedure for inference about the state of nature ϕ_t. The key notion here is that given the data $Y_t = (Y_t, \ldots Y_1)$ the inference about ϕ_t can be carried out through a direct application of a Bayes's theorem.

Prob {State of Nature | Data= Prob (Data | State of Nature}

which can be written as

$$P(\phi_t | Y_t) = P(Y_t | \phi_t, Y_{t-1}) \times P(\phi_t | Y_{t-1}) \quad (4)$$

Where the notion of P (A | B) denotes the probability of occurrence of even A given that (or conditional on) event B has occurred. Note that the expression on the left side of (4) denotes the *posterior* distribution for ϕ at time t, whereas the first and second expression on the right side denotes the likelihood and the prior distribution for ϕ, respectively.

The recursive procedure can best be explained if we focus attention on time point t-1, t = 1, 2, and the observed data until then, $(Y_{t-1}, Y_{t-2 \ldots} Y_1))$. In what follows, we use matrix manipulation in allowing for Y and / or, ϕ to be vectors without explicitly noting them as such.

At t-1, our state of knowledge without ϕ_{t-1} is embodied in the following probability statement for ϕ_{t-1}

$$(\phi_{t-1} | Y_{t-1} \rightarrow N(\hat{\phi}_{t-1}, \Sigma_{t-1}) \quad (5)$$

where $\hat{\phi}$ and Σ_{t-1} are the expectation and variance of $\phi_{t-1} | Y_{t-1}$ In effect (5) represents *posterior* distribution of ϕ_{t-1}; its evaluation will become clear in the subsequent text.

It is helpful to remark here that the recursive procedure is stated off at a time 0 by choosing $\hat{\phi}_0$ and Σ_0 to be our best guess about the mean and the variance of ϕ_0, respectively.

We now look forward to time t but in two stages[5]

1. Prior to observing Y_t and
2. After observing Y_t

[5] Kalman filters are ideal for systems which are continuously changing. They have the advantage that they are light on memory (they don't need to keep any history other than the previous state), and they are very fast, making them well suited for real time problems and embedded systems. For a Monte Carlo Sampling Method for Bayesian Filter see [3] Sequential Bayesian filtering is the extension of the Bayesian estimation for the case when the observed value changes in time. It is a method to estimate the real value of an observed variable that evolves in time. See [11]

Stage 1

Prior to choosing Y_t our best choice for ϕ_t is governed by the system equation (2) and is given by $\Psi\,\phi_{t-1} + \zeta_t$. Since ϕ_{t-1} is described by (5) and state of knowledge above ϕ_t is embodied in the probability statement

$$(\phi_{t-1} \mid Y_{t-1} \rightarrow N(\psi_t, \hat{\phi}_{t-1},\ \Theta = \Psi_t \Sigma_{t-1}\psi' + \zeta_t) \quad (6)$$

This is our prior distribution.

In observing (6) which represents our prior for ϕ_t in the next cycle of (4), we use the well -known result that for any constant c

$$X \rightarrow N(\mu, \Sigma) = CX \rightarrow N(C\mu,\ C\Sigma C')$$

Where C denotes the transpose of C

Stage 2

On observing Y_t our goal is to complete the posterior of ϕ_t using(4). However, to do this, we need to know the likelihood $\Re(\phi_t \mid Y_t)$ or equivalently P (Y_t) the determination of which is undertaken via the following arguments.
Let e_t denote the error in predicting Y_t from the point t-1; thus

$$e_t = Y_t - \hat{Y}_t = Y_t - \omega_t \psi_t \hat{\phi}_{t-1} \quad (7)$$

Since ω_t, ψ_t and $\hat{\phi}_{t-1}$ are all known, observing Y_t is equivalent to observing e_t. Thus (4) can be written as

$$P(\phi_t \mid Y_t, Y_{t-1}) = P(\phi_t \mid e_{t-1}) = P(e_t \mid \phi_t, Y_{t-1})\,\mathbf{X}$$
$$P(\phi_t \mid Y_{t-1}) \quad (8)$$

with P $(e_t \mid \phi_t, Y_{t-1})$ being the likelihood[6].

Using the fact that $Y_t = \omega_t\,\varphi_t + \upsilon_t$ (7) can be written as $e_t = \omega_t(\phi_t - \psi_t\,\phi_{t-1}) + \upsilon_t$ so that $\Sigma(e_t \mid \phi_t, Y_{t-1}) = \omega_t(\phi_t - \psi_t\hat{\phi}_{t-1})$

Since $\upsilon_t \rightarrow N(0, \upsilon_t)$ it follows that the likelihood function is described by

$$(e_t \mid \phi_t, Y_{t-1}) \rightarrow N(\upsilon_t(\phi_t - \psi_t\,\hat{\phi}_{t-1}), \upsilon_t \quad (9)$$

We can now use Bayes's theorem (eel 8) to obtain

$$P(\phi_t \mid Y_t, Y_{t-1}) = \frac{P(e_t \mid \phi_t, Y_{t-1}) \text{ x } P(\phi_t \mid Y_{t-1})}{\int_{\text{all } e_t} P(e_t, \phi_t \mid Y_{t-1})\, d\phi_1} \quad (10)$$

and this best describes our state of knowledge about ϕ_t at time t. Once $P(\phi_t \mid Y_t, Y_{t-1})$ is continued, we can go back to (5) for the next cycle of the recursive procedure. Therefore, Kalman filter can be a very effective forecasting tool. It should be useful in a

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[6] The opportunity exists to proclaim an inherent equivalence of the least square estimation and Kalman filter theory, See [3] See also [2]

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wide variety of situations. [9]developeda complete numerical procedure called 'state-space forecasting' for predicting future values of a multivariate stationary process Y_t given past values. The procedure involves basically two main stages.

a. First fit a canonical state-space model to the given observation using Akaike's canonical correlations technique to determine the dimensions of the state vector and to provide estimates of the non-zero elements of the matrix ω. A multivariate AR model is also fitted to the observations(using AIC to determine the order) to provide estimates of Σ_t and the impulse response matrices.
b. Having filtered and estimated ω, ψ, Σ, the procedures are computed recursively using Kalman's algorithm[7]. Practical applications are given in the paper by Mehra.

IV. Conclusion

The note presents a mathematical theory of Kalman filtering. The filtering techniques is discussed as a problem in Bayesian inference in a series of elementary steps, enabling the optimality of the process to be understood. The style of the note is informal and the mathematics elementary but rigorous, making it accessible to all those with a minimal knowledge of linear algebra and systems theory Many other topics related to Kalman filtering are ignored (for example, Wavelet) although occasionally we referred to them inside the text.

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[7] In addition to the Kalman filtering algorithms there are other time domain algorithms available in literature. Perhaps the most exciting ones are the so-called wavelet algorithms. Wavelets were first introduced by [6].Wavelets are based on translation W(x) $\rightarrow W(x+1)$ and above all on dilation (w (x) $\rightarrow (2x)$ The basic dilation is a two-scale difference equation $\Phi(x) = \Sigma c_k \Phi(2x-k)$ ... We look for a solution normalized by $\int \Phi \, dx = 1$ The first requirement on the coefficients c_k comes from multiplying by 2 and integrating $2\int \Phi \, dx = \Sigma c_k \int \Phi(2x-k)\, d(2x-k)$ yields $\Sigma c_k = 2$. Uniqueness of Φ is ensured by $\Sigma c_k = 2$

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Electromagnetic theory from standpoint of the General Theory of Relativity in Y^n - Space

By Yaremenko Mikola (Nikolay) Ivanovich
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Abstract- In this article we consider a long standing problem concerning of unification of electromagnetic and gravitational field concepts under single geometrical point of view. We discussed possibilities of obtaining of Maxwell's equations from the equations of unified field theory in Y^n - space. This equation we derived from the variation principle of least action in Y^n space and they are the analog of Einstein-Hilbert equation in Y^n - space. We present the theory that unifies electromagnetic and gravitational interactions.

Keywords: *electromagnetism, gravitation, torsion, connection, field equations.*

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I. Introduction

In 1928 Albert Einstein made an attempt to unify theories of electromagnetism and gravity by using the new mathematical concepts of absolute or distant parallelism in non flat spaces, known as teleparallelism.

According to Albert Einstein Riemannian Geometry has led to a physical description of the gravitational field in the theory of general relativity, but it did not provide concepts that can be attributed to the electromagnetic field [12]. So, A. Einstein generalized Riemannian geometry to new structure and called it distant parallelism or teleparallelism, which described by a tetrad field that make possible the distant comparison of vector at different points. But the geometry of teleparallelism isn't natural and all attempts to create unified field theory that base on Riemannian metrics and distant parallelism weren't successful.

In this article we develop new theory of general relativity base on geometrical structure of Y^n- space. Classical gravitational theory combine geometry and gravity but doesn't contain Maxwell's theory. So in previous works we developed new theory of relativity in Y^n- space [21-23] and obtained analog of gravitational-electromagnetic equations in general case. The geometric structure of Y^n- space described by metric and torsion tensors hence the field equations in Y^n- space determine metric and torsion by distribution of energy-matter. This equations contained classical gravitational equations as a special case, when tensor $S_{ik}^{j} = \mathbf{0}$, in classical form $R_{ik} - \frac{1}{2} g_{ik} R = \mathbf{0}$ [23].

Ref

12. Einstein A. Riemannian Geometry with Maintaining the Notion of Distant Parallelism. Session Report of the Prussian Academy of Sciences, June 7th, 1928 pp. 217-221.

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The general relativity theory build on Y^n- space is different from theory with teleparallelism structure.

The main assumption of theory of general relativity is that matter and energy determine the structure of space-time continuum [4, 7]. If we presume interconnection between space-time structure and distribution of energy-matter, we can obtain A. Einstein equations of gravity. But what we understand under the structure of space-time from physical point of view? The only way to perceive and observe the geometry of the real physical world is electromagnetic wave i.e. by light. So, roughly speaking ideal relativistic theory must interconnect two fundamental concept energy-matter (gravity) and propagation of light or geometrical structure of the space that is the same.

In our previous works we believed that our theory contains also Maxwell's theory as a special case with weak gravity, in this article we show that theory of relativity in Y^n- space, indeed, include Maxwell's theory of electromagnetism.

This paper is organized as follows. In Sect. 1 some properties of Y^n- space structure are briefly discussed. In Sect. 2 classical electromagnetic field theory reviewed. These results are using in next sect. In Sect. 3 we obtained the field equations in case of Y^n- space. In Sect. 4 we show that the field equations, which we obtained in sect. 3 include Maxwell's theory of electromagnetism.

II. Mathematical Description of The Space Structure of Y^n

We define invariant differential quadratic form $g_{ik}\,dx^i dx^k$ determined on the manifold and satisfying the conditions $Det\,|\,g_{ik}\,|\neq \mathbf{0}$, $g_{ik}=g_{ki}$. As a consequence of the invariance of the form:

$$ds^2 = g_{ik}\,dx^i dx^k \tag{1.1}$$

we find that the coefficients g_{ik} are forming a tensor field i.e. the transformation from one coordinate system x^i to another $x^{i'}$ subjected to the law: $g_{i'j'} = g_{ij}\dfrac{\partial x^i}{\partial x^{i'}}\dfrac{\partial x^j}{\partial x^{j'}}$.

The connection $\Gamma^i_{jk}(M)$ is a geometric object on a manifold and is subjected to the law of the transformation from one coordinate system x^i to another $x^{i'}$ in the form of:

$$\Gamma^{i'}_{j'k'} = \Gamma^i_{jk}\frac{\partial x^{i'}}{\partial x^i}\frac{\partial x^j}{\partial x^{j'}}\frac{\partial x^k}{\partial x^{k'}} + \frac{\partial^2 x^i}{\partial x^{j'}\partial x^{k'}}\frac{\partial x^{i'}}{\partial x^i}, \tag{1.2}$$

where the functions Γ^i_{jk} are sufficiently smooth.

Let along a curve $x^i = x^i(t), t\in[a,b]\subset R$ given tensor field $A^i = A^i(t)$, if for each infinitesimal displacement tensor $A^i(t)$ coordinates is changing by the law:

$$dA^i = -\Gamma^i_{jk}A^j dx^k, \tag{1.3}$$

then we say that the tensor A^i is transported parallel to the curve t.

Definition 1: A vector A^i along a curve t is called parallel along t if

$$\frac{dA^i}{dt} = -\Gamma^i_{jk}A^j\frac{dx^k}{dt}.$$

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Ref

4. Alpher R. A., Herman R. C. Evolution of the universe, Nature 1948, 162 (4124): 774–775.

It can be shown that connection define by formula

$$\Gamma^p_{kl} = \frac{1}{2} g^{pi}\left(g_{ik,l} + g_{li,k} - g_{kl,i} + g_{km}S^m_{li} + g_{lm}S^m_{ki}\right) + \frac{1}{2}S^p_{kl} \quad . \tag{1.4}$$

Then we introduce the notation and from the last formula we see that

$$\mathrm{P}^p_{kl} = \frac{1}{2} g^{pi}\left(g_{ik,l} + g_{li,k} - g_{kl,i}\right) \tag{1.5}$$

is geometric object.

$$L^p_{kl} \equiv \frac{1}{2}S^p_{kl} + \frac{1}{2} g^{pi}\left(g_{km}S^m_{li} + g_{lm}S^m_{ki}\right) \tag{1.6}$$

is tensor.
It is not difficult to prove the relation:

$$\Gamma^p_{pl} = \frac{1}{2} g_{ip,l} g^{ip} = \frac{1}{\sqrt{g}} \frac{\partial \sqrt{g}}{\partial x^l}, \text{ where } g = \det\left|g_{ik}\right|.$$

Next, we consider the difference of the second order derivatives:

$$u_{i;l;k} - u_{i;k;l} = R^p_{kli} u_p + S^q_{kl} u_{i;q} \tag{1.7}$$

where we identified R^p_{kli} is curvature tensor

$$R^p_{kli} \equiv \Gamma^p_{ik,l} - \Gamma^p_{il,k} + \Gamma^p_{ql}\Gamma^q_{ik} - \Gamma^p_{qk}\Gamma^q_{il} . \tag{1.8}$$

Similarly, we have

$$u^i_{;l;k} - u^i_{;k;l} = -R^i_{klp} u^p + S^q_{kl} u^i_{;q} \tag{1.9}$$

Then we obtain

$$R^p_{ikl} = \mathrm{P}^p_{li,k} - \mathrm{P}^p_{lk,i} + \mathrm{P}^p_{qk}\mathrm{P}^q_{li} - \mathrm{P}^p_{qi}\mathrm{P}^q_{lk} + L^p_{li,k} - L^p_{lk,i} + \mathrm{P}^p_{qk}L^q_{li} + \mathrm{P}^q_{li}L^p_{qk} - \mathrm{P}^p_{qi}L^q_{lk} - \mathrm{P}^q_{lk}L^p_{qi} + \\ + L^p_{qk}L^q_{li} - L^p_{qi}L^q_{lk} .$$

It is easy to prove the equations $S^i_{jp}S^p_{ki} + S^i_{kp}S^p_{ij} = 0,\quad S^i_{ip}S^p_{jk} = 0;$ and as a consequence, we obtain the equation:

$$S^i_{jp}S^p_{ki} + S^i_{kp}S^p_{ij} + S^i_{ip}S^p_{jk} = 0 .$$

III. Classical Electromagnetic Field Theory

In four-dimensional space-time continuum we have an electromagnetic field tensor, which is defined as

$$F_{ik} = \begin{bmatrix} 0 & -E_1 & -E_2 & -E_3 \\ E_1 & 0 & B_3 & -B_2 \\ E_2 & -B_3 & 0 & B_1 \\ E_3 & B_2 & -B_1 & 0 \end{bmatrix} = A_{i;k} - A_{k;i}$$

Notes

where A_i is electromagnetic four-potential equal $A_i = \left(\phi, \tilde{A}_1, \tilde{A}_2, \tilde{A}_3\right)$, which is related to E and B as $E = -\nabla\phi - \frac{1}{c}\frac{\partial\tilde{A}}{\partial t}$ and $B = \nabla\times\tilde{A}$, ϕ is the electric potential and $\tilde{A} = \left(\tilde{A}_1, \tilde{A}_2, \tilde{A}_3\right)$ is the magnetic potential.

Electromagnetic field is defined by vector potential $A_i = \left(\phi, \tilde{A}_1, \tilde{A}_2, \tilde{A}_3\right)$ and can be written in form of the electromagnetic tensor F_{ik} in form

$$F_{ik} = A_{i;k} - A_{k;i} = A_{i,k} - A_{k,i} - S_{ik}^{\ p}A_p =$$
$$= A_{i,k} - A_{k,i} + S_{ki}^{\ p}A_p .$$

Presume that the Lagrangian density of the electromagnetic field is $L = -\frac{1}{4}F^{ik}F_{ik}$, we obtain the energy-momentum tensor for electromagnetic field equal

$$T_{ik} = F_i^{\ p}F_{pk} - \frac{1}{4}g_{ik}F^{pq}F_{pq} .$$

We have that Maxwell-Bianchi equations can be rewritten as

$$F_{ik,l} + F_{li,k} + F_{kl,i} = 0$$

is the system of four equations, since the electromagnetic tensor is antisymetric, and

$$F^{ik}{}_{;k} = \frac{4\pi}{c}J^i$$

where $J^i = \left(\rho, J^1, J^2, J^3\right)$ is four-current vector. These are the main equations of the theory that describes electromagnetic phenomena. So, essentially we use the Lagrangian density $L = -\frac{1}{4}F^{ik}F_{ik}$ and obtain field equations in form

$$F^{ik}{}_{;k} = 0$$

in vacuum, its electromagnetic field equations in vacuum.

IV. The Field Equations in the Y^n Space

We will derive the field equations from the variation principle of least action, by varying the function S_{jk}^i and g_{ik} independently.

We form the scalar density as $\left(R_{ik} + S_{im}^n S_{kn}^m\right)g^{ik}\sqrt{-g}$ and postulate that all the variations of the integral:

$$\int\left(R_{ik} + S_{im}^n S_{kn}^m\right)g^{ik}\sqrt{-g}\,dV$$

with respect to S_{jk}^i and g_{ik} as the independent variables are zero (at the boundaries they do not vary)

$$\delta\int (R_{ik}g^{ik}\sqrt{-g} + S_{im}^n S_{kn}^m g^{ik}\sqrt{-g})d \;\; = 0V.$$

Now, we obtain some preliminaries results. For variation the second term we have formula

Notes

$$\delta(S^n_{im}S^m_{kn}) = (S^q_{kl}\delta^p_i + S^q_{il}\delta^p_k)\delta(S^l_{pq})$$

and

$$\delta\int S^n_{im}S^m_{kn}g^{ik}\sqrt{-g}dV = \int(S^q_{kl}\delta^p_i + S^q_{il}\delta^p_k)\delta(S^l_{pq})g^{ik}\sqrt{-g}dV.$$

Notes

Recalling that $\Gamma^j_{ik} = P^j_{ik} + L^j_{ik}$ where P^j_{ik} are function only of g_{ik} and tensor L^j_{ik} are function of S^i_{jk} and g_{ik} we have $\delta(\Gamma^j_{ik}) = \delta(L^j_{ik})$. Then, it easy to obtain

$$\delta(L^j_{ik}) = \frac{1}{2}(\delta^p_i\delta^q_k\delta^j_l + g^{jq}g_{il}\delta^p_k + g^{jq}g_{kl}\delta^p_i)\delta(S^l_{pq}).$$

We can rewrite

$$R_{ik} = S^n_{in,k} + \frac{1}{2}(g_{nm,i}g^{nm})_{,k} - (P^n_{ik} + L^n_{ik})_{,n} + P^n_{mk}P^m_{in} + P^n_{mk}L^m_{in} +$$

$$+L^n_{mk}P^m_{in} + L^n_{mk}L^m_{in} - P^n_{mn}P^m_{ik} - P^n_{mn}L^m_{ik} - L^n_{mn}P^m_{ik} - L^n_{mn}L^m_{ik}.$$

and

$$\delta\int R_{ik}g^{ik}\sqrt{-g}dV = \int(-(g^{ik}\sqrt{-g})_{,k}\delta(S^n_{in}) + (g^{ik}\sqrt{-g})_{,n}\delta(L^n_{ik}) +$$

$$+[P^n_{mk}\delta(L^m_{in}) + P^m_{in}\delta(L^n_{mk}) + L^m_{in}\delta(L^n_{mk}) + L^n_{mk}\delta(L^m_{in}) -$$

$$-P^n_{mn}\delta(L^m_{ik}) - P^m_{ik}\delta(L^n_{mn}) - L^n_{mn}\delta(L^m_{ik}) - L^m_{ik}\delta(L^n_{mn})]g^{ik}\sqrt{-g})d\ \ .$$

Then we calculate

$$\delta\int\left(R_{ik} + S^n_{im}S^m_{kn}\right)g^{ik}\sqrt{-g}dV =$$

$$= \frac{1}{2}\int\Big(-2\ g^{pk}\sqrt{-g})_{,k}\delta^q_l + (g^{pq}\sqrt{-g})_{,l} + (g^{ip}\sqrt{-g})_{,n}g^{nq}g_{il} +$$

$$+(g^{pk}\sqrt{-g})_{,n}g^{nq}g_{kl} + g^{ik}\sqrt{-g}[P^q_{lk}\delta^p_i + P^p_{mk}g^{mq}g_{il} + P^n_{mk}g^{mq}g_{nl}\delta^p_i +$$

$$+P^p_{il}\delta^q_k + P^m_{in}g^{nq}g_{ml}\delta^p_k + P^p_{in}g^{nq}g_{kl} + L^q_{lk}\delta^p_i + L^p_{mk}g^{mq}g_{il} + L^n_{mk}g^{mq}g_{nl}\delta^p_i -$$

$$-P^n_{ln}\delta^p_i\delta^q_k - P^n_{mn}g^{mq}g_{il}\delta^p_k - P^n_{mn}g^{mq}g_{kl}\delta^p_i -$$

$$-P^p_{ik}\delta^q_l - P^m_{ik}g^p\ g_{ml} - P^p_{ik}\delta^q_l - L^n_{ln}\delta^p_i\delta^q_k - L^n_{mn}g^{mq}g_{il}\delta^p_k -$$

$$-L^n_{mn}g^{mq}g_{kl}\delta^p_i - L^p_{ik}\delta^q_l - L^m_{ik}g^p\ g_{ml} - L^p_{ik}\delta^q_l + (S^q_{kl}\delta^p_i + S^q_{il}\delta^p_k)]\Big)(\delta S^l_{pq})dV = 0.$$

By using the principle of variational calculus, we have the field equations

$$-2(g^{pk}\sqrt{-g})_{,k}\delta_l^q+(g^{pq}\sqrt{-g})_{,l}+(g^{ip}\sqrt{-g})_{,n}g^{nq}g_{il}+$$

$$+(g^{pk}\sqrt{-g})_{,n}g^{nq}g_{kl}+g^{ik}\sqrt{-g}[P_{lk}^q\delta_i^p+P_{mk}^p g^{mq}g_{il}+P_{mk}^n g^{mq}g_{nl}\delta_i^p+$$

$$+P_{il}^p\delta_k^q+P_{in}^m g^{nq}g_{ml}\delta_k^p+P_{in}^p g^{nq}g_{kl}+L_{lk}^q\delta_i^p+L_{mk}^p g^{mq}g_{il}+L_{mk}^n g^{mq}g_{nl}\delta_i^p-$$

$$-P_{ln}^n\delta_i^p\delta_k^q-P_{mn}^n g^{mq}g_{il}\delta_k^p-P_{mn}^n g^{mq}g_{kl}\delta_i^p-$$

$$-P_{ik}^p\delta_l^q-P_{ik}^m g^p g_{ml}-P_{ik}^p\delta_l^q-L_{ln}^n\delta_i^p\delta_k^q-L_{mn}^n g^{mq}g_{il}\delta_k^p-$$

$$-L_{mn}^n g^{mq}g_{kl}\delta_i^p-L_{ik}^p\delta_l^q-L_{ik}^m g^p g_{ml}-L_{ik}^p\delta_l^q+(S_{kl}^q\delta_i^p+S_{il}^q\delta_k^p)]=0.$$

and we can rewrite

$$-2(g^{pk}\sqrt{-g})_{,k}\delta_l^q+(g^{pq}\sqrt{-g})_{,l}+(g^{ip}\sqrt{-g})_{,n}g^{nq}g_{il}+(g^{pk}\sqrt{-g})_{,n}g^{nq}g_{kl}+$$

$$+\sqrt{-g}[P_{lm}^q g^{pm}+3P_{lm}^p g^{mq}+2P_{mk}^n g^{mq}g_{nl}g^{pk}-P_{ln}^n g^{pq}-2P_{mn}^n g^{mq}\delta_l^p-$$

$$-2P_{ik}^p\delta_l^q g^{ik}-P_{ik}^m g^{pq}g_{ml}g^{ik}+L_{lk}^q g^{kp}+L_{ml}^p g^{mq}+L_{mk}^n g^{mq}g_{nl}g^{pk}-L_{ln}^n g^{pq}-$$

$$-2L_{mn}^n\delta_l^p g^{mq}-2L_{nm}^p\delta_l^q g^{nm}-L_{ik}^m g^{pq}g_{ml}g^{ik}+2S_{nl}^q g^{pn}]=0.$$

So we obtained the first system of field equations by variation of the torsion tensor.

Remark: We could transform it by using formulas $g_{,l}=\dfrac{\partial g}{\partial x^l}=gg^{ik}g_{ik,l}=-gg_{ik}g^{ik}_{,l}$ and $(g^{ik}\sqrt{-g})_{,l}=(g^{ik}_{,l}-\dfrac{1}{2}g^{ik}g^{pq}g_{pq,l})\sqrt{-g}$, then they would be free from $\sqrt{-g}$.

We will derive the field equations from the variation principle of least action, but now by varying the function g_{ik}.

We form the scalar density as $\left(R_{ik}+S_{im}^n S_{kn}^m\right)g^{ik}\sqrt{-g}$ and postulate that all the variations of the integral by varying the function g_{ik} are equal zero.

By standard calculations, we have:

$$\delta\int\left(R_{ik}+S_{il}^j S_{kj}^l\right)g^{ik}\sqrt{-g}dV=\int\Big(R_{ik}\sqrt{-g}\delta g^{ik}+R_{ik}g^{ik}\delta\sqrt{-g}+$$

$$+g^{ik}\sqrt{-g}\delta R_{ik}+S_{im}^n S_{kn}^m\sqrt{-g}\delta g^{ik}+S_{im}^n S_{kn}^m g^{ik}\delta\sqrt{-g}\Big)dV,$$

and

$$R_{ik}g^{ik}\delta\sqrt{-g}=-\frac{1}{2}R_{pq}g^{pq}g_{ik}\sqrt{-g}\delta g^{ik}.$$

Similarly, we obtain:

Notes

$$S_{im}^{n}S_{kn}^{m}g^{ik}\delta\sqrt{-g}=-\frac{1}{2}S_{pm}^{n}S_{qn}^{m}g^{pq}g_{ik}\sqrt{-g}\delta g^{ik}.$$

Now we compute $g^{ik}\sqrt{-g}\delta R_{ik}$ directly by using the definition, thus obtain two types of summands, the first have the standard form $g^{ik}\left(\delta\Gamma_{ki}^{l}\right)_{,l}-g^{ik}\left(\delta\Gamma_{kl}^{l}\right)_{,i}=\left(g^{ik}\delta\Gamma_{ki}^{l}-g^{il}\delta\Gamma_{lp}^{p}\right)_{,l}$ and by Stokes' theorem turns into zeros. The term of the second type exists due to the absence of symmetry connection and then we express the connection coefficients via the metric and torsion, after a calculation, we obtain:

Notes

$$g^{ik}\delta R_{ik}=g^{ik}\delta\left(\Gamma_{qk}^{p}\Gamma_{ip}^{q}-\Gamma_{qp}^{p}\Gamma_{ik}^{q}\right)=$$

$$=\frac{1}{4}g^{ik}\delta(2g^{pn}g_{is}S_{kn}^{m}S_{pm}^{s}+g^{pn}g_{km}g^{qt}g_{is}S_{qn}^{m}S_{pt}^{s}-2g^{pn}g_{is}S_{pm}^{m}S_{kn}^{s}-2g^{pn}g_{ks}S_{pm}^{m}S_{in}^{s}=$$

$$=\frac{1}{4}(4S_{im}^{m}S_{kn}^{n}+2S_{nk}^{m}S_{im}^{n}+2g_{is}g^{pn}S_{nk}^{m}S_{pm}^{s}+2g_{ms}g^{pn}S_{pk}^{m}S_{in}^{s}+$$

$$+2g_{km}g_{is}g^{pn}g^{qt}S_{qn}^{m}S_{tp}^{s}+2g_{is}g^{pn}S_{pm}^{m}S_{kn}^{s}+2g_{ks}g^{pn}S_{pm}^{m}S_{in}^{s})\delta g^{ik}.$$

Thus, we have:

$$\delta\int\left(R_{ik}+S_{im}^{n}S_{kn}^{m}\right)g^{ik}\sqrt{-g}dV=\int\Bigg(R_{ik}-\frac{1}{2}g_{ik}R+$$

$$+\frac{1}{2}(2S_{im}^{m}S_{kn}^{n}+S_{nk}^{m}S_{im}^{n}+g_{is}g^{pn}S_{nk}^{m}S_{pm}^{s}+g_{ms}g^{pn}S_{pk}^{m}S_{in}^{s}+$$

$$+g_{km}g_{is}g^{pn}g^{qt}S_{qn}^{m}S_{tp}^{s}+g_{is}g^{pn}S_{pm}^{m}S_{kn}^{s}+g_{ks}g^{pn}S_{pm}^{m}S_{in}^{s})+$$

$$+S_{im}^{n}S_{kn}^{m}-\frac{1}{2}S_{pm}^{n}S_{qn}^{m}g^{pq}g_{ik}\Bigg)\sqrt{-g}\delta g^{ik}dV=0.$$

Then we obtain the conclusions:

$$R_{ik}-\frac{1}{2}g_{ik}R+\frac{1}{2}(2S_{im}^{m}S_{kn}^{n}+S_{kn}^{m}S_{im}^{n}+g_{is}g^{pn}S_{nk}^{m}S_{pm}^{s}+g_{ms}g^{pn}S_{pk}^{m}S_{in}^{s}+$$

$$+g_{km}g_{is}g^{pn}g^{qt}S_{qn}^{m}S_{tp}^{s}+g_{is}g^{pn}S_{pm}^{m}S_{kn}^{s}+g_{ks}g^{pn}S_{pm}^{m}S_{in}^{s})-\frac{1}{2}S_{pm}^{n}S_{qn}^{m}g^{pq}g_{ik}=0.$$

We have obtained the system of field equations where g_{ik} and S_{ik}^{j} are unknown functions, these equations must be solved together. They determine the metric tensor and torsion tensor of space-time for a given arrangement of energy and matter in the space-time.

It is a set of non-linear partial differential equations with regard to g_{ik} and S_{ik}^{j}. The solutions of these E.Q. are the components of the metric and torsion tensors. These metric and torsion together describe the structure of the space-time including the inertial motion of objects and electromagnetic fields in the space-time.

Now consider free point particles in Y^n displaying by Einstein model and compare results with classical theory of relativity.

The system of free point particles or light is characterized by a four-vector velocity v and by scalar the proper density ρ, and n is a baryon number flux vector density. Since the considerations and calculations aren't distinguished from classical we give only the schema of output.

Let us considered Lagrangian functions of a perfect fluid as the energy scalar that is proper density ρ, so $L=-\rho$. If the pressure equal to zero we obtain the energy-momentum tensor in a form $T_{ik}=\rho v_i v_k$. The motion of free point particles is described by the equation of the energy-momentum conservation law in divergent form $T^{ik}_{;k}=\mathbf{0}$, so we have $\left(\rho v^i v^k\right)_{;k}=\mathbf{0}$, then by using identity $g^{ik}v_i v_k=\mathbf{1}$, obtain $v^i_{;k}v_i=\mathbf{0}$ and multiply $v^i\left(\rho v^k\right)_{;k}+v^i{}_{;k}\rho v^k=\mathbf{0}$ by v_i follow $\left(\rho v^k\right)_{;k}=\mathbf{0}$, we have the equation of motion for free point particles $v^k v^i{}_{;k}=\mathbf{0}$, or $v^k v^i_{,k}+\Gamma^i_{jk}v^j v^k=\mathbf{0}$.

Let us remember that v^k is a velocity vector so by the definition is a tangent vector of motion trajectory and $v^k=\dfrac{dx^k}{ds}$, so

$$\frac{d^2x^i}{ds^2}+\Gamma^i_{jk}\frac{dx^j}{ds}\frac{dx^k}{ds}=\mathbf{0} \tag{3.1}$$

these are the equation of geodesic. So, similarly to Einstein-Riemannian space relativistic theory, we obtain that free particles move in Y^n space along geodesics lines, but this line don't coincide with geodesics in Riemannian space, we have (3.1) in form

$$\frac{d^2x^i}{ds^2}-\Gamma^i_{jk}\frac{dx^j}{ds}\frac{dx^k}{ds}=$$

$$=\frac{d^2x^i}{ds^2}+\left(\mathrm{P}^i_{jk}+M^i_{jk}\right)\frac{dx^j}{ds}\frac{dx^k}{ds}=$$

$$=\frac{d^2x^i}{ds^2}+\frac{\mathbf{1}}{\mathbf{2}}\left(g^{ip}\left(g_{pj,k}+g_{kp,j}-g_{jk,p}\right)+g^{ip}\left(g_{jm}S^m_{kp}+g_{km}S^m_{jp}\right)\right)\frac{dx^j}{ds}\frac{dx^k}{ds}=\mathbf{0}$$

here g_{ik} is gravitation-metric tensor and S^i_{pq} is torsion tensor that describe non gravitation fields (electromagnetic fields).

IV. The Maxwell's Theory in Y^n with Euclidean Metric

Let us consider the equation:

$$S^i_{jk;p;q}-S^i_{jk;q;p}=R^t_{qpj}S^i_{tk}+R^t_{qpk}S^i_{jt}-R^i_{qpt}S^t_{jk}+S^t_{qp}S^i_{jk;t},$$

or

$$S^i_{jk;p;q}-S^i_{jk;q;p}-S^t_{qp}S^i_{jk;t}=R^t_{qpj}S^i_{tk}+R^t_{qpk}S^i_{jt}-R^i_{qpt}S^t_{jk},$$

Notes

we contract this tensors by an indices $i,\ \ q$; then the left side of this equation can be transformed into:

$$S^i_{jk;p;i}-S^i_{jk;i;p}-S^t_{ip}S^i_{jk;t}=\left(S^i_{jk;p}-S^i_{qp}S^q_{jk}\right)_{;i}-S^i_{jk;i;p}-S^i_{pq;i}S^q_{jk}.$$

then, we contract this equation by an indices $k,\ \ p$ and raising the index j, we obtained:

Notes

$$\left(g^{kp}g^{js}S^i_{sk;p}-g^{kp}g^{js}S^i_{qp}S^q_{sk}\right)_{;i}-g^{kp}g^{js}S^i_{sk;i;p}-g^{kp}g^{js}S^i_{pq;i}S^q_{sk}=$$

$$=g^{kp}g^{js}R^t_{ips}S^i_{tk}+g^{kp}g^{js}R^t_{ipk}S^i_{st}-g^{kp}g^{js}R^i_{ipt}S^t_{sk},$$

we introduce the notation:

$$H^{ji}=g^{kp}g^{js}S^i_{sk;p}-g^{kp}g^{js}S^i_{qp}S^q_{sk},$$

$$F^{jp}=g^{kp}g^{js}S^i_{sk;i}.$$

Then, without any loss of generality, we obtain the relations:

$$H^{ji}_{;i}-F^{ji}_{;i}-g^{kp}g^{js}S^q_{sk}F_{pq}=g^{kp}g^{js}R^t_{ips}S^i_{tk}+g^{kp}g^{js}R^t_{ipk}S^i_{st}-g^{kp}g^{js}R^i_{ipt}S^t_{sk}, \tag{4.1}$$

where $F_{pq}=S^i_{pq;i}$.

Suppose now that the identity Ricci – Jacobi run in a standard form, $R^p_{ikl}+R^p_{kli}+R^p_{lik}=\mathbf{0}$, hence:

$$S^p_{ik;l}+S^p_{kl;i}+S^p_{li;k}+S^p_{lq}S^q_{ik}+S^p_{kq}S^q_{li}+S^p_{iq}S^q_{kl}=\mathbf{0}.$$

we contract this equation by an indices $p,\ \ l$ we found identity:

$$S^p_{ik;p}+S^p_{kp;i}+S^p_{pi;k}=\mathbf{0}.$$

Next, we is assuming that $S^p_{ip}=\varphi_i$ and taking into account the identity $S^p_{ij}S^q_{pq}=\mathbf{0}$, we obtain the following expression:

$$S^p_{ij;p}=\varphi_{i,j}-\varphi_{j,i}.$$

Next if we put $S^p_{ij;p}=\mathbf{0}$, then it follows that $\varphi_{i,j}-\varphi_{j,i}=\mathbf{0}$ and hence the value S^p_{ip} can be expressed in terms of the partial derivative of the scalar $S^p_{ip}=\varphi_i=\left(\ln\psi\right)_{,i}$. System (4.1) takes the form:

$$H^{ji}_{;i}=g^{kp}g^{js}R^t_{ips}S^i_{tk}+g^{kp}g^{js}R^t_{ipk}S^i_{st}-g^{kp}g^{js}R^i_{ipt}S^t_{sk},$$

$$F^{ij}=\mathbf{0}.$$

We consider the tensor

$$C^{ijk}=g^{pj}g^{qk}S^i_{pq}+g^{pk}g^{qi}S^j_{pq}+g^{pi}g^{qj}S^k_{pq},$$

obvious that it is asymmetric in any pair of indices.
We have the equality:

$$H^{jk} - H^{kj} = C^{ikj}_{;i} + F^{jk} + g^{kp}g^{qs}S^{t}_{pq}S^{j}_{ts} - g^{jp}g^{qs}S^{t}_{pq}S^{k}_{ts},$$

By direct calculations we can conclude that

$$g^{kp}g^{qs}S^{t}_{pq}S^{j}_{ts} - g^{jp}g^{qs}S^{t}_{pq}S^{k}_{ts} = \frac{1}{2}\left(C^{jpq}S^{k}_{pq} - C^{kpq}S^{j}_{pq}\right),$$

hence

$$H^{jk} - H^{kj} = C^{ikj}_{;i} + F^{jk} + \frac{1}{2}\left(C^{jpq}S^{k}_{pq} - C^{kpq}S^{j}_{pq}\right).$$

We calculate the covariant derivative

$$C^{ikj}_{;i} = -C^{ijk}_{;i} = -\left(C^{ijk}_{,i} + \Gamma^{j}_{pi}C^{ipk} + \Gamma^{k}_{pi}C^{ijp} + \Gamma^{i}_{pi}C^{pkj}\right).$$

By virtue of the fact that tensor $C^{ijk} = g^{pj}g^{qk}S^{i}_{pq} + g^{pk}g^{qi}S^{j}_{pq} + g^{pi}g^{qj}S^{k}_{pq}$ is asymmetric, we have:

$$\Gamma^{j}_{pi}C^{ipk} = \Gamma^{j}_{ip}C^{ipk} = \frac{1}{2}\left(\Gamma^{j}_{ip}C^{ipk} + \Gamma^{j}_{pi}C^{pik}\right) = \frac{1}{2}S^{j}_{ip}C^{pki} = -\frac{1}{2}S^{j}_{pi}C^{kpi},$$

similarly, we obtain

$$\Gamma^{k}_{pi}C^{ijp} = \Gamma^{k}_{ip}C^{pji} = \frac{1}{2}C^{jpi}\left(\Gamma^{k}_{ip} - \Gamma^{k}_{pi}\right) = \frac{1}{2}S^{k}_{ip}C^{jpi} = \frac{1}{2}S^{k}_{pq}C^{jqp}.$$

Then we write,

$$C^{ikj}_{;i} = -C^{ijk}_{;i} = -C^{ijk}_{,i} - \frac{1}{2}S^{j}_{pq}C^{kpq} + \frac{1}{2}S^{k}_{pq}C^{jpq} - \Gamma^{q}_{pq}C^{pkj},$$

and

$$H^{jk} - H^{kj} = -C^{ijk}_{,i} + \frac{1}{2}S^{j}_{pq}C^{kpq} - \frac{1}{2}S^{k}_{pq}C^{jpq} - \Gamma^{q}_{pq}C^{pkj} + F^{jk} + \frac{1}{2}\left(C^{jpq}S^{k}_{pq} - C^{kpq}S^{j}_{pq}\right),$$

$$H^{jk} - H^{kj} = -C^{ijk}_{,i} - \Gamma^{q}_{pq}C^{pkj} + F^{jk}.$$

We will compute Γ^{p}_{lp}, for this, we recall that $\Gamma^{p}_{pl} = \frac{1}{2}g_{ip,l}g^{ip} = \frac{1}{\sqrt{g}}\frac{\partial\sqrt{g}}{\partial x^{l}}$ and $\Gamma^{p}_{lp} = \Gamma^{p}_{pl} + S^{p}_{lp}$, obtain:

$$\Gamma^{p}_{pl} = \frac{1}{\sqrt{-g}}\frac{\partial\sqrt{-g}}{\partial x^{l}} + \left(\ln\psi\right)_{,l} = \left(\ln\left(\psi\sqrt{-g}\right)\right)_{,l}.$$

Then we obtain

$$H^{jk} - H^{kj} - F^{jk} = -C^{ijk}_{,i} - \left(\ln\left(\psi\sqrt{-g}\right)\right)_{,i}C^{ikj}.$$

We multiple by $\psi\sqrt{-g}$, have

Notes

$$\psi\sqrt{-g}\left(H^{jk}-H^{kj}-F^{jk}\right)=-\psi\sqrt{-g}\left(C^{ijk}_{,i}+\left(\ln\left(\psi\sqrt{-g}\right)\right)_{,i}C^{ikj}\right),$$

$$\psi\sqrt{-g}\left(H^{jk}-H^{kj}-F^{jk}\right)=-\left(\psi\sqrt{-g}C^{ijk}\right)_{,i}.$$

We differentiate the last equality, in view of the antisymmetry of the tensors, we obtain the next important equality:

Notes

$$\left(\psi\sqrt{-g}\left(H^{jk}-H^{kj}-F^{jk}\right)\right)_{,k}=\mathbf{0},$$

where $S^p_{ip}=\varphi_i=\left(\ln\psi\right)_{,i}$.

Now, let us assume that we are situated in space with very weak gravity in such space we can introduce diagonal Euclidean metric $g_{ik}=\delta_{ik}$ and then curvature tensor is equal to $R^p_{ikl}=L^p_{li,k}-L^p_{lk,i}+L^p_{qk}L^q_{li}-L^p_{qi}L^q_{lk}$, where $L^p_{kl}\equiv\frac{\mathbf{1}}{\mathbf{2}}S^p_{kl}+\frac{\mathbf{1}}{\mathbf{2}}\delta^{pi}\left(\delta_{km}S^m_{li}+\delta_{lm}S^m_{ki}\right)$. Let introduce tensor $W^p_{ikl}=R^p_{ikl}$ for this special case of Riemannian tensor and we can written $u_{i;l;k}-u_{i;k;l}=W^p_{kli}u_p+S^q_{kl}u_{i;q}$, for tensor W^p_{ikl} we obtain the analog of Ricci – Jacobi identity

$$W^p_{ikl}+W^p_{kli}+W^p_{lik}=S^p_{ik;l}+S^p_{kl;i}+S^p_{li;k}+S^p_{lq}S^q_{ik}+S^p_{kq}S^q_{li}+S^p_{iq}S^q_{kl}$$

and remind that $S^i_{jp}S^p_{ki}+S^i_{kp}S^p_{ij}=\mathbf{0}$, $S^i_{ip}S^p_{jk}=\mathbf{0}$; $S^i_{jp}S^p_{ki}+S^i_{kp}S^p_{ij}+S^i_{ip}S^p_{jk}=\mathbf{0}$.
We have

$$H^{ji}_{;i}-F^{ji}_{;i}-\delta^{kp}\delta^{js}S^q_{sk}F_{pq}=\delta^{kp}\delta^{js}R^t_{ips}S^i_{tk}+\delta^{kp}\delta^{js}R^t_{ipk}S^i_{st}-\delta^{kp}\delta^{js}R^i_{ipt}S^t_{sk}.$$

The last equation is true in any Y^n space with Euclid metric, now assume that right part is equal to zero, so we obtain

$$H^{ji}_{;i}-F^{ji}_{;i}-\delta^{kp}\delta^{js}S^q_{sk}F_{pq}=\mathbf{0}.$$

Now, we consider more specific case. The geometrical structure of space without gravity different from Euclid space only torsion, which we will be associated with electromagnetism. Next, we presume $S^p_{ip}=\varphi_i$ and by definition $C^{ijk}=g^{pj}g^{qk}S^i_{pq}+g^{pk}g^{qi}S^j_{pq}+g^{pi}g^{qj}S^k_{pq}$ or $C^{ijk}=\delta^{pj}\delta^{qk}S^i_{pq}+\delta^{pk}\delta^{qi}S^j_{pq}+\delta^{pi}\delta^{qj}S^k_{pq}$, $F_{pq}=S^i_{pq;i}$, and taking into account the identity $S^p_{ij}S^q_{pq}=\mathbf{0}$, we obtain equation:

$$S^p_{ij;p}=\varphi_{i,j}-\varphi_{j,i}.$$

It can be obtained

$$H^{ji}_{;i}=\delta^{kp}\delta^{js}R^t_{ips}S^i_{tk}+\delta^{kp}\delta^{js}R^t_{ipk}S^i_{st}-\delta^{kp}\delta^{js}R^i_{ipt}S^t_{sk},$$

$$F^{jp}=\delta^{kp}\delta^{js}S^i_{sk;i},\qquad F^{ij}=\mathbf{0}.$$

Next if we put $S^p_{ij;p}=\mathbf{0}$, then it follows that $\varphi_{i,j}-\varphi_{j,i}=\mathbf{0}$ and hence the value S^p_{ip} can be expressed in terms of the partial derivative of the scalar $S^p_{ip}=\varphi_i=\left(\ln\psi\right)_{,i}$.

We will compute Γ^p_{lp}, for this, we recall that $\Gamma^p_{pl} = \frac{1}{2} g_{ip,l} g^{ip} = \frac{1}{\sqrt{g}} \frac{\partial \sqrt{g}}{\partial x^l} = 0$ and $\Gamma^p_{lp} = S^p_{lp} = 0$, obtain $\psi = \alpha$ and $S^p_{ip} = 0$. Then we obtain

$$H^{jk} - H^{kj} - F^{jk} = -C^{ijk}_{,i} .$$

So, we have the equations

$$\left(H^{jk} - H^{kj} - F^{jk}\right)_{,k} = 0$$

its system of divergent type. If we note that $\Im^{jk} = H^{jk} - H^{kj} - F^{jk}$ is asymmetrical tensor and for tensor $\Im^{jk}$ we have the system of true divergent type

$$\Im^{jk}{}_{,k} = 0 . \quad (4.2)$$

Notes

Now, we assume the existence of vector in Y^n such that $\Im_{ik} = A_{i;k} - A_{k;i} = A_{i,k} - A_{k,i} - S^p_{ik} A_p$ and have analog of Bianchi equation

$$\Im_{ik,l} + \Im_{li,k} + \Im_{kl,i} = 0 . \quad (4.3)$$

Equation (4.2) is describe electromagnetic field and is classical Maxwell's theory that was obtained from theory of relativity in Y^n, so new theory of relativity contained Maxwell's theory.

V. Conclusion

In this article we obtained new relativistic theory which is included the Maxwell's theory as a special case. We unified electromagnetic and gravitational field concepts in one theory.

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Mathematical Modeling of a Predator-Prey Model with Modified Leslie-Gower and Holling-type II Schemes

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Abstract- In this paper, a two-dimensional continuous predator-prey system with Holling type II functional response and modified of Leslie –Gower type dynamics incorporating constant proportion of prey refuge compared by the model without refuge is proposed and analyzed. In both cases, by non-dimensionalize the system, the fixed points are computed and condition for local and global asymptotic stability of the system are obtained. Moreover, the global asymptotic stability of the system is proved by defining appropriate Dulac function. Numerical simulations are also carried out to verify the analytical results.

Keywords: *prey, predator, refuge, stability, holling type II functional response, modified leslie-gower dynamics.*

GJSFR-F Classification: *MSC 2010: 00A69*

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Strictly as per the compliance and regulations of :

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Ahmed Buseri Ashine $^{\alpha}$ & Dawit Melese Gebru (Ph.D.) $^{\sigma}$

Abstract- In this paper, a two-dimensional continuous predator-prey system with Holling type II functional response and modified of Leslie –Gower type dynamics incorporating constant proportion of prey refuge compared by the model without refuge is proposed and analyzed. In both cases, by non-dimensionalize the system, the fixed points are computed and condition for local and global asymptotic stability of the system are obtained. Moreover, the global asymptotic stability of the system is proved by defining appropriate Dulac function. Numerical simulations are also carried out to verify the analytical results.

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Chapter One

I. Introduction

a) *Background*

A predator is an organism that eats another organism. The prey is the organism which the predator eats. Some examples of predator and prey are lion and zebra, fish and shark, shark and fish, fox and rabbit, and bat and moth. The words “predator” and “prey” are almost always used to mean only animals that eat animals, but the same concept also applies to plants: Bear and berry, rabbit and lettuce, grasshopper and leaf.

The dynamic relationships between species and their complex properties are at the heart of many ecological and biological processes [1]. One of such relationships is the dynamical relationship between a predator and their prey which has long been and will continue to be one of the dominant themes in both ecology and mathematical ecology due to its universal existence and importance [2].

Nature can provide some degree of protection to a given number of prey populations by providing refuges. Such refugia can help in prolonging prey- predator interactions by reducing the chance of extinction due to predation [3, 4] and damp prey predator oscillations [5, 6]. The effects of prey refuges on the population dynamics are very complex in nature, but for modeling purposes, it can be considered as constituted by two components: the first effects, which affect positively the growth of prey and negatively that of predators, comprise the reduction of prey mortality due to decrease in predation success. The second one may be the trade-offs and by-products of the

Ref

1. Camara, B. I. and Aziz-Alaoui, M. A., Complexity in a prey predator Model, International Conference in Honor of Claude Lobry, (2007).

Author α σ: Degree of Master's of Science in Mathematics, Adama Science and Technology University, School of Graduate Studies, Department of Mathematics, Adama, Ethiopia.

hiding behavior of prey which could be advantageous or detrimental for all the interacting populations [7, 8].

The goal of this paper is to introduce and to give a first study of a two-dimensional system of autonomous differential equations modeling a predator-prey system. This model incorporates a modified version of the Leslie-Gower functional response as well as that of the Holling-type II functional response.

The dynamic relationship between predators and their prey has long been and will continue to be one of the dominant themes in both ecology and mathematical ecology due to its universal existence and importance [9-11]. These problems may appear to be simple mathematically at first sight, they are, in fact, often very challenging and complicated. Although the predator–prey theory has seen much progress in the last 40 years, many long standing mathematical and ecological problems remain open [12–17].

Species interaction in natural and wildlife environments is unavoidable. Different species occupying similar ecosystems and living amongst each other will regularly interact. These leads to the question, how does this primal concern on survival impact the way in which species interact? Mathematical population models have been used to study the dynamics of prey predator systems since Lotka (1925) and Volterra (1927) proposed the simple model of prey predator interactions now called the Lotka-Volterra model. Since then, many mathematical models, some reviewed in this study, have been constructed based on more realistic explicit and implicit biological assumptions.

Some of the aspects that need to be critically considered in a realistic and plausible mathematical model include; carrying capacity which is the maximum number of prey that the ecosystem can sustain in absence of predator, competition among prey and functional responses of the species.

The main aim of this paper is to study the effect of prey refuge on the stability property of the coexistence equilibrium point a prey predator system with Holling type II functional response and the modified Leslie-Gower type dynamics incorporating constant proportion of prey refuge.

b) *Basic Definitions*

i. *Stability Analysis*

Stability refers to system's ability to resist small perturbations. Stability analysis is an acceptable tool for studying long time behavior of dynamical systems. Stability concept has been studied extensively for almost one hundred years [6, 14, 16]. There are many kinds of stability concepts which are frequently associated with a dynamical system: local stability, global stability and Lyapunov stability of equilibrium points. The local stability of non-linear system gives the behavior of the system in a small neighborhood of an equilibrium point. As such it does not predict the overall behavior of the system. However, global stability captures the behavior of the non-linear system far from an equilibrium point.

ii. *Local asymptotical Stability*

Consider the following system of differential equations for n-interacting species:

$$\frac{dU}{dT} = f(U, s) \qquad (1.1)$$

where $U = (U_1, U_2, ..., U_n)^t$ is a state variable and s is the control parameter. The equilibrium point, say U_e, of the system (1.1) is the point of satisfying $f(U_e, s) = 0$.

Ref

7. M. E. Hochberg and R. D. Holt, "Refuge evolution and the population dynamics of coupled Host parasitoid associations," Evolutionary Ecology, vol. 9.

Notes

Linear stability is used to study the stability of the steady state U_e in a small neighborhood. The system (1.1) can be linearized at U_e as

$$\frac{dU}{dT} = JU \tag{1.2}$$

where the the matrix **J** is the Jacobian matrix of the system (1.2) with elements defined as $a_{ij} = \left(\frac{\partial f_i}{\partial U_j}\right)|_{U_e}$. The polynomial characteristic equation corresponding to the system (1.2) is obtained as

$$\lambda^n + b_1\lambda^{n-1} + b_2\lambda^{n-2} + \ldots + b_n = 0 \tag{1.3}$$

The stability of the steady state U_e depends on the sign of the real parts of the eigenvalues˘ λ_i of **J**. If one or more of the real parts of the eigenvalues λ_i are positive, the state is unstable. The stationary states can be classified further according to the imaginary parts and signs of the corresponding eigenvalues. For example, in a two-dimensional phase space there are five types of fixed points: stable and unstable foci (imaginary eigenvalues with non-zero real part), stable and unstable nodes (real eigenvalues) and saddle-nodes (real eigenvalues with different sign).

iii. *Routh-Hurwitz Stability Criterion*

The Routh-Hurwitz Stability Criterion provides the necessary and sufficient conditions under which all the roots of the polynomial equation (1.3) lie in the left half of the complex plane:

$$b_1 > 0, \begin{vmatrix} b_1 & b_3 \\ 1 & b_2 \end{vmatrix} > 0, \begin{vmatrix} b_1 & b_3 & b_5 \\ 1 & b_2 & b_4 \\ 0 & b_1 & b_3 \end{vmatrix} > 0, \ldots, \begin{vmatrix} b_1 & b_3 & b_5 & \ldots & 0 \\ 1 & b_2 & b_4 & \ldots & 0 \\ 0 & b_1 & b_3 & \ldots & 0 \\ & & & \ldots & \\ 0 & 0 & 0 & \ldots & b_n \end{vmatrix} > 0$$

In particular, the Routh-Hurwitz criterion for n = 2 is:

n=2; $\quad b_1 = -trace\, J > 0\,,\ b_2 = \det J > 0$

iv. *Globally Asymptotical Stability*

If a limit set contains more than one equilibrium, then it must also contain orbits joining these equilibria. In essence, we can say that a bounded solution tends either to an equilibrium or to limit cycle, overlooking such ”unlikely coincidences” as the possibility of a running from a saddle point to itself. Thus, if we can show that, for a given system, all solutions are bounded but there are no asymptotically stable equilibrium points, we can deduce that there must be at least one periodic orbit.

If there is only one periodic orbit, then it must be globally asymptotically stable in the sense that every orbit tends to it. If there is more than one periodic orbit, each must be asymptotically stable from at least one side: orbits may spiral toward it from the inside, from the outside, or both.

v. *Hopf Bifurcation*

Hopf bifurcation is defined as the appearance or disappearance of a periodic orbit through a local change in the stability properties of a steady point. It is named in the memory of the mathematician Eberhard Hopf. In a dynamical system, Hopf bifurcation occurs when the system loses its stability in the form of complex conjugate eigenvalues of the linearization around the fixed point which crosses the imaginary axis of the complex plane.

Notes

c) Statement of the problem

Many of the most interesting dynamics in the biological world have to do with interactions between species. In ecology, predation explains a biological interaction where a predator forages on its prey. The predator-prey relationship is substantial in maintaining the equilibrium between various animal species. The focus of the thesis is that in the realm of biological mathematics, it is possible to mathematically represent the population variations of a predation relationship to a certain extent of accuracy. This will be done using the modified Leslie-Gower dynamics and Holling type II functional response incorporating prey refuge.

d) Objectives

i. *General Objectives*

The general objective of this study is to establish and study a predator prey model with modified Leslie Gower type dynamics and Holling type II functional response incorporating prey refuge.

ii. *Specific objectives*

The specific objectives of this study are:-

- To formulate a mathematical model of modified Leslie Gower and Holling type II with prey refuge.
- To investigate the effect of prey refuge on the predator-prey population in Leslie-Gower and Holling type II model.
- To establish several sufficient conditions on the stability of a positive equilibrium.

e) Significance of the study

It is hoped that if the effect of parameters such as intrinsic growth rate, carrying capacity of the environment, prey refuge, etc, on the long term stable co-existence of the 2 species is known, this will enable the environment authorities to manage the population of the prey, the predator in the that area, especially taking well established measures to avoid extinction of any of the species.

f) Methodology

The model in the thesis is nonlinear 1^{st} order ordinary differential equation. First of all, we will convert our model into non-dimensional form. This will reduce the number of parameters. Linear stability analysis and bifurcation analysis will be carried out to see the dynamical behavior of the system under consideration. Finally, numerical simulations will be done by using MATLAB.

g) Structure and Presentation

This paper is presented in four chapters. Chapter 1 gives the background to the study, statement of the problem, objectives of the study and significance of the study. Chapter 2 presents the literature review, focusing on Leslie Gower and Holing type II prey-predator systems and mathematical models. Chapter 3 deals with formulating and qualitative analysis of the model while, Chapter 4 includes discussion of results and conclusion.

Chapter Two

II. Review Literature

a) *Historical Background*

One of the important interactions among species is the predator-prey relationship and it has been extensively studied because of its universal existence. There are many factors affecting the dynamics of predator-prey models. One of the familiar factors is the functional response, referring to the change in the density of prey attached per unit time per predator as the prey density changes. In the classical Lotka-Volterra model, the functional response is linear, which is valid first-order approximations of more general interaction.

In the literatures studies show that refuges have both stabilizing [15] and destabilizing effect [18, 19]. The traditional ways in which the effect of refuge used by the preys has been incorporated in predator- prey models is to consider either a constant number or a constant proportion of the prey population being protected from predation [15, 19, 22]. Hassel [15] notes that in reality refugia fall between these two extremes. It is pointed out that those protecting a proportion of the prey population appearing to be more common [6]. However, as pointed out by the authors [17, 21], the refuges, which protect a constant number of preys, have a stronger stabilizing effect on population dynamics than the refuges, which protect a constant proportion of prey. For more biological backgrounds and results on the effects of a prey refuge, one could refer to [4, 5, 6, 17] and the references therein.

Mathematical modeling and analysis of multiple species ecological problems was first done by Volterra (1927). Volterra had been introduced to an ecological problem that in the years after the first World War, the proportion of the predatory fishes caught in the Upper Adriatic Sea was found to be considerably higher than in the years before the war, whereas the proportion of prey fishes was down. In order to come out with an explanation to this ecological problem, Volterra formulated and analyzed a system of ordinary differential equations which is represented as below:

$$\dot{x} = x(a - by)$$

$$\dot{y} = y(-c + dx),$$

where x and y were the densities of the prey and predator respectively. This system of differential equations was also studied independently by Lotka (1925) in the context of chemical kinetics and is now known as the Lotka-Volterra model. Volterra's study showed that the steady state for the co-existence of the prey and predatory was periodic and that a pause of fishery would indeed lead to an increase of the predators and a decrease in the prey.

Recently, many mathematical models incorporating diverse areas of interest such as Holling Type functional responses, ratio-dependent functional responses, bio-economic exploitation or harvesting, delayed harvesting and age-structured models have been formulated and analyzed.

b) *Predator-prey Models*

It has been observed that the inclusion of the logistic growth and Holling Type II functional response in the model, guarantees permanence. This showed the importance of incorporating logistic growth in prey-predator models.

In the Holling type II functional response, the rate of prey consumption by a predator rises as prey density increases, but eventually levels off at a plateau (or

Ref

18. McNair, J. N., The effects of refuges on predator-prey interactions: A reconsideration, Theor. Popul. Biol., 29(1986), 38-63.

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asymptote) at which the rate of consumption remains constant regardless of increases in prey density. Holling's disk equation, named after experiments he performed in which a blindfolded assistant picked up sandpaper disks from a table, describes the type II functional response.

Holling (1959) studied predation of small mammals on pine sawflies, and he found that predation rates increased with increasing prey population density. This resulted from 2 effects: (1) each predator increased its consumption rate when exposed to a higher prey density, and (2) predator density increased with increasing prey density. Holling considered these effects as 2 kinds of responses of predator population to prey density: (1) the functional response and (2) the numerical response.

In the late 1980s, a credible, simple alternative to the Lotka-Volterra predator-prey model (and its common prey dependent generalizations) emerged, the ratio dependent or Arditi-Ginzburg model. The two are the extremes of the spectrum of predator interference models.

Notes

Chapter Three

II. Model Formulation and Analysis

In this chapter, we present model description, formulation and analysis. Let $X(t)$, and $Y(t)$ represent the population of the prey and predator species at any time t. The main feature of the model is that the interaction of species affects both populations. Terms representing logistic growth of the prey species in the absence of the predator are included in the prey equations. The Leslie-Gower formulation of the model is based on the assumption that reduction in a predator population has a reciprocal relationship with per capita availability of its preferred food. The model has two non-linear autonomous ordinary differential equations describing how the population densities of the two species would vary with time.

a) Assumptions

The following assumptions are made in order to construct the model:

- The species live in an ecosystem where external factors such as droughts, fires, epidemics are stable or have a similar effect on the interacting species.
- The predator is completely dependent on the prey as the only favorite food source
- The prey species have an unlimited food supply.
- There is logistic growth of the prey in absence of the predator or human poaching of the prey. That is the population of the prey would increase (or decrease) exponentially until it reaches the maximum density of the living area, which is its carrying capacity.
- There is no threat to the prey besides the predator species being studied.
- Some of the preys were reserved that they are free from predator.

b) Model Variables and Parameters

The following variables and parameters are used in the model:

i) X(t) - the population of the prey at time t.
ii) Y(t) - the population of the predator at time t.
iii) r is per capita intrinsic growth rates for prey.
iv) s is gives the maximal per-capita growth rate of predator.
v) K is the carrying capacity of the environment.
vi) k_1 measures the extent of which environment provides protection to prey.

vii) k_2 measures the extent of which environment provides protection to predator.

viii) c_1 is the maximum value which per capita reduction rate of prey.

ix) c_2 is the crowding effect for the predator.

c) The Mathematical Model

Notes

Besides the model description, assumptions and definition of variables and parameters in section 3.1, it is assumed that a constant proportion $m \in [0,1)$ of the prey can take refuge to avoid predation, this leaves $(1-m)X$ of the prey available for predation.

Thus, the model under the above assumptions with Holling type II functional response and the modified Leslie-Gower type predator dynamics is given by:-

$$\begin{cases} \dfrac{dX}{dT} = r\left(1-\dfrac{X}{K}\right)X - \dfrac{c_1(1-m)XY}{k_1+(1-m)X} \\ \dfrac{dY}{dT} = Y\left(s - \dfrac{c_2 Y}{k_2+(1-m)X}\right) \end{cases} \tag{3.1}$$

where all the parameters in the model assumes positive values and with initial value $X(0) \geq 0$ and $Y(0) \geq 0$. The following non-dimensional state variables and parameters are chosen.

$$x = \frac{X}{K} \qquad y = \frac{Y}{K} \qquad t = rT$$

$$\alpha = \frac{c_1}{r} \qquad \beta = \frac{k_1}{K} \qquad \gamma = \frac{s}{r} \qquad \sigma = \frac{c_2}{r} \qquad \omega = \frac{k_2}{K}$$

The system (3.1) takes the following non-dimensional form

$$\begin{cases} \dfrac{dx}{dt} = (1-x)x - \left(\dfrac{\alpha(1-m)xy}{\beta+(1-m)x}\right) \equiv F(x,y) \\ \dfrac{dy}{dt} = y\left(\gamma - \dfrac{\sigma y}{\omega+(1-m)x}\right) \equiv G(x,y) \end{cases} \tag{3.2}$$

$x(0) = x_0 \geq 0$; $y(0) = y_0 \geq 0$.

Theorem 3.1: All the solutions $(x(t), y(t))$ of the system (3.2) are nonnegative. i.e $x(t) \geq 0$ $y(t) \geq 0$ for all $t \geq 0$.

Proof: Considering the biological significance, we investigate the dynamical system (3.2). From first equation of (3.2) it follows that x=0 is an invariant subset. i.e $x \equiv 0$ if and only if x =0 for some time t. This implies that $x(t) > 0 \quad \forall t$ if $x(0) > 0$. The same argument follows the second equation the system (3.2). i.e any trajectory in $\Re_+^2$, cannot cross the coordinate planes.

i. *Boundedness of the solution*

Theorem 3.2: All the solution $(x(t), y(t))$ of the system (3.2) are bounded.

Proof: The first equation of (3.2) gives us

$$\frac{dx}{dt} = x\left(1 - x - \frac{\alpha(1-m)y}{\beta + (1-m)x}\right)$$
$$< \quad x(1-x)$$

Therefore, $\limsup_{t\to\infty} x(t) < 1$. Hence, $x(t)$ is always bounded.

Similarly,

$$\frac{dy}{dt} = y\left(\gamma - \frac{\sigma y}{\omega + (1-m)x}\right)$$
$$\leq y\left(\gamma - \frac{\sigma y}{\omega + 1 - m}\right)$$
$$= \gamma y\left(1 - \frac{y}{\omega + 1 - m}\frac{\sigma}{\gamma}\right)$$
$$= \gamma y\left(1 - \frac{y}{\omega + 1 - m}\lambda\right)$$
$$= y\left(1 - \frac{y}{(\omega + 1 - m)/\lambda}\right)$$

Therefore, we have $y(t) \leq \max\left\{\frac{\omega + 1 - m}{\lambda}, y(0)\right\} \equiv M_2$, $\lambda = \frac{\sigma}{\gamma}$.

Hence, the solutions $(x(t), y(t))$ of the system (3.2) with the given initial conditions are bounded.

ii. *Equilibrium points and their stability*

One can see that the system (3.2) has three boundary equilibrium points, $E_1(0,0)$, $E_2\left(0, \frac{\omega\gamma}{\sigma}\right)$ and $E_3(1,0)$. Besides these three boundary equilibrium points the system (3.2) has one positive equilibrium points, say $E(x^*, y^*)$. $E(x^*, y^*)$ is obtained by solving the following simultaneous equation

$$\frac{\alpha(1-m)y^*}{\beta + (1-m)x^*} = 1 - x^*$$
$$y^* = \frac{\gamma(\omega + (1-m)x^*)}{\sigma} \tag{3.3}$$

One can easily see that x^* satisfies the quadratic equation

$$Ax^{*2} + Bx^* + C = 0 \tag{3.4}$$

$$A = (1-m)\sigma, B = \alpha\gamma m^2 + (\sigma - 2\alpha\gamma)m + \alpha\gamma + \sigma\beta - \sigma, \; C = \alpha\gamma\omega - \alpha\gamma\omega m - \sigma\beta.$$

Proposition 3.1: The system (3.2) has a unique interior equilibrium $E^*\left(x^*, y^*\right)$ (i.e $x^* > 0$ and $y^* > 0$) provided

Notes

$$m > 1 - \frac{\sigma\beta}{\alpha\gamma\omega}$$

Notes

Proof: From the second equation of (3.3), we can see that y^* is positive whenever x^* is. x^* is the unique positive root of the quadratic equation (3.4). Equation (3.4) will have a unique positive root when $C < 0$. Hence, $C < 0$ whenever $m > 1 - \frac{\sigma\beta}{\alpha\gamma\omega}$. Therefore, $E^*(x^*, y^*)$ exists uniquely whenever $m > 1 - \frac{\sigma\beta}{\alpha\gamma\omega}$.

The unique equilibrium density x^* can be explicitly given as

$$x^* = \frac{-B + \sqrt{B^2 - 4AC}}{2A}$$

The Jacobean matrix of the system (3.2) at the equilibrium points E_i is given as

$$J(E_i) = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} \frac{\partial F}{\partial x}(x_i, y_i) & \frac{\partial F}{\partial y}(x_i, y_i) \\ \frac{\partial G}{\partial x}(x_i, y_i) & \frac{\partial G}{\partial y}(x_i, y_i) \end{bmatrix};$$

where

$$\frac{\partial F}{\partial x}(x_i, y_i) = 1 - 2x - \frac{\alpha\beta(1-m)y}{(\beta + (1-m)x)^2}$$

$$\frac{\partial F}{\partial y}(x_i, y_i) = -\frac{\alpha(1-m)x}{\beta + (1-m)x}$$

$$\frac{\partial G}{\partial x}(x_i, y_i) = \frac{\sigma(1-m)y^2}{(\omega + (1-m)x)^2}$$

$$\frac{\partial G}{\partial y}(x_i, y_i) = \gamma - \frac{2\sigma y}{\omega + (1-m)x}$$

Proposition 3.2: The equilibrium point $E_1(0,0)$ is unstable node.

Proof: At $E_1(0,0)$, the Jacobean matrix becomes

$$J(E_1) = \begin{pmatrix} 1 & 0 \\ 0 & \gamma \end{pmatrix}$$

The eigen values of this matrix are $\lambda_1 = 1$ and $\lambda_2 = \gamma$, which are both positive. Hence $E_1(0,0)$ is an unstable node.

Proposition 3.3: The equilibrium point $E_2\left(0, \frac{\omega\gamma}{\sigma}\right)$ is locally asymptotically stable for

$$m < 1 - \frac{\sigma\beta}{\alpha\gamma\omega}.$$

Proof: At $E_2\left(0, \frac{\omega\gamma}{\sigma}\right)$, the Jacobean matrix is

$$J(E_2) = \begin{pmatrix} 1 - \frac{\alpha\gamma\omega(1-m)}{\sigma\beta} & 0 \\ \frac{\gamma^2}{\sigma} & -\gamma \end{pmatrix}$$

The eigenvalues of the matrix J(E_2) are

$$\lambda_1 = 1 - \frac{\alpha\gamma\omega(1-m)}{\sigma\beta}, \qquad \lambda_2 = -\gamma < 0$$

For E_2 to be locally asymptotically stable, we should have $\lambda_1 < 0$, since $\lambda_2 < 0$. This implies that $m < 1 - \frac{\sigma\beta}{\alpha\gamma\omega}$.

Proposition 3.4: The equilibrium point $E_3(1,0)$ is unstable.
Proof: At $E_3(1,0)$, the Jacobean matrix becomes

$$J(E_3) = \begin{bmatrix} -1 & \frac{-\alpha(1-m)}{\beta + (1-m)} \\ 0 & \gamma \end{bmatrix}$$

The eigenvalues are $\lambda_1 = -1 < 0$, $\lambda_2 = \gamma > 0$
Thus the equilibrium point $E_3(1,0)$ is a saddle point. i.e. Unstable.

Proposition 3.5: The coexistence equilibrium point $E^*(x^*, y^*)$ is locally asymptotically stable
provided

$$\gamma > \left(1 - 2x^* - \frac{\alpha\beta(1-m)y^*}{(\beta + (1-m)x^*)^2}\right). \qquad (3.5)$$

Proof: At $E^*(x^*, y^*)$, the Jacobean matrix takes the form

$$J(E^*) = \begin{pmatrix} x^*\left(-1 + \frac{\alpha(1-m)^2 y^*}{(\beta + (1-m)x^*)^2}\right) & \frac{-\alpha(1-m)x^{\circ}}{\beta + (1-m)x^*} \\ \frac{(1-m)\gamma^2}{\sigma} & -\gamma \end{pmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$trace(J(E^*)) = a_{11} + a_{22} = x^*\left(-1 + \frac{\alpha(1-m)^2 y^*}{(\beta + (1-m)x^*)^2}\right) - \gamma$$

Thus, $trace(J(E^*) < 0$ if and only if $\gamma > x^*\left(-1 + \frac{\alpha(1-m)^2 y^*}{(\beta + (1-m)x^*)^2}\right)$.

Notes

$$\det(J(E^*)) = a_{11}a_{22} - a_{12}a_{21} = \gamma x^*\left(\frac{(\beta+(1-m)x^*)^2\sigma + \alpha\gamma(1-m)^2(\beta-\omega)}{(\beta+(1-m)x^*)^2\sigma}\right) > 0$$

Hence, the equilibrium point E^* is locally asymptotically stable provided $\gamma > -x^* - \dfrac{\alpha(1-m)^2 x^* y^*}{(\beta+(1-m)x^*)^2}$.

Notes

iii. *Global Stability*

Proposition 3.6. The system (3.2) does not admit any periodic solution for $m > 1-\beta$.

Proof: Let $(x(t), y(t))$ be solutions of the system (3.2). Define Dulac function

$$H(x,y) = \frac{\beta+(1-m)x}{xy}$$

Then

$$Q = \frac{\partial(HF)}{\partial x} + \frac{\partial(HG)}{\partial y}$$

$$= -\left(\frac{(\beta-1+m)+2(1-m)x}{y} + \frac{(x(1-m)+\beta)\delta}{x((1-m)x+\omega)}\right)$$

It is observed that $Q<0$ for $m>1-\beta$. Therefore, by Dulac criterion, the system (3.2) has no non-trivial periodic solutions.

Theorem 3.3. If $m>1-\beta$ then the local asymptotical stability of the system (3.2) ensures its global asymptotical stability around the unique positive interior equilibrium point $E^*(x^*, y^*)$.

Proof: The unique equilibrium point $E^*(x^*, y^*)$ is the only stable point in the xy plane. The boundedness of the solutions of the system together with the non-existence of periodic solutions establishes the global asymptotical stability.

d) *Mathematical Model without prey refuge*

If it is assumed that all the prey are accessible to the predator species, our mathematical model becomes,

$$\begin{cases} \dfrac{dX}{dT} = r\left(1-\dfrac{X}{K}\right)X - \dfrac{c_1XY}{k_1+X} \\ \dfrac{dY}{dT} = Y\left(s - \dfrac{c_2Y}{k_2+X}\right) \end{cases} \tag{3.6}$$

where all the parameters in the model are positive.

The following non-dimensional state variables and parameters are chosen.

$$x = \frac{X}{k} \qquad y = \frac{Y}{k} \qquad t = rT$$

$$\alpha = \frac{c_1}{r} \qquad \beta = \frac{k_1}{K} \qquad \gamma = \frac{s}{r} \qquad \sigma = \frac{c_2}{r} \qquad \omega = \frac{k_2}{K}$$

The system (3.6) takes the following non-dimensional form

$$\begin{cases} \dfrac{dx}{dt} = \left(1 - x - \dfrac{\alpha y}{\beta + x}\right)x \equiv F(x,y) \\ \dfrac{dy}{dt} = y\left(\gamma - \dfrac{\sigma y}{\omega + x}\right) \equiv G(x,y) \end{cases} \quad (3.7)$$

$$x(0) = x_0 \geq 0;\ y(0) = y_0 \geq 0$$

Notes

i. *Equilibrium Points*

We now study the existence of equilibria of system (3.7). All possible equilibria are

(i) The trivial equilibrium $E_0(0,0)$

(ii) Equilibrium in the absence of predator (y = 0) $E_1(1,0)$

(iii) Equilibrium in the absence of prey (x= 0) $E_2\left(0, \dfrac{\omega\gamma}{\sigma}\right)$

(iv) The interior (positive) equilibrium $E_3(x^*, y^*)$ where x^* is the unique positive root of the quadratic equation

$$\sigma x^{*2} + (\alpha\gamma + \sigma\beta - \sigma)x^* + \alpha\gamma\omega - \sigma\beta = 0;$$

$$x^* = \frac{-B + \sqrt{B^2 - 4\sigma C}}{2\sigma}, \quad y^* = \frac{\gamma(\omega + x^*)}{\sigma}$$

where $B = \alpha\gamma + \sigma\beta - \sigma$, $C = \alpha\gamma\omega - \sigma\beta$

ii. *Local stability of the equilibrium points*

The local asymptotic stability of each equilibrium point is studied by computing the Jacobean matrix and finding the eigenvalues evaluated at each equilibrium point. For stability of the equilibrium points, the real parts of the eigenvalues of the Jacobean matrix must be negative. From equations (3.7), the Jacobean matrix of the system is given by

$$J(E_i) = \begin{bmatrix} \dfrac{\partial F}{\partial x} & \dfrac{\partial F}{\partial y} \\ \dfrac{\partial G}{\partial x} & \dfrac{\partial G}{\partial y} \end{bmatrix}$$

which gives

$$J(E_i) = \begin{bmatrix} 1 - 2x - \dfrac{\alpha\beta y}{(\beta + x)^2} & -\dfrac{\alpha x}{\beta + x} \\ \dfrac{\sigma y^2}{(\omega + x)^2} & \gamma - \dfrac{2\sigma y}{\omega + x} \end{bmatrix}$$

The local asymptotic stability for each equilibrium point is analyzed as below:

i. $E_0(0,0)$ is unstable point.

The Jacobean matrix evaluated at E_0 gives,

$$J_0 = \begin{bmatrix} 1 & 0 \\ 0 & \gamma \end{bmatrix}$$

The eigen values of $J(E_0)$ are 1 and γ. $E_0(0,\ 0)$ is an unstable node, since 1 and γ are always positive.

Notes

ii. $E_1(1,0)$ is unstable point.

The Jacobean matrix evaluated at E_1 gives;

$$J_1 = \begin{bmatrix} -1 & \dfrac{-\alpha}{\beta+1} \\ 0 & \gamma \end{bmatrix}$$

The eigenvalues of matrix $J(E_1)$ are -1 and γ. The eigenvalues above are one negative and the other positive. Hence, the equilibrium point $E_1(1,0)$ is unstable, saddle.

iii. $E_2\left(0, \dfrac{\omega\gamma}{\sigma}\right)$ is locally asymptotically stable for $\gamma > \dfrac{\sigma\beta}{\alpha\omega}$

The Jacobean matrix evaluated at E_2 gives,

$$J_2 = \begin{pmatrix} 1 - \dfrac{\alpha\gamma\omega}{\sigma\beta} & 0 \\ \dfrac{\gamma^2}{\sigma} & -\gamma \end{pmatrix}$$

The eigen values of the matrix $J(E_2)$ are, $1 - \dfrac{\alpha\gamma\omega}{\sigma\beta}$ and $-\gamma$. Both the eigenvalues are negative if

$$\gamma > \frac{\sigma\beta}{\alpha\omega}.$$

For a positive equilibrium E_3 (x^*, y^*), $J(E_3)$ can be simplified to

$$J(E_3) = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

where

$$a_{11} = 1 - 2x^* - \frac{\alpha\gamma b x^*}{\beta + x^*} \qquad a_{12} = \frac{-\alpha x^*}{\beta + x^*},\quad a_{21} = \frac{\gamma^2}{\sigma},\quad a_{22} = -\gamma$$

Proposition 3.7: The unique positive equilibrium point $E_3(x^*, y^*)$ is locally asymptotically stable provided $\gamma > a_{11}$.

Proof: The characteristic equation is $\lambda^2 - (a_{11} + a_{22})\lambda + (a_{11}a_{22} - a_{21}a_{12}) = 0$.

The equilibrium point $E_3(x^*, y^*)$ is stable when $trace(J(E^*) = a_{11} + a_{22} < 0$ and $\det(J(E^*) = a_{11}a_{22} - a_{12}a_{21} > 0$

Now

$$trace(J(E^*) = a_{11} + a_{22}$$
$$= a_{11} - \gamma < 0$$
$$\Rightarrow \gamma > a_{11}$$

$$\det(J(E^*) = a_{11}a_{22} - a_{12}a_{21}$$

$$= \frac{2\,\delta x^3 + (\gamma\,\alpha - \delta + 4\,\beta\,\delta)x^2 + 2\beta\,(\,\alpha\gamma - \delta + \beta\delta)x + \beta(\alpha\gamma\omega - \beta\delta)}{(x+\beta)^2\,\delta} > 0$$

Hence, the unique positive equilibrium point $E_3(x^*, y^*)$ is locally asymptotically stable provided $\gamma > a_{11}$.

iii. *Global Stability*

Proposition 3.7: The system (3.7) does not admit any periodic solution for $\beta > 1$.

Proof: Let $(x(t), y(t))$ be solutions of the system (3.7). Define Dulac function

$$H(x,y) = \frac{\beta + x}{xy}$$

Then

$$Q = \frac{\partial(HF)}{\partial x} + \frac{\partial(HG)}{\partial y}$$

$$= -\left(\frac{(\beta-1)+2x}{y} + \frac{(x+\beta)\delta}{x(x+\omega)}\right)$$

It is observed that $Q < 0$ for $\beta > 1$. Therefore, by Dulac criterion, the system (3.7) has no non-trivial periodic solutions.

Theorem 3.4: If $\beta > 1$ then the local asymptotical stability of the system (3.7) ensures its global asymptotical stability around the unique positive interior equilibrium point $E^*(x^*, y^*)$.

Proof: The unique equilibrium point $E^*(x^*, y^*)$ is the only stable point in the xy plane. The boundedness of the solutions of the system together with the non-existence of periodic solutions establishes the global asymptotical stability.

Chapter Four

VI. Numerical Simulation

Many areas of science and engineering relays on quantitative analysis, as more complex mathematical models of the real world phenomena become available. Since most of these models don't have a closed form exact solution, numerical approximations are the only tools available for analyzing them. In this chapter we will solve the system equation (3.2) and (3.7) by using the in-built ordinary differential equation solver MatLab function ode45.

Notes

Notes

For solving system (3.2), we took the following parametric values. $\alpha = 1,\ \gamma = 0.2,\ \omega = 0.2,\ \sigma = 0.1,\ \beta = 0.2$ in appropriate units. For these values of parameter, we simplify the existence and stability properties of the equilibrium for the system.

For the given parametric values, it is found that the coexistence equilibrium point exists for $m > 0.5$. Hence, in our simulation we took the values of m in the range $0.5 < m < 1$.

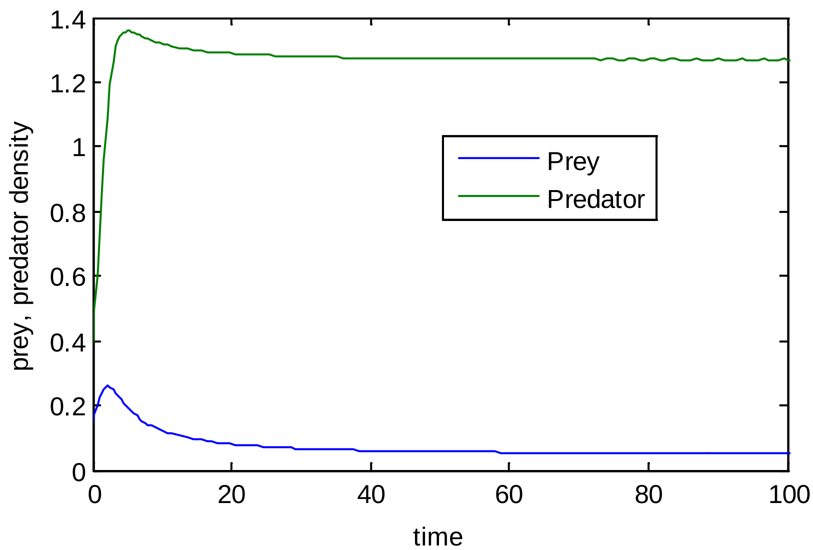

Figure 1: Times series plot of prey and predator at m=0.55

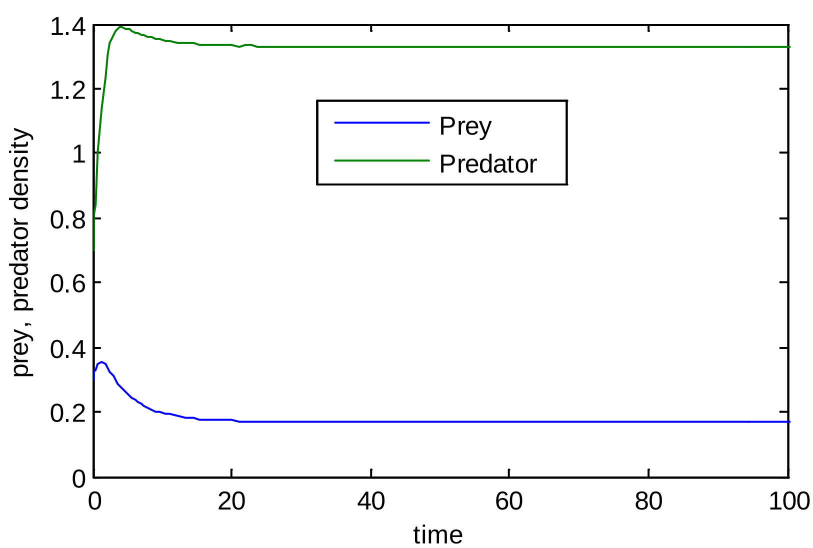

Figure 2: Time series plot of prey and predator at m=0.6.

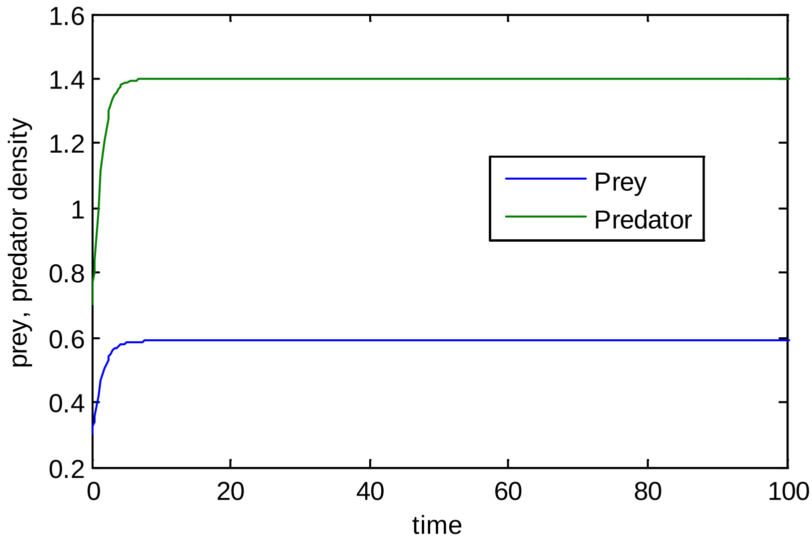

Figure 3: Time series plot of prey and predator at m=0.8.

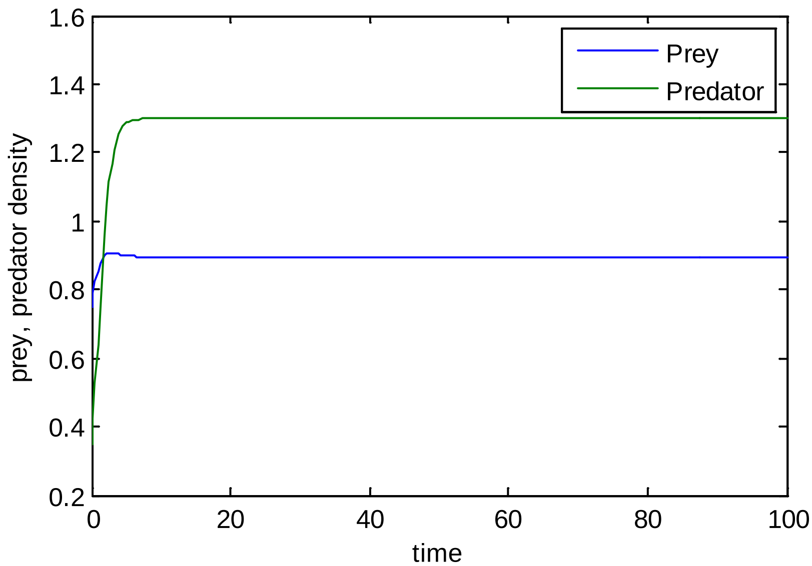

Figure 4: Time series plot of prey and predator at m=0.95.

For the system equation (3.7), that is the system in the absence of prey refuge, we have used the following parametric values as fixed and the parameter γ as a control parameter. These values are $\alpha = 1,\ \omega = 0.2,\ \sigma = 0.1,\ \beta = 0.2$.

For these set of parametric values the coexistence equilibrium point exists whenever $\gamma < 0.1$.

The coexistence equilibrium point is locally asymptotically stable for $\gamma < 0.651234$ and hence unstable otherwise.

Figures 5-7 shows the stability of the coexistence equilibrium point. i.e. the solution, trajectory, of the prey and predator species approaches to the coexistence equilibrium point.

Notes

Notes

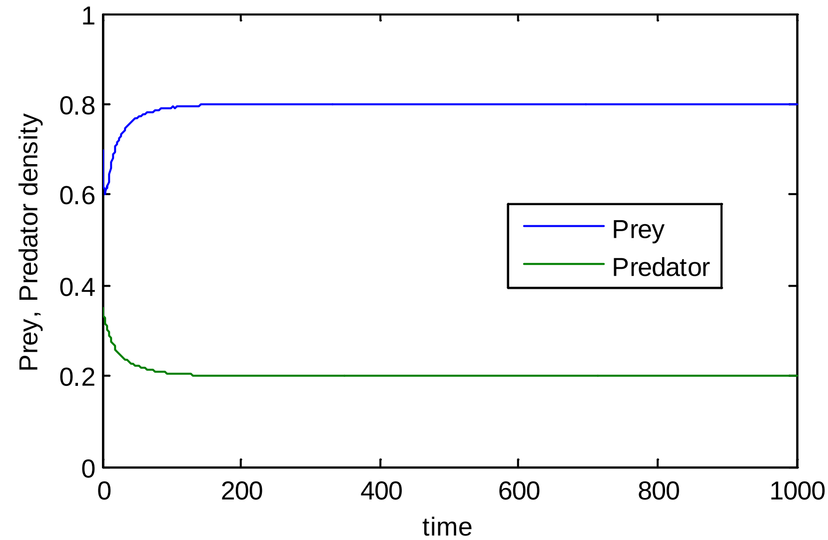

Figure 5: Time series plot of the prey and predator at $\gamma = 0.02$

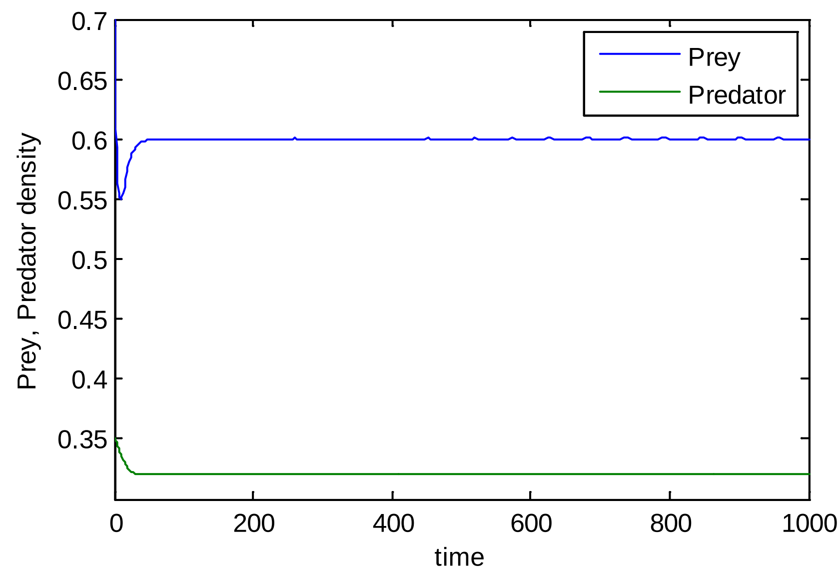

Figure 6: Time series plot of prey and predator at $\gamma = 0.04$

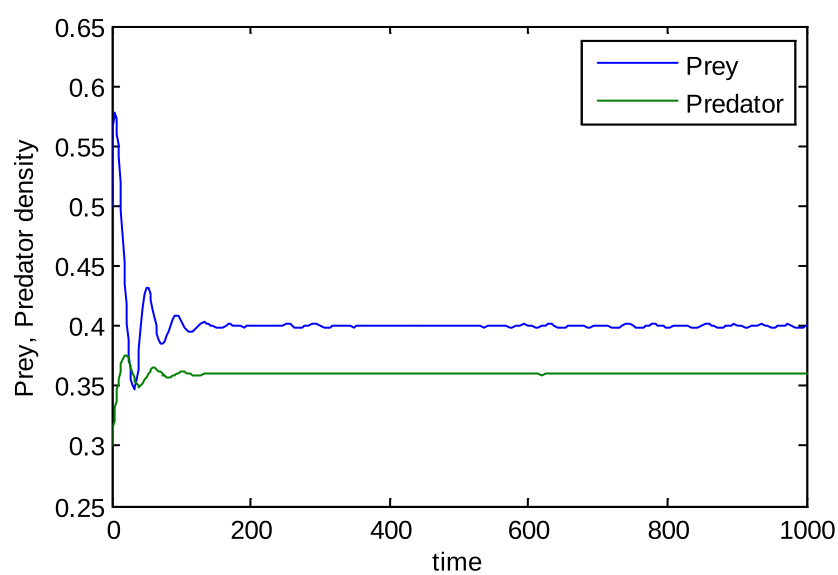

Figure 7: Time series plot of prey and predator at $\gamma = 0.06$

Notes

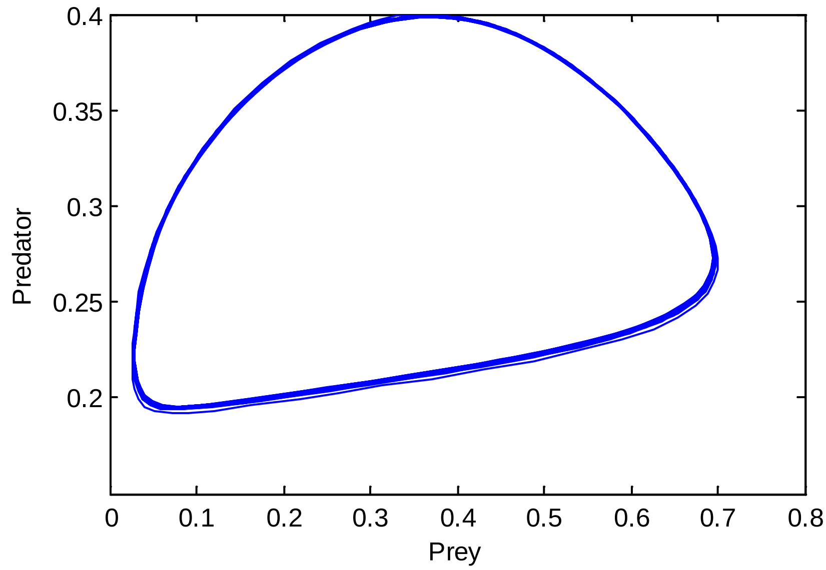

Figure 8: Phase portrait of prey and predator at $\gamma = 0.07$.

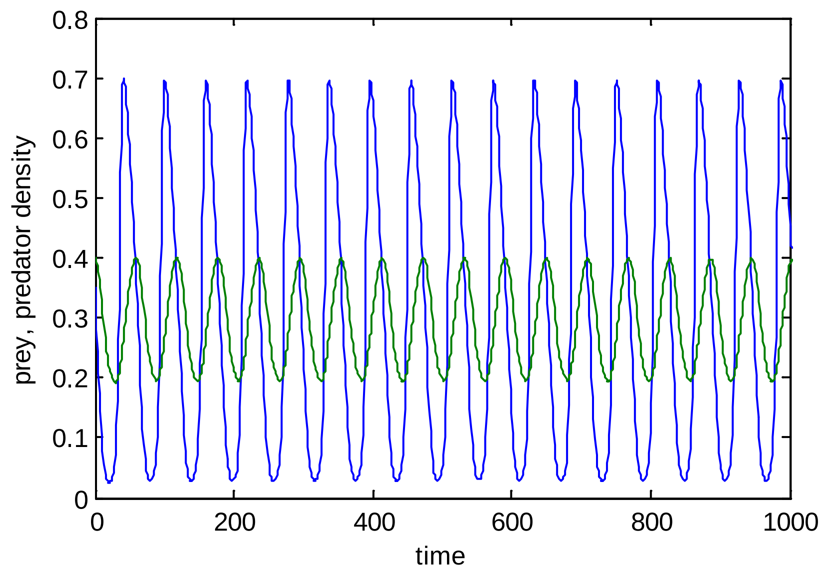

Figure 9: Time series plot of prey and predator at $\gamma = 0.07$.

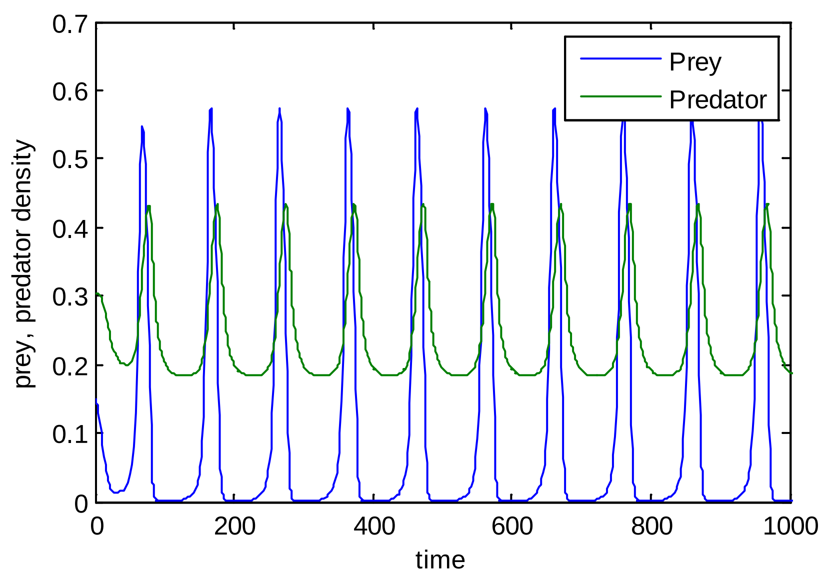

Figure 10: Time series plot of prey and predator at $\gamma = 0.09$.

Figures 8 shows the existence of a limit cycle, periodic solution. Figure 9 also shows the oscillatory nature of the predator prey system. Figure 10 represents the instability of the coexistence equilibrium point.

Chapter Five

V. Conclusion

Notes

This paper presents a prey-predator model with Holling type II functional response and modified Leslie Gower incorporating a constant proportion of prey refuge. Incorporating a refuge into system (3.2) provides a more realistic model. Refugi, therefore, can be considered as, areas in which the predator is not successfully controlling the prey and important for the biological control of a predator. The main focus of this paper was to introduce new mathematical models of biological systems and techniques for their analysis. Local asymptotic stability of the positive equilibrium implies its global asymptotic stability. Moreover, we established some new results such as the existence of stable or unstable equilibrium points under suitable values of parameters in the models. Two species can coexist in the case of stable condition; otherwise they might be extinct in the case of unstable condition.

Declaration

I, undersigned declare that this research entitled with "Mathematical modeling of a prey-predator Model with Modified Leslie-Gower and Holling -Type II incorporating prey refuge" is original and it has not been submitted to any institution elsewhere for the award of any academic degree or like, where other sources of information have been used, they have been acknowledged.

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I would like to appreciate Dr. Dawit Melese (my advisor) for his painstakingly gone through all my research from scratch to the last submission.

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Comparison of Different Volatility Model on Dhaka Stock Exchange

By Imran Parvez, Md. Moyazzem Hossain & Masudul Islam

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Abstract- Dhaka stock exchange (DSE) is a very hazardous market, so its volatility forecasting would be a very difficult and necessary task. The behaviour of Stock Market is different from market to market. So a unique time series model couldn't be a best forecasting technique for all stock market because of their varying nature. There are various types of time series model like Expected weighted moving average model, GARCH-type models, Moving Average models, Exponential smoothing model and so on. In this paper our objective is to compare the ability of different types of models to forecast volatility of Dhaka stock exchange.

Keywords: DSE, volatility, forecasting, model comparison.

GJSFR-F Classification: MSC 2010: 13P25

COMPARISONOFDIFFERENTVOLATILITYMODELONDHAKASTOCKEXCHANGE

Strictly as per the compliance and regulations of :

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Abstract- Dhaka stock exchange (DSE) is a very hazardous market, so its volatility forecasting would be a very difficult and necessary task. The behaviour of Stock Market is different from market to market. So a unique time series model couldn't be a best forecasting technique for all stock market because of their varying nature. There are various types of time series model like Expected weighted moving average model, GARCH-type models, Moving Average models, Exponential smoothing model and so on. In this paper our objective is to compare the ability of different types of models to forecast volatility of Dhaka stock exchange.

Keywords: DSE, volatility, forecasting, model comparison.

I. Introduction

Volatility in financial markets, particularly stock and foreign exchange markets is an important issue that concerns government policy makers, market analysis, corporate managers, and financial managers and it has held the attention of academics and practitioners over the last two decades.

Volatility is not the same as risk. When it is interpreted as uncertainty, it becomes a key input to many investment decisions and portfolio creations. Investors and portfolio managers have certain levels of risk which they can bear. A good forecast of the volatility of asset process over the investment holding period is a good starting point for assessing investment risk. Volatility is the most important variable in the pricing of derivative securities, whose trading volume has quadrupled in recent years. To price an option, we need to know the volatility of the underlying asset from now until the option expires. In fact, the market convention is to list option prices in terms of volatility units. Nowadays, one can buy derivatives that are written in volatility itself, in which case the definition and measurement of volatility will be clearly specified in the derivative contracts. In these new contracts, volatility now becomes the underlying "asset." So a volatility forecast is needed to price such derivative contracts.[12] Policy makers often rely on market estimates of volatility as a barometer for the vulnerability of financial markets and the economy. So, for the economic development of Bangladesh, the volatility forecasting is a very important issue.

Many econometric models have been used. The Auto Regressive Conditional Heteroscedasticity (ARCH) model introduced by Engel and Bollerslev's Generalized

Ref

12. Yang, X; "Forecasting volatility in Stock Markets Using Garch Models"

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ARCH (GARCH) model conveniently accounted for time varying volatility. In ARCH, the conditional variance is equal to a linear function of past squared errors. The GARCH specification allows the current conditional variance to be a function of past variance as well. The GARCH models have been applied to study stock market volatility by poon and Taylor, Engel and Ng and Kearns and Pagan.

The conditional variance of the current error in the GARCH model is specified as a function of past conditional variances and past errors. Only the magnitude of the errors affects volatility, but their signs do not. The character of asymmetry in the distribution of stock returns allows an unexpected positive return to cause less volatility then an unexpected negative return of the same size. The GARCH model cannot explain asymmetry in distribution of stock returns. A new model is needed to make allowances for asymmetric distribution of stock returns that the GARCH model fails to capture. Nelson proposed the Exponential GARCH (EGARCH) model, Glosten et al. proposed the GJR-GARCH model. A comparison of the GARCH, EGARCH GTJR-GARCH models by Engel and Ng on daily Japanese stock index return data suggests that GJR-GARCH IS the best one. Using monthly US stock returns pagan and Schwert find better explanatory power from the EGARCH model. [11]

However, despite the appeal of complexity and despite their popularity, it is by no means agreed that complex models such as GARCH provide superior forecasts of return volatility. Dimson and Marsh (1990) is a notable example in which simple models have prevailed – although it should be pointed out that ARCH models were not included in their analysis. Specifically, Dimson and Marsh apply five different types of forecasting model to a set of UK equity data, namely, (a) a random walk model; (b) a long-term mean model; (c) a moving average model; (d) an exponential smoothing model; and (e) regression models. They recommend the final two of these models and, in so doing, sound an early warning in this literature that the best forecasting models may well be the simple ones. Other papers however spell out a mixed set of findings on this issue. For example, Akgiray (1989) found in favor of a GARCH (1, 1) model (over more traditional counterparts) when applied to monthly US data. Brailsford and Faff (1996) investigate the out-of-sample predictive ability of several models of monthly stock market volatility in Australia. In the measurement of the performance of the models, in addition to symmetric loss functions, they use asymmetric loss functions to penalize under/over-prediction. They conclude that the ARCH class of models and a simple regression model provide superior forecast of volatility. However, the various model rankings are shown to be sensitive to the error statistics used to assess the accuracy of the forecasts. [1]

Bangladesh is a developing country here the capital formation is so much important for the economic development. For this capital formation the Stock Market plays a crucial role. But the Stock Market of Bangladesh is not an efficient market. So to make the market efficient and to reduce the uncertainty of the investor to invest, the volatility forecast is necessary step for the government and policy makers.

A volatility model must be able to forecast volatility; this is the central requirement in almost all financial applications. In this study, we try to forecast the daily volatility of Dhaka stock market by different well recognized models: Moving Average, GARCH (1, 1), Exponential Smoothing, E-GARCH, GJR-GARCH and rank their ability by different error measurement tools like Mean Sum Square of Error (MSE), Mean Absolute Error (MAE), Theil-U, Linex Loss Functions etc.

Ref

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Notes

II. DSE General Index

a) *Return Series*

Here we use the DSE General Index from 1st January 2002 to 19th June 2012, 2639 daily data. In following Fig-1 we depict the return series of DSE General Index, reveals that, as expected, volatility is not constant over time and moreover tends to cluster. Periods of high volatility can be distinguished from low volatility periods. Here the maximum change occurs in 16th November 2009, which is 23 per cent rise. The second largest rise is the 15.5 per cent return on 11th January 2011 and the largest fall is 9.3 per cent return in 10th January 2011.

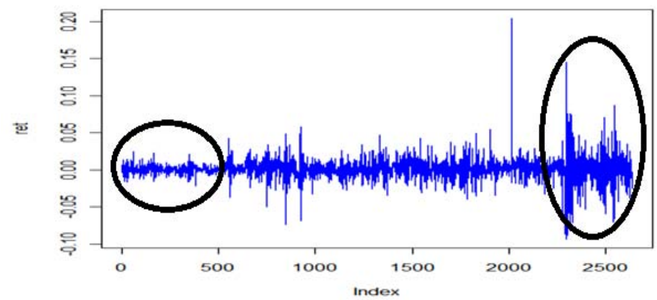

Fig. 1: Return Series. The Oval indicate a low volatility period and a high volatility period

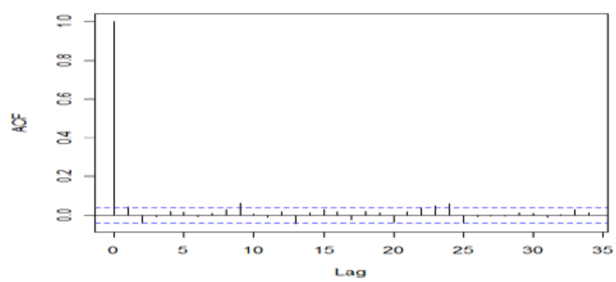

Fig. 2: Correlogram of Return Series

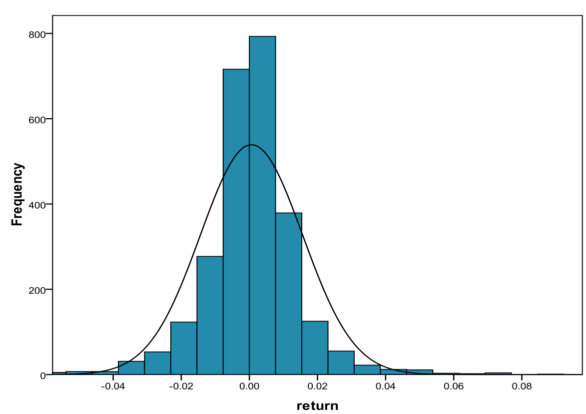

Fig. 3: Histogram of return with normal curve

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Notes

Table 1: Descriptive Statistics of return

		Statistic
r	Mean	0.00063
	Std. deviation	0.015
	Skewness	0.913
	Kurtosis	20.431
	N	2639

The mean daily return is 0.063 per cent. The standard deviation of the daily returns is 0.015. The series also exhibits a positive skewness of 0.913 and excess kurtosis of 20.431, indicating the absence of normality which also reveals from Q-Q plot, Kolmogorov smirnov test and Shapiro-Wilk normality test. It can be easily seen from the histogram that there are many returns which are above four standard deviation is highly unlikely with the normal distribution.

Using the augmented Dickey-Fuller (ADF) unit root test we can clearly reject, as expected, the hypothesis of a unit root in the return process. The ADF t-statistic is −12.92 which rejects the unit root hypothesis with a confidence level of more than 99 per cent.

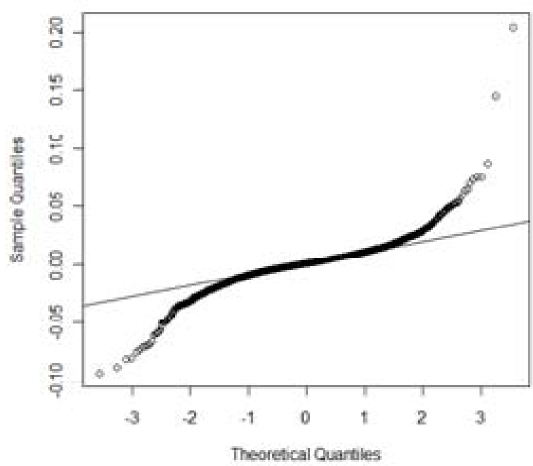

Fig. 4: Normal Q-Q plot of returnTable 2: Normality Test of return

	Kolmogorv Smirnov		**Shapiro-Wilk**	
r	Statistic	Sig.	Statistic	Sig.
	5.134	0.00	.8621	0.00
	ADF Test			
	Statistic		Sig	
	-12.92		0.01	

b) Volatility Series

We obtain the daily volatility simply squaring the return series. i.e., $\sigma_t^2 = r_t^2$, Where $r_t$is the daily return on a day t. From the following Fig-5 we can easily point out the huge volatility periods which are 16th November 2009, 11th January 2011 and the 10th January 2011 which also focuses from the return series. The table-3 and the Fig-6 both show that the first five autocorrelations are significantly different

Notes

from zero which suggests that the daily volatility series is apart from randomness and hence predictable. To test for possible unit roots the augmented Dickey-Fuller (ADF) statistic is calculated and the results are also presented in table-3. The ADF statistic for the entire sample is -10.13 with p-value 0.01. Hence the hypothesis that the daily volatility in the DSE General index over the period 1st January 2002 to 19th June 2012 has a unit root has to be rejected.

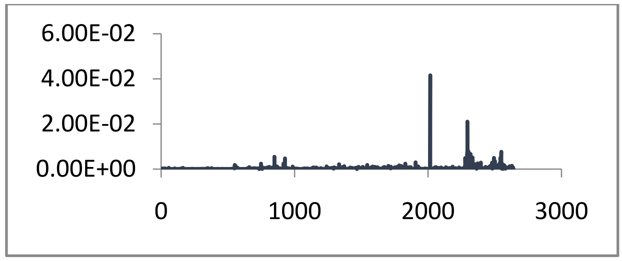

Fig. 5: Daily volatility series

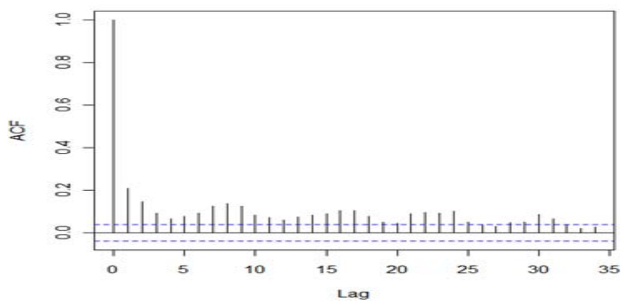

Fig. 6: Correlogram of Daily Volatility

Table 3: Summary statistics and normality test of Daily volatility series

Mean	Maximum	Skewness	Kurtosis	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	ρ_6	ρ_7
0.0002	0.04	25.128	868.254	.208	0.146	.093	.066	.078	.093	.124

ADF Test		Shapiro-Wilk Normality Test	
Test Statistic	P-value	Test Statistic	P-Vlaue
-10.13	0.01	0.1508	0.000

III. Volatility Forecasting Techniques

a) Moving average

According to the historical average model, all past observations receive equal weight. In the moving average model, however, more recent observations receive more weight. In the paper, two moving average models are used: a five-year and a six-year moving average. The five-year model is defined as

$$\hat{\boldsymbol{\sigma}}^2_{\boldsymbol{T+1}} = \sum_{\boldsymbol{i=1}}^{\boldsymbol{1825}} \hat{\boldsymbol{\sigma}}^2_{\boldsymbol{T+1-i}} \, T = 2286, \ \dots 2638[13].$$

b) Exponential smoothing

Exponential smoothing is a simple method of adaptive forecasting. Unlike forecasts from regression models which use fixed coefficients, forecasts from exponential smoothing methods adjust based upon past forecast errors. Single exponential smoothing forecast is given by $\hat{\sigma}^2_{T+1} = (1-\alpha)\hat{\sigma}^2_T + \alpha\sigma^2_T$ where $0 < \alpha < 1$ is the damping (or smoothing factor). By repeated substitution, the recursion can be rewritten as $\hat{\sigma}^2_{T+1} = \alpha \sum_{t=1}^{T} (1-\alpha)^t \sigma^2_{T+1-t}$, 2286, ... ,2638 This shows why this method is called exponential smoothing - the forecast of σ^2_{T+1} is a weighted average of the past values of σ^2_{T+1-t}, where the weights decline exponentially with time. The value of α is chosen to produce the best fit by minimizing the sum of the squared in sample-forecast errors. Dimson and Marsh (1990) and BF select the optimal α annually.[13]

c) Generalized Arch (GARCH)

For the ARCH (q) model, in most empirical studies, q has to be large. This motivates Bollerslev (1986) to use the GARCH (p; q) specification which is defined as

$$\begin{cases} r_t = \mu + \sigma_t \epsilon_t \\ \sigma_t^2 = \lambda + \sum_{j=1}^{q} \alpha_j \left(r_{t-j} - \mu\right)^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \end{cases}$$

Define $s_t = r_t - \mu$, $m = max\{p,q\}$, $\alpha_i = 0$ for $i > q$ and $\beta_i = 0$ for $i > p$. Following Baillie and Bollerslev (1992), the optimal h-day ahead forecast of the volatility can be calculated by iterating on

$\hat{\sigma}^2_{t+h} = \lambda + \sum_{i=1}^{m}(\alpha_i + \beta_i)\hat{\sigma}^2_{t+h-i} - \beta_h \widehat{w}_t - \cdots - \beta_m \widehat{w}_{t+h-m}$, for $h = 1,2,\ldots,p$

$\hat{\sigma}^2_{t+h} = \lambda + \sum_{i=1}^{m}(\alpha_i + \beta_i)\hat{\sigma}^2_{t+h-i}$ for $h = p+1, p+2, \ldots$

$\hat{\sigma}^2_\tau = s^2_\tau \text{for} 0 \leq \tau \leq t,$

$\hat{\sigma}^2_\tau = s^2_\tau = T^{-1}\sum_{i=1}^{T} s_i^2,$ for $\tau \leq 0$

$\widehat{w}_\tau = s^2_\tau - E(s^2_\tau / I_{\tau-1}),\ for\ 0 < \tau \leq t,$

$\widehat{w}_\tau = 0, \quad for \tau \leq 0$

With the daily volatility forecasts across all trading days in each month. Again, the selection of p and q is an important empirical question. Here BIC is used to choose p and q. The GARCH (1, 1) model has been found to be adequate in many applications and hence is used as a candidate model. [13]

d) GJR-GARCH

In order to accommodate the possible differential impact on conditional volatility from positive and negative shocks, Glosten et al. (1992) extended the GARCH model to capture possible asymmetries between the effects of positive and negative shocks of the same magnitude on the conditional variance. The GJR(p, q) model is given by:

$$h_t = \omega + \sum_{j=1}^{p} \alpha_j \varepsilon^2_{t-j} + \gamma I\left(\varepsilon_{t-1}\right)\varepsilon^2_{t-1} + \sum_{i=1}^{q} \beta_i h_{t-i}$$

where the indicator variable, I(x) is defined as:

$$I(x) = \begin{cases} 1, & x \leq 0 \\ 0, & x > 0 \end{cases}$$

for the case $p = q = 1,\ \omega > 0,\ \alpha_1 > 0,\ \alpha_1 + \gamma > 0,\ \beta_1 \geq 0$ are sufficient conditions to ensure a strictly positive conditional variance, $h_t > 0$ The indicator variable distinguishes between positive and negative shocks, where the asymmetric effect $(\gamma > 0)$ measures the

Ref

13. Yu, J., (2002), "Forecasting Volatility in the New Zealand Stock Market", *Applied Financial Economics*, Vol. 12, pp.193-202.

contribution of shocks to both short-run persistence $\alpha_1+\gamma/2$ and long-run persistence $\alpha_1+\beta_1+\gamma/2$. The GJR model has also had some important theoretical developments. In the case of symmetry of η_t, the regularity condition for the existence of the second moment of GJR(1, 1) is $\alpha_1+\beta_1+\gamma/2<1$. Moreover, the weak log-moment condition for GJR(1, 1),

$$E\left(\log\left[\left(\alpha_1+\gamma I\left(\eta_t\right)\right)\eta_t^2+\beta_1\right]\right)<0$$

is sufficient for the consistency and asymptotic normality of the QMLE. In empirical examples, the parameters in the regularity condition are replaced by their QMLE, the standardized residuals, η_t are replaced by the estimated residuals from the GJR model, for t = 1,. . . n, and the expected value is replaced by their sample mean. Gonzalez-Rivera (1998) developed the Logistic Smooth Transition GARCH (LSTGARCH) which allows for a gradual change of threshold parameter in GJR. This model is given by:

$$h_t=\omega+\sum_{j=1}^{p}\alpha_j\varepsilon_{t-j}^2+\gamma L\left(\varepsilon_{t-1}\right)\varepsilon_{t-1}^2+\sum_{i=1}^{q}\beta_i h_{t-i}$$

where the Logistic smooth transition variable, L(x) is defined as:

$$L\left(x\right)=\frac{1}{1+\exp\left(-\theta x\right)}$$

Sufficient conditions to ensure a strictly positive conditional variance $h_t>0$ are the same as those for GJR. The Logistic variable takes any value between zero and one and measures the magnitude of positive or negative shocks.[7]

e) EGARCH

Nelson (1991) proposed the exponential GARCH model, which is given as:

$$\log\left(h_t\right)=\omega+\sum_{i=1}^{p}\alpha_i\left|\frac{\varepsilon_{t-i}}{h_{t-i}}\right|+\sum_{k=1}^{r}\gamma_k\frac{\varepsilon_{t-k}}{h_{t-k}}+\sum_{j=1}^{k}\beta_j\log\left(h_{i-j}\right)$$

As the range of log(ht) is the real number line, the EGARCH model does not require any parametric restrictions to ensure that the conditional variances are positive. Furthermore, the EGARCH specification is able to capture several stylized facts, such as small positive shocks having a greater impact on conditional volatility than small negative shocks, and large negative shocks having a greater impact on conditional volatility than large positive shocks. Such features in financial returns and risk are often cited in the literature to support the use of EGARCH to model the conditional variances. Unlike the EWMA, ARCH, GARCH and GJR models, EGARCH uses the standardized rather than the unconditional shocks. Moreover, as the standardized shocks have finite moments, the moment conditions of EGARCH are straightforward and may be used as diagnostic checks of the underlying models. However, the statistical properties of EGARCH have not yet been developed formally. If the standardized shocks are independently and identically distributed, the statistical properties of EGARCH are likely to be natural extensions of (possibly vector) ARMA time series processes [8]

Ref

8. McAleer, M. (2005), "Automated Inference and Learning in Modeling Financial Volatility", Econometric Theory, 21, 232-261.

i. *Specification on Conditional Variance*

The strict form of the error distribution plays an important role in estimating the EGARCH(1,1) formulation. We conducted two different functional forms of the conditional density $\psi(.)$, the Gaussian Normal distribution and the standardized Student-t distribution.

IV. Evaluation of Measures

Notes

Four measures are used to evaluate the forecast accuracy, namely, the root means square error (RMSE), the root mean absolute error (MAE), the Theil-U statistic and the LINEX loss function. They are defined by

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\hat{\sigma}_i^2 - \sigma_i^2\right)}$$

$$MAE = \frac{1}{n}\sum_{i=1}^{n}\left|\hat{\sigma}_i^2 - \sigma_i^2\right|$$

$$Theil - U = \frac{\sum_{i=1}^{n}\left(\hat{\sigma}_i^2 - \sigma_i^2\right)^2}{\sum_{i=1}^{n}\left(\sigma_{i-1}^2 - \sigma_i^2\right)^2}$$

$$LINEX = \frac{1}{n}\sum_{i=1}^{n}\left[\exp\left(-a\left(\hat{\sigma}_i^2 - \sigma_i^2\right)\right) + a\left(\hat{\sigma}_i^2 - \sigma_i^2\right) - 1\right]$$

where a in the LINEX loss function is a given parameter.

The RMSE and MAE are two of the most popular measures to test the forecasting power of a model. Despite their mathematical simplicity, however, both of them are not invariant to scale transformations. Also, they are symmetric, a property which is not very realistic and inconceivable under some circumstances (See BF).

In the Theil-U statistic, the error of prediction is standardized by the error from the random walk forecast. For the random walk model, which can be treated as the bench mark model, the Theil-U statistic equal. Of course, the random walk is not necessarily a naÏve competitor, particularly for many economic and financial variables, so that the value of the Theil-U statistic close 1 is not necessarily and indication of bad forecasting performance. Several authors, such as Armstrong and Fildes (1995), have advocated using U-statistic and close relatives to evaluate the accuracy of various forecasting methods. One advantage of using U-statistic is that it is invariant to scalar transformation. The Theil-U statistic is symmetric, however.

In the LINEX loss function, positive errors are weighted differently from the negative errors. If $a > 0$, for example, the LINEX loss function is approximately linear for $\hat{\sigma}_t^2 - \sigma_t^2 > 0$ ('ove-predictions') and exponential for $\hat{\sigma}_t^2 - \sigma_t^2 < 0$ ('under predictions'). Thus, negative errors receive more weight tan positive errors. In the context of volatility forecasts, this implies that an under-prediction of volatility needs to be taken into consideration more seriously. Similarly, negative errors receive less weight than positive errors than positive errors when $a < 0$. BF argue an underestimate of the call option price, which corresponds an under-prediction of stock price volatility, is more likely to be of greater concern to a seller than a buyer and the reverse should be true of the over-predictions. Christo Versen and Diebold (1997) provide the analytical expression for the optimal LINEX prediction under assumption that the process is conditional normal. Using a series of annual volatilities in the UK stock market, Hwang et al. (1999) show that the LINEX forecasts outperform the conventional forecasts with an appropriate LINEX parameter, a. BF also adopt asymmetric loss functions to evaluate forecasting performance. An important reason why the LINEX function is more popular in the literature is it provides the analytical solution for the optimal prediction under conditional normality, while the same argument cannot be applied to the loss functions used by BF.[13]

Notes

13. Yu, J., (2002), "Forecasting Volatility in the New Zealand Stock Market", *Applied Financial Economics*, Vol. 12, pp.193-202.

V. Results

Table-4: Forecasting performance of competing models under symmetric loss

	RMSE	Rank	MAE	Rank	Theil-U	Rank
GARCH(1,1)_Student-t	0.001771	1	0.00000314	1	0.958604	1
GARCH(11)_Skew Normal	0.002834	10	0.00000805	10	2.461194	10
GARCH(1,1)_Normal	0.001858	6	0.00000345	6	1.055114	6
MA(5)	0.001817	4	0.0000033	4	1.009108	4
MA(6)	0.001825	5	0.00000333	5	1.017771	5
EGARCH(1,1)_Student-t	0.001883	7	0.00000354	7	1.083849	9
EGARCH(1,1)_Skewed Normal	0.001883	8	0.00000354	8	1.083846	7
EGARCH(1,1)_Normal	0.001883	9	0.00000354	9	1.083847	8
GJR-GARCH(1,1)_Student-t	0.001773	2	0.00000314	2	0.96097	2
GJR-GARCH(1,1)_Skew Normal	0.002844	11	0.00000809	11	2.473141	11
GJR-GARCH(1,1)_Normal	0.00309	12	0.00000955	12	2.918154	12
Exponential Smoothing	0.001789	3	0.0000032	3	0.978268	3

Table-5: Forecasting performance of competing models under asymmetric loss

	Linex	Rank	Linex	Rank	Linex	Rank	Linex	Rank
	$a = 10$		$a = -10$		$a = 20$		$a = -20$	
GARCH(1,1)_Student-t	0.000162669	1	0.000151388	1	0.000676337	1	0.000585645	1
GARCH(1,1)_Skew Normal	0.000405247	10	0.000400391	10	0.001634105	10	0.001594809	10
GARCH(1,1)_Normal	0.000178935	6	0.000166714	6	0.000743415	6	0.000645185	6
MA(5)	0.000171325	4	0.000159274	4	0.000712632	4	0.000615759	4
MA(6)	0.000172781	5	0.000160654	5	0.000718625	5	0.000621147	5
EGARCH(1,1)_Student-t	0.000183852	9	0.000171217	7	0.000764041	9	0.000662482	9
EGARCH(1,1)_Skew Normal	0.000183851	7	0.000171217	8	0.000764039	7	0.00066248	7
EGARCH(1,1)_Normal	0.000183851	8	0.000171217	9	0.00076404	8	0.00066248	8
GJR-GARCH(1,1)_Student-t	0.000163044	2	0.000151787	2	0.000677786	2	0.000587286	2
GJR-GARCH(1,1)_Skew Normal	0.000407189	11	0.000402359	11	0.001641835	11	0.001602747	11
GJR-GARCH(1,1)_Normal	0.000478697	12	0.000476454	12	0.001922861	12	0.001904477	12
Exponential Smoothing	0.000166082	3	0.000154411	3	0.000690787	3	0.000596981	3

Here we rank our models with four evaluation of measures: RMSE, MAE, Theil-U and LINEX loss functions. From the above results it is noted all the evaluation method indicates that the GARCH (1, 1) model with conditional distribution Student-t provides the most accurate forecasts and the GJR-GARCH (1, 1) model which is also with conditional distribution Student-t which ranks two.

In RMSE, Exponential Smoothing which has placed third forecasts 1.01 per cent and 0.9 per cent less accurately then GARCH (1, 1) _Student-t and GJR-GARCH (1, 1) _Student-t respectively. Where the GJR-GARCH (1, 1) _Normal has placed last and forecast 74.47 per cent and 74.28 per cent less accurately then the first and second superior model. So, the GJR-GARCH (1, 1) _Student-t model provides the worse forecast.

In MAE, Theil-u and in all the Linex loss functions the first, second and the third are the same one which are GARCH(1,1)_Student-t, GJR-GARCH(1,1)_Student-t and Exponential Smoothing respectively. The GJR-GARCH(1,1)_Normal is placed last by all the evaluation methods. It forecasts 204.14 per cent, 204.42 per cent, 194.28 per cent, 214.72 per cent, 184.35 per cent and 225.19 per cent less accurately then the GARCH (1, 1) _Student in MAE, Theil-u, Linex (a=10), Linex (a=-10), Linex (a=20) and Linex (a=-20) respectively. Which shows great evidence that the GJR-GARCH (1, 1) forecasts worse among the twelve competing models for the Dhaka Stock Exchange.

Here all the linex loss functions evaluates similarly except few cases such as EGARCH(1,1)–Skew Normal and EGARCH(1,1)–Normal have placed seven and eight in all the Linex loss functions except Linex (a=-10) etc.

VI. Conclusion

The stock market is a pivotal institution in the financial system of a country. In the stock market, when share prices fall below the normal, a warning is given out that the economy is running down and may approach the points of collapse. On the other hand, when the prices are abnormally high, it is the indication of fever in the system and danger of possible death. And these ups and downs are the root cause of what is called volatility which indicates the fickleness or the instability of the stock prices in the market.

The volatility is the important issue for the financial market particularly for the stock market. So in this study our main object is to forecast the future volatility by the best model for the Dhaka Stock Exchange. There is a large literature on forecasting volatility, Many econometric models have been used. However, no single model is superior. Using US Stock data, for example, Akgiray (1989), pagan and Schwert (1989) and Brooks (1998) finds the GARCH models outperform most competitors. Brailsford and Fafi (1996) (hereafter BF) find that the GARCH models are slightly superior to most simple models for forecasting Australian monthly stock index volatility. Using data sets from Japanese and Singaporean markets respectively, however, Tse (1991) and Tse and Tung (1992) find that the Exponential Weighted Moving Average models provide more accurate forecasts that the GARCH model Oimson and Marsh (1990) find in the UK equity market more parsimonious models such as the Smoothing and Regression models perform better than less parsimonious models, although the GARCH models are not among the set of competing models considered.

This paper examined twelve univariate models for forecasting stock market volatility of the DSE General Index. After comparing the forecasting performance of all twelve models, it was found that the GARCH (1, 1) with conditional distribution student-t model is superior according to the RMSE, MAE, Theil-U and four asymmetric loss functions.

Notes

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Notes

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Stability Analysis of A Ratio-Dependent Predator-Prey Model with Disease in the Prey

By Ahmed Buseri Ashine
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Abstract- Ahmed [1], considered study of prey-predator model with predator in disease and harvesting. In this article, a ratio-dependent predator-prey model with disease in the prey population is formulated and analyzed. Assuming that prey populations suffered from epidemics and the predator population will prefer only infected population for their diet as those are more vulnerable. Dynamical behaviours such as boundedness, local and global stabilities are discussed.

Keywords: *eco-epidemiological model, global stability.*

GJSFR-F Classification: *MSC 2010: 92B05*

Stability Analysis of a Ratio-Dependent Predator-Prey Model with Disease in the Prey

Ahmed Buseri Ashine

Abstract- Ahmed [1], considered study of prey-predator model with predator in disease and harvesting. In this article, a ratio-dependent predator-prey model with disease in the prey population is formulated and analyzed. Assuming that prey populations suffered from epidemics and the predator population will prefer only infected population for their diet as those are more vulnerable. Dynamical behaviours such as boundedness, local and global stabilities are discussed.

Keywords: eco-epidemiological model, global stability.

I. Introduction

The dynamic relationship between predator and their prey has long been and will continue to be one of the dominant topics in both applied mathematics and theoretical ecology due to its universal existence and importance. These problems may appear to be simple mathematically at first sight, they are, in fact, often very challenging and complicated.

Since the pioneering work of Kermack-Mckendrick on SIRS [2], epidemiological models have received much attention from scientists. Mathematical models have become important tools in analyzing the spread and control of infectious disease. It is of more biological significance to consider the effect of interacting species when we study the dynamical behaviors of epidemiological models. Eco epidemiology which is a relatively new branch of study in theoretical biology, tackles such situations by dealing with both ecological and epidemiological issues. It can be viewed as the coupling of an ecological predator-preymodel and an epidemiological SI, SIS, or SIRS model. Following Anderso and May [3] who were the first to propose an eco-epidemiological model by merging the ecological predator-prey model introduced by Lotka and Volterra, the effect of disease in ecological system is an important issue from mathematical and ecological point of view. Many works have been devoted to the study of the effects of a disease on a predator-prey system [2–7]. In this paper, the dynamical behaviourof a ratio-dependent predator-prey systems with infection in prey populationis investigated. Here we have studied the boundedness, permanence, local and global stabilities of the non-equilibrium points of this system.

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II. The Mathematical Model

Consider the following mathematical model:

$$\begin{cases} \dfrac{dS}{dt} = r_1(1-\dfrac{S+I}{K})S - \beta SI \\ \dfrac{dI}{dt} = \beta SI - dI - \dfrac{bIY}{aY+I} \\ \dfrac{dY}{dt} = -cY + \dfrac{pbIY}{aY+I} \end{cases} \qquad (1)$$

With initial data $S(0) \geq 0$, $I(0) \geq 0$, $Y(0) \geq 0$

We make the following assumptions in formulating the mathematical model of apredator-prey system with disease in the prey population:

1) In the absence of disease, the prey population grows logistically with carrying capacity $K \in \Re_+$ and intrinsic birth rate $r \in \Re_+$.
2) In the presence of virus, the prey population is divided into two groups, namely susceptible prey denoted by S(T) and infected prey denoted by I(T). Therefore at time T, the total population is N(T) = S(T) + I(T).
3) The disease is not genetically inherited. The infected populations do not recover or become immune. We assume that the disease transmission follows the simple law of mass action βS(T)I(T) with β as the transmission rate.
4) The infected prey I(T) is removed by death (say, its death rate is positive constant c) or by predation before having the possibility of reproducing. However, the infected prey population I(T) still contribute with S(T) towards the carrying capacity of the system.
5) The infected prey is more vulnerable than susceptible prey. We assume that thepredator population consumes only infected prey with ratio-dependent Michaelis–Menten functional response function

$$\mu(I,Y) = \frac{IY}{aY+I}, \quad (a>0)$$

It is assumed that the predator has the death rate constant d (c> 0), and the predation coefficient b (b> 0). The coefficient in conversing prey into predator is p $(0 < p \leq 1)$.

To reduce the number of parameters and to determine which combinations of parameters control the behaviour of the system, we nondimensionalize system (2). We choose

$$s = \frac{S}{K}, \qquad i = \frac{I}{K}, \qquad q = \frac{aY}{K}, \qquad t = \beta KT$$

Then, after some simplification, the system (1) takes the form

$$\begin{aligned} \frac{ds}{dt} &= r(1-(s+i))s - si \\ \frac{di}{dt} &= si - ei - \frac{qiy}{y+i} \\ \frac{dy}{dt} &= -wy + \frac{pqiy}{y+i} \end{aligned} \qquad (2)$$

Notes

With initial data $s(0) \geq 0$, $i(0) \geq 0$, $y(0) \geq 0$

Where $r = \frac{r_r}{\beta K}$, $w = \frac{d}{\beta K}$, $q = \frac{b}{a\beta K}$, $e = \frac{c}{a\beta K}$

III. Boundedness

Ref

Theorem 3.1. Any solution of system (2) is uniformly bounded in $\Re^3_+$.

Proof. Let $(s(t), i(t), y(t))$ be any solution of the system (2). Since, $\frac{ds}{dt} = rs(1-s)$

We have, $$\limsup_{t\to\infty} s(t) \leq r$$

$$V = \frac{s}{1+r} + i + \frac{y}{p}. \text{ Then,}$$

$$\frac{dV}{dt} = \frac{r}{1+r}s(1-s) - wi - \frac{e}{p}y \leq \frac{r}{1+r}s - wi - \frac{e}{p}y$$

$\frac{dV}{dt} \leq \frac{2r}{1+r} - \eta V$; where $\eta = \min\{1, w, e\}$

Therefore, $\frac{dV}{dt} + \eta V \leq \frac{2r}{1+r}$. Applying theorem on differential inequalities [8], we obtain $0 \leq V(s,i,y) \leq \frac{2r}{(1+r)\eta} + \frac{V(s(0), i(0), y(0))}{e^{\eta t}}$ and as $t \to \infty$, $0 \leq V \leq \frac{2r}{(1+r)\eta}$

Thus, all the solution of (2) enter into the region

$$D = \left\{ (s,i,y) : 0 \leq V \leq \frac{2r}{(1+r)\eta} + \varepsilon \ for \quad any \quad \varepsilon > 0 \right\}$$

Hence the theorem.

IV. Equilibrium Points and Stability Analysis

The equilibrium points are obtained by solving $\frac{ds}{dt} = \frac{di}{dt} = \frac{dy}{dt} = 0$. It is found that the system (2) has two boundary equilibrium $E_0(0,0,0)$, the axial equilibrium $E_1(1,0,0)$, the predator-free equilibrium point $E_2(\bar{s}, \bar{i}, 0)$, where $\bar{s} = w$ and $\bar{i} = \frac{r(1-w)}{1+r}$, and the interior equilibrium $E^*(s^*, i^*, y^*)$

where $s^* = \frac{pw + (pq-e)}{p}$, $i^* = \frac{r}{p(1+r)}\big(p(1-w) - (pq-e)\big)$,

and $y^* = \frac{r(pq-e)}{ep(1+r)}\big(p(1-w) - (pq-w)\big)$

The system (2) cannot be linearized at $E_0(0,0,0)$ and $E_1(1,0,0)$ and therefore local stability of E_0 and E_1 cannot be studied[9].

Lemma 4.1. The predator-free equilibrium point $E_2(\bar{s}, \bar{i}, 0)$ exists if and only if $w < 1$

Note: In terms of the original parameters of the system, the condition $w < 1$ becomes $d < \beta K$.

This implies that if the ratio of the death rate of the infected prey to the carrying capacity (d/K) is less than the transmission rate β, then the predator become extinct and conversely.

The Jacobean Matrix at the equilibrium point E_2 is given by

$$J(E_2) = \begin{pmatrix} -r\bar{s} & -(1+r)\bar{s} & 0 \\ \bar{i} & 0 & -q \\ 0 & 0 & -e+pq \end{pmatrix}$$

The characteristics equation of $J(E_2)$ is $(\lambda^2 + B\lambda + C)(\lambda - pq + e) = 0$,

Where $B = r\bar{s} > 0$ and $C = (1+r)\bar{s}\bar{i} > 0$.

The eigenvalues are $\lambda_{1,2} = \dfrac{-B \pm \sqrt{B^2 - 4C}}{2}$ and $\lambda_3 = pq - e$

Since, $B > 0$ and $C > 0$, therefore, the signs of the real parts λ_1 and λ_2 are negative. This implies that E_2 is locally asymptotically stable in the si-plane. Now E_2 is asymptotically stable in the y-direction if and only if $pq - e < 0$.

Lemma 4.2. The interior equilibrium $E^*(s^*, i^*, y^*)$ of the system (2) existsif and only if the following conditions hold:

(a) $pq > e$

(b) $p(1-w) - (pq - e) > 0$

In terms of the original parameters of the system, the conditions (a) and (b) respectively become $pb > c$ and $ap(\beta K - d) > pb - c$, which are the necessary and sufficient conditions for the co-existence of the susceptible prey, infected prey and the predator.

Local Stability of E^*

$$J(E^*) = \begin{pmatrix} j_{11} & j_{12} & 0 \\ j_{21} & j_{22} & j_{23} \\ 0 & j_{32} & j_{33} \end{pmatrix} \text{ where}$$

$$j_{11} = -rs^*,\ j_{12} = -(1+r)s^*,\ j_{21} = i^*,\ j_{22} = \frac{qi^* y^*}{(i^* + y^*)^2},$$

$$j_{23} = \frac{qi^{*2}}{(i^* + y^*)^2},\ j_{32} = \frac{pqy^{*2}}{(i^* + y^*)^2},\ j_{33} = -\frac{pqi^* y^*}{(i^* + y^*)^2}$$

The characteristics is $\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0$

$$a_1 = -trJ(E^*) = -j_{11} - j_{22} - j_{33} = rs^* - \frac{q(1-p)i^* y^*}{(y^* + i^*)^2}$$

Notes

Notes

$$a_1 = \frac{\Gamma}{qp^2};\ \Gamma = rqp^2 w + pe(pq-e)(pq-e)rpq-e)$$

$$a_2 = j_{11}j_{22} + j_{11}j_{33} + j_{22}j_{33} - j_{23}j_{32} - j_{12}j_{21}$$

$$= \quad i^* s^* \left\{(1+r) - \frac{rq(1-p)y^*}{(y^*+i^*)^2}\right\}$$

$$a_3 = -\det[J(E^*)] = j_{11}j_{23}j_{32} + j_{12}j_{21}j_{33} - j_{11}j_{22}j_{33}$$

$$= \frac{pq(1+r)s^* y^* i^{*2}}{(y^*+i^*)^2}$$

Now,

$$\Delta = a_1 a_2 - a_3$$

$$= -(j_{11})^2 j_{22} - (j_{11})^2 j_{33} + j_{11}j_{12}j_{21} - (j_{22})^2 j_{33} - (j_{22})^2 j_{11}$$

$$-2j_{11}j_{22}j_{33} + j_{22}j_{12}j_{21} + j_{23}j_{32}j_{22} - j_{22}(j_{33})^2 - j_{11}(j_{33})^2 + j_{23}j_{32}j_{33}$$

$$\Delta \quad = \quad i^* s^* \left\{(1+r)rs^* - \frac{r^2 q(1-p)s^* y^*}{(y^*+i^*)^2} + \frac{rq^2(1-p)^2 i^* y^{*2}}{(y^*+i^*)^4} - \frac{q(1+r)i^* y^*}{(y^*+i^*)^2}\right\}$$

Theorem 4.3. E^* is locally asymptotically stable if and only if $\Gamma > 0$ and $\Delta > 0$

Proof. Note that

i) $\Gamma > 0$ if and only if $a_1 > 0$.

ii) $a_3 > 0$ for all value of the parameters., and

iii) $\Delta = a_1 a_2 - a_3 > 0$.

Hence, from Routh Hurwitz criterion the theorem holds.

Theorem 4.4. Existence of positive equilibrium of the system (2) implies its global stability around the positive interior equilibrium.

V. Conclusion

In this paper, an eco-epidemiological model with disease in the prey population which is governed by modified logistic equation is studied. It is shown(in Theorem 3.1) that the non-dimensionalized system (2) is uniformly bounded, which in turn, implies that the system is biologically well behaved. In deterministic situation, theoretical epidemiologists are usually guided by an implicit assumption that most epidemic models we observe in nature correspond to stable equilibria of the models. From this viewpoint, we have presented the most important equilibrium point $E^*(s^*, i^*, y^*)$. The stability criteria given in Lemma 4.2 and Theorem 4.3 are the conditions for stable coexistence of the susceptible prey population, infected prey population and predator population.

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New Approach for Similarity of Trapezoids

By Getachew Abiye Salilew[1]
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Abstract- Chaudhary and Getachew [1] recently introduce a new technique for identification of the nature of trapezoid. In this paper, the author presented presumably new technique to explain similarity of trapezoids. The author presented general conditions that make trapezoids [1] are similar by using known results.

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GJSFR-F Classification: *MSC: 51A15, 51M20, 51M30*

New Approach for Similarity of Trapezoids

Getachew Abiye Salilew[1]

Abstract- Chaudhary and Getachew [1] recently introduce a new technique for identification of the nature of trapezoid. In this paper, the author presented presumably new technique to explain similarity of trapezoids. The author presented general conditions that make trapezoids [1] are similar by using known results.

Keywords: similarity, trapezoids, triangles.

I. Introduction

Trapezoid is one of the interested areas in geometry, for the researcher since ancient time. But major contributions have been made on it during 19^{th} centuries, which were initiated since 17^{th} century. On this topic several contributions have been already done by the mathematician, but we believe that the approach for dealing current problems described in this article is different than others. Triangle is the simplest polygon with three edges and three vertices. It is one of the basic shapes in geometry. In Euclidean geometry any three points, when non- collinear, determine a unique triangle and a unique plane. The basic elements of any triangle are its sides and angles. Triangles are classified depending on relative sizes of their elements. In Euclidean geometry, a convex quadrilateral with at least one pair of parallel sides is referred to as a trapezoid in American and Canadian English but as a trapezium in English outside North America. A trapezium in Proclus' sense is a quadrilateral having one pair of its opposite sides parallel[2- 14].

In this study we consider a trapezoid which has only one pair of parallel sides. The parallel sides are called the bases, while the other sides are called the legs or lateral sides. The larger base side of a trapezoid used as simply the base of a trapezoid. When the legs have the same length and the base angles have the same measure then the trapezoid is acute angle trapezoid. If the two adjacent angles are right angle, then the trapezoid is a right angle trapezoid. If the trapezoid has no sides of equal measures, it is called a scalene trapezoid. Two triangles are said to be similar if and only if they have the same measure of corresponding angles and also the proportional ratio of their corresponding sides is the same. We can apply this technique for presenting similarity of trapezoids. In this paper, we presented the general conditions that make trapezoids are similar by using known results.

II. Main Theorem

Theorem 1: Find necessary conditions, which enable to identify similarity of trapezoids.

Proof: Our approach to derive necessary conditions, for the similarity of trapezoids, motivated by recent work [1], and also by using known results. Let us consider two

Ref

14. B. Scimemi, *Paper – folding and Euler's theorem revisited*, Forum Geo., 2(2002) 93 – 104.

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similar triangles that is, ΔABE is similar to $\Delta A'B'E'$ ($\Delta ABE \sim \Delta A'B'E'$), as given in figure 1 below.

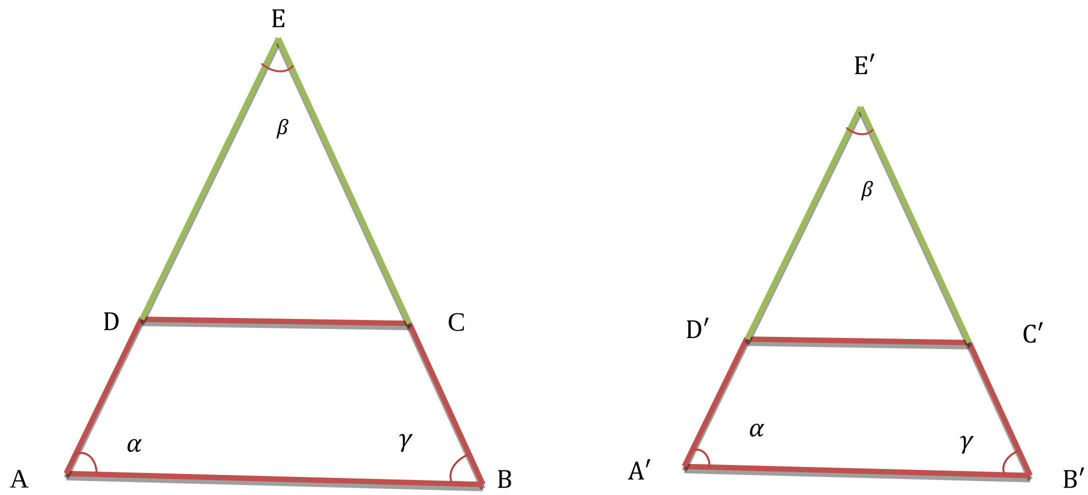

Figure 1: Two similar triangles

Since $\Delta ABE \sim \Delta A'B'E' \Leftrightarrow \angle BAE \cong \angle B'A'E'$, $\angle ABE \cong \angle A'B'E'$, $\angle AEB \cong \angle A'E'B'$ and $\frac{AB}{A'B'} = \frac{AE}{A'E'} = \frac{BE}{B'E'}$. We consider parallel segments $AB \parallel DC$ and $A'B' \parallel D'C'$ in triangles ΔABE and $\Delta A'B'E'$ respectively. Then using the recent work done [1], we have $\Delta ABE \sim \Delta DCE$ and $\Delta A'B'E' \sim \Delta D'C'E'$. Thus we obtain

$$\frac{DC}{AB} = \frac{CE}{BE} = \frac{DE}{AE} \quad (1)$$

$$\frac{D'C'}{A'B'} = \frac{C'E'}{B'E'} = \frac{D'E'}{A'E'} \quad (2)$$

$$\frac{AB}{A'B'} = \frac{AE}{A'E'} = \frac{BE}{B'E'} \quad (3)$$

From (1) and (2), we have

$$DE = \frac{AD \times DC}{AB - DC} \text{ and } D'E' = \frac{A'D' \times D'C'}{A'B' - D'C'} \quad (4)$$

$$CE = \frac{DC \times BC}{AB - DC} \text{ and } C'E' = \frac{D'C' \times B'C'}{A'B' - D'C'} \quad (5)$$

Using (4) and (5) in (3), we obtain similarity properties for trapezoids ABCD and $A'B'C'D'$ as follows:

$$1 = \frac{AD}{A'D'} \times \frac{A'B' - D'C'}{AB - DC} = \frac{BC}{B'C'} \times \frac{A'B' - D'C'}{AB - DC}$$

$$\Leftrightarrow \frac{AB - DC}{A'B' - D'C'} = \frac{AD}{A'D'} = \frac{BC}{B'C'} \quad (6)$$

Then $\angle ADC \cong \angle A'D'C'$ and $\angle DCB \cong \angle D'C'B'$ with (6) show that trapezoids ABCD and $A'B'C'D'$ in figure 1 have the same shape but not necessarily the same size are

Ref

1. M. P. Chaudhary and Getachew Abiye Salilew, *Note on the properties of trapezoid*, Global J. of Sci. Frontier Res., 16(2016) 53–58.

similar. Therefore, we obtained that trapezoid ABCD is similar to trapezoid $A'B'C'D'$. Thus like similarity of triangles, we have:

$ABCD \sim A'B'C'D' \Leftrightarrow \angle BAE \cong \angle B'A'E'$, $\angle ABE \cong \angle A'B'E'$, $\angle BCD \cong \angle B'C'D'$, $\angle ADC \cong \angle A'D'C'$, and also

$$\frac{AB - DC}{A'B' - D'C'} = \frac{AD}{A'D'} = \frac{BC}{B'C'}$$

Notes

III. Conclusions

We have found the following conditions from our main result. Trapezoid ABCD is similar to trapezoid $A'B'C'D'$ which is denoted as $ABCD \sim A'B'C'D'$ if and only if $\angle BAE \cong \angle B'A'E'$, $\angle ABE \cong \angle A'B'E'$, $\angle BCD \cong \angle B'C'D'$, $\angle ADC \cong \angle A'D'C'$, and

$$\frac{AB - DC}{A'B' - D'C'} = \frac{AD}{A'D'} = \frac{BC}{B'C'}$$

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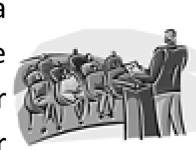

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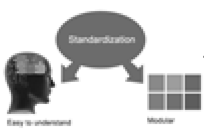

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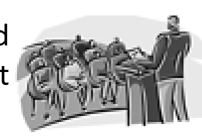

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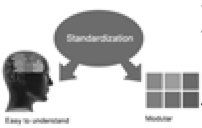

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Note :

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- In future, if the board feels the necessity to change any board member, the same can be done with the consent of the chairperson along with anyone board member without our approval.
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2. Ethical Guidelines,

3. Submission of Manuscripts,

4. Manuscript’s Category,

5. Structure and Format of Manuscript,

6. After Acceptance.

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28. Make colleagues: Always try to make colleagues. No matter how sharper or intelligent you are, if you make colleagues you can have several ideas, which will be helpful for your research.

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Key points to remember:

- Submit all work in its final form.
- Write your paper in the form, which is presented in the guidelines using the template.
- Please note the criterion for grading the final paper by peer-reviewers.

Final Points:

A purpose of organizing a research paper is to let people to interpret your effort selectively. The journal requires the following sections, submitted in the order listed, each section to start on a new page.

The introduction will be compiled from reference matter and will reflect the design processes or outline of basis that direct you to make study. As you will carry out the process of study, the method and process section will be constructed as like that. The result segment will show related statistics in nearly sequential order and will direct the reviewers next to the similar intellectual paths throughout the data that you took to carry out your study. The discussion section will provide understanding of the data and projections as to the implication of the results. The use of good quality references all through the paper will give the effort trustworthiness by representing an alertness of prior workings.

Writing a research paper is not an easy job no matter how trouble-free the actual research or concept. Practice, excellent preparation, and controlled record keeping are the only means to make straightforward the progression.

General style:

Specific editorial column necessities for compliance of a manuscript will always take over from directions in these general guidelines.

To make a paper clear

· Adhere to recommended page limits

Mistakes to evade

- Insertion a title at the foot of a page with the subsequent text on the next page
- Separating a table/chart or figure - impound each figure/table to a single page
- Submitting a manuscript with pages out of sequence

In every sections of your document

· Use standard writing style including articles ("a", "the," etc.)

· Keep on paying attention on the research topic of the paper

· Use paragraphs to split each significant point (excluding for the abstract)

· Align the primary line of each section

· Present your points in sound order

· Use present tense to report well accepted

· Use past tense to describe specific results

· Shun familiar wording, don't address the reviewer directly, and don't use slang, slang language, or superlatives

· Shun use of extra pictures - include only those figures essential to presenting results

Title Page:

Choose a revealing title. It should be short. It should not have non-standard acronyms or abbreviations. It should not exceed two printed lines. It should include the name(s) and address (es) of all authors.

Abstract:

The summary should be two hundred words or less. It should briefly and clearly explain the key findings reported in the manuscript--must have precise statistics. It should not have abnormal acronyms or abbreviations. It should be logical in itself. Shun citing references at this point.

An abstract is a brief distinct paragraph summary of finished work or work in development. In a minute or less a reviewer can be taught the foundation behind the study, common approach to the problem, relevant results, and significant conclusions or new questions.

Write your summary when your paper is completed because how can you write the summary of anything which is not yet written? Wealth of terminology is very essential in abstract. Yet, use comprehensive sentences and do not let go readability for briefness. You can maintain it succinct by phrasing sentences so that they provide more than lone rationale. The author can at this moment go straight to shortening the outcome. Sum up the study, with the subsequent elements in any summary. Try to maintain the initial two items to no more than one ruling each.

- Reason of the study - theory, overall issue, purpose
- Fundamental goal
- To the point depiction of the research
- Consequences, including definite statistics - if the consequences are quantitative in nature, account quantitative data; results of any numerical analysis should be reported
- Significant conclusions or questions that track from the research(es)

Approach:

- Single section, and succinct
- As a outline of job done, it is always written in past tense
- A conceptual should situate on its own, and not submit to any other part of the paper such as a form or table
- Center on shortening results - bound background information to a verdict or two, if completely necessary
- What you account in an conceptual must be regular with what you reported in the manuscript
- Exact spelling, clearness of sentences and phrases, and appropriate reporting of quantities (proper units, important statistics) are just as significant in an abstract as they are anywhere else

Introduction:

The **Introduction** should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable to comprehend and calculate the purpose of your study without having to submit to other works. The basis for the study should be offered. Give most important references but shun difficult to make a comprehensive appraisal of the topic. In the introduction, describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will have no attention in your result. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here. Following approach can create a valuable beginning:

- Explain the value (significance) of the study
- Shield the model - why did you employ this particular system or method? What is its compensation? You strength remark on its appropriateness from a abstract point of vision as well as point out sensible reasons for using it.
- Present a justification. Status your particular theory (es) or aim(s), and describe the logic that led you to choose them.
- Very for a short time explain the tentative propose and how it skilled the declared objectives.

Approach:

- Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done.
- Sort out your thoughts; manufacture one key point with every section. If you make the four points listed above, you will need a least of four paragraphs.

- Present surroundings information only as desirable in order hold up a situation. The reviewer does not desire to read the whole thing you know about a topic.
- Shape the theory/purpose specifically - do not take a broad view.
- As always, give awareness to spelling, simplicity and correctness of sentences and phrases.

Procedures (Methods and Materials):

This part is supposed to be the easiest to carve if you have good skills. A sound written Procedures segment allows a capable scientist to replacement your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt for the least amount of information that would permit another capable scientist to spare your outcome but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section. When a technique is used that has been well described in another object, mention the specific item describing a way but draw the basic principle while stating the situation. The purpose is to text all particular resources and broad procedures, so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step by step report of the whole thing you did, nor is a methods section a set of orders.

Materials:

- Explain materials individually only if the study is so complex that it saves liberty this way.
- Embrace particular materials, and any tools or provisions that are not frequently found in laboratories.
- Do not take in frequently found.
- If use of a definite type of tools.
- Materials may be reported in a part section or else they may be recognized along with your measures.

Methods:

- Report the method (not particulars of each process that engaged the same methodology)
- Describe the method entirely
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures
- Simplify - details how procedures were completed not how they were exclusively performed on a particular day.
- If well known procedures were used, account the procedure by name, possibly with reference, and that's all.

Approach:

- It is embarrassed or not possible to use vigorous voice when documenting methods with no using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result when script up the methods most authors use third person passive voice.
- Use standard style in this and in every other part of the paper - avoid familiar lists, and use full sentences.

What to keep away from

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings - save it for the argument.
- Leave out information that is immaterial to a third party.

Results:

The principle of a results segment is to present and demonstrate your conclusion. Create this part a entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Carry on to be to the point, by means of statistics and tables, if suitable, to present consequences most efficiently.You must obviously differentiate material that would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matter should not be submitted at all except requested by the instructor.

Content

- Sum up your conclusion in text and demonstrate them, if suitable, with figures and tables.
- In manuscript, explain each of your consequences, point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation an exacting study.
- Explain results of control experiments and comprise remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or in manuscript form.

What to stay away from

- Do not discuss or infer your outcome, report surroundings information, or try to explain anything.
- Not at all, take in raw data or intermediate calculations in a research manuscript.
- Do not present the similar data more than once.
- Manuscript should complement any figures or tables, not duplicate the identical information.
- Never confuse figures with tables - there is a difference.

Approach

- As forever, use past tense when you submit to your results, and put the whole thing in a reasonable order.
- Put figures and tables, appropriately numbered, in order at the end of the report
- If you desire, you may place your figures and tables properly within the text of your results part.

Figures and tables

- If you put figures and tables at the end of the details, make certain that they are visibly distinguished from any attach appendix materials, such as raw facts
- Despite of position, each figure must be numbered one after the other and complete with subtitle
- In spite of position, each table must be titled, numbered one after the other and complete with heading
- All figure and table must be adequately complete that it could situate on its own, divide from text

Discussion:

The Discussion is expected the trickiest segment to write and describe. A lot of papers submitted for journal are discarded based on problems with the Discussion. There is no head of state for how long a argument should be. Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implication of the study. The purpose here is to offer an understanding of your results and hold up for all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of result should be visibly described. Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved with prospect, and let it drop at that.

- Make a decision if each premise is supported, discarded, or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."
- Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work
- You may propose future guidelines, such as how the experiment might be personalized to accomplish a new idea.
- Give details all of your remarks as much as possible, focus on mechanisms.
- Make a decision if the tentative design sufficiently addressed the theory, and whether or not it was correctly restricted.
- Try to present substitute explanations if sensible alternatives be present.
- One research will not counter an overall question, so maintain the large picture in mind, where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

- When you refer to information, differentiate data generated by your own studies from available information
- Submit to work done by specific persons (including you) in past tense.
- Submit to generally acknowledged facts and main beliefs in present tense.

The Administration Rules

Please carefully note down following rules and regulation before submitting your Research Paper to Global Journals Inc. (US):

Segment Draft and Final Research Paper: You have to strictly follow the template of research paper. If it is not done your paper may get rejected.

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- To guard yourself and others from possible illegal use please do not permit anyone right to use to your paper and files.

Criterion for Grading a Research Paper (Compilation)
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Topics	Grades		
	A-B	**C-D**	**E-F**
Abstract	Clear and concise with appropriate content, Correct format. 200 words or below	Unclear summary and no specific data, Incorrect form Above 200 words	No specific data with ambiguous information Above 250 words
Introduction	Containing all background details with clear goal and appropriate details, flow specification, no grammar and spelling mistake, well organized sentence and paragraph, reference cited	Unclear and confusing data, appropriate format, grammar and spelling errors with unorganized matter	Out of place depth and content, hazy format
Methods and Procedures	Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads	Difficult to comprehend with embarrassed text, too much explanation but completed	Incorrect and unorganized structure with hazy meaning
Result	Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake	Complete and embarrassed text, difficult to comprehend	Irregular format with wrong facts and figures
Discussion	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

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