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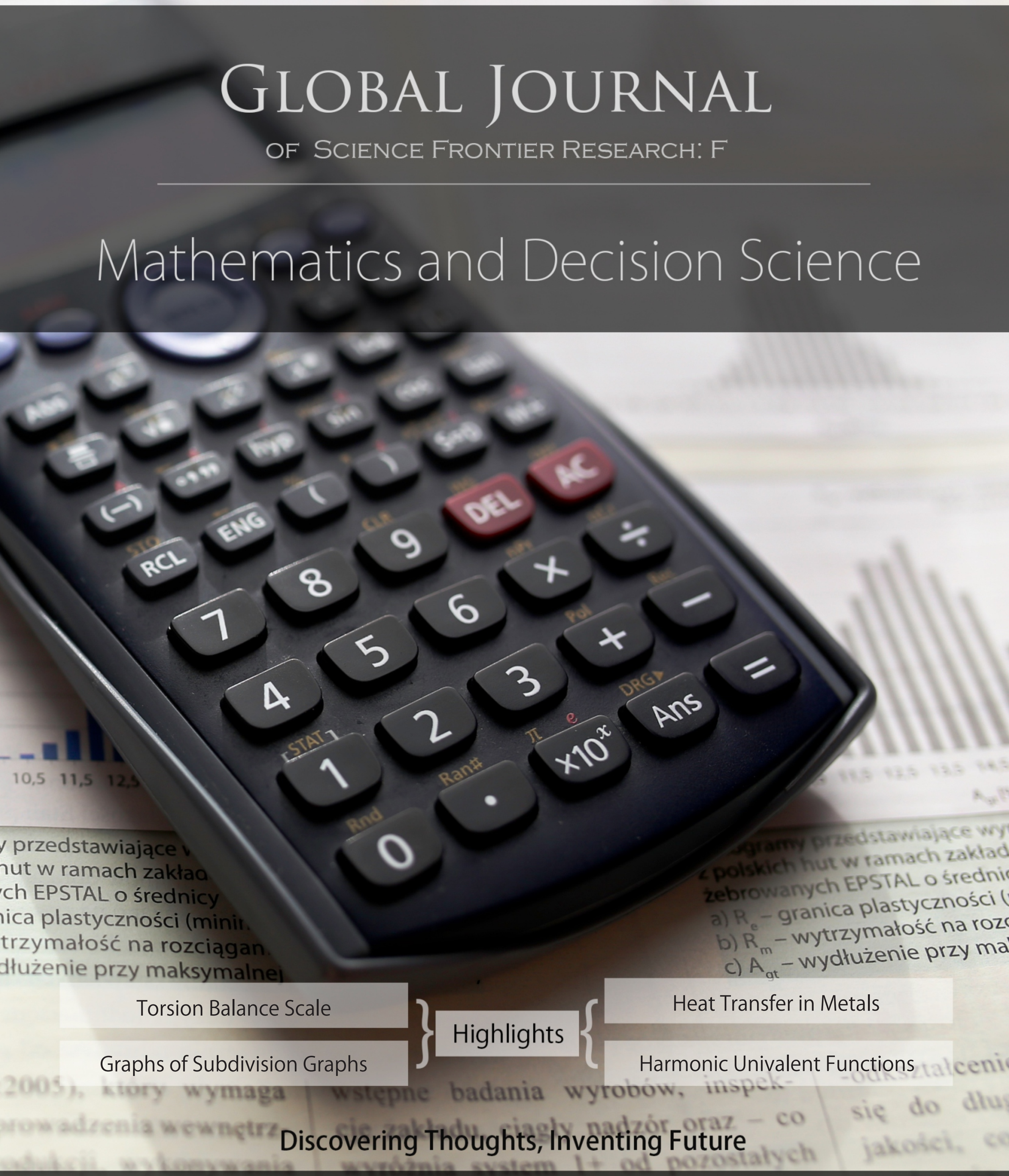

Highlights

- Torsion Balance Scale
- Graphs of Subdivision Graphs
- Heat Transfer in Metals
- Harmonic Univalent Functions

Discovering Thoughts, Inventing Future

VOLUME 17 ISSUE 4 VERSION 1.0

GLOBAL JOURNAL OF SCIENCE FRONTIER RESEARCH: F
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The Temperature Dependence of Gravitation for the Metallic Balls - Measured with a Torsion Balance Scale

By C. Y. Lo
Applied and Pure Research Institute, United States

Abstract- We use a torsion balance scale to measure the attraction between two large lead balls and two smaller brass balls, connected with an up-side down T bar that is hung with a string. The vertical part of the T bar is attached with a mirror that reflects a laser beam to provide a light spot that shows how much the T bar has turned. It is observed that the gravitational forces between the lead balls and the brass balls are reduced when the temperature of the lead balls increased. Thus, this experiment shows clearly the existence of a repulsive gravitational force that increases as the temperature of the lead balls increase. This supports that the charge-mass interaction is the reason that the theories of Galileo, Newton, and Einstein failed to explain the Anomaly of the Space-Probes and flybys, and the fact that not all the neutral subjects necessarily fall with the same acceleration. In other words, the Newtonian law of gravitation is only approximately valid. Thus, the attempts such as J. Luo (罗俊)'s to obtain an accurate gravitational coupling constant with just improved skill are futile. The physical picture of Galileo, Newton, and Einstein on gravitation needs to be improved.

Keywords: *repulsive gravitation; invalidity of* $E = mc^2$.

GJSFR-F Classification: *MSC 2010: 13C12*

THETEMPERATUREDEPENDENCEOFGRAVITATIONFORTHEMETALLICBALLSMEASUREDWITHATORSIONBALANCESCALE

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2. D. Yu. Tsipenyuk, V. A. Andreev, Physical Interpretations of the Theory of Relativity Conference (Bauman Moscow State Technical University, Moscow 2005).

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C. Y. Lo

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I. INTRODUCTION

Gravitation is the first force in nature discovered by mankind. Due to the limitation of our knowledge, gravitation was thought of as a force between masses only. Galileo was the first to show experimentally that the gravitational acceleration on a piece of neutral matter is independent of its mass. However, in Einstein's time. it was discovered from special relativity that mass and energy can be related. Therefore the relation between gravitation and energy must be investigated. Einstein [1] claimed that $E = mc^2$ is generally valid and thus an increment of energy on matter must have led to an increment of its weight.

Experimentally, in contrast to Einstein's claim, it is known that a charged metal ball has reduced weight [2], and a charged capacitor has reduced weight [3] and a piece of heated-up metal also has reduced weight [4, 5]. Theoretically, we also derived that the weight reduction is due to the existence of a repulsive force toward mass [4]. However, the skeptics, believing in the relation $E = mc^2$, are not convinced. Although it is apparent that part of the mass has converted to energy in the atomic bomb, the proof for the conversion of a single type of energy to mass has never been proven. In fact, Einstein [6] tried very hard from 1905 to 1909 to show this, but failed. Moreover, it is found that the gravitational acceleration of a neutral object is not necessarily the same [7].

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One might argue that the photonic energy can be converted into mass [8]. Moreover, such a conversion is supported by the fact that the π_0 meson can decay into two photons. However, since the addition of electromagnetic energies is still an electromagnetic energy, the electromagnetic energy, which has a traceless electromagnetic energy-stress tensor cannot be converted into a mass, which has a non-zero trace energy-stress tensor. Moreover, the formula $E = mc^2$ is also inconsistent with the Einstein equation, because it implies that the traceless electromagnetic energy-stress tensor cannot affect the Ricci curvature R, but the mass can [9]. This conflict is resolved because the photonic energy must include gravitational energy [10. 11].

Some argued that the weight reduction of a charged metal could be due to electric effects, but the static electric effect can only increase the weight; and that the weight reduction of a charged capacitor could be due to the leaked out electricity, but the charged capacitor is in a metal box. Moreover, metals such as gold has nothing to lose.

Recently an experiment on the temperature dependence of gravitation was done by Li Hua-Wang [12] inside a large vacuum can in China. Although it confirms the dependence of the gravitational force, the oscillating of the two brass balls make an elaborated theoretical explanation necessary. Here, we present a verification conducted in the air. Moreover, this verification is direct and thus there is no room for doubt (see Appendix A).

Prof. A. Napier attempted to measure the change of gravitation due to temperature increment. However, owing to the external interference for the passing of a subway near by, the data of the periodic changes is not reliable. Thus, this paper on the existence of gravitation reduction is published first. An accurate reduction due to temperature increase will be published in the future.

Ref

9. C. Y. Lo, J. of Advances in Physics, Vol. 10, No. 3, 2874-2885 (Oct. 2015).

II. THE APPARATUS OF LI AND MEASUREMENTS

The inner radius of Li's can is 1.8 meter and an inner height of 1.8 meters as show in Figure 1.

Figure 1: The big vacuum can that contains the torsion balance scale (courtsey of Mr. Li)

This can has two windows. The round one is for observations inside the can, and the meter next to it is the vacuum meter connected to a powerful vacuum pump. The rectangle window is for the in and out of a light beam.

This setup can isolate influences such as air movements. Moreover, the can is located in a insolated chamber inside a cave, and thus effectively also eliminates the outside vibrations and temperature changes. To control the temperature of the chamber, the chamber is isolated and equipped with an air heater and a powerful air cooler.

In comparison, our equipment is a bare torsion balance scale, but the large lead balls are placed in a carrying bar with the length of the T bar that connects the two brass balls. The carrying bar can be rotated around the mirror to adjust the distance between that large lead ball and the smaller brass ball. An advantage of a bare torsion balance scale is that the oscillation of the brass balls can be controlled easier.

To avoid the influence of out-side disturbances; the experiment was conducted in an isolated room that does not have the interference of air flows. It is fortunate that our method of experiment can resist the interference of a subway passing near by.

a) The Torsion Balance Scale

We also use the torsion balance scale as Li except that the distance of the two brass balls is extended from 30 cm to 40 cm. The forces between brass balls and lead balls are measured with a torsion balance scale as follows:

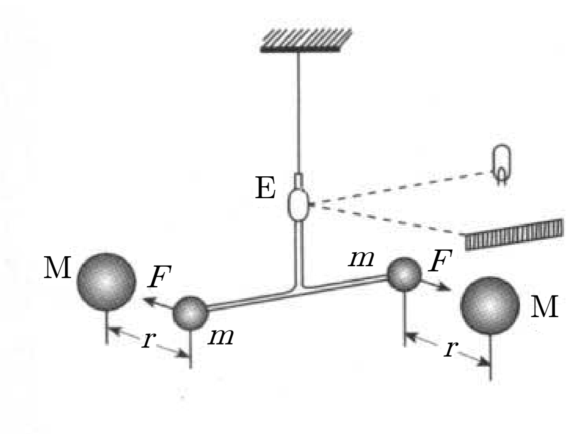

Figure 2: The torsion balance scale

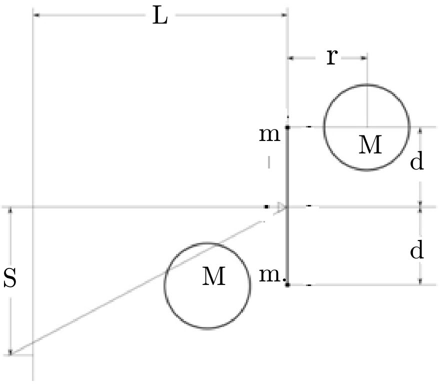

Figure 3: Details of the distances

1. The small brass ball has a mass m = 0.575kg and the large lead ball has a mass M = 1.5kg.
2. The two brass balls are connected with a bar of 2d = 0.40 meter, and suspended from the middle in a horizon orientation by a fine wire ("torsion balance") as shown in Fig. 2 and Fig. 3.
3. A mirror E is attached to the bar to reflect a light beam shined in the mirror.
4. A white board is placed at distance L = 10.3 meter from the mirror as shown in Figure 3.
5. A light spot is shown in the board at a distance S from the middle (the moving distance of the light spot).
6. The distance between the center of the brass ball and the lead ball is r = 10 cm as shown in Figure 3.
7. The natural period of the torsion balance is T.

Then, according to Newtonian approximate theory, the gravitational force is $F = \pi^2 mdS/T^2L$. Thus, the torsion balance can be very sensitive since the sensitivity will increase with the distance L. Then, the Newtonian coupling constant would be $G = \pi^2 r^2 dS/MT^2L$. However, since Newtonian theory is only approximately valid, this formula is not useful to deal with the repulsive gravitation.

Although the period T could increase as the temperature increases, it is difficult to measure an accurate T due to external interferences. Besides the small difference could be mistakenly considered as due to experimental errors.2) Thus, we decided to do the experiment in two ways. First, we prove directly the existence of a reduction of gravitation due the increment of temperature of the lead balls. Second, we used the traditional method to see the differences between the periods between before and after the lead balls are heated. However, the latter is not accurate enough to be conclusive.

b) Experimental Steps

So, a new way must be found the verify the gravitational reduction due to temperature increase. Our method is to find a balance point between gravitation and the torsion force first. Then, we show that the gravitational force is reduced when the lead balls are heated-up. The steps are as follows:

A) Determine the balance position

1) Take away the lead balls from the scale and leave the two brass balls in the T bar.
2) Adjust the torsion to the minimum.
3) Reduce the oscillation of the light spot at the screen to the minimum to about 15 cm.

B) Measuring the gravitational force in the room temperature.

4) Place the large lead balls inside thermal containers on the carrying bar, which is in a perpendicular position to the T bar.
5) Turn the carrying bar so that the lead balls are in a close position with the brass balls. Then, fix the position of the carrying bar in this angle. Adjust the torsion such that the brass balls barely touch the lead balls. Thus, the gravitational force is in an almost balance position with the torsion force.

C) Measuring the gravitational force after the lead balls are heated-up in boiling water, which is about 97^0C.

6) Take the lead balls away from the scale and heat them up in boiling water. After about 20 minutes, put the lead balls back to the thermal containers. Then place them in the previous positions.
7) Then one would find the brass balls cannot be in touch positions with the lead balls. This shows that the gravitational force has been reduced because the temperature of the lead balls has increased. These successful operations were repeated five times.

Thus, the experiment shows the temperature dependence of that gravitational force that would reduce the gravitational forces between the brass and the lead balls as temperature increases. The merit of this experiment is that it shows the temperature reduces the gravitational force directly even the temperature increment is small.3) Another merit of this method that it is not influenced by the normal air movements.

This method also resists the influence of a passing subway near by. Moreover, since the electromagnetic force is not involved, the existence of repulsive gravitational force seems to be the only possibility.

III. Conclusions and Discussions

Now, it is clear that the attractive gravitational force reduces as the temperature increases. However, while the experiment confirms the temperature dependence of the gravitational force, it did not provide a numerical value for such changes. Nevertheless, it confirms the existence of a repulsive gravitational force. For the numerical value of such a repulsive gravitational force, whose existence can be derived from the static Einstein equation [13], is complicated and thus requires to design a new experiment. For the related theoretical consideration (see the Appendix A).

In the experiment of Li [12], the temperature of everything is changed simultaneously, and thus the temperature effects on the brass balls and the hanging string must be considered although they are neglegible. In our approach, only the temperature of the lead balls has been changed.

However, the experiments on the weight reductions of charged metal ball and the charged capacitor, had not been completely trusted because of the involvement of other forces [3, 4, 14]. The weight reductions of the heated-up metals were mistaken as a reduction of mass [4, 5]. (The pendulum experiments would show that mass does not change [7].) From this experiment, the existence of repulsive gravitation is no longer questionable although the details of such a repulsion force due to heat is not yet clear. To explain this, one must understand the charge-mass interaction [4, 15].

Thus, the American Physical Society (APS) should have recognized that they are lacking behind in the field of gravitation. However, in the April 2015 APS Meeting, the top executive still does not know that there are three experiments that support the existence of repulsive gravitation and invalidity of $E = mc^2$ [14] because Eric J. Weinberg, the editor of the Physical Review D has not accepted them. Because of inadequacy in mathematics and physics, it never occurred to him that the gravitation is actually a combination of effects [4] (see Appendix A).

Moreover, the 2016 award of APS Medal for Exceptional Achievement in Research was awarded to E. Witten, without the necessary supports of any experiment [16]. Apparently, the APS Awarding Committee does not know that just as Yau [17], Witten [18] has made serious mathematical and physical errors on general relativity [19]. Thus, it is clear that APS has a problem on her competence in mathematics and physics of gravitation [14].

At the beginning of this research, we could not find a physicist who is willing to do the experiments that would show Einstein wrong. This is so because the physical community generally but incorrectly believed that Einstein could not be wrong in classical theory!! It turns out, however, many experiments that are, in fact, against Einstein's predictions have been done because those who did such experiments did not know their actually physical meaning [4, 16]. Moreover, we must be grateful to the US Department of Defense, who allows the publication of the experimental results done under their contracts [20].

Many theorists, just as Einstein, have a blind faith on $E = mc^2$ [3, 4] because of the atomic bomb. 4) Another problem is that they do not understand pure mathematics, and non-linear mathematics in particular [9, 14]. According to Dr. Daniel Kulp, Editorial Director of APS, no editor of APS has a background in pure mathematics. In fact, there are incorrect papers on general relativity published in the Proceeding of the Royal Society A, Classical and Quantum Gravity, General Relativity and Gravitation, and the Annals of Physics, in addition to the Physical Review. They all are at best speculations without rigorous theoretical or experimental supports. These problems in

Ref

12. C. Y. Lo, The Observable Temperature Dependence of Gravitation, Phys. Essays, **29**, 3, 247 (2016).

gravitation have been pointed out and our APS CEO Kate Kirby has promised to deal with these issues in this year (see Appendix B).

These journals accepted that the Einstein equation has dynamic solutions because they have mistaken that linearization of the Einstein equation would produce approximate solution for the Einstein equation even for the dynamic case [21]. Due to such confusion, the claim of Christodoulou and Klainerman [22] was incorrectly accepted [23]. Thus, the announcement of the 1993 Nobel Committee for Physics [24] contains invalid statements [25].

Newton's inverse-square law of gravitation is the oldest mathematical description of a fundamental interaction. Experimental tests of gravity's distance-dependence define a frontier between our understanding of gravity and many proposed forms of new physics. The invited talk of Charles Hagedorn [26] by APS surveys the past, present, and near-future of the experimental field, with substantial emphasis on precision sub-millimeter laboratory experiments. However, Hagedorn also did not know that the measured weight of testing matter actually depends on its temperature [3, 4]. Although Faller [27] is aware that error budgets in the measurements of the Newtonian coupling constant are fundamentally flawed because one cannot make allowances for error sources that have not been thought of. However, he also did not know that the current measurements to obtain the Big G could not be accurate due to ignorance on the influence of heat to weight [3, 4].

In conclusion, a common error in gravitational measurement is that many failed to see that the gravitational effect is not a single effect as many believed, but is a combined effects of at least three factors, i.e. 1) the mass-mass attractive interaction, 2) the charge-mass repulsive interaction,5) and 3) the current-mass attractive interaction. Thus, the measurement of gravitation is still in its infancy.

For instance, believing in that the most accurate G can be obtained by current method of measurements alone, J. Luo (罗俊) hopes to be able to obtain the most accurate G soon [28]. However, such efforts are futile because of the temperature dependency, but he can direct his efforts to measure the temperature dependence of gravitation. However, to verify this gravitational force reduction is due to the charge-mass interaction, more experiments are needed to see the nature of such gravitational force reduction.

Based on the notion that gravity is always attractive, a popular speculation on gravitation is the existence of black holes. Now, since it is proven that there exists repulsive gravitation, naturally the notion of black holes is questionable. Moreover, because the Einstein equation is not valid for the dynamic case [14] there is actually no theoretical basis for the existence of black holes.6)

IV. ACKNOWLEDGEMENTS

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Appendix A: Some Theoretical Considerations Related to the Repulsive Gravitation

The first theoretical existence of the repulsive gravitation actually comes from a solution of the static Einstein equation for a charged particle Q, the Reissner-Nordstrom metric [29] as follows:

Ref

27. James Faller: Why is it so difficult to measure big G? APS April Meeting 2015, S11. 2.

$$ds^2 = \left(1 - \frac{2M}{r} + \frac{q^2}{r^2}\right) dt^2 - \left(1 - \frac{2M}{r} + \frac{q^2}{r^2}\right)^{-1} dr^2 - r^2 d\Omega^2 , \tag{A1}$$

(with c = 1) where q and M are the charge and mass of a particle, and r is the radial distance (in terms of the Euclidean-like structure [30]) from the particle center. In metric (A1), the gravitational components generated by electricity have not only a very different radial coordinate dependence but also a different sign that makes it a new repulsive gravity in general relativity.

However, owing to the belief that the electric energy had a mass equivalence, theorists including Einstein, consider the mass M would include the electric energy, i.e.,

$$M = m(r_0) + q^2/r_0 \tag{A2}$$

where $m(r_0)$ is the mass of the particle and q^2/r_0 is the electric energy of the particle outside the radius r_0 of the particle. Thus, in the net effect, there would be no repulsive gravitation since

$$\frac{1}{2}\frac{\partial}{\partial r}\left(1 - \frac{2M}{r} + \frac{q^2}{r^2}\right) = \left(M - \frac{q^2}{r}\right)\frac{1}{r^2} = \left(m(r_0) + q^2\left(\frac{1}{r_0} - \frac{1}{r}\right)\right)\frac{1}{r^2} > 0 . \tag{A3}$$

Nevertheless, Tsipenyuk & Andreev [13] observed a weight reduction of a charged metal ball.

Thus, the existence of repulsive gravitation is confirmed by experiments. This mistake in (A2) [31] is due to that the effect of the electric energy has been incorrectly counted twice in the Reissner-Nordstrom metric.

a) *The Charge-Mass Repulsive Force and Unification*

Another problem for the existence of the repulsive gravitation in the Reissner-Nordstrom metric is that it makes clear that general relativity is incomplete. To show the static repulsive effect of a charged particle, one needs to consider only g_{tt} in metric (A1). According to Einstein [8], the equation of motion is

$$\frac{d^2x^\mu}{ds^2} + \Gamma^\mu{}_{\alpha\beta}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds} = 0, \quad \text{where} \quad \Gamma^\mu{}_{\alpha\beta} = (\partial_\alpha g_{\nu\beta} + \partial_\beta g_{\nu\alpha} - \partial_\nu g_{\alpha\beta}) g^{\mu\nu}/2 \tag{A4}$$

and $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$. Consider the static case. (One need not worry whether the gauge is physically valid because the gauge affects only the second order approximation of g_{tt} [32].) The force on a test particle P with mass m at **r** is

$$\left(-m\frac{M}{r^2} + m\frac{q^2}{r^3}\right)\hat{r} \quad \text{where} \quad \hat{r} \text{ is a unit vector} \tag{A5}$$

in the first order approximation because $g^{rr} \cong -1$. Thus, the second term is a repulsive force.

If the particles are at rest, then the force generated by p acting on the charged particle Q would be

$$\left(m\frac{M}{r^2} - m\frac{q^2}{r^3}\right)\hat{r}, \quad \text{where} \quad \hat{r} \text{ is a unit vector} \tag{A6}$$

because the action and reaction forces are equal and in the opposite directions. However, for the motion of the charged particle with mass M, if one calculates the metric

Ref

30. C. Y. Lo, Chinese J. of Phys., Vol. 41, No. 4, 332-342 (2003).

according to the particle P of mass m, only the first term is obtained. Thus, the geodesic equation is inadequate for the equation of motion.

Thus, it is necessary to have a repulsive force with the coupling q^2 to the charged particle Q in a gravitational field generated by masses. It thus follows that, force (A6) to particle Q is beyond current theoretical framework of gravitation + electromagnetism. Lo, Goldstein, & Napier [33] predicted that general relativity leads to a realization of the inadequacy of general relativity, just as electricity and magnetism lead to the exposition of their shortcomings.

The charge-mass repulsive force $\boldsymbol{mq^2/r^3}$ for two point-like particles is inversely proportional to the cube power (instead of the square) of the distances between the two particles. This would mean that such a repulsive force is much weaker faster than gravity at long distance, although it would be much stronger at very small distance. Moreover, this force is proportional to the square of the charge q, and thus is independent of the charge sign. Such characteristics would make [34] a concentration of charged particles would increase such repulsion.

The repulsive force in (A1) comes from the electric energy [31]. An immediate question would be whether such a charge-mass repulsive force $\boldsymbol{mq^2/r^3}$ is subjected to electromagnetic screening. It is conjectured that this force, being independent of a charge sign, would not be subjected to such a screening although it should be according to general relativity. From the viewpoint of physics, this force can be considered as a result of a field created by the mass m and the field interacts with the q^2. Thus such a field is independent of the electromagnetic field.

b) Extension of Einstein's Equivalence Principle and the Five-Dimensional Relativity

If we consider the coupling with q^2, this naturally leads to a five dimensional space. Kaluza [35] proposed a five-dimensional general relativity, and his cylindrical condition reduced the five variables to four. This maintains the equation of motion as being a geodesic equation, and this theory reproduces the Einstein equation and the Maxwell equation if the "extra" metric elements are considered as constant or negligible. Subsequently, Einstein and Pauli [36] wrote a paper to continue the work of Kaluza. However, their five-dimensional relativity does not have the coupling with the square of a charge since the "extra" metric elements are neglected. This theory also cannot account for the radiation reaction force because the cylindrical condition is imposed [33].

One may ask what the physical meaning of the fifth dimension is. Many theorists claimed that those high dimensions are curl up. Their position [33] is that the physical meaning of the fifth dimension is not yet very clear, except some physical meaning is given in the equation, $dx^5/d\tau = q/Mc^2K$ where M and q are respectively the mass and charge of a test particle, and K is a constant. This equation relates the fifth variable x^5 to τ.

The fifth dimension is assumed [33] as part of the physical reality, and the metric signature is (+, −, −, −, −). We shall denote the fifth axis as the w-axis (w stands for "wunderbar", in memorial of Kaluza), and thus the coordinates are (t, w, x, y, z). Our approach is to find out the full meaning of the w-axis as our understanding gets deeper.

For a static case, we have the forces on the charged particle Q in the ρ-direction

$$-\frac{mM}{\rho^2} \approx \frac{Mc^2}{2}\frac{\partial g_{tt}}{\partial \rho}\frac{dct}{d\tau}\frac{dct}{d\tau}g^{\rho\rho}, \qquad \text{and} \qquad \frac{mq^2}{\rho^3} \approx -\Gamma_{\rho,55}\frac{1}{K^2}\frac{q^2}{Mc^2}\,g^{\rho\rho} \tag{A7a}$$

Ref

33. C. Y. Lo, G. R. Goldstein and A. Napier, Electromagnetic Radiation Reaction Force and Radiation Potential in General Five-Dimensional Relativity, Hadronic Journal **12** (2) 75-89 (1989).

and

$$\Gamma_{k,55}\frac{q}{KMc^{2}}\frac{dx^{k}}{d\tau}=0, \qquad \text{where} \qquad \Gamma_{k,55}\equiv\frac{\partial g_{k5}}{\partial x^{5}}-\frac{1}{2}\frac{\partial g_{55}}{\partial x^{k}}=-\frac{1}{2}\frac{\partial g_{55}}{\partial x^{k}} \tag{A7b}$$

in the (-*r*)-direction. The meaning of (A7b) is the energy momentum conservation. It is interesting that the same force would come from a different type of metric element depending on the test particle used. Thus,

$$g_{tt}=1-\frac{2m}{\rho c^{2}}, \qquad \text{and} \qquad g_{55}=\frac{mMc^{2}}{\rho^{2}}K^{2}+\text{constant。} \tag{A8}$$

In other words, g_{55} is a repulsive potential. Because g_{55} depends on M, it is a function of local property, and this is different from the metric element $g_{t\,t}$ that depends on a distant source of mass m.

On the other hand, because g_{55} is independent of q, $(\partial g_{55}/\partial \rho)/M$ depends only on the distant source m. Thus, this force, though acting on a charged particle, would penetrate electromagnetic screening. From the above, it is possible that a charge-mass repulsive potential would exist for a metric based on the mass M of the charged particle Q. However, because P is neutral, there is no charge-mass repulsion force on P.

That the repulsive gravitational potential to charge can be generated from a mass, would explain the fact that a charged capacitor can also have the repulsive force [34], but such a force is absent from the current four-dimensional theory. This is why many theorists would not accept the existence of the repulsive gravitation. They are so involved in current theoretical consideration that they forget that physics is based on experiments.

c) The Charge-Mass Interaction and the Question of Weight

It is found that a charge may generate a gravitational static field that repulses a mass [3]. Since the discovery and the prediction are based on general relativity, Einstein's theory would have another important confirmation to be verified [13]. Thus, there is a new neutral charge-mass interaction that is beyond electromagnetism and gravitation, and thus Einstein's unification is a necessity [31].

In general relativity, there is no field that couples with the square of a charge. Moreover, since this new force is independent of the charge sign, it should not be subjected to electromagnetic screening although general relativity would imply that any electromagnetic interaction does. Nevertheless, such a coupling exists in the five-dimensional relativity of Lo, Goldstein and Napier [33]. In addition, their theory would support that such a neutral force is not subjected to electromagnetic screening. It thus follows that the existence of this static neutral repulsive force can be tested by weighing a capacitor to see whether its weight is reduced after being charged [3]. The existence of such a force on a capacitor was first verified by Liu [3] although such weight reduction has been found much earlier [37].

Thus, it is found that a weight reduction of a neutral object is not due to a reduction of mass, but a neutral repulsive force, which was unknown to Galileo, Newton, and Einstein [7].

If the electric energy leads to a repulsive force toward a mass, according to general relativity, the magnetic energy would lead to an attractive force from a current toward a mass [38]. The existence of such a current-mass attractive force has been verified by Martin Tajmar and Clovis de Matos [39] from the European Space Agency.

Ref

3. C. Y. Lo, Gravitation, Physics, and Technology, Physics Essays, **25** (4), 553-560 (Dec. 2012).

They found that a spinning ring of superconducting material increases its weight much more than expected. Thus, they incorrectly believed that general relativity had been proven wrong. However, according to quantum theory, spinning superconductors should produce a weak magnetic field. Thus, they are also measuring the interaction between an electric current and the earth.

The existence of the current-mass attractive force would solve a puzzle, i.e., why a charged capacitor exhibits the charge-mass repulsive force since a charged capacitor has no additional electric charges? In a normal situation, the charge-mass repulsive force would be cancelled by other forms of the current-mass force as Galileo, Newton and Einstein implicitly assumed. This general force is related to the static charge-mass repulsive force in a way similar to the Lorentz force is related to the Coulomb force. One may ask what is the formula for the current-mass force? However, unlike the static charge-mass repulsive force, which can be derived from general relativity, this general force would be beyond general relativity since a current-mass interaction would involve the acceleration of a charge, this force would be time-dependent and generates electromagnetic radiation. Moreover, when the radiation is involved, the radiation reaction force and the variable of the fifth dimension must be considered [33].

Thus, we are not ready to derive the current-mass interaction yet. Nevertheless, we may assume that, for a charged capacitor, the resulting force is the interaction of net macroscopic charges with the mass [34].

d) Weight Reduction by Heat

This current-mass interaction also explains a phenomenon, which is also reported by Liu [3, 4] that it takes time for a capacitor to recover its weight after being discharged. A discharged capacitor needs time to dissipate the heat generated by discharging. Then, the motion of its charges would recover to normal.

Thus, it should be expected that the heated-up metals would reduce their weight. It is conjectured that the heat would additionally convert some orbital electrons to random motion, but the increased mass due to heat energy is negligible as Einstein [1] pointed out. If this explanation of weight reduction is valid, then a metal would reduce its weight as the temperature increases. This should be further tested such that the physics can be understood in depth.

Moreover, since a heated-up metal is a solid, one can in principle test its mass by acceleration. (Another way to do this is to compare the periods of a pendulum before and after the metal is heated.) One can also verify the existence of the repulsive gravitation by measuring the reduction of attractive gravitation with a torsion balance scale after the metal is heated. Note, however, that the reduction of gravitation by heat also depends on the metal [4].

Appendix B: An Open letter to Kate Kirby, CEO of American Physical Society

Dear Dr. Kate Kirby:

It was a pleasure to meet you in the APS Meeting at New Orleans, LA in March 16, 2017. It is remarkable that you promise to improve the mathematics of the APS editors and related problems in gravitation. As shown in my report ”American Physical Society and Errors in Gravitation” to the APS March meeting, there are ten major problems and errors in gravitation. I admire your exceptional ability to lead and have the courage to deal with problems and errors in gravitation of over 100 years old. I do not know where such a strength of your comes from, I guess that

Ref

34. C. Y. Lo, The Weight Reduction of Charged Capacitors, Charge-Mass Interaction, and Einstein's Unification, Journal of Advances in Physics, Vol. 7, No 3, 1959-1969 (Feb. 2015).

this must come from years of your experience in leadership. Here, I would like to introduce myself related to gravitation since I am the one who discovers and raises the problems to APS. This information would help you to understand where my strength comes from.

I earned my undergraduate degree in physics. However, I was very unsatisfied on the mathematical education for physicists because it essentially makes physicists in the dark when a problem related to pure mathematics appears. I believe that this problem must be fixed since I am interested theoretical physics. A current problem of most physicists is that they are not aware this problem could be serious as shown by the American Society in the area of gravitation which I pointed out in my article presented to the APS Business Meeting, 2017. Since this is not a problem of American physicists alone, it is a world wide problem. (For example, none of the books in general relativity, except Einstein's own [1], gives the correct assertion to Einstein's equivalence principle. The reason is that, for instance, Editor of Physical Review D, Eric J. Weinberg as well as W. Pauli 1) did not understand the mathematics related to this principle [2].) Consequently, almost all the physicists of world make the similar mistakes in gravitation without knowing their errors. This would, of course, involve the 1993 Nobel Committee [3]; and surprisingly also the Fields Medal committee for mathematics [4]. It should be noted that the errors of the Fields Medal Committee is not in mathematics, but they deepen the errors because they do not understand physics and thus used the wrong conclusion of the physicists as their starting assumption. For instance, the misleading positive mass theorem of Schoen and Yau [5] had prevented the progress of general relativity for at least 13 years. 2)

Luckily, I had my opportunity to learn pure mathematics from the start in Canada before I went to MIT. I was exceptionally fortunate to have my pure mathematical training under Professor I. Halperin for my degree in mathematics. Professor Halperin is a Fellow of the Canadian Royal Society (CRSF), who got his degree from the worldly famous mathematician John von Neumann. Because of my rigorous training and my very careful nature, I have never made any mistakes in my published work in mathematics. Moreover, I have been known to correct obscure errors of pure mathematicians all over the world, and calculation errors of well-known physicists in MIT and Harvard. These corrected calculation results have been published in the Physical Review [6, 7] because the ability of the APS editors in applied mathematics is generally pretty good. Then, I was known for being an applied mathematician with exceptional ability who is better than anybody in long calculations. However, the exception did occur because the ability in pure mathematics of APS editors such as Eric J. Weinberg is very poor, he even failed to understand Einstein's equivalence principle [2]. When I realized that this is not an individual problem of APS alone but a general problem of physics, I started to publish my papers in other journals.

My first break though in gravitation is my paper [8], " *Einstein's Radiation Formula and Modifications to the Einstein Equation* " published in 1995 in Astrophysics Journal, when Nobel Laureate, S. Chandrasekhar was the editor-in-chief. Based on this, I subsequently show that there is no dynamic solution for the Einstein equation [9]. For this, I was questioned by Professor P. Morrison of MIT for a month. Then he went to Princeton University to question J. H. Taylor on his justification for his

Ref

1. Einstein, '$E = MC^2$' (1946), *Ideas and Opinions*, p. 337 (Crown, New York, 1982).

calculation on gravitational radiation. When Taylor admits that he was unable to justify their calculation as expected, Morrison advised me to write a book on this issue [10]. However, I feel that we must solve also related problems. One reason is that many physicists had claimed that they have dynamic solutions for the Einstein equation. I went through all such claims and find they are due to various mathematical mistakes or errors in physics [11]. What surprised me is that some errors such as those from Misner, Thorne and Wheeler x) are in the undergraduate level of calculus [2]. What is more strange is that these three gentlemen used Einstein's abandoned 1911 invalid assumption of equivalence of acceleration to gravitation [12] 3) and Pauli's misinterpretation as the references of Einstein's equivalence principle, but they ignored Einstein's 1916 paper and his book, both of which stated Einstein's equivalence principle clearly.

It is even more surprising that the 2016 Award of APS Medal for Exceptional Achievement in Research was given to E. Witten, whose errors in mathematics is well-known because he does not understand even Einstein's equivalence principle.4) Moreover, this award of achievements in mathematical physics actually has no support from the experiments [13]. Since the award committee consists of the mathematical talents of APS, it is clear that APS has big trouble in mathematics.

The second break though on gravitation is my discovery of the repulsive gravitation [14] that Einstein rejected because he incorrectly believed in $E= mc^2$ as generally valid. 5) An unexpected problem was that no experimental physicists was willing to try any experiment that would show that Einstein was wrong in classical physics. Fortunately, such experiments have been done with the support of the US Defense Department although these experiments have been interpreted wrong [15, 16]. Thus, a new revolution of physics has born. More important, Einstein's claim of unification between gravitation and electromagnetism has been proven correct [15, 16].

Unfortunately, the APS executives were not aware of these new developments mainly because the Editor of Physical Review D, Eric. J. Weinberg is incompetent in mathematics as well as in recent developments in physics. For instance, the Einstein equation and the formula $E = mc^2$ are not consistent.6) In fact, I have pointed out these in my 2015 paper [17]. It seems to be necessary to have a statistics on the wrong papers that APS published each year.

I have promised you to keep the errors of APS on gravitation private and to wait for sometime such that APS can respond properly. However, more than a month have been passed and I have not heard anything from you. Moreover, I am going to participate in a conference in gravitational wave in Korea in May 22-26. This means that some errors of APS in gravitation must be known to the world.

For your perusal, the draft of my speech is attached herewith. Any comments and suggestions that you may have will be appreciated. Thank you.

Now, it should be clear that it is not easy to correct the errors in gravitation of APS and the world. I considered myself very fortunate that I have discovered these problems. I hope that you are just as fortunate enough that you can solve these

Ref

11. C. Y. Lo, "*Completing Einstein's Proof of $E = mc^2$*," Progress in Phys., Vol. 4, 14-18 (2006).

issues for the APS. If you have any questions, I shall be happy to answer them. I am looking forward to hearing from you.

Sincerely yours,

C. Y. Lo

c.c.:

- Laura Greene, 2017 President; email: kroberts@magnet.fsu.edu
- James Hollenhorst, 2017 Treasurer
- Daniel Kleppner, Speaker of the Council; email: kleppner@mit.edu
- Pierre Meystre, Editor in Chief; email: pmeystre@aps.org.
- Matthew Salter, Publisher; email: Matthew Salter @MSalt69

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5. A. L. Dmitriev, E. M. Nikushchenko, and V. S. Snegov, Influence of the Temperature of a Body on its Weight, Measurement Techniques, Vol. 46, No. 2, 115-120 (2003).

Endnotes:

1. W. Pauli and Eric J. Weinberg cannot tell the difference between the existence of a local Minkowski space at a point and the existence of local Minkowski space in a very small neighborhood.
2. Schoen and Yau [5] assumed the metric is asymptotically flat without realizing that this would exclude all the dynamic solutions since the Einstein equation implies that all the dynamic solutions are unbounded.
3. The 1911 assumption of Einstein has been proven incorrect by observation.
4. Witten made the same mistakes as Schoen and Yau [5]. They both were awarded the Fields Medal in 1982 and 1990 because the mathematicians do not understand physics.
5. There exist at least three types of experiments that shows $E = mc^2$ is not valid. They are: 1) the weight reduction of a charged metal ball; 2) the weight reduction of a charged capacitor; 3) the weight reduction of a piece of heated-up metal.
6. The mass and the electromagnetic energy have very different effects according to the Einstein equation.

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Endnotes:

1) Dmitriev, Nikushchenko & Snegov [4] has mistaken that the weight reduction is due to a reduction of mass.
2) The period T of our torsion balance is about 91.2 seconds. It is difficult to see the small changes for T after the lead balls have been heated-up.
3) The merit of Li's experiment is that the temperature can be changed continuously. However, we can also do this by heating the lead balls directly.
4) One should not expect that a theorist understands what he proposed fully. Einstein is also not an exception.
5) Since the charge mass interaction is absent form current quantum mechanics, quantum mechanic is not a complete theory just as Einstein claimed.
6) A popular error in general relativity is the belief that general relativity has superseded Newtonian gravity. This is not true since the Einstein equation has no two-body solution whereas Newtonian gravity does [40].

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On a New Mechanics Systems on the Standard Cliffordian $Kähler$ Manifolds using A Canonical Local Basis

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Abstract- In this paper we obtained a canonical local basis $\{J_i\}, i = 1,2,3$ of vector bundle V on the Standard Cliffordian $Kähler$ Manifold (R^8, V). The paths of semispray on the Standard Cliffordian $Kähler$ Manifold are in fact the solutions of Euler-Lagrange equations.

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Abstract- In this paper we obtained a canonical local basis $\{J_i\}$, $i = 1,2,3$ of vector bundle V on the Standard Cliffordian $K\ddot{a}hler$ Manifold (R^8, V). The paths of semispray on the Standard Cliffordian $K\ddot{a}hler$ Manifold are in fact the solutions of Euler-Lagrange equations.

I. INTRODUCTION

Modern differential geometry explains explicitly the dynamics of Lagrangians. So, we say that if M is an $m-dimensional$ configuration manifold and $L: TM \rightarrow R$ is a regular Lagrangian function, then there is a unique vector field ξ on TM such that dynamics equations is given by:

$$i_\xi \Phi_L = dE_L \qquad \rightarrow \qquad (1)$$

Where Φ_L indicates the symplectic form and E_L is the energy associated to L.

The triple (TM, Φ_L, ξ) is called Lagrangian system on the tangent bundle TM.

In literature, there are a lot of studies about Lagrangian mechanics, formalisms, systems and equations [1,2,3] and there in. There are real, complex, paracomplex and other analogues. It is possible to produce different analogues in different spaces. Finding new dynamics equations is both a new expansion and contribution to science to explain physical events.

Quaternions were invented by Sir William Rowan Hamilton as an extension to the complex number. Hamilton's defining relation is most succinctly written as:

$$i^2 = j^2 = k^2 = ijk = -1 \qquad \rightarrow \qquad (2)$$

If it is compared to the calculus of vectors, quaternions have slipped into the realm of obscurity.

They do however still find use in the computation of rotations. A lot of physical laws in classical, relativistic, and quantum mechanics can be written pleasantly by means of quaternions. Some physicists hope they will find deeper understanding of the universe by restating basic principles in terms of quaternion algebra. It is well-known that quaternions are useful for representing rotations in both quantum and classical mechanics [4]. Cliffordian manifold is a quaternion manifold. The above properties yield also for Cliffordian manifold.

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II. Preliminaries

Throughout this paper, all mathematical objects and mappings are assumed to be smooth, i.e. infinitely differentiable and Einstein convention of summarizing is adopted. $\mathcal{F}(M), \mathcal{X}(M)$ and $\Lambda^1(M)$ denote the set of functions on M, the set of vector fields on M and the set of 1-forms on M, respectively.

Notes

a) Theorem

Let f be differentiable ϕ ,ψ are 1-form,then [5]:

- $d(f\phi) = df\wedge\phi + fd\phi$
- $d(\phi\wedge\psi) = d\phi\wedge\psi - \phi\wedge d\psi$

b) Definition (Kronecker's delta)

Kronecker's delta denote by δ and defined as follows [6,7]:

$$\delta_i^j = \begin{cases} 1 \;\; ; & i = j \\ 0 \;\; ; & i \neq j \end{cases}$$

c) Cliffordian Kähler Manifolds

Here, we recalled the main concepts and structures given in [8,9]. Let M be a real smooth manifold of dimension m. Suppose that there is a 6-dimensional vector bundle V consisting of $F_i(i = 1,2, \dots ,6)$ tensors of type(1,1) over M. Such a local basis $\{F_1, F_2, \dots , F_6\}$ is called a canonical local basis of the bundle V in neighborhood $U \, of \, M$. Then V is called an almost Cliffordian structure in M. The pair (M, V) is named an almost Cliffordian manifold with V. Hence, an almost Cliffordian manifold M is of $dimension \, m = 8n$. If there exists on (M, V) a global basis $\{F_1, F_2, \dots , F_6\}$,$then$ (M, V) is said to be an almost Cliffordian manifold; the basis $\{F_1, F_2, \dots , F_6\}$ is called a global basis for V.

An almost Cliffordian connection on the almost Cliffordian manifold(M, V) is a linear connection $\nabla \, on \, M$ which preserves by parallel transport the vector bundle V. This means that if Φ is a cross-section (local-global) of the bundle V ,$then$ $\nabla_X\Phi$ is also a cross-section (local-global, respectively) of V ,X being an arbitrary vector field of M.

If for any canonical basis $\{J_1, J_2, \dots , J_6\}$ of V in a coordinate neighborhood, the identities

$$g(J_iX \,, J_iY) = g(X \,, Y), \;\; \forall X, Y \in \mathcal{X}(M) \,, \;\; i = 1,2, \dots ,6 \qquad \rightarrow \tag{3}$$

Hold, the triple (M, g, V) is named an almost Cliffordian Hermitian manifold or metric Cliffordian denoting by V an almost Cliffordian structure $V \, and \, by \, g$ a Riemannian metric and by (g, V) an almost Cliffordian metric structure.

Since each $J_i(i = 1,2, \dots ,6)$ is almost Hermitian structure with respect to g, setting

$$\Phi_i(X, Y) = g(J_iX, Y), \quad i = 1,2, \dots ,6 \qquad \rightarrow \tag{4}$$

For any vector fields and Y, we see that Φ_i are 6 local 2-forms.

If the Levi-Civita connection $\nabla = \nabla^g$ on (M, g, V) preserves the vector bundle V by parallel transport, then (M, g, V) is called a Cliffordian $Kähler$ manifold, and an almost

Cliffordian structure Φ_i *of* M is called a Cliffordian *Kähler* structure. A Clifford *Kähler* manifold is Riemannian manifold (M^{8n}, g). For example, we say that R^{8n} is the simplest example of Clifford *Kähler* manifold. Suppose that let $\{x_i, x_{n+i}x_{2n+i}, x_{3n+i}, x_{4n+i}, x_{5n+i}, x_{6n+i}, x_{7n+i}\}$, $i = \overline{1,n}$ be a real coordinate system on R^{8n}. Then we define by $\left\{\frac{\partial}{\partial x_i}, \frac{\partial}{\partial x_{n+i}}, \frac{\partial}{\partial x_{2n+i}}, \frac{\partial}{\partial x_{3n+i}}, \frac{\partial}{\partial x_{4n+i}}, \frac{\partial}{\partial x_{5n+i}}, \frac{\partial}{\partial x_{6n+i}}, \frac{\partial}{\partial x_{7n+i}}\right\}$ and $\{dx_i, dx_{n+i}, dx_{2n+i}, dx_{3n+i}, dx_{4n+i}, dx_{5n+i}, dx_{6n+i}, dx_{7n+i}\}$ be natural bases over R of the tangent space $T(R^{8n})$ and the cotangent space $T^*(R^{8n})$ *of* R^{8n} , respectively.

By structures J_1 ,J_2 ,J_3, the following expressions are obtained

$$\begin{array}{lll}
J_1\left(\frac{\partial}{\partial x_i}\right) = \frac{\partial}{\partial x_{n+i}} & J_2\left(\frac{\partial}{\partial x_i}\right) = \frac{\partial}{\partial x_{2n+i}} & J_3\left(\frac{\partial}{\partial x_i}\right) = \frac{\partial}{\partial x_{3n+i}} \\
J_1\left(\frac{\partial}{\partial x_{n+i}}\right) = -\frac{\partial}{\partial x_i} & J_2\left(\frac{\partial}{\partial x_{n+i}}\right) = -\frac{\partial}{\partial x_{4n+i}} & J_3\left(\frac{\partial}{\partial x_{n+i}}\right) = -\frac{\partial}{\partial x_{5n+i}} \\
J_1\left(\frac{\partial}{\partial x_{2n+i}}\right) = \frac{\partial}{\partial x_{4n+i}} & J_2\left(\frac{\partial}{\partial x_{2n+i}}\right) = -\frac{\partial}{\partial x_i} & J_3\left(\frac{\partial}{\partial x_{2n+i}}\right) = -\frac{\partial}{\partial x_{6n+i}} \\
J_1\left(\frac{\partial}{\partial x_{i+3n}}\right) = \frac{\partial}{\partial x_{i+5n}} & J_2\left(\frac{\partial}{\partial x_{i+3n}}\right) = \frac{\partial}{\partial x_{i+6n}} & J_3\left(\frac{\partial}{\partial x_{i+3n}}\right) = -\frac{\partial}{\partial x_i} \\
J_1\left(\frac{\partial}{\partial x_{i+4n}}\right) = -\frac{\partial}{\partial x_{i+2n}} & J_2\left(\frac{\partial}{\partial x_{i+4n}}\right) = \frac{\partial}{\partial x_{i+n}} & J_3\left(\frac{\partial}{\partial x_{i+4n}}\right) = \frac{\partial}{\partial x_{i+7n}} \\
J_1\left(\frac{\partial}{\partial x_{i+5n}}\right) = -\frac{\partial}{\partial x_{i+3n}} & J_2\left(\frac{\partial}{\partial x_{i+5n}}\right) = -\frac{\partial}{\partial x_{i+7n}} & J_3\left(\frac{\partial}{\partial x_{i+5n}}\right) = \frac{\partial}{\partial x_{i+n}} \\
J_1\left(\frac{\partial}{\partial x_{6n+i}}\right) = \frac{\partial}{\partial x_{7n+i}} & J_2\left(\frac{\partial}{\partial x_{6n+i}}\right) = -\frac{\partial}{\partial x_{3n+i}} & J_3\left(\frac{\partial}{\partial x_{6n+i}}\right) = \frac{\partial}{\partial x_{2n+i}} \\
J_1\left(\frac{\partial}{\partial x_{7n+i}}\right) = -\frac{\partial}{\partial x_{6n+i}} & J_2\left(\frac{\partial}{\partial x_{7n+i}}\right) = \frac{\partial}{\partial x_{5n+i}} & J_3\left(\frac{\partial}{\partial x_{7n+i}}\right) = -\frac{\partial}{\partial x_{4n+i}}
\end{array} \quad \rightarrow \quad (5)$$

III. Lagrangian Mechanics

In this section, we obtain Euler-Lagrange equations for quantum and classical mechanics by means of a canonical local basis $\{J_1, J_2, J_3\}$ *of* V on the standard Cliffordian *Kähler* manifold (R^{8n}, V).

First:

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_i}\right) + \frac{\partial L}{\partial x_{n+i}} = 0 , \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{n+i}}\right) - \frac{\partial L}{\partial x_i} = 0 , \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{2n+i}}\right) + \frac{\partial L}{\partial x_{4n+i}} = 0,$$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{3n+i}}\right) + \frac{\partial L}{\partial x_{5n+i}} = 0 , \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{4n+i}}\right) - \frac{\partial L}{\partial x_{2n+i}} = 0, \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{5n+i}}\right) - \frac{\partial L}{\partial x_{3n+i}} = 0,$$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{6n+i}}\right) + \frac{\partial L}{\partial x_{7n+i}} = 0 , \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{7n+i}}\right) - \frac{\partial L}{\partial x_{6n+i}} = 0.$$

Second:

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_i}\right)+\frac{\partial L}{\partial x_{2n+i}}=0\ ,\ \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{n+i}}\right)-\frac{\partial L}{\partial x_{4n+i}}=0\ ,\ \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{2n+i}}\right)-\frac{\partial L}{\partial x_i}=0\ ,$$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{4n+i}}\right)+\frac{\partial L}{\partial x_{n+i}}=0\ ,\ \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{3n+i}}\right)+\frac{\partial L}{\partial x_{6n+i}}=0\ ,\ \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{5n+i}}\right)-\frac{\partial L}{\partial x_{7n+i}}=0\ ,$$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{6n+i}}\right)-\frac{\partial L}{\partial x_{3n+i}}=0\ ,\ \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{7n+i}}\right)+\frac{\partial L}{\partial x_{5n+i}}=0$$

Thirdly, let J_3 take a local basis component on the standard Cliffordian *Kähler* manifold (R^{8n}, V) and $\{x_i, x_{n+i}x_{2n+i}, x_{3n+i}, x_{4n+i}, x_{5n+i}, x_{6n+i}, x_{7n+i}\}$, $i=\overline{1,n}$ be its coordinate functions.

Let semispray be the vector field ξ determined by:

$$\xi = X^i\frac{\partial}{\partial x_i}+X^{n+i}\frac{\partial}{\partial x_{n+i}}+X^{2n+i}\frac{\partial}{\partial x_{2n+i}}+X^{3n+i}\frac{\partial}{\partial x_{3n+i}}+X^{4n+i}\frac{\partial}{\partial x_{4n+i}}$$

$$+X^{5n+i}\frac{\partial}{\partial x_{5n+i}}+X^{6n+i}\frac{\partial}{\partial x_{6n+i}}+X^{7n+i}\frac{\partial}{\partial x_{7n+i}} \qquad \rightarrow \qquad (6)$$

Where

$$X^i=\dot{x}_i\ ,X^{n+i}=\dot{x}_{n+i}\ ,X^{2n+i}=\dot{x}_{2n+i}\ ,X^{3n+i}=\dot{x}_{3n+i}\ ,X^{4n+i}=\dot{x}_{4n+i}$$

$$X^{5n+i}=\dot{x}_{5n+i}\ ,X^{6n+i}=\dot{x}_{6n+i}\ ,X^{7n+i}=\dot{x}_{7n+i}.$$

This equation (6) can be written concise manner

$$\xi=\sum_{a=0}^{7}X^{an+i}\frac{\partial}{\partial x_{an+i}} \qquad \rightarrow \qquad (7)$$

And the dot indicates the derivative with respect to time t. The vector field defined by

$$V_{J_3}=J_3(\xi)=X^i\frac{\partial}{\partial x_{3n+i}}-X^{n+i}\frac{\partial}{\partial x_{5n+i}}-X^{2n+i}\frac{\partial}{\partial x_{6n+i}}-X^{3n+i}\frac{\partial}{\partial x_i}+X^{4n+i}\frac{\partial}{\partial x_{7n+i}}$$

$$+\ X^{5n+i}\frac{\partial}{\partial x_{n+i}}+\ X^{6n+i}\frac{\partial}{\partial x_{2n+i}}-\ X^{7n+i}\frac{\partial}{\partial x_{4n+i}} \qquad \rightarrow \qquad (8)$$

Is called Liouville vector field on the standard Cliffordian *Kähler* manifold (R^{8n},V). The maps given by $T\ ,P\colon R^{8n}\rightarrow R$ such that:

$$T=\frac{1}{2}m_i(\dot{x}_i^2+\dot{x}_{n+i}^2+\dot{x}_{2n+i}^2+\dot{x}_{3n+i}^2+\dot{x}_{4n+i}^2+\dot{x}_{5n+i}^2+\dot{x}_{6n+i}^2+\dot{x}_{7n+i}^2)$$

$$\therefore\ T=\frac{1}{2}m_i\sum_{a=0}^{7}\dot{x}_{an+i}^2 \qquad , \quad P=m_igh$$

Are called the kinetic energy and the potential energy of the system, respectively. Here m_i , g *and* h stand for mass of a mechanical system having m particles, the gravity acceleration and distance to the origin of a mechanical system on the standard Cliffordian *Kähler* manifold (R^{8n},V), respectively.

Then $L: R^{8n} \to R$ is a map that satisfies the conditions:

i) $L = T - P$ is a Lagrangian function.

ii) the function given by $E_L^{J_3} = V_{J_3}(L) - L$, is energy function.

The operator i_{J_3} induced by J_3 and given by:

$$i_{J_3}\omega(X_1, X_2, \dots, X_r) = \sum_{i=1}^{r} \omega\,(X_1, \dots, J_3X_i, \dots, X_r) \qquad \to \qquad (9)$$

Is said to be vertical derivation, where $\omega \in \Lambda^r R^{8n}, X_i \in \mathcal{X}(R^{8n})$. The vertical differentiation d_{J_3} is defined by:

$$d_{J_3} = [i_{J_3}, d] = i_{J_3}d - di_{J_3} \qquad \to \qquad (10)$$

Where d is the usual exterior derivation. For J_3, the closed Cliffordian *Kähler* form is the closed 2-form given by $\Phi_L^{J_3} = -dd_{J_3}L$ such that

$$d_{J_3} = \frac{\partial}{\partial x_{3n+i}}dx_i - \frac{\partial}{\partial x_{5n+i}}dx_{n+i} - \frac{\partial}{\partial x_{6n+i}}dx_{2n+i} - \frac{\partial}{\partial x_i}dx_{3n+i} + \frac{\partial}{\partial x_{7n+i}}dx_{4n+i}$$

$$+ \frac{\partial}{\partial x_{n+i}}dx_{5n+i} + \frac{\partial}{\partial x_{2n+i}}dx_{6n+i} - \frac{\partial}{\partial x_{4n+i}}dx_{7n+i}$$

Defined by operator

$$d_{J_3} : \mathcal{F}(M) \to \Lambda^1 R^{8n} \qquad \to \qquad (11)$$

Then

$$\Phi_L^{J_3} = -\frac{\partial^2 L}{\partial x_j \partial x_{3n+i}}dx_j \wedge dx_i + \frac{\partial^2 L}{\partial x_j \partial x_{5n+i}}dx_j \wedge dx_{n+i} + \frac{\partial^2 L}{\partial x_j \partial x_{6n+i}}dx_j \wedge dx_{2n+i} +$$

$$\frac{\partial^2 L}{\partial x_j \partial x_i}dx_j \wedge dx_{3n+i} - \frac{\partial^2 L}{\partial x_j \partial x_{7n+i}}dx_j \wedge dx_{4n+i} - \frac{\partial^2 L}{\partial x_j \partial x_{n+i}}dx_j \wedge dx_{5n+i} - \frac{\partial^2 L}{\partial x_j \partial x_{2n+i}}dx_j \wedge dx_{6n+i}$$

$$+\frac{\partial^2 L}{\partial x_j \partial x_{4n+i}}dx_j \wedge dx_{7n+i} - \frac{\partial^2 L}{\partial x_{n+j} \partial x_{3n+i}}dx_{n+j} \wedge dx_i + \frac{\partial^2 L}{\partial x_{n+j} \partial x_{5n+i}}dx_{n+j} \wedge dx_{n+i} +$$

$$+\frac{\partial^2 L}{\partial x_{n+j} \partial x_{6n+i}}dx_{n+j} \wedge dx_{2n+i} + \frac{\partial^2 L}{\partial x_{n+j} \partial x_i}dx_{n+j} \wedge dx_{3n+i} - \frac{\partial^2 L}{\partial x_{n+j} \partial x_{7n+i}}dx_{n+j} \wedge dx_{4n+i}$$

$$-\frac{\partial^2 L}{\partial x_{n+j} \partial x_{n+i}}dx_{n+j} \wedge dx_{5n+i} - \frac{\partial^2 L}{\partial x_{n+j} \partial x_{2n+i}}dx_{n+j} \wedge dx_{6n+i} + \frac{\partial^2 L}{\partial x_{n+j} \partial x_{4n+i}}dx_{n+j} \wedge dx_{7n+i}$$

$$-\frac{\partial^2 L}{\partial x_{2n+j} \partial x_{3n+i}}dx_{2n+j} \wedge dx_i + \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{5n+i}}dx_{2n+j} \wedge dx_{n+i} + \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{6n+i}}dx_{2n+j} \wedge dx_{2n+i}$$

$$+\frac{\partial^2 L}{\partial x_{2n+j} \partial x_i}dx_{2n+j} \wedge dx_{3n+i} - \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{7n+i}}dx_{2n+j} \wedge dx_{4n+i} - \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{n+i}}dx_{2n+j} \wedge dx_{5n+i} -$$

$$\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{2n+i}}dx_{2n+j}\wedge dx_{6n+i}+\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{4n+i}}dx_{2n+j}\wedge dx_{7n+i}-\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{3n+i}}dx_{3n+j}\wedge dx_{i}+$$

$$\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{5n+i}}dx_{3n+j}\wedge dx_{n+i}+\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{6n+i}}dx_{3n+j}\wedge dx_{2n+i}+\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{i}}dx_{3n+j}\wedge dx_{3n+i}-$$

$$\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{7n+i}}dx_{3n+j}\wedge dx_{4n+i}-\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{n+i}}dx_{3n+j}\wedge dx_{5n+i}-\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{2n+i}}dx_{3n+j}\wedge dx_{6n+i}+$$

$$\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{4n+i}}dx_{3n+j}\wedge dx_{7n+i}-\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{3n+i}}dx_{4n+j}\wedge dx_{i}+\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{5n+i}}dx_{4n+j}\wedge dx_{n+i}+$$

$$\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{6n+i}}dx_{4n+j}\wedge dx_{2n+i}+\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{i}}dx_{4n+j}\wedge dx_{3n+i}-\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{7n+i}}dx_{4n+j}\wedge dx_{4n+i}-$$

$$\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{n+i}}dx_{4n+j}\wedge dx_{5n+i}-\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{2n+i}}dx_{4n+j}\wedge dx_{6n+i}+\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{4n+i}}dx_{4n+j}\wedge dx_{7n+i}-$$

$$\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{3n+i}}dx_{5n+j}\wedge dx_{i}+\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{5n+i}}dx_{5n+j}\wedge dx_{n+i}+\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{6n+i}}dx_{5n+j}\wedge dx_{2n+i}+$$

$$\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{i}}dx_{5n+j}\wedge dx_{3n+i}-\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{7n+i}}dx_{5n+j}\wedge dx_{4n+i}-\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{n+i}}dx_{5n+j}\wedge dx_{5n+i}-$$

$$\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{2n+i}}dx_{5n+j}\wedge dx_{6n+i}+\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{4n+i}}dx_{5n+j}\wedge dx_{7n+i}-\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{3n+i}}dx_{6n+j}\wedge dx_{i}+$$

$$\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{5n+i}}dx_{6n+j}\wedge dx_{n+i}+\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{6n+i}}dx_{6n+j}\wedge dx_{2n+i}+\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{i}}dx_{6n+j}\wedge dx_{3n+i}-$$

$$\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{7n+i}}dx_{6n+j}\wedge dx_{4n+i}-\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{n+i}}dx_{6n+j}\wedge dx_{5n+i}-\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{2n+i}}dx_{6n+j}\wedge dx_{6n+i}+$$

$$\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{4n+i}}dx_{6n+j}\wedge dx_{7n+i}-\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{3n+i}}dx_{7n+j}\wedge dx_{i}+\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{5n+i}}dx_{7n+j}\wedge dx_{n+i}+$$

$$\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{6n+i}}dx_{7n+j}\wedge dx_{2n+i}+\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{i}}dx_{7n+j}\wedge dx_{3n+i}-\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{7n+i}}dx_{7n+j}\wedge dx_{4n+i}-$$

$$\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{n+i}}dx_{7n+j}\wedge dx_{5n+i}-\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{2n+i}}dx_{7n+j}\wedge dx_{6n+i}+\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{4n+i}}dx_{7n+j}\wedge dx_{7n+i}$$

Let ξ be the second order differential equation by given Eq(1) and defined by Eq(6) and

$$i_\xi\Phi_L^{J_3}=-X^i\frac{\partial^2 L}{\partial x_j\partial x_{3n+i}}\delta_i^j\,dx_i+X^i\frac{\partial^2 L}{\partial x_j\partial x_{3n+i}}dx_j+X^i\frac{\partial^2 L}{\partial x_j\partial x_{5n+i}}\delta_i^j\,dx_{n+i}-X^{n+i}\frac{\partial^2 L}{\partial x_j\partial x_{5n+i}}dx_j+$$

$$X^i\frac{\partial^2 L}{\partial x_j\partial x_{6n+i}}\delta_i^j\,dx_{2n+i}-X^{2n+i}\frac{\partial^2 L}{\partial x_j\partial x_{6n+i}}dx_j+X^i\frac{\partial^2 L}{\partial x_j\partial x_i}\delta_i^j\,dx_{3n+i}-X^{3n+i}\frac{\partial^2 L}{\partial x_j\partial x_i}dx_j-$$

$$X^i\frac{\partial^2 L}{\partial x_j\partial x_{7n+i}}\delta_i^j\,dx_{4n+i}+X^{4n+i}\frac{\partial^2 L}{\partial x_j\partial x_{7n+i}}dx_j-X^i\frac{\partial^2 L}{\partial x_j\partial x_{n+i}}\delta_i^j\,dx_{5n+i}+X^{5n+i}\frac{\partial^2 L}{\partial x_j\partial x_{n+i}}dx_j-$$

$$X^i\frac{\partial^2 L}{\partial x_j\partial x_{2n+i}}\delta_i^j\,dx_{6n+i}+X^{6n+i}\frac{\partial^2 L}{\partial x_j\partial x_{2n+i}}dx_j+X^i\frac{\partial^2 L}{\partial x_j\partial x_{4n+i}}\delta_i^j\,dx_{7n+i}-X^{7n+i}\frac{\partial^2 L}{\partial x_j\partial x_{4n+i}}dx_j-$$

$$X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{3n+i}}\delta_{n+i}^{n+j}\,dx_i+X^i\frac{\partial^2 L}{\partial x_{n+j}\partial x_{3n+i}}dx_{n+j}+X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{5n+i}}\delta_{n+i}^{n+j}\,dx_{n+i}-$$

$$X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{5n+i}}dx_{n+j}+X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{6n+i}}\delta_{n+i}^{n+j}\,dx_{2n+i}-X^{2n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{6n+i}}dx_{n+j}+$$

$$X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_i}\delta_{n+i}^{n+j}\,dx_{3n+i}-X^{3n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_i}dx_{n+j}-X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{7n+i}}\delta_{n+i}^{n+j}\,dx_{4n+i}+$$

$$X^{4n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{7n+i}}dx_{n+j}-X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{n+i}}\delta_{n+i}^{n+j}\,dx_{5n+i}+X^{5n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{4n+i}}dx_{n+j}-$$

$$X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{2n+i}}\delta_{n+i}^{n+j}\,dx_{6n+i}+X^{6n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{2n+i}}dx_{n+j}+X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{4n+i}}\delta_{n+i}^{n+j}\,dx_{7n+i}-$$

$$X^{7n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{4n+i}}dx_{n+j}-X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{3n+i}}\delta_{2n+i}^{2n+j}\,dx_i+X^i\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{3n+i}}dx_{2n+j}+$$

$$X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{5n+i}}\delta_{2n+i}^{2n+j}\,dx_{n+i}-X^{n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{5n+i}}dx_{2n+j}+X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{6n+i}}\delta_{2n+i}^{2n+j}\,dx_{2n+i}-$$

$$X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{6n+i}}dx_{2n+j}+X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_i}\delta_{2n+i}^{2n+j}\,dx_{3n+i}-X^{3n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_i}dx_{2n+j}-$$

$$X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{7n+i}}\delta_{2n+i}^{2n+j}\,dx_{4n+i}+X^{4n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{7n+i}}dx_{2n+j}-X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{n+i}}\delta_{2n+i}^{2n+j}\,dx_{5n+i}$$

$$+X^{5n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{n+i}}dx_{2n+j}-X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{2n+i}}\delta_{2n+i}^{2n+j}\,dx_{6n+i}+X^{6n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{2n+i}}dx_{2n+j}+$$

$$X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{4n+i}}\delta_{2n+i}^{2n+j}\,dx_{7n+i}-X^{7n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{4n+i}}dx_{2n+j}-X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{3n+i}}\delta_{3n+i}^{3n+j}\,dx_i+$$

$$X^i\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{3n+i}}dx_{3n+j}+X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{5n+i}}\delta_{3n+i}^{3n+j}\,dx_{n+i}-X^{n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{5n+i}}dx_{3n+j}+$$

$$X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{6n+i}}\delta_{3n+i}^{3n+j}\,dx_{2n+i}-X^{2n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{6n+i}}dx_{3n+j}+X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_i}\delta_{3n+i}^{3n+j}\,dx_{3n+i}$$

$$-X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_i}dx_{3n+j}-X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{7n+i}}\delta_{3n+i}^{3n+j}\,dx_{4n+i}+X^{4n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{7n+i}}dx_{3n+j}-$$

$$X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{n+i}}\delta_{3n+i}^{3n+j}\,dx_{5n+i}+X^{5n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{n+i}}dx_{3n+j}-X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{2n+i}}\delta_{3n+i}^{3n+j}\,dx_{6n+i}+$$

Notes

$$X^{6n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{2n+i}}dx_{3n+j}+X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{4n+i}}\delta_{3n+i}^{3n+j}dx_{7n+i}-X^{7n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{4n+i}}dx_{3n+j}-$$

$$X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{3n+i}}\delta_{4n+i}^{4n+j}dx_{i}+X^{i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{3n+i}}dx_{4n+j}+X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{5n+i}}\delta_{4n+i}^{4n+j}dx_{n+i}-$$

$$X^{n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{5n+i}}dx_{4n+j}+X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{6n+i}}\delta_{4n+i}^{4n+j}dx_{2n+i}-X^{2n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{6n+i}}dx_{4n+j}+$$

$$X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{i}}\delta_{4n+i}^{4n+j}dx_{3n+i}-X^{3n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{i}}dx_{4n+j}-X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{7n+i}}\delta_{4n+i}^{4n+j}dx_{4n+i}$$

$$+X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{7n+i}}dx_{4n+j}-X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{n+i}}\delta_{4n+i}^{4n+j}dx_{5n+i}+X^{5n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{n+i}}dx_{4n+j}-$$

$$X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{2n+i}}\delta_{4n+i}^{4n+j}dx_{6n+i}+X^{6n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{i}}dx_{4n+j}+X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{4n+i}}\delta_{4n+i}^{4n+j}dx_{7n+i}-$$

$$X^{7n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{4n+i}}dx_{4n+j}-X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{3n+i}}\delta_{5n+i}^{5n+j}dx_{i}+X^{i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{3n+i}}dx_{5n+j}+$$

$$X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{5n+i}}\delta_{5n+i}^{5n+j}dx_{n+i}-X^{n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{5n+i}}dx_{5n+j}+X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{6n+i}}\delta_{5n+i}^{5n+j}dx_{2n+i}-$$

$$X^{2n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{6n+i}}dx_{5n+j}+X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{i}}\delta_{5n+i}^{5n+j}dx_{3n+i}-X^{3n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{i}}dx_{5n+j}-$$

$$X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{7n+i}}\delta_{5n+i}^{5n+j}dx_{4n+i}+X^{4n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{7n+i}}dx_{5n+j}-X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{n+i}}\delta_{5n+i}^{5n+j}dx_{5n+i}$$

$$+X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{n+i}}dx_{5n+j}-X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{2n+i}}\delta_{5n+i}^{5n+j}dx_{6n+i}+X^{6n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{2n+i}}dx_{5n+j}+$$

$$X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{4n+i}}\delta_{5n+i}^{5n+j}dx_{7n+i}-X^{7n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{4n+i}}dx_{5n+j}-X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{3n+i}}\delta_{6n+i}^{6n+j}dx_{i}$$

$$+X^{i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{3n+i}}dx_{6n+j}+X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{5n+i}}\delta_{6n+i}^{6n+j}dx_{n+i}-X^{n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{5n+i}}dx_{6n+j}+$$

$$X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{6n+i}}\delta_{6n+i}^{6n+j}dx_{2n+i}-X^{2n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{6n+i}}dx_{6n+j}+X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{i}}\delta_{6n+i}^{6n+j}dx_{3n+i}$$

$$-X^{3n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{i}}dx_{6n+j}-X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{7n+i}}\delta_{6n+i}^{6n+j}dx_{4n+i}+X^{4n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{7n+i}}dx_{6n+j}-$$

$$X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{n+i}}\delta_{6n+i}^{6n+j}dx_{5n+i}+X^{5n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{n+i}}dx_{6n+j}-X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{2n+i}}\delta_{6n+i}^{6n+j}dx_{6n+i}$$

$$+X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{2n+i}}dx_{6n+j}+X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{4n+i}}\delta_{6n+i}^{6n+j}dx_{7n+i}-X^{7n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{4n+i}}dx_{6n+j}-$$

$$X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{3n+i}}\delta_{7n+i}^{7n+j}dx_i + X^i\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{3n+i}}dx_{7n+j} + X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{5n+i}}\delta_{7n+i}^{7n+j}dx_{n+i} -$$

$$X^{n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{5n+i}}dx_{7n+j} + X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{6n+i}}\delta_{7n+i}^{7n+j}dx_{2n+i} - X^{2n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{6n+i}}dx_{7n+j} +$$

$$X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_i}\delta_{7n+i}^{7n+j}dx_{3n+i} - X^{3n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_i}dx_{7n+j} - X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{7n+i}}\delta_{7n+i}^{7n+j}dx_{4n+i}$$

$$+X^{4n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{7n+i}}dx_{7n+j} - X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{n+i}}\delta_{7n+i}^{7n+j}dx_{5n+i} + X^{5n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{n+i}}dx_{7n+j} -$$

$$X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{2n+i}}\delta_{7n+i}^{7n+j}dx_{6n+i} + X^{6n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{2n+i}}dx_{7n+j} + X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{4n+i}}\delta_{7n+i}^{7n+j}dx_{7n+i}$$

$$- X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{4n+i}}dx_{7n+j}$$

Since the closed standard Cliffordian *Kähler* form $\Phi_L^{J_3}\ on\,(R^{8n},V)$ is the symplectic structure, it is found

$$E_L^{J_3} = V_{J_3}(L) - L = X^i\frac{\partial L}{\partial x_{3n+i}} - X^{n+i}\frac{\partial L}{\partial x_{5n+i}} - X^{2n+i}\frac{\partial L}{\partial x_{6n+i}} - X^{3n+i}\frac{\partial L}{\partial x_i} +$$

$$X^{4n+i}\frac{\partial L}{\partial x_{7n+i}} + X^{5n+i}\frac{\partial L}{\partial x_{n+i}} + X^{6n+i}\frac{\partial L}{\partial x_{2n+i}} - X^{7n+i}\frac{\partial L}{\partial x_{4n+i}} - L \quad \rightarrow \tag{12}$$

And hence

$$dE_L^{J_3} = X^i\frac{\partial^2 L}{\partial x_j\partial x_{3n+i}}dx_j - X^{n+i}\frac{\partial^2 L}{\partial x_j\partial x_{5n+i}}dx_j - X^{2n+i}\frac{\partial^2 L}{\partial x_j\partial x_{6n+i}}dx_j - X^{3n+i}\frac{\partial^2 L}{\partial x_j\partial x_i}dx_j$$

$$+X^{4n+i}\frac{\partial^2 L}{\partial x_j\partial x_{7n+i}}dx_j + X^{5n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{n+i}}dx_j + X^{6n+i}\frac{\partial^2 L}{\partial x_j\partial x_{2n+i}}dx_j - X^{7n+i}\frac{\partial^2 L}{\partial x_j\partial x_{4n+i}}dx_j$$

$$+X^i\frac{\partial^2 L}{\partial x_{n+j}\partial x_{3n+i}}dx_{n+j} - X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{5n+i}}dx_{n+j} - X^{2n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{6n+i}}dx_{n+j} -$$

$$-X^{3n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_i}dx_{n+j} + X^{4n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{7n+i}}dx_{n+j} + X^{5n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{n+i}}dx_{n+j} +$$

$$X^{6n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{2n+i}}dx_{n+j} - X^{7n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{4n+i}}dx_{n+j} + X^i\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{3n+i}}dx_{2n+j} -$$

$$X^{n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{5n+i}}dx_{2n+j} - X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{6n+i}}dx_{2n+j} - X^{3n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_i}dx_{2n+j} +$$

$$X^{4n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{7n+i}}dx_{2n+j} + X^{5n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{n+i}}dx_{2n+j} + X^{6n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{2n+i}}dx_{2n+j} -$$

Notes

$$X^{7n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{4n+i}}dx_{2n+j}+X^{i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{3n+i}}dx_{3n+j}-X^{n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{5n+i}}dx_{3n+j}-$$

$$X^{2n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{6n+i}}dx_{3n+j}-X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{i}}dx_{3n+j}+X^{4n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{7n+i}}dx_{3n+j}+$$

$$X^{5n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{n+i}}dx_{3n+j}+X^{6n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{2n+i}}dx_{3n+j}-X^{7n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{4n+i}}dx_{3n+j}+$$

$$X^{i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{3n+i}}dx_{4n+j}-X^{n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{5n+i}}dx_{4n+j}-X^{2n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{6n+i}}dx_{4n+j}-$$

$$X^{3n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{i}}dx_{4n+j}+X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{7n+i}}dx_{4n+j}+X^{5n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{n+i}}dx_{4n+j}+$$

$$X^{6n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{2n+i}}dx_{4n+j}-X^{7n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{4n+i}}dx_{4n+j}+X^{i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{3n+i}}dx_{5n+j}-$$

$$X^{n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{5n+i}}dx_{5n+j}-X^{2n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{6n+i}}dx_{5n+j}-X^{3n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{i}}dx_{5n+j}+$$

$$X^{4n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{7n+i}}dx_{5n+j}+X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{n+i}}dx_{5n+j}+X^{6n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{2n+i}}dx_{5n+j}-$$

$$X^{7n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{4n+i}}dx_{5n+j}+X^{i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{3n+i}}dx_{6n+j}-X^{n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{5n+i}}dx_{6n+j}-$$

$$X^{2n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{6n+i}}dx_{6n+j}-X^{3n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{i}}dx_{6n+j}+X^{4n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{7n+i}}dx_{6n+j}+$$

$$X^{5n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{n+i}}dx_{6n+j}+X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{2n+i}}dx_{6n+j}-X^{7n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{4n+i}}dx_{6n+j}+$$

$$X^{i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{3n+i}}dx_{7n+j}-X^{n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{5n+i}}dx_{7n+j}-X^{2n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{6n+i}}dx_{7n+j}-$$

$$X^{3n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{i}}dx_{7n+j}+X^{4n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{7n+i}}dx_{7n+j}+X^{5n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{n+i}}dx_{7n+j}+$$

$$X^{6n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{2n+i}}dx_{7n+j}-X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{4n+i}}dx_{7n+j}-\frac{\partial L}{\partial x_{j}}dx_{j}-\frac{\partial L}{\partial x_{n+j}}dx_{n+j}$$

$$-\frac{\partial L}{\partial x_{2n+j}}dx_{2n+j}-\frac{\partial L}{\partial x_{3n+j}}dx_{3n+j}-\frac{\partial L}{\partial x_{4n+j}}dx_{4n+j}-\frac{\partial L}{\partial x_{5n+j}}dx_{5n+j}$$

$$-\frac{\partial L}{\partial x_{6n+j}}dx_{6n+j}-\frac{\partial L}{\partial x_{7n+j}}dx_{7n+j}$$

With the use of Eq.(1), the following expressions can be obtained:

$$-X^{i}\frac{\partial^2 L}{\partial x_j \partial x_{3n+i}}\delta_i^j\,dx_i+X^{i}\frac{\partial^2 L}{\partial x_j \partial x_{5n+i}}\delta_i^j\,dx_{n+i}+X^{i}\frac{\partial^2 L}{\partial x_j \partial x_{6n+i}}\delta_i^j\,dx_{2n+i}+X^{i}\frac{\partial^2 L}{\partial x_j \partial x_{i}}\delta_i^j\,dx_{3n+i}$$

$$-X^{i}\frac{\partial^2 L}{\partial x_j \partial x_{7n+i}}\delta_i^j\,dx_{4n+i}-X^{i}\frac{\partial^2 L}{\partial x_j \partial x_{n+i}}\delta_i^j\,dx_{5n+i}-X^{i}\frac{\partial^2 L}{\partial x_j \partial x_{2n+i}}\delta_i^j\,dx_{6n+i}+X^{i}\frac{\partial^2 L}{\partial x_j \partial x_{4n+i}}\delta_i^j\,dx_{7n+i}$$

$$-X^{n+i}\frac{\partial^2 L}{\partial x_{n+j} \partial x_{3n+i}}\delta_{n+i}^{n+j}\,dx_i+X^{n+i}\frac{\partial^2 L}{\partial x_{n+j} \partial x_{5n+i}}\delta_{n+i}^{n+j}\,dx_{n+i}+X^{n+i}\frac{\partial^2 L}{\partial x_{n+j} \partial x_{6n+i}}\delta_{n+i}^{n+j}\,dx_{2n+i}+$$

$$X^{n+i}\frac{\partial^2 L}{\partial x_{n+j} \partial x_{i}}\delta_{n+i}^{n+j}\,dx_{3n+i}-X^{n+i}\frac{\partial^2 L}{\partial x_{n+j} \partial x_{7n+i}}\delta_{n+i}^{n+j}\,dx_{4n+i}-X^{n+i}\frac{\partial^2 L}{\partial x_{n+j} \partial x_{n+i}}\delta_{n+i}^{n+j}\,dx_{5n+i}-$$

$$X^{n+i}\frac{\partial^2 L}{\partial x_{n+j} \partial x_{2n+i}}\delta_{n+i}^{n+j}\,dx_{6n+i}-X^{n+i}\frac{\partial^2 L}{\partial x_{n+j} \partial x_{4n+i}}\delta_{n+i}^{n+j}\,dx_{7n+i}-X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j} \partial x_{3n+i}}\delta_{2n+i}^{2n+j}\,dx_i+$$

$$X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j} \partial x_{5n+i}}\delta_{2n+i}^{2n+j}\,dx_{n+i}+\;X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j} \partial x_{6n+i}}\delta_{2n+i}^{2n+j}\,dx_{2n+i}+$$

$$X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j} \partial x_{i}}\delta_{2n+i}^{2n+j}\,dx_{3n+i}-\;X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j} \partial x_{7n+i}}\delta_{2n+i}^{2n+j}\,dx_{4n+i}-$$

$$X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j} \partial x_{n+i}}\delta_{2n+i}^{2n+j}\,dx_{5n+i}-X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j} \partial x_{2n+i}}\delta_{2n+i}^{2n+j}\,dx_{6n+i}+$$

$$X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j} \partial x_{4n+i}}\delta_{2n+i}^{2n+j}\,dx_{7n+i}\;-\;X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j} \partial x_{3n+i}}\delta_{3n+i}^{3n+j}\,dx_i\;+$$

$$X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j} \partial x_{5n+i}}\delta_{3n+i}^{3n+j}\,dx_{n+i}\;+\;X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j} \partial x_{6n+i}}\delta_{3n+i}^{3n+j}\,dx_{2n+i}\;+$$

$$X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j} \partial x_{i}}\delta_{3n+i}^{3n+j}\,dx_{3n+i}\;-\;X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j} \partial x_{7n+i}}\delta_{3n+i}^{3n+j}\,dx_{4n+i}\;-$$

$$X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j} \partial x_{n+i}}\delta_{3n+i}^{3n+j}\,dx_{5n+i}-X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j} \partial x_{2n+i}}\delta_{3n+i}^{3n+j}\,dx_{6n+i}$$

$$+X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j} \partial x_{4n+i}}\delta_{3n+i}^{3n+j}\,dx_{7n+i}-X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j} \partial x_{3n+i}}\delta_{4n+i}^{4n+j}\,dx_i\;+$$

$$X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j} \partial x_{5n+i}}\delta_{4n+i}^{4n+j}\,dx_{n+i}\;+\;X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j} \partial x_{6n+i}}\delta_{4n+i}^{4n+j}\,dx_{2n+i}+$$

$$X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j} \partial x_{i}}\delta_{4n+i}^{4n+j}\,dx_{3n+i}-X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j} \partial x_{7n+i}}\delta_{4n+i}^{4n+j}\,dx_{4n+i}\;-$$

$$X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j} \partial x_{n+i}}\delta_{4n+i}^{4n+j}\,dx_{5n+i}-\;X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j} \partial x_{2n+i}}\delta_{4n+i}^{4n+j}\,dx_{6n+i}+$$

$$X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j} \partial x_{4n+i}}\delta_{4n+i}^{4n+j}\,dx_{7n+i}-X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j} \partial x_{3n+i}}\delta_{5n+i}^{5n+j}\,dx_i+$$

$$X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{5n+i}}\delta_{5n+i}^{5n+j}dx_{n+i} + X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{6n+i}}\delta_{5n+i}^{5n+j}dx_{2n+i} +$$

$$X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{i}}\delta_{5n+i}^{5n+j}dx_{3n+i} - X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{7n+i}}\delta_{5n+i}^{5n+j}dx_{4n+i} -$$

$$X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{n+i}}\delta_{5n+i}^{5n+j}dx_{5n+i} - X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{2n+i}}\delta_{5n+i}^{5n+j}dx_{6n+i} +$$

$$X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{4n+i}}\delta_{5n+i}^{5n+j}dx_{7n+i} - X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{3n+i}}\delta_{6n+i}^{6n+j}dx_{i} + X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{5n+i}}\delta_{6n+i}^{6n+j}dx_{n+i}$$

$$+ X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{6n+i}}\delta_{6n+i}^{6n+j}dx_{2n+i} + X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{i}}\delta_{6n+i}^{6n+j}dx_{3n+i} -$$

$$X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{7n+i}}\delta_{6n+i}^{6n+j}dx_{4n+i} - X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{n+i}}\delta_{6n+i}^{6n+j}dx_{5n+i} -$$

$$X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{2n+i}}\delta_{6n+i}^{6n+j}dx_{6n+i} + X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{4n+i}}\delta_{6n+i}^{6n+j}dx_{7n+i} -$$

$$X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{3n+i}}\delta_{7n+i}^{7n+j}dx_{i} + X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{5n+i}}\delta_{7n+i}^{7n+j}dx_{n+i} +$$

$$X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{6n+i}}\delta_{7n+i}^{7n+j}dx_{2n+i} + X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{i}}\delta_{7n+i}^{7n+j}dx_{3n+i} -$$

$$X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{7n+i}}\delta_{7n+i}^{7n+j}dx_{4n+i} - X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{n+i}}\delta_{7n+i}^{7n+j}dx_{5n+i} -$$

$$X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{2n+i}}\delta_{6n+i}^{6n+j}dx_{6n+i} + X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{4n+i}}\delta_{7n+i}^{7n+j}dx_{7n+i} + \frac{\partial L}{\partial x_j}dx_j + \frac{\partial L}{\partial x_{n+j}}dx_{n+j}$$

$$+\frac{\partial L}{\partial x_{2n+j}}dx_{2n+j} + \frac{\partial L}{\partial x_{3n+j}}dx_{3n+j} + \frac{\partial L}{\partial x_{4n+j}}dx_{4n+j} + \frac{\partial L}{\partial x_{5n+j}}dx_{5n+j} + \frac{\partial L}{\partial x_{6n+j}}dx_{6n+j} +$$

$$\frac{\partial L}{\partial x_{7n+j}}dx_{7n+j} = 0$$

If a curve denoted by $\alpha : R \rightarrow R^8$ is considered to be an integral curve of ξ, then we calculate the following equation:

$$-X^{i}\frac{\partial^2 L}{\partial x_j\partial x_{3n+i}}dx_j - X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{3n+i}}dx_j - X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{3n+i}}dx_j - X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{3n+i}}dx_j$$

$$-X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{3n+i}}dx_j - X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{3n+i}}dx_j - X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{3n+i}}dx_j -$$

$$X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{3n+i}}dx_j + X^{i}\frac{\partial^2 L}{\partial x_j\partial x_{5n+i}}dx_{n+j} + X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{5n+i}}dx_{n+j} +$$

$$X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{5n+i}}dx_{n+j}+X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{5n+i}}dx_{n+j}+X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{5n+i}}dx_{n+j}+$$

$$X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{5n+i}}dx_{n+j}+X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{5n+i}}dx_{n+j}+X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{5n+i}}dx_{n+j}+$$

$$X^{i}\frac{\partial^2 L}{\partial x_{j}\partial x_{6n+i}}dx_{2n+j}+X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{6n+i}}dx_{2n+j}+X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{6n+i}}dx_{2n+j}+$$

$$X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{6n+i}}dx_{2n+j}+X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{6n+i}}dx_{2n+j}+X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{6n+i}}dx_{2n+j}+$$

$$X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{6n+i}}dx_{2n+j}+X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{6n+i}}dx_{2n+j}+X^{i}\frac{\partial^2 L}{\partial x_{j}\partial x_{i}}dx_{3n+j}+$$

$$X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{i}}dx_{3n+j}+X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{i}}dx_{3n+j}+X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{i}}dx_{3n+j}+$$

$$X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{i}}dx_{3n+j}+X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{i}}dx_{3n+j}+X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{i}}dx_{3n+j}+$$

$$X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{i}}dx_{3n+j}-X^{i}\frac{\partial^2 L}{\partial x_{j}\partial x_{7n+i}}dx_{4n+j}-X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{7n+i}}dx_{4n+j}-$$

$$X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{7n+i}}dx_{4n+j}-X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{7n+i}}dx_{4n+j}-X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{7n+i}}dx_{4n+j}-$$

$$X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{7n+i}}dx_{4n+j}-X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{7n+i}}dx_{4n+j}-X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{7n+i}}dx_{4n+j}-$$

$$X^{i}\frac{\partial^2 L}{\partial x_{j}\partial x_{n+i}}dx_{5n+j}-X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{n+i}}dx_{5n+j}-X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{n+i}}dx_{5n+j}-$$

$$X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{n+i}}dx_{5n+j}-X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{n+i}}dx_{5n+j}-X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{n+i}}dx_{5n+j}-$$

$$X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{n+i}}dx_{5n+j}-X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{n+i}}dx_{5n+j}-X^{i}\frac{\partial^2 L}{\partial x_{j}\partial x_{2n+i}}dx_{6n+j}-$$

$$X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{2n+i}}dx_{6n+j}-X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{2n+i}}dx_{6n+j}-X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{2n+i}}dx_{6n+j}-$$

$$X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{2n+i}}dx_{6n+j}-X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{2n+i}}dx_{6n+j}-X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{2n+i}}dx_{6n+j}-$$

$$X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{2n+i}}dx_{6n+j}+X^{i}\frac{\partial^2 L}{\partial x_{j}\partial x_{4n+i}}dx_{7n+j}+X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{4n+i}}dx_{7n+j}+$$

$$X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{4n+i}}dx_{7n+j}+X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{4n+i}}dx_{7n+j}+X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{4n+i}}dx_{7n+j}+$$

$$X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{4n+i}}dx_{7n+j}+X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{4n+i}}dx_{7n+j}+X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{4n+i}}dx_{7n+j}+\frac{\partial L}{\partial x_j}dx_j+$$

$$\frac{\partial L}{\partial x_{n+j}}dx_{n+j}+\frac{\partial L}{\partial x_{2n+j}}dx_{2n+j}+\frac{\partial L}{\partial x_{3n+j}}dx_{3n+j}+\frac{\partial L}{\partial x_{4n+j}}dx_{4n+j}+\frac{\partial L}{\partial x_{5n+j}}dx_{5n+j}+$$

$$\frac{\partial L}{\partial x_{6n+j}}dx_{6n+j}+\frac{\partial L}{\partial x_{7n+j}}dx_{7n+j}=0 \qquad \rightarrow \qquad (13)$$

Or

$$-[X^{i}\frac{\partial^2 L}{\partial x_j\partial x_{3n+i}}+X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{3n+i}}+X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{3n+i}}+X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{3n+i}}+X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{3n+i}}$$

$$+X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{3n+i}}+X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{3n+i}}+X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{3n+i}}]\,dx_j+\frac{\partial L}{\partial x_j}dx_j+$$

$$[X^{i}\frac{\partial^2 L}{\partial x_j\partial x_{5n+i}}+X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{5n+i}}+X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{5n+i}}+X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{5n+i}}+X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{5n+i}}$$

$$+X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{5n+i}}+X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{5n+i}}+X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{5n+i}}]\,dx_{n+j}+\frac{\partial L}{\partial x_{n+j}}dx_{n+j}+$$

$$[X^{i}\frac{\partial^2 L}{\partial x_j\partial x_{6n+i}}+X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{6n+i}}+X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{6n+i}}+X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{6n+i}}+X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{6n+i}}$$

$$+X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{6n+i}}+X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{6n+i}}+X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{6n+i}}]\,dx_{2n+j}+\frac{\partial L}{\partial x_{2n+j}}dx_{2n+j}+$$

$$[X^{i}\frac{\partial^2 L}{\partial x_j\partial x_i}+X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_i}+X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_i}+X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_i}+X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_i}$$

$$+X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_i}+X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_i}+X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_i}]\,dx_{3n+j}+\frac{\partial L}{\partial x_{3n+j}}dx_{3n+j}-$$

$$[X^{i}\frac{\partial^2 L}{\partial x_j\partial x_{7n+i}}+X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{7n+i}}+X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{7n+i}}+X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{7n+i}}+X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{7n+i}}$$

$$+X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{7n+i}}+X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{7n+i}}+X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{7n+i}}]\,dx_{4n+j}+\frac{\partial L}{\partial x_{4n+j}}dx_{4n+j}-$$

$$[X^{i}\frac{\partial^2 L}{\partial x_j\partial x_{n+i}}+X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{n+i}}+X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{n+i}}+X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{n+i}}+X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{n+i}}$$

$$+X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{n+i}}+X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{n+i}}+X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{n+i}}]\,dx_{5n+j}+\frac{\partial L}{\partial x_{5n+j}}dx_{5n+j}-$$

$$[X^{i}\frac{\partial^2 L}{\partial x_j\partial x_{2n+i}}+X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{2n+i}}+X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{2n+i}}+X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{2n+i}}+X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{2n+i}}$$

$$+X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{2n+i}} + X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{2n+i}} + X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{2n+i}}]\,dx_{6n+j} + \frac{\partial L}{\partial x_{6n+j}}dx_{6n+j} +$$

$$[X^{i}\frac{\partial^2 L}{\partial x_j\partial x_{4n+i}} + X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{4n+i}} + X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{4n+i}} + X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{4n+i}} + X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{4n+i}}$$

$$+X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{n4+i}} + X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{4n+i}} + X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{4n+i}}]\,dx_{7n+j} + \frac{\partial L}{\partial x_{7n+j}}dx_{7n+j} = 0$$

In this equation can be concise manner

$$-\sum_{a=0}^{7}X^{an+i}\frac{\partial^2 L}{\partial x_{an+j}\partial x_{3n+i}}dx_j + \frac{\partial L}{\partial x_j}dx_j + \sum_{a=0}^{7}X^{an+i}\frac{\partial^2 L}{\partial x_{an+j}\partial x_{5n+i}}dx_{n+j} + \frac{\partial L}{\partial x_{n+j}}dx_{n+j} +$$

$$\sum_{a=0}^{7}X^{an+i}\frac{\partial^2 L}{\partial x_{an+j}\partial x_{6n+i}}dx_{2n+j} + \frac{\partial L}{\partial x_{2n+j}}dx_{2n+j} + \sum_{a=0}^{7}X^{an+i}\frac{\partial^2 L}{\partial x_{an+j}\partial x_{i}}dx_{3n+j} +$$

$$\frac{\partial L}{\partial x_{3n+j}}dx_{3n+j} - \sum_{a=0}^{7}X^{an+i}\frac{\partial^2 L}{\partial x_{an+j}\partial x_{7n+i}}dx_{4n+j} + \frac{\partial L}{\partial x_{4n+j}}dx_{4n+j} -$$

$$\sum_{a=0}^{7}X^{an+i}\frac{\partial^2 L}{\partial x_{an+j}\partial x_{n+i}}dx_{5n+j} + \frac{\partial L}{\partial x_{5n+j}}dx_{5n+j} - \sum_{a=0}^{7}X^{an+i}\frac{\partial^2 L}{\partial x_{an+j}\partial x_{2n+i}}dx_{6n+j} + \frac{\partial L}{\partial x_{6n+j}}dx_{6n+j}$$

$$+\sum_{a=0}^{7}X^{an+i}\frac{\partial^2 L}{\partial x_{an+j}\partial x_{4n+i}}dx_{7n+j} + \frac{\partial L}{\partial x_{7n+j}}dx_{7n+j} = 0 \qquad \rightarrow \tag{14}$$

Then we obtain the equations

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_i}\right) + \frac{\partial L}{\partial x_{3n+i}} = 0, \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{n+i}}\right) - \frac{\partial L}{\partial x_{5n+i}} = 0, \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{2n+i}}\right) - \frac{\partial L}{\partial x_{6n+i}} = 0,$$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{3n+i}}\right) - \frac{\partial L}{\partial x_i} = 0, \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{4n+i}}\right) + \frac{\partial L}{\partial x_{7n+i}} = 0, \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{5n+i}}\right) + \frac{\partial L}{\partial x_{n+i}} = 0,$$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{6n+i}}\right) + \frac{\partial L}{\partial x_{2n+i}} = 0, \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{7n+i}}\right) - \frac{\partial L}{\partial x_{4nn+i}} = 0 \qquad \rightarrow \tag{15}$$

Thus equations obtained in Eq(15) are called Euler-Lagrange equations structured by means of means of $\Phi_L^{J_3}$ on the standard Cliffordian *Kähler* manifold (R^8, V) and So, the triple$(R^8, \Phi_L^{J_3}, \xi)$ is said to be a mechanical system on the standard Cliffordian *Kähler* manifold (R^8, V).

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Notes

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Model of High Temperature Heat Transfer in Metals

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Abstract- Based on the analysis of the atomic – ionic radii and volume characteristics new the approach (exchange – fluctuation) is considered under band blurring. It is shown that atomic space between atomic and ionic radii is divided into the ready Kλ cells (where K - nearest neighbours, λ = h/mc). The heat energy transfer in this space is represented in the model of fluxes, exchanging by the electron density in the ±Kλ mode.

Keywords: ionic and atomic radii, electronic density, fluctuation density.

GJSFR-F Classification: FOR Code: 82D35

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I. INTRODUCTION

Despite greate progress in physic of metals the high temperature formation nature of heat transfere remains inexplicable. Main reason is band structure degradate. Because a nontraditional approach was chosen: instead of impulse (p=hk, its value and direction) the electron density distribution (its probability) was analyzed in the coordination (pozition) space between ionic cores, using atomic (r_a) and ionic (r_i) radius on the one hand and using values λ_F and Z (a valency) of condition electrons on the other hand [1,2].

II. INITIAL DATA

Main assumption is as follows: electron density fluctuanions appear, when the density of probability is distributed in the pozition space between ionic cores (in coordinated space) at the pseudo – potential approximation. Differences in states of separate electrons can be described as mean value of all posible fluctuation, using a half - width distribution of probability per r at the level of the maximum electron density fluctuation. This level has been distinguished by ultimate possible fluctuations of potencial energies appear at $r_i \approx R_c$ and r_a (Its min. and max.), while one half these values is a mean ultimate fluctuation (It is $1/2\ (r_a + R_c)$). Based on conception of free electrons and approximation of non-screen Coulomb interaction we are obtained:

$$U(r) = (e^2/4\pi\varepsilon_0)\ Z/K[1/2(\ r_a + R_c)]$$

Where K is number of nearest neighbors, $R_c \approx r_i$ (where r_i – crystal-chemical ionic radius, R_c – a radius of atomic core in metals), Z/K - the density of the charge, $1/2(\ r_a + R_c) = R$ is the mean value between of ultimate fluctuations by R_c and r_a, U(r) is the bond energy [1,2]. If R is considered as a main

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dimensional feature, then a chain of R interactions with co-ordinated (pozitions) space and the number nearest neighbors (K) can be obtained:

1) as an orbital: $1/2(r_a + R_c) = R$;
2) as a volume spheare in pseudo- potential field: $4/3\pi(r_a^3 - r_i^3)/K = R^3$ and also: $4/3\pi R^3 = 4/3\pi r_i^3 + r_s^3 Z$, when $r_s = 1{,}92\ \lambda_F/2\pi$;
3) as a surfase: $4\pi R^2 (K\lambda) = (r_a^3 - r_i^3)\ Z^{1/3}$ (where $\lambda = h/mc = 0{,}0242\acute{Å}$);
4) as a characteristic of melting: $V_l - V_0 = R^3$ (where V_l and V_0 are volumes of liquid state at T_{melt} and at 0K [1,2];
5) as the characteristic SAP (statistical atomic packing, having $K_{SAP} = 1/12(1+2+\ldots+12) = 6{,}5$ [2]): $4/3\pi[(\sigma/2)^3 - r_i^3]/K_{SAP} = R^3$ (where σ – the diameter of hard sphere in number decidings of Eq. Percus-Yevick and hence it follows: $\sigma/2 - r_i = R - R_c$);
6) at last, on the other hand the value R can be obtained from characteristics of conduction electrons, using wave lengths $\lambda_F/2\pi$: $2\pi R = 2\pi R_c + n(\lambda_F/2\pi)$, where $2\pi R_c = 2\pi r_i + \lambda_F/2\pi$ and where n =1,2. Here, this standing wave formation at $R_c - r_i$ as self-closing orbital appears considering $\lambda_F/2\pi$ (a cicle of radius r comprises a whole number of de-Broglie wave lengths: $2\pi r = n\lambda$).

Conclusion: Coordinated (pozition) spase between ions can be represented by the value R, using all variants: as an orbital, as spheres and surfase in the fluctuation approximation. There is a reson – the best use of a space principle admiting the band structure degrades at high temperature. Consequently, there are grounds to the analisis of the high temperature processes, which has been represented by number nearest neighbors (K) and shortest distances (r_a, r_i) by using the parametr R. Here the value R is a coordinate per r of maximum probability at the electron density distribution between r_a and R_c in pseudo-potencial approximation. From here we can conclude the following: main progress is determined of the value R. This parameter is formed by dimensial rations in the coordinate space, where main role is consisted in the definition of the ionic core as function $\lambda_F/2\pi$ (its formed of a standing wave by self -closed orbital: $2\pi R_c = 2\pi r_i + \lambda_F/2\pi$). Here we can answer on question: why are we considered all rations by means of R: it is determined by the energy bond; it is represented by all variants: as a continual, as contactial spheres and a surface of a sphere into exchengeal intereaction on $\lambda = h/mc$, using both line-space values (r_a, r_i, R_c) and physical values (λ, λ_F, Z).

III. Model of the Fluctuanion- Exchange of Heat Transfer in Solid and Liquid States

The identified melting characteristics of the model R, R_c, r_a and r_i associated with the electron density fluctuanion in the coordinate space can be applied to the heat transfer model within the following assumptions: The heat energy flux is associated with the electron – photon interaction and, respectively, with $\lambda = h/mc$; and there is empirical fact: $R_c - r_i = K\lambda$ ffiλ , as result of which the coordinate atomic space has an excess $+K\lambda$ or a deficit $- K\lambda$ in the interaction mode (photon + electron $\rightarrow\lambda$) between the emitter and absorber

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2. E. S. Filippov, Izv. Vyssh. Uchebn. Zaved. Chern. Metallurg., №5, 3-6 (2009).

photons [3]. The size of such an exchange – fluctuation cell must be limited to the value of λ_F :

$$R_c = r_i + K\lambda/Z \quad \text{and} \quad 2\pi R_c = 2\pi r_i + \lambda_F/2\pi \tag{1}$$

Hence: $$k_F = 1/2\pi[1/(K\lambda/Z\mathrm{ffi}n\lambda)] \tag{2}$$

Where at n = 0,1,2 there is a complete correspondence between $K\lambda/Z$ and k_F for the alkali and alkali-earth metals – Pb,Al,In….Justification of the Kλ value also follows from mentioned relation: $r_a^3 - r_i^3 \approx 4\pi R^2 (K\lambda)$, where the geometric volume in pseudo-potential approximation is coincided with the physical volume.

A mirror reflection – Kλ should correspond to the fluctuation level + Kλ (similar to the 2λ- model in the formation of the hcp- structure, where according to [1], the elongation by 2λ along the [a] – axis corresponds to the compression by 2λ along [c]- axis). Hence, in equilibrium – vibrational mode $(R - K\lambda) \leftrightarrow (R + K\lambda)$, we have the equality of fluxes (photon+ electron→λ) of the emitter and detector. This equality of fluxes we write by the following equations using the parametr R corresponding to the maximum level of the electron density fluctuation and determined as $1/2\ (R_c + r_a)$:

$$R + K\lambda = r_a - K\lambda/Z \tag{3}$$

$$R - K\lambda = R_c + K\lambda/Z \tag{4}$$

$$R_c = r_i + K\lambda/Z \tag{5}$$

Here, we take into account the number of valence electrons Z per photon absorption under the Kλ exchange interaction in the range from r_a to r_i.

Equations (3-5) are fulfilled accurate to (1,2) λ for 14 of metals under study: alkali and alkiline earth, Pb,Al,In,Cd and Zn.

For hexagonal structure (6+6) Cd and Zn, we find (Å/atom):

$$\text{For Cd} \qquad r_a^{max} - r_a^{min} = 0{,}152 \approx K\lambda;$$

$$\text{For Zn} \qquad r_a^{max} - r_a^{min} = 0{,}140 \approx K\lambda,$$

where $K\lambda = 6\lambda = 0{,}145$ and r_a^{max} and r_a^{min} are determined from the lattice parameters [a] and [c].

These data allow us to conclude that the value of Kλ corresponds to the maximum level of probability for the exchange interaction of conduction electrons of K atoms according to the scheme: electron + photon →λ . Combining Eqs.(3-5) and using the relation for determined R, we get:

$$R - R_c = K\lambda(1/Z + 1); \quad R - r_i = K\lambda(2/Z + 1); \quad r_a - R_c = 2K\lambda(1/Z + 1); \quad r_a - R = K\lambda(1 + 1/Z);\ R_c - r_i = K\lambda/Z. \tag{6}$$

These data confirm by the balans of line correlation between r_a, r_i and Kλ:

$$r_a = r_i + K\lambda(1/Z + 1) + K\lambda(1/Z + 1) + K\lambda\ \mathrm{ffi}\ n\lambda, \tag{7}$$

where n = 0,1,2 for 14 metals.

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Thus, it can be assumed that the atomic space between r_a and r_i is divided into the ready Kλ cells, whose number could not be less than the maximum possible fluctuations of the electron density in the coordinate space. This number must be discret for Kλ. Therefore, the heat energy transfer (the change in the intensity of the atomic-vibrational mode) in this space is represented in the model of two fluxes exchanging by fluctuations of the electron density in the ffiKλ mode.

Along the radius R corresponding to the maximum of the electron density, two fluctuation fluxes before R and after R are separated and regulated: heating – cooling (here, R is determined by the bond energy). Heating mode is the fluctuation flux from the atomic periphery r_a to the ionic core R_c. Cooling mode – vice versa. The equilibrium state is the compensation of the electron density fluctuations (ffiKλ).

IV. LIQUID STATE

A liquid aggregate state precisely fits into this model scheme of heat transfer, since according to [1,2] and Eqs. (3-6), we have:

$$\sigma/2 \;-\; r_i \;=\; r_a - R_c \;=\; 2\,K\lambda\,(1/Z + 1), \qquad (8)$$

where σ/2 is a semi-diameter of the hard sphere (Percus-Yevick).

For the statistical packing of atoms (SAP), where the average statistical number of nearest neigbors is $K_{SAP} = 6{,}5$ we obtain:

$$\sigma/2 \;=\; R \;+\; K_{SAP}\,\lambda \qquad (9)$$

Equation (9) is fulfilled with an accuracy of 1-3% for the 14 of the studied metals.

These data for the liquid state confirm the initially assumed model of heat transfer via the exchange fluctuation quantities Kλ between atomic spheres being in the thermal vibrational mode.

V. ANALISIS OF VOLUME CORRELATIONS AT THE EXCHANGE- FLUCTUATION MODEL OF HEAT TRANSFER

Here we have follows initial data: $K\lambda/Z = R_c - r_i$; $K\lambda(1/Z + 1) = \Delta R \equiv r_a - R \equiv R - R_c$ from Eqs.(3-7) and also for the coefficient packing $k_p = 0{,}68 - 0{,}74$:

$$k_p = V_a / (V_a + V_{void}) = r_a^3 / (r_a^3 + R^3) \text{ and also: } k_p = R^3 / (R^3 + R_c^3), \qquad (10)$$

Hence it follows:

$$R^3 / r_a^3 = R_c^3 / R^3 \qquad (11)$$

where values R and R_c are determined as: $2\pi R_c = 2\pi r_i + \lambda_F/2\pi$ and $2\pi R = 2\pi R_c + n\,\lambda_F/2\pi$ and n = 1,2. Here, we must mark that its new discovery correlation ($V_{voids} = R^3$) is connecting link between values R^3 and R_c^3 and by two principles: the uncertainty and the best use of the space- interatomic voids.

Main assuption is as follows: atomic sphere (r_a = const.) is transformed into a sphere, having variable radii but in definit limits: $r_a - r_i$ or $R - r_a$, $R - r_i$

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for which it is determined average value $(r_a - r_i)/2$, $(R - r_i)/2$ ets. However, atomic volume and the interatomic voids are remained by nonchangeable and also the coefficient packing (k_p). Besides, we have the ground for the approach to the problem of high temperature heat transfer into coordinate space, using the volume in different forms:

$$4\pi R^2 K\lambda = (r_a^3 - r_i^3) Z^{1/3} ; \qquad (12)$$

$$4\pi R_c^2 K\lambda = R^3 Z^{1/3} .$$

Equations (12) is fulfilled with an accuracy of $Z^{1/3}$ according to [1,2].

The value $K\lambda$ is the binding unit of two forms of atomic-ionic volumes in fact.

Here we must consider the balance of volumes. The physical volume should correspond to the geometrical volume. Here a radius can not has fixed orbital. It is average value in according to the principle uncertainty:

$\Delta V_{\text{emitter}} = 4\pi[1/2(r_a + r_i)]^2 K\lambda$; $\Delta V_{\text{absortion}} = 4\pi[1/2(r_a + r_i)]^2 K\lambda/Z$ and other relations.

$$\Delta V_{\text{emitter}} + \Delta V_{\text{absorption}} = r_a^3/k_p , \; r_a^{3=} , \; R^3 , \; R^3 /k_p$$

Thus we have the ground for the combination of double form the volume, assuming that the coordinate space from r_a to r_i is divided into the ready $K\lambda$ cells. Consequantly, proceeding from these states and Eq.(12), we may write four balance relations for heat transfer by using assumption of ready $K\lambda$- cells into the coordinate space:

1) $4\pi[1/2(r_a + r_i)]^2 K\lambda (1/Z + 1) = r_a^3 / k_p = r_a^3 + R^3$ (where $R^3 = V_{\text{voids}}$); (13)

2) $4\pi[1/2(r_a + r_i) - K\lambda/Z]^2 K\lambda/Z = R^3$; (14)

3) $4\pi[1/2(R + r_i)]^2 K\lambda (1/Z + 1) = r_a^3$; (15)

4) $4\pi r_i^2 K\lambda (1/Z + 1) = R^3/ k_p = R^3 + R_c^3$ (where $R_c^3 = V_{\text{voids}}$). (16)

Eqs.(13-16) are fulfilled with an accuracy of ffi4% in average or the 14 of metals.

Here geometrical volume is a continual, it is without $4/3\pi$, assuming that interatomic space can has different forms in according to the principle uncertanty.

Hence it follows that the mechanism heat transfer may to present by the substitution R^3 in Eq.(13) on the same value R^3 from Eq.(14). Hovewer the value R^3 in Eq.(13) is the volume of interatomic voids and the value R^3 in Eq.(14) is the characteristic of the transfer $K\lambda$ in the direct of opposite situation (- $K\lambda$) instead of (+ $K\lambda$) in Eq.(13). Besaides the value R^3 in Eq.(13) may substitude by the value R^3 from Eq.(16), using Eq.(11). Here we have the substitution R^3/r_a^3 into R_c^3/ R^3. This chain of substitutions is allowed by the transference the volume of voids from r_i to r_a and vs and by heat transference of cells ($K\lambda$) at variable radii which may form from r_a to r_i . Here the principle uncertainty is formed by variable radii: $\Delta r = r_a - r_i$ and other and the principle of the best use of a space is determined by the coefficient of the packing (k_p).

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These mutual substitutions R^3 and r_a^3 in Eqs.(13-16) are the model of equilibrium process between the heating and the cooling. Hence, it may assume that heat transference of cells (ffiKλ) is connected with the transference of volumes voids from r_a to r_i and ves.

VI. CONCLUSION

The maximum possible value of the electron density fluctuation is theoretically determined as ffi Kλ and the heat transfer process is modeled as a λ exchange between the detector fluctuation and the fluctuation of the emitter of the photon – electron interaction. The main conclusion is: the atomic coordinate space corresponds to the maximum possible fluctuations of electron density for the exchange of + Kλ (emitted) by -Kλ (absorbed) in the photon – electron interaction. This space is discrete for placing Kλ or it quatized by Kλ. On the whole, the analysis of high- temperature processeses in coordinate space using the linear (R_c, R…) and volumetric (R^3, $4\pi R^2$…) characteristics reveals the processes of heat transference, the structure formation and phase transitions, which still did not have a simple explanation [1,2]

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On Topological Properties of the Line Graphs of Subdivision Graphs of Certain Nanostructures - II

By Sunilkumar M. Hosamani
Rani Channamma University

Abstract- In this note, we give expressions for the first(second) Zagreb coindex, second Zagreb index(coindex), third Zagreb index and first hyper-Zagreb index of the line graphs of subdivision graphs of 2D-lattice, nanotube and nanotorus of $TUC_4C_8[p, q]$ and obtain upper bounds for Wiener index and degree-distance index of these graphs. This note continue the program of computing certain topological indices of the line graphs of subdivision graphs of 2D-lattice, nanotube and nanotorus of $TUC_4C_8[p, q]$ [25] [M. F. Nadeem, S. Zafar, Z. Zahid, *On topological properties of the line graphs of subdivision graphs of certain nanostructures*, *Appl. Math. Comput.* 273(2016) 125{130].

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ONTOPOLOGICALPROPERTIESOFTHELINEGRAPHSOFSUBDIVISIONGRAPHSOFCERTAINNANOSTRUCTURESII

Strictly as per the compliance and regulations of :

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I. INTRODUCTION

Let G be a simple graph. The order of a graph is $|V(G)|$, its number of vertices denoted by n. The size of a graph is $|E(G)|$, its number of edges denoted by m. The degree of a vertex v, denoted by $d_G(v)$. The complement of a graph G, denoted by $\overline{G}$, is a simple graph on the same set of vertices $V(G)$ in which two vertices u and v are connected by an edge uv, if and only if they are not adjacent in G. Obviously, $E(G) \cup E(\overline{G}) = E(K_n)$ where K_n is complete graph of order n, and $|E(\overline{G})| = \frac{n(n-1)}{2} - m$. The subdivision graph $S(G)$ is the graph attained from G by replacing each of its edges by a path of length 2. The line graph $L(G)$ of a graph is the graph derived from G in such a way that the edges in G are replaced by vertices in $L(G)$ and two vertices in $L(G)$ are connected whenever the corresponding edges in G are adjacent [19].

The Zagreb indices were first introduced by Gutman [17], they are important molecular descriptors and have been closely correlated with many chemical properties [29] and defined as:

$$M_1(G) = \sum_{u \in V(G)} d_G(u)^2 \qquad \text{and} \tag{1}$$

$$M_2(G) = \sum_{uv \in E(G)} d_G(u)\, d_G(v), \tag{2}$$

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In fact, one can rewrite the first Zagreb index as

$$M_1(G) = \sum_{uv \in E(G)} \big[d_G(u) + d_G(v)\big].$$

Noticing that contribution of nonadjacent vertex pairs should be taken into account when computing the weighted Wiener polynomials of certain composite graphs (see [6]) defined first Zagreb coindex and second Zagreb coindex as

$$\overline{M_1}(G) = \sum_{uv \notin E(G)} \big[d_G(u) + d_G(v)\big] \qquad \text{and}$$

$$\overline{M_2}(G) = \sum_{uv \notin E(G)} d_G(u)\, d_G(v), \tag{4}$$

respectively.

The third Zagreb index was first introduced by Fath-Tabar [13]. This index is defined as follows:

$$M_3(G) = \sum_{uv \in E(G)} |d_G(u) - d_G(v)| \tag{5}$$

The hyper-Zagreb index was first introduced in [28]. This index is defined as follows:

$$HM(G) = \sum_{uv \in E(G)} (d_G(u) + d_G(v))^2 \tag{6}$$

In fact the idea of topological index appears from work done by Wiener [31] in 1947 although he was working on boiling point of paraffin. He called this index as Wiener index then theory of topological index started. The Wiener index of graph G is defined as

$$W(G) = \frac{1}{2}\sum_{(u,v)} d(u,v) \tag{7}$$

where (u, v) is any ordered pair of vertices in G and $d(u, v)$ is $u - v$ geodesic.

The degree distance index for graphs developed by Dobrynin and Kochetova [12] and Gutman [14] as a weighted version of the Wiener index. The degree distance of G, denoted by $DD(G)$, is defined as follows

$$DD(G) = \sum_{\{u,v\} \subseteq V(G)} d(u,v)[d_G(u) + d_G(v)]. \tag{8}$$

For more details on the topological indices we refer to the articles [2–5,17,20,21,24,30,32].

Ref

6. T. Došlić, Vertex-weighted Wiener polynomials for composite graphs, Ars Mathematica Contemporanea 1 (2008) 66–80.

II. NANOSTRUCTURES

In a series of papers, Diudea and co-authors studied the structure and topological indices of some chemical graphs related to some nanostructures [1,7–11,23]. Rajani et. al derived the expressions for the Shultz indices of the subdivision graphs of the tadpole graph, wheel, helm and ladder graphs [27]. The expressions for the line graphs of subdivision graphs of the tadpole, wheel and ladder graphs can be seen in [26] and [14]. Recently Nadeem et. al [25] obtained expressions for certain topological indices for the line graph of subdivision graphs 2D-lattice, nanotube and nanotorus of $TUC_4C_8[p,q]$, where p and q denote the number of squares in a row and the number of rows of squares, respectively in 2D-lattice, nanotube and nanotorus.

Ref

27. P.S.Ranjini, V.Lokesha, M.A.Rajan, On the Shultz index of the subdivision graphs, Adv. Stud. Contemp. Math. 21(3)(2011) 279-290.

In Fig. 1, 2D-lattice, nanotube and nanotorus of $TUC_4C_8[p,q]$ are depicted. The order and size of 2D-lattice, nanotube and nanotorus of $TUC_4C_8[p,q]$ are given in the Table 1.

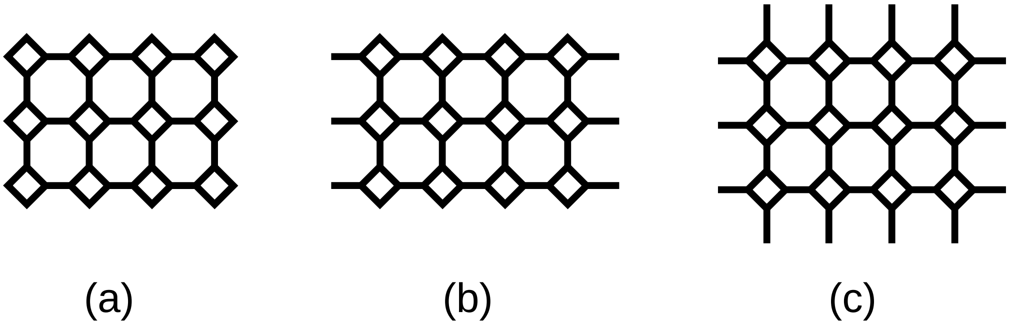


Figure 1: (a) 2D−lattice of $TUC_4C_8[4,3]$; (b) $TUC_4C_8[4,3]$ nanotube; (c) $TUC_4C_8[4,3]$ nanotorus

Table 1: Order and size

Graph	Order	Size
$2D$ − lattice of $TUC_4C_8[p,q]$	$4pq$	$6pq-p-q$
$TUC_4C_8[p,q]$ nanotube	$4pq$	$6pq-p$
$TUC_4C_8[p,q]$ nanotorus	$4pq$	$6pq$

The goal of this paper is to continue this program to compute the first (second) Zagreb coindex, second Zagreb index (coindex), third Zagreb index and first hyper-Zagreb index of the line graphs of subdivision graphs of 2D-lattice, nanotube and nanotorus of $TUC_4C_8[p,q]$ and to obtain upper bounds for Wiener index and degree-distance index of these graphs.

III. MAIN RESULTS

We begin with the following straightforward, previously known, auxiliary results.

Lemma 1. [18] *For any graph G of order n and size m, the subdivision graph $S(G)$ of G is a graph of order $n+m$ and size $2m$.*

Lemma 2. [18] *Let G be a graph of order n and size m, then the line graph $L(G)$ of G is a graph of order m and size $\frac{1}{2}M_1(G)-m$.*

Theorem 1. [15] *Let G be a graph of order n and size m. Then*

$$M_1(\overline{G}) = M_1(G) + n(n-1)^2 - 4m(n-1) \tag{9}$$

$$\overline{M_1}(G) = 2m(n-1) - M_1(G) \tag{10}$$

$$\overline{M_1}(\overline{G}) = 2m(n-1) - M_1(G) \tag{11}$$

Theorem 2. [16] *Let G be a graph of order n and size m. Then*

$$M_2(\overline{G}) = \frac{1}{2}n(n-1)^3 - 3m(n-1)^2 + 2m^2 + \frac{2n-3}{2}M_1(G) - M_2(G) \tag{12}$$

$$\overline{M_2}(G) = 2m^2 - \frac{1}{2}M_1(G) - M_2(G) \tag{13}$$

$$\overline{M_2}(\overline{G}) = m(n-1)^2 - (n-1)M_1(G) + M_2(G) \tag{14}$$

Theorem 3. [22] *Let G be a graph of order n and size m. Then*

$$\overline{M_1}(G) \geq 2W(G) - 2M_1(G) + 6m(n-1) - n^3 + n^2 \tag{15}$$

Theorem 4. [22] *Let G be a nontrivial graph of diameter $d \geq 2$. Then*

$$\overline{M_1}(G) \leq \frac{DD(G) - M_1(G)}{2} \tag{16}$$

with equality if and only if $d = 2$.

IV. TOPOLOGICAL INDICES OF LINE GRAPH OF THE SUBDIVISION GRAPH OF 2D-LATTICE OF $TUC_4C_8[P, Q]$

In Fig. 2 (b) the line graph of the subdivision graph of 2D-lattice of $TUC_4C_8[p,q]$ is depicted.

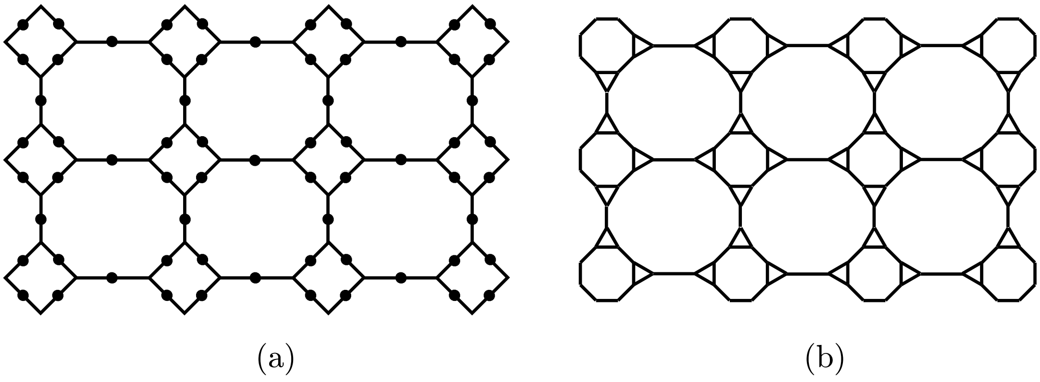


Figure 2: (a) Subdivision of 2*D*-lattice of $TUC_4C_8[4;3]$; (b) Line graph of the subdivision graph of 2*D*-lattice of $TUC_4C_8[4;3]$.

Theorem 5. *Let G be the line graph of the subdivision graph of 2D-lattice of $TUC_4C_8[p,q]$. Then*

1. $M_3(G) = 4(p+q-2);$
2. $HM(G) = 648pq - 264(p+q) + 8;$
3. $\overline{M_1}(G) = \overline{M_1}(\overline{G}) = 2[18pq-5p-5q][12pq-2p-2q-1] + 38(p+q) - 108pq;$
4. $M_1(\overline{G}) = 2[6pq-p-q][4(6pq-p-q)^2 - 4(6pq-p-q)+1] - 2[36pq-10(p+q)][6pq-p-q-1] + 108pq - 38(p+q);$
5. $M_2(G) = 162pq - 67(p+q) + 4;$
6. $M_2(\overline{G}) = (6pq-p-q)[12pq-2p-2q-1]^3 - 3(18pq-5p-5q)[12pq-2p-2q-1]^2 + (24pq-4p-4q-3)[8(p+q)+27(2pq-p-q)] + 2[18pq-5p-5q]^2 - (162pq-67(p+q)+4)^2;$
7. $\overline{M_2}(G) = 2[18pq-5p-5q]^2 + 86(p+q) - 216pq - 4;$
8. $\overline{M_2}(\overline{G}) = (18pq-5p-5q)[12pq-2p-2q-1]^2 - (12pq-2p-2q-1)[108pq-38(p+q)] + 162pq - 67(p+q) + 4;$
9. $W(G) \leq (18pq-5p-5q)(12pq-2p-2q-1) - 3(18pq-5p-5q)(12pq-2p-2q-1) + 4(12pq-2p-2q-1)[6pq-p-q]^2 + 54pq - 19(p+q);$
10. $DD(G) \leq 4(18pq-5p-5q)(12pq-2p-2q-1) - 32pq + 114(p+q).$

Proof. The 2D-lattice of $TUC_4C_8[p,q]$ is a graph of order $4pq$ and size $6pq-p-q$. Then by Lemma 1, the subdivision graph of 2D-lattice of $TUC_4C_8[p,q]$ have order $10pq-p-q$ and size $2[6pq-p-q]$ (see Fig. 2 (a)). Therefore by Lemma 2, G will have order $2[6pq-p-q]$ and size $18pq-5p-5q$. Further notice that in a graph G there are $4(p+q)$ vertices are of degree 2 and remaining all the vertices of degree 3. Hence we can partition the edge set of a graph G as shown in Table 2.

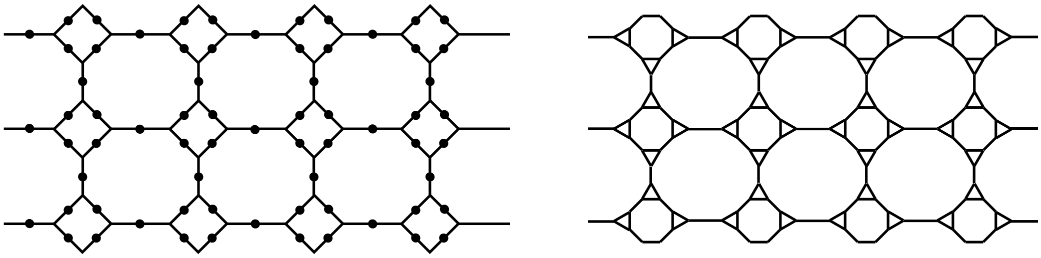
Table 2: The edge partition of the graph G

(d_u, d_v) where $uv \in E(G)$	$(2,2)$	$(2,3)$	$(3,3)$
Number of edges	$2p+2q+4$	$4p+4q-8$	$18pq-11p-11q+4$

We apply Formulas (1)-(8) and by employing the Equations (9)-(16) we can obtain the required results. □

V. TOPOLOGICAL INDICES OF LINE GRAPH OF THE SUBDIVISION GRAPH OF $TUC_4C_8[P,Q]$ NANOTUBE

In Fig. 3 (b), the line graph of the subdivision graph of $TUC_4C_8[p,q]$ nanotube is depicted.



(a) (b)

Figure 3: (a) Subdivision of $TUC_4C_8[4, 3]$ nanotube; (b) Line graph of subdivision of $TUC_4C_8[4, 3]$ nanotube

Theorem 6. *Let G be the line graph of the subdivision graph of $TUC_4C_8[p, q]$ of nanotube. Then*

1. $M_3(G) = 4p$;
2. $HM(G) = 648pq - 264p$;
3. $\overline{M_1}(G) = \overline{M_1}(\overline{G}) = 2[18pq - 5p][12pq - 2p - 1] + 38p - 108pq$;
4. $M_1(\overline{G}) = 2(6pq - p - q)[12pq - 2p]^2 - 4(18pq - 5p)(12pq - 2p - 1) + 120pq - 38p$;
5. $M_2(G) = 162pq - 67p$;
6. $M_2(\overline{G}) = (6pq - p)[12pq - 2p - 1]^3 - 3(18pq - 5p)[12pq - 2p - 1]^2 + (24pq - 4p - 3)[57pq - 19p] + 2[18pq - 5p]^2 - 162pq + 67p$;
7. $\overline{M_2}(G) = 2[18pq - 5p]^2 + 86p - 216pq$;
8. $\overline{M_2}(\overline{G}) = (18pq - 5p)[12pq - 2p - 1]^2 - (12pq - 2p - 1)[108pq - 38p)] + 162pq - 67p$;
9. $W(G) \leq (18pq - 5p)(12pq - 2p - 1) - 3(18pq - 5p)(12pq - 2p - 1) + (12pq - 2p)^2(12pq - 2p - 1) + 54pq - 19p$;
10. $DD(G) \leq 4(18pq - 5p)(12pq - 2p - 1) - 108pq + 6p$.

Proof. The $TUC_4C_8[p, q]$ of nanotube is a graph of order $4pq$ and size $6pq - p$. Then by Lemma 1, the subdivision graph of $TUC_4C_8[p, q]$ of nanotube of order $10pq - p$ and size $12pq - 2p$ (see Fig. 3 (a)). Therefore by Lemma 2, G will have order $12pq - 2p$ and size $18pq - 5p$. Further notice that in a graph G there are $4p$ vertices are of degree 2 and remaining all the vertices of degree 3. Hence we can partition the edge set of a graph G as shown in Table 3.

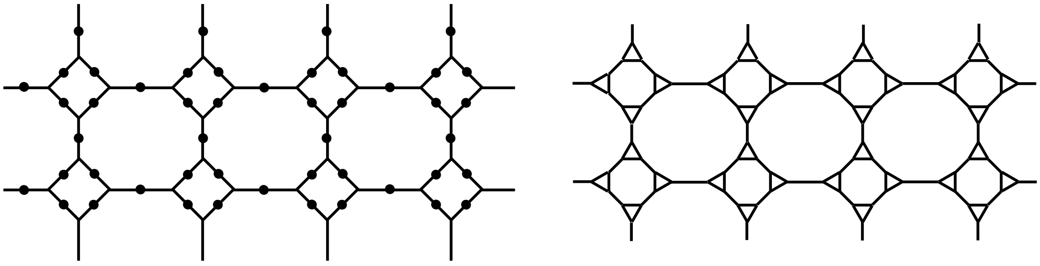
Table 3: The edge partition of the graph G

(d_u, d_v) where $uv \in E(G)$	$(2,2)$	$(2,3)$	$(3,3)$
Number of edges	$2p$	$4p$	$18pq - 11p$

We apply Formulas (1)-(8) and by employing the Equations (9)-(16) we can obtain the required results. □

VI. Topological Indices of Line Graph of the Subdivision Graph of TUC_4C_8 [P, Q] Nanotorus

In Fig. 4 (b) the line graph of the subdivision graph of $TUC_4C_8[p,q]$ nanotorus is depicted.



(a) (b)

Figure 4: (a) Subdivision of TUC_4C_8[4, 2] nanotorus; (b) Line graph of Subdivision of TUC_4C_8[4, 2] nanotorus

Theorem 7. *Let G be the line graph of the subdivision graph of $TUC_4C_8[p,q]$ nanotorus. Then*

1. $M_3(G) = 0$;
2. $HM(G) = 648pq$;
3. $\overline{M_1}(G) = \overline{M_1}(\overline{G}) = 432p^2q^2 - 144pq$;
4. $M_1(\overline{G}) = 12pq(12pq-1)^2 - 72pq(12pq-1) + 108pq$;
5. $M_2(G) = 162pq$;
6. $M_2(\overline{G}) = 6pq[12pq-1]^3 - 64pq[12pq-1]^2 + 54pq(24pq-3) + 324p^2q^2 - 162pq$;
7. $\overline{M_2}(G) = 648p^2q^2 - 216pq$;
8. $\overline{M_2}(\overline{G}) = (18pq(12pq-1)^2 - (12pq-1)108pq + 162pq$;
9. $W(G) \leq 6p^2q^2[288pq+12] - 54pq(12pq-1) + 36pq$;
10. $DD(G) \leq 864p^2q^2 - 396pq$.

Proof. The $TUC_4C_8[p,q]$ of nanotorus is a graph of order $4pq$ and size $6pq$. Then by Lemma 1, the subdivision graph of $TUC_4C_8[p,q]$ of nanotorus have order $10pq$ and size $12pq$ (see Fig. 4 (a)). Therefore by Lemma 2, G will have $12pq$ vertices and $18pq$ edges. Further note that the degree of each vertex is 3 in G. Hence we can partition the edge set of a graph G as shown in Table 4.

Table 4: The edge partition of the graph G

(d_u, d_v) where $uv \in E(G)$	$(3,3)$
Number of edges	$18pq$

We apply Formulas (1)-(8) and by employing the Equations (9)-(16) we can obtain the required results. □

Conclusion: In this paper, we continue the study certain degree based topological indices for the line graph of subdivision graph of 2D-lattice, nanotube and nanotorus of $TUC_4C_8[p,q]$ and obtained upper bounds for Wiener index and degree distance index of 2D-lattice, nanotube and nanotorus of $TUC_4C_8[p,q]$ respectively.

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Variations for 9

By Dr. Zoltan Istvan Szabo

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Introduction- If we multiply the number n=3529411764705882 by 2, then the result m=2n=7058823529411764 will contain the same digits. Moreover, we can cut n into two section $n=\underline{\underline{bd}}$ with $\underline{b}=\underline{3529411764}$ and $\underline{\underline{d}}=\underline{\underline{705882}}$ such that m contains the same sections in reverse order: $m=\underline{\underline{db}}$.

GJSFR-F Classification: *NLMC Code: 11L20*

Variations for 9

Dr. Zoltan Istvan Szabo

I. INTRODUCTION

If we multiply the number n=3529411764705882 by 2, then the result m=2n=7058823529411764 will contain the same digits. Moreover, we can cut n into two section n=$\underline{\underline{bd}}$ with $\underline{b}$=$\underline{3529411764}$ and $\underline{\underline{d}}$=$\underline{\underline{705882}}$ such that m contains the same sections in reverse order: m=$\underline{\underline{db}}$.

Furthermore, if we cut n into two sections of the same length, then the sum of these sections is

$$\begin{array}{r} 35294117 \\ + \; 64705882 \\ \hline 99999999 \end{array}$$

The same applies to m:

$$\begin{array}{r} 70588235 \\ + \; 29411764 \\ \hline 99999999 \end{array}$$

Interestingly, n and m contain each decimal digit twice, expect the multiples of 3; 0, 3, 6 and 9 appear only once.

Surprisingly, one can continue to multiply

N=m=7058823529411764 by 2, and a similar rule can be observed.
N can be divided into three sections N=$\underline{\underline{B}}9\underline{\underline{D}}$ with
B=$\underline{705882352}$ and $\underline{\underline{D}}$=$\underline{\underline{411764}}$, while the number
M=2N=14117647058823528 contains $\underline{\underline{B}}$ and $\underline{\underline{D}}$ in reverse order.

More precisely, M=2N=1$\underline{\underline{DB}}$8, where the sum of the first and the last numbers gives the "middle" number of N (1+8=9).

On the other hand, M can be divided into two parts the sum of which is

$$\begin{array}{r} 141176470 \\ + \; 58823528 \\ \hline 199999998 \end{array}$$

The sum of the digits which are different from 9 is 1+8=9.

Interestingly, some regularity can be noticed if we continue to multiply n=M=14117647058823528 by 2. N can be divided into five sections: n=1$\underline{b}$8$\underline{\underline{d}}$8, and its double as well:

m=2n=2$\underline{\underline{82352}}9\underline{411764705}$6=2$\underline{\underline{d}}9\underline{b}$6, where $\underline{b}$=$\underline{411764705}$ and

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$\underline{\underline{d}}=\underline{\underline{82352}}$. The "middle" number 8 in n is the sum of the first and the last numbers of m(2+6=8), and the "middle" number 9 in m is the sum of the first and the last numbers of n(1+8=9).

Furthermore, we can cut m into two sections the sum of which is

$$\begin{array}{r} 282352941 \\ +\ \ 17647056 \\ \hline 299999997 \end{array}$$

The sum of the digits which are different from 9 is 2+7=9.

We can continue doubling the number m just obtained and, very much surprisingly, the interesting feature of the previous numbers can be observed in many more steps. We can describe these interesting properties of those numbers as follows.

In each step, we multiply the number n by 2 to obtain m=2n. the numbers n and m can be divided into five sections as follows:

$$n=a\ \underline{b}\ e\ \underline{\underline{d}}\ c \qquad m=p\ \underline{\underline{d}}\ q\ \underline{b}\ r$$

such that a+c=q and p+r=e

(i.e. the middle section of m is the sum of the first and the last sections of n, and conversely). Thus, the numbers n and m(=2n) can be written in the form

$$n=a\ \underline{b}(p+r)\underline{\underline{d}}\ c \qquad m=p\ \underline{\underline{d}}(a+c)\underline{b}\ r.$$

On the other hand, n can be divided into two parts the sum of which has the form

K999….99L

with K+L=99…9. The same applies to m.

This formal description applies to our first few steps too, provided empty sections are allowed.

In the first step, when n=3529411764705882, we had $\underline{b}=\underline{3529411764}$ $\underline{\underline{d}}=\underline{\underline{705882}}$
a=e=c=p=q=r=K=L=empty section

In the second step, when n=7058823529411764, we had $\underline{b}=\underline{705882352}$ $\underline{\underline{d}}=\underline{\underline{411764}}$ e=9
p=1 r=8 a=c=q= K=L=empty

In the third step, when n=14117647058823528, we have $\underline{b}=\underline{411764705}$ $\underline{\underline{d}}=\underline{\underline{82352}}$ a=1 e=8
c=8 p=2 q=9 r=6 K=1 L=8 (since

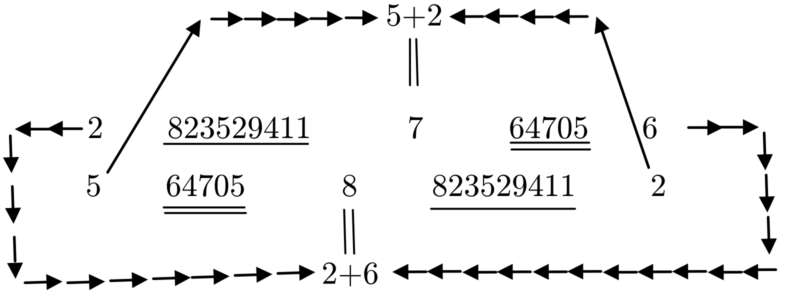
$$\begin{array}{r} 141176470 \\ +\ \ 58823528 \\ \hline 199999998 \end{array}).$$

Let us shorten the description of some further steps as follows. In each step, we will write down the values of the following sections (in this order):

(n=) a b e d c two sections of n
(m=) p d q b r the sum of which is
"almost" 99…99

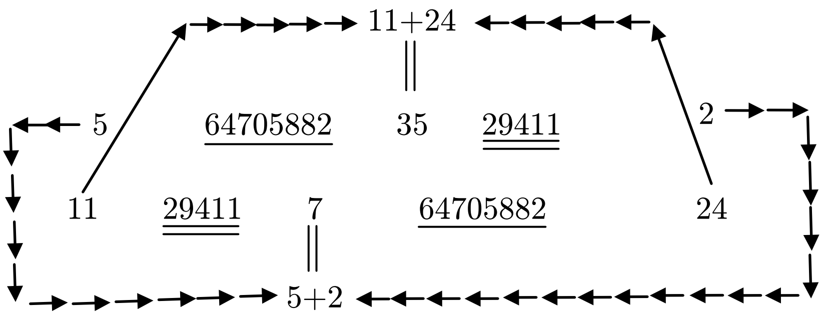
(In addition, in steps 4 and 5, the connections between sections are denoted by arrows.)

Step 4.



$$\begin{array}{r} 282352941 \\ +17647056 \\ \hline 299999997 \end{array}$$
(2+9=9)

Step 5.



$$\begin{array}{r} 564705882 \\ +\ 35294112 \\ \hline 599999994 \end{array}$$

(5+4=9)

Step 6.

$$\begin{array}{ccccc} 11 & \underline{29411764} & 70 & \underline{\underline{5882}} & 24 \\ 22 & \underline{\underline{5882}} & 35 & \underline{29411764} & 48 \end{array}$$

$$\begin{array}{r} 1129411764 \\ +\ 70588224 \\ \hline 1199999988 \end{array}$$

(11+88=99)

Step 7.

$$\begin{array}{ccccc} 22 & \underline{5882352} & 941 & \underline{\underline{1764}} & 48 \\ 45 & \underline{\underline{1764}} & 70 & \underline{5882352} & 896 \end{array}$$

$$\begin{array}{r} 2258823529 \\ +\ 41176448 \\ \hline 2299999977 \end{array}$$

(22+77=99)

Step 8.

$$\begin{array}{ccccc} 45 & \underline{1764705} & 882 & \underline{\underline{352}} & 896 \\ 90 & \underline{\underline{352}} & 941 & \underline{1764705} & 792 \end{array}$$

$$\begin{array}{r} 4517647058 \\ +\ 82352896 \\ \hline 4599999954 \end{array}$$

(45+54=99)

Step 9.

$$\begin{array}{ccccc} 90 & \underline{3529411} & 764 & \underline{\underline{705}} & 792 \\ 180 & \underline{\underline{705}} & 882 & \underline{3529411} & 584 \end{array}$$

$$\begin{array}{r} 9035294117 \\ +\ 64705792 \\ \hline 9099999909 \end{array}$$

(90+09=99)

Step 10.

$$\begin{array}{ccccc} 180 & \underline{7058823} & 529 & \underline{\underline{411}} & 584 \\ 361 & \underline{\underline{411}} & 764 & \underline{7058823} & 168 \end{array}$$

$$\begin{array}{r} 18070588235 \\ +\ 29411584 \\ \hline 18099999819 \end{array}$$

(180+819=999)

Step 11.

$$\begin{array}{ccccc} 361 & \underline{411764} & 7058 & \underline{\underline{823}} & 168 \\ 722 & \underline{\underline{823}} & 529 & \underline{411764} & 6336 \end{array}$$

$$\begin{array}{r} 36141176470 \\ +\ 58823168 \\ \hline 36199999638 \end{array}$$

(361+638=999)

Step 12.

$$\begin{array}{ccccc} 722 & \underline{823529} & 4117 & \underline{\underline{64}} & 6336 \\ 1445 & \underline{\underline{64}} & 7058 & \underline{823529} & 2672 \end{array}$$

$$\begin{array}{r} 72282352941 \\ +\ 17646336 \\ \hline 72299999277 \end{array}$$

(722+277=999)

Step 13.

$$\begin{array}{ccccc} 1445 & \underline{647058} & 8235 & \underline{\underline{29}} & 2672 \\ 2891 & \underline{\underline{29}} & 4117 & \underline{647058} & 5344 \end{array}$$

$$\begin{array}{r} 144564705882 \\ +\ 35292672 \\ \hline 144599998554 \end{array}$$

(1445+8554=9999)

Step 14.

$$\begin{array}{ccccc} 2891 & \underline{294117} & 6470 & \underline{\underline{58}} & 5344 \\ 5782 & \underline{\underline{58}} & 8235 & \underline{294117} & 0688 \end{array}$$

$$\begin{array}{r} 289129411764 \\ +\ \underline{70585344} \\ 289199997108 \end{array}$$

(2891+7108=9999)

Step 15.

$$\begin{array}{ccccc} 5782 & \underline{58823} & 52941 & \underline{\underline{17}} & 0688 \\ 11565 & \underline{\underline{17}} & 6470 & \underline{58823} & 41376 \end{array}$$

$$\begin{array}{r} 578258823529 \\ +\ \underline{41170688} \\ 578299994217 \end{array}$$

(5782+4217=9999)

Step 16.

$$\begin{array}{ccccc} 11565 & \underline{1764} & 705882 & \underline{\underline{3}} & 41376 \\ 23130 & \underline{\underline{3}} & 52941 & \underline{1764} & 682752 \end{array}$$

$$\begin{array}{r} 1156517647058 \\ +\ \underline{82341376} \\ 1156599988434 \end{array}$$

(11565+88434=99999)

Step 17. (From this step, section d becomes empty.)

$$\begin{array}{ccccc} 23130 & \underline{3529} & 411764 & \underline{\underline{\mathrm{empty}}} & 682752 \\ 46260 & \underline{\underline{\mathrm{empty}}} & 705882 & \underline{3529} & 365504 \end{array}$$

$$\begin{array}{r} 2313035294117 \\ +\ \underline{64682752} \\ 2313099976869 \end{array}$$

(2130+76869=99999)

Step 18.

$$\begin{array}{ccccc} 46260 & \underline{7058} & 823529 & \underline{\underline{\mathrm{empty}}} & 365504 \\ 92521 & \underline{\underline{\mathrm{empty}}} & 411764 & \underline{7058} & 731008 \end{array}$$

$$\begin{array}{r} 4626070588235 \\ +\ \underline{29365504} \\ 4626099953739 \end{array}$$

(46260+53739=99999)

Step 19.

$$\begin{array}{ccccc} 92521 & \underline{4117} & 647058 & \underline{\underline{\mathrm{empty}}} & 731008 \\ 185042 & \underline{\underline{\mathrm{empty}}} & 823529 & \underline{4117} & 462016 \end{array}$$

$$\begin{array}{r} 9252141176470 \\ +\ \underline{58731008} \\ 9252199907478 \end{array}$$

(92521+07478=99999)

Step 20.

$$\begin{array}{ccccc} 185042 & \underline{823} & 5294117 & \underline{\underline{\mathrm{empty}}} & 462016 \\ 370085 & \underline{\underline{\mathrm{empty}}} & 647058 & \underline{823} & 4924032 \end{array}$$

$$\begin{array}{r} 18504282352941 \\ +\ \underline{17462016} \\ 18504299814957 \end{array}$$

(185042+814957=999999)

Step 21. 37008564705882349240 32 x 2=

=74017129411764698480 64

and the amazing rule breaks up at this step.

But we can go to the opposite direction: If we divide our original number 3529411764705882 by 2, then we get 1764705882352941. Interestingly, the well-known regularity occurs in this, say,

Step 0.

$$\begin{array}{cc} \underline{1764705882} & \underline{\underline{352941}} \\ \underline{\underline{352941}} & \underline{1764705882} \end{array}$$

$$\begin{array}{r} 17647058 \\ +\ \underline{82352941} \\ 99999999 \end{array}$$

Unexpectedly, if we continue dividing by 2 and leave the decimal points, then we obtain the series of numbers with remarkable regularity:

$$\text{Step -1.}\quad \begin{array}{ll} \underline{882352941} & \underline{\underline{1764705}} \\ \underline{\underline{1764705}} & \underline{882352941} \end{array} \qquad \begin{array}{r} 88235294 \\ +\ \underline{11764705} \\ 99999999 \end{array}$$

$$\text{Step 2.}\quad \begin{array}{llll} 4 & \underline{411764705} & \underline{\underline{882352}} & 5 \\ \underline{\underline{882352}} & 9 & \underline{411764705} & \end{array} \qquad \begin{array}{r} 441176470 \\ +\ \underline{58823525} \\ 499999995 \\ (4+5=9) \end{array}$$

$$\text{Step 3.}\quad \begin{array}{lllll} 22 & \underline{05882352} & 9 & \underline{\underline{41176}} & 25 \\ 4 & \underline{\underline{41176}} & 47 & \underline{05882352} & 5 \end{array} \qquad \begin{array}{r} 2205882352 \\ +\ \underline{94117625} \\ 2299999977 \\ (22+77=99) \end{array}$$

(In Steps 0 and -1, a=e=c=p=q=r=K=L=empty section
[like in step 1];
in step -2, e=p=r=empty
[in some sense, this is a mirror image of step 2];
In step -3, there is no empty section.)

In the next step, # -4, the rule breaks up.
Let us mention here some other interesting facts.
In the original number 3529411764705882,

the number of digits is 16 = 2**x**2**x**2**x**2, and

the sum of digits is 72 = (2+2+2+2) **x** 9.

If we now consider the "positive" step only, then the regularity practically breaks up, when the section $\underline{\underline{\text{d}}}$ disappears, i.e. after 16 steps!

If we started the entire calculation at step -3 with the smallest number of our series 220588235294117.625, then the discovered rule would stop after 24=2**x**2**x**2**x**3 steps.

Actually, the prime factors of our original number are 2 x 3^3 x 11 x 73 x 101 x 137 x 5882353. Surprisingly, the last prime factor almost occurs in the original number $\underline{352}9411764705\underline{5882}$. In fact, the last four positions and, when rounded to 353, the first three positions contain that factor.

Interestingly, each odd prime factor and its odd neighbour

3 , 5
11 , 13
71 , 73
101 , 103
137 , 139
5882351 , 5882353 from TWIN PRIMES.

If we multiply the number n=3529411764705882 by 2, then the result m=2n=7058823529411764 will contain the same digits. Moreover, we can cut n into two sections n=$\underline{\underline{\text{bd}}}$ with $\underline{\text{b}}$=$\underline{3529411764}$ and $\underline{\underline{\text{d}}}$=$\underline{\underline{705882}}$ such that m contains the same sections in reverse order: m=$\underline{\underline{\text{db}}}$.

Furthermore, if we cut n into two sections of the same length, then the sum of these sections is

$$\begin{array}{r} 35294117 \\ +\;64705882 \\ \hline 99999999 \end{array}$$

The same applies to m:

$$\begin{array}{r} 70588235 \\ +\;29411764 \\ \hline 99999999 \end{array}$$

Interestingly, n and m contain each decimal digit twice, except the multiples of 3 ; 0, 3, 6 and 9 appear only once.

Surprisingly, one can continue to multiply
N=m=7058823529411764 by 2, and a similar rule can be observed.
N can be divided into three sections N=$\underline{\text{B}}$9$\underline{\underline{\text{D}}}$ with
$\underline{\text{B}}$=$\underline{705882352}$ and $\underline{\underline{\text{D}}}$=$\underline{\underline{411764}}$, while the number
M=2N=14117647058823528 contain $\underline{\text{B}}$ and $\underline{\text{D}}$ in reverse order.

More precisely, M=2N=1$\underline{\underline{\text{DB}}}$8, where the sum of the first and the last numbers gives the “middle” number of N (1+8=9).
On the other hand, M can be divided into two parts the sum of which is

$$\begin{array}{r} 141176470 \\ +\;\;58823528 \\ \hline 199999998 \end{array}$$

The sum of the digits which are different from 9 is 1+8=9.

Interestingly, some regularity can be noticed if we continue to multiply n=M=14117647058823528 by 2. n can be divided into five sections: n=1$\underline{\text{b}}$8$\underline{\underline{\text{d}}}$8, and its double as well.

m=2n=2$\underline{\underline{82352}}9\underline{411764705}$6=2$\underline{\underline{\text{d}}}9\underline{\text{b}}$6, where $\underline{\text{b}}$=$\underline{411764705}$ and $\underline{\underline{\text{d}}}$=$\underline{\underline{82352}}$. The “middle” number 8 in n is the sum of the first and last numbers of m (2 + 6 = 8), and the “middle” number 9 in m is the sum of the first and the last numbers of n(1+8=9).

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A New Subclass of Harmonic Univalent Functions Defined by q-Calculus

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Abstract- In this paper we study a new subclass of harmonic univalent functions defined by q-calculus coefficient inequalities, distortion, bounds, extreme point, convolution, convex combination are determined for this class. Finally we discuss a class preserving integral operator and q- Jackson's type integral for this class.

Keywords: *analytic function, harmonic function, q-calculus, univalent function, integral operator.*

GJSFR-F Classification: *MSC 2010: 11K70*

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A New Subclass of Harmonic Univalent Functions Defined by *q*-Calculus

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Abstract- In this paper we study a new subclass of harmonic univalent functions defined by q-calculus coefficient inequalities, distortion, bounds, extreme point, convolution, convex combination are determined for this class. Finally we discuss a class preserving integral operator and q- Jackson's type integral for this class.

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I. INTRODUCTION

A continuous complex-valued function $f = u + iv$ is said to be harmonic in a simply connected domain D if both u and v are real harmonic in D. In any simply connected domain. We can write $f = h + \overline{g}$, where h and g are analytic in D. We call h the analytic part and g the co-analytic part of f.

A necessary and sufficient condition for f to be locally univalent and sence preserving in D is that $|h'(z)| > |g'(z)|,\ z \in D$ see Clunie and Shiel-small [7].

Let S_H denote the class of functions $f = h+\overline{g}$ that are harmonic univalent and sense-preserving in the open unit disc $U = \{z : |z| < 1\}$ for which $f(0) = f_z(0) - 1 = 0$. Then for $f = h + \overline{g} \in S_H$ we may express the analytic functions h and g as,

$$h(z) = z + \sum_{k=2}^{\infty} a_k z^k, \quad g(z) = \sum_{k=1}^{\infty} b_k z^k, \quad |b_k| < 1. \tag{1.1}$$

Note that S_H reduces to class S of normalized analytic univalent functions if the co-analytic part of its member is zero.

After the appearance of the paper of Clune and Sheil-Small [10] several researchers for example (Silverman [6], Jahangiri [11], Dixit and Porwal [13], Dixit et al. [14], Frasin [4], Kumar et al. [21]) presented a systematic and unified study of various sub classes of harmonic univalent function.

Now, we recall the concept of q-calculus which may be found in [2], for $n \in N$, the q-number is defined as follows:

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$$[K]_q = \frac{1-q^k}{1-q}, \quad 0<q<1. \tag{1.2}$$

Hence, $[K]_q$ can be expressed as a geometric series $\sum_{i=0}^{k-1} q^i$, when $k \to \infty$ the series converges to $\frac{1}{1-q}$. As $q \to 1$, $[k]_q \to k$ and this is the bookmark of a q-analogus the limit as $q \to 1$ recovers the classical object.

The q-derivative of a function f is defined by

$$D_q(f(z)) = \frac{f(qz)-f(z)}{(q-1)z} \quad q \neq 1, \quad z \neq 0$$

and $D_q(f(0)) = f'(0)$ provided $f'(0)$ exists.

For a function $h(z) = z^k$ observe that

$$D_q(h(z)) = D_q(z^k) = \frac{1-q^k}{1-q} z^{k-1} = [k]_q z^{k-1}.$$

Then

$$\lim_{q\to 1} D_q(h(z)) = \text{limit}_{q\to 1}[k]_q z^{k-1} = kz^{k-1} - h'(z)$$

where h' is the ordinary derivative.

The q-Jackson definite integral of the function f is defined by

$$\int_0^z f(t) d_q t = (1-q)z \sum_{n=0}^{\infty} f(zq^n)q^n, \quad z \in C.$$

Now for $1 < \beta < \frac{4}{3}$, $0 \leq \lambda \leq 1$, $0 < q < 1$.

Suppose that $M_H[\lambda, q, \beta]$ denote the family of harmonic function of the form $f = h + \overline{g}$ (1.1).

Satisfying the condition

$$Re\left[\frac{z(zD_q h(z))' - \overline{z(zD_q g(z))'}}{\lambda[z(zD_q h(z))' - \overline{z(zD_q g(z))'}] + (1-\lambda)[h(z) + \overline{(g(z))}]}\right] < \beta. \tag{1.3}$$

Further let M_H the subclasses of S_H consisting of functions of the form,

$$f(z) = z + \sum_{k=2}^{\infty} |a_k| z^k - \sum_{k=1}^{\infty} |b_k| \overline{z}^k \tag{1.4}$$

Further, we define $M_H(\lambda, q, \beta) = N_H(\lambda, q, \beta) \cap M_H$.

Notes

In this paper, we obtain coefficient bound, extreme point, distortion bound, convolution, convex combination for the class $M_H(\lambda, q, \beta)$. We also discuss a class preserving integral operator.

II. MAIN RESULTS

Theorem 2.1 Let the function $f = h + \overline{g}$ be given by (1.1). If

$$\sum_{k=2}^{\infty} \frac{k[k]_q(1-\beta\lambda) - (1-\lambda)\beta}{\beta - 1}|a_k| + \sum_{k=1}^{\infty} \frac{k[k]_q(1-\beta\lambda) - (1-\lambda)\beta}{\beta - 1}|b_k| \leq 1 \tag{2.1}$$

where $1 < \beta \leq \frac{4}{3}$, $0 \leq \lambda \leq 1$, then $f \in N_H(\lambda, q, \beta)$.

Proof. Let

$$\sum_{k=2}^{\infty} \frac{k[k]_q(1-\beta\lambda) - (1-\lambda)\beta}{\beta - 1}|a_k| + \sum_{k=1}^{\infty} \frac{k[k]_q(1-\beta\lambda) + (1-\lambda)\beta}{\beta - 1}|b_k| \leq 1$$

It suffices to show that,

$$\left| \frac{\dfrac{z(zD_qh(z))' - \overline{z(zD_qg(z))'}}{\lambda[z(zD_qh(z))' - \overline{z(zD_qg(z))'}] + (1-\lambda)[h(z) + \overline{g(h)}]} - 1}{\dfrac{z(zD_qh(z))' - \overline{z(zD_qg(z))'}}{\lambda[z(zD_qh(z))' - \overline{z(zD_qg(z))'}] + (1-\lambda)[h(z) + \overline{g(h)}]} - (2\beta - 1)} \right| < 1$$

$$\leq \left| \frac{\dfrac{z + \sum_{k=2}^{\infty} k[k]_q a_k z^k - \overline{\sum_{k=1}^{\infty} k[k]_q b_k z^k}}{z + \sum_{k=2}^{\infty}(\lambda k[k]_q + 1 - \lambda)a_k z^k + \sum_{k=1}^{\infty}(\lambda k[k]_q - 1 + \lambda)\bar{b}_k \bar{z}^k} - 1}{\dfrac{z + \sum_{k=2}^{\infty} k[k]_q a_k z^k - \overline{\sum_{k=1}^{\infty} k[k]_q b_k z^k}}{z + \sum_{k=2}^{\infty}(\lambda k[k]_q + 1 - \lambda)a_k z^k + \sum_{k=1}^{\infty}(\lambda k[k]_q - 1 + \lambda)\bar{b}_k \bar{z}^k} - (2\beta - 1)} \right|$$

$$\leq \frac{\sum_{k=2}^{\infty}[k[k]_q(1-\lambda) - (1-\lambda)]|a_k|\ |z|^{k-1} + \sum_{k=1}^{\infty}[k[k]_q(1-\lambda) + (1-\lambda)]|b_k|\ |z|^{k-1}}{\begin{array}{c} 2(\beta-1) - \sum_{k=2}^{\infty}[k[k]_q(1-\lambda(2\beta-1)) - (2\beta-1)(1-\lambda)]|a_k|\ |z|^{k-1} \\ - \sum_{k=1}^{\infty}[k[k]_q(1-\lambda(2\beta-1)) + (2\beta-1)(1-\lambda)]|b_k|\ |z|^{k-1} \end{array}}.$$

This last expression is bounded above by,

$$\sum_{k=2}^{\infty}[k[k]_q(1-\lambda)-(1-\lambda)]|a_k|+\sum_{k=1}^{\infty}[k[k]_q(1-\lambda)+(1-\lambda)]|b_k|$$

$$\leq 2(\beta-1)-\sum_{k=2}^{\infty}[k[k]_q(1-\lambda(2\beta-1))-(2\beta-1)(1-\lambda)]|a_k|$$

$$-\sum_{k=1}^{\infty}[k[k]_q(1-\lambda(2\beta-1))+(2\beta-1)-(1-\lambda)]|b_k|$$

$$\sum_{k=2}^{\infty}[k[k]_q(1-\lambda)-(1-\lambda)]|a_k|+\sum_{k=2}^{\infty}[k[k]_q(1-\lambda(2\beta-1))-(2\beta-1)(1-\lambda)]|a_k|$$

$$+\sum_{k=1}^{\infty}[k[k]_q(1-\lambda)+(1-\lambda)]|b_k|+\sum_{k=1}^{\infty}[k[k]_q(1-\lambda(2\beta-1))+(2\beta-1)(1-\lambda)]|b_k|\leq 2(\beta-1)$$

$$2\sum_{k=2}^{\infty}[k[k]_q(1-\lambda\beta)-(1-\lambda)\beta]|a_k|+2\sum_{k=1}^{\infty}[k[k]_q(1-\lambda\beta)+(1-\lambda)\beta]|b_k|\leq 2(\beta-1)$$

which is equivalent to

$$\sum_{k=2}^{\infty}\frac{[k[k]_q(1-\lambda\beta)-(1-\lambda)\beta]}{\beta-1}|a_k|+\sum_{k=1}^{\infty}\frac{[k[k]_q(1-\lambda\beta)+(1-\lambda)\beta]}{\beta-1}|b_k|\leq 1.$$

Hence,

$$\left|\frac{\dfrac{z(zD_qh(z))'-\overline{z(zD_qg(z))'}}{\lambda[z(zD_qh(z))'-\overline{z(zD_qg(z))'}]+(1-\lambda)[h(z)+\overline{g(h)}]}-1}{\dfrac{z(zD_qh(z))'-\overline{z(zD_qg(z))'}}{\lambda[z(zD_qh(z))'-\overline{z(zD_qg(z))'}]+(1-\lambda)[h(z)+\overline{g(h)}]}-(2\beta-1)}\right|<1,$$

$z\in U$, and the theorem is proved. □

Theorem 2.2 A function of the form (1.4) is in $M_H(\lambda,q,\beta)$ if and only if,

$$\sum_{k=2}^{\infty}\frac{k[k]_q(1-\beta\lambda)-(1-\lambda)\beta}{\beta-1}|a_k|+\sum_{k=2}^{\infty}\frac{k[k]_q(1-\beta\lambda)+(1-\lambda)\beta}{\beta-1}|b_k|\leq 1. \tag{2.2}$$

Proof. Since $M_H(\lambda,q,\beta)\subset N_H(\lambda,q,\beta)$, we only need to prove the "only iff" Part of the theorem. For this we show that $f\in M_H(\lambda,q,\beta)$ if the above condition does not hold. Note that a necessary and sufficient condition for $f=h+\overline{g}$ given by (1.4) is in $M_H(\lambda,q,\beta)$

$$Re\left\{\frac{z(zD_qh(z))'-\overline{z(zD_qg(z))'}}{\lambda[zD_qh(z))'-\overline{z(zD_qg(z))'}]+(1-\lambda)[h(z)+\overline{g(z)}}\right\}<\beta,$$

is equivalent to

$$Re\left\{\frac{(\beta-1)z-\sum_{k=2}^{\infty}[k[k]_q(1-\lambda\beta)-(1-\lambda)\beta]|a_k|z^k-\sum_{k=1}^{\infty}[k[k]_q(1-\beta\lambda)+(1-\lambda)\beta]|b_k|\overline{z}^k}{z+\sum_{k=2}^{\infty}[\lambda k[k]_q+(1-\lambda)]|a_k|z^k+\sum_{k=1}^{\infty}[\lambda k[k]_q-(1-\lambda)]\,|b_k|\overline{z}^k}\right\}\geq 0$$

The above condition must hold for all values of z, $|z|=r<1$, upon choosing the values of z on the positive real axis where $0\leq z=r<1$, we must have

$$\left\{\frac{(\beta-1)z-\sum_{k=2}^{\infty}k[k]_q(1-\lambda\beta)-(1-\lambda)\beta|a_k|r^{k-1}-\sum_{k=1}^{\infty}k[k]_q(1-\lambda\beta)+(1-\lambda)\beta|b_k|r^{k-1}}{1+\sum_{k=2}^{\infty}\lambda k[k]_q+(1-\lambda)|a_k|r^{k-1}-\sum_{k=1}^{\infty}\lambda k[k]_q-(1-\lambda)|b_k|r^{k-1}}\right\}\geq 0 \tag{2.3}$$

If the condition (2.2) does not hold then the numerator of (2.3) is negative for r sufficiently close to 1. Thus there exist a $z_0=r_0$ in (0,1) for which the quotient in (2.3) is negative. This contradicts the required condition for $f\in M_H(\lambda,q,\beta)$ and so the proof is complete.

Next we determine the extreme points of the closed convex hulls of $M_H(\lambda,q,\beta)$ denoted by clco $M_H(\lambda,q,\beta)$ □

Theorem 2.3 If $f\in$ clco $M_H(\lambda,q,\beta)$, if and only if

$$f(z)=\sum_{k=1}^{\infty}\{x_kh_k(z)+y_kg_k(z)\}, \tag{2.4}$$

where

$$h_1(z)=z,\ \ h_k(z)=z+\frac{\beta-1}{k[k]_q(1-\beta\lambda)-(1-\lambda)\beta}z^k,\quad k=(2,3,...)$$

and

$$g_k(z)=z-\frac{\beta-1}{k[k]_q(1-\lambda\beta)+(1-\lambda)\beta}z^k,\quad k=(2,3,...),$$

$$\sum_{k=1}^{\infty}(x_k+y_k)=1,\ \ x_k\geq 0,\ \ y_k\geq 0$$

In particular extreme points of $M_H(\lambda,q,\beta)$ are $\{h_k\}$ and $\{g_k\}$.

Proof. For functions f of the form (1.4), we have,

$$f(z)=\sum_{k=2}^{\infty}[x_k h_k(z)+y_k g_k(z)]$$

$$=z+\sum_{k=2}^{\infty}\frac{\beta-1}{k[k]_q(1-\lambda\beta)-(1-\lambda)\beta}x_k z^k-\sum_{k=1}^{\infty}\frac{\beta-1}{k[k]_q(1-\lambda\beta)+(1-\lambda)\beta}y_k\overline{z}^k$$

Then by theorem (2.1)

$$\sum_{k=2}^{\infty}\frac{k[k]_q(1-\beta\lambda)-(1-\lambda)\beta}{\beta-1}\left\{\frac{\beta-1}{k[k]_q(1-\beta\lambda)-(1-\lambda)\beta}x_k\right\}$$

$$+\sum_{k=1}^{\infty}\frac{k[k]_q(1-\beta\lambda)+(1-\lambda)\beta}{\beta-1}\left\{\frac{\beta-1}{k[k]_q(1-\beta\lambda)+(1-\lambda)\beta}y_k\right\}$$

$$=\sum_{k=2}^{\infty}x_k+\sum_{k=1}^{\infty}y_k$$

$$=1-x_1\leq 1,$$

and so $f\in$ clco $M_H(\lambda,q,\beta)$ Set $x_k=\dfrac{k[k]_q(1-\beta\lambda)-(1-\lambda)\beta}{\beta-1}|a_k|,$ k=2,3,4,...

and $y_k=\dfrac{k[k]_q(1-\beta\lambda)+(1-\lambda)\beta}{\beta-1}|b_k|,\quad k=1,2,3,....$

Then note that by Theorem 2.2, $0\leq x_k\leq 1,(k=1,2,3,...)$.

We define $x_1=1-\sum_{k=2}^{\infty}x_k-\sum_{k=1}^{\infty}y_k$ and by Theorem 2.2, $x_1\geq 0$.

Consequently, we obtain $f(z)=\sum_{k=1}^{\infty}\{x_k h_k(z)+y_k g_k(z)\}$ as required. ☐

Theorem 2.4 Let $f\in M_H(\lambda,q,\beta)$. Then for $|z|=r<1$, we have,

$$|f(z)|\leq(1+|b_1|)r+\left(\frac{\beta-1}{2[2]_q(1-\lambda\beta)-(1-\lambda)\beta}-\frac{\beta+1}{2[2]_q(1-\beta\lambda)-(1-\lambda)\beta}|b_1|\right)r^2$$

and

$$|f(z)|\geq(1-|b_1|)r-\left(\frac{\beta-1}{2[2]_q(1-\lambda\beta)-(1-\lambda)\beta}-\frac{\beta+1}{2[2]_q(1-\beta\lambda)-(1-\lambda)\beta}|b_1|\right)r^2.$$

Proof. We only prove the right hand inequality. The proof for left hand inequality is similar and will be omitted. Let $f(z)\in M_H(\lambda,q,\beta)$, taking the absolute value of f, we have,

$$|f(z)| \le (1+|b_1|)r + \sum_{k=2}^{\infty}(|a_k|+|b_k|)r^k$$

$$\le (1+|b_1|)r + \sum_{k=2}^{\infty}(|a_k|+|b_k|)r^2$$

$$= (1+|b_1|)r + r^2\sum_{k=2}^{\infty}(|a_k|+|b_k|)$$

$$= (1+|b_1|)r + r^2\frac{\beta-1}{2[2]_q(1-\beta\lambda)-(1-\lambda)\beta}\sum_{k=2}^{\infty}\frac{2[2]_q(1-\beta\lambda)-(1-\lambda)\beta}{\beta-1}(|a_k|+|b_k|)$$

$$= (1+|b_1|)r + r^2\frac{\beta-1}{2[2]_q(1-\beta\lambda)-(1-\lambda)\beta}\sum_{k=2}^{\infty}\frac{k[k]_q(1-\beta\lambda)-(1-\lambda)\beta}{\beta-1}(|a_k|+|b_k|)$$

$$\le (1+|b_1|)r + r^2\frac{\beta-1}{2[2]_q(1-\beta\lambda)-(1-\lambda)\beta}$$

$$\sum_{k=2}^{\infty}\left(\frac{k[k]_q(1-\beta\lambda)-(1-\lambda)\beta}{\beta-1}|a_k| + \frac{k[k]_q(1-\beta\lambda)+(1-\lambda)\beta}{\beta-1}|b_k|\right)$$

$$= (1+|b_1|)r + r^2\frac{\beta-1}{2[2]_q(1-\beta\lambda)-(1-\lambda)\beta}\left(1-\frac{1+\beta-2\beta\lambda}{\beta-1}|b_1|\right)$$

$$= (1+|b_1|)r + r^2\left(\frac{\beta-1}{2[2]_q(1-\beta\lambda)-(1-\lambda)\beta} - \frac{1+\beta-2\beta\lambda}{2[2]_q(1-\beta\lambda)-(1-\lambda)\beta}|b_1|\right).$$

Thus the proof of Theorem 2.4 is established. ☐

Theorem 2.5 For $1 < \alpha \le \beta \le \frac{4}{3}, 0 \le \lambda \le 1$, let $f \in M_H(\lambda,q,\alpha)$, and $F \in M_H(\lambda,q,\beta)$ then $f * F \in M_H(\lambda,q,\alpha) \subseteq M_H(\lambda,q,\beta)$.

Proof. Let $f(z) = z + \sum_{k=2}^{\infty}|a_k|z^k - \sum_{k=1}^{\infty}|b_k|\overline{z}^k$ be in $M_H(\lambda,q,\alpha)$ and

$$F(z) = z + \sum_{k=2}^{\infty}|A_k|z^k - \sum_{k=1}^{\infty}|B_k|\overline{z}^k \text{ be in } M_H(\lambda,q,\beta).$$

Then the convolution $f * F$ is given by

$$(f*F)(z) = f(z)*F(z)$$

$$= z + \sum_{k=2}^{\infty}|a_kA_k|z^k - \sum_{k=1}^{\infty}|b_kB_k|\overline{z}^k.$$

We wish to show that the coefficient of $f*F$ satisfy the required condition in Theorem 2.2 for $F(z) \in M_H(\lambda,q,\beta)$ we note that $|A_k| < 1$ and $|B_K| < 1$. Now for the convolution function $f * F$, we obtain,

$$\begin{aligned}
&\sum_{k=2}^{\infty}\frac{k[k]_q(1-\beta\lambda)-(1-\lambda)\beta}{\beta-1}|a_kA_k|+\sum_{k=1}^{\infty}\frac{k[k]_q(1-\lambda\beta)-(1-\lambda)\beta}{\beta-1}|b_kB_k|\\
&\leq \frac{k[k]_q(1-\beta\lambda)-(1-\lambda)\beta}{\beta-1}|a_k|+\sum_{k=1}^{\infty}\frac{k[k]_q(1-\lambda\beta)-(1-\lambda)\beta}{\beta-1}|b_k|\\
&\leq 1 \qquad \text{Since } f(z)\in M_H(\lambda,q,\beta).
\end{aligned}$$

Therefore, $f*F\in M_H(\lambda,q,\alpha)\subseteq M_H(\lambda,q,\beta)$.

Thus the proof of the Theorem 2.5 is established. □

A family of class Preserving Integral Operator

Let $f(x)=h(x)+\overline{g(x)}$ be defined by (1.1). Let us defined $F(z)$ by the relation,

$$F(z)=\frac{c+1}{z^c}\int_0^z t^{c-1}h(t)dt+\overline{\frac{c+1}{z^c}\int_0^z t^{c-1}g(t)dt},\quad (c>-1). \tag{2.5}$$

Theorem 2.6 Let $f(z)=h(z)+\overline{g(z)}\in S_H$ be given by (1.4) and $f\in M_H(\lambda,q,\beta)$ where $1<\beta\leq\frac{4}{3}$, $0<\lambda\leq 1$. Then $F(z)$ defined by (2.5) is also in the class $M_H(\lambda,q,\beta)$.

Proof. Let $f(z)=z+\sum_{k=2}^{\infty}|a_k|z^k-\sum_{k=1}^{\infty}|b_k|\overline{z}^k$ be in $M_H(\lambda,q,\beta)$ then by Theorem 2.2, we have

$$\sum_{k=2}^{\infty}\frac{k[k]_q(1-\lambda\beta)-(1-\lambda)\beta}{\beta-1}|a_k|+\sum_{k=1}^{\infty}\frac{k[k]_q(1-\lambda\beta)+(1-\lambda)\beta}{\beta-1}|b_k|\leq 1.$$

From the representation (2.5) of $F(z)$, it follows that:

$$F(z)=z+\sum_{k=2}^{\infty}\frac{c+1}{c+k}|a_k|z^k-\sum_{k=1}^{\infty}\frac{c+1}{c+k}|b_k|\overline{z}^k.$$

Now,

$$\begin{aligned}
&\sum_{k=2}^{\infty}\frac{k[k]_q(1-\lambda\beta)-(1-\lambda)\beta}{\beta-1}\left(\frac{c+1}{c+k}\right)|a_k|\\
&+\sum_{k=1}^{\infty}\frac{k[k]_q(1-\lambda\beta)+(1-\lambda)\beta}{\beta-1}\left(\frac{c+1}{c+k}\right)|b_k|\\
&\leq\sum_{k=2}^{\infty}\frac{k[k]_q(1-\lambda\beta)-(1-\lambda)\beta}{\beta-1}|a_k|+\sum_{k=1}^{\infty}\frac{k[k]_q(1-\lambda\beta)+(1-\lambda)\beta}{\beta-1}|b_k|\\
&\leq 1.
\end{aligned}$$

Thus $F(z)\in M_H(\lambda,q,\beta)$.
The proof of following Theorem 2.6 is complete. □

Definition 2.1 Let $f = h + \overline{g}$ be defined, by (1.1); then the q-integral operator $F_q : H \to H$ is defined by the relation,

$$F_q(z) = \frac{[c]_q}{z^{c+1}} \int_0^z t^c h(t) d_q t + \frac{[c]_q}{z^{c+1}} \int_0^z t^c g(t) d_q t, \tag{2.6}$$

where $[a]_q$ is the q-number defiend by (1.2) and H is the class of functions of the form (1.1) which are harmonic in U.

Theorem 2.7 Let $f(z) = h(z) + \overline{g(z)}$ be given by (1.3) and $f \in M_H(\lambda, q, \beta)$ where $1 < \beta \leq \frac{4}{3}$, $0 < q < 1, 0 \leq \lambda \leq 1$. Then $F_q(z)$ defined by (2.6) is also in the class $M_H(\lambda, q, \beta)$.

Proof. Let $f(z) = z + \sum_{k=2}^{\infty} |a_k| z^k - \sum_{k=1}^{\infty} |b_k| \overline{z}^k$ be in $M_H(\lambda, q, \beta)$ then by Theorem 2.2. We have,

$$\sum_{k=2}^{\infty} \frac{k[k]_q(1-\lambda\beta) - (1-\lambda)\beta}{\beta - 1} |a_k| + \sum_{k=1}^{\infty} \frac{k[k]_q(1-\lambda\beta) + (1-\lambda)\beta}{\beta - 1} |b_k| \leq 1.$$

From the representation (2.6) of $F_q(z)$, it follows that

$$F_q(z) = z + \sum_{k=2}^{\infty} \frac{[c]_q}{[k+c+1]_q} |a_k| z^k - \sum_{k=1}^{\infty} \frac{[c]_q}{[k+c+1]_q} |b_k| \overline{z}^k.$$

Since

$$[k+c+1]_q - [c]_q$$

$$= \sum_{i=0}^{k+c} q^i - \sum_{i=0}^{c-1} q^i = \sum_{i=c}^{k+c} q^i > 0$$

$$[k+c+1]_q > [c]_q,$$

or

$$\frac{[c]_q}{[k+c+1]_q} < 1.$$

Now

$$\sum_{k=2}^{\infty} \frac{k[k]_q(1-\lambda\beta) - (1-\lambda)\beta}{\beta - 1} \frac{[c]_q}{[k+c+1]_q} |a_k| + \sum_{k=1}^{\infty} \frac{k[k]_q(1-\lambda\beta) + (1-\lambda)\beta}{\beta - 1} \frac{[c]_q}{[k+c+1]_q} |b_k|$$

$$\leq \sum_{k=2}^{\infty} \frac{k[k]_q(1-\lambda\beta) - (1-\lambda)\beta}{\beta - 1} |a_k| + \sum_{k=1}^{\infty} \frac{k[k]_q(1-\lambda\beta) + (1-\lambda)\beta}{\beta - 1} |b_k|$$

$$\leq 1.$$

Thus the proof of the Theorem 2.7 is established. ☐

Theorem 2.8. The class $M_H(\lambda, q, \beta)$ is closed under convex function.

Proof. For $i = \{1, 2, 3, ...\}$, let $f_i(z) \in M_H(\lambda, q, \beta)$ where $f_i(z)$ is given by

$$f_i(z) = z + \sum_{k=2}^{\infty} |a_{k_i}| z^k - \sum_{k=1}^{\infty} |b_{k_i}| \overline{z}^k.$$

Then by Theorem 2.2,

$$\sum_{k=2}^{\infty} \frac{k[k]_q(1-\lambda\beta) - (1-\lambda)\beta}{\beta - 1} |a_k| + \sum_{k=1}^{\infty} \frac{k[k]_q(1-\lambda\beta) + (1-\lambda)\beta}{\beta - 1} |b_k| \le 1. \tag{2.7}$$

For $\sum_{i=1}^{\infty} t_i = 1,\ 0 \le t_i \le 1$ the convex combination of f_i may be written as,

$$\sum_{i=1}^{\infty} t_i f_i(z) = z + \sum_{k=2}^{\infty} \left(\sum_{i=1}^{\infty} t_i |a_{k_i}| \right) z^k - \sum_{k=1}^{\infty} \left(\sum_{i=1}^{\infty} t_i |b_{k_i}| \right) \overline{z}^k.$$

Then by (2.2), we have,

$$\sum_{k=2}^{\infty} \frac{k[k]_q(1-\lambda\beta) - (1-\lambda)\beta}{\beta - 1} \left(\sum_{i=1}^{\infty} t_i |a_{k_i}| \right) + \sum_{k=1}^{\infty} \frac{k[k]_q(1-\lambda\beta) + (1-\lambda)\beta}{\beta - 1} \left(\sum_{i=1}^{\infty} t_i |b_{k_i}| \right)$$

$$= \sum_{i=1}^{\infty} t_i \left(\sum_{k=2}^{\infty} \frac{k[k]_q(1-\lambda\beta) - (1-\lambda)\beta}{\beta - 1} |a_{k_i}| + \sum_{k=1}^{\infty} \frac{k[k]_q(1-\lambda\beta) + (1-\lambda)\beta}{\beta - 1} |b_{k_i}| \right)$$

$$\le \sum_{i=1}^{\infty} t_i = 1.$$

This is the condition required by Theorem 2.8 and so $\sum_{i=1}^{\infty} t_i f_i(z) \in M_H(\lambda, q, \beta)$. The proof of the following Theorem 2.8 is complete. ☐

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Notes

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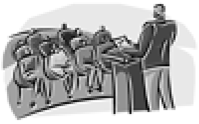

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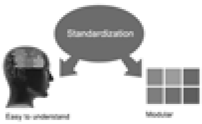

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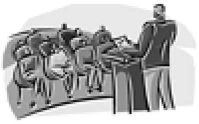

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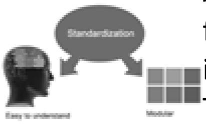

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1. GENERAL

Before submitting your research paper, one is advised to go through the details as mentioned in following heads. It will be beneficial, while peer reviewer justify your paper for publication.

Scope

The Global Journals Inc. (US) welcome the submission of original paper, review paper, survey article relevant to the all the streams of Philosophy and knowledge. The Global Journals Inc. (US) is parental platform for Global Journal of Computer Science and Technology, Researches in Engineering, Medical Research, Science Frontier Research, Human Social Science, Management, and Business organization. The choice of specific field can be done otherwise as following in Abstracting and Indexing Page on this Website. As the all Global

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Complete support for both authors and co-author is provided.

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Based on potential and nature, the manuscript can be categorized under the following heads:

Original research paper: Such papers are reports of high-level significant original research work.

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Research articles: These are handled with small investigation and applications

Research letters: The letters are small and concise comments on previously published matters.

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The recommended size of original research paper is less than seven thousand words, review papers fewer than seven thousands words also.Preparation of research paper or how to write research paper, are major hurdle, while writing manuscript. The research articles and research letters should be fewer than three thousand words, the structure original research paper; sometime review paper should be as follows:

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(b) A brief Summary, "Abstract" (less than 150 words) containing the major results and conclusions.

(c) Up to ten keywords, that precisely identifies the paper's subject, purpose, and focus.

(d) An Introduction, giving necessary background excluding subheadings; objectives must be clearly declared.

(e) Resources and techniques with sufficient complete experimental details (wherever possible by reference) to permit repetition; sources of information must be given and numerical methods must be specified by reference, unless non-standard.

(f) Results should be presented concisely, by well-designed tables and/or figures; the same data may not be used in both; suitable statistical data should be given. All data must be obtained with attention to numerical detail in the planning stage. As reproduced design has been recognized to be important to experiments for a considerable time, the Editor has decided that any paper that appears not to have adequate numerical treatments of the data will be returned un-refereed;

(g) Discussion should cover the implications and consequences, not just recapitulating the results; conclusions should be summarizing.

(h) Brief Acknowledgements.

(i) References in the proper form.

Authors should very cautiously consider the preparation of papers to ensure that they communicate efficiently. Papers are much more likely to be accepted, if they are cautiously designed and laid out, contain few or no errors, are summarizing, and be conventional to the approach and instructions. They will in addition, be published with much less delays than those that require much technical and editorial correction.

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Metric SI units are supposed to generally be used excluding where they conflict with current practice or are confusing. For illustration, 1.4 l rather than 1.4 × 10-3 m3, or 4 mm somewhat than 4 × 10-3 m. Chemical formula and solutions must identify the form used, e.g. anhydrous or hydrated, and the concentration must be in clearly defined units. Common species names should be followed by underlines at the first mention. For following use the generic name should be constricted to a single letter, if it is clear.

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A major linchpin in research work for the writing research paper is the keyword search, which one will employ to find both library and Internet resources.

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Search engines for most searches, use Boolean searching, which is somewhat different from Internet searches. The Boolean search uses "operators," words (and, or, not, and near) that enable you to expand or narrow your affords. Tips for research paper while preparing research paper are very helpful guideline of research paper.

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- It may take the discovery of only one relevant paper to let steer in the right keyword direction because in most databases, the keywords under which a research paper is abstracted are listed with the paper.
- One should avoid outdated words.

Keywords are the key that opens a door to research work sources. Keyword searching is an art in which researcher's skills are bound to improve with experience and time.

Numerical Methods: Numerical methods used should be clear and, where appropriate, supported by references.

Acknowledgements: Please make these as concise as possible.

References

References follow the Harvard scheme of referencing. References in the text should cite the authors' names followed by the time of their publication, unless there are three or more authors when simply the first author's name is quoted followed by et al. unpublished work has to only be cited where necessary, and only in the text. Copies of references in press in other journals have to be supplied with submitted typescripts. It is necessary that all citations and references be carefully checked before submission, as mistakes or omissions will cause delays.

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TECHNIQUES FOR WRITING A GOOD QUALITY RESEARCH PAPER:

1. Choosing the topic: In most cases, the topic is searched by the interest of author but it can be also suggested by the guides. You can have several topics and then you can judge that in which topic or subject you are finding yourself most comfortable. This can be done by asking several questions to yourself, like Will I be able to carry our search in this area? Will I find all necessary recourses to accomplish the search? Will I be able to find all information in this field area? If the answer of these types of questions will be "Yes" then you can choose that topic. In most of the cases, you may have to conduct the surveys and have to visit several places because this field is related to Computer Science and Information Technology. Also, you may have to do a lot of work to find all rise and falls regarding the various data of that subject. Sometimes, detailed information plays a vital role, instead of short information.

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3. Think Like Evaluators: If you are in a confusion or getting demotivated that your paper will be accepted by evaluators or not, then think and try to evaluate your paper like an Evaluator. Try to understand that what an evaluator wants in your research paper and automatically you will have your answer.

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7. Use right software: Always use good quality software packages. If you are not capable to judge good software then you can lose quality of your paper unknowingly. There are various software programs available to help you, which you can get through Internet.

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15. Use of direct quotes: When you do research relevant to literature, history or current affairs then use of quotes become essential but if study is relevant to science then use of quotes is not preferable.

16. Use proper verb tense: Use proper verb tenses in your paper. Use past tense, to present those events that happened. Use present tense to indicate events that are going on. Use future tense to indicate future happening events. Use of improper and wrong tenses will confuse the evaluator. Avoid the sentences that are incomplete.

17. Never use online paper: If you are getting any paper on Internet, then never use it as your research paper because it might be possible that evaluator has already seen it or maybe it is outdated version.

18. Pick a good study spot: To do your research studies always try to pick a spot, which is quiet. Every spot is not for studies. Spot that suits you choose it and proceed further.

19. Know what you know: Always try to know, what you know by making objectives. Else, you will be confused and cannot achieve your target.

20. Use good quality grammar: Always use a good quality grammar and use words that will throw positive impact on evaluator. Use of good quality grammar does not mean to use tough words, that for each word the evaluator has to go through dictionary. Do not start sentence with a conjunction. Do not fragment sentences. Eliminate one-word sentences. Ignore passive voice. Do not ever use a big word when a diminutive one would suffice. Verbs have to be in agreement with their subjects. Prepositions are not expressions to finish sentences with. It is incorrect to ever divide an infinitive. Avoid clichés like the disease. Also, always shun irritating alliteration. Use language that is simple and straight forward. put together a neat summary.

21. Arrangement of information: Each section of the main body should start with an opening sentence and there should be a changeover at the end of the section. Give only valid and powerful arguments to your topic. You may also maintain your arguments with records.

22. Never start in last minute: Always start at right time and give enough time to research work. Leaving everything to the last minute will degrade your paper and spoil your work.

23. Multitasking in research is not good: Doing several things at the same time proves bad habit in case of research activity. Research is an area, where everything has a particular time slot. Divide your research work in parts and do particular part in particular time slot.

24. Never copy others' work: Never copy others' work and give it your name because if evaluator has seen it anywhere you will be in trouble.

25. Take proper rest and food: No matter how many hours you spend for your research activity, if you are not taking care of your health then all your efforts will be in vain. For a quality research, study is must, and this can be done by taking proper rest and food.

26. Go for seminars: Attend seminars if the topic is relevant to your research area. Utilize all your resources.

27. Refresh your mind after intervals: Try to give rest to your mind by listening to soft music or by sleeping in intervals. This will also improve your memory.

28. Make colleagues: Always try to make colleagues. No matter how sharper or intelligent you are, if you make colleagues you can have several ideas, which will be helpful for your research.

29. Think technically: Always think technically. If anything happens, then search its reasons, its benefits, and demerits.

30. Think and then print: When you will go to print your paper, notice that tables are not be split, headings are not detached from their descriptions, and page sequence is maintained.

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32. Never oversimplify everything: To add material in your research paper, never go for oversimplification. This will definitely irritate the evaluator. Be more or less specific. Also too, by no means, ever use rhythmic redundancies. Contractions aren't essential and shouldn't be there used. Comparisons are as terrible as clichés. Give up ampersands and abbreviations, and so on. Remove commas, that are, not necessary. Parenthetical words however should be together with this in commas. Understatement is all the time the complete best way to put onward earth-shaking thoughts. Give a detailed literary review.

33. Report concluded results: Use concluded results. From raw data, filter the results and then conclude your studies based on measurements and observations taken. Significant figures and appropriate number of decimal places should be used. Parenthetical remarks are prohibitive. Proofread carefully at final stage. In the end give outline to your arguments. Spot out perspectives of further study of this subject. Justify your conclusion by at the bottom of them with sufficient justifications and examples.

34. After conclusion: Once you have concluded your research, the next most important step is to present your findings. Presentation is extremely important as it is the definite medium though which your research is going to be in print to the rest of the crowd. Care should be taken to categorize your thoughts well and present them in a logical and neat manner. A good quality research paper format is essential because it serves to highlight your research paper and bring to light all necessary aspects in your research.

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Key points to remember:

- Submit all work in its final form.
- Write your paper in the form, which is presented in the guidelines using the template.
- Please note the criterion for grading the final paper by peer-reviewers.

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A purpose of organizing a research paper is to let people to interpret your effort selectively. The journal requires the following sections, submitted in the order listed, each section to start on a new page.

The introduction will be compiled from reference matter and will reflect the design processes or outline of basis that direct you to make study. As you will carry out the process of study, the method and process section will be constructed as like that. The result segment will show related statistics in nearly sequential order and will direct the reviewers next to the similar intellectual paths throughout the data that you took to carry out your study. The discussion section will provide understanding of the data and projections as to the implication of the results. The use of good quality references all through the paper will give the effort trustworthiness by representing an alertness of prior workings.

Writing a research paper is not an easy job no matter how trouble-free the actual research or concept. Practice, excellent preparation, and controlled record keeping are the only means to make straightforward the progression.

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To make a paper clear

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- Separating a table/chart or figure - impound each figure/table to a single page
- Submitting a manuscript with pages out of sequence

In every sections of your document

· Use standard writing style including articles ("a", "the," etc.)

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· Align the primary line of each section

· Present your points in sound order

· Use present tense to report well accepted

· Use past tense to describe specific results

· Shun familiar wording, don't address the reviewer directly, and don't use slang, slang language, or superlatives

· Shun use of extra pictures - include only those figures essential to presenting results

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Choose a revealing title. It should be short. It should not have non-standard acronyms or abbreviations. It should not exceed two printed lines. It should include the name(s) and address (es) of all authors.

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The summary should be two hundred words or less. It should briefly and clearly explain the key findings reported in the manuscript--must have precise statistics. It should not have abnormal acronyms or abbreviations. It should be logical in itself. Shun citing references at this point.

An abstract is a brief distinct paragraph summary of finished work or work in development. In a minute or less a reviewer can be taught the foundation behind the study, common approach to the problem, relevant results, and significant conclusions or new questions.

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- Fundamental goal
- To the point depiction of the research
- Consequences, including definite statistics - if the consequences are quantitative in nature, account quantitative data; results of any numerical analysis should be reported
- Significant conclusions or questions that track from the research(es)

Approach:

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- A conceptual should situate on its own, and not submit to any other part of the paper such as a form or table
- Center on shortening results - bound background information to a verdict or two, if completely necessary
- What you account in an conceptual must be regular with what you reported in the manuscript
- Exact spelling, clearness of sentences and phrases, and appropriate reporting of quantities (proper units, important statistics) are just as significant in an abstract as they are anywhere else

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The **Introduction** should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable to comprehend and calculate the purpose of your study without having to submit to other works. The basis for the study should be offered. Give most important references but shun difficult to make a comprehensive appraisal of the topic. In the introduction, describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will have no attention in your result. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here. Following approach can create a valuable beginning:

- Explain the value (significance) of the study
- Shield the model - why did you employ this particular system or method? What is its compensation? You strength remark on its appropriateness from a abstract point of vision as well as point out sensible reasons for using it.
- Present a justification. Status your particular theory (es) or aim(s), and describe the logic that led you to choose them.
- Very for a short time explain the tentative propose and how it skilled the declared objectives.

Approach:

- Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done.
- Sort out your thoughts; manufacture one key point with every section. If you make the four points listed above, you will need a least of four paragraphs.

- Present surroundings information only as desirable in order hold up a situation. The reviewer does not desire to read the whole thing you know about a topic.
- Shape the theory/purpose specifically - do not take a broad view.
- As always, give awareness to spelling, simplicity and correctness of sentences and phrases.

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This part is supposed to be the easiest to carve if you have good skills. A sound written Procedures segment allows a capable scientist to replacement your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt for the least amount of information that would permit another capable scientist to spare your outcome but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section. When a technique is used that has been well described in another object, mention the specific item describing a way but draw the basic principle while stating the situation. The purpose is to text all particular resources and broad procedures, so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step by step report of the whole thing you did, nor is a methods section a set of orders.

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- Explain materials individually only if the study is so complex that it saves liberty this way.
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- Do not take in frequently found.
- If use of a definite type of tools.
- Materials may be reported in a part section or else they may be recognized along with your measures.

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- Report the method (not particulars of each process that engaged the same methodology)
- Describe the method entirely
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures
- Simplify - details how procedures were completed not how they were exclusively performed on a particular day.
- If well known procedures were used, account the procedure by name, possibly with reference, and that's all.

Approach:

- It is embarrassed or not possible to use vigorous voice when documenting methods with no using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result when script up the methods most authors use third person passive voice.
- Use standard style in this and in every other part of the paper - avoid familiar lists, and use full sentences.

What to keep away from

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- Skip all descriptive information and surroundings - save it for the argument.
- Leave out information that is immaterial to a third party.

Results:

The principle of a results segment is to present and demonstrate your conclusion. Create this part a entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Carry on to be to the point, by means of statistics and tables, if suitable, to present consequences most efficiently.You must obviously differentiate material that would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matter should not be submitted at all except requested by the instructor.

Content

- Sum up your conclusion in text and demonstrate them, if suitable, with figures and tables.
- In manuscript, explain each of your consequences, point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation an exacting study.
- Explain results of control experiments and comprise remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or in manuscript form.

What to stay away from

- Do not discuss or infer your outcome, report surroundings information, or try to explain anything.
- Not at all, take in raw data or intermediate calculations in a research manuscript.
- Do not present the similar data more than once.
- Manuscript should complement any figures or tables, not duplicate the identical information.
- Never confuse figures with tables - there is a difference.

Approach

- As forever, use past tense when you submit to your results, and put the whole thing in a reasonable order.
- Put figures and tables, appropriately numbered, in order at the end of the report
- If you desire, you may place your figures and tables properly within the text of your results part.

Figures and tables

- If you put figures and tables at the end of the details, make certain that they are visibly distinguished from any attach appendix materials, such as raw facts
- Despite of position, each figure must be numbered one after the other and complete with subtitle
- In spite of position, each table must be titled, numbered one after the other and complete with heading
- All figure and table must be adequately complete that it could situate on its own, divide from text

Discussion:

The Discussion is expected the trickiest segment to write and describe. A lot of papers submitted for journal are discarded based on problems with the Discussion. There is no head of state for how long a argument should be. Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implication of the study. The purpose here is to offer an understanding of your results and hold up for all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of result should be visibly described. Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved with prospect, and let it drop at that.

- Make a decision if each premise is supported, discarded, or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."
- Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work
- You may propose future guidelines, such as how the experiment might be personalized to accomplish a new idea.
- Give details all of your remarks as much as possible, focus on mechanisms.
- Make a decision if the tentative design sufficiently addressed the theory, and whether or not it was correctly restricted.
- Try to present substitute explanations if sensible alternatives be present.
- One research will not counter an overall question, so maintain the large picture in mind, where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

- When you refer to information, differentiate data generated by your own studies from available information
- Submit to work done by specific persons (including you) in past tense.
- Submit to generally acknowledged facts and main beliefs in present tense.

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Introduction	Containing all background details with clear goal and appropriate details, flow specification, no grammar and spelling mistake, well organized sentence and paragraph, reference cited	Unclear and confusing data, appropriate format, grammar and spelling errors with unorganized matter	Out of place depth and content, hazy format
Methods and Procedures	Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads	Difficult to comprehend with embarrassed text, too much explanation but completed	Incorrect and unorganized structure with hazy meaning
Result	Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake	Complete and embarrassed text, difficult to comprehend	Irregular format with wrong facts and figures
Discussion	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

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