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Dynamics of Two Coupled Van der Pol Oscillators with Delay Coupling Revisited

By Mark Gluzman & Richard Rand
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Abstract- The problem of two van der Pol oscillators coupled by velocity delay terms was studied by Wirkus and Rand in 2002 [5]. The small-ε analysis resulted in a slow flow which contained delay terms. To simplify the analysis, Wirkus and Rand followed a common procedure of replacing the delay terms by non-delayed terms, a step said to be valid for small ε, resulting in a slow flow which was an ODE rather than a DDE (delay-differential equation). In the present paper we consider the same problem but leave the delay terms in the slow flow, there by offering an evaluation of the approximate simplification made in [5].

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4. Sah, S.M. and Rand, R.H. (2016) *Delay Terms in the Slow Flow*, Journal of Applied Nonlinear Dynamics 5(4):471-484.

Dynamics of Two Coupled Van der Pol Oscillators with Delay Coupling Revisited

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Abstract- The problem of two van der Pol oscillators coupled by velocity delay terms was studied by Wirkus and Rand in 2002 [5]. The small-ε analysis resulted in a slow flow which contained delay terms. To simplify the analysis, Wirkus and Rand followed a common procedure of replacing the delay terms by non-delayed terms, a step said to be valid for small ε, resulting in a slow flow which was an ODE rather than a DDE (delay-differential equation). In the present paper we consider the same problem but leave the delay terms in the slow flow, thereby offering an evaluation of the approximate simplification made in [5].

I. Introduction

In a recent paper by Sah and Rand [4], it was shown that a class of nonlinear oscillators with delayed self-feedback gave rise via a perturbation solution to a DDE slow flow. An exact solution was obtained to the DDE slow flow and was compared to the approximate solution resulting from the common approach of replacing the delayed variables in the slow flow with non-delayed variables, thereby replacing the DDE slow flow with an easier to solve ODE slow flow.

The paper by Sah and Rand [4] was motivated by numerous papers in the literature of nonlinear dynamics in which the DDE slow flow is replaced by an approximate ODE slow flow, for example [1], [2], [5].

In particular, in 2002 Wirkus and Rand wrote a paper entitled "The Dynamics of Two Coupled van der Pol Oscillators with Delay Coupling" [5] in which averaging was used to derive a slow flow which governed the stability of the in-phase mode. In studying the resulting slow flow, Wirkus and Rand replaced certain delayed quantities with non-delayed versions in order to simplify the analysis. In this paper we reexamine the previously studied system with the idea of leaving the delayed quantities in the slow flow.

II. Delay-Coupled Van der Pol Oscillators

The governing equations are:

$$\ddot{x}_1 + x_1 - \varepsilon(1 - x_1^2)\dot{x}_1 = \varepsilon\alpha\dot{x}_2(t - T) \tag{1}$$

$$\ddot{x}_2 + x_2 - \varepsilon(1 - x_2^2)\dot{x}_2 = \varepsilon\alpha\dot{x}_1(t - T) \tag{2}$$

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This system admits an in-phase mode in which $x_1 = x_2 = y(t)$ where

$$\ddot{y} + y - \varepsilon(1 - y^2)\dot{y} = \varepsilon\alpha\dot{y}(t - T) \tag{3}$$

In order to investigate the stability of the in-phase mode $y(t)$, we will first obtain an approximate expression for it using Lindstedt's method. We replace independent variable t by stretched time τ:

$$\tau = \omega t, \qquad \text{where } \omega = 1 + \varepsilon k + O(\varepsilon^2) \tag{4}$$

so that eq.(3) becomes:

$$\omega^2 y'' + y - \varepsilon(1 - y^2)\omega y' = \varepsilon\alpha\omega y'(\tau - \omega T) \tag{5}$$

where primes represent differentiation with respect to τ. We expand y in a power series in ε,

$$y = y_0 + \varepsilon y_1 + \cdots \tag{6}$$

and substitute (6) into (5). After collecting terms, we get

$$y_0'' + y_0 = 0 \tag{7}$$

$$y_1'' + y_1 = -2ky_0'' + (1 - y_0^2)y_0' + \alpha y_0'(\tau - T) \tag{8}$$

We take the solution of (7) to be

$$y_0 = R\cos\tau \tag{9}$$

and substitute (9) into (8), giving

$$y_1'' + y_1 = (2kR + \alpha R\sin T)\cos\tau + (-R + \frac{R^3}{4} - \alpha R\cos T)\sin\tau + \frac{R^3}{4}\sin(3\tau) \tag{10}$$

Removing secular terms in (10), we obtain

$$k = -\frac{\alpha}{2}\sin T \qquad \text{and} \qquad R = 2\sqrt{1 + \alpha\cos T} \tag{11}$$

Thus the in-phase mode $x_1 = x_2 = y(t)$, the stability of which is desired, is given by the approximation

$$y(t) = R\cos\omega t = 2\sqrt{1 + \alpha\cos T}\cos\left(1 - \frac{\alpha}{2}\varepsilon\sin T\right)t \tag{12}$$

III. Stability of the in-Phase Mode

In order to study the stability of the motion (12), we set

$$x_1 = y(t) + w_1 \qquad \text{and} \qquad x_2 = y(t) + w_2 \tag{13}$$

where w_1 and w_2 are deviations off of the in-phase mode $x_1 = x_2 = y(t)$. Substituting (13) into (1) and (2) and using (3), we obtain the following equations on w_1 and w_2, linearized about $w_1 = w_2 = 0$:

$$\ddot{w}_1 + (1 + 2\varepsilon y\dot{y})w_1 - \varepsilon(1 - y^2)\dot{w}_1 = \varepsilon\alpha\dot{w}_2(t - T) \tag{14}$$

$$\ddot{w}_2 + (1 + 2\varepsilon y\dot{y})w_2 - \varepsilon(1 - y^2)\dot{w}_2 = \varepsilon\alpha\dot{w}_1(t - T) \tag{15}$$

Equations (14) and (15) can be uncoupled by setting

$$z_1 = w_1 + w_2 \qquad \text{and} \qquad z_2 = w_1 - w_2 \tag{16}$$

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Giving

$$\ddot{z}_1 + (1 + 2\varepsilon y\dot{y})z_1 - \varepsilon(1 - y^2)\dot{z}_1 = \varepsilon\alpha\dot{z}_1(t - T) \tag{17}$$

$$\ddot{z}_2 + (1 + 2\varepsilon y\dot{y})z_2 - \varepsilon(1 - y^2)\dot{z}_2 = -\varepsilon\alpha\dot{z}_2(t - T) \tag{18}$$

The only difference between these two equations is the sign of the right-hand side, which may be absorbed into a new coefficient, call it β, which equals either 1 or -1:

$$\ddot{u} + (1 + 2\varepsilon y\dot{y})u - \varepsilon(1 - y^2)\dot{u} = \varepsilon\beta\alpha\dot{u}(t - T) \tag{19}$$

where $u = z_1$ for $\beta = 1$ and $u = z_2$ for $\beta = -1$. In eq.(19), $y(t)$ is given by eq.(12). In order to study the boundedness of solutions to eq.(19), we return to using $\tau = \omega t$ as independent variable, where $\omega = 1 - \frac{\alpha}{2}\varepsilon \sin T$:

$$\omega^2 u'' + (1 + 2\varepsilon\omega y y')u - \varepsilon(1 - y^2)\omega u' = \varepsilon\beta\alpha\omega u'(\tau - \omega T) \tag{20}$$

We study (20) by using the two variable perturbation method [3]. We let $\xi = \tau$ and $\eta = \varepsilon\tau$, giving:

$$\omega^2(u_{\xi\xi} + 2u_{\xi\eta}) + (1 + 2\varepsilon\omega y y_\xi)u - \varepsilon(1 - y^2)\omega u_\xi = \varepsilon\beta\alpha\omega u_\xi(\xi - \omega T, \eta - \varepsilon\omega T) \tag{21}$$

where $y(\xi) = R\cos\xi$, $R = 2\sqrt{1 + \alpha\cos T}$, $\omega = 1 - \frac{\alpha}{2}\varepsilon \sin T$ and where we have neglected terms of $O(\varepsilon^2)$. Now we expand $u = u_0 + \varepsilon u_1 + O(\varepsilon^2)$ and collect terms, giving:

$$u_{0_{\xi\xi}} + u_0 = 0 \tag{22}$$

$$\begin{aligned} u_{1_{\xi\xi}} + u_1 = -2u_{0_{\xi\eta}} - \alpha\sin T\ u_{0_{\xi\xi}} + 8(1 + \alpha\cos T)\cos\xi\sin\xi\ u_0 \\ -4(1 + \alpha\cos T)\cos^2\xi\ u0_\xi + \alpha\beta u_{0_\xi}(\xi - T, \eta - \varepsilon T) \end{aligned} \tag{23}$$

We take the solution of (22) in the form

$$u_0 = A(\eta)\cos\xi + B(\eta)\sin\xi \tag{24}$$

Note that $u_0(\xi - T, \eta - \varepsilon T) = A_d\cos(\xi - T) + B_d\sin(\xi - T)$ where

$$A_d = A(\eta - \varepsilon T) \quad \text{and} \quad B_d = B(\eta - \varepsilon T) \tag{25}$$

Substituting (24),(25) into (23) and eliminating secular terms gives the slow flow

$$\frac{dA}{d\eta} = -A - \frac{3\,\alpha\,A\,\cos T}{2} + \frac{\alpha\,B\,\sin T}{2} + \frac{\alpha\,A_d\,\beta\,\cos T}{2} - \frac{\alpha\,\beta\,B_d\,\sin T}{2} \tag{26}$$

$$\frac{dB}{d\eta} = -\frac{\alpha\,A\,\sin T}{2} - \frac{\alpha\,B\,\cos T}{2} + \frac{\alpha\,A_d\,\beta\,\sin T}{2} + \frac{\alpha\,\beta\,B_d\,\cos T}{2} \tag{27}$$

The slow flow (26),(27) is a system of linear delay-differential equations (DDEs). A common approach to treating such a system is to replace the delayed variables by non-delayed variables, as in $A_d = A$ and $B_d = B$. To support such a step, it is often argued that since a Taylor expansion gives $A(\eta - \varepsilon T) = A(\eta) + O(\varepsilon)$, the replacement of A_d by A is an approximation valid for small ε [1],[2],[4]. Let us follow this procedure and see what we get, and then compare results with what we would get by treating the system as a DDE.

Replacing A_d and B_d by A and B, we obtain the ODE system:

$$\frac{d}{d\eta}\begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} -1 + \frac{\alpha}{2}(\beta - 3)\cos T & \frac{\alpha}{2}(1 - \beta)\sin T \\ -\frac{\alpha}{2}(1 - \beta)\sin T & -\frac{\alpha}{2}(1 - \beta)\cos T \end{pmatrix}\begin{pmatrix} A \\ B \end{pmatrix} \tag{28}$$

Ref

1. Atay, F.M. (1998) *Van der Pol's oscillator under delayed feedback*, Journal of Sound and Vibration 218(2): 333-339.

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Recall from eqs.(17)-(19) that there are two cases, $\beta = +1$ and $\beta = -1$. In the case that $\beta = -1$, the system (28) becomes:

$$\frac{d}{d\eta}\begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} -1 - 2\alpha\cos T & \alpha\sin T \\ -\alpha\sin T & -\alpha\cos T \end{pmatrix}\begin{pmatrix} A \\ B \end{pmatrix} \tag{29}$$

This system of ODEs exhibits both Hopf and saddle-node bifurcations. The Hopf bifurcations occur when the trace $= -1 - 3\alpha\cos T = 0$ with positive determinant, i.e. when

$$\text{Hopf bifurcations:} \quad \alpha = -\frac{1}{3\cos T} \tag{30}$$

The saddle-node bifurcations occur when the determinant $= \alpha^2 + \alpha\cos T + \alpha^2\cos^2 T = 0$, i.e. when

$$\text{saddle-node bifurcations:} \qquad \alpha = 0 \quad \text{and} \quad \alpha = -\frac{\cos T}{1+\cos^2 T} \tag{31}$$

Now let us consider the other case, $\beta = +1$. In this case the ODE system (28) becomes:

$$\frac{d}{d\eta}\begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} -1 - \alpha\cos T & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} A \\ B \end{pmatrix} \tag{32}$$

From eq.(11) we see that the amplitude of the in-phase mode, whose stability we are investigating, is given by $R = 2\sqrt{1+\alpha\cos T}$. Inspection of (32) shows that that system exhibits the birth of the in-phase mode when

$$\text{birth of the in-phase mode:} \quad \alpha = -\frac{1}{\cos T} \tag{33}$$

We note that all of the bifurcations (30),(31),(33) were observed by Wirkus and Rand [5] in their original work on this system.

In what follows, we return to the DDE system (26)-(27), but we do not make the simplifying assumption of replacing A_d by A and B_d by B. In order to treat the DDE system, we set:

$$A = Pe^{\lambda\eta}, \quad B = Qe^{\lambda\eta}, \quad A_d = Pe^{\lambda(\eta - \varepsilon T)}, \quad B_d = Qe^{\lambda(\eta-\varepsilon T)} \tag{34}$$

where P and Q are constants. We are particularly interested in the effect of the DDE slow flow on Hopf bifurcations, eq.(30). Thus we restrict ourselves to the case $\beta = -1$, whereby we obtain the following pair of algebraic equations on P and Q:

$$\begin{pmatrix} -\frac{\alpha e^{-\lambda\varepsilon T}\cos T}{2} - \frac{3\alpha\cos T}{2} - \lambda - 1 & \frac{\alpha e^{-\lambda\varepsilon T}\sin T}{2} + \frac{\alpha\sin T}{2} \\ -\frac{\alpha e^{-\lambda\varepsilon T}\sin T}{2} - \frac{\alpha\sin T}{2} & -\frac{\alpha e^{-\lambda\varepsilon T}\cos T}{2} - \frac{\alpha\cos T}{2} - \lambda \end{pmatrix}\begin{pmatrix} P \\ Q \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{35}$$

For a nontrivial solution, the determinant must vanish:

$$\alpha\cos T\,\lambda\, e^{-\varepsilon T\lambda} - \frac{\alpha^2\sin^2 T\, e^{-\varepsilon T\lambda}}{2} + \frac{\alpha\cos T\, e^{-\varepsilon T\lambda}}{2} + \alpha^2 e^{-\varepsilon T\lambda}$$

$$+\frac{\alpha^2 e^{-2\varepsilon T\lambda}}{4} + \lambda^2 + 2\alpha\cos T\,\lambda + \lambda - \frac{\alpha^2\sin^2 T}{2} + \frac{\alpha\cos T}{2} + \frac{3\alpha^2}{4} = 0 \tag{36}$$

Notes

5. Wirkus, S. and Rand, R. (2002) *Dynamics of Two Coupled van der Pol Oscillators with Delay Coupling*, Nonlinear Dynamics 30:205-221.

For ε=0 and $\beta = -1$ the DDE system (26), (27) reduces to the ODE system (29), which exhibits a Hopf bifurcation (30) when $\alpha = -\frac{1}{3\cos T}$. By determining the location of the associated Hopf bifurcation in (36) for $\varepsilon > 0$ we may assess the accuracy of the often made approximation based on replacing the delayed variables in the slow flow with non-delayed variables.

In the case of the ODE system (29), we obtained the conditions for a Hopf bifurcation, namely that there be a pair of pure imaginary eigenvalues, by requiring the trace of the matrix (29) to be zero. In the case of the DDE system (26), (27), we seek a Hopf bifurcation by setting $\lambda = i\Omega$ in (36).

We seek a solution to (36) in the form of a perturbation series in ε by expanding

$$T = T_0 + \varepsilon T_1 + \varepsilon^2 T_2 + \cdots \tag{37}$$

$$\Omega = \Omega_0 + \varepsilon \Omega_1 + \varepsilon^2 \Omega_2 + \cdots \tag{38}$$

Separating (36) into real and imaginary parts and collecting like powers of ε allows us to obtain the following expressions:

$$\cos T_0 = -\frac{1}{3\alpha} \tag{39}$$

$$\Omega_0 = \frac{\sqrt{9\,\alpha^2 - 2}}{3} \tag{40}$$

$$T_1 = -\frac{\sqrt{9\,\alpha^2 - 1}\;T_0}{9} \tag{41}$$

$$\Omega_1 = -\frac{(18\,\alpha^2 - 5)\;T_0}{54\sqrt{9\,\alpha^2 - 2}} \tag{42}$$

$$T_2 = \frac{\sqrt{9\,\alpha^2 - 1}\;(27\,\alpha^2 - 6)\;{T_0}^2 + (162\,\alpha^4 - 36\,\alpha^2 + 2)\;T_0}{1458\,\alpha^2 - 162} \tag{43}$$

$$\Omega_2 = \frac{\sqrt{9\alpha^2 - 2}\left((-8019\,\alpha^6 + 5346\,\alpha^4 - 1206\alpha^2 + 91)\,{T_0}^2 + \sqrt{9\alpha^2 - 1}\,(648\alpha^4 - 324\,\alpha^2 + 40)\,T_0\right)}{157464\,\alpha^4 - 69984\,\alpha^2 + 7776} \tag{44}$$

IV. Results and Conclusions

To summarize our treatment of the original coupled van der Pol eqs.(1),(2), we first used Lindstedt's method to obtain an approximate expression for the in-phase mode, eq.(12). Then we studied the stability of the in-phase mode by applying the two variable perturbation method to eq.(20). This resulted in the DDE slow flow (26), (27). We investigated this system of equations in two ways:

1) First we followed a number of other works [1],[2],[4] by replacing the delayed variables A_d and B_d by non-delayed variables A and B. This resulted in a system of ODEs (28) which possessed Hopf and saddle-node bifurcations, in agreement with the earlier work of Wirkus and Rand [5].
2) Then we treated the slow flow system (26),(27) as DDEs which resulted in a transcendental characteristic equation (36) which is harder to solve than the more familiar polynomial characteristic equations of ODEs. We sought a series solution (37), (38) and obtained the results listed in eqs.(39)-(44).

The results are plotted in Fig.1 where we show the critical delay for Hopf bifurcation T_{Hopf} versus coupling strength α for $\varepsilon = 0.5$. The dashed curve is the analytical

Ref

2. Morrison, T.M. and Rand, R.H. (2007) 2:1 *Resonance in the delayed nonlinear Mathieu equation*, Nonlinear Dynamics 50: 341-352.

approximation based on replacing the delay terms in the slow flow (26),(27) by non-delayed variables, as in $A_d = A$ and $B_d = B$. Thus the dashed curve corresponds to $\varepsilon = 0$. The dash-dot curve and the solid curve correspond to the analytical approximations respectively given by 2- and 3-term truncations of eq.(37). The + signs represent stability transitions obtained by numerical integration of the DDEs (26)-(27).

The comparison between the various approximations for the critical delay for Hopf bifurcation shown in Fig.1 is further explored in Table 1, where we list the errors obtained using 1-, 2- and 3-term truncations of eq.(37) compared to values obtained by numerical integration of the DDEs (26)-(27). The maximum error is computed over the set $\alpha \in [\frac{\sqrt{2}}{3}, 1]$. (Here $\alpha = \frac{\sqrt{2}}{3}$ is chosen because $(\alpha, T) = (\frac{\sqrt{2}}{3}, \frac{3\pi}{4})$ is a point of intersection of the bifurcation curves $\alpha = -\frac{\cos T}{1+\cos^2 T}$ and $\alpha = -\frac{1}{3\cos T}$, compare eqs.(30),(31).) We consider *absolute error* $\max\limits_{\alpha \in [\frac{\sqrt{2}}{3},\, 1]} |T(\alpha) - T_n(\alpha)|$, *relative error* $\max\limits_{\alpha \in [\frac{\sqrt{2}}{3},\, 1]} |\frac{T(\alpha)-T_n(\alpha)}{T(\alpha)}|$, and *percent error* $\max\limits_{\alpha \in [\frac{\sqrt{2}}{3},\, 1]} |\frac{T(\alpha)-T_n(\alpha)}{T(\alpha)}| \times 100\%$, where $T(\alpha)$ is the value we get by numerical integration of the DDEs (26)-(27) and $T_n(\alpha)$ is the value given by the n-term truncation of eq.(37). We also note that the maximum in all three cases was attained at $\alpha = 1$.

As previously noted, replacing the delay terms in the slow flow (26),(27) by non delayed variables means that we take only the first term in eq.(37) and $n = 1$. In this case, the error incurred by omitting the delay terms in the slow flow is generally about 3 to 15% for small values of ε. On the other hand, the 3-term truncation of eq.(37) typically has the percent error less than 1%.

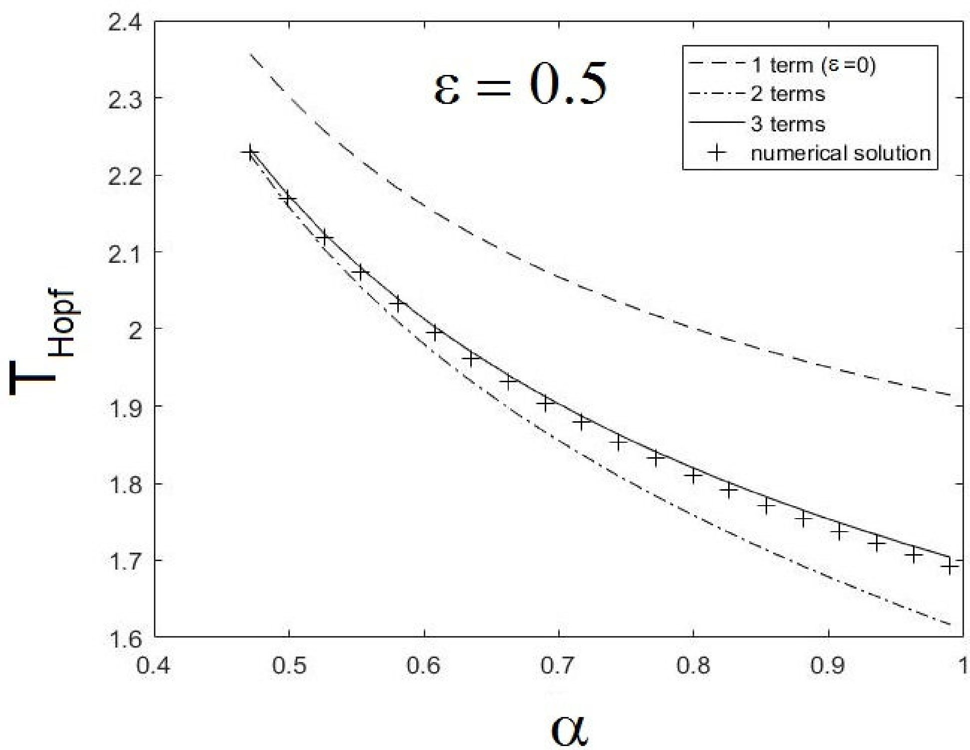

Figure 1: Critical delay for Hopf bifurcation versus α for $\varepsilon = 0.5$. Dashed curve is the analytical approximation based on replacing the delay terms in the slow flow (26), (27) by non-delayed variables, as in $A_d = A$ and $B_d = B$. Thus the dashed curve corresponds to $\varepsilon = 0$. The dash-dot curve and the solid curve correspond to the analytical approximations respectively given by 2- and 3-term truncations of eq.(37). The + signs represent stability transitions obtained by numerical integration of the DDEs (26)-(27).

Notes

Notes

Table 1: Errors in critical delay for Hopf bifurcation produced by 1-, 2- and 3-term truncations of eq.(37) as compared to values obtained by numerical integration of the slow flow (26)-(27). The errors are defined as *absolute error* $\max_{\alpha\in[\frac{\sqrt{2}}{3},\,1]} |T(\alpha)-T_n(\alpha)|$, *relative error* $\max_{\alpha\in[\frac{\sqrt{2}}{3},\,1]} |\frac{T(\alpha)-T_n(\alpha)}{T(\alpha)}|$, and *percent error* $\max_{\alpha\in[\frac{\sqrt{2}}{3},\,1]} |\frac{T(\alpha)-T_n(\alpha)}{T(\alpha)}|\times 100\%$, where $T(\alpha)$ is the value we get by numerical integration of the DDEs (26)-(27) and $T_n(\alpha)$ is the value given by the n-term truncation of eq.(37).

	$\varepsilon = 0.1$			$\varepsilon = 0.3$			$\varepsilon = 0.5$		
n terms in (37)	n=1	n=2	n=3	n=1	n=2	n=3	n=1	n=2	n=3
absolute error	0.056	0.0025	0.001	0.1499	0.0285	0.003	0.2218	0.0756	0.012
relative error	0.0307	0.0013	0.0005	0.085	0.0162	0.0017	0.1311	0.0447	0.0071
percent error	3.07%	0.13%	0.05%	8.5%	1.62%	0.17%	13.11%	4.47%	0.71%

V. Acknowledgement

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Restricted Three-Body Problem with Albedo Effect when Smaller Primary is an Oblate Spheroid

By M. Javed Idrisi & M. Shahbaz Ullah
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Abstract- The present paper deals with the existence and stability of libration points in restricted three-body problem with Albedo effect when less massive primary is an oblate spheroid. Since, the spacecraft is affected by both radiations i.e radiation pressure as well as Albedo. In this paper this is investigated how Albedo perturbed the libration points and its stability? It is found that there exist five libration points, three collinear and two non-collinear, the non-collinear libration points are stable for a critical value of mass parameter $0 < \mu < \mu_c$ where μ_c = 0.0385208965 ... - (0.00891747 + 0.222579k) α -0.0627796 σ but collinear libration points are still unstable. Also, an example of Sun-Earth system is taken in the last as a real application.

Keywords: restricted three-body problem, radiation pressure, albedo effect, libration points, stability.

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Strictly as per the compliance and regulations of:

Restricted Three-Body Problem with Albedo Effect when Smaller Primary is an Oblate Spheroid

M. Javed Idrisi [α] & M. Shahbaz Ullah [σ]

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Keywords: *restricted three-body problem, radiation pressure, albedo effect, libration points, stability.*

I. Introduction

The restricted three-body problem is one of well known problem in the field of celestial mechanics in which two finite bodies called primaries move around their center of mass in circular or elliptic orbits under the influence of their mutual gravitational attraction and a third body of infinitesimal mass is moving in the plane of the primaries which is attracted by the primaries and influenced by their motion but not influencing them. In classical case there exist five libration points out of which three are collinear and two are non-collinear. The collinear libration points L_1, L_2 and L_3 are unstable for $0 \le \mu \le \frac{1}{2}$ and the non collinear libration points $L_{4,5}$ are stable for a critical value of mass parameter $\mu < \mu_c = 0.03852\ldots$, Szehebely (1967). Some studies related to the equilibrium points in R3BP or ER3BP, taken into account the oblateness and triaxiality of the primaries, Coriolis and Centrifugal forces, Yarkovsky effect, variation of the masses of the primaries and the infinitesimal mass etc. are discussed by Danby (1964); Vidyakin (1974); Sharma (1975); Choudhary R. K. (1977); Subbarao and Sharma (1975); Cid R. et. al. (1985); El-Shaboury (1991); Bhatnagar et al. (1994); Selaru D. et.al. (1995); Markellos et al. (1996); Subbarao and Sharma (1997); Khanna and Bhatnagar (1998, 1999); Roberts G.E. (2002); Oberti and Vienne (2003); Perdiou et. al. (2005); Sosnytskyi (2005); Ershkov (2012); Arredondo et.al. (2012); Idrisi and Taqvi (2013); Idrisi (2014); Idrisi and Amjad (2015). The photo-gravitational restricted three-body problem arises from the classical problem if one or both primaries is an intense emitter of radiation, formulated by Radzievskii (1950). He has considered only the central forces of gravitation and radiation pressure on the particle of infinitesimal mass without considering the other two components of light pressure field and studied this problem for three specific

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bodies; the Sun, a planet and a dust particle. The radiation repulsive force F_p exerted on a particle can be represented in terms of gravitational attraction F_g (Radzievskii, 1950) as $F_p = F_g(1-q)$, where $q = 1 - \frac{F_p}{F_g}$, a constant for a given particle, is a reduction factor expressed in terms of the particle radius a, density δ and radiation-pressure efficiency factor x (in c.g.s. system) as:

$$q = 1 - \frac{5.6*10^{-3}}{a\delta}x$$

The assumption that q is a constant implies that the fluctuations in the beam of solar radiation and the effect of planets shadow are neglected. Typical values for diameter of IDP (Interplanetary Dust Particles) are in the range of 50 - 500 μm and their densities range is $1 - 3g/cm^3$ with an average density of $2g/cm^3$. As the size of the particles increases, their density decreases (Grűn et.al. 2001). Some of the notable research in PRTBP are carried by Chernikov (1970); Bhatnagar and Chawla (1979); Schuerman D.W (1980); Simmons et. al. (1985); Kunitsyn and Tureshbaev (1985); Lukyanov (1988), Sharma (1987); Xuetang et.al. (1993); Ammar (2008); Singh and Leke (2010); Douskos (2010); Katour et.al. (2014) etc. In 2012, S. V. Ershkov studied the Yarkovsky effect in generalized photogravitational 3-body problem and proved the existence of maximally 256 different non-planar equilibrium points when second primary is non-oblate spheroid. The main contribution of the natural radiation pressure on the satellite is due to the direct solar radiation and the second main contribution of radiation forces is due to the Earth reflected radiation known as the Albedo studied by Anselmo et.al. (1983); Nuss (1998); McInnes (2000); Bhanderi (2005); Pontus (2005); MacDonald (2011) etc. Albedo effect is one of the most interesting non-gravitational force having significant effects on the motion of infinitesimal mass. Albedo is the fraction of solar energy reflected diffusely from the planet back into space (Harris and Lyle, 1969). It is the measure of the reflectivity of the planets surface. Therefore, the Albedo can be defined as the fraction of incident solar radiation returned to the space from the surface of the planet (Rocco, 2009) as

$$Albedo = \frac{radiation\ reflected\ back\ to\ space}{incident\ radiation}$$

In this paper the Albedo effect on the existence and stability of the libration points when smaller primary is a homogeneous ellipsoid has been studied. This paper is divided into five sections. In section-2, the equations of motion are derived. The existence of non-collinear and collinear libration points is shown in section-3. In section-4, the stability of non-collinear and collinear libration points is discussed. In section-5, a real application to Sun-Earth system has shown. In the last section, all the results are discussed.

II. Equations of Motion

Let m_2 be an oblate spheroid with axes a, b and c $(a = b > c)$ and m_1 a point mass and a source of radiation such that $m_1 > m_2$, are moving in the circular orbits around their center of mass O. An infinitesimal mass m_3 $<< 1$, is moving in the plane of motion of m_1 and m_2. The distances of m_3 from m_1, m_2 and O are r_1, r_2 and r respectively. F_1 and F_2 are the gravitational forces acting on m_3 due to m_1 and m_2 respectively, F_p is the solar radiation pressure on m_3 due to m_1 and F_A is the Albedo force (solar radiation reflected by m_2 in space) on m_3 due to m_2 (Fig. 1). Also, let us consider that the principal axes of spheroid remains parallel to the synodic axes $Oxyz$ throughout the motion and the equatorial plane of m_2 is coincide with the plane of motion of m_1 and m_2. Let the line joining m_1 and m_2 be taken as $X - axis$ and O their center of mass as origin. Let the line passing through O and perpendicular to OX and lying in the plane of motion m_1 and m_2 be the $Y - axis$. Let us consider a synodic system of co-ordinates $Oxyz$ initially coincide with the inertial system $OXYZ$, rotating with angular

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velocity ω about $Z-axis$ (the $z-axis$ is coincide with $Z-axis$). We wish to find the equations of motion of m_3 using the terminology of Szebehely (1967) in the synodic co-ordinate system and dimensionless variables *i.e.* the distance between the primaries is unity, the unit of time t is such that the gravitational constant $G=1$ and the sum of the masses of the primaries is unity *i.e.* $m_1+\ m_2=1$.

The force acting on m_3 due to m_1 and m_2 is F_1 (1 - F_p/F_1)=$F_1(1-\alpha)$ and F_2 (1 - F_A/F_2) = F_2 ($1-\beta$) respectively, where $\alpha=\ F_p/F_1\ <<1$ and $\beta=\ F_A/F_2\ <<1$. Also, α and β can be expressed as:

$$\alpha=\frac{L_1}{2\pi G\ m_1 c\sigma};\ \ \beta=\frac{L_2}{2\pi G\ m_2 c\sigma}$$

where L_1 is the luminosity of the large primary m_1, L_2 is the luminosity of small primary m_2, G is the gravitational constant, c is the speed of velocity

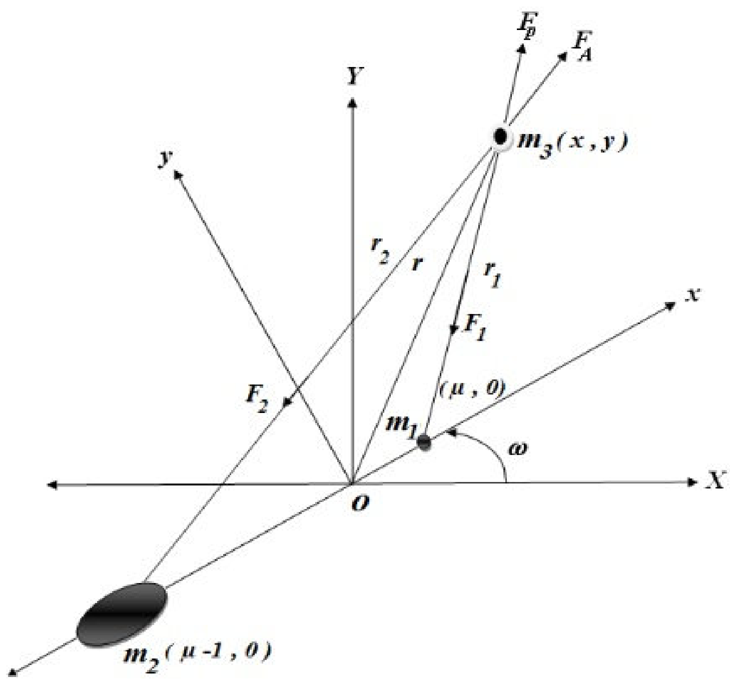

Fig. 1: Configuration of the R3BP under Albedo effect when m_2 is an oblate spheroid

and σ is mass per unit area. Now,

$$\frac{\beta}{\alpha}=\frac{L_2}{L_1}\frac{m_1}{m_2}\Rightarrow\beta=\alpha\left(\frac{1-\mu}{\mu}\right)k;k=\frac{L_2}{L_1}=constant. \tag{1}$$

The equations of motion of the infinitesimal mass m_3 in the synodic coordinate system and dimensionless variables are given by

$$\ddot{x}-2n\dot{y}=\Omega_x;\ddot{y}+2n\dot{x}=\Omega_y \tag{2}$$

where

$$\Omega=\frac{n^2}{2}\{(1-\mu)\ r_1^2\}+\frac{(1-\mu)(1-\alpha)}{r_1}+\frac{\mu(1-\beta)}{r_2}\left(1+\frac{\sigma}{2r_2^2}\right)$$

$$\Omega_x=\ n^2x-\frac{(1-\mu)(x-\mu)(1-\alpha)}{r_1^3}-\frac{\mu(x+1-\mu)(1-\beta)}{r_2^3}\left(1+\frac{3\sigma}{2r_2^2}\right)$$

$$\Omega_y=y\left\{n^2-\frac{(1-\mu)(1-\alpha)}{r_1^3}-\frac{\mu(1-\beta)}{r_2^3}\left(1+\frac{3\sigma}{2r_2^2}\right)\right\}$$

$$n^2 = 1 + \frac{3\sigma}{2} \; is\ the\ mean\ motion\ of\ the\ primaries, \tag{3}$$

$$\sigma = \frac{a^2 - c^2}{5} \; is\ the\ oblateness\ factor,$$

$$r_1^2 = (x-\mu)^2 + y^2, \tag{4}$$

$$r_2^2 = (x+1-\mu)^2 + y^2, \tag{5}$$

$$0 < \mu = \frac{m_2}{m_1 + m_2} < \frac{1}{2} \Rightarrow m_1 = 1-\mu;\ m_2 = \mu.$$

III. Libration Points

At the libration points all the derivatives are zero $i.e.$

$$\dot{x} = 0, \dot{y} = 0, \ddot{x} = 0, \ddot{y} = 0, \Omega_x = 0, \Omega_y = 0.$$

Therefore, the libration points are the solutions of the equations

$$\Omega_x = n^2 x - \frac{(1-\mu)(x-\mu)(1-\alpha)}{r_1^3} - \frac{\mu(x+1-\mu)(1-\beta)}{r_2^3}\left(1 + \frac{3\sigma}{2r_2^2}\right) = 0$$

$$\Omega_y = y\left\{n^2 - \frac{(1-\mu)(1-\alpha)}{r_1^3} - \frac{\mu(1-\beta)}{r_2^3}\left(1 + \frac{3\sigma}{2r_2^2}\right)\right\} = 0$$

a) Non-collinear Libration Points

The non-collinear libration points are the solution of the Equations $\Omega_x = 0$ and $\Omega_y = 0,\ y \neq 0$ $i.e.$

$$n^2 x - \frac{(1-\mu)(x-\mu)(1-\alpha)}{r_1^3} - \frac{\mu(x+1-\mu)(1-\beta)}{r_2^3}\left(1 + \frac{3\sigma}{2r_2^2}\right) = 0 \tag{6}$$

$$n^2 - \frac{(1-\mu)(1-\alpha)}{r_1^3} - \frac{\mu(1-\beta)}{r_2^3}\left(1 + \frac{3\sigma}{2r_2^2}\right) = 0 \tag{7}$$

On substituting $\sigma = 0$, $\alpha = 0$ and $\beta = 0$, the solution of Eqns. (6) and (7) is $r_1 = 1$, $r_2 = 1$ and from Eqn. (3), $n = 1$. Now we assume that the solution of Eqns. (6) and (7) for $\sigma \neq 0$, $\alpha \neq 0$ and $\beta \neq 0$ as $r_1 = 1 + \xi_1$, $r_2 = 1 + \xi_2$, $\xi_1, \xi_2 << 1$. Substituting these values of r_1 and r_2 in the Eqns. (4) and (5), we get

$$x = \mu - \frac{1}{2} + \xi_2 - \xi_1;\ y = \pm\frac{\sqrt{3}}{2}\left\{1 + \frac{2}{3}(\xi_2 + \xi_1)\right\} \tag{8}$$

Table 1: Non-collinear Libration Points $L_{4,5}(x \pm y)$ for $\mu = 0.1$ and $\sigma = 10^{-3}$

	$k=0$	$k=0$	$k=0.01$	$k=0.01$	$k=0.1$	$k=0.1$
α	x	$\pm y$	x	$\pm y$	x	$\pm y$
0.0	−0.399512	0.865737	−0.399512	0.865737	−0.399512	0.865737
0.1	−0.366167	0.846492	−0.369167	0.844761	−0.396167	0.829171
0.2	−0.332833	0.827247	−0.338833	0.823783	−0.392833	0.792606
0.3	−0.299512	0.808002	−0.308513	0.800806	−0.389512	0.756041
0.4	−0.266167	0.788757	−0.278167	0.781828	−0.386167	0.719475
0.5	−0.232833	0.769512	−0.247833	0.760851	−0.382833	0.682909
0.6	−0.199511	0.750267	−0.217502	0.739874	−0.379510	0.646344
0.7	−00.166167	0.731022	−0.187167	0.718897	−0.376167	0.609778
0.8	−0.132833	0.711777	−0.156833	0.697921	−0.372833	0.573213
0.9	−0.099512	0.692532	−0.126502	0.676943	−0.369514	0.536647

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Now, substituting the values of $x,\ y$ from Eqns. (8), $r_1 = 1+\xi_1$ and $r_2 = 1+\xi_2$ in the Eqns. (6) and (7) and neglecting higher order terms, we obtain

$$\xi_1 = -\frac{\alpha}{3} + \frac{1}{2}\left(1 - \frac{\mu}{1-\mu}\right)\sigma;\ \xi_2 = -\frac{\beta}{3}$$

Thus, the coordinates of the non-collinear libration points $L_{4,5}$ are

$$x = \mu - \frac{1}{2} + \frac{1}{3}(\alpha - \beta) + \frac{\sigma}{2},$$

$$y = \pm\frac{\sqrt{3}}{2}\left[1 - \frac{2}{3}\left\{\frac{1}{3}(\alpha+\beta) + \frac{\sigma}{2}\right\}\right]$$

Using the relation (1) *i.e.* $\beta = \alpha(1-\mu)k/\mu$, we have

$$x = \mu - \frac{1}{2} + \frac{1}{3}\left\{1 - \frac{(1-\mu)k}{\mu}\right\}\alpha + \frac{\sigma}{2}, \tag{9}$$

$$y = \pm\frac{\sqrt{3}}{2}\left[1 - \frac{2}{3}\left\{\frac{\alpha}{3}\left(1 + \frac{(1-\mu)k}{\mu}\right)\alpha + \frac{\sigma}{2}\right\}\right] \tag{10}$$

Thus, we conclude that there exist two non-collinear libration points $L_{4,5}$ and these points are affected by oblateness as well as Albedo effect (Fig. 2), also these points form scalene triangle with the primaries as $r_1 \neq r_2$. The numerical location of $L_{4,5}$ is also calculated in Table 1 for $\mu = 0.1,\ \sigma = 0.001$ and different values of α and k and it is found that the abscissa and ordinate of non-collinear libration points are the decreasing functions of α and k *i.e.* as α and k increases, x and y decreases. For $\alpha = 0$, the results are in conformity with those of Bhatnagar and Hallan (1979). If $\alpha = 0$ and $\sigma = 0$, the classical case of the restricted three body problem is verified (Szebehely, 1967).

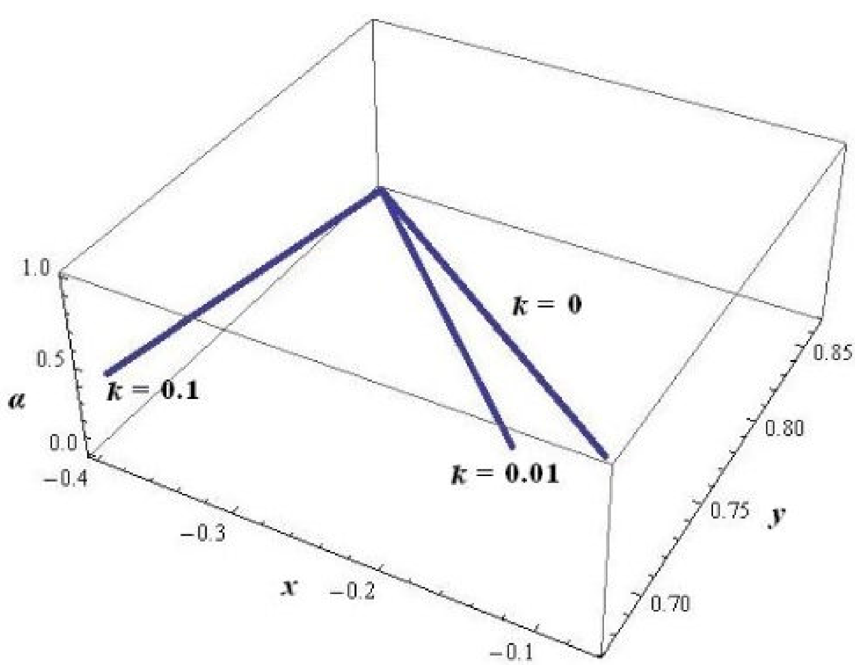

Fig. 2: L_4 verses α; $\mu = 0.1,\ \sigma = 10^{-3}$

b) Collinear Libration Points

The collinear libration points are the solution of the Equations $\Omega_x = 0$ and $y = 0$ *i.e.*

$$f(x) = n^2 x - \frac{(1-\mu)(x-\mu)(1-\alpha)}{r_1^3} - \frac{\mu(x+1-\mu)(1-\beta)}{r_2^3}\left(1+\frac{3\sigma}{2r_2^2}\right) = 0 \tag{11}$$

where $r_i = |x - x_i|, i = 1, 2$ is a seventh degree equation in x.

Since $f(x) > 0$ in each of the open interval $(-\infty, \mu-1)$, $(\mu-1, \mu)$ and (μ, ∞), the function $f(x)$ is strictly increasing in each of them. Also, $f(x) \to -\infty$ as $x \to -\infty$, $(\mu-1)+0$ or $(\mu+0)$ and $f(x) \to \infty$ as $x \to \infty$, $(\mu-1)-0$ or $(\mu-0)$. Therefore, there exists only one value of x in each of the open intervals $(-\infty, \mu-1)$, $(\mu-1, \mu)$ and (μ, ∞) such that $f(x) = 0$. Further, $f(\mu-2) < 0$, $f(0) \geq 0$ and $f(\mu+1) > 0$. Therefore, there are only three real roots lying in each of the intervals $(\mu-2, \mu-1)$, $(\mu-1, 0)$ and $(\mu, \mu+1)$. Thus there are only three collinear libration points.

From the Fig. 3, this is observed that the first collinear libration point L_1 always lie at the right of the primary m_2, the second libration point L_2 lies

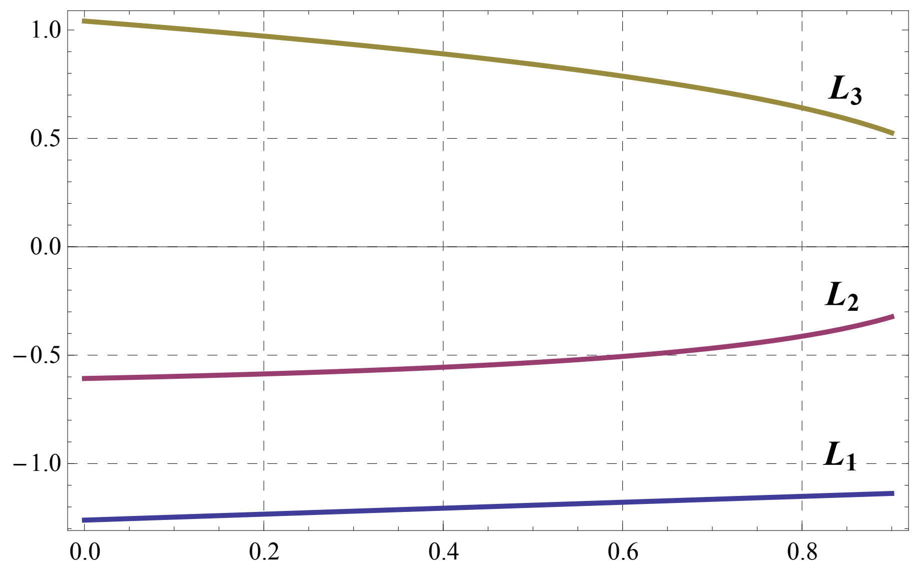

Fig. 3: α verses $L_i, i = 1, 2, 3;$ $\mu = 0.1,$ $\sigma = 10^{-3}$

between the center of mass of the primaries O and m_1 and the third libration point L_3 lies at the right of the primary m_1. This is also observed that the libration points L_1 move away from the center of mass as α increases while the second and third libration points L_2 and L_3 move toward the center of mass as α increases.

The Equation (11) determines the location of the collinear libration points $L_1(x_1, 0)$ $L_2(x_2, 0)$ and $L_3(x_3, 0)$ lie in the intervals $(-\infty, \mu-1)$, $(\mu-1, \mu)$ and (μ, ∞) respectively, where

$$x_1 = \mu - 1 - \xi_1,$$
$$x_2 = \mu - 1 + \xi_2,$$
$$x_3 = \mu + \xi_3. \tag{12}$$

Since the libration point L_1 lies in the interval $(-\infty, \mu-1)$ *i.e.* left to the smaller primary, we have $r_1 = \mu - x_1$ and $r_2 = \mu - 1 - x_1$ which when substituted in Equation (11), gives

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$$n^2x + \frac{(1-\mu)(x-\mu)(1-\alpha)}{r_1^3} + \frac{\mu(x+1-\mu)(1-\beta)}{r_2^3}\left(1+\frac{3\sigma}{2r_2^2}\right) = 0 \quad (13)$$

Similarly, for $L_2(x_2, 0)$ and $L_3(x_3, 0)$ the Equation (11) becomes

$$n^2x + \frac{(1-\mu)(x-\mu)(1-\alpha)}{r_1^3} - \frac{\mu(x+1-\mu)(1-\beta)}{r_2^3}\left(1+\frac{3\sigma}{2r_2^2}\right) = 0 \quad (14)$$

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$$n^2x - \frac{(1-\mu)(x-\mu)(1-\alpha)}{r_1^3} - \frac{\mu(x+1-\mu)(1-\beta)}{r_2^3}\left(1+\frac{3\sigma}{2r_2^2}\right) = 0 \quad (15)$$

Table 2: Collinear Libration Points $L_i(x_{i,0})(i = 1, 2, 3)$ for $k = 0$

	$\mu = .1, \sigma = 10^{-3}, k = 0$	$\mu = .1, \sigma = 10^{-3}, k = 0$	$\mu = .1, \sigma = 10^{-3}, k = 0$
α	L_1	L_2	L_3
0.0	−1.26086	−0.607519	1.04112
0.1	−1.25296	−0.594138	1.00813
0.2	−1.24529	−0.578763	0.972618
0.3	−1.23783	−0.560863	0.934029
0.4	−1.23061	−0.539686	0.891595
0.5	−1.22361	−0.514114	0.844181
0.6	−1.21684	−0.482382	0.789997
0.7	−1.21029	−0.441432	0.725923
0.8	−1.20396	−0.385085	0.645599
0.9	−1.19786	−0.296465	0.531473

Table 3: Collinear Libration Points $L_i(x_{i,0})(i = 1, 2, 3)$ for $k = 0.01$

	$\mu = .1, \sigma = 10^{-3}, k = .01$	$\mu = .1, \sigma = 10^{-3}, k = .01$	$\mu = .1, \sigma = 10^{-3}, k = .01$
α	L_1	L_2	L_3
0.0	−1.26086	−0.607519	1.04112
0.1	−1.25182	−0.594889	1.00806
0.2	−1.24295	−0.580271	0.972458
0.3	−1.23431	−0.563125	0.933778
0.4	−1.22588	−0.542681	0.891246
0.5	−1.21768	−0.517794	0.843722
0.6	−1.20971	−0.486666	0.789412
0.7	−1.20197	−0.446175	0.725189
0.8	−1.19446	−0.390047	0.644678
0.9	−1.18717	−0.301225	0.530279

The solution of Equations (13), (14) and (15) is given in Table 2.

For $k = 0$, the solutions obtained for the Equations (13), (14) and (15) are the libration points $L_i, (i = 1, 2, 3)$ in the photogravitational restricted three-body problem when smaller primary is an oblate body but if $k \neq 0$ the libration points $L_i, (i = 1, 2)$ are affected by the Albedo and this effect displaced the libration points from its actual position as shown in Fig. 4 and 5 but L_3 not much affected by albedo, see Fig. 6.

IV. Stability of Libration Points

The variational equations are obtained by substituting $x = x_0 + \xi$ and $y = y_0 + \eta$ in the equations of motion (2), where (x_0, x_0) are the coordinates of L_4 or L_5 and $\xi, \eta << 1$ *i.e.*

$$\ddot{\xi} - 2n\dot{\eta} = \xi\Omega^0_{xx} + \eta\Omega^0_{xy},$$

$$\ddot{\eta} + 2n\dot{\xi} = \xi\Omega^0_{xy} + \eta\Omega^0_{yy}. \quad (16)$$

Here we have taken only linear terms in ξ and η. The subscript in Ω indicates the second partial derivative of Ω and superscript o indicates that the

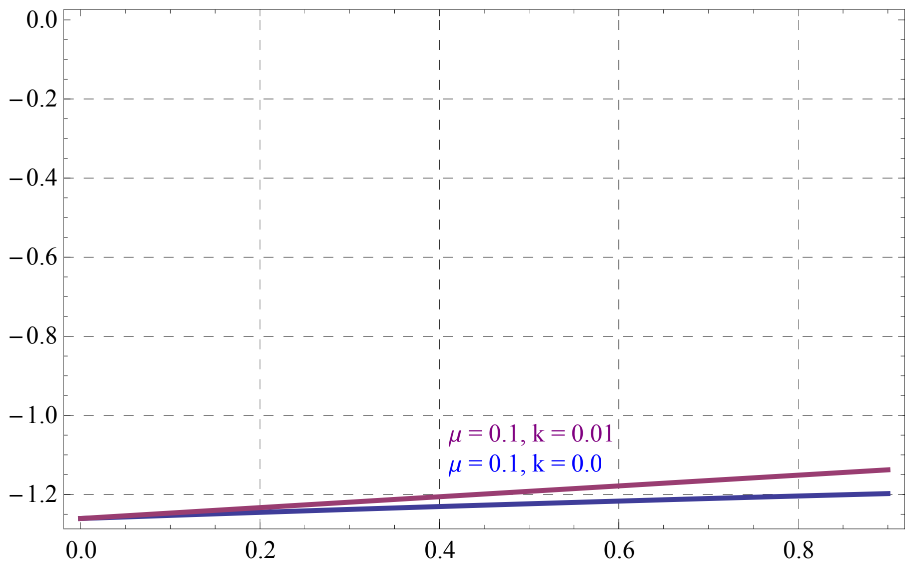

Fig. 4: α versus L_1; $\sigma = 10^{-3}$

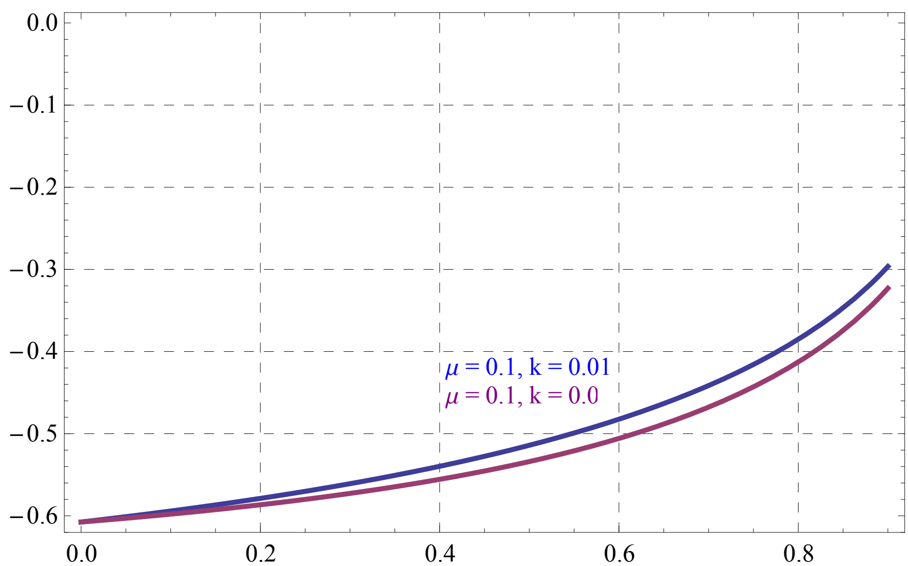

Fig. 5: α versus L_2; $\sigma = 10^{-3}$

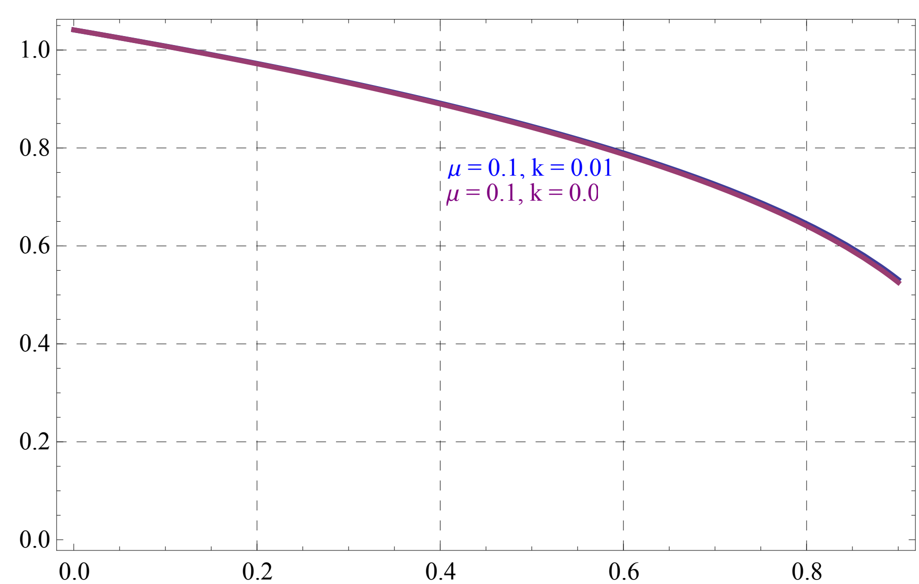

Fig. 6: α versus L_3; $\sigma = 10^{-3}$

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derivative is to be evaluated at the libration point (x_0, y_0). The characteristic equation corresponding to Eqn. (16) is

$$\lambda^4 + \left(4n^2 - \Omega^0_{xx} - \Omega^0_{yy}\right)\lambda^2 + \Omega^0_{xx}\Omega^0_{yy} - (\Omega^0_{xy})^2 = 0 \tag{17}$$

where

$$\Omega^0_{xx} = \frac{3}{4}\left\{1 - \frac{2}{3}(1-3\mu)\alpha + \frac{2}{3}(2-3\mu)\beta + \left(\frac{13}{2} - \frac{23\mu}{4} - \frac{2\mu^2}{1-\mu}\right)\sigma_1\right\}$$

$$\frac{3}{4}\left\{\left(-6 + \frac{39}{4}\mu + \frac{2\mu^2}{1-\mu}\right)\sigma_2\right\}$$

$$\Omega^0_{xy} = \frac{3\sqrt{3}}{2}\left\{\frac{1}{9}(1+\mu)\alpha + \left(-\frac{25}{12} + \frac{85}{24}\mu - \frac{\mu}{6(1-\mu)} - \frac{\mu^2}{6(1-\mu)}\right)\sigma_1\right\}$$

$$+\frac{3\sqrt{3}}{2}\left\{\left(\mu - \frac{1}{2}\right) - \frac{1}{9}(2-\mu)\beta + \left(\frac{3}{2} - \frac{11}{8}\mu + \frac{\mu}{6(1-\mu)} + \frac{\mu^2}{6(1-\mu)}\right)\sigma_2\right\}$$

$$\Omega^0_{yy} = \frac{9}{4} + \frac{1}{2}(1-3\mu)\alpha + \left(\frac{33}{8} + \frac{135\mu}{16} - \frac{33\mu}{8(1-\mu)} + \frac{45\mu^2}{8(1-\mu)}\right)\sigma_1$$

$$-\frac{1}{2}(2-3\mu)\beta + \left(\frac{135\mu}{16} + \frac{33\mu}{8(1-\mu)} - \frac{45\mu^2}{8(1-\mu)}\right)\sigma_2.$$

a) Stability of Non-collinear Libration points

Let $\lambda^2 = \Pi$, therefore Equation (17) becomess

$$\Pi^2 + q_1\Pi + q_2 = 0 \tag{18}$$

which is a quadratic equation in Π and its roots are given by

$$\Pi_{1,2} = \frac{1}{2}(-q_1 \pm \sqrt{D}) \tag{19}$$

where $q_1 = 4n^2 - \Omega^0_{xx} - \Omega^0_{yy}; q_2 = \Omega^0_{xx}\Omega^0_{yy} - (\Omega^0_{xy})^2;\ D = q_1^2 - 4q_2$.
The motion near the Libration point (x_0, y_0) is said to be bounded if $D \geq 0$ *i.e.*

$$1 - 27\mu + 27\mu^2 - 6(1-\mu)\left\{\mu + (1-\mu)k\right\}\alpha + \tfrac{3}{4}\left(8 - 237\mu + 267\mu^2\right)\sigma_1$$
$$+\tfrac{3}{4}(-4 + 37\mu - 111\mu^2)\sigma_2 \geq 0 \tag{20}$$

The Equation (20) is quadratic in μ, on solving it we have

$$\mu_{1,2} = \frac{-p_2 \pm \sqrt{p_2^2 - 4p_1p_3}}{2p_1} \tag{21}$$

where

$$p_1 = 108 + 24(1-k)\alpha + 468\sigma,$$

$$p_2 = -108 - 24(1-2k)\alpha - 492\sigma,$$

$$p_3 = 1 - 6k\alpha + 3\sigma.$$

From the Fig. 7, $\mu_1 > 1/2$ and $\mu_2 < 1/2$ for all values of α. Thus the critical value of mass parameter μ_c for which the non-collinear libration points $L_{4,5}$ are stable is

$$\mu_c = \frac{-p_2 - \sqrt{p_2^2 - 4p_1p_3}}{2p_1} < 1/2 \tag{22}$$

If $\alpha = 0, \sigma = 0$, $\mu_c = 0.038520896504551...$ which is the critical value of mass parameter for classical case. Also, if $\alpha = 0, \sigma = 0$ then $\mu = \mu_0$ is the solution of the Equation (21) where $\mu_0 = 0.0385208965...$ (Szebehely, 1967). When $\alpha \neq 0, \sigma \neq 0$, we suppose that $\mu_c = \mu_0 + \xi_1\alpha + \xi_2\sigma_1 + \xi_3\sigma_2$ as the root of the equation (20), where ξ_1, ξ_2, ξ_3 are to be determined in a manner such that $D = 0$. Therefore we have

$$\xi_1 = -\frac{2\left(k+\mu_0-2k\mu_0-\mu_0^2+k\mu_0^2\right)}{9(1-2\mu_0)},$$

$$\xi_2 = \frac{(8-237\mu_0+267\mu_0^2)}{36(1-2\mu_0)},$$

$$\xi_3 = \frac{(-4+73\mu_0-111\mu_0^2)}{36(1-2\mu_0)}.$$

Therefore

$$\mu_c = 0.0385208965... - (0.00891747 + 0.222579k)\alpha - 0.0627796\sigma \tag{23}$$

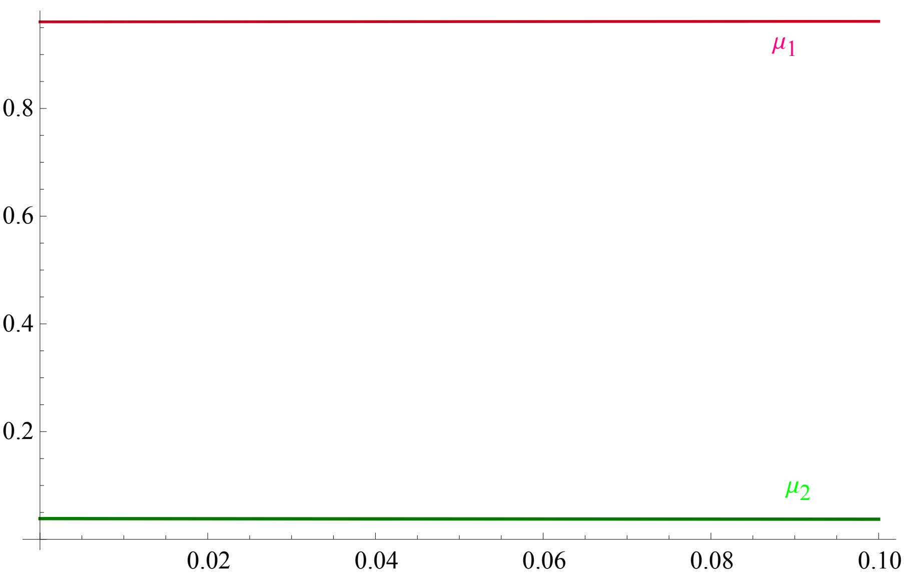

Fig. 7: α versus $\mu_i (i = 1, 2)$; $k = 0.01, \sigma = 10^{-3}$

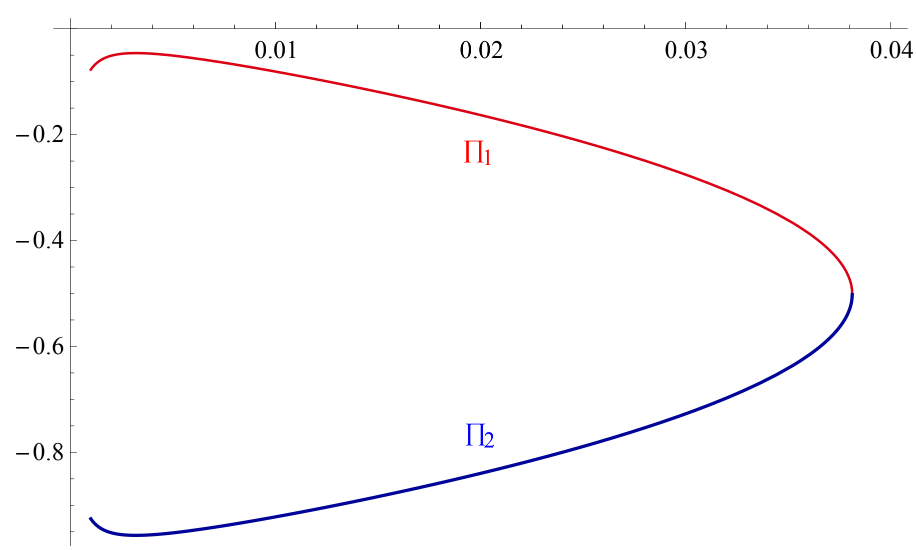

Fig. 8: μ versus Π; $\alpha = 0.01$, $k = 0.01$, $\sigma_1 = 10^{-3}$, $\sigma_2 = 10^{-4}$

Notes

Thus, the non-collinear libration points $L_{4,5}$ are stable for the critical mass parameter $0 < \mu \leq \mu_c$, where μ_c is defined in Equation (23).

As shown in the Fig. 8, $\Pi_{1,2} < 0$ for $\mu \leq \mu_c$. Thus the eigen-values of characteristic Equation (17) are given by $\lambda_{1,2} = \pm i\sqrt{\Pi_1}$, $\lambda_{3,4} = \pm i\sqrt{\Pi_2}$, therefore, the non-collinear libration points $L_{4,5}$ are periodic and bounded and hence stable for the critical mass parameter $\mu \leq \mu_c$, where μ_c is defined in Equation (23).

Notes

b) Stability of Collinear Libration points

First we consider the point lying in the interval $(\mu - 2, \mu - 1)$. For this point, $r_2 < 1$, $r_1 > 1$ and

$$\Omega^0_{xx} = n^2 + \frac{2(1-\mu)(1-\alpha)}{r_1^3} + \frac{2\mu(1-\beta)}{r_2^3} + \frac{6\mu\sigma}{r_2^5} > 0, \Omega^0_{xx} = 0$$

$$\Omega^0_{yy} = \mu\left(\frac{1}{r_2} - \frac{1}{r_1}\right)\left(r_2 - \frac{1}{r_2^2}\right) + \frac{\mu}{r_2}\left(\frac{1}{r_2} - \frac{1}{r_1}\right)\beta + \frac{3\mu\sigma}{r_1} + \frac{3\mu\sigma}{2r_1r_2^4} - \frac{3\mu\sigma}{2r_2^5} < 0$$

Similarly, for the points lying in the interval $(\mu-1, 0)$ and $(\mu, \mu+1)$, $\Omega^0_{xx} > 0$, $\Omega^0_{xy} = 0$, $\Omega^0_{yy} < 0$ Since the discriminant of Equation (19) is positive and the four roots of the characteristic equation (17) can be written as $\lambda_{1,2} = \pm s$ and $\lambda_{3,4} = \pm t$ (s and t are real). Hence the motion around the collinear libration points is unbounded and consequently the collinear libration points are unstable.

c) Application to Real System

Let us consider an example of the Sun-Earth system in the restricted three-body problem in which the smaller primary m_2 (ellipsoid) is taken as the Earth and the bigger one m_1 as Sun. From the astrophysical data we have: Mass of the Sun (m_1) $= 1.9891 \times 10^{30} kg$; Mass of the Earth (m_2) $= 5.9742 \times 10^{24} kg$; Axes of the Earth: $a = 6378.140km$, $c = 6356.755km$; Mean distance of Earth from the Sun $= 1AU = 1.5 \times 10^{11} m$; Luminosity of Sun $= 3.9 \times 10^{26} W$; Flux received on Earth by the Sun $= 1379W/m^2$; Albedo of Earth $= 0.3$ *i.e.* 30 percentage of energy reflected back to space by the Earth, therefore the luminosity of Earth $= 5.2 \times 10^{16} W$.

In dimensionless system

$\mu = 0.00000300346$, $a = 0.0000426352$, $c = 0.0000424923$, $k = 1.3 \times 10^{-10}$. Therefore $\beta = 0.0000443931\alpha$, $\sigma_1 = 2.43294 \times 10^{-12}$, $\sigma_2 = 1.2793 \times 10^{-12}$, $n = 1.0000000000018248$. From the Equations (9), (10), (13), (14) and (15), the libration points obtained in Sun-Earth system are the given in Table 3.

Table 4: Libration Points in Sun-Earth System

α	L_1	L_2	L_3	$L_{4,5}(x, \pm y)$
0.0	−1.01003	−0.990027	1.000001	(−0.499997, ±0.866025)
0.1	−1.00512	−0.964684	0.965491	(−0.466665, ±0.846781)
0.2	−1.00378	−0.928121	0.928319	(−0.433333, ±0.827534)
0.3	−1.00312	−0.887822	0.887905	(−0.400001, ±0.808288)
0.4	−1.00272	−0.843389	0.843434	(−0.366671, ±0.789042)
0.5	−1.00244	−0.793674	0.793699	(−0.333338, ±0.769796)
0.6	−1.00223	−0.736789	0.736808	(−0.300006, ±0.750551)
0.7	−1.00206	−0.669421	0.669432	(−0.266674, ±0.731304)
0.8	−1.00193	−0.584795	0.584805	(−0.233342, ±0.712058)
0.9	−1.00182	−0.464153	0.464161	(−0.200011, ±0.692813)

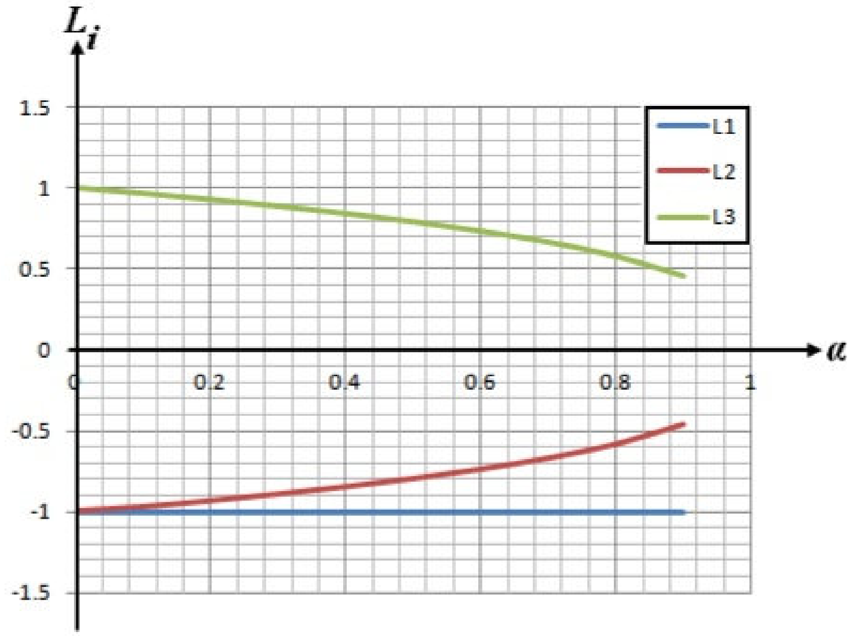

Fig. 9: α versus Collinear Libration Points $L_i(i = 1, 2, 3.)$ in Sun-Earth system

Since all the libration points in Sun-Earth system are the functions of α, so as α increases in the interval $0 \le \alpha < 1$, the first collinear libration point L_1 slightly displaced while second and third libration points L_2 and L_3 move toward center of mass (Fig. 9). The abscissa and ordinate of non-collinear libration points also decreases as α increases and hence the shape of the scalene triangle formed by $L_{4,5}$ reduces (Fig. 10).

Since $\Pi_1 < 0$ in the interval $0 \le \alpha \le 0.00785$ and $\Pi_2 < 0$ in $0 \le \alpha < 1$, the roots of the characteristic equation (17) are pure imaginary in the interval

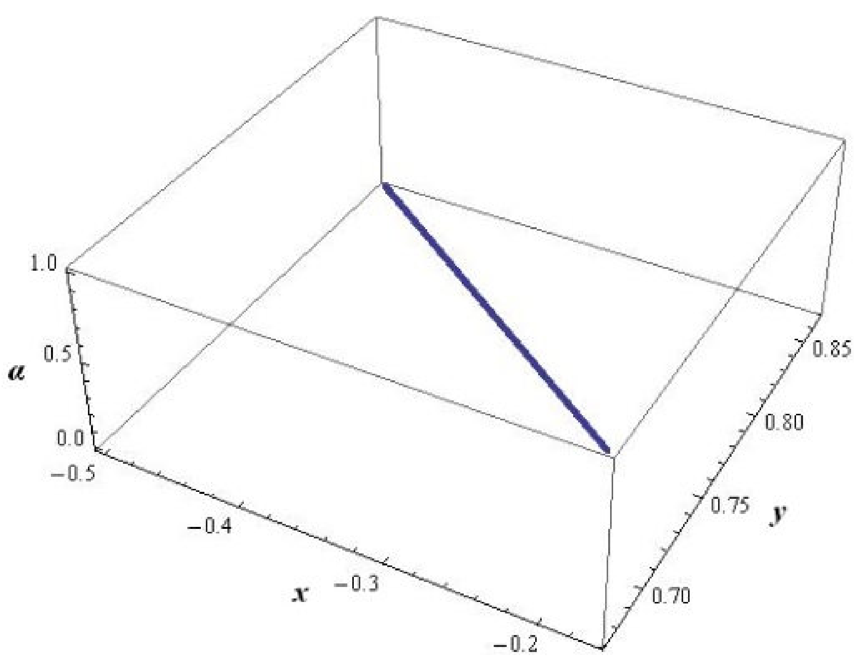

Fig. 10: α versus Non-collinear Libration Points $L_j(i = 1, 2.)$ in Sun-Earth system

Notes

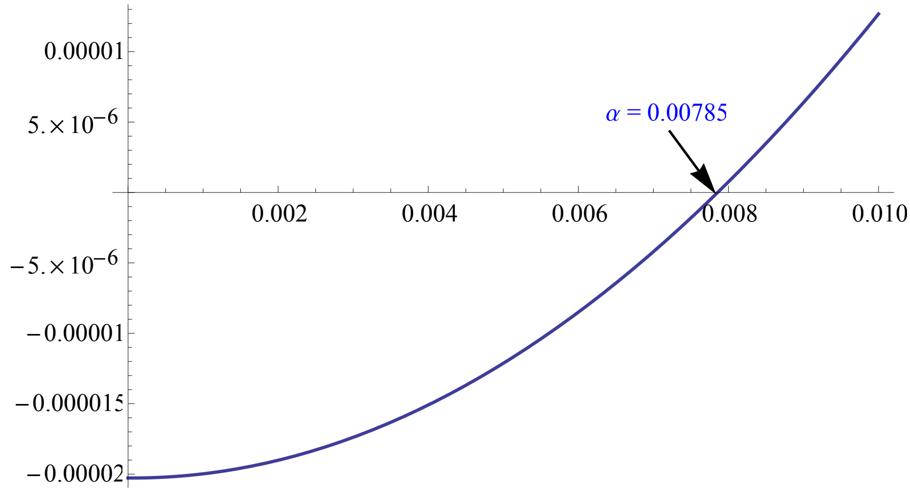

Fig. 11: α versus Π_1

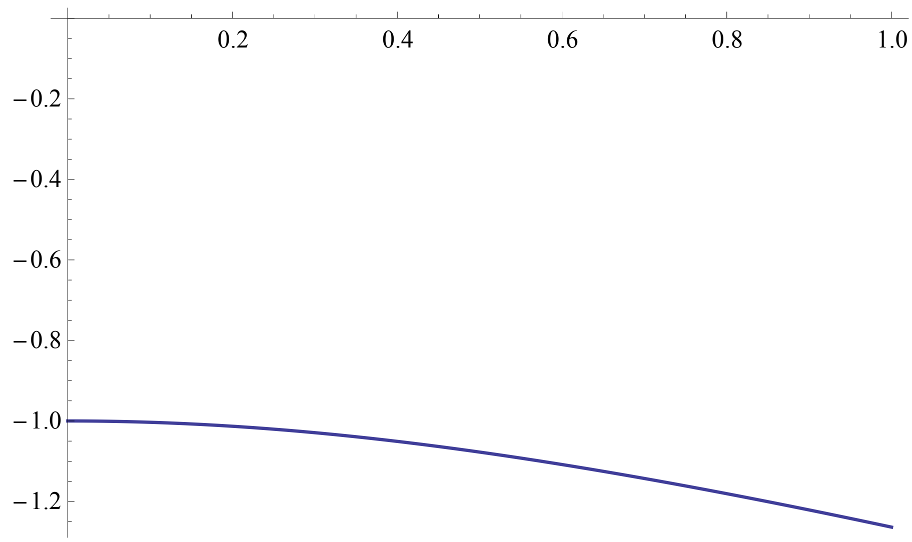

Fig. 12: α versus Π_2

$0 \leq \alpha \leq 0.00785$. Thus the non-collinear libration points $L_{4,5}$ in Sun-Earth system are stable if $0 \leq \alpha \leq 0.00785$.
For the collinear libration points $L_i (i = 1, 2, 3)$ in Sun-Earth system, $\Omega^0_{xx} > 0$, $\Omega^0_{xy} = 0$, $\Omega^0_{yy} < 0$, in $0 \leq \alpha < 1$. Since the discriminant of Equation (19) is positive and the four roots of the characteristic equation (17) an be written as $\lambda_{1,2} = \pm s$ and $\lambda_{3,4} = \pm t$ (s and t are real). Hence the motion around the collinear libration points is unbounded and consequently the collinear libration points are unstable.

V. Conclusion

In the present paper, the existence and stability of libration points in circular restricted three-body problem has been studied under Albedo effect when smaller primary is an oblate spheroid. The equations of motion in case of Albedo effect are derived, Eqn. (2). For For $\beta = 0$, the problem reduces to photogravitational restricted three-body problem when smaller primary is an oblate spheroid. It is found that there exist five libration points, three collinear and two non-collinear. The first collinear libration point L_1 lie at the right of the primary m_2, the second libration point L_2 lies between the center of mass of the primaries O and m_1 and the third libration point L_3 lies at the right of the primary m_1. The libration points L_i (i = 1, 2, 3) are affected by the Albedo effect and this effect displaced the libration points from its actual position as shown in Figs. 4, 5 and 6. Also, there exist two non-collinear libration points $L_{4,5}$ and these points are affected by triaxiality as well as Albedo effect Figs. 3 and 4, these points form scalene triangle with the primaries as $r_1 \neq r_2$. The numerical location of $L_{4,5}$ is also calculated in Table 1 for $\mu = 0.1$ and different values of α and k and it is found that the abscissa and ordinate of non-collinear libration points are the decreasing functions of α and k *i.e.* as α and k increases, x and y decreases. For $\alpha = 0$, the results are in conformity with those of Bhatnagar and Hallan (1979). If $\alpha = 0$ and $\sigma = 0$, the classical case of the restricted three body problem is verified (Szebehely, 1967). The non-collinear libration points are stable for a critical value of mass parameter $\mu \leq \mu_c$ where $\mu_c = 0.0385208965 \ldots$ - (0.00891747 + 0.222579k) α - 0.0627796 σ but collinear libration points are still unstable. Also, an example of Sun-Earth system is taken in Section-5 as a real application and this is found that all the libration points in Sun-Earth system are the functions of α, so as α increases in the interval $0 \leq \alpha < 1$, the first collinear libration point L_1 slightly displaced while second and third libration points L_2 and L_3 move toward center of mass (Fig. 9). The abscissa and ordinate of non-collinear libration points also decreases as α increases and hence the shape of the scalene triangle formed by $L_{4,5}$ reduces (Fig. 10). The non-collinear libration points $L_{4,5}$ in Sun-Earth system are stable for $0 \leq \alpha < 0.00785$ but collinear libration points are unstable.

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A Note on Chebyshev Inequality: To Explain or to Predict

By Amaresh Das
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Abstract- The question is: What proportion of the total probability of a random varriable *X* lies within a certain interval of the mea μ? What is the probability of being hit by a meteor greater in size than five times the standard deviation above the mean? Because it can be applied to completely arbitrary distributions(unknown except for mean and variables), the inequality generally gives a poor bound compared to what might be deduced if more aspects are known about the distribution involved.

Keywords: *eucladian norm, monotonic function, jensen inequality.*

GJSFR-F Classification: *MSC 2010: 11Y16*

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A Note on Chebyshev Inequality: To Explain or to Predict

Amaresh Das

Abstract- The question is: What proportion of the total probability of a random variable X lies within a certain interval of the mea μ? What is the probability of being hit by a meteor greater in size than five times the standard deviation above the mean? Because it can be applied to completely arbitrary distributions(unknown except for mean and variables), the inequality generally gives a poor bound compared to what might be deduced if more aspects are known about the distribution involved.

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I. Introduction

This note introduces a useful inequality called the Chebyshev (often called Tchebysheff) inequality but its explanation requires a good familiarity in calculus. One of the most difficult and important tasks before the statistician is to discover the probability distribution that may be involved in any problem. If one is unsure what the underlying distribution is, it is comforting to know that there are more universal inequalities that may give us some useful information; At the very least, the Chebyshev inequality allows one to bound how far away from the mean the random variable could be.

II. Exploring the Inequality

Theorem 1

Let $\mu(X)$ be a nonnegative function of the random variable X. If $E[\mu(\mathrm{x})]$ exists, then, for every positive constant C

$$\Pr\,[\,\mu(X) \geq C\,] \leq \frac{[\mu(\mathrm{x})]}{\mathrm{C}} \tag{1}$$

The proof[1] is given when the random variable[2] X of the continuous type, but, the proof can be adapted to the discrete case if we replace integrals by sums. Let $A = [\mathrm{x};\ \mu(\mathrm{x}) \geq \mathrm{C}\,]$ and let $f(\mathrm{x})$ denote the pdf. of X Then

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[1] See any standard text, for example, Mood and Grsybill [5]

[2] Chebyshev's inequality is usually stated for random variables, but can be generalized to a statement about measure spaces.

Fix t and let A_t be defined as $A_t = [\,x \in X \mid f\,(\mathrm{x}) \geq t\,]$ and $\mathrm{I_A}$ be the indicator function of the set $\mathrm{A_t}$. Then it is not difficult to see for any t, $0 < \mathrm{g(t)}\ \mathrm{I_A} \leq g\,(\,f(x)\ \mathrm{I_A}$ since g is not decreasing on the range of t and, therefore, $\mathrm{g}(t)\,\mu\,(A_t) = \int_A g \circ f\, d\mu \leq \int_x g \circ f\, d\mu$. The desired monotonicity follows from dividing the above inequality by $g(t)$

$$E\mu(x)] = \int_{\infty}^{\infty} \mu(x)\ f(x)\ dx = f\ (x)\ dx = \int_A \mu(x)\ f(x)\,dx + \int_{A^*} \mu(x)\, f\,(x0\,dx \tag{2}$$

Since each of the integrals in the extreme right-hand member of the preceding equation is nonnegative, the left-hand side member is greater than or equal to either of them. In particular,

$$E\ [\mu(X)] \geq \int_A \mu(x) f(x)\,dx \tag{3}$$

However, if $x \in A$ then $\mu(x) \geq C$; accordingly, the right-hand member of the preceding inequality is not increased if we replace $\mu(x)$ by C. Thus

$$E[\ \mu(x)\ ] \geq C \int_A f(x)\ dx, \tag{4}$$

Since

$$\int_A f(x)\,dx = \Pr\ (X \in A) = Pr[\mu(X) \geq C \tag{5}$$

It follows that

$$E[\mu(X)] \geq C\ \Pr\ [\mu(X) \geq C] \tag{6}$$

which is the desired result.

The theorem is a generalization of an inequality which is often called Chebyshev's inequality.[3] One can establish the inequality this way[4]:

Theorem 2

Let the random variable X have a distribution of probability about which we assume only that there is a finite variance σ^2. This of course, implies that there is a mean μ. Then for every $K > 0$

$$\Pr\ (|X - \mu| \geq k\sigma) \leq \frac{1}{k^2} \tag{7}$$

In theorem 1 take $\mu(X) = (X - \mu)^2$ and $C = k^2 \sigma^2$ Then we have

$$\Pr[\ (X - \mu)^2 \geq k^2\sigma^2] \geq k^2\sigma^2 \leq \frac{E[(X-\mu)^2]}{k^2\sigma^2} \tag{8}$$

---

[3] Ferentinos[1] has shown that for a vector $X = (x_1, x_2 \ldots)\ \mu = (\mu_1, \mu_2 \ldots)$ variance $\boldsymbol{\sigma}^2 = (\sigma_1{}^2, \sigma_2{}^2 \ldots\ )$ and The Eucladian norm $\|.\ \|$ that $\Pr\ (\|x - \mu\| \geq k\ \|\sigma\| \leq \frac{1}{k^2}$

[4] Symmetry of the distribution decreases the inequality's bounds by a factor of 2 while unimodality sharpens the bounds by a factor of 4/9.Because the mean and the mode in a unimodal distribution differ by at most √3 standard deviations at most 5% of a symmetrical unimodal distribution lies outside (2√10 + 3√3)/3 standard deviations of the mean (approximately 3.840 standard deviations). This is sharper than the bounds provided by the Chebyshev inequality (approximately 4.472 standard deviations).These bounds on the mean are less sharp than those that can be derived from symmetry of the distribution alone which shows that at most 5% of the distribution lies outside approximately 3.162 standard deviations of the mean. The known Vysochanskiï–Petunin inequality further sharpens this bound by showing that for such a distribution that at most 5% of the distribution lies outside 4√5/3 (approximately 2.981) standard deviations of the mean. See Kotz *et al* [3]

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Since the numerator of the right-hand side member of the preceding inequality is σ^2, the inequality may be written as

$$\Pr\left(|X - \mu| \geq k\sigma\right) \leq \frac{1}{k^2} \tag{9}$$

which is the desired result. Obviously we should take the positive number k to be greater than one to have the inequality of consequence. Because it can be applied to completely arbitrary distributions (unknown except for mean and variance), the inequality generally gives a poor bound compared to what might be deduced if more aspects are known about the distribution

Table 1: Chebyshev Inequality

k	Min. % within k stan deviations of mean	Max. % beyond k stand deviations from mean
1	0%	100%
$\boxed{\sqrt{2}}$	50%	50%
1.5	55.56%	44.44%
2	75%	25%
3	88.8889%	11.1111%
4	93.75%	6.25%
5	96%	4%
6	97.2222%	2.7778%
7	97.9592%	2.0408%
8	98.4375%	1.5625%
9	98.7654%	1.2346%
10	99%	1%

Although Chebyshev inequality enables you to find an answer to the questions we raised at the very outset, it comes to the rescue in offering at least an appropriate answer.

Table 2: Chebyshev Inequality Bounds and Actual Bounds

K value	Chebyshev	Gaussian	Chi-square	t
2	.45	.05412	.05241	.05312
3	10	.00231	.00511	.03121
3	.07	.00001	.00313	.00613.

How tight is the broadly applicable inequality? We can calculate Chebyshev inequality and contrast that value with the exact calculation obtained from knowing

[5] There are many extensions to Chebyshev inequality, for example, Chebyshev inequality of exponential version. Inequalities for bounded variables, inequalities in the multivariate case, or its use in infinite dimensional case; see [Stellato, *et al*. [7], Lal [4]] There may be integral inequality, too. An extension to higher moments is also possible.

If $f.g;[a,b] \to R$ are less monotonic functions of the same monotonicity, then

$$\frac{1}{b-a}\int_a^b dx \geq \left[\frac{1}{b-a}\int_a^b f(x)\,dx\right]\left[\frac{1}{b-a}\right]\int_a^b g(x)\,dx$$

the probability function as indicated as follows: The resilts are tabulated in the above Table 2.

$$\Pr\{\,|x| \geq k\,\sigma \;=\; \int_{-\alpha}^{-k\sigma} df\,(x) + \int_{k\sigma}^{\infty} df\,(x) \qquad (11)$$

The Chebyshev theorem typically provides rather loose bounds[6]. However, these bounds cannot in general (remaining true for arbitrary distributions) be improved upon. The bounds are sharp for the following example: for any $k \geq 1$,

$$x = \left[\begin{array}{l} 1 \text{ with probability } \dfrac{1}{2k^2} \\ 0 \text{ with probability } 1 - \dfrac{1}{2k^2} \\ 1 \text{ with probability } \dfrac{1}{2k^2} \end{array} \right] \qquad (12)$$

Exercise 1

Is it possible to find an upper bound for this integral?

$$\int_0^A (\,A - x\,)\; P(\,x0\,dx \qquad (13)$$

Hint; Lower is easy to find by using Markow's Inequality but how to find the Upper bound?

III. Concluding Remark

This note discusses the Chebyshev inequality as a very app approximate but universally applicable upper bound on probability. The Chebyshev inequality allows us to bound how far away from the mean the random variable could be. It is rather remarkable that one can find inequalities on probability that will hold for any distribution.

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If *f* and *g* are of opposite monotonicity, then the above) inequality works in the opposite way. This inequality is related to Jensen's inequality or Kantorovich's inequality.

[6] Although Chebyshev's inequality may not be necessarily true for finite samples. Samuelson's inequality states that all values of a sample will lie within $\sqrt{(N-1)}$ standard deviations of the mean. Chebyshev's bound improves as the sample size increases. When N = 10, Samuelson's inequality states that all members of the sample lie within 3 standard deviations of the mean: in contrast Chebyshev's states that 99.5% of the sample lies within 13.5789 standard deviations of the mean. When N = 100, Samuelson's inequality states that all members of the sample lie within approximately 9.9499 standard deviations of the mean: Chebyshev's states that 99% of the sample lies within 10 standard deviations of the mean. See Dasgupta[1]. DasGupta has determined a set of best possible bounds for a normal distribution for this inequality. Steliga and Szynal [6] have extended these bounds to the Pareto distribution.

Notes

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The Truly Marvellous Proof for n=4

By Leszek W. Guła

Abstract- The truly marvellous proof of the Fermat's Last Theorem for $n=4$.

Keywords: diophantine equations, diophantine inequalities, fermat equation, greatest common divisor, newton binomial formula.

GJSFR-F Classification: MSC 2010: 11D45

THETRULYMARVELLOUSPROOFFORN4

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1. Guła, L. W.: Disproof of the Birch and Swinnerton-Dyer Conjecture – http://pubs.sciepub.com/EDUCATION/4/7/1/index.html

The Truly Marvellous Proof for n=4

Leszek W. Guła

Abstract- The truly marvellous proof of the Fermat's Last Theorem for *n*=4.

Keywords: *diophantine equations, diophantine inequalities, fermat equation, greatest common divisor, newton binomial formula.*

I. Introduction

The Fermat's Last Theorem is the famous theorem. Here we have the truly marvellous proof for n=4.

II. The Truly Marvellous Proof of The Fermat's Last Theorem For N=4

Theorem 1: (FLT for n=4). *The equation*

$$A^4 + B^4 = C^4$$

has no primitive solutions $[A,B,C]$ *in* $\{1,2,3,\ldots\}$.

Proof. Suppose that the $A^4+B^4=C^4$ equation has primitive solutions $[A, B, C]$ in $\{1, 2, 3,\ldots\}$. [1], [2]

Then A, B and C are co-prime. Without loss for this proof we can assume that A is odd. Then B is even and C is odd, which is obviously.

For some C, $A \in \{1,3,5,\ldots\}$ and for some $B \in \{4,6,8,\ldots\}$:

$$(C-A+A)^4 - A^4 = B^4 \Longrightarrow (C-A)^3 + 4(C-A)^2A + 6(C-A)A^2 + 4A^3 = \frac{B^4}{C-A}.$$

Notice that

$$(C-A)^3 + 4(C-A)^2A + 6(C-A)A^2 + 4A^3 = \frac{C^4 - A^4}{C-A} = \frac{(C^2+A^2)(C+A)(C-A)}{C-A}.$$

Hence – For some $k \in \{1,2,3,\ldots\}$ and for some e, c, $d \in \{1,3,5,\ldots\}$ such that e, c and d are co-prime:

$$\frac{\left(2^k ecd\right)^4}{C-A} = \frac{\left(2^k ecd\right)^4}{2^{4k-2}d^4} = 4(ec)^4 = \frac{B^4}{C-A}.$$

Therefore – For some relatively prime e, c, $\in \{1,3,5,\ldots\}$ such that e> c:

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$$4(ec)^4 = (C^2 + A^2)(C + A) \Longrightarrow (C^2 + A^2 = 2e^4 \wedge C + A = 2c^4) \Longrightarrow$$

$$(C = x + y \wedge A = x - y \wedge C + A = 2x = 2c^4 \wedge x = c^4 \wedge x^2 + y^2 = e^4 \wedge x = c^4 = u^2 - v^2 \wedge y = 2uv \wedge e^2 = u^2 + v^2 \wedge e = p^2 + q^2 \wedge u = p^2 - q^2 \wedge v = 2pq) \Longrightarrow \{x^2 = [(p^2 - q^2)^2 - (2pq)^2]^2 = c^8 \equiv \mathbf{0} \wedge y^2 = 4(p^2 - q^2)^2(2pq)^2 \wedge e^4(p^2 + q^2)^4 \wedge [(p^2 - q^2)^2 - (2pq)^2]^2 + 4(p^2 - q^2)^2(2pq)^2 = (p^2 + q^2)^4 \equiv \mathbf{1}\}.$$

where $(p^2 - q^2)^2 - (2pq)^2 > 4(p^2 - q^2)pq$.

The above last sentence is false inasmuch as on the strength of the Guła's Theorem [1] we have

$$(2pq)^2 = (p^2 - q^2)^2 - (c^2)^2 \Longrightarrow p^2 - q^2 = \frac{(2pq)^2 + (2q^2)^2}{2(2q^2)} = p^2 + q^2 \equiv \mathbf{0}.$$

This is the proof.

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Revised Estimation of New Drug Product Approval Probabilities in Phased Clinical Trials

By Matthew Chukwuma Michael, Oyeka I. C. A. & Ajibade Bright

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Abstract- This paper proposes and presents a method for the estimation of approval probabilities of new drug or product. The proposed method assumes that three evaluation communities are used to assess and evaluate the quality of a new drug or product and that the evaluation is done by the committees in three period phased clinical trials of the drug or product using matched samples of subjects at each phase. Estimates of absolute and conditional approval probabilities by various combination evaluation committees at each phase of clinical trials are provided. Test statistics are also developed testing desired hypothesis at each of the phased clinical trials. The proposed method is illustrated with some sample data. It is shown in terms of estimated probability that it is more difficult for all three evaluation committees to be in complex agreement to approve or not approve a new drug or product than for fewer evaluation committees to grant approval.

Keywords: evaluation committees, product, volunteer, probabilities, phased controlled clinical trials, diagnostic screening tests.

GJSFR-F Classification: MSC 2010: 97K80

REVISEDESTIMATIONOFNEWDRUGPRODUCTAPPROVALPROBABILITIESINPHASEDCLINICALTRIALS

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Revised Estimation of New Drug Product Approval Probabilities in Phased Clinical Trials

Matthew Chukwuma Michael [α], Oyeka I. C. A. [σ] & Ajibade Bright [ρ]

Abstract- This paper proposes and presents a method for the estimation of approval probabilities of new drug or product. The proposed method assumes that three evaluation communities are used to assess and evaluate the quality of a new drug or product and that the evaluation is done by the committees in three period phased clinical trials of the drug or product using matched samples of subjects at each phase. Estimates of absolute and conditional approval probabilities by various combination evaluation committees at each phase of clinical trials are provided. Test statistics are also developed testing desired hypothesis at each of the phased clinical trials. The proposed method is illustrated with some sample data. It is shown in terms of estimated probability that it is more difficult for all three evaluation committees to be in complex agreement to approve or not approve a new drug or product than for fewer evaluation committees to grant approval.

Keywords: evaluation committees, product, volunteer, probabilities, phased controlled clinical trials, diagnostic screening tests.

I. Introduction

As observed in Onyiora et al (2013) most health care professionals would want their patients to have the best available clinical care but the problem these professional often have is the inability to clearly identify the optimum drug or interventionist procedure to adopt in patient treatment and management and often rely on own experience or those of colleagues in actual practice. However, health professionals are increasingly relying on evidence based medical and health practices hinged on a systematic review, evaluation, assessment and application of clinical research findings (Rising, Bacchetti and Baro, 2009; Chow and Liu, 2004). In medical practice and health management, erroneous and misguided approval of a new drug or product is often hazardous and costly in human and material resources (Gobburn and Leske, 2009). Following a sequence of clinical trials often conducted in phases by evaluation bodies or committees, approval of a new drug or product for use in a population may be granted if the drug or product satisfies some set of predetermined criteria for use (Haff, 2003). In controlled clinical trials of new drug or product using cross sectional, prospective or retrospective study methods, the trials are usually conducted in phases using usually test animals and subsequently volunteer human subjects (Onyiora et al, 2013; Lipkovic et al, 2008). Approval for use of a new drug product in a population is granted only after the phased clinical trials the proportion of subjects improving with the new drug or product is higher than the proportion improving with the standard drug under all or most of the evaluation committees involved in the phased clinical trials.

Following the phased clinical trial procedures, specifically using three period phased clinical trials by three evaluation committees. Onyiora et al (2013) proposed and

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developed a probability model that would enable the calculation of the proportion or probabilities of approving or not approving new drug or product by none, some or all the evaluation committees.

The probability estimation model developed by the authors is however most useful if the probabilities'*a-g*' are given or already known and hence can readily be used in the estimation of the probabilities of possible outcomes including the outcomes listed in the authors' Table 2.The method under reference does not however provide a method to use in the a-priori estimation of the probabilities '*a-g*' if not already given and are not known, and must be estimated from sample data obtained in relevant phased clinical trials of a new drug or product.

In this paper we propose to develop a more generalized method for the estimation of probabilities of outcomes in phased controlled clinical trials of a drug or product by three evaluation committees. The present method would readily enable one estimate probabilities of approval or non-approval of a new drug or product using sample data obtained in three phased clinical trials by three evaluation committees: in a cross-sectional, prospective or retrospective clinical trials conducted in three phases.

II. Proposed Method

To develop a method for use in estimating probabilities that may help in the assessment and evaluation of a new drug or product for possible approval for use in a population when these probabilities are not a-priori given, we may assume following Onyiorah et al (2013) (that three mutually co-operating evaluation bodies or committees x, y and z co-operating in the sense that they employ the same evaluation criteria used for the drug or product quality assessment or evaluation) in phased controlled clinical trials. The evaluation would be done using controlled cross-sectional comparative either prospective or retrospective study in clinical trials conducted in three phases. Now to conduct the clinical trials, matched random samples of consenting subjects or volunteers matched by age, sex, body weight and other demographic characteristics are to be used. If the study is a retrospective one then the required data would of course be obtained from case history files of the study participants. Suppose in the first phase of the controlled clinical trials each of the evaluation committees tests, screens or administers a new drug or product to a different but comparable sample of such matched samples of subjects of equal sizes, n_1. In the second phase of the clinical trials samples of three equal size n_2 matched pairs of subjects matched on the same demographic characteristics as in the first phase of the trials are used. Pairs of the three co-operating evaluation committees are assigned to test, screen or treat members in one of each of the three paired samples of matched subjects, with one evaluation committee in each pair testing the first members say of each paired sample of subjects and the other member of the paired evaluation committees testing the second members, say of the paired sample of subjects assigned to that evaluating committee.

In the third and last phase of the clinical trials matched triples of size n_3 subjects are used. That is matched triples of sizen_3 samples each of three matched subjects are used. One subject in each matched triple, that is one subject in each of three matched subjects is tested, screened or treated by one of the three evaluation committees.

Now as in Onyiorah et al (2013), suppose A and $\bar{A}$are respectively the events that evaluation committee X approves and does not approve a new drug or product for use; B and $\bar{B}$ are respectively the events that evaluation committee Y approves and does not approve a new drug or product for use; and C and $\bar{C}$ are respectively the events that evaluation committee Z approves and does not approve a new drug or product for use in a population. The resulting set of all possible outcomes, that is the sample space, S, of the experiment; namely, three-phased Clinical trials by three evaluation committees is then,

Notes

$$S = \{ABC, AB\bar{C}, A\bar{B}C, A\bar{B}\bar{C}, \bar{A}BC, \bar{A}B\bar{C}, \bar{A}\bar{B}C, \bar{A}\bar{B}\bar{C},\}$$

To develop a method for the estimation of new drug or product approval probability assuming that three mutually cooperating evaluation committees X, Y and Z are used in clinical trials conducted in three phases to assess the quality of the drug or product for possible approval, we may proceed as follows: For the first phase of the clinical trials, considering drug assessment by evaluation committees X, say, we may let

Notes

$$u_{ix} = \begin{cases} 1, if\ the\ i^{th}\ subject\ tested, screened\ or\ administered\ a\ new \\ \quad drug\ by\ committee\ X\ responds\ positive. \\ o, therwise \end{cases} \quad (1)$$

for $i = 1, 2, \ldots, n_1$
Let

$$\pi_x^+ = P(u_{ix} = 1) \quad (2)$$

Also define

$$W_x = \sum_{i=1}^{n_1} u_{ix} \quad (3)$$

Now the expected value and variance of u_{ix} are respectively

$$E(u_{ix}) = \pi_x^+; Var(u_{ix}) = \pi_x^+(1 - \pi_x^+) \quad (4)$$

Also, the expected value and variance of W_x are respectively,

$$E(W_x) = \sum_{i=1}^{n_i} E(u_{ix}) = n_1\pi_x^+; Var(W_x) = \sum_{i=1}^{n_1} Var\,(u_{ix}) = n_1.\pi_x^+(1 - \pi_x^+) \quad (5)$$

Now π_x^+ is the proportion or probability that on the average subjects tested, screened or treated by evaluation committee X responds positive. Its sample estimate is

$$\hat{\pi}_x^+ = p_x = \frac{W_x}{n_1} = \frac{f_x^+}{n_1} \quad (6)$$

where f_x^+ is the number of subjects responding positive under evaluation committee X, that is when tested by evaluation committee X. Thus f_x^+ is the total number of 1s in the frequency distribution of then_1 values of 0's and 1's in u_{ix} for $i = 1, 2, \ldots, n_1$.
The sample estimate of the variance of $\hat{\pi}_x^+$ is from Equation 5

$$Var(\pi_x^+) = \frac{Var\,(W_x)}{n_1^2} = \frac{\hat{\pi}_x^+(1-\hat{\pi}_x^+)}{n_1} \quad (7)$$

A null hypothesis that may often be of interest could be that the proportion π_x^+ of subjects responding positive under evaluation committee X is at most some value , π_{x0}or symbolically

$$H_0: \pi_x^+ \le \pi_{x0} \text{Versus}\, H_1: \pi_x^+ > \pi_{x0} (0 \le \pi_{x0} \le 1) \quad (8)$$

The null hypothesis H_0 of Equation 8 may be tested using the test statistic

$$\chi^2 = \frac{(W_x - n_1.\pi_{x0})^2}{Var\,(W_x)} = \frac{n_1(\hat{\pi}_x^+ - \pi_{x0})^2}{\hat{\pi}_x^+(1-\hat{\pi}_x^+)} \quad (9)$$

Which under H_0 has approximately the chi-square distribution with 1 degree of freedom for sufficiently large n_1. The null hypothesis H_0 of equation 8 is rejected at α level of significance if

$$\chi^2 \geq \chi^2_{1-\alpha;1} \tag{10}$$

Otherwise H_0 is accepted. To estimate the probability of approval of a new drug or product by evaluation committee Y during the first phase of clinical trials we may let

$$u_{ix} = \begin{cases} 1 \; if \; the \; subject \; tested, screened \; or \; treated \; with \; a \; new \; drug \; product \\ \; by \; evaluation \; committee \; or \; approval \; agency \; Y \; responds \; positive \\ 0, otherwise \end{cases} \tag{11}$$

Notes

for all $i = 1, 2, \ldots, n_1$.
Let

$$\pi_y^+ = P(u_{iy} = 1) \tag{12}$$

and

$$W_y = \sum_{i=1}^{n_1} u_{iy} \tag{13}$$

now,

$$E(u_{iy}) = \pi_y^+; Var(u_{iy}) = \pi_y^+(1 - \pi_y^+) \tag{14}$$

and

$$E(W_y) = n_1.\pi_y^+; Var(W_y) = n_1.\pi_y^+(1 - \pi_y^+) \tag{15}$$

For evaluation committee or approval agency, Y, $\pi_y^+$is the proportion of subjects responding positive when tested, screened or administered by evaluation committee Y during the first phase of clinical trials. Its sample estimate is

$$\hat{\pi}_y^+ = p_y = \frac{W_y}{n_1} = \frac{f_y^+}{n_1} \tag{16}$$

where f_y^+ is the number of subjects responding positive to evaluation committee Y in the first phase of clinical trials which is the total number of 1s in u_{iy}, $i = 1, 2, \ldots, n_1$. The corresponding sample variance is

$$Var(\hat{\pi}_y^+) = \frac{Var\,(W_y)}{n_1^2} = \frac{\hat{\pi}_y^+(1-\hat{\pi}_y^+)}{n_1} \tag{17}$$

A null hypothesis similar to that of Equation 17 for evaluation committee X may also be stated and tested for evaluation committee approval agency, Y. Following similar approaches as above, we also develop sample estimate of approval probability π_z^+ for evaluation committee agency Z as

$$\hat{\pi}_z^+ = p_z = \frac{W_z}{n_1} = \frac{f_z^+}{n_1} \tag{18}$$

Where f_z^+ is the number of subjects responding positive when tested, screened or administered a new drug or products by evaluation committee or approval agency Z during the first phase of clinical trials. The corresponding sample variance is similarly estimated.

Note that π_x^+, π_y^+ and π_z^+ are respectively the equivalence of A, B and c in Onyiorah et al (2013).

To estimate conditional probabilities of approval of a new drug or product by any pair of evaluation committees X and Y, say, during the second phase of clinical trials, we may first suppose that of the n_2 matched paired samples of subjects used in this phase of trials $n_{y,x}$ and $n_{z,x}$ subjects respond positive to the drug or product when tested by evaluation committee Y and Z respectively; and $n_{z,y}$ subjects respond positive under evaluation committees Y when paired with evaluation committee Z.

To estimate conditional probabilities of approval of a new drug or product by any pair of evaluation committees $X$and Y say during the second phase of clinical trials, we may let

Notes

$$u_{iy,x} = \begin{cases} 1, \textit{if the } i^{th} \textit{pair of subjects tested by evaluation committees X and Y} \\ \textit{during the second phase of trials, the subjects tested by evaluation} \\ \textit{committee Y responds positive given that the correspo nding subject} \\ \textit{in the pair tested by evaluation committee X has also responded} \\ \textit{positive} \\ 0, \textit{otherwise} \end{cases} \quad (19)$$

for $i = 1, 2, \ldots, n_{y.x}$

Let

$$\pi^{+}_{y.x} = P(u_{iy.x} = 1) \quad (20)$$

and

$$W_{y.x} = \sum_{i=1}^{n_{y.x}} u_{iy.x} \quad (21)$$

The expected value and variance of $u_{iy.x}$ are respectively

$$E(u_{iy.x}) = \pi^{+}_{y.x}; \quad Var(u_{iy.x}) = \pi^{+}_{y.x}(1 - \pi^{+}_{y.x}) \quad (22)$$

Also the expected value and variance of $W_{y.x}$ are respectively

$$E(W_{y.x}) = \sum_{i=1}^{n_{y.x}} E(u_{iy.x}) = n_{y.x}\,\pi^{+}_{y.x}; \; Var(W_{y.x}) = \sum_{i=1}^{n_{y.x}} Var(u_{iy.x}) = n_{y.x}\pi^{+}_{y.x}(1 - \pi^{+}_{y.x}) \quad (23)$$

Now $\pi^{+}_{y.x}$ is the proportion or probability that on the average in the pairs of subjects tested by evaluation committees X and Y during the second phase of clinical trials subjects tested by evaluation committee Y respond positive given that the corresponding subjects tested by evaluation committee X have also responded positive to the new drug product. Its sample estimate is

$$\hat{\pi}^{+}_{y.x} = P_{y.x} = \frac{W_{y.x}}{n_{y.x}} = \frac{f^{+}_{y.x}}{n_{y.x}} \quad (24)$$

Where $f^{+}_{y.x}$ is the number of pairs of subjects for which subjects tested in the pairs by evaluation committee Y respond positive given that the corresponding subjects in the same pairs treated by evaluation committee X have also responded positive to the drug or product in the second phase of clinical trials. Thus $f^{+}_{y.x}$is the total number of 1s in the frequency distribution of the $n_{y.x}$ values of 0s and 1s in $u_{iy.x}$ for $i = 1, 2, \ldots, n_{y.x}$.

The sample variance of $\hat{\pi}^{+}_{y.x}$ is from Equation 23

$$Var(\hat{\pi}^{+}_{y.x}) = \frac{Var\,(W_{y.x})}{n^{2}_{y.x}} = \frac{\hat{\pi}^{+}_{y.x}(1-\hat{\pi}^{+}_{y.x})}{n_{y.x}} \quad (25)$$

For the second phase of clinical trials, the null hypothesis that may be of interest concerning evaluation committees or approval agencies Y and Y say may be that the proportion of subjects responding positive when tested by evaluation committee Y given positive response under evaluation committee agency X is at least some value $\pi^{+}_{y.x_0}$ that is the null hypothesis

$$H_0: \pi^+_{y.x} \geq \pi^+_{y.x_0} \; versus \; H_1: \pi^+_{y.x} < \pi^+_{y.x_0}, \left(0 \leq \pi^+_{y.x_0} \leq 1\right) \tag{26}$$

The null hypothesis H_0 of Equation 26 may be tested using the test statistic

$$\chi^2 = \frac{\left(W_{y.x} - n_x \pi_{y.x_0}\right)^2}{Var\,(W_{y.x})} = \frac{n_{y.x}\left(\hat{\pi}^+_{y.x} - \pi^+_{y.x_0}\right)^2}{\hat{\pi}^+_{y.x}\left(1 - \hat{\pi}^+_{y.x}\right)} \tag{27}$$

which has approximately the chi-square distribution with 1 degree of freedom for sufficiently large $n_{y.x}$. The null hypothesis H_0 is rejected at the α level of significance if Equation 10 is satisfied otherwise H_0 is accepted.

To estimate conditional probability of positive response under evaluation committees X and Z we may let

$$u_{iz.x} = \begin{cases} 1, if\ for\ the\ i^{th}\ pair\ of\ subjects\ tested\ by\ evaluation\ committees\ X\ and\ Z \\ in\ the\ second\ phase\ of\ trials, the\ subject\ tested\ by\ evaluation\ committee \\ Z\ responds\ positve\ given\ that\ the\ corresponding\ subject\ tested\ by\ evalu \\ ation\ committee\ X\ has\ also\ responded\ positive \\ 0, otherwise \end{cases}$$

for

$$i = 1, 2, \ldots, n_{z.x} \tag{28}$$

Let

$$\pi^+_{z.x} = P(u_{iz.x} = 1) \tag{29}$$

and

$$W_{z.x} = \sum_{i=1}^{n_{z.x}} u_{iz.x} \tag{30}$$

Now,

$$E(u_{iz.x}) = \pi^+_{z.x}; Var(u_{iz.x}) = \pi^+_{z.x}(1 - \pi^+_{z.x}) \tag{31}$$

and

$$E(W_{z.x}) = n_{z.x}.\pi^+_{z.x};\ \ Var(W_{z.x}) = n_{z.x}.\pi^+_{z.x}(1 - \pi^+_{z.x}) \tag{32}$$

Note that $\pi^+_{z.x}$ is the proportion or conditional probability that in the paired samples of subjects tested by evaluation committees X and Z the subjects testedby evaluation committee Z respond positive given that the corresponding subjects tested by evaluation committee X have also responded positive to the new drug or product during the second phase of clinical trials. Its sample estimate is

$$\hat{\pi}^+_{z.x} = P_{z.x} = \frac{W_{z.x}}{n_{z.x}} = \frac{f^+_{z.x}}{n_{z.x}} \tag{33}$$

Where $f^+_{z.x} = W_{z.x}$is the number of pairs of subjects in which the subjects tested by evaluation committee Z respond positive given that the corresponding subject tested by evaluation committee X have also tested positive. Thus, $f^+_{z.x}$ is the total number of 1's in the frequency distribution of the $n_{z.x}$ values of 0's and 1's in $u_{iz.x}$, for $i = 1, 2, \ldots, n_{z.x}$.

The sample variance of $\hat{\pi}^+_{z.x}$ is

$$Var(\hat{\pi}^+_{z.x}) = \frac{Var\,(W_{z.x})}{n^2_{z.x}} = \frac{\hat{\pi}^+_{z.x}\left(1 - \hat{\pi}^+_{z.x}\right)}{n_{z.x}} \tag{34}$$

If desired, a null hypothesis similar to that of Equation 26 for $\pi^+_{y.x}$ may also be tested for $\pi^+_{z.x}$. Similar procedure as above are also followed to obtain sample estimate of the conditional probability $\pi^+_{z.y}$ of positive approval by evaluation committee Z given

Notes

positive approval by evaluation committee Y during the second phase of clinical trials. This would yield a sample estimate of the conditional approval probability $\pi^+_{z.y}$ of positive approval by evaluation committee Z given positive approval by evaluation committee Y as

$$\hat{\pi}^+_{z.y} = P_{z.y} = \frac{W_{z.y}}{n_{z.y}} = \frac{f^+_{z.y}}{n_{z.y}} \qquad (35)$$

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where $f^+_{z.y}$ is the number of pairs of subjects tested by evaluation committees Y and Z in which subjects tested by evaluation committee Z respond positive given that the corresponding subjects tested by evaluation committee Y have also responded positive to the drug or product. The sample variance of $\hat{\pi}^+_{z.y}$ is given by

$$Var\left(\hat{\pi}^+_{z.y}\right) = \frac{Var\,(W_{z.y})}{n^2_{z.y}} = \hat{\pi}^+_{z.y}\frac{(1-\hat{\pi}^+_{z.y})}{n_{z.y}} \qquad (36)$$

If of research interest a null hypothesis similar to Equation 26 for $\pi^+_{y.x}$ may also be stated and tested for $\pi^+_{z.y}$. Note that $\pi^+_{y.x}$, $\pi^+_{z.x}$ and $\pi^+_{z.y}$ are respectively the equivalence of a, c and f in Onyiora et al (2013).

Note also that by the above specifications the sample estimates of marginal and conditional probabilities of positive approval by the three evaluation committees X, Y and Z are respectively

$$P(A) = \hat{\pi}^+_x = P_x; P(B) = \hat{\pi}^+_y = P_y; P(C) = \hat{\pi}^+_z = P_z \qquad (37)$$

and

$$P(B/A) = \hat{\pi}^+_{y.x} = P_{y.x}; P(C/A) = \hat{\pi}^+_{z.x} = P_{z.x}; P(C/B) = \hat{\pi}^+_{z.y} = P_{z.y} \qquad (38)$$

In practice there may actually be no need to estimate such conditional probabilities of positive approval as the conditional probability of positive approval by evaluation committee X given positive approval by evaluation committee Y, that is $P(A/B)$; conditional probability of positive approval by evaluation committee Y given positive approval by evaluation committee Z, namely $P(B/C)$; etc. This is because by the rule of conditional probability and algebraic manipulations, we have for example that

$$P(A/B) = \frac{P(B/A).P(A)}{P(B)} = \frac{P_{y.x}.P_x}{P_y} \qquad (39)$$

Finally, to obtain sample estimates of approval probabilities by three evaluation committees X, Y and Z during the third and last phase of controlled clinical trials, we would proceed as follows:

Suppose each of the n_3 matched triples of subjects that is of each sample of three matched subjects screened or administered a new drug or product by three evaluation committees in the third phase of clinical trials, $n_{z.xy}$ subjects respectively under evaluation committees X and Y in comparison with evaluation committee Z, $n_{y.xz}$ subject respond positive under evaluation committees X and Z in comparison with evaluation committee Y, and $n_{x.yz}$ subjects respond positive under evaluation committees Y and Z in comparison with evaluation committee X.

Now to estimate conditional probability of positive approval by say approval evaluation committee Z given positive approval by evaluation committees X and Y during the third and last phase of clinical trials we may let

$$u_{iz.xy} = \begin{cases} 1, if\ for\ the\ i^{th}\ matched\ sample\ of\ three\ subjects, that\ is\ for\ the\ i^{th} match \\ ed\ tripple\ of\ subjects, the\ subject\ tested\ by\ evaluation\ committee\ Z\ res \\ pond\ pisitive\ given\ that\ the\ corresponding\ subjects\ in\ the\ tripple\ tested \\ by\ evaluation\ committees\ X\ and\ Y\ respectively\ have\ also\ tested\ positive. \\ 0, otherwise \end{cases}$$

for $$i = 1, 2, \ldots, n_{z.xy} \tag{40}$$

Let

$$\pi^{+}_{z.xy} = P\left(u_{iz.xy} = 1\right) \tag{41}$$

and

$$W_{z.xy} = \sum_{i=1}^{n_{z.xy}} u_{iz.xy} \tag{42}$$

Now,

$$E\left(u_{iz.xy}\right) = \pi^{+}_{z.xy};\ \ Var\left(u_{iz.xy}\right) = \pi^{+}_{z.xy}\left(1 - \pi^{+}_{z.xy}\right) \tag{43}$$

Also,

$$E\left(W_{z.xy}\right) = n_{z.xy}.\pi^{+}_{z.xy};\ Var\left(W_{z.xy}\right) = n_{z.xy}.\pi^{+}_{z.xy}\left(1 - \pi^{+}_{z.xy}\right) \tag{44}$$

Here $\pi^{+}_{z.xy}$ is the proportion or conditional probability that in the third phase of clinical trials for the matched samples of three subjects tested by three evaluation committees, subjects tested by evaluation committee Z respond positive given that the other two subjects in the triple separately tested by evaluation committees X and Y have also responded positive to a new drug or product. Its sample estimate is

$$\hat{\pi}^{+}_{z.xy} = P_{z.xy} = \frac{W_{z.xy}}{n_{z.xy}} = \frac{f^{+}_{z.xy}}{n_{z.xy}} \tag{45}$$

where $f^{+}_{z.xy}$ is the number of matched samples of three subject, that is the number of matched triples of subjects in which subjects in the triple tested by evaluation committee Z respond positive given that the other subjects in the same triple tested respectively by evaluation committees X and Y have also responded positive.

In other words, $f^{+}_{z.xy}$ is the total number of 1's in the frequency distribution of the $n_{z.xy}$ values 0's and 1's in $u_{iz.xy}$; $i = 1, 2, \ldots, n_{z.xy}$.

The corresponding sample variance of $\hat{\pi}^{+}_{z.xy}$ from Equation 44

$$Var\left(\hat{\pi}^{+}_{z.xy}\right) = \frac{Var\left(W_{z.xy}\right)}{n^{2}_{z.xy}} = \frac{\hat{\pi}^{+}_{z.xy}\left(1 - \hat{\pi}^{+}_{z.xy}\right)}{n_{z.xy}} \tag{46}$$

Although hypothesis testing may not be as important as the need to determine whether most or all the evaluation committees are able to grant positive approval to a new drug or product for use in a population after a series of phased controlled clinical trials, one may nevertheless wish to test any desired null hypothesis. For example, one may wish to test the null hypothesis that the probability of positive approval of a new drug or product by a given evaluation committee, Z say, assuming that positive approval has been granted by evaluation committees X and Y say, is not more than some value, $\pi_{z.xy0}$. That is, the null hypothesis;

$$H_0: \pi^{+}_{z.xy} \leq \pi_{z.xy0}\ versus\ H_1: \pi^{+}_{z.xy} > \pi_{z.xy0}\left(0 \leq \pi_{z.xy0} \leq 1\right)$$

which is tested using the test statistic

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$$\chi^2 = \frac{\left(W_{z.xy} - n_{z.xy}.\pi_{z.xy\,0}\right)^2}{Var\left(W_{z.xy}\right)} = n_{z.xy}\frac{\left(\hat{\pi}^+_{z.xy} - \pi_{z.xy\,0}\right)^2}{\hat{\pi}^+_{z.xy}\left(1-\hat{\pi}^+_{z.xy}\right)} \tag{48}$$

The null hypothesis of Equation 47 is rejected at the α −level of significance if Equation 10 is satisfied; otherwise the null hypothesis is accepted. Note that $\pi^+_{z.xy}$ is the equivalence of g in Onyiora et al (2013).

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To obtain a sample estimate of the probability of positive response under evaluation committee Y given positive approval by evaluation committees X and Z we may define

$$u_{iy.xz} = \begin{cases} 1, \textit{if for the } i^{th} \textit{ matched sample of three subjects of the } i^{th} \textit{ matched tripple of} \\ \textit{subjects tested by three evaluation committees } X, Y \textit{ and } Z, \textit{the subjects tested by} \\ \textit{evaluation committee } Y \textit{ responds positive given that the other two subjects in} \\ \textit{the matched tripple tested respectively by evaluation committees } X \textit{ and } Z \textit{ also} \\ \textit{responded positve to a new drug or product at the third phase of trials.} \\ 0, \textit{otherwise.} \end{cases}$$

For $$i = 1, 2, \ldots, n_{y.xz} \tag{49}$$

Let

$$\pi^+_{y.xz} = P\left(u_{iy.xz} = 1\right) \tag{50}$$

and

$$W_{y.xz} = \sum_{i=1}^{n_{y.xz}} u_{iy.xz} \tag{51}$$

Now,

$$E\left(u_{iy.xz}\right) = \pi^+_{y.xz}; Var\left(u_{iy.xz}\right) = \pi^+_{y.xz}\left(1 - \pi^+_{y.xz}\right) \tag{52}$$

and

$$E\left(W_{y.xz}\right) = n_{y.xz}.\pi^+_{y.xz}; Var\left(W_{y.xz}\right) = n_{y.xz}.\pi^+_{y.xz}\left(1 - \pi^+_{y.xz}\right) \tag{53}$$

Note that $\pi^+_{y.xz}$ is the proportion or conditional probability that for the matched samples of three subjects, that is for the matched triples of subjects tested by three evaluation committees, subjects tested by evaluation committee Y respond positive given that the corresponding two subjects separately tested by evaluation committees X and Z respectively also respond positive to the drug or product during the third phase of clinical trials. Its sample estimate is

$$\hat{\pi}^+_{y.xz} = P_{y.xz} = \frac{W_{y.xz}}{n_{y.xz}} = \frac{f^+_{y.xz}}{n_{y.xz}} \tag{54}$$

where $f^+_{y.xz}$ is the number of matched triples of subjects in which the subjects tested by evaluation committee Y respond positive given that the other two subjects in the matched triples tested by evaluation committees X and Z respectively have also responded positive, which is also really the total number of 1's in $u_{iy.xz}, i = 1, 2, \ldots, n_{y.xz}$. The sample estimate of the variance of $\hat{\pi}^+_{y.xz}$ is

$$Var\left(\hat{\pi}^+_{y.xz}\right) = \frac{Var\left(W_{y.xz}\right)}{n^2_{y.xz}} = \frac{\hat{\pi}^+_{y.xz}\left(1-\hat{\pi}^+_{y.xz}\right)}{n_{y.xz}} \tag{55}$$

Again if of research interest a null hypothesis similar to that of Equation 47 may be stated and similarly tested for $\hat{\pi}^+_{y.xz}$. Similar procedures as above would enable us to also obtain sample estimate of the conditional probability $\hat{\pi}^+_{y.xz}$, the proportion or conditional probability that during the third phase of clinical trials by evaluation committees X, Y and Z for subjects in the matched triples of subjects tested by these committees, the subjects tested by evaluation committee X respond positive given that corresponding subjects in the matched triples tested by evaluation committees Y and Z respectively also respond positive. This conditional probability is estimated as

$$\hat{\pi}_{x.yz}^{+} = P_{x.yz} = \frac{W_{x.yz}}{n_{x.yz}} = \frac{f_{x.yz}^{+}}{n_{x.yz}} \quad (56)$$

where $W_{x.yz} = f_{x.yz}^{+}$ is the number of matched triples of subjects; that is, matched samples of three subjects in which subjects tested by evaluation committee X respond positive given that the other two subjects in the matched triples tested by evaluation committees Y and Z respectively also tested positive to the new drug or product in the third phase of controlled clinical trials. The sample estimate of the variance of $\hat{\pi}_{y.xz}^{+}$ is similarly obtained as

$$Var\left(\hat{\pi}_{y.xz}^{+}\right) = \frac{Var\,(W_{x.yz})}{n_{x.yz}^{2}} = \frac{\hat{\pi}_{y.xz}^{+}(1-\hat{\pi}_{y.xz}^{+})}{n_{x.yz}} \quad (57)$$

Again if research interest a null hypothesis similar to that of Equation 47 may also be stated and similarly tested for $\pi_{x.yz}^{+}$. Note again that by the specifications adopted above conditional probabilities $P(C/AB), P(B/AC)$ and $P(A/BC)$ namely; $\pi_{z.xy}^{+}, \pi_{y.xz}^{+}$ and $\pi_{x.yz}^{+}$ are estimated as respectively

$$\hat{\pi}_{z.xy}^{+} = P_{z.xy};\ \hat{\pi}_{y.xz}^{+} = P_{y.xz};\ \hat{\pi}_{x.yz}^{+} = P_{x.yz} \quad (58)$$

Other conditional probabilities may be similarly estimated as desired.

If stringencies in terms of high approval probability is a desired and preferred criterion for new drug or product approval, then in the third phase of clinical trials the outcome or event C/AB, say is more desirable and preferable to eventB/AC, say if and only if$P(C/A) > P(B/A)$. This is because if event $C/AB$is more preferable to event B/AC, then

$$P(C/AB) = \frac{P(ABC)}{P(AB)} > \frac{P(ABC)}{P(AC)} = P(B/AC)$$

So that

$$\frac{1}{P(AB)} = \frac{1}{P(A).P(B/A)} > \frac{1}{P(A).P(C/A)} = \frac{1}{P(AC)}$$

Hence

$$P(C/A) > P(B/A)$$

On the other hand if $P(C/A) > P(B/A)$ then clearly $P(C/AB) > P(B/AC)$.

Stated in terms of sample estimates of probabilities, this would mean that in the third phase of three phased controlled clinical trials of a new drug or product by these evaluation committees $X, Y$and$Z$. $P_{z.xy} > P_{y.xz}$ if and only if in the second phase of clinical trials $P_{z.x} > P_{y.x}$.

Other conditional probabilities may be similarly estimated as desired.

Now we have so far presented the probability estimation procedures generally under the assumption that all three evaluation committees are equally competent in experience or otherwise to assess and evaluate new drug or product. In reality however some evaluation committees may be better qualified, experienced, with higher expertise, better equipped etc., than others and hence may play supervisory roles and be able to obtain more reliable results. Hence we may but without loss of generality assume that three evaluation committees used here can be ordered in terms of experience and

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seniority in assessment, evaluation and approval of new drugs or products ranked from the most senior down to the least senior. Thus we may again but without loss of generality assume that evaluation committee X is the most senior followed by evaluation committees Y and Z in this order. This would in effect mean that any drug or product approved by evaluation committee Z would be subjected to further approvals by evaluation committee Y and finally by evaluation committee X. Under these assumptions the probabilities already estimated above would be sufficient to estimate the required overall approval probability after the third and last phase of controlled clinical trials.

Never the-less, the proposed probability estimation model would enable the estimation of the probabilities of all events that can possibly be obtained in the event space of all conceivable outcomes in phased controlled clinical trials. For example the probability that say evaluation committees X and Y do not approve a new drug or product given that evaluation committee Z approves, is the probability of the event $(\bar{A}\bar{B}/C)$ which is

$$P(\bar{A}\bar{B}/C) = \frac{P(C) - P(A).P(C/A) - P(B).P(C/B) + P(C/AB).P(B/A).P(A)}{P(C)}$$

or In terms of estimated probabilities,

$$P(\bar{A}\bar{B}/C) = P_z - P_x.P_{z.x} - P_y P_{z.y} + P_{z.xy}.P_{y.x}.P_x$$

However for the purpose of this paper, if interest is only in the estimation of the probabilities of the events in table 1 of Onyira et al (2013) which we obtained using the marginal and conditional probabilities already estimated above, namely

$$P(A) = a = P_x; P(B) = b = P_y; P(C) = c = P_z \quad (59)$$

and

$$P(B/A) = d = P_{y.x}; P(C/A) = e = P_{z.x}; P(C/B) = f = P_{z.y}; P(C/AB) = g = P_{z.xy} \quad (60)$$

With these results the probability that all the three evaluation committees X, Y and Z approved a new drug or product is the probability of the event$S_3 = (ABC)$ which is estimated using sample values obtained above as

$$P(ABC) = P(C/AB).P(B/A).P(A) = P_{z.xy};\ P_{y.x};\ P_x \quad (61)$$

If at least two evaluation committees must approve a new drug or product before use, then the corresponding events set is $S_2 = (ABC, AB\bar{C}, \bar{A}BC)$ whose probability is easily shown to be

$$P(S_2) = P_x.P_{y.x} + P_x.P_{z.x} + P_y.P_{z.y} - 2P_{z.xy}.P_{y.x}.P_x \quad (62)$$

If there is a supervising evaluation committee such as evaluation committee X who must approve in addition to at least one other evaluation committee before a new drug or product is considered approved for use, then the required events set is $S_x = (ABC, AB\bar{C}, A\bar{B}C)$ whose sample estimate is

$$P(S_x) = P_x.P_{y.x} + P_x.P_{z.x} - P_{z.xy}.P_{yx}.P_x \quad (63)$$

The probability that evaluation committees Y and Z approve a drug or product but evaluation committee X does not approve it is the probability $S_{yz} = (\bar{A}BC)$which is estimated as

$$P(S_{yz}) = P(\bar{A}.BC) = (1 - P(A/BC)).P(BC) = P(C/B).P(B) - P(ABC)$$

which when expressed in terms of sample probabilities becomes

$$P(S_{yz}) = P_y.P_{zy} - P_{z.xy}.P_{y.x}.P_x \tag{64}$$

The probability that none of the evaluation committees approves the drug or product for use is the probability of the event $S_0 = (\bar{A}\bar{B}\bar{C})$ which is

$$P(S_0) = P(\bar{A}\bar{B}\bar{C}) = 1 - \left(P(A) + P(B) + P(C) - P(B/A).P(A) - P(C/A).P(A) - P(C/B).P(B) + P(ABC)\right)$$

Which when evaluated in terms of sampled estimates becomes

$$P(S_0) = 1 - \left(P_x + P_y + P_z - P_x.P_{y.x} - P_x.P_{z.x} - P_y.P_{z.y} + P_{z.xy}.P_{y.x}P_x\right) \tag{65}$$

Other probabilities are similarly estimated. The results are shown in Table 1

Table 1: Sample Estimates of New Drug or Product Approval Probabilities by three Evaluation in Phased Clinical Trials

S/No	Event	Approval Probability
1	ABC	$P_{z.xy}.P_{y.x}P_x$
2	$AB\bar{C}$	$P_x.P_{y.x} - P_{z.xy}.P_{y.x}.P_x$
3	$A\bar{B}C$	$P_x.P_{z.x} - P_{z.xy}P_{y.x}.P_x$
4	$A\bar{B}\bar{C}$	$P_x - P_x.P_{z.x} - P_xP_{y.x} + P_{z.xy}P_{y.x}.P_x$
5	$\bar{A}BC$	$P_y.P_{z.y} - P_{z.xy}.P_{yx}P_x$
6	$\bar{A}B\bar{C}$	$P_y - P_y.P_{z.y} - P_x.P_{y.x} + P_{z.xy}.P_{y.x}.P_x$
7	$\bar{A}\bar{B}C$	$P_z - P_x.P_{z.y} - P_y.P_{z.y} + P_{z.xy}.P_{y.x}.P_{y.x}$
8	$\bar{A}\bar{B}\bar{C}$	$1 - (P_x + P_y + P_x.P_{yx} - P_x.P_{zx} - P_y.P_{z.y}$ $+ P_{z.xy}.P_{y.x}.P_x)$
9	S_2(at least two Evaluation; committees)	$P_x.P_{y.x} + P_x.P_{z.x} + P_y.P_{z.y} - 2P_{z.xy}.P_{y.x}.P_x$
10	S_x(Evaluation Committee X; and at least one other)	$P_x.P_{y.x} + P_x.P_{z.x} - P_{z.xy}.P_{y.x}.P_x$
11	S_y(Evaluation Committee Y and at least one other)	$P_x.P_{y.x} + P_y.P_{z.y} - P_{z.xy}.P_{y.x}.P_x$
12	S_z(Evaluation Committee Z and at least one other)	$P_y.P_{z.y} + P_x.P_{z.x} - P_{z.xy}.P_{y.x}.P_x$

III. Illustrative Example

Teams of research scientists in the Department of Pharmacology of three Universities X, Y and Z were interested in conducting phased controlled prospective clinical trials on a certain herb product believed by a local population to be effective in the treatment of malaria. In the first phase of clinical trials the three research teams collected three random samples each of size 40 of volunteer malaria patients matched on age, gender and body mass index(BMI),and each research team or committee team administered appropriately determined dosages of the malaria herb product each on patients in only one of the three matched samples.

In the second phase of clinical trials three matched pairs of patients each of size 30 were used. The three research teams were also then paired. Each pair of the research team administered dosages of the herb product to one paired sample of patients with one research team administering the dosage to say the first patient in each pair and the other research team administering the dosage to the remaining patient in the pair.

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In the third phase of the clinical trials, 25 samples of matched triples of patients, that is 25 samples each of three matched patients were used. The three research teams each administered dosages of the malaria herb product to only one patient in each of the 25 matched triples of patients. At the end of each phase of the clinical trials the research scientists assesses the malaria patients as either recovered (R) or not recovered (N) obtaining the results shown in tables 2 – 4.

Table 2: Patient Response in Phase One Clinical Trials of Anti-Malaria Herb Product by Three Research Teams

S/No	Team 1 (Sample 1)	Team 2 (Sample 2)	Team 3 (Sample 3)
	X	Y	Z
1	R	N	R
2	R	R	N
3	R	N	N
4	N	R	R
5	R	R	R
6	R	N	R
7	N	R	N
8	N	N	N
9	N	N	R
10	N	R	R
11	N	N	N
12	R	N	R
13	N	R	R
14	R	R	R
15	N	N	R
16	R	N	N
17	R	R	N
18	R	R	R
19	N	R	R
20	R	N	R
21	N	N	N
22	R	R	R
23	N	R	N
24	R	R	R
25	N	R	N
26	R	N	R
27	R	R	R
28	R	R	N
29	R	R	N
30	R	R	N
31	N	R	R
32	R	N	R
33	N	R	N
34	R	R	N
35	N	R	R
36	N	N	N
37	N	N	R
38	R	N	N
39	R	N	R
40	R	N	N
n_i	40	40	40
f_i^+	23(f_x^+)	22(f_y^+)	22
$\hat{\pi}_i^+$	0.575$(\hat{\pi}_x^+)$	0.55	0.55

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Table 3: Patient Response in Phase Two Clinical Trials of Malaria Herb Product by Three Research Teams

	Matched Pair Team 1		Matched Pair Team 2		Matched Pair Team 3	
	X	Y	X	Z	Y	Z
1	N	N	R	N	N	N
2	R	R	N	R	R	R
3	R	R	R	R	N	R
4	N	R	R	N	R	R
5	N	R	N	R	R	R
6	R	N	R	R	N	R
7	N	R	N	R	R	N
8	N	N	N	N	R	R
9	R	N	R	R	R	R
10	N	N	R	N	N	N
11	N	N	R	R	N	R
12	R	N	N	R	N	N
13	R	N	N	N	R	N
14	R	R	R	R	N	N
15	N	R	N	R	N	N
16	N	N	R	N	R	R
17	R	N	R	R	R	R
18	N	N	R	R	R	N
19	N	R	N	N	N	N
20	N	N	N	N	N	N
21	N	R	R	N	N	N
22	R	N	R	N	N	R
23	N	N	R	R	R	N
24	R	R	R	N	R	R
25	N	R	N	N	N	R
26	N	R	N	R	N	R
27	N	R	N	N	R	N
28	R	N	R	N	R	R
29	N	N	R	R	R	R
30	R	N	R	R	R	R
$n_{K.J}$	$12(n_{y.x})$		$18(n_{x.z})$		$16(n_{y.z})$	
$f^+_{k.j}$	$4(f^+_{y.x})$		$10(f^+_{x.z})$		$11(f^+_{y.z})$	
$\hat{\pi}^+_{k\ i}$	$0333(\hat{\pi}^+_{y.x})$		$0.556(\hat{\pi}^+_{x.x})$		$0.688(\hat{\pi}^+_{y.z})$	

Table 4: Patient Response in Phase Three Clinical Trials of Anti-Malaria Herb Product by Three Research Teams

Matched Triple	Research Team X	Research Team Y	Research Team Z
1	N	R	N
2	R	N	N
3	R	R	N
4	N	R	R
5	N	N	R
6	R	R	R
7	N	N	N
8	N	R	N
9	R	R	R
10	N	N	N
11	N	N	N
14	R	N	N
15	N	N	N
16	R	R	R
17	N	N	R
18	R	N	R
19	R	N	N

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20	N	N	R
21	N	R	N
22	N	R	N
23	R	R	R
24	N	R	R
25	N	R	R
$n_{k.lj}$	$8(n_{x.yz})$	$5(n_{y.xz})$	$6(n_{z.xy})$
$f_{k.lj}^+$	$4(f_{x.yz}^+)$	$4(f_{y.xz}^+)$	$4(f_{z.xy}^+)$
$\hat{\pi}_{k.lj}^+$	$0.500(\hat{\pi}_{x.yz}^+)$	$0.800(\hat{\pi}_{y.xz}^+)$	$0.667(\hat{\pi}_{z.xy}^+)$

We here use the sample data of Table 2-4 to illustrate the present probability estimation method. Thus applying the methods to the data we have as shown at the bottom of Table 2 with $n = n_1 = 40$, that

$$f_x^+ = 23;\ f_y^+ = 22\ and\ f_z^+ = 22; so\ that\ \hat{\pi}_x^+ = P_x = 0.575\ (= a)$$

$$\hat{\pi}_y^+ = P_y = 0.550(= b); and\ \hat{\pi}_z^+ = P_z = 0.550\ (= c)$$

From Table 3 we have that

$$n_{y.x} = 12;\ n_{z.x} = 18\ and\ n_{z.y} = 16$$

Also,

$$f_{y.x}^+ = 4;\ f_{z.x}^+ = 10\ and\ f_{z.y}^+ = 11$$

Hence,

$$\hat{\pi}_{y.x}^+ = P_{y.x} = 0.333\ (= d);\ \hat{\pi}_{z.x}^+ = P_{z.x} = 0.556\ (= e); and \hat{\pi}_{z.y}^+ = P_{z.y} = 0.688\ (= f).$$

Finally from Table 4 we have that

$$n_{z.xy} = 6\ and\ f_{z.y}^+ = 4$$

Hence,

$$\hat{\pi}_{z.xy}^+ = P_{z.xy} = 0.667\ (= g)$$

Note also from Table 4 that

$$n_{y.xz} = 5, n_{x.yz} = 8;\ f_{y.xz}^+ = f_{x.yz}^+ = 4\ so\ that\ \hat{\pi}_{y.xz}^+ = P_{y.xz} = 0.800; and\ \hat{\pi}_{x.yz}^+ = P_{x.yz} = 0.500$$

These probability estimates are now used with Table 2 to obtain sample estimates of some of possibilities of outcomes in three phased controlled clinical trials of a product, namely anti-malaria herb product. The estimates are presented in Table 5.

Table 5: Sample Estimates of Probabilities of the events of Table 1 for anti-malaria herb product

S/No	Event	Estimated Approval Probability
1	ABC	0.127
2	$AB\bar{C}$	0.064
3	$A\bar{B}C$	0.193
4	ABC	0.191
5	ABC	0.251
6	$\bar{A}B\bar{C}$	0.108
7	$\bar{A}\bar{B}C$	0.021
8	$\bar{A}\bar{B}\bar{C}$	0.079
9	S_2 (at least two evaluation committees)	0.635
10	S_x (evaluation committee and at least one other)	0.384
11	S_v(evaluation committee and at least one other)	0.442
12	S_z (evaluation committee and at least one other)	0.571

Notes

It is seen from Table 2 that in the first phase of controlled clinical trials, evaluation committee X approved the anti-malaria herb product with an estimated probability of 0.575 while evaluation committees$Y$ and Z approved the drug with equal probability of 0.550.

In the second phase of clinical trials (Table 3 given that evaluation committee X has approved the drug, evaluation committees Y and Z are found to approve the drug with estimated probabilities of 0.333 and 0.556 respectively while if evaluation committee Y has already approved the drug, then evaluation committee Z would be expected to approve the drug with probability 0.688.

In the third phase of clinical trials (Table 4) it is seen that if evaluation committees X and Y have already approved the drug, then evaluation committee Z would approve the drug with an estimated probability of 0.667 while evaluation committee Y would approve with estimated probability of 0.800 if evaluation committees X and Z have already granted the approval. From Table 5, it is seen that if all three evaluation committees are required to grant approval before a new drug or product (anti-malaria herb product) can be approved for use in a population then the estimated probability of such an approval being granted is only 12.7 percent, which is relatively more stringent compared with when only two evaluation committees are required to grant approval with an estimated probability of which is relatively more liberal.

$$(0.575)(0.333) + (0.575)(0.556) + (0.530)(0.688) - 3(0.127) = 0.889 - 0.381 = 0.508$$

Note from Table 5 that at the end of the third phase of clinical trials if the drug must be approved by at least one evaluation committee as the supervisory committee, then evaluation committee X is seen to be the most stringent with an estimated overall probability of approval of only 38.4 percent while evaluation committee Z is the most liberal with an estimated overall probability of approval of as high as 57.1 percent. It is found that just as the probability of three evaluation committees completely agreeing approve drug after the third phase of clinical trials is rather small at 0.127, the probability of three committees being in complete agreement not to approve the drug is even much smaller with an estimated value of only 7.9 percent.

IV. Summary and Conclusion

We have in this paper developed and presented statistical method that would enable the estimation of probabilities of approving and not approving a new drug or product for possible use in a population under the assumption that three evaluation committees are used to assess and evaluate the drug or product in clinical trials conducted in three phases. At each phase of clinical trials evaluation committees used matched samples of subjects for drug or product quality evaluation or assessment.

Test statistics were developed for testing any desired hypothesis about approval probabilities each phase of clinical trials. The proposed method was illustrated with some sample data and the results show that the probabilities of three evaluation committees being in complete agreement to approve and not approve a new drug or product are likely to be much smaller than the probabilities that only some of the three evaluation committees approve the drug or product.

Notes

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The Study of Effects of Gravity Modulation on Double Diffusive Convection in Oldroyd-B Liquids

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Abstract- The effect of gravity modulation is analysed in Oldroyd-B liquids subjected to double diffusive convection. Both linear and non-linear analysis has been done. A regular perturbation technique has been employed to arrive at the thermal Rayleigh number. The results show that stress relaxation destabilises the system whereas strain retardation parameter and Lewis number stabilises the system. Truncated Fourier series expansion gives a system of Lorentz equations that represent the non-linear analysis. Nusselt and Sherwood numbers are used to quantify the heat and mass transfer respectively. It is observed that Lewis number and strain retardation parameter decreases heat and mass transfer and stress relaxation parameter increases them. It is seen that modulation gives rise to sub-critical motion.

Keywords: double diffusive convection, gravity modulation, oldroyd-B liquids, rayleigh-bénard convection.

GJSFR-F Classification: *MSC 2010: 83C27*

THESTUDYOFEFFECTSOFGRAVITYMODULATIONONDOUBLEDIFFUSIVECONVECTIONINOLDROYDBLIQUIDS

Strictly as per the compliance and regulations of:

The Study of Effects of Gravity Modulation on Double Diffusive Convection in Oldroyd-B Liquids

R. K. Vanishree [α] & K. Anjana [σ]

Abstract- The effect of gravity modulation is analysed in Oldroyd-B liquids subjected to double diffusive convection. Both linear and non-linear analysis has been done. A regular perturbation technique has been employed to arrive at the thermal Rayleigh number. The results show that stress relaxation destabilises the system whereas strain retardation parameter and Lewis number stabilises the system. Truncated Fourier series expansion gives a system of Lorentz equations that represent the non-linear analysis. Nusselt and Sherwood numbers are used to quantify the heat and mass transfer respectively. It is observed that Lewis number and strain retardation parameter decreases heat and mass transfer and stress relaxation parameter increases them. It is seen that modulation gives rise to sub-critical motion.

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I. Introduction

The study of convection in non-Newtonian liquids has been a topic of interest due to its usage as a working media in many engineering and industrial applications. Viscoelastic fluids exhibits both solid and liquid properties and find application in diverse fields such as geothermal energy modeling, crystal growth, solar receivers etc. Some other applications include chemical industry, bioengineering, petroleum industry and so on. These liquids are defined by constitutive equations which include complex differential operators. They also include relaxation and retardation times. As they possess both elastic (property of solids) and viscosity (property of liquids) leading to a unique instability patterns such as overstability which is not observed in Newtonian fluids. This is the main reason why many researchers have studied Rayleigh-Benard convection in a rectangular layer of viscoelastic fluid heated from below (Vest and Apaci [1], Sokolov and Tanner[2, Green[3], Siddheshwar *et. al.* [4]).

Oldroyd-B liquid is a type of viscoelastic fluid. The study of stationary and oscillatory convection in viscoelastic fluids gave information about the formation of pattern in these fluids (Li and Khayat [5,6]). It was also found that a thin layer of fluid when heated from below sets up oscillatory convection. Siddheshwar and Krishna [7] investigated Rayleigh– Bénard convection in a viscoelastic fluid and found that the ratio of strain retardation parameter to the stress relaxation parameter should be less than one for convection to set in.

The nonlinear stability analysis under the influence of gravity modulation in viscoelastic fluids was studied by Siddheshwar [8] who found that modulation helps in controlling the onset of convection. The destabilizing and stabilizing effects of rotation

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Ref

1. Charles M. Vest, Vedat S. Arpaci, 1969, *“Stability of natural convection in a vertical slot”*, J. Fluid Mech., 36, pp. 1-15.

on Oldroyd-B liquids were found by Sharma [9]. In spite of these studies not many literature exists on non-linear convection in Oldroyd-B liquids.

The classical Rayleigh– Bénard convection arises due to temperature gradient alone. In most practical cases convection may be caused not just due to one gradient, but multiple gradients. Double diffusion occurs when there are two components whose rates of diffusion are different. These varying diffusivities give rise to unpredictable movement of fluid particles, thus making these problems interesting. In majority of cases the two varying components are temperature and solute (Mojtabi and Charrier-Mojtabi [10]). In such fluids density variations depend on both thermal and solutal gradients which diffuse at different rates. This leads to the formation of salt fingers or oscillations in the fluid layer. Malashetty and Swamy [11] found that there is a competition between the processes of thermal diffusion, solute diffusion and viscoelasticity that causes the convection to set in through oscillatory mode rather than stationary. A common example of double diffusive convection is found in the ocean (Stommel *et al.* [12]). Stommel noticed that with the decrease in solute quantity there was a large amount of potential energy available. Further study on this was done by Stern [13, 14] who made the observation that if there are two diffusing components in a system, then the behaviour of the system depend on whether the solute component is stabilizing or destabilizing. Siddheshwar and Pranesh [15] studied the effects of temperature modulation and g-gitter on magneto-convection in a weak electrically conducting fluid with internal angular momentum. The effects of temperature modulation on double diffusion were found by Bhadauria [16]. Double diffusive magneto convection in viscoelastic fluids was investigated by Narayana *et. al.* [17]. A stability analysis of chaotic and oscillatory magneto-convection in binary viscoelastic fluids with gravity modulation was done by Bhadauria and Kiran [18]. A Ginzburg–Landau model was adopted to find the effects of the parameters. It was found that gravity modulation can be used to either advance or delay convection by varying its frequency. Siddheshwar *et. al.* [19] analyzed the heat transport by stationary magneto-convection in Newtonian liquids under g-gitter and temperature modulation and obtained similar. Kiran [20] used the Darcy model the porous medium to study the nonlinear thermal convection in a porous medium saturated with viscoelastic nanofluids and found that frequency of modulation can be varied to get the desired results with respect to onset of convection.

In this paper we use linear and non-linear stability analysis to investigate the effects of gravity modulation on double diffusive convection in Oldroyd-B liquid.

II. Mathematical Formulation

Consider a layer of Oldroyd-B liquid held between two parallel plates at z = 0 and z = d. The two plates are maintained at two different temperatures with the difference in temperatures and solute concentrations ΔT and ΔS respectively. This causes variable heating of the fluid particles and hence, variable movements. That is, a temperature gradient arises and in turn gives rise to convection. The fluid density is assumed to be a linear function of temperature, T, and solute concentration, S. A Cartesian co-ordinate system is taken with origin in the lower boundary and z-axis vertically upwards (fig 1).

Ref

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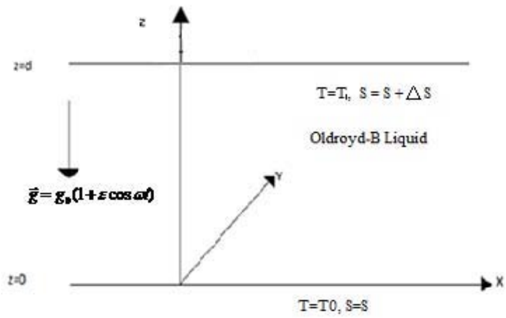

Fig.: Physical Configuration

Nomenclature

d	thickness of the liquid
k	dimensionless wave number
pr	Prandtl number
q	velocity
Ra	thermal Rayleigh number
Rs	solutal Rayleigh number
t	time
T	temperature
T_o	constant temperature of the upper boundary
T_R	reference temperature
Le	Lewis number
Greek symbols	
α	thermal expansion coefficient
ε	amplitude of modulation
κ	thermal diffusivity
κ_s	solutal diffusivity
λ_1	stress relaxation coefficient
λ_2	strain retardation coefficient
Λ	elastic ratio(λ_2 / λ_1)
μ	viscosity
ω	frequency of modulation
ρ	density

Thus the governing equations for Rayleigh-Benard situation of an Oldroyd-B liquid are:

Continuity Equation:

$$\nabla.\vec{q} = 0 \tag{1}$$

Conservation of momentum:

$$\rho_0\left(\frac{\partial \vec{q}}{\partial t}\right) = -\nabla p + \rho \vec{g}(t) + \nabla.\tau' \tag{2}$$

Rheological Equation:

$$\left(1+\lambda_1 \frac{\partial}{\partial t}\right)\tau' = \left(1+\lambda_2 \frac{\partial}{\partial t}\right)\left(\nabla \vec{q} + \nabla \vec{q}^{\,tr}\right) \tag{3}$$

Conservation of Energy:

$$\frac{\partial T}{\partial t} + (\vec{q}.\nabla)T = \kappa \nabla^2 T \tag{4}$$

Conservation of Species:

$$\frac{\partial S}{\partial t} + (\vec{q}.\nabla)S = \kappa_s \nabla^2 S \tag{5}$$

Energy Equation:

$$\rho = \rho_0\left(1-\alpha(T_b - T_0) + \alpha_s(S_b - S_0)\right) \tag{6}$$

The variation of gravity with time is given by

$$\vec{g}(t) = g_0(1+\delta\varepsilon \cos \omega t)\vec{k} \tag{7}$$

Where δ is the amplitude of gravity modulation and ε is a small quantity indicative of weak variation.

III. Basic State

In the basic state the fluid is at rest. Therefore, the velocity is zero and the other parameters are function of z alone.

$$\vec{q} = \vec{q}(b) = 0, \quad p = p_b(z), \quad \rho = \rho_b(z), \qquad S = S_b(z), \quad T = T_b(z) \tag{8}$$

The temperature T_b, pressure p_b and density ρ_b satisfy

$$\frac{dp}{dz} + \rho g(1+\varepsilon \cos \omega t) = 0 \tag{9}$$

$$\frac{\partial T_b}{\partial t} = \kappa \frac{\partial^2 T_b}{\partial z^2} \tag{10}$$

Using the boundary condition, the above equation yields

$$T_b = -\frac{\Delta T}{d} + T_0 \tag{11}$$

$$\rho = \rho_0\left(1-\alpha(T_b - T_0) + \alpha_s(S_b - S_0)\right) \tag{12}$$

The rheological equation takes the form

$$\left(1+\lambda_1 \frac{\partial}{\partial t}\right)\left[\rho_0 \frac{\partial \vec{q}}{\partial t} + \nabla p + \rho g(1+\varepsilon \cos \omega t)\right]$$

Notes

$$= \mu\left(1+\lambda_2 \frac{\partial}{\partial t}\right)\nabla^2 \vec{q} \tag{13}$$

IV. Stability Analysis

Notes

The infinitesimal perturbations on the basic state are superimposed to study the stability of the system. The basic state is slightly perturbed by an infinitesimal perturbation as given in eq. (14). The primes denote the perturbations.

$$\vec{q} = \vec{q}', p = p_b + p', \rho = \rho_b + \rho', T = T_b + T', S = S_b + S' \tag{14}$$

Substituting eq. (14) in the governing equations and with the help of the basic state solutions, we get eq. (15) – (17) for the perturbations

$$\frac{\partial T'}{\partial t} + w'\frac{\partial T'}{\partial z} + w'\frac{\partial T_b}{\partial z} = \kappa \nabla^2 T' \tag{15}$$

$$\frac{\partial S'}{\partial t} + w'\frac{\partial S'}{\partial z} + w'\frac{\partial S_b}{\partial z} = \kappa_s \nabla^2 S' \tag{16}$$

$$\left(1+\lambda_1 \frac{\partial}{\partial t}\right)\begin{bmatrix} \rho_0 \dfrac{\partial(\nabla^2 w')}{\partial t} - \alpha\rho_0 g(1+\varepsilon\cos\omega t)\nabla_1^2 T' \\ +\alpha_s \rho_0 g(1+\varepsilon\cos\omega t)\nabla_1^2 S' \end{bmatrix}$$

$$= \mu\left(1+\lambda_2 \frac{\partial}{\partial t}\right)\nabla^4 w' \tag{17}$$

Eq. (20) is used to arrive at the non-dimensional form of the above equations.

$$w^* = \frac{w'}{\kappa/d}, \ t^* = \frac{t}{d^2/\kappa}, \ \theta = \frac{T'}{\Delta T}, \ \phi = \frac{S'}{\Delta S},$$

$$\nabla^* = d\nabla, \ (x^*, y^*, z^*) = \left(\frac{x}{d}, \frac{y}{d}, \frac{z}{d}\right) \tag{18}$$

Since we consider only two-dimensional disturbances, we introduce the stream function ψ such that

$$u = \frac{\partial \psi}{\partial z}, w = -\frac{\partial \psi}{\partial x} \tag{19}$$

and all the terms are independent of y. The resulting non-dimensional equations are:

$$\left[-\left(1+\Lambda_1 \frac{\partial}{\partial t}\right)\frac{1}{\Pr}\nabla^2 \frac{\partial}{\partial t} + \left(1+\Lambda_2 \frac{\partial}{\partial t}\right)\nabla^4\right]\frac{\partial \psi}{\partial x} =$$

$$\left(1+\Lambda_1 \frac{\partial}{\partial t}\right)\left[Ra(1+\varepsilon\cos\omega t)\nabla_1^2\theta + Rs(1+\varepsilon\cos\omega t)\nabla_1^2\phi\right], \tag{20}$$

$$\left(\frac{\partial}{\partial t} - \nabla^2\right)\theta + (\vec{q}.\nabla)\theta = \frac{\partial \psi}{\partial x}, \tag{21}$$

$$\left(\frac{\partial}{\partial t}-\frac{1}{Le}\nabla^2\right)\phi+(\vec{q}.\nabla)\phi=-\frac{\partial\psi}{\partial x}. \tag{22}$$

The non-dimensional parameters appearing in eq. (20) – (22) are the Prandtl number, Thermal Rayleigh number, Solutal Rayleigh number, stress relation parameter and strain retardation parameter, which are given in equation (23).

$$\Lambda_1=\frac{\lambda_1\kappa}{d^2},\ \Lambda_2=\frac{\lambda_2\kappa}{d^2},\ Le=\frac{\kappa}{\kappa_s},\quad \Pr=\frac{\mu}{\rho_0\kappa},$$

$$Ra=\frac{\alpha\rho_0 g\Delta T d^3}{\mu\kappa},\quad Rs=\frac{\alpha_s\rho_0 g\Delta S d^3}{\mu\kappa} \tag{23}$$

V. Linear Stability Analysis

In this section, we discuss the linear stability analysis by considering marginal and over-stable states. The solution of this analysis is of great utility in the local non-linear stability analysis discussed in the later sections. The linearlized equations after neglecting the non linear terms are:

$$\left[-\left(1+\Lambda_1\frac{\partial}{\partial t}\right)\frac{1}{\Pr}\nabla^2\frac{\partial}{\partial t}+\left(1+\Lambda_2\frac{\partial}{\partial t}\right)\nabla^4\right]\frac{\partial\psi}{\partial x}=$$

$$\left(1+\Lambda_1\frac{\partial}{\partial t}\right)\left[Ra(1+\varepsilon\cos\omega t)\nabla_1^2\theta+Rs(1+\varepsilon\cos\omega t)\nabla_1^2\phi\right], \tag{24}$$

$$\left(\frac{\partial}{\partial t}-\nabla^2\right)\theta=\frac{\partial\psi}{\partial x}, \tag{25}$$

$$\left(\frac{\partial}{\partial t}-\frac{1}{Le}\nabla^2\right)\phi=-\frac{\partial\psi}{\partial x}. \tag{26}$$

Eq. (24) – (26) are reduced to a single equation by eliminating θ and φ to get an equation in terms of the stream function, ψ.

$$\left(\frac{\partial}{\partial t}-\nabla^2\right)\left(\frac{\partial}{\partial t}-\frac{1}{Le}\nabla^2\right)\left[\begin{array}{l}\left(1+\Lambda_1\frac{\partial}{\partial t}\right)\frac{1}{pr}\frac{\partial}{\partial t}\\ \qquad -\left(1+\Lambda_2\frac{\partial}{\partial t}\right)\nabla^2\end{array}\right]\nabla^2\psi$$

$$=\left(1+\Lambda_1\frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial t}-\nabla^2\right)\left(Ra(1+\varepsilon\cos\omega t)\frac{\partial^2\psi}{\partial x^2}\right)$$

$$-\left(1+\Lambda_1\frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial t}-\frac{1}{Le}\nabla^2\right)Rs(1+\varepsilon\cos\omega t)\frac{\partial^2\psi}{\partial x^2} \tag{27}$$

VI. Perturbation Procedure

We seek the eigenfunction, ψ, and eigenvalue Ra of eq. (27) for the basic temperature distribution that departs from the linear profile by using quantities of order ε. Thus, the eigenvalues of the present problem differ from those of double diffusive

Notes

convection in Oldroyd-B liquids by quantities of ε. The solution of eq. (27) is sought in the form

$$\psi = \psi_0 + \varepsilon\psi_1 + \varepsilon^2\psi_2 + ...$$

$$Ra = Ra_0 + \varepsilon Ra_1 + \varepsilon^2 Ra_2 + ... \qquad (28)$$

Malkus and Veronis [21] first used this type of expansion in connection with the study of finite amplitude convection. Here ψ_0 and Ra_0 are the eigenfunction and eigenvalue respectively of the unmodulated system and (ψ_i, Ra_i), i>1 are the corrections due to modulation of ψ_0 and Ra_0.

These expansions are used in eq. (28) and the coefficients of various powers of ε are equated to obtain the following system of equations

$$L\psi_0 = 0 \qquad (29)$$

$$L\psi_1 = \left(\frac{\partial}{\partial t} - \frac{1}{Le}\nabla^2\right)\left(1+\Lambda_1\frac{\partial}{\partial t}\right)\left(Ra_1\frac{\partial^2\psi_0}{\partial x^2} + Ra_0\cos\Omega t\frac{\partial^2\psi_0}{\partial x^2}\right)$$

$$+\left(\frac{\partial}{\partial t} - \nabla^2\right)\left(1+\Lambda_1\frac{\partial}{\partial t}\right)Rs\cos\Omega t\frac{\partial^2\psi_0}{\partial x^2} \qquad (30)$$

$$L\psi_2 = \left(\frac{\partial}{\partial t} - \frac{1}{Le}\nabla^2\right)\left(1+\Lambda_1\frac{\partial}{\partial t}\right)$$

$$\left(Ra_0\cos\Omega t\frac{\partial^2\psi_1}{\partial x^2} + Ra_1\frac{\partial^2\psi_1}{\partial x^2} + Ra_1\cos\Omega t\frac{\partial^2\psi_1}{\partial x^2} + Ra_2\frac{\partial^2\psi_0}{\partial x^2}\right) \qquad (31)$$

$$-\left(\frac{\partial}{\partial t} - \nabla^2\right)\left(1+\Lambda_1\frac{\partial}{\partial t}\right)Rs\cos\Omega t\frac{\partial^2\psi_1}{\partial x^2}$$

Where,

$$L = \left(\frac{\partial}{\partial t} - \nabla^2\right)\left(\frac{\partial}{\partial t} - \frac{1}{Le}\nabla^2\right)$$

$$\left[\left(1+\Lambda_1\frac{\partial}{\partial t}\right)\frac{1}{pr}\frac{\partial}{\partial t} - \left(1+\Lambda_2\frac{\partial}{\partial t}\right)\nabla^2\right]\nabla^2$$

$$-\left(1+\Lambda_1\frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial t} - \nabla^2\right)Ra_0\frac{\partial^2}{\partial x^2}$$

$$+\left(1+\Lambda_1\frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial t} - \frac{1}{Le}\nabla^2\right)Rs\frac{\partial^2}{\partial x^2} \qquad (32)$$

Each ψ_n is required to satisfy the boundary condition

$$\psi = \nabla^2\psi = \nabla^4\psi = 0 \text{ at } z = 0, 1. \qquad (33)$$

a) Solution to the zeroth order problem

The absence of gravity modulation is equivalent to the zeroth order problem. The general solution of eq. (29), obtained at $o(\varepsilon^0)$, is the marginally stable solution of the zeroth order problem. The marginally stable solutions are

Ref

21. W.V.R. Malkus, G. Veronis, 1958, *"Finite amplitude cellular convection"*, J. Fluid Mech., 4, pp. 225-260.

$$\psi_0 = \sin(\pi\alpha x)\sin(\pi z) \tag{34}$$

with the corresponding eigenvalue R_{a0} given by

$$Ra_0 = \frac{\delta^6}{\pi^2 a^2} + LeR_s + \varepsilon^2 Ra_2 \tag{35}$$

b) Solution to the first order problem

Substituting Eq. (34) in Eq. (30), we get

$$\begin{aligned} L\psi_1 &= \frac{-\pi^2\alpha^2 k^2}{Le} Ra_0 \cos\Omega t\psi_0 + Ra_0\pi^2\alpha^2\Omega\sin\Omega t\psi_0 \\ &+ Ra_0\pi^2\alpha^2\Lambda_1\Omega^2\cos\Omega t\psi_0 + Rs\pi^2\alpha^2 k^2\cos\Omega t\psi_0 \\ &- Rs\pi^2\alpha^2\Omega\sin\Omega\psi_0 - Rs\pi^2\alpha^2\Lambda_1\Omega^2\cos\Omega t\psi_0 \end{aligned} \tag{36}$$

Where,

$$L(\omega, n) = Y_1 + iY_2 \tag{37}$$

$$\begin{aligned} Y_1 &= \frac{-k^8}{Le} + \frac{k^6\Lambda_1\Omega^2}{\Pr} + k^4\Omega^2 - \frac{k^2\Lambda_1\Omega^4}{\Pr} + \frac{k^4\Omega^2}{\Pr} \\ &+ k^6\Lambda_2\Omega^2 + \frac{k^4\Omega^2}{Le\Pr} + \frac{k^6\Lambda_2\Omega^2}{Le} + \pi^2\alpha^2 Ra\left(\frac{k^2}{Le} - \Lambda_1\Omega^2\right) \\ &- \pi^2\alpha^2 Rs\left(k^2 - \Lambda_1\Omega^2\right) \end{aligned} \tag{38}$$

$$\begin{aligned} Y_2 &= \frac{k^6\Omega}{Le\Pr} + \frac{k^8\Lambda_2\Omega}{Le} - \frac{k^2\Omega^3}{\Pr} - k^4\Lambda_2\Omega^3 + k^6\Omega \\ &- \frac{k^4\Lambda_1\Omega^3}{\Pr} + \frac{k^6\Omega}{Le} - \frac{k^4\Lambda_1\Omega^3}{Le\Pr} - \pi^2\alpha^2 Ra\left(\Omega + \frac{k^2\Lambda_1\Omega}{Le}\right) \\ &+ \pi^2\alpha^2 Rs\left(\Omega + k^2\Omega\right) \end{aligned} \tag{39}$$

Eq. (36) is inhomogeneous and its solution poses a problem since it contains resonance terms The solvability condition requires that the first non-zero correction to R_0. The steady part of eq. (36) is orthogonal to sin πz. We take the time average and get the following expression for the correction Rayleigh number.

$$Ra_{2C} = \frac{Le}{2\pi^2\alpha^2 k^2 \,|\, L(\Omega, n)\,|^2}\begin{bmatrix} (X_1X_3\Omega + X_2X_4 + X_3^2)Y_1 \\ +(X_1X_4\Omega + X_2X_3 - X_3X_4)Y_2 \end{bmatrix} \tag{40}$$

Where,

$$X_1 = -Ra_0\pi^2\alpha^2 - \frac{Ra_0\pi^2\alpha^2 k^2\Lambda_1}{Le} + Rs\pi^2\alpha^2 + Rs\pi^2\alpha^2 k^2\Lambda_1 \tag{41}$$

$$\begin{aligned} X_2 &= -\frac{Ra_0\pi^2\alpha^2 k^2}{Le} + Ra_0\pi^2\alpha^2\Omega^2\Lambda_1 + Rs\pi^2\alpha^2 k^2 \\ &- Rs\pi^2\alpha^2\Omega^2\Lambda_1 \end{aligned} \tag{42}$$

$$X_3 = Ra_0\pi^2\alpha^2\Omega - Rs\pi^2\alpha^2\Omega \tag{43}$$

Notes

$$X_4 = \frac{-Ra_0\pi^2\alpha^2k^2}{Le} + Rs\pi^2\alpha^2k^2 + Ra_0\pi^2\alpha^2\Omega^2\Lambda_1 \tag{44}$$

$$-Rs\pi^2\alpha^2\Omega^2\Lambda_1$$

Notes

The linear theory predicts only the condition for the onset of convection and is silent about the heat and mass transport. We now embark on a non-linear analysis by means of truncated representation of Fourier series to find the effects of various parameters on finite amplitude convection and to know the amount of heat and mass transport.

VII. Non Linear Theory

A non-linear analysis is done to study the amount of heat and mass transfer due to the various parameters. Using the stream functions given by eq. (19)

$$\left(1+\Lambda_1\frac{\partial}{\partial t}\right)\left[\frac{1}{PR}\frac{\partial}{\partial t}\left(\nabla^2\psi\right)+Ra\frac{\partial\theta}{\partial x}-Rs\frac{\partial\varphi}{\partial x}\right]$$

$$=\left(1+\Lambda_1\frac{\partial}{\partial t}\right)\nabla^4\psi \tag{45}$$

$$\frac{\partial\theta}{\partial t} - J(\psi,\theta) + (1-\varepsilon f)\frac{\partial\psi}{\partial x} = \frac{1}{Le}\nabla^2\theta \tag{46}$$

$$\frac{\partial\varphi}{\partial t} - J(\psi,\phi) + \frac{\partial\psi}{\partial x} = \frac{1}{Le}\nabla^2\phi \tag{47}$$

An infinite series representation is used to find the solutions to eq. (45)– (47). The amplitudes depend only on time. Only one term in the Fourier representation for the stream function may be retained with two terms in the temperature expressions to retain some nonlinearity.
Eq. (45) is decomposed into two first order equations since it is a second order equation.

$$\frac{1}{\Pr}\frac{\partial}{\partial t}(\nabla^2\psi) = -Ra\frac{\partial\theta}{\partial x} + Rs\frac{\partial\phi}{\partial x} + \Lambda\nabla^4\psi + M \tag{48}$$

where
$$\Lambda = \frac{\Lambda_2}{\Lambda_1} \tag{49}$$

and
$$\Lambda_1\frac{\partial M}{\partial t} = -M + (1-\Lambda)\nabla^4\psi \tag{50}$$

The stream function, ψ, the temperature distribution, θ, concentration distribution, φ and M are represented as follows:

$$\psi = A(t)\sin(\pi\alpha x)\sin(\pi z) \tag{51}$$

$$\theta = B(t)\cos(\pi\alpha x)\sin(\pi z) + C(t)\sin(2\pi z) \tag{52}$$

$$\phi = E(t)\cos(\pi\alpha x)\sin(\pi z) + F(t)\sin(2\pi z) \tag{53}$$

$$M = G(t)\sin(\pi\alpha x)\sin(\pi z) \tag{54}$$

where A(t), B(t), C(t), E(t), F(t) and, G(t) are the amplitudes to be determined from the dynamics of the system.

Projecting eq. (46), (47), (48) and (50) onto the modes (51) - (54) and following the standard orthogonalization procedure, we obtain the following non-linear autonomous system of differential equations (generalized Lorenz model [22]):

$$\dot{A}(t) = \frac{-Ra\,\Pr \pi\alpha}{k^2} B(t) + \frac{Rs\,\Pr \pi\alpha}{k^2} E(t) - \Lambda \Pr k^2 A(t) - \frac{\Pr}{k^2} G(t) \tag{55}$$

$$\dot{B}(t) = (\varepsilon f - 1)\pi\alpha A(t) - k^2 B(t) \tag{56}$$

$$\dot{C}(t) = \frac{\pi^2 \alpha}{2} A(t)B(t) - 4\pi^2 C(t) \tag{57}$$

$$\dot{E}(t) = -\pi\alpha A(t) - \frac{k^2}{Le} E(t) \tag{58}$$

$$\dot{F}(t) = \frac{\pi^2 \alpha}{2} A(t)E(t) - \frac{4\pi^2}{Le} F(t) \tag{59}$$

$$\dot{G}(t) = \frac{-1}{\Lambda_1} G(t) + \frac{(1-\Lambda)}{\Lambda_1} k^4 A(t) \tag{60}$$

Where the over dot denotes the time derivative with respect to t. More modes other than the minimal ones have not been considered in the study in view of the observation by Siddheshwar and Titus [23] that additional modes do not significantly alter the results on the onset of convection as well as transport.

VIII. HEAT TRANSPORT

In non-linear study of convection, the heat transport across the layers of fluid is important. The onset of convection can be easily determined by analyzing the increase and decrease in heat transport. In the basic state, transfer of heat takes place only due to convection.

If H_T is the rate of heat transfer / unit area, then

$$H_T = -\chi \left\langle \frac{\partial T_{total}}{\partial z} \right\rangle_{z=0}, \tag{61}$$

where the bracket corresponds to a horizontal average and

$$T_{total} = \left[T_0 - \frac{\Delta T}{d} z \right] + T(x,z)$$

The first term is the temperature distribution due to conduction state prevalent before convection sets in. The second term represents the convective heat transport.

The Nusselt number Nu is defined by

$$Nu = \frac{H_T}{\kappa \Delta T / d}. \tag{62}$$

Alternately, Nu may be directly defined in terms of the non-dimensional quantities as follows:

Ref

23. P.G. Siddheshwar, P.S. Titus, 2013, *"Nonlinear Rayleigh–Bénard convection with variable heat source"*, J. Heat transfer, 135, pp. 122502.

$$Nu = \frac{\left[\frac{k_c}{2\pi}\int_0^{2\pi/k_c}(1-z+T)_z\,dx\right]}{\left[\frac{k_c}{2\pi}\int_0^{2\pi/k_c}(1-z)_z\,dx\right]} = 1 + \frac{4k_c^2A^2}{4\delta^2}. \tag{63}$$

Notes

The solutal gradient, arising due to double diffusive convection, causes a transfer of mass in the fluid system. This is quantified using Sherwood number given by

$$Sh = 1 + \frac{4k_c^2 Le^2 A^2}{4\delta^2} \tag{64}$$

We use these expressions to determine the effects of various parameters of the problem on heat and mass transfer.

III. Results and Discussions

In this paper an attempt is made to study the effects of gravity modulation on double diffusive convection in Oldroyd-B liquids. The following effects on the classical Rayleigh-Benard problem are considered

i) Stress relaxation parameter.
ii) Strain retardation parameter.
iii) Lewis number.
iv) Frequency of modulation.

These are represented by Λ_1, Λ_2, Le and ω. The effects of these parameters on heat and mass transfer are also analyzed. In the case of thermal modulation the amplitude, ε, is small compared with the imposed steady temperature difference. The validity of the results obtained here depends on the value of modulating frequency, ω. When ω < 1, the period of modulation is large and hence, the disturbance grows to such an extent as to make finite amplitude effects important. When $\omega \to \infty$, $Ra_{2c} \to 0$. Thus, modulation becomes small. Therefore, we choose moderate values of ω

Graphs of Ra_{2c} versus ω are plotted for varying values of the parameters which represent the linear part of the problem (figs (2) – (5)). The effects of gravity modulation on non-linear stability analysis are also discussed using graphs of Nusselt number versus time and Sherwood number versus time. The non – autonomous Lorenz model obtained is solved numerically. The parameters of the system are Lewis number, Le, Stress relaxation parameter, Λ_1, Strain retardation parameter, Λ_2, Prandtl number, Pr, Solutal Rayleigh number, Rs and frequency of modulation, ω, which influence the heat and mass transfer.

The linear stability analysis is discussed through graphs of correction Rayleigh number, Ra_{2c}, as a function of frequency of modulation, ω. Figures (2) – (6) are the corresponding graphs.

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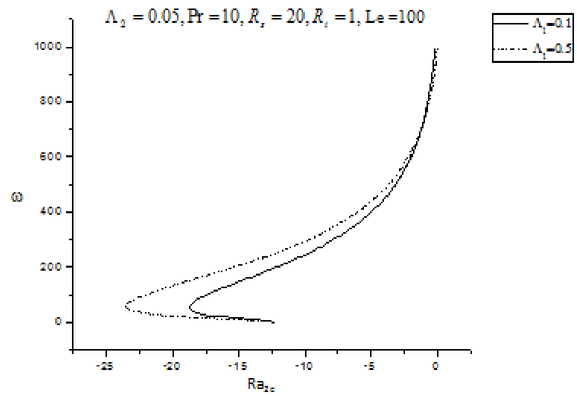

Fig. 2: Graph of Ra_{2c} vs t for different values of Λ_1

Fig (2) is the graphs for different values of the stress relaxation parameter, Λ_1, for fixed values of other parameters. It is evident from the graph that increase in Λ_1, causes a decrease in the value of Ra_{2c}. This, in turn, causes acceleration in the onset of convection, thus, destabilizing the system

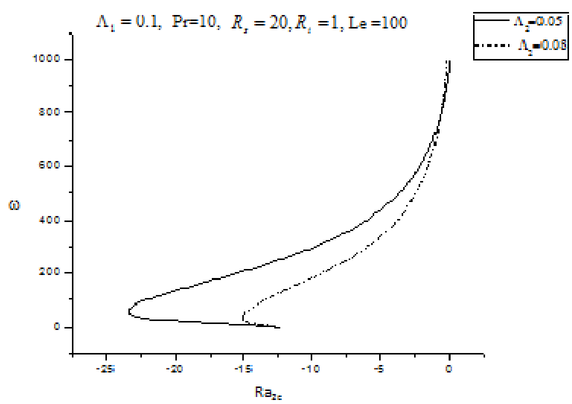

Fig. 3: Graph of Ra_{2c} vs t for different values of Λ_2

Fig (3) is the graphs of Ra_{2c} versus ω for varying values of Λ_2. The other parameters remain fixed. It can be seen that Λ_2 causes an effect opposite to that of Λ_1. With increase in Λ_2, Ra_{2c} also increases, thus delaying the convective process

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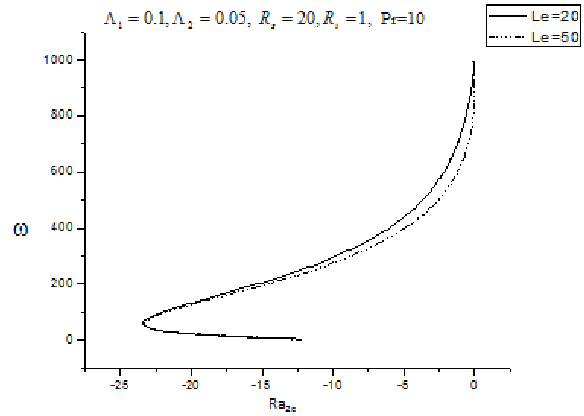

Fig. 4: Graph of Ra_{2c} vs t for different values of Le

Fig (4) is a graph of Ra_{2c} versus ω for different values of Le. As can be seen from the graph the increasing values of Le results in the increase in Ra_{2c}. This delays the onset of convection. Le is the ratio of thermal to solutal diffusivities. As Le increases the Solutal diffusivity decreases and the thermal diffusivity increases. This results in more heat transfer

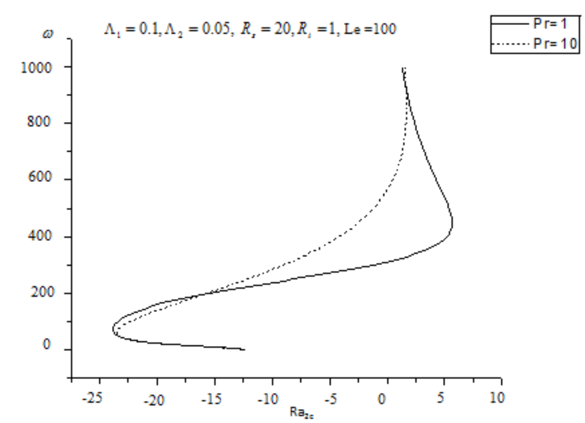

Fig. 5: Graph of Ra_{2c} vs t for different values of Pr

Figure (5) is the graph varying the values of Pr. It is appropriate here to note that Pr does not significantly affect the values of Ra_{2c}. The graphs in fig (2)-(5) depict sub critical motion. There is a steady line as well as a parabolic profile. This parabolic part is subject to finite amplitude instabilities

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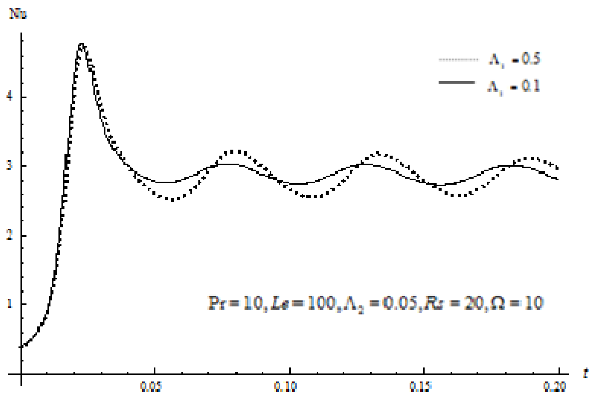

Fig. 6: Graph of Nu vs t for different values of Λ_1

Figures (6) – (15) are the graphs of heat and mass transfer. They represent the non-linear theory. Nusselt number, Nu, and Sherwood number, Sh, are used to plot these graphs as functions of time. Figures (6) – (10) show the effects of the different parameters on the Nusselt number. Fig (6) shows that Λ_1 causes an increase in the Nusselt number and in turn, the heat transport. This is obvious as Λ_1 causes a decrease in Ra_{2c}

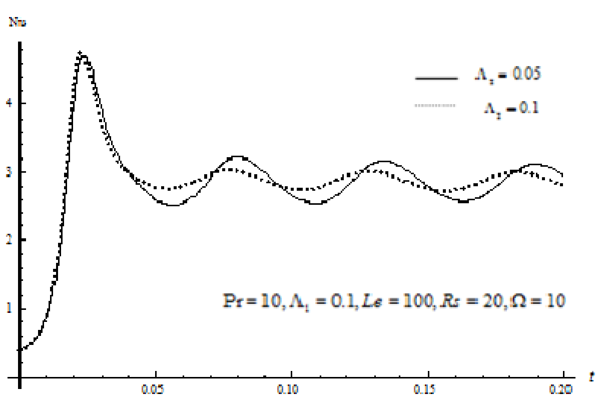

Fig. 7: Graph of Nu vs t for different values of Λ_2

The opposite result is seen for Λ_2 (Fig (7)). That is, as Λ_2 increases Nusselt number decreases, thus reducing the heat transfer. This is again an expected result as Λ_2 was found to cause an increase in the value of Ra_{2c}. Therefore, its stabilizing effects are affirmed here

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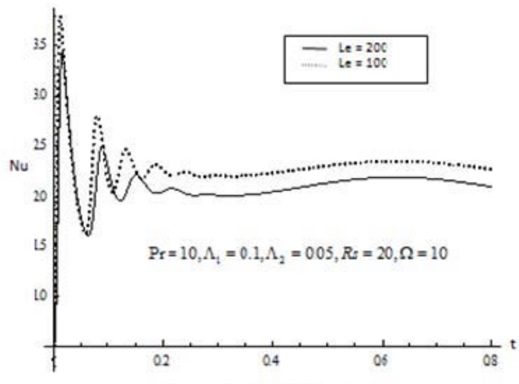

Fig. 8: Graph of Nu vs t for different values of Le

Fig (8) Shows the graphs of Nusselt number versus time for varying values of Le. It can be seen that the increasing values of Le decreases Nu, thus reducing heat transfer.

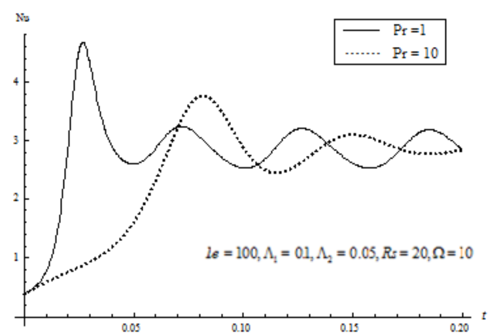

Fig. 9: Graph of Nu vs t for different values of Pr

Fig (9) Shows the pattern in Nu when Prandtl number is varied. The variation in the values of Nu can be observed for smaller values of Pr. As the value of Pr increases largely the values of Nu becomes more or less similar. Pr is the property of the type of fluid and hence smaller values of it differentiates the fluids

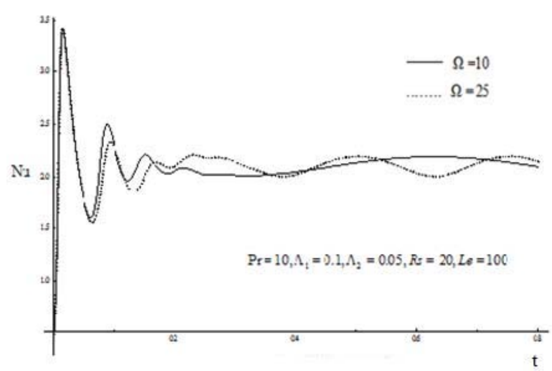

Fig. 10: Graph of Nu vs t for different values of Ω

Fig (10) is the graphs of Nusselt number versus time for varying values of the frequency of modulation, ω. It is evident that the increase in ω results in the decrease of Nu, Therefore, higher the frequency of modulation, lesser is the transport of heat. Thus, the frequency can be controlled to get desirable results in the system

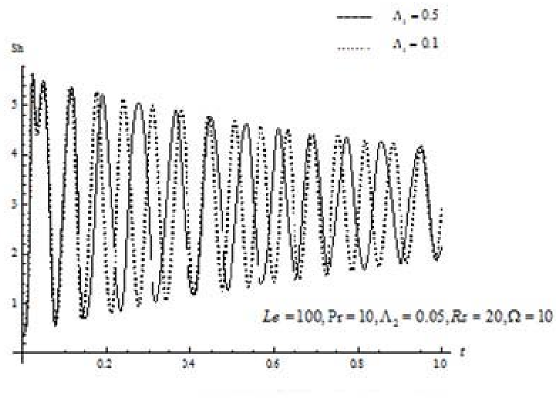

Fig. 11: Graph of Sh vs t for different values of Λ_1

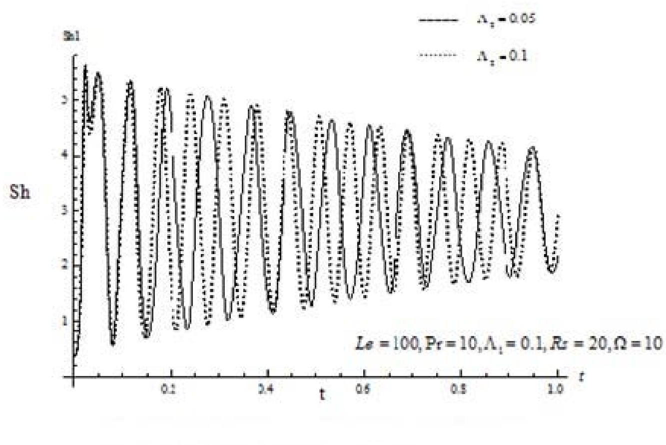

Fig. 12: Graph of Sh vs t for different values of Λ_2

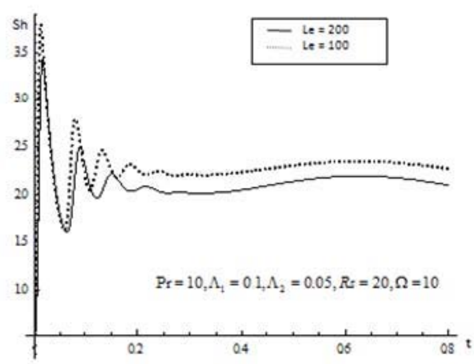

Fig. 13: Graph of Sh vs t for different values of Le

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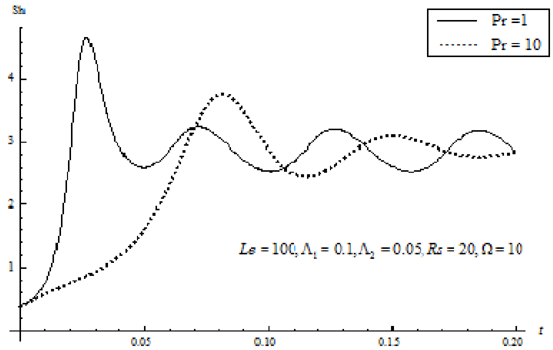

Fig. 14: Graph of Sh vs t for different values of Pr

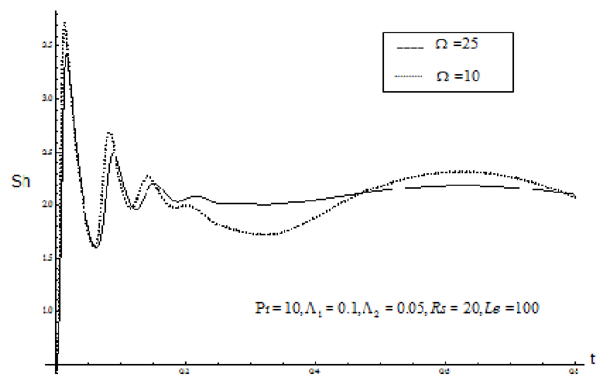

Fig. 15: Graph of Sh vs t for different values of Ω

Figures (11) – (15) are graphs of Sherwood number versus time for the same parameters mentioned above. These graphs show a pattern similar to that of Nusselt number. Therefore, heat and mass transfer show same type of variations for all the parameters.

IV. Conclusions

1. The stress relaxation parameter, Λ_1, and strain retardation parameter, Λ_2, have opposing effects on the stability with Λ_1 destabilizing the system and thereby increasing the heat transfer.
2. Lewis number, Le, stabilizes the system thereby decreasing the heat transfer.
3. Effect of the frequency of modulation, ω, is to decrease the heat transfer.
4. Sherwood number behaves in a way similar to Nusselt number.
5. Modulation can be used as effective means of controlling convection.

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Ricci Solitons on *CR*-Submanifolds of Maximal *CR* Dimension of a Complex Space Form

By Z. Nazari & E. Abedi

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Abstract- We study Ricci solitons on CR-submanifolds of maximal CR di-mension Mn of a complex space form $\mathbb{C}^{\frac{n+p}{2}}$ such that the shape operator A has only one eigenvalue. We prove that Ricci soliton on *CR*-submanifolds of maximal *CR* dimension M^n with eigenvalue zero is expanding and with eigen-value nonzero is expanding and shrinking.

Finally, we study Ricci soliton on *CR*-hypersurfaces M^n of a complex space form $M^{\frac{n+1}{2}}$ (4k) such that the shape operator A has exactly two distinct eigen-values and show that a Ricci soliton (M, g, V, λ) for $k < 0$ is shrinking and expanding and for $k > 0$ is shrinking.

Keywords: *Ricci soliton, complex space form, CR-submanifolds of maximal CR dimension.*

GJSFR-F Classification: *MSC 2010: 53C25, 53C15, 53c40.*

RICCISOLITONSONCRSUBMANIFOLDSOFMAXIMALCRDIMENSIONOFACOMPLEXSPACEFORM

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Keywords: Ricci soliton, complex space form, CR-submanifolds of maximal CR dimension.

I. Introduction

A Ricci soliton is defined on a Riemannian manifold (M, g) by

$$\frac{1}{2}\mathcal{L}_V g + Ric - \lambda g = 0 \qquad (1.1)$$

where $L_V g$ is the Lie-derivative of the metric tensor g with respect to V and λ is a constant on M. The Ricci soliton is a natural generalization of an Einstein metric. The Ricci soliton is said to be shrinking, steady and expanding according as $\lambda > 0$, $\lambda = 0$ and $\lambda < 0$, respectively. Compact Ricci solitons are the fixed points of the Ricci flow:

$$\frac{\partial}{\partial t} g(t) = -2Ric(g(t)) \qquad (1.2)$$

projected from the space of metrics onto its quotient modulo diffeomorphisms and scalings and often arise as blow-up limits for the Ricci flow on compact manifolds. We denote a Ricci soliton by $(M, g, V; \lambda)$ and call the vector field V the potential vector field of the Ricci soliton. A trivial Ricci soliton is one for which V is Killing or zero. If its potential field $V = \nabla f$ such that f is some smooth function on M then a Ricci soliton $(M, g, V; \lambda)$ is called a gradient Ricci soliton and the smooth function f is called the potential function. It was proved by Grigory Perelman in [13] that any compact Ricci soliton is the sum of a gradient of some smooth function f up to the addition of a Killing field. Thus compact Ricci solitons are gradient Ricci solitons. In particular, Perelman applied Ricci solitons to solve the long standing Poincare conjecture posed in 1904.

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Hamilton[7] and Ivey [9] proved that a Ricci soliton on a compact manifold has constant curvature in dimension 2 and 3, respectively. In [10], Ki proved that there are no real hypersurfaces with parallel Ricci tensor in a complex space form $\overline{M}^n(c)$ with $c \neq 0$ when $n \geq 3$. Kim [11] proved that when $n = 2$, this is also true. In particular, these results give that there is not any Einstein real hypersurfaces in a non-flat complex space form.

In [2], Chen studied important results on Ricci solitons which occur obviously on some Riemannian submanifolds. He presented several recent new criterions of trivial compact shrinking Ricci solitons.

Cho and Kimura [3] studied on Ricci solitons of real hypersurfaces in a non-flat complex space form and showed that a real hypersurface M in a non-flat complex space form $\overline{M}^n(c \neq 0)$ does not admit a Ricci soliton such that the Reeb vector field ξ is potential vector field. They defined so called η-Ricci soliton, such that satisfies

$$\frac{1}{2}\mathcal{L}_V g + Ric - \lambda g - \mu\eta \otimes \eta = 0 \tag{1.3}$$

where λ, μ are constants. They first proved that a real hypersurface M of a non-flat complex space form $\overline{M}^n(c)$ which accepts an η-Ricci soliton is a Hopf-hypersurface and classified that η-Ricci soliton real hypersurfaces in a non-flat complex space form.

We study Ricci solitons on CR-submanifolds of maximal CR dimension M^n of a complex space form $\mathbb{C}^{\frac{n+p}{2}}$ such that the shape operator A has only one eigenvalue. We prove that Ricci soliton on CR-submanifolds of maximal CR dimension M^n with eigenvalue zero is expanding and with eigenvalue nonzero is expanding and shrinking.

Finally, we study Ricci solitons on CR-hypersurfaces M^n with exactly two distinct eigenvalues of a complex space form $\overline{M}^{\frac{n+1}{2}}(4k)$ and show that a Ricci soliton (M, g, V, λ) for $k < 0$ is shrinking and expanding and for $k > 0$ is shrinking.

II. Preliminaries

Let $\overline{M}^{\frac{n+p}{2}}$ be a complex Kähler manifold with the natural almost complex structure J. A Kähler manifold $\overline{M}^{\frac{n+p}{2}}$ is called a complex space form if it has constant holomorphic sectional curvature. The Riemannian curvature tensor $\overline{R}$ of a complex space form is given by

$$\begin{aligned} \overline{R}(X,Y)Z &= k\{\overline{g}(Y,Z)X - \overline{g}(X,Z)Y \\ &+ \overline{g}(JY,Z)JX - \overline{g}(JX,Z)JY - 2\overline{g}(JX,Y)JZ\}. \end{aligned} \tag{2.1}$$

A CR-submanifold is a submanifold M^n tangent to ξ that admits an invariant distribution D whose orthogonal complementary distribution $D^\perp$ is anti-invariant, that is, $TM = D \oplus D^\perp$ with condition $\varphi(D_p) \subset D_p$ for all $p \in M$ and $\varphi(D_p^\perp) \subset T_p^\perp M$ for all $p \in M$, where $D = span\{X_1, ..., X_m, \varphi X_1, ..., \varphi X_m\}$ and $D^\perp = span\{\xi\}$ such that $m = \frac{n-1}{2}$.

Therefore, there exists a vector subbundles anti-invariant ν and J-invariant $\nu^\perp$ of the normal bundle such that

$$\begin{aligned} &J\nu_p \subset T_pM, \\ &J\nu_p^\perp \subset \nu_p^\perp, \end{aligned} \tag{2.2}$$

for $p \in M$, where $\nu^\perp = span\{N_1, ..., N_q, N_{1^*} = JN_1, ..., N_{q^*} = JN_q\}, q = \frac{p-1}{2}$ and $\nu = span\{N\}$ and $T^\perp M = \nu \oplus \nu^\perp$.

Ref

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Notes

If M^n is an CR-submanifolds of maximal CR dimension of $\overline{M}^{\frac{n+p}{2}}$, then at each point $p \in M$, the real dimension of $JT_p(M) \cap T_p(M) = n-1$.

Let $\overline{\nabla}$ and ∇ are the Riemannian connections of $\overline{M}$ and M, respectively and $\nabla^{\perp}$ is the normal connection induced from $\overline{\nabla}$ in the normal bundle $T^{\perp}(M)$.

Let M^n be a CR-submanifolds of maximal CR dimension of a complex space form $\overline{M}^{\frac{n+p}{2}}$ with constant holomorphic sectional curvature $4k$ and the normal vector field N be parallel with respect to normal conniction $\nabla^{\perp}$. We can write

$$\nabla^{\perp}_X N = \sum_{a=1}^{q}\{s_a(X)N_a + s_{a^*}(X)N_{a^*}\} \tag{2.3}$$

by the relation 2.3, we have the following lemma

Lemma 2.1. [6] *for a CR- submanifold of maximal CR dimension, the vector field N is parallel with respect to the normal connection $\nabla^{\perp}$, if and only if $s_a = s_{a^*} = 0$ for $a = 1, ..., q$.*

We define a metric g on CR-submanifolds M^n of maximal CR dimension by

$$g(X,Y) = \overline{g}(\iota X, \iota Y),$$

for any $X, Y \in TM$. The Riemannian metric g is said the induced metric from $\overline{g}$ on $\overline{M}^{n+1}(4k)$ and the ι is called an isometric immersion.

For any vector field $X \in \chi(M)$ the decomposition holds:

$$JX = \varphi X + \eta(X)N \tag{2.4}$$

where, φ is an endomorphism acting on $T(M)$ and η is one-form on M and N is a unit normal vector field on M^n such that $JN = -\xi$. The structure (φ, η, ξ, g) is an almost contact metric structure on M^n such that

$$\varphi^2 = -Id + \eta \otimes \xi, \qquad \eta(\xi) = 1, \qquad \varphi\xi = 0, \qquad \eta o \varphi = 0. \tag{2.5}$$

and

$$\overline{g}(\varphi X, \varphi X) = \overline{g}(X,Y) - \eta(X)\eta(Y), \qquad \eta(X) = \overline{g}(X,\xi). \tag{2.6}$$

Now, the Gauss formula are given by

$$\overline{\nabla}_X Y = \nabla_X Y + h(X,Y), \tag{2.1}$$

for any $X, Y \in \chi(M)$. Where, the h is the second fundamental form such that

$$\begin{aligned} h(X,Y) &= g(AX,Y)N \\ &+ \sum_{a=1}^{q}\{g(A_aX,Y)N_a + g(A_{a^*}X,Y)N_{a^*}\}. \end{aligned} \tag{2.8}$$

Moreover, the Weingarten formulae can be written as follows

$$\begin{aligned} \overline{\nabla}_X N &= -AX + \nabla^{\perp}_X N \\ &= -AX + \sum_{a=1}^{q}\{s_a(X)N_a + s_{a^*}(X)N_{a^*}\}, \end{aligned} \tag{2.9}$$

$$\begin{aligned} \overline{\nabla}_X N_a &= -A_aX + \nabla^{\perp}_X N_a \\ &= -A_aX - s_a(X)N + \sum_{b=1}^{q}\{s_{ab}(X)N_b + s_{ab^*}(X)N_{b^*}\}, \end{aligned} \tag{2.10}$$

$$\begin{aligned} \overline{\nabla}_X N_{a^*} &= -A_{a^*}X + \nabla^{\perp}_X N_{a^*} \\ &= -A_{a^*}X - s_{a^*}(X)N + \sum_{b=1}^{q}\{s_{a^*b}(X)N_b + s_{a^*b^*}(X)N_{b^*}\}, \end{aligned} \quad (2.11)$$

where A, A_a, A_{a^*} are the shape operators for the normals N, N_a, N_{a^*}, respectively, and s's are called the coefficients of the third fundamental form of M in $\overline{M}$.

Therefore, taking the covariant derivative of $N_{a^*} = JN_a$ and using (2.4), (2.10), (2.11) and $JN = -\xi$, we compute

$$A_{a^*}X = \varphi A_a X - s_a(X)\xi, \quad (2.12)$$

$$A_a X = -\varphi A_{a^*}X + s_{a^*}(X)\xi, \quad (2.13)$$

$$s_{a^*}(X) = \eta(A_a X) = g(A_a\xi, X), \quad (2.14)$$

$$s_a(X) = -\eta(A_{a^*}X) = -g(A_{a^*}\xi, X), \quad (2.15)$$

$$s_{a^*b^*} = s_{ab}, \qquad s_{a^*b} = -s_{ab^*}. \quad (2.16)$$

for all $X, Y \in TM$ and $a, b = 1, ..., q$. Further, since φ is skew-symmetric and $A_a, A_{a^*}, a = 1, ..., q$ are symmetric, using relations (2.12) and (2.13), we compute

$$trace A_{a^*} = \sum_{i=1}^{n} g(A_{a^*}e_i, e_i) = s_a(\xi),$$

$$trace A_a = s_{a^*}(\xi). \quad (2.17)$$

By the note the vector field N is parallel with respect to the normal connection $\nabla^{\perp}$, using Lemma 2.1 and relations (2.12)- (2.15), we conclude

$$A_a\xi = 0, \qquad A_{a^*}\xi = 0,$$

$$A_a X = -\varphi A_{a^*}X, \qquad A_{a^*}X = \varphi A_a X, \quad (2.18)$$

for all $X \in TM$ and all $a = 1, ..., q$. Further, we differentiate (2.4) and $JN = -\xi$ covariantly and compare the tangential part and the normal part. Then we obtain

$$\begin{aligned} &(\nabla_X \varphi)Y = \eta(Y)AX - g(AY, X)\xi, \\ &\nabla_X \xi = \varphi AX. \end{aligned} \quad (2.19)$$

Then from (2.4), The Gauss equation are written as follow:

$$\begin{aligned} R(X,Y)Z &= k\{g(Y,Z)X - g(X,Z)Y \\ &+ g(\varphi Y, Z)\varphi X - g(\varphi X, Z)\varphi Y - 2g(\varphi X, Y)\varphi Z\} \\ &+ g(AY, Z)AX - g(AX, Z)AY \\ &+ \sum_{a=1}^{q}\{g(A_aY, Z)A_aX - g(A_aX, Z)A_aY \\ &+ g(A_{a^*}Y, Z)A_{a^*}X - g(A_{a^*}X, Z)A_{a^*}Y\}, \end{aligned} \quad (2.20)$$

by Lemma 2.1, the Codazzi equation become

$$(\nabla_X A)Y - (\nabla_Y A)X = k\{\eta(X)\varphi Y - \eta(Y)\varphi X - 2g(\varphi X, Y)\xi\}, \quad (2.21)$$

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hence, by the relations (2.17), (2.18), Ricci tensor is obtained as

$$\begin{aligned} Ric(X,Y) &= k\{(n+2)g(X,Y) - 3\eta(X)\eta(Y)\} \\ &+ (traceA)g(AX,Y) - g(AX,AY) \\ &- 2\sum_{a=1}^{q} g(A_aX, A_aY). \end{aligned} \tag{2.22}$$

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for any tangent vector fields X, Y, Z on M, where R and Ric are the curvature and Ricci tensors of M, respectively.

III. Ricci Soliton on CR Hypersurfaces

Let M^n be a CR-submanifolds of maximal CR dimension of a complex space form $\overline{M}^{\frac{n+p}{2}}$ with the vector field N be parallel with respect to normal connction $\nabla^\perp$ such that the shape operator A for unit normal vector field N has only one eigenvalue. Let $\{e_1, ..., e_{n-1}, \xi\}$ be a local orthonormal fram field such that $D^\perp = span\{\xi\}$ and $D = span\{e_1, ..., e_m, e_{m+1} = \varphi e_1, ..., e_{2m=n-1} = \varphi e_m\}$ such that $m = \frac{n-1}{2}$.

In [6], proved that

Theorem 3.1. If the shape operator A with respect to unit normal vector field N of M^n has only one eigenvalue, then $\overline{M}^{\frac{n+p}{2}}$ is a complex Euclidean space.

According to the assumption, it follows that $A = 0$ or $AX = \alpha X$ for all $X \in T(M)$ such that $\alpha \neq 0$.

Let $AX = \alpha X$, therefore by the relation (2.22), we obtain

$$Ric(e_i, e_j) = \{(n-1)\alpha^2\}\delta_{ij} - 2\sum_{a=1}^{q} g(A_ae_i, A_ae_j), \qquad i, j = 1, ..., n-1, \tag{3.1}$$

$$Ric(\xi, \xi) = (n-1)\alpha^2, \tag{3.2}$$

$$Ric(e_i, \xi) = 0, \qquad i = 1, ..., n-1. \tag{3.3}$$

We consider CR-submanifolds of maximal CR dimension of a complex space form $\mathbb{C}^{\frac{n+p}{2}}$ satisfying Ricci soliton equation

$$\frac{1}{2}\mathcal{L}_V g + Ric - \lambda g = 0 \tag{3.4}$$

with respect to potential vector field V on M for constant λ.

Putting

$$V := f\xi, \qquad (f : M \to \mathbb{R}, f \neq 0) \tag{3.5}$$

Then definition of Lie derivative and second relation (2.19) imply

$$(\mathcal{L}_{f\xi}g)(X,Y) = df(X)\eta(Y) + df(Y)\eta(X). \tag{3.6}$$

We compute

$$(\mathcal{L}_{f\xi}g)(\xi,\xi) = 2df(\xi), \tag{3.7}$$

$$(\mathcal{L}_{f\xi}g)(\xi, e_i) = df(e_i), \qquad (i = 1, ..., n-1), \tag{3.8}$$

$$(\mathcal{L}_{f\xi}g)(e_i, e_j) = 0 \qquad (i, j = 1, ..., n-1). \tag{3.9}$$

Using relations (3.1)-(3.3) and (3.7)-(3.9), Ricci soliton equation (3.4) is equivalent to

$$df(\xi) = \lambda - (n-1)\alpha^2, \tag{3.10}$$

$$df(e_i) = 0, \qquad (i = 1, ..., n-1), \tag{3.11}$$

$$\{(n-1)\alpha^2 - \lambda\}\delta_{ij} - 2\sum_{a=1}^{q} g(A_a e_i, A_a e_j) = 0, \quad (i, j = 1, ..., n-1). \tag{3.12}$$

By the relation (3.12), for $i = j$ we have $\lambda = (n-1)\alpha^2 - 2\sum_{a=1}^{q} g(A_a e_i, A_a e_i)$ and thus the following theorem holds:

Theorem 3.2. Let M^n be a CR-submanifolds of maximal CR dimension of a complex space form $\mathbb{C}^{\frac{n+p}{2}}$ with $AX = \alpha X$. Then a Ricci soliton (M, g, V, λ) with potential field $V := f\xi$ is

(a) shrinking Ricci soliton if $(n-1)\alpha^2 > 2\sum_{a=1}^{q} g(A_a e_i, A_a e_i)$.

(b) expanding Ricci soliton if $(n-1)\alpha^2 < 2\sum_{a=1}^{q} g(A_a e_i, A_a e_i)$.

Now, let $A = 0$, using relation (2.22), it follows that

$$Ric(e_i, e_j) = -2\sum_{a=1}^{q} g(A_a e_i, A_a e_j), \quad i, j = 1, ..., n-1, \tag{3.13}$$

$$Ric(\xi, \xi) = 0, \tag{3.14}$$

$$Ric(e_i, \xi) = 0, \qquad i = 1, ..., n-1. \tag{3.15}$$

CR-submanifolds of maximal CR dimension M^n $(n \geq 3)$ is considered in a complex space form $\mathbb{C}^{\frac{n+p}{2}}$ satisfying Ricci soliton equation with potential vector field $f\xi$. From relations (3.13)-(3.15) and (3.7)-(3.9), Ricci soliton equation (3.4) is equivalent to

$$df(\xi) = \lambda, \tag{3.16}$$

$$df(e_i) = 0, \qquad (i = 1, ..., n-1), \tag{3.17}$$

$$(-\lambda)\delta_{ij} - 2\sum_{a=1}^{q} g(A_a e_i, A_a e_j) = 0, (i, j = 1, ..., n-1). \tag{3.18}$$

Using the relation (3.18), it follows $\lambda = -2\sum_{a=1}^{q} g(A_a e_i, A_a e_i)$ and hence

Theorem 3.3. Let M^n be a CR-submanifolds of maximal CR dimension of complex space form $\mathbb{C}^{\frac{n+p}{2}}$ with $A = 0$. Then a Ricci soliton (M, g, V, λ) with potential field $V := f\xi$ is expanding Ricci soliton.

Let M^n $(n \geq 3)$ is a CR-hypersurface in a complex space form $\overline{M}^{\frac{n+1}{2}}$. We assume that the shape operator A with respect to N has exactly two distinct eigenvalues α and β. The following lemma holds[6]

Lemma 3.4. Let $\overline{M}^{\frac{n+1}{2}}$ be a Kähler manifold of constant holomorphic sectional curvature $4k$, with $k \neq 0$. If the shape operator A has exactly two distinct eigenvalues, then ξ ia an eigenvector of A.

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By the lemma above, let $A\xi = \alpha\xi$. Differentiating $A\xi = \alpha\xi$ covariantly and the second relation (2.19) imply

$$(\nabla_X A)\xi = \alpha\varphi AX - A\varphi AX + (X\alpha)\xi$$

The Codazzi equation is obtained as

$$(\nabla_\xi A)X = k\varphi X + \alpha\varphi AX - A\varphi AX + (X\alpha)\xi$$

Since $\nabla_\xi A$ is self-adjoint, we conclude the relation:

$$\begin{aligned} 0 &= -2g(A\varphi AX, Y) + 2kg(\varphi X, Y) + \alpha g((A\varphi + \varphi A)X, Y) \\ &+ (X\alpha)\eta(Y) - (Y\alpha)\eta(X) \end{aligned} \tag{3.19}$$

Substituting Y for ξ in (3.19) and using of the fact that α is an eigenvalue of A, it follow that $(X\alpha) = \eta(X)\xi\alpha$. Similarly by substituting X for ξ in (3.19), we get $(Y\alpha) = \eta(Y)\xi\alpha$. It follows

$$2A\varphi AX - 2k\varphi X = \alpha(A\varphi + \varphi A)X \tag{3.20}$$

We assume that $AX = \beta X$ for any vector field $X \in D, \|X\| = 1$. Then

$$A\varphi X = \frac{(\alpha\beta + 2k)}{(2\beta - \alpha)}\varphi X. \tag{3.21}$$

Therefore, φX is an eigenvector corresponding to the eigenvalue

$$\gamma = \frac{(\alpha\beta + 2k)}{(2\beta - \alpha)} \tag{3.22}$$

As A has exactly two distinct eigenvalues, we have the following three cases:
If $\alpha = \beta$, we conclude that $\gamma = \frac{(\alpha^2+2k)}{(\alpha)}$ and $traceA = \frac{n\alpha^2 + k(n-1)}{\alpha}$.
Since the shape operator A is self-adjoint, for any $X, Y \in D$

$$\alpha g(\varphi X, Y) = g(AX, \varphi Y) = g(X, A\varphi Y) = \gamma g(X, \varphi Y) \tag{3.23}$$

therefore, $\alpha = \beta = \gamma$, which is a contradiction since the shape operator A with respect to N has exactly two distinct eigenvalues.

Now, if $\gamma = \alpha$, we conclude that $\alpha\beta - \alpha^2 = 2k$ and $traceA = \frac{n-1}{2}\beta + \frac{n+1}{2}\alpha$.
By the note the shape operator A is self-adjoint, we have

$$\alpha g(X, \varphi Y) = g(X, A\varphi Y) = g(AX, \varphi Y) = \beta g(X, \varphi Y) \tag{3.24}$$

therefore, $\gamma = \alpha = \beta$, which is a contradiction since the shape operator A with respect to N has exactly two distinct eigenvalues. Thus the multiplicity of the eigenvalue α corresponding to the eigenvector ξ is one.

Therefore, we suppose that the shape operator A has exactly two distinct eigenvalues, α, $\beta = \gamma$. Then it follows that $\beta^2 - \alpha\beta = k$ and $A\varphi = \varphi A$ and $traceA = \alpha + (n-1)\beta$.

Hence, by the relation (2.22), Ricci tensor related to a CR-hypersurface (M^n, g) is written as

$$Ric(e_i, e_j) = \{2kn + (n-1)\beta^2\}\delta_{ij}, \qquad (i, j = 1, ..., n-1), \tag{3.25}$$

$$Ric(\xi, \xi) = (n-1)(k + \beta^2), \tag{3.26}$$

$$Ric(e_i, \xi) = 0, \qquad (i = 1, ..., n-1), \tag{3.27}$$

We consider a CR-hypersurface M^n $(n \geq 3)$ in complex space form $\overline{M}^{\frac{n+1}{2}}(4k)$ that satisfying Ricci soliton

$$\frac{1}{2}\mathcal{L}_V g + Ric - \lambda g = 0 \tag{3.28}$$

with respect to potential vector field V on M for constant λ.
We put

$$V := f\xi, \qquad (f : M \to \mathbb{R}, f \neq 0) \tag{3.29}$$

Definition of Lie derivative and the second relation (2.19) imply

$$(\mathcal{L}_{f\xi}g)(X,Y) = df(X)\eta(Y) + df(Y)\eta(X). \tag{3.30}$$

We obtain

$$(\mathcal{L}_{f\xi}g)(\xi,\xi) = 2df(\xi), \tag{3.31}$$

$$(\mathcal{L}_{f\xi}g)(\xi,e_i) = df(e_i), \qquad (i = 1, ..., n-1), \tag{3.32}$$

$$(\mathcal{L}_{fU}g)(e_i,e_j) = 0, \qquad (i,j = 1, ..., n-1). \tag{3.33}$$

Using relations (3.25)-(3.27) and (3.31)-(3.33), Ricci soliton equation (3.28) follows

$$\lambda = 2kn + (n-1)\beta^2, \tag{3.34}$$

$$df(\xi) = k(n+1), \tag{3.35}$$

$$df(e_i) = 0, \qquad (i = 1, ..., n-1), \tag{3.36}$$

Theorem 3.5. Let M be a CR-hypersurface of complex space form $\overline{M}^{\frac{n+1}{2}}(4k)$. If $k > 0$, then a Ricci soliton (M, g, V, λ) with potential field $V := f\xi$ is shrinking Ricci soliton.

Theorem 3.6. Let M be a CR-hypersurface of complex space form $\overline{M}^{\frac{n+1}{2}}(4k)$ with $k < 0$.

a) If $|k| > \frac{(n-1)\beta^2}{2n}$. Then a Ricci soliton (M, g, V, λ) with potential field $V := f\xi$ is expanding Ricci soliton.

b)If $|k| < \frac{(n-1)\beta^2}{2n}$. Then a Ricci soliton (M, g, V, λ) with potential field $V := f\xi$ is shrinking Ricci soliton.

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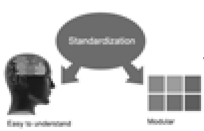

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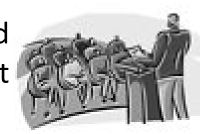

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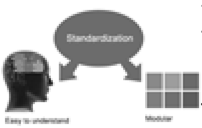

After nomination of your institution as “Institutional Fellow” and constantly functioning successfully for one year, we can consider giving recognition to your institute to function as Regional/Zonal office on our behalf.
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Note :

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- In future, if the board feels the necessity to change any board member, the same can be done with the consent of the chairperson along with anyone board member without our approval.
- In case, the chairperson needs to be replaced then consent of 2/3rd board members are required and they are also required to jointly pass the resolution copy of which should be sent to us. In such case, it will be compulsory to obtain our approval before replacement.
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2. Ethical Guidelines,

3. Submission of Manuscripts,

4. Manuscript’s Category,

5. Structure and Format of Manuscript,

6. After Acceptance.

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Key points to remember:

- Submit all work in its final form.
- Write your paper in the form, which is presented in the guidelines using the template.
- Please note the criterion for grading the final paper by peer-reviewers.

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A purpose of organizing a research paper is to let people to interpret your effort selectively. The journal requires the following sections, submitted in the order listed, each section to start on a new page.

The introduction will be compiled from reference matter and will reflect the design processes or outline of basis that direct you to make study. As you will carry out the process of study, the method and process section will be constructed as like that. The result segment will show related statistics in nearly sequential order and will direct the reviewers next to the similar intellectual paths throughout the data that you took to carry out your study. The discussion section will provide understanding of the data and projections as to the implication of the results. The use of good quality references all through the paper will give the effort trustworthiness by representing an alertness of prior workings.

Writing a research paper is not an easy job no matter how trouble-free the actual research or concept. Practice, excellent preparation, and controlled record keeping are the only means to make straightforward the progression.

General style:

Specific editorial column necessities for compliance of a manuscript will always take over from directions in these general guidelines.

To make a paper clear

· Adhere to recommended page limits

Mistakes to evade

- Insertion a title at the foot of a page with the subsequent text on the next page
- Separating a table/chart or figure - impound each figure/table to a single page
- Submitting a manuscript with pages out of sequence

In every sections of your document

· Use standard writing style including articles ("a", "the," etc.)

· Keep on paying attention on the research topic of the paper

· Use paragraphs to split each significant point (excluding for the abstract)

· Align the primary line of each section

· Present your points in sound order

· Use present tense to report well accepted

· Use past tense to describe specific results

· Shun familiar wording, don't address the reviewer directly, and don't use slang, slang language, or superlatives

· Shun use of extra pictures - include only those figures essential to presenting results

Title Page:

Choose a revealing title. It should be short. It should not have non-standard acronyms or abbreviations. It should not exceed two printed lines. It should include the name(s) and address (es) of all authors.

Abstract:

The summary should be two hundred words or less. It should briefly and clearly explain the key findings reported in the manuscript--must have precise statistics. It should not have abnormal acronyms or abbreviations. It should be logical in itself. Shun citing references at this point.

An abstract is a brief distinct paragraph summary of finished work or work in development. In a minute or less a reviewer can be taught the foundation behind the study, common approach to the problem, relevant results, and significant conclusions or new questions.

Write your summary when your paper is completed because how can you write the summary of anything which is not yet written? Wealth of terminology is very essential in abstract. Yet, use comprehensive sentences and do not let go readability for briefness. You can maintain it succinct by phrasing sentences so that they provide more than lone rationale. The author can at this moment go straight to shortening the outcome. Sum up the study, with the subsequent elements in any summary. Try to maintain the initial two items to no more than one ruling each.

- Reason of the study - theory, overall issue, purpose
- Fundamental goal
- To the point depiction of the research
- Consequences, including definite statistics - if the consequences are quantitative in nature, account quantitative data; results of any numerical analysis should be reported
- Significant conclusions or questions that track from the research(es)

Approach:

- Single section, and succinct
- As a outline of job done, it is always written in past tense
- A conceptual should situate on its own, and not submit to any other part of the paper such as a form or table
- Center on shortening results - bound background information to a verdict or two, if completely necessary
- What you account in an conceptual must be regular with what you reported in the manuscript
- Exact spelling, clearness of sentences and phrases, and appropriate reporting of quantities (proper units, important statistics) are just as significant in an abstract as they are anywhere else

Introduction:

The **Introduction** should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable to comprehend and calculate the purpose of your study without having to submit to other works. The basis for the study should be offered. Give most important references but shun difficult to make a comprehensive appraisal of the topic. In the introduction, describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will have no attention in your result. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here. Following approach can create a valuable beginning:

- Explain the value (significance) of the study
- Shield the model - why did you employ this particular system or method? What is its compensation? You strength remark on its appropriateness from a abstract point of vision as well as point out sensible reasons for using it.
- Present a justification. Status your particular theory (es) or aim(s), and describe the logic that led you to choose them.
- Very for a short time explain the tentative propose and how it skilled the declared objectives.

Approach:

- Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done.
- Sort out your thoughts; manufacture one key point with every section. If you make the four points listed above, you will need a least of four paragraphs.

- Present surroundings information only as desirable in order hold up a situation. The reviewer does not desire to read the whole thing you know about a topic.
- Shape the theory/purpose specifically - do not take a broad view.
- As always, give awareness to spelling, simplicity and correctness of sentences and phrases.

Procedures (Methods and Materials):

This part is supposed to be the easiest to carve if you have good skills. A sound written Procedures segment allows a capable scientist to replacement your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt for the least amount of information that would permit another capable scientist to spare your outcome but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section. When a technique is used that has been well described in another object, mention the specific item describing a way but draw the basic principle while stating the situation. The purpose is to text all particular resources and broad procedures, so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step by step report of the whole thing you did, nor is a methods section a set of orders.

Materials:

- Explain materials individually only if the study is so complex that it saves liberty this way.
- Embrace particular materials, and any tools or provisions that are not frequently found in laboratories.
- Do not take in frequently found.
- If use of a definite type of tools.
- Materials may be reported in a part section or else they may be recognized along with your measures.

Methods:

- Report the method (not particulars of each process that engaged the same methodology)
- Describe the method entirely
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures
- Simplify - details how procedures were completed not how they were exclusively performed on a particular day.
- If well known procedures were used, account the procedure by name, possibly with reference, and that's all.

Approach:

- It is embarrassed or not possible to use vigorous voice when documenting methods with no using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result when script up the methods most authors use third person passive voice.
- Use standard style in this and in every other part of the paper - avoid familiar lists, and use full sentences.

What to keep away from

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings - save it for the argument.
- Leave out information that is immaterial to a third party.

Results:

The principle of a results segment is to present and demonstrate your conclusion. Create this part a entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Carry on to be to the point, by means of statistics and tables, if suitable, to present consequences most efficiently.You must obviously differentiate material that would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matter should not be submitted at all except requested by the instructor.

Content

- Sum up your conclusion in text and demonstrate them, if suitable, with figures and tables.
- In manuscript, explain each of your consequences, point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation an exacting study.
- Explain results of control experiments and comprise remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or in manuscript form.

What to stay away from

- Do not discuss or infer your outcome, report surroundings information, or try to explain anything.
- Not at all, take in raw data or intermediate calculations in a research manuscript.
- Do not present the similar data more than once.
- Manuscript should complement any figures or tables, not duplicate the identical information.
- Never confuse figures with tables - there is a difference.

Approach

- As forever, use past tense when you submit to your results, and put the whole thing in a reasonable order.
- Put figures and tables, appropriately numbered, in order at the end of the report
- If you desire, you may place your figures and tables properly within the text of your results part.

Figures and tables

- If you put figures and tables at the end of the details, make certain that they are visibly distinguished from any attach appendix materials, such as raw facts
- Despite of position, each figure must be numbered one after the other and complete with subtitle
- In spite of position, each table must be titled, numbered one after the other and complete with heading
- All figure and table must be adequately complete that it could situate on its own, divide from text

Discussion:

The Discussion is expected the trickiest segment to write and describe. A lot of papers submitted for journal are discarded based on problems with the Discussion. There is no head of state for how long a argument should be. Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implication of the study. The purpose here is to offer an understanding of your results and hold up for all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of result should be visibly described. Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved with prospect, and let it drop at that.

- Make a decision if each premise is supported, discarded, or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."
- Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work
- You may propose future guidelines, such as how the experiment might be personalized to accomplish a new idea.
- Give details all of your remarks as much as possible, focus on mechanisms.
- Make a decision if the tentative design sufficiently addressed the theory, and whether or not it was correctly restricted.
- Try to present substitute explanations if sensible alternatives be present.
- One research will not counter an overall question, so maintain the large picture in mind, where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

- When you refer to information, differentiate data generated by your own studies from available information
- Submit to work done by specific persons (including you) in past tense.
- Submit to generally acknowledged facts and main beliefs in present tense.

The Administration Rules

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Topics	Grades		
	A-B	**C-D**	**E-F**
Abstract	Clear and concise with appropriate content, Correct format. 200 words or below	Unclear summary and no specific data, Incorrect form Above 200 words	No specific data with ambiguous information Above 250 words
Introduction	Containing all background details with clear goal and appropriate details, flow specification, no grammar and spelling mistake, well organized sentence and paragraph, reference cited	Unclear and confusing data, appropriate format, grammar and spelling errors with unorganized matter	Out of place depth and content, hazy format
Methods and Procedures	Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads	Difficult to comprehend with embarrassed text, too much explanation but completed	Incorrect and unorganized structure with hazy meaning
Result	Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake	Complete and embarrassed text, difficult to comprehend	Irregular format with wrong facts and figures
Discussion	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

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