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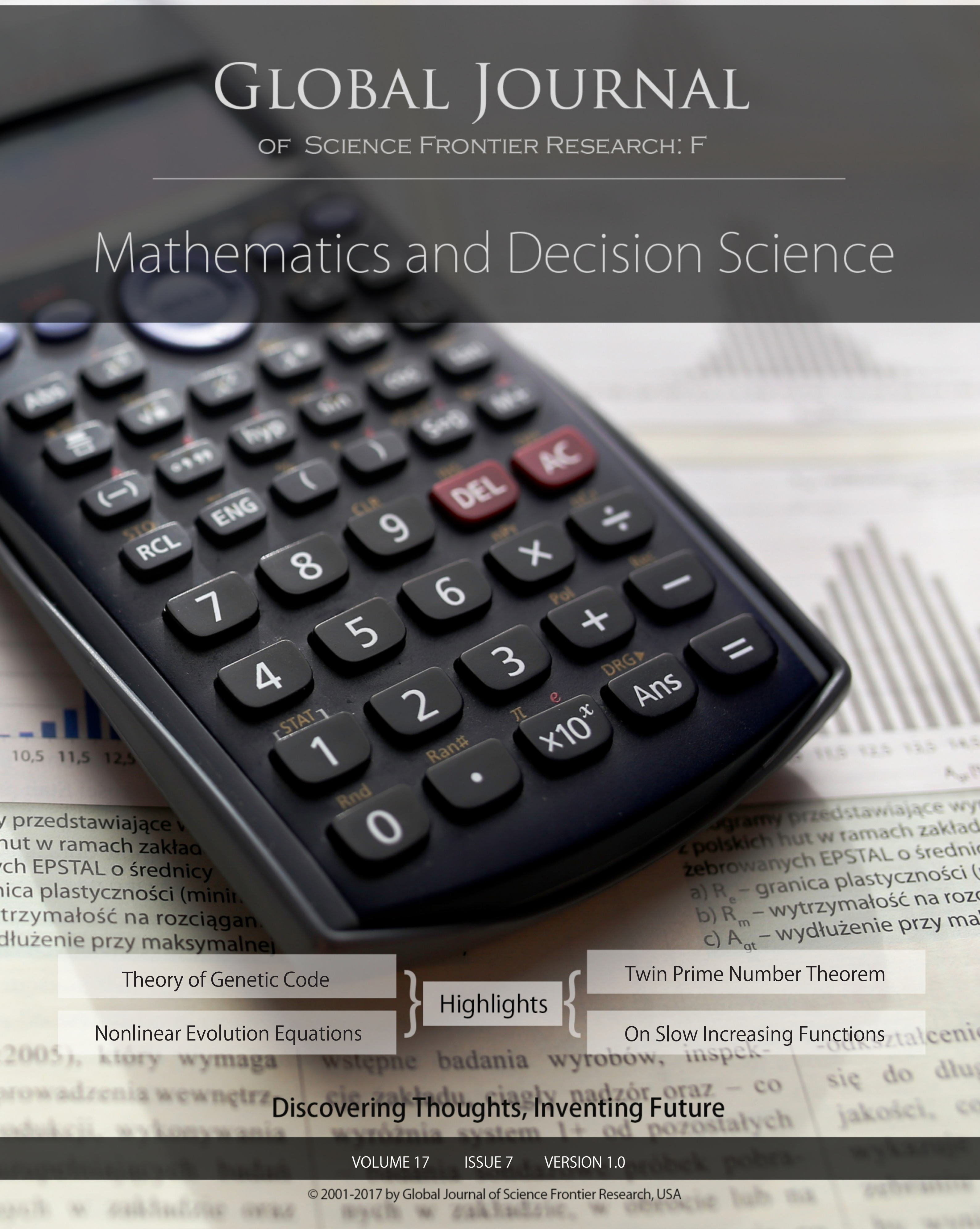

Global Journal

of Science Frontier Research: F

Mathematics and Decision Science

Highlights

- Theory of Genetic Code
- Nonlinear Evolution Equations
- Twin Prime Number Theorem
- On Slow Increasing Functions

Discovering Thoughts, Inventing Future

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Mathematics & Decision Sciences

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Theory of Classical Gaussian Observer

By Henrik Stenlund
Visilab Signal Technologies Oy

Abstract- This paper treats the concept of the Gaussian probability distribution both for the target and observer. The resulting observations become Gaussian distributions as well. The time coordinate gets an equal setting as any physical quantity. This treatment is purely classical with no essential reference to quantum mechanics nor to theory of relativity.

Keywords: *measurement, observation of physical quantity, gaussian observer, classical observer.*

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4. J. A. Wheeler, W. H. Zurek, *Quantum Theory and Measurement*, Princeton University Press 1983.

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Keywords: measurement, observation of physical quantity, gaussian observer, classical observer.

I. Introduction

a) Background

The motivation for this paper has been the need to define the general classical physical observation in a satisfactory way. The system consists of a Gaussian measuring instrument (observer) and a target system (target) with a Gaussian distribution. This point of view seems to be overlooked and considered trivial in spite of its importance. The literature around this subject seems to be little. The author dares to complete this issue and put the tile on its place.

The corresponding quantum mechanical aspects have been treated in a great number of articles but even that problem has not found a final solution. These two topics must have some correspondence and common points. Lamb [1], Reece [2] and Zeh [3] are most notable of the recent studies with a well covered list of references therein. In Wheeler [4] is an excellent collection of all important articles on the subject up to 1983. The treatments do not seem to have a consistent handling of observation of probability distributions with Gaussian profiles and the classical point of view has no weight.

In the following is presented a theory of observation of classical physical quantities by using a Gaussian model and based on elements of probability. One is talking interchangeably of measurement and observation. The subject of the measuring system affecting the target system's behavior is not treated since that is mainly a phenomenon of the quantum world and outside of the topic of this paper.

b) General

The general conception in earlier, a bit outdated, articles is that a human is active in the observation process and one has to take into account his brain functioning and other biological processes, like eye sight. This misconception is completely outside the topic since observations can be made by automatic measuring systems, robots and satellites without any human intervention. The incorrect view exists in the 1930's to 1950's in many articles attempting to connect the classical or macroscopic world and quantum mechanics. In the following the human aspect is completely ignored and this is treated as a pure observation irrespective of specific observer details.

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i. *The Process of Observation*

Focusing more closely into observing the value of some physical quantity in a target system, one will soon realize that it is more complicated than advertised. For instance, in spite of the apparent simplicity from the point of view of a physicist, image analysis and pattern recognition in industrial processes are seldom accurate. They contain lots of distorting factors destroying any ideal model [5]. As further examples, to measure visible spectral contents from a galaxy or the fluorescent radiation coming from a single molecule implanted in a crystal, the process becomes very complicated. The actual measuring process usually goes as follows, with one or more aspects dominating the others.

- Locate the target system to be measured, in the spatial dimensions. The need to scan some volume or coordinate range of the space to locate the target system is recognized.
- Identify the target since there may be others similar in the vicinity, within the volume. Some sort of pattern matching is required to ascertain which object one would be dealing with.
- Make the actual measurement to the accuracy allowed by the instruments, of the variables intended. The measurement process itself is usually complicated since there are no perfect instruments for measuring any physical quantity. Many types of noise contributions must be eliminated with run-time filtering and post-processing.
- The process will require some time forcing the time to become one of the coordinates. Also very often the target has an interesting temporal dependence (event) requiring simultaneity of the measurement and the event to succeed. The measuring time spent consists of time windowing for analysis, sampling or acceptance time, phase-locking time, sensor rise times etc, depending on the system in question. Claiming that some measurement is an infinitely short delta-function type event is totally false. Time is in the same category as any other measurable quantity in the system.
- Interpret the measurement results correctly. This is self-evident but is not always trivial.
- Repeat the measurements in a completely different way creating results independent of the first ones, if any doubt appears of their validity.

It is now obvious that one would be interested in measuring simultaneously the position and some observable and time. To simplify the initial analysis, in the following a one-dimensional model is set up for making simple measurements and that model is used as a basis for generalization to three dimensions and to adopting an arbitrary quantity for measurement. Quantum mechanical and relativistic phenomena at all stages are ignored. That is done in spite of knowing that quantum physics is generally considered more profound than classical physics. This starting point is justified until the quantum mechanical measurement problem has a complete solution, possibly extendable to a macroscopic system and classical variables.

II. Physics of Observation

a) *The Observer*

The observer function $g(x, t)$ describes the ability of the observing instrument to measure a specific observable x and is blurred around the peaking value at the origin, no matter how accurate instruments there are. They always contain noise and drift of different types in varying frequency bands, generated by many physical phenomena. In addition, other unwanted signals are affecting the end result. Traditionally, an instrument does have a Gaussian distribution

Ref

5. Nelson Bridwell, *Maximizing product quality and yield using vision systems*, Vision Systems Design, Feb 2016.

in its observables. The uncertainty spreads to the time coordinate too since no system is able to make measurements in zero time. Often delta-function like measurements have been assumed and the preceding fact ignored. The physical division between the target and observer is reasonable to be made immediately outside the target since the target is what is required to be measured, not anything that affects the measurements outside of it. The external phenomena do not belong to the target variable and must be isolated.

Things get more complex when smaller targets are studied and approach the microscopic and atomic world. The variables measured can be practically any physical quantities like position, momentum, radiative content with extensive analysis etc. but actual quantum mechanical phenomena are left outside the scope. Position is considered a fundamental variable in many systems; therefore it is picked up for our examples.

For observing dynamic phenomena, like the velocity of a target, the observer is acting in its own inertial frame of coordinates. It should not be subject to significantly interfering interactions with the rest of the world. The observer is not part of the laws of physics in the events of the target. The observer only obeys its own laws mostly associated with the observation itself. Things change gradually when the target size becomes of microscopic order. Observer's influence on the target will become more perceivable if it needs to send some excitation to the target of atomic magnitude.

The observation needs to be complicated with the following common realities. The target and observer may be in accelerating curvilinear relative motion. Also the medium (e.g. gases) carrying the primary measurement signals (usually electromagnetic radiation or acoustic waves) may be in motion relative to the target. The medium's volume may consist of complex flows and rotors and be most inhomogenous in consistency. The medium itself may generate disturbing radiation without external excitation or be selectively absorbing. These facts will affect measurements directly in many practical cases.

There is no perfect observer nor instrument and never will be. This is illustrated in Fig. 1 as a placeholder for an ideal instrument covered with a blurring wall separating it from the target.

b) The Target System

The target system is here referring to an object whose particular physical quantity one intends to measure. The measurement can be focusing on one quantity only but can cover a great number of them as well, to be measured either simultaneously or independently. As an important example is taken the coordinate of the target in one dimension. It is common to treat the target position as an ideal point or its outline dimensions like a hard-core stable object. In reality, the target's variable will have a blurred distribution $y(x,t)$ in the coordinate due to various reasons. The coordinate of a classical object is not so accurate as one might expect (specified as the center of gravity). This thought was suggested already by Heisenberg [6] and Scrödinger [7]. A recent discussion of this was by Mehdipour [8] pointing out the possibility of having Gaussian distributions.

The object may have a varying velocity due to a number of external forces (e.g. Brownian movement), thermal expansions in its volume, extra atomic layers on top of it (e.g. a monolayer of water molecules). It may be rotating at a fast rate or have an inaccurate volume boundary and a complicated varying three-dimensional structure rendering difficult the exact specification of its position. It may be losing or gaining energy for some unexpected reason and numerous other interactions may affect. The exact location of the center of gravity is not stable in a macroscopic object and surely has a distribution. The smaller in the size of the target particle one goes, the relatively more blurred it becomes due to interactions with the surroundings. A good example is a small molecule whose atoms are vibrating and it is impossible to exactly set its center

Ref

8. S. Hamid Mehdipour, *Entropic force law in the presence of noncommutative inspired spacetime for solar system scale*, arXiv:1605.03409v1 [physics.gen-ph], 2 May 2016.

of gravity, even in a crystal lattice. Similar change, while going to the small, may happen to all other physical variables, some are more vulnerable than the others. Obviously, some of the facts listed may as well be overlapping with the features of the imperfect observer itself. One cannot always draw a clear borderline between the two sources of uncertainty.

A great example of an observable which always has a significant uncertainty is the temperature of an object. It has both a distribution inside macroscopic objects and temporal fluctuations and may be subject to endothermic or exothermic processes. One would need a precise way of defining the target temperature, irrespective of the apparent triviality. The measurement itself would be based on infrared radiation from the surface or on some indirect method, like a Platinum resistor mounted inside. They are both far from being perfect in absolute precision although they can offer a fair repeatability and resolution with a relatively low noise. This fact is immediately reflected on the distribution of the variable itself.

Notes

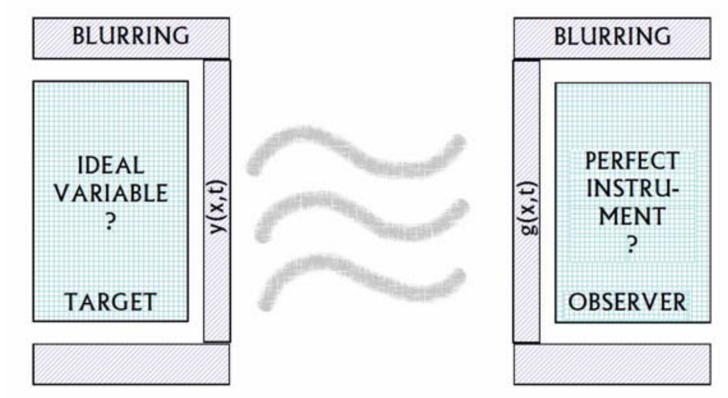

Figure 1: Observation with blurring

i. *Uncertainty Relations*

One could argue that physical quantities themselves are ideal to measure and have no distribution but this has not yet been proven. On the contrary, not even on the classical level can be stated that all, if any, variables would be ideal. When the atomic scale is approached, the particles are acknowledged to have distributions of probability instead of precise ideal values. In the microscopic world the Heisenberg and other uncertainty relations give estimates and conditions for variables' limiting accuracy. For instance, infrared radiation at $\lambda = 10\ \mu m$ whose frequency one needs to measure from one or a few photons. One insists on having a fair accuracy of 15 digits. The Heisenberg uncertainty relation suggests an uncertainty in time of the order of a few seconds, while using a perfect measuring instrument. It would not be reasonable to suggest making a zero-time delta-function type measurement of this observable. While measuring spectra of atomic emission having broad peaks, one can easily have a situation where the target is restricting the measurement's accuracy and cannot be made any better even with a perfect instrument, if there would be any. Even the spectral line width of a freely radiating cold atom is not zero. It can easily be calculated.

All this unavoidably brings to mind that there is some sort of internal uncertainty associated with each variable, including time, affecting the measurements but being independent of the observer. Traditionally, it is expected that things are relatively more accurate with a growing target mass. That is partly true but other phenomena start to creep in. There is no such thing as an ideal variable. Refer to the Fig. 1. There is a placeholder for it behind a blurring wall.

c) Constructing the Observation

i. Distributions for the Target and Observer

The conclusion from the facts in the preceding paragraphs is that probability distributions for each observable exist, including the time, and for the observer. The resulting observation becomes a probability distribution. No quantum mechanical effects as such are taken into account. In astronomical measurements one would be limited by restrictions caused by the event horizon due to extremely long distances and possibly high velocities.

Notes

ii. Distribution for Observation

The fact that there is only one kind of target in the volume one is interested in, is assumed. In the following one is concentrating on measuring the coordinate of the target. Also it is assumed that the range of interest for the spatial coordinate will be $(-L, L)$ and for the temporal coordinate $(-T, T)$. The observation can be performed in one dimension or variable at a time as a process of summing the contribution of infinitesimal parts throughout the volume. Simultaneously one runs through with the observer function and progress from positive to negative direction. The infinitesimal probabilities for the simultaneous measurements in x' and t' are $\Delta p'_x$ and $\Delta p'_t$ respectively with corresponding infinitesimal widths $\Delta x'$ and $\Delta t'$

$$\Delta p'_x \Delta p'_t = \Delta x' \Delta t' y(x', t') g(x - x', t - t') \tag{1}$$

Summing the infinitesimal probabilities along x' and t' will lead to a double integral forming the observation at (x, t)

$$z(x, t) = \int_{-L}^{L} dx' \int_{-T}^{T} dt' y(x', t') g(x - x', t - t') \tag{2}$$

The $g(x, t)$ function is normalized properly for both integrations. $g(x, t)$ will be independent on the details of the target function $y(x, t)$ and determined by the measuring instrument and by the details of the measurement process.

iii. Three-Dimensional Distribution for Observation

In three dimensions there is a straightforward extension to

$$z(\vec{r}, t) = \int_V d\vec{r'} \int_{-T}^{T} dt' y(\vec{r'}, t') g(\vec{r} - \vec{r'}, t - t') \tag{3}$$

The functions z, y are scalar functions of vectors but can be vector functions of vectors in vectorized cases and the multiplication specified properly.

III. The Gaussian Model

a) One-dimensional Model

In the following a simple Gaussian peaking observation function and a basic single-variable target having the same nature are prepared. The distribution functions can accept other than Gaussian forms but will not likely cause significant qualitative changes in equations, except add some mathematical inconvenience. One requirement is that the distribution approaches zero quickly after a few half-widths away from the peak, with both functions. The use of a Gaussian is well established in statistical processes and it brings to the analysis certain easiness in integration without having to fall back on piecewise integration or complicated approximation methods.

The observer's and target's distribution functions can be multipeaking, according to the system's specific requirements. The systems may consist, for instance, of multiple states and the exact state is not predictable. Thus a multipeaking Gaussian may be justified for the target which can be approximated well with exponential functions allowing easy integrability.

i. *The Observer*

The observer function is expected to behave as a Gaussian around the origin in both coordinates (x, t) as

$$g(x,t) = \frac{1}{MN} e^{-\kappa x^2 - \xi t^2} \tag{4}$$

x and t are coordinates in the range within which the target lies and which are an active part of the observation process. Here M, N are normalization constants, evaluated with a constant target distribution y. Normalization will give a unity observation if the $y(x,t)$ is unity, indicating that the target is within the volume but one cannot say where and when. The peak width in x-coordinate of this distribution is $1/\sqrt{\kappa}$ and the temporal width is $1/\sqrt{\xi}$.

ii. *The Target*

The target has a Gaussian distribution of probability of the position x and time t

$$y(x,t) = e^{-\beta(x-\hat{x})^2 - \eta(t-\hat{t})^2} \tag{5}$$

Here $\hat{x}$ is the position variable's expectation value which is the ideal variable having an infinite accuracy if ever possible. Correspondingly, $\hat{t}$ indicates the ideal (expectation) value for the time when the target can be located at the point $\hat{x}$. See the Figure 2. below. The resulting observation of the Gaussian particle in one dimension will be the following

$$z(x,t) = \frac{1}{MN} \int_{-L}^{L} dx' \int_{-T}^{T} dt' e^{-\beta(x'-\hat{x})^2 - \eta(t'-\hat{t})^2} e^{-\kappa(x-x')^2 - \xi(t-t')^2} \tag{6}$$

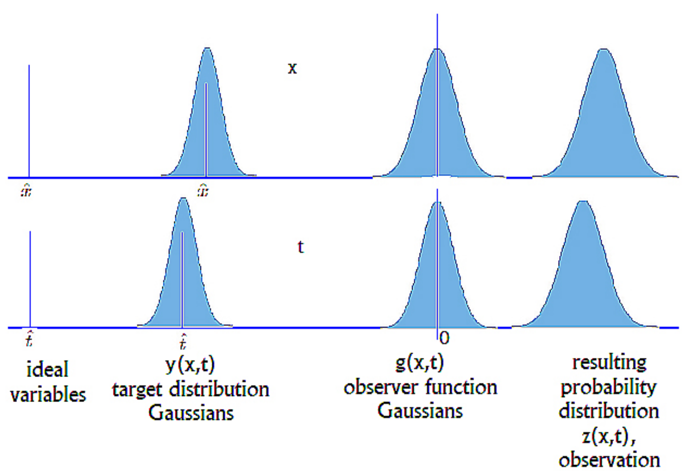

Figure 2: A crude sketch of the observation process with Gaussian distributions. To the left are the Dirac delta function distributions of the ideal variables and while proceeding to the right through each stage the distributions become wider

iii. *Infinite Ranges*

As agreed above, the target distribution and the observer functions fall rapidly to zero outside the peak and therefore one can let the limits of integration L and T to go to infinity, since it is expected not to make observations near the boundaries.

Notes

$$z(x,t) = \frac{1}{MN}\int_{-\infty}^{\infty} dx' \int_{-\infty}^{\infty} dt' e^{-\beta(x'-\hat{x})^2-\eta(t'-\hat{t})^2} e^{-\kappa(x-x')^2-\xi(t-t')^2} \tag{7}$$

In this model the normalizations for x'- and t'-integrations will become

$$\frac{1}{M} = \sqrt{\frac{\kappa}{\pi}} \tag{8}$$

$$\frac{1}{N} = \sqrt{\frac{\xi}{\pi}} \tag{9}$$

Thus one gets after integration

$$z(x,t) = \sqrt{\frac{\kappa\xi}{(\kappa+\beta)(\eta+\xi)}} e^{-\frac{\kappa\beta(x-\hat{x})^2}{\kappa+\beta}-\frac{\xi\eta(t-\hat{t})^2}{\eta+\xi}} \tag{10}$$

In the following is studied limiting cases for this expression.

iv. *Accurate Observer Limit*

If the observer's Gaussian is narrow compared to the target's Gaussian ($\beta << \kappa, \eta << \xi$), one expects to get rather accurate results. The observation becomes

$$z(x,t) \approx e^{-\beta(x-\hat{x})^2-\eta(t-\hat{t})^2} \tag{11}$$

which is what traditionally is expected of this measurement. The instrument's capability is not restrictive in this case.

v. *Inaccurate Observer Limit*

In case the observer's Gaussian is broad compared to the target's Gaussian ($\beta >> \kappa, \eta >> \xi$), one gets

$$z(x,t) \approx \sqrt{\frac{\kappa\xi}{\beta\eta}} e^{-\kappa(x-\hat{x})^2-\xi(t-\hat{t})^2} \tag{12}$$

The observation distribution has flattened wider compared to the more accurate case above.

vi. *Dirac Delta Function*

It is interesting to note that our observer function

$$g(x,t) = \frac{\sqrt{\xi\kappa}}{\pi} e^{-\kappa x^2-\xi t^2} \tag{13}$$

is precisely the definition of the Dirac delta function in the limit of growing κ and ξ, treated separately.

$$\lim_{\kappa\to\infty,\ \xi\to\infty} g(x-x', t-t') \to \delta(x-x')\delta(t-t') \tag{14}$$

This gives some justification for the traditional assumption of infinitely fast and accurate measurements, in the limit of extremely sharp Gaussian of the observer, both in time and spatial coordinates. The Dirac delta function will let the $y(x,t)$ to emerge from the integrals (7) offering it as the result of measurement.

vii. *Accurate Target Limit*

If the target's Gaussian becomes narrow to the limit of Dirac delta function, it will push out the $g(x,t)$ from the double integral (10)

Notes

$$y(x,t) = \delta(x - \hat{x})\delta(t - \hat{t}) \tag{15}$$

$$z(x,t) = g(x,t) \tag{16}$$

The result will be the observer's distribution. The Gaussian $y(x,t)$ does not become a Dirac delta function automatically just by narrowing its Gaussian width but must in that case be the distribution of the target as with the observer function, with a multiplier of $\sqrt{\beta}$ and/or $\sqrt{\eta}$.

b) *Adding a Simultaneous Variable for Measurement*

Suppose there is a physical quantity u and the target distribution is the following

$$y(x,t) = e^{-\beta(x-\hat{x})^2 - \eta(t-\hat{t})^2 - \gamma(u-\hat{u})^2 - \rho(t-\hat{T})^2} \tag{17}$$

One has added a new time $\hat{T}$ indicating the moment of proper measurement of the variable u having a specific ideal value $\hat{u}$. To test if the added time Gaussian has some meaning one calculates the observation with the observer function

$$g(x,t) = \frac{1}{MNK} e^{-\kappa x^2 - \xi t^2 - \alpha u^2} \tag{18}$$

and perform the integration to get

$$z(x,t) = \sqrt{\frac{\kappa\xi\alpha}{(\kappa+\beta)(\eta+\xi+\rho)(\alpha+\gamma)}} e^{-\frac{\kappa\beta(x-\hat{x})^2}{\kappa+\beta} - \frac{\alpha\gamma(u-\hat{u})^2}{\alpha+\gamma} - \frac{\xi\eta(t-\hat{t})^2 + \xi\rho(t-\hat{T})^2 + \rho\eta(\hat{t}-\hat{T})^2}{\eta+\xi+\rho}} \tag{19}$$

One can immediately see that this expression is nonzero only if $\hat{t} \approx \hat{T}$. It is equivalent to having exactly the same measuring time for all simultaneous measurements. The contribution of simultaneous observation of the variable u is with the common temporal term shown

$$z(u,t) = \sqrt{\frac{\alpha}{\alpha+\gamma}} e^{-\frac{\alpha\gamma(u-\hat{u})^2}{\alpha+\gamma} - \frac{\xi\eta(t-\hat{t})^2}{\eta+\xi}} \tag{20}$$

This is peaking nicely at $\hat{u}$ as it is supposed to. The width of the observational distribution is affected by α. If an added measurement is independent of the original measurement performed, the end result of the observation is additive. For simultaneous dependent measurements, it is multiplicative.

IV. Discussion

The classical physical quantities behaving according to the laws of physics is one thing and measuring them is another. The measurement results can approach accurate values if the measuring conditions are favorable and the instruments have suitable properties, i.e. their Gaussian widths are extremely narrow approaching Dirac delta functions in form. However, they are not the same except by chance, since no perfect instruments exist and the target's variable will also have a Gaussian distribution due to its own uncertainties. The results of measuring classical quantities will always have probability distributions based both on uncertainties of the target system and on imperfections in the observer. Ideal variables are good for theories but exist only in the minds of physicists; they are affected by blurring.

One takes into use a Gaussian distribution both for the observer and for the target system's variable to be measured. It will give a model which is closer to reality than hard core type objects and Dirac delta-function type measurements which are ideal and nonexistent. The observation is a Gaussian in many cases.

The main results of this work are equations (2) and (10).

Notes

Notes

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A Sufficient Condition for the Uniform Convergence of Truncated Cardinal Functions Whittaker Inside the Interval

By A. Yu. Trynin
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Abstract- In terms of the one-sided module of continuity and positive (negative) module of change obtain sufficient conditions for the uniform convergence of truncated cardinal functions Whittaker inside the interval.

Keywords: *sinc approximation, interpolation functions, uniform approximation, cardinal functions Whittaker.*

GJSFR-F Classification: *MSC 2010: 41A05; 65D05, 65T60.*

ASUFFICIENTCONDITIONFORTHEUNIFORMCONVERGENCEOFTRUNCATEDCARDINALFUNCTIONSWHITTAKERINSIDETHEINTERVAL

A Sufficient Condition for the Uniform Convergence of Truncated Cardinal Functions Whittaker Inside the Interval

A. Yu. Trynin

Abstract- In terms of the one-sided module of continuity and positive (negative) module of change obtain sufficient conditions for the uniform convergence of truncated cardinal functions Whittaker inside the interval.

Keywords: sinc approximation, interpolation functions, uniform approximation, cardinal functions Whittaker.

I. Introduction and Preliminaries

E. Borel and E.T. Whittaker introduced the notion of a truncated cardinal function, whose restriction on the segment $[0, \pi]$ reads as follows:

$$C_\Omega(f,x) = \sum_{k=0}^{n} \frac{\sin\left(\Omega x - k\pi\right)}{\Omega x - k\pi} f\left(\frac{k\pi}{\Omega}\right) = \sum_{k=0}^{n} \frac{(-1)^k \sin \Omega x}{\Omega x - k\pi} f\left(\frac{k\pi}{\Omega}\right), \tag{1.1}$$

here $\Omega > 0$ and $n = [\Omega]$ is integer part $\Omega \in \mathbb{R}$. The function $\frac{\sin(\Omega x)}{\Omega x}$ called sinc-function. Up to now, a fairly well-studied problem is the one concerning sinc approximations of an analytic function on the real axis decreasing exponentially at infinity. The most complete survey of the results obtained in this direction by 1993 be found in [1].

Sinc approximations have wide applications in mathematical physics, in constructing various numerical methods and the approximation theory for the functions of both one and several variables [2], [3] [4], [5], [6] [1], [7], in theory of quadrature formulae [8], [1], in theory of wavelets or wavelet-transforms in [9, Ch. 2], [10], [11].

One test for the uniform convergence on the axis for Whitteker cardinal functions were provided in [12], [13]. Another important sufficient condition for convergence of sinc approximations was obtained in [14]. It was established that for some subclasses of functions absolutely continuous together with their derivatives on the interval $(0, \pi)$ and having a bounded variation on the whole axis $\mathbb{R}$ Kotel'nikov series (or cardinal Whitteker functions) converge uniformly inside the interval $(0, \pi)$. In [15] was obtained by an upper bound for the best possible approximations of sincs. In book [16] designated perspective directions of development of sinc approximations. In papers [17] there were obtained estimates for

Ref

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the error of approximations of uniformly continuous and bounded on $\mathbb{R}$ functions by the values of various operators being combinations of sincs. Unfortunately, while approximating continuous functions on a segment by means of (1.1) and many other operators, Gibbs phenomenon arises in the vicinity of the segment end-points, see, for instance [18]. In [19] and [18] various estimates for the error of approximation of analytic in a circle functions by sinc-approximations (1.1) (when $\Omega = n$) were obtained.

In paper [19] sharp estimates were established for the functions and Lebesgue constants of operator (1.1) (when $\Omega = n$). Works [20], [21] were devoted to obtaining necessary and sufficient conditions of pointwise and uniform in interval $(0, \pi)$ convergence of values operators (1.1) (when $\Omega = n$) for functions $f \in C[0, \pi]$. In [22] there was constructed an example of continuous function vanishing at the end-points of the segment $[0, \pi]$ for which the sequence of the values of operators (1.1) (when $\Omega = n$) diverges unboundedly everywhere on the interval $(0, \pi)$. Work [23] was denoted to studying approximative properties of interpolation operators constructed by means of solutions to the Cauchy problems with second order differential expressions. Papers [24] and [25] were devoted to applications of considered in [23] Lagrange-Sturm-Liouville interpolation processes. In [26] the results of work [23] were applied for studying approximative properties of classical Lagrange interpolation processes with the matrix of interpolation nodes, whose each row consists of zeroes of Jacobi polynomials $P_n^{\alpha_n, \beta_n}$ with the parameters depending on n. In the works [27], [28], [29] of construction of new operators sinc approximations. They allow you to uniformly approximate any continuous function on the segment.

II. Results and Discussion

In the present work we follow the lines of publications [33], [34], [35], [36], [30], [37], [38], [39], [31], [32], [40] and we obtain sufficient conditions approximations of continuous on the segment $[0, \pi]$ functions inside interval $(0, \pi)$ by means of truncated cardinal function (1.1) (in case $\Omega > 0$).

Fix $\rho_\lambda = o\left(\frac{\sqrt{\lambda}}{\ln \lambda}\right)$ as $\lambda \to +\infty$, let $h(\lambda) \in \mathbb{R}$, and for each nonnegative λ let q_λ be arbitrary function in the ball $V_{\rho_\lambda}[0, \pi]$ of radius ρ_λ in the space of functions with bounded variation vanishing at the origin, so that

$$V_0^\pi[q_\lambda] \le \rho_\lambda, \quad \rho_\lambda = o\left(\frac{\sqrt{\lambda}}{\ln \lambda}\right), \quad \text{as } \lambda \to \infty, \quad q_\lambda(0) = 0. \tag{2.1}$$

For a potential $q_\lambda \in V_{\rho_\lambda}[0, \pi]$, where $\lambda \to +\infty$, the zeros of solution of the Cauchy problem

$$\begin{cases} y'' + \big(\lambda - q_\lambda(x)\big)y = 0, \\ y(0, \lambda) = 1, \quad y'(0, \lambda) = h(\lambda), \end{cases} \tag{2.2}$$

or, provided that $h(\lambda) \ne 0$

$$V_0^\pi[q_\lambda] \le \rho_\lambda, \quad \rho_\lambda = o\left(\frac{\sqrt{\lambda}}{\ln \lambda}\right), \text{ as } \lambda \to \infty, \quad q_\lambda(0) = 0, \quad h(\lambda) \ne 0, \tag{2.3}$$

Ref

18. Trynin A. Yu., Sklyarov V. P. *Error of sinc approximation of analytic functions on an interval*, Sampling Theory in Signal and Image Processing 7 (3), 263-270 (2008).

the zeros of Cauchy problem

$$\begin{cases} y'' + \big(\lambda - q_\lambda(x)\big)y = 0, \\ y(0,\lambda) = 0, \quad y'(0,\lambda) = h(\lambda), \end{cases} \tag{2.4}$$

which lie in $[0,\pi]$ and are numbered in ascending order, will be denoted by

$$0 \leq x_{0,\lambda} < x_{1,\lambda} < \ldots < x_{n(\lambda),\lambda} \leq \pi \quad (x_{-1,\lambda} < 0, x_{n(\lambda)+1,\lambda} > \pi). \tag{2.5}$$

(Here $x_{-1,\lambda} < 0$, and $x_{n(\lambda)+1,\lambda} > \pi$ are the zeros of the extension of solution of the Cauchy problem (2.2) or (2.4) corresponding to some extension of function q_λ outside $[0,\pi]$ having similar bounds for the variation).

In [23] the properties of the Lagrange type approximation investigated. The operators which include the solution of the Cauchy problem of the form (2.4) or (2.5) and the continuous function which bind

$$S_\lambda(f,x) = \sum_{k=0}^{n} \frac{y(x,\lambda)}{y'(x_{k,\lambda},\lambda)(x - x_{k,\lambda})} f(x_{k,\lambda}) = \sum_{k=0}^{n} s_{k,\lambda}(x) f(x_{k,\lambda}); \tag{2.6}$$

it interpolates f at the nodes $\{x_{k,\lambda}\}_{k=0}^{n}$.

Let $C_0[0,\pi] = \{f : f \in C[0,\pi],\ f(0) = f(\pi) = 0\}$. When approximation using sinc approximations (1.1) function $f \in C[0,\pi] \setminus C_0[0,\pi]$ near the endpoints of the Gibbs phenomenon occurs. This problem can be solved with the help of the reception that was used in the construction of the operator [23, formula (1.9)]

$$T_\lambda(f,x) = \sum_{k=0}^{n} \frac{y(x,\lambda)}{y'(x_{k,\lambda})(x - x_{k,\lambda})} \left\{ f(x_{k,\lambda}) - \frac{f(\pi) - f(0)}{\pi} x_{k,\lambda} - f(0) \right\} + \frac{f(\pi) - f(0)}{\pi} x + f(0), \tag{2.7}$$

where $y(x,\lambda)$ – solution problem Cauchy (2.2) or (2.4) and $x_{k,\lambda}$ – the zeros of the solutions.

III. Sufficient Conditions of Sinc Approximations within the Interval of Uniform Convergence $(0, \pi)$

Let $\mathbf{\Omega}$ set of real continuous non decreasing convex up on $[0, b-a]$, vanishing at zero functions ω. Let $C(\omega^l, [a,b])$ and $C(\omega^r, [a,b])$ is the set of elements of $C[a,b]$ such that for any x and $x+h$ $(a \leq x < x+h \leq b)$ we have the equalities

$$f(x+h) - f(x) \geq -K_f\omega(h) \text{ or } f(x+h) - f(x) \leq K_f\omega(h), \tag{3.1}$$

accordingly. Where $\omega \in \mathbf{\Omega}$. Selecting positive constants K_f may depend only on the function f. In this case the function $\omega(h)$ is sometimes referred to, accordingly, the left-hand or right-hand continuity module. In principle, the definition of a unilateral module of continuity could be considered any functions $\hat{\omega}(h)$ vanishing at zero, continuous on $[0, b-a]$ or $[0,\infty)$. The wording of all the results of this work in this case, would remain in force. Without loss of generality, in the definition of unilateral modulus of continuity (3.1) can be considered $\omega \in \mathbf{\Omega}$.

Classic modulus of continuity $f \in C[a,b]$ denoted as usual $\omega(f,\delta) = \sup\limits_{|h|<\delta; x,x+h\in[a,b]} |f(x+h) - f(x)|$. The module of continuity of $f \in C[0,\pi]$, if $a = 0$, $b = \pi$ will denote $\omega_1(f,\delta) = \sup\limits_{|h|<\delta; x,x+h\in[0,\pi]} |f(x+h) - f(x)|$. Module of change of f on the interval $[a,b]$ is called function defined by the equation

$$v(n,f) = \sup_{T_n} \sum_{k=0}^{n-1} |f(t_{k+1}) - f(t_k)|,$$

where $T_n = \{a \le t_0 < t_1 < t_2 < \cdots < t_{n-1} < t_n \le b\}$, $n \in \mathbb{N}$. Take a non-negative, non-decreasing convex up function of a natural argument to $v(n)$. If a module of changes of function f on the interval $[a,b]$, such that $v(n,f) = O(v(n))$ with $n \to \infty$, then we say that f belongs to the class $V(v)$. Here, also, the choice of uniformity of the constants o-symbolism can only depend on f.

By analogy with the positive (negative) change of function will be called positive (negative) module of change of function f on the interval $[a,b]$, accordingly, the function of a natural argument type

$$v^+(n,f) = \sup_{T_n} \sum_{k=0}^{n-1} \big(f(t_{k+1}) - f(t_k)\big)_+ \text{ and } v^-(n,f) = \inf_{T_n} \sum_{k=0}^{n-1} \big(f(t_{k+1}) - f(t_k)\big)_-,$$

where $z_+ = \frac{z+|z|}{2}$ and $z_- = \frac{z-|z|}{2}$ and $T_n = \{a \le t_0 < t_1 < t_2 < \cdots < t_{n-1} < t_n \le b\}$, $n \in \mathbb{N}$. We say that f belongs to the class of $V^+(v)$ or $V^-(v)$, if there exists a constant M_f, that for any natural n true inequality

$$v^+(n,f) \le M_f v(n) \text{ or } v^-(n,f) \ge -M_f v(n)$$

accordingly.

Unless otherwise stated, suppose that for each $\lambda > 1$, $n := [\sqrt{\lambda}]$, $\Omega := \sqrt{\lambda}$ and $x_{k,\lambda} := k\pi/\sqrt{\lambda}$ and $l_{k,\lambda}(x) := \frac{(-1)^k \sin \Omega x}{\Omega x - k\pi}$.

Theorem 3.1. Let $f \in C[0,\pi]$, $0 \le a < b \le \pi$, $0 < \varepsilon < (b-a)/2$. *If a non-decreasing concave function of a natural argument* $v(n)$ *and the function* $\omega \in \mathbf{\Omega}$ *such that*

$$\lim_{n\to\infty} \min_{1\le m\le k_2-k_1-1} \left\{ \omega\left(\frac{\pi}{\sqrt{\lambda}}\right) \sum_{k=1}^{m} \frac{1}{k} + \sum_{k=m+1}^{k_2-k_1-1} \frac{v(k)}{k^2} \right\} = 0, \qquad (3.2)$$

where k_1 $k_2 + 1$ *— the smallest and largest number of nodes* $x_{k,\lambda} = k\pi/\Omega$, *falling in the interval* $[a,b]$, *then for any continuous on* $[0,\pi]$, *the function* $f \in C(\omega^l[a,b]) \cap V^-(v)$ $(f \in C(\omega^r[a,b]) \cap V^+(v))$ *is performed*

$$\lim_{\Omega\to\infty} \|f - C_\Omega(f,\cdot)\|_{C[a+\varepsilon, b-\varepsilon]} = 0. \qquad (3.3)$$

Here operator $C_\Omega(f,\cdot)$ *defined in* (1.1).

Remark 3.2 On the set $[0,\pi] \setminus [a,b]$ ratio (1.1) can be not performed (See [22]).

We present auxiliary results, which will be used in the future.

Ref

22. Trynin A.Yu. *On divergence of sinc-approximations everywhere on* $(0,\pi)$, Algebra i Analiz, 22:4 (2010), 232-256 English transl. St. Petersburg Math. J. 22 (2011), 683-701.

Proposition 3.3 ([23, Proposition 9]). *Let $y(x,\lambda)$ be the solution of Cauchy problem (2.4) or (2.5) and assume that in case of the Cachy problem (2.4) relations (2.1) hold, while in the case of (2.5) relations (2.3) hold. If $f \in C_0[0,\pi]$, then*

$$\lim_{\lambda\to\infty}\Big(f(x) - S_\lambda(f,x) - \frac{1}{2}\sum_{k=0}^{n-1}\big(f(x_{k+1,\lambda}) - f(x_{k,\lambda})\big)s_{k,\lambda}(x)\Big) = 0, \tag{3.4}$$

Remark 3.4. From the Proposition 3.3 follows that values operators

$$A_\lambda(f,x) = \frac{1}{2}\sum_{k=0}^{n-1}\big(f(x_{k+1,\lambda}) + f(x_{k,\lambda})\big)s_{k,\lambda}(x),$$

$$B_\lambda(f,x) = \frac{1}{2}\sum_{k=1}^{n}\big(f(x_{k-1,\lambda}) + f(x_{k,\lambda})\big)s_{k,\lambda}(x)$$

or

$$C_\lambda(f,x) = \frac{1}{4}\sum_{k=1}^{n-1}\big(f(x_{k-1,\lambda}) + 2f(x_{k,\lambda}) + f(x_{k+1,\lambda})\big)s_{k,\lambda}(x)$$

give an opportunity approximations every function $f \in C_0[0,\pi]$.

For any $0 \le a < b \le \pi$, $0 < \varepsilon < (b-a)/2$ denoted

$$Q_\lambda(f,[a,b],\varepsilon) := \max_{p_1\le p\le p_2}\left|\sum_{m=m_1}^{m_2}{}' \frac{f(x_{2m+1,\lambda}) - f(x_{2m,\lambda})}{p-2m}\right|. \tag{3.5}$$

Here the dashes on the summation signs in (3.5) mean that are no terms with zero denominator. Where p_1, p_2, m_1 and m_2 are the indices of the zeros determined by the inequalities

$$x_{p_1,\lambda} \le a+\varepsilon < x_{p_1+1,\lambda}, \quad x_{p_2,\lambda} \le b-\varepsilon < x_{p_2+1,\lambda},$$

$$x_{k_1-1,\lambda} < a \le x_{k_1,\lambda}, \quad x_{k_2+1,\lambda} \le b < x_{k_2+2,\lambda},$$

$$m_1 = \left[\frac{k_1}{2}\right] + 1, \quad m_2 = \left[\frac{k_2}{2}\right].$$

Here $[z]$ denote the integer part z.

Proposition 3.5. If function $f \in C[0,\pi]$, then from a ratio

$$\lim_{\lambda\to\infty} Q_\lambda(f,[a,b],\varepsilon) = 0 \tag{3.6}$$

follows (3.3).

Proof of Proposition 3.5. We denote

$$\psi_{k,\lambda} = f(x_{k+1,\lambda}) - f(x_{k,\lambda}) \quad k_1 \le k \le k_2;\ \lambda > 0. \tag{3.7}$$

We take into account that we have the estimate

$$|\psi_{k,\lambda}| = |f(x_{k+1,\lambda}) - f(x_{k,\lambda})| \leq \omega\Big(f, \frac{\pi}{\sqrt{\lambda}}\Big) \quad \text{for all } k_1 \leq k \leq k_2; \lambda > 0. \quad (3.8)$$

We fix an arbitrary $x \in [a+\varepsilon, b-\varepsilon]$. Choose index $p = p(x,\lambda)$, so that $x \in [x_{p,\lambda}, x_{p+1,\lambda})$. Then $x = x_{p,\lambda} + \frac{\alpha\pi}{\sqrt{\lambda}}$, where $\alpha = \alpha(x,\lambda) \in [0,1)$

$$x - x_{k,\lambda} = \frac{p-k+\alpha}{\sqrt{\lambda}}\pi.$$

From (3.8) for all $x \in [a+\varepsilon, b-\varepsilon]$ we have the estimate

$$\Big|\sum_{\substack{k:k_1\leq k\leq k_2;\\ |p-k|\geq 3;}} \frac{(-1)^k\psi_{k,\lambda}}{p-k+\alpha} - \sum_{\substack{k:k_1\leq k\leq k_2;\\ |p-k|\geq 3;}} \frac{(-1)^k\psi_{k,\lambda}}{p-k}\Big| \leq$$

$$\omega\left(f, \frac{\pi}{\sqrt{\lambda}}\right) \sum_{\substack{k:k_1\leq k\leq k_2;\\ |p-k|\geq 3;}} \frac{\alpha}{|p-k|(|p-k|-1)} \leq \omega\Big(f, \frac{\pi}{\sqrt{\lambda}}\Big). \quad (3.9)$$

Notice, that if $h(\lambda) = \sqrt{\lambda}$, $q_\lambda \equiv 0$ solution of the Cauchy problem (2.4) is $y(x,\lambda) = \sin\sqrt{\lambda}x$.

We take into account (3.7). We decompose the sum in (3.4) as follows:

$$\frac{1}{2}\sum_{k=k_1}^{k_2}\big(f(x_{k+1,\lambda}) - f(x_{k,\lambda})\big)l_{k,\lambda}(x) + \frac{1}{2}\sum_{k\in[0,\lambda-1]\setminus[k_1,k_2]}\big(f(x_{k+1,\lambda}) - f(x_{k,\lambda})\big)l_{k,\lambda}(x) =$$

$$\frac{1}{2}\sum_{\substack{k:k_1\leq k\leq k_2;\\ |p-k|\geq 3;}} \psi_{k,\lambda}l_{k,\lambda}(x) + \frac{1}{2}\sum_{\substack{k:k_1\leq k\leq k_2;\\ |p-k|<3}} \psi_{k,\lambda}l_{k,\lambda}(x) + \frac{1}{2}\sum_{k\in[0,\lambda-1]\setminus[k_1,k_2]} \psi_{k,\lambda}l_{k,\lambda}(x). \quad (3.10)$$

Now, using the triangle inequality, of (3.7), (3.9) uniformly for all $x \in [a+\varepsilon, b-\varepsilon]$ the estimate

$$\Big|\frac{1}{2}\sum_{k=k_1}^{k_2}\big(f(x_{k+1,\lambda}) - f(x_{k,\lambda})\big)l_{k,\lambda}(x) - \frac{\sin\sqrt{\lambda}x}{2\pi}\sum_{k=k_1}^{k_2}{}' \frac{(-1)^k\psi_{k,\lambda}}{p-k}\Big| \leq$$

$$\frac{1}{2\pi}\Big|\sum_{k:|p-k|\geq 3} \frac{(-1)^k\psi_{k,\lambda}}{p-k+\alpha} - \sum_{k:|p-k|\geq 3}\frac{(-1)^k\psi_{k,\lambda}}{p-k}\Big| +$$

$$\frac{1}{2\pi}\sum_{k:|p-k|<3}\Big|\psi_{k,\lambda}l_{k,\lambda}(x)\Big| + \frac{1}{2\pi}\sum_{k:|p-k|<3}{}' \frac{|\psi_{k,\lambda}|}{|p-k|} \leq \frac{5}{\pi}\omega\Big(f, \frac{\pi}{\sqrt{\lambda}}\Big). \quad (3.11)$$

There are a constant C and number $n_0 \in \mathbb{N}$ independent of function $f \in C[0,\pi]$, $0 \leq a < b \leq \pi$ and $0 < \varepsilon < (b-a)/2$, such that for all $x \in [a+\varepsilon, b-\varepsilon]$ and $n > n_0$ the inequality is fair

Notes

$$\left|\frac{1}{2}\sum_{k\in[0,n-1]\setminus[k_1,k_2]}\psi_{k,\lambda}l_{k,\lambda}(x)\right| \leq \frac{\omega_1\left(f,\frac{\pi}{\sqrt{\lambda}}\right)}{2}\sum_{k\in[0,n-1]\setminus[k_1,k_2]}|l_{k,\lambda}(x)| \leq$$

$$C\omega_1\left(f,\frac{\pi}{\sqrt{\lambda}}\right)\ln\frac{2\pi}{\varepsilon}.$$

Notes

Thence, by (3.11) (3.4) we have for all $x\in[a+\varepsilon,b-\varepsilon]$ ratio

$$\lim_{n\to\infty}\quad f(x)-C_{\Omega}(f,x)-\frac{\sin\sqrt{\lambda}x}{2\pi}\sum_{k=k_1}^{k_2}{}'\frac{(-1)^k\psi_{k,\lambda}}{p-k}\Bigg)=0. \tag{3.12}$$

We estimate the last term in (3.12) by means of ratio (3.8) and triangle inequality

$$\left|\frac{\sin\sqrt{\lambda}x}{2\pi}\sum_{k=k_1}^{k_2}{}'\frac{(-1)^k\psi_{k,\lambda}}{p-k}\right|\leq 2\left|\frac{1}{2\pi}\sum_{m=m_1}^{m_2}{}'\frac{\psi_{2m,\lambda}}{p-2m}\right|+\left|\frac{1}{2\pi}\sum_{k=k_1}^{k_2}{}'\frac{\psi_{k,\lambda}}{p-k}\right|+O\left(\omega\left(f,\frac{1}{\sqrt{\lambda}}\right)\right). \tag{3.13}$$

By the continuity of f there exists a sequence of positive integers $\{l_n\}_{n=1}^{\infty}$, such that

$$l_n=o(n),\quad \lim_{n\to\infty}l_n=\infty,\quad \lim_{\lambda\to\infty}\omega\left(f,\frac{1}{\sqrt{\lambda}}\right)\sum_{k=1}^{l_n}\frac{1}{k}=0,\quad n:=[\lambda]. \tag{3.14}$$

We estimate the second sum in (3.13)

$$\left|\frac{1}{2\pi}\sum_{k=k_1}^{k_2}{}'\frac{\psi_{k,\lambda}}{p-k}\right|\leq\left|\frac{1}{2\pi}\sum_{k:|p-k|\leq l_n}{}'\frac{\psi_{k,\lambda}}{p-k}\right|+\left|\frac{1}{2\pi}\sum_{k:|p-k|>l_n}{}'\frac{\psi_{k,\lambda}}{p-k}\right|. \tag{3.15}$$

From here and inequalities (3.8) follows

$$\left|\frac{1}{2\pi}\sum_{k:|p-k|\leq l_n}{}'\frac{\psi_{k,\lambda}}{p-k}\right|\leq\frac{1}{2\pi}\sum_{k:|p-k|\leq l_n}{}'\left|\frac{\psi_{k,\lambda}}{p-k}\right|\leq\frac{1}{\pi}\omega\left(f,\frac{\pi}{\sqrt{\lambda}}\right)\sum_{k=1}^{l_n}\frac{1}{k}. \tag{3.16}$$

Hence by (3.15) after taking the Abel transform in case $k\in[k_1,k_2]:|p-k|>l_n$ we obtain the estimate

$$\left|\frac{1}{2\pi}\sum_{k:|p-k|>l_n}{}'\frac{\psi_{k,\lambda}}{p-k}\right|\leq\frac{4\|f\|_{C[a,b]}}{l_n+1}+4\|f\|_{C[a,b]}\sum_{k=l_n}^{\infty}\frac{1}{k(k+1)}.$$

Hence by (3.14), (3.15) and (3.16) we obtain the uniform estimate for all $x\in[a+\varepsilon,b-\varepsilon]$

$$\left|\frac{1}{2\pi}\sum_{k=k_1}^{k_2}{}'\frac{\psi_{k,\lambda}}{p-k}\right|=o(1). \tag{3.17}$$

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Notice, that if $h(\lambda) = \sqrt{\lambda}$, $q_\lambda \equiv 0$ solution of the Cauchy problem (2.4) is $y(x,\lambda) = \sin\sqrt{\lambda}x$. Then by (3.4), (3.5), (3.12), (3.13), (3.17) and triangle inequality we obtain the relation

$$\left|f(x) - C_\Omega(f,x)\right| \leq$$

$$\left|f(x) - C_\Omega(f,x) - \frac{\sin\sqrt{\lambda}x}{2\pi}\sum_{k=k_1}^{k_2}{}' \frac{(-1)^k\psi_{k,\lambda}}{p-k}\right| +$$

$$\left|\frac{1}{\pi}\sum_{m=m_1}^{m_2}{}' \frac{\psi_{2m,\lambda}}{p-2m}\right| + \left|\frac{1}{2\pi}\sum_{k=k_1}^{k_2}{}' \frac{\psi_{k,\lambda}}{p-k}\right| + O\left(\omega\left(f,\frac{1}{\sqrt{\lambda}}\right)\right) \leq$$

$$\frac{1}{\pi}Q_\lambda(f,[a,b],\varepsilon) + o(1).$$

From which it follows the sufficiency (3.6) for uniform convergence (3.3). Proposition 3.5 proved.

For all $0 \leq a < b \leq \pi$, $0 < \varepsilon < (b-a)/2$ denoted

$$Q^*_\lambda(f,[a,b],\varepsilon) := \max_{p_1\leq p\leq p_2}\sum_{m=m_1}^{m_2}{}' \left|\frac{f(x_{2m+1,\lambda}) - f(x_{2m,\lambda})}{p-2m}\right|. \tag{3.18}$$

Proposition 3.6. If function $f \in C[0,\pi]$, then the ratio of

$$\lim_{n\to\infty} Q^*_\lambda(f,[a,b],\varepsilon) = 0 \tag{3.19}$$

implies (3.3).

Proof. Indeed, by Proposition 3.5 satisfy the condition (3.19) implies truth of the saying (3.6) and therefore, the ratio (3.3).

Remark 3.7. Propositions 3.5 and 3.6 are analogues of known signs of A.A. Privalov uniform convergence of trigonometric polynomial and algebraic interpolations polynomial Lagrange with the matrix of interpolation nodes P.L. Chebyshev [33].

Proof of the Theorem 3.1 Let the function $v\ \ \omega$ satisfies the condition (3.2) and $f \in C(\omega^l[a,b]) \cap V^-(v)$. We show that the relation (3.19) is true. By virtue of the uniform continuity and boundedness of f, for any positive $\tilde{\epsilon}$ there exist natural numbers $\nu\ \ n_1$ such that for all $\lambda \geq n_1$ $(\lambda \in \mathbb{R})$ simultaneously take place two inequalities

$$\omega\left(f,\frac{\pi}{\sqrt{\lambda}}\right)\sum_{k=1}^{\nu}\frac{1}{k} < \frac{\tilde{\epsilon}}{6} \tag{3.20}$$

and

$$24\|f\|_{C[a,b]} < \tilde{\epsilon}\nu. \tag{3.21}$$

Notes

Let $\lambda \geq n_1$. We find p_0, depending on n, a, b, ε and f at which the maximum in the definition (3.18)

$$Q_\lambda^*(f,[a,b],\varepsilon) = \sum_{m=m_1}^{m_2}{}' \left| \frac{f(x_{2m+1,\lambda}) - f(x_{2m,\lambda})}{p_0 - 2m} \right|.$$

Notes

Assuming that

$$Q_\lambda^{**}(f,[a,b],\varepsilon) := \sum_{k=k_1}^{k_2}{}' \left| \frac{f(x_{k+1,\lambda}) - f(x_{k,\lambda})}{p_0 - k} \right|.$$

The value of $Q_\lambda^{**}(f,[a,b],\varepsilon)$ is obtained from $Q_\lambda^*(f,[a,b],\varepsilon)$ by the addition of non-negative terms, therefore is fair the inequality

$$Q_\lambda^*(f,[a,b],\varepsilon) \leq Q_\lambda^{**}(f,[a,b],\varepsilon). \tag{3.22}$$

We divide $Q_\lambda^{**}(f,[a,b],\varepsilon)$ into two terms

$$Q_\lambda^{**}(f,[a,b],\varepsilon) = \sum_{k=k_1}^{k_2}{}' \frac{f(x_{k+1,\lambda}) - f(x_{k,\lambda})}{|p_0 - k|} -$$

$$2\sum_{k=k_1}^{k_2}{}'' \frac{f(x_{k+1,\lambda}) - f(x_{k,\lambda})}{|p_0 - k|} = S_1(p_0) + S_2(p_0), \tag{3.23}$$

where two strokes mean that in the sum are absent non-negative summands and with index $k = p_0$.

First, we estimate the first sum. Representing it in the form

$$S_1(p_0) = \sum_{\substack{k\,:\,k\in[k_1,k_2],\\ 0<|p_0-k|<\nu}} \frac{f(x_{k+1,\lambda}) - f(x_{k,\lambda})}{|p_0 - k|} +$$

$$\sum_{\substack{k\,:\,k\in[k_1,k_2],\\ 0<|p_0-k|\geq\nu}} \frac{f(x_{k+1,\lambda}) - f(x_{k,\lambda})}{|p_0 - k|} = S_{1,1}(p_0) + S_{1,2}(p_0). \tag{3.24}$$

In the case $\{k : k \in [k_1,k_2], 0 < |p_0 - k| \geq \nu\} = \varnothing$ believe that the second term is zero.

From the inequality (3.20) have

$$|S_{1,1}(p_0)| \leq 2\omega\left(f, \frac{\pi}{\sqrt{\lambda}}\right) \sum_{k=1}^{\nu} \frac{1}{k} < \frac{\tilde{\epsilon}}{3}. \tag{3.25}$$

We now estimate the amount $S_{1,2}(p_0)$. If p_0 such that inequalities are fair $k_1 \leq p_0 - \nu < p_0 < p_0 + \nu \leq k_2$, then ratios take place $p_0 - k_1 \geq \nu$ $k_2 - p_0 \geq \nu$. Hence by (3.21), after taking the Abel transform we obtain estimate

$$|S_{1,2}(p_0)| \le \left|\sum_{k=k_1}^{p_0-\nu} \frac{f(x_{k+1,\lambda}) - f(x_{k,\lambda})}{p_0 - k}\right| + \left|\sum_{k=p_0+\nu}^{k_2} \frac{f(x_{k+1,\lambda}) - f(x_{k,\lambda})}{k - p_0}\right| \le$$

$$\left|\sum_{k=k_1}^{p_0-\nu-1} \frac{f(x_{k+1,\lambda}) - f(x_{k_1,\lambda})}{(p_0 - k)(p_0 - k - 1)}\right| + \left|\frac{f(x_{p_0-\nu+1,\lambda}) - f(x_{k_1,\lambda})}{p_0 - k_1}\right| +$$

$$\left|\sum_{k=p_0+\nu}^{k_2-1} \frac{f(x_{k+1,\lambda}) - f(x_{p_0+\nu,\lambda})}{(k - p_0)(k + 1 - p_0)}\right| + \left|\frac{f(x_{k_2,\lambda}) - f(x_{p_0+\nu,\lambda})}{k_2 - p_0}\right| \le$$

$$4\|f\|_{C[a,b]} \sum_{i=\nu}^{\infty} \frac{1}{i(i+1)} + \frac{4\|f\|_{C[a,b]}}{\nu} \le \frac{8\|f\|_{C[a,b]}}{\nu} < \frac{\tilde{\epsilon}}{3}. \tag{3.26}$$

Similarly we prove (3.26), if p_0 would be so, that will be inequality $p_0 - \nu < k_1 \le p_0 < p_0 + \nu \le k_2$ or inequality $k_1 \le p_0 - \nu < p_0 \le k_2 < p_0 + \nu$. Of the possible variant remained only when $p_0 - \nu < k_1 \le p_0 \le k_2 < p_0 + \nu$. In this situation, we have $|S_{1,2}(p_0)| = 0$.

From (3.24), (3.25) end (3.26) we obtain inequality

$$|S_1(p_0)| \le \frac{2\tilde{\epsilon}}{3} \tag{3.27}$$

for all $\lambda \ge n_1$.

Let's move on to the study of the properties of the sum $S_2(p_0)$. Take any integer $m : 1 \le m \le k_2 - k_1 - 2$ and represented $S_2(p_0)$ in the form

$$0 \le S_2(p_0) = -2 \sum_{\substack{k\,:\,k \in [k_1, k_2], \\ |p_0 - k| \le m}} {}'' \frac{f(x_{k+1,\lambda}) - f(x_{k,\lambda})}{|p_0 - k|} -$$

$$2 \sum_{\substack{k\,:\,k \in [k_1, k_2], \\ |p_0 - k| > m}} {}'' \frac{f(x_{k+1,\lambda}) - f(x_{k,\lambda})}{|p_0 - k|} =$$

$$S_{2,1}(p_0) + S_{2,2}(p_0). \tag{3.28}$$

Function $f \in C(\omega^l[a, b])$, therefore by definition (3.1) we have relation

$$f(x_{k+1,\lambda}) - f(x_{k,\lambda}) \ge -K_f \omega\left(\frac{\pi}{\sqrt{\lambda}}\right).$$

Therefore

$$0 \le S_{2,1}(p_0) = -2 \sum_{\substack{k\,:\,k \in [k_1, k_2], \\ |p_0 - k| \le m}} {}'' \frac{f(x_{k+1,\lambda}) - f(x_{k,\lambda})}{|p_0 - k|} \le$$

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$$4K_f\omega\Big(\frac{\pi}{\sqrt{\lambda}}\Big)\sum_{k=1}^{m}\frac{1}{k}. \tag{3.29}$$

We estimate the amount $S_{2,2}(p_0)$.

$$0 \le S_{2,2}(p_0) = -2 \sum_{\substack{k\,:\,k\in[k_1,k_2],\\ |p_0-k|>m}} {}'' \frac{f(x_{k+1,\lambda})-f(x_{k,\lambda})}{|p_0-k|} \le$$

$$2\sum_{k=k_1}^{p_0-m-1}\frac{-(f(x_{k+1,\lambda})-f(x_{k,\lambda}))_-}{p_0-k}+2\sum_{k=p_0+m+1}^{k_2}\frac{-(f(x_{k+1,\lambda})-f(x_{k,\lambda}))_-}{k-p_0}. \tag{3.30}$$

Note that $p_0-m\le k_1$ or $p_0+m\ge k_2$, then in (3.30) disappears respectively, the first or second term. In case $p_0-m<k_1<k_2<p_0+m$, sum $S_{2,2}(p_0)$ in (3.28) absent. Take into account that $f\in V(v)$. We will apply Abel's transformation in estimate (3.30)

$$0 \le S_{2,2}(p_0) \le$$

$$2\Bigg(\frac{\sum_{k=k_1}^{p_0-m-1}-(f(x_{k+1,\lambda})-f(x_{k,\lambda}))_-}{p_0-k_1}+\sum_{k=k_1+1}^{p_0-m-1}\frac{\sum_{j=k}^{p_0-m-1}-(f(x_{j+1,\lambda})-f(x_{j,\lambda}))_-}{(p_0-k)(p_0-k+1)}+$$

$$\frac{\sum_{k=p_0+m+1}^{k_2}-(f(x_{k+1,\lambda})-f(x_{k,\lambda}))_-}{k_2-p_0}+\sum_{k=p_0+m+1}^{k_2-1}\frac{\sum_{j=p_0+m+1}^{k}-(f(x_{j+1,\lambda})-f(x_{j,\lambda}))_-}{(p_0-k)(p_0-k-1)}\Bigg) \le$$

$$2\Bigg(\frac{\big((p_0-k_1)-m-1\big)K_f\omega\Big(\frac{\pi}{\sqrt{\lambda}}\Big)}{p_0-k_1}+M_f\sum_{k=k_1+1}^{p_0-m-1}\frac{v(p_0-m-k)}{(p_0-k)(p_0-k+1)}+$$

$$\frac{\big((k_2-p_0)-m-1\big)K_f\omega\Big(\frac{\pi}{\sqrt{\lambda}}\Big)}{k_2-p_0}+M_f\sum_{k=p_0+m+1}^{k_2-1}\frac{v(k-p_0-m)}{(p_0-k)(p_0-k-1)}\Bigg) \le$$

$$2M_f\Bigg(\sum_{k=m+1}^{p_0-k_1-1}\frac{v(k-m)}{k(k+1)}+\sum_{k=m+1}^{k_2-p_0-1}\frac{v(k-m)}{k(k+1)}\Bigg)+4K_f\omega\Big(\frac{\pi}{\sqrt{\lambda}}\Big) \le$$

$$4M_f\sum_{k=m+1}^{k_2-k_1-1}\frac{v(k)}{k^2}+4K_f\omega\Big(\frac{\pi}{\sqrt{\lambda}}\Big).$$

Hence (3.28), (3.29) and (3.30) we have

$$0 \leq S_2(p_0) \leq 4K_f\omega\Big(\frac{\pi}{\sqrt{\lambda}}\Big)\sum_{k=1}^{m}\frac{1}{k} + 4M_f\sum_{k=m+1}^{k_2-k_1-1}\frac{v(k)}{k^2} + 4K_f\omega\Big(\frac{\pi}{\sqrt{\lambda}}\Big).$$

Conditions (3.2), due to the non-negativity of both summands, equivalent to

$$\lim_{n\to\infty}\min_{1\leq m\leq k_2-k_1-1}\max\Big\{\omega\Big(\frac{\pi}{\sqrt{\lambda}}\Big)\sum_{k=1}^{m}\frac{1}{k}, \sum_{k=m+1}^{k_2-k_1-1}\frac{v(k)}{k^2}\Big\} = 0.$$

Therefore exists an $n_2 \in \mathbb{N}, n_2 \geq n_1$, that for avery $\lambda \geq n_2$ there are $m : 1 \leq m \leq k_2 - k_1 - 1$ for which the inequality

$$0 \leq S_2(p_0) \leq \frac{\tilde{\epsilon}}{3}. \tag{3.31}$$

As result of by (3.22), (3.23), (3.24), (3.27) and (3.31) we get that for any $\tilde{\epsilon} > 0$ exists an $n_2 \in \mathbb{N}$, that for every $\lambda > n_2 > n_1$ there exists an $m : 1 \leq m \leq k_2 - k_1 - 2$, that performed the inequalities

$$Q^*_\lambda(f, [a, b], \varepsilon) \leq Q^{**}_\lambda(f, [a, b], \varepsilon) < \tilde{\epsilon}.$$

Now Theorem 3.1 follows from Proposition 3.6.

To prove the theorem 3.1 if $f \in C(\omega^r[a, b]) \cap V^+(v)$ is sufficient to note that if $f \in C(\omega^r[a, b]) \cap V^+(v)$, then $-f \in C(\omega^l[a, b]) \cap V^-(v)$ and operator $C_\Omega(f, \cdot)$ — linear. Theorem 3.1 proved.

Remark 3.8. In the case when $f \in C(\omega^l[a, b]) \cap V(v)$ or $f \in C(\omega^r[a, b]) \cap V(v)$ (v is the majorant classic module change $v(n, f)$) in [33] *proved that the conditions of the form* (3.2) *are sufficient for the uniform convergence of trigonometric interpolation processes and sequences of classical Lagrange interpolation polynomials with the matrix of interpolation nodes P.L. Chebyshev.*

The paper [34] *set uniform convergence of trigonometric Fourier series for the $2\pi-$periodic, functions of the class $f \in C(\omega[a, b]) \cap V(v)$, where functions ω v are majorants classical modulus of continuity $\omega(f, \delta)$ and module changes $v(n, f)$.*

Remark 3.9. From Theorem 3.1 it follows that if $f_1 \in C(\omega^r_1[a, b]) \cap V^+(v_1)$, and $f_2 \in C(\omega^l_2[a, b]) \cap V^-(v_2)$, and the two pairs of functions (v_i, ω_i), where $i = 1, 2$, satisfy the relation (3.2)*, that, although a linear combination of $f = \alpha f_1 + \beta f_2$ can non-belong to any of classes, however because of the linearity of the operator $C_\Omega(f, \cdot)$, will have the relate* (3.3)*.*

Remark 3.10. Each of the classes of functions: Dini-Lipschitz $\lim\limits_{n\to\infty}\omega(f, 1/n)\ln n =$ 0 *(see.,* [20, Corollary 2]*), and satisfying the condition of Krylov (continuous function of bounded variation), is a subset of functional class, described by the terms* (3.2)*.*

Remark 3.11. If $f \in C[0, \pi]$, there are the relations

$$v^+(n, f) \leq v(n, f) \leq 2\left(v^+(n, f) + \|f\|_{C[0,\pi]}\right),$$

$$-v^-(n, f) \leq v(n, f) \leq 2\left(-v^-(n, f) + \|f\|_{C[0,\pi]}\right).$$

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Corollary 3.12. *From Theorem 3.1 follow that* $\lim_{n\to\infty} \omega^l(f,1/n)\ln n = 0$ *or* $\lim_{n\to\infty} \omega^r(f,1/n)\ln n = 0$ *ensure fairness* (3.3).

Corollary 3.13. *If a non-decreasing, concave function of natural argument* v *such that*

$$\sum_{k=1}^{\infty} \frac{v(k)}{k^2} < \infty, \tag{3.32}$$

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then for any function $f \in C[0,\pi] \cap V^{\pm}(v)$ *is true ratio* (3.3).

Proof. Indeed, from the continuity of f implies the existence of a sequence of positive integers $\{m_n\}_{n=1}^{\infty}$ such that $\lim_{n\to\infty} m_n = \infty$ and $\lim_{n\to\infty} \omega(f,1/n)\ln m_n = 0$. Therefore, the convergence of series (3.32) ensures that the condition (3.2) for any function f, belonging to at one the classes of $C[0,\pi] \cap V^+(v)$ or $C[0,\pi] \cap V^-(v)$. The proof is complete.

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Soliton-Like Solutions for Some Nonlinear Evolution Equations through the Generalized Kudryashov Method

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Abstract- In this present article, we apply the generalized Kudryashov method for constructing ample new exact traveling wave solutions of the (2+1)-dimensional Breaking soliton (BS) equation, (2+1)-dimensional Burgers equation and (2+1)-dimensional Boussinesq equation. We attain successfully numerous new exact traveling wave solutions. This method is candid and concise, and it can be also applied to other nonlinear evolution equations in mathematical physics and engineering sciences. Moreover, some of the newly attained exact solutions are demonstrated graphically.

Keywords: generalized kudryashov method, BS equation, burgers equation, boussinesq equation, NLEEs.

GJSFR-F Classification: MSC 2010: 34A34

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Soliton-Like Solutions for Some Nonlinear Evolution Equations through the Generalized Kudryashov Method

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Abstract- In this present article, we apply the generalized Kudryashov method for constructing ample new exact traveling wave solutions of the (2+1)-dimensional Breaking soliton (BS) equation, (2+1)-dimensional Burgers equation and (2+1)-dimensional Boussinesq equation. We attain successfully numerous new exact traveling wave solutions. This method is candid and concise, and it can be also applied to other nonlinear evolution equations in mathematical physics and engineering sciences. Moreover, some of the newly attained exact solutions are demonstrated graphically.

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I. Introduction

At the present time, investigating exact solutions of nonlinear evolution equations (NLEEs) are largely used as models to characterize physical phenomena in several fields of science and engineering, especially in biology, solid state physics, plasma, physics and fluid mechanics. Ultimately all the fundamental equations of physics are nonlinear and in general it's very complicate to solve explicitly these types of NLEEs. To solve the inherent nonlinear problems advance nonlinear techniques are very momentous; for the most part of those are involving dynamical system and related areas. Nonetheless, in the last few decades important development has been made and many influential methods for attaining exact solutions of NLEEs have been recommended in the works. Most of the methods found in the literature include, the tanh-sech method [1], simplest equation method [2],the homotopy perturbation method [3,4], Modified method of simplest equation [5,6], Bäcklund Transformations method [7],the (G'/G)-expansion method [8-13], the generalized Kudryashov Method [14,15], the Exp-function method [16,17], the $\exp(-\Phi(\xi))$-expansion method [18], the modified simple equation method [19], Improved F-expansion method [20-23] and so on.

In this article, we would like to discuss further (2+1)-dimensional Breaking Soliton equation, (2+1)-dimensional Burgers equation and (2+1)-dimensional Boussinesq equation by the generalized Kudryashov method. Consequently, more new exact traveling wave solutions have found through these three NLEEs. The (2+1)-dimensional Boussinesq describe the propagation of long waves in shallow water under gravity propagating in both directions. It also arises in other physical applications Such as nonlinear lattice waves, iron sound waves in plasma, and in vibrations in a nonlinear string. The Burgers equation is one of the fundamental model equations in fluid

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mechanics. It is also used to describe the structure of shock waves, traffic flow, and acoustic transmission. Burgers equation is completely integrable. The wave solutions of Burgers equation are single and multiple-front solutions.

The plan of this paper is as follows. In Sec. 2, we designate momentarily the generalized Kudryashov method. In Sec. 3, we apply the method to (2+1) -dimensional breaking soliton equation, (2+1)-dimensional Burgers equation and (2+1)-dimensional Boussinesq equation. In sec. 4, graphical representation of particular attained solutions and in sec. 5 Conclusions will be presented finally.

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II. Algorithm of the Generalized Kudryashov Method

In this segment, we elect the generalized Kudryashovmethod looking for the exact traveling wave solutions of some NLEEs.

We consider the NLEEs of the form

$$\Psi(u,\frac{\delta u}{\delta t},\frac{\delta u}{\delta x},\frac{\delta u}{\delta y},\frac{\delta u}{\delta z},\frac{\delta^2 u}{\delta x^2},\frac{\delta^2 u}{\delta y^2},\frac{\delta^2 u}{\delta z^2},\cdots)=0\,,x\in\Psi\,,t>0\,, \tag{1}$$

where $u=u(x,y,z,t)$ is an unfamiliar function, Ψ is a polynomial in u and its innumerable partial derivatives, in which the highest order derivatives and nonlinear terms are engaged. The generalized Kudryashov method carries the following steps [24].

Step 1: The traveling wave transformation $u(x,y,t)=u(\eta),\eta=x+y-ct$ transform Eq. (1) into an ordinary differential equation

$$\mathrm{T}(u,\frac{du}{d\eta},\frac{d^2u}{d\eta^2},\cdots)=0\,, \tag{2}$$

Step 2: Assume that the solution of Eq. (3) has the following form

$$u(\eta)=\frac{\sum_{i=0}^{N}a_iQ^i(\eta)}{\sum_{j=0}^{M}b_jQ^j(\eta)}\,, \tag{3}$$

where $a_i\,(i=0,1,2,\ldots,N)$ and $b_j\,(j=0,1,2,\ldots,M)$ are constants to be determined later such $a_N\neq 0$ and $b_M\neq 0$, and $Q=Q(\eta)$ satisfies the ordinary differential equation

$$\frac{dQ(\eta)}{d\eta}=Q^2(\eta)-Q(\eta)\,. \tag{4}$$

The solutions of Eq. (4) are as follows:

$$Q(\eta)=\frac{1}{1\pm A\exp(\eta)}\,. \tag{5}$$

Step 3: Using the homogeneous balance method between the highest order derivatives and the nonlinear terms in Eq. (2),determine the positive integer numbers *N* and *M* in Eq. (3).

Step 4: Substituting Eqs. (3) and (4) into Eq. (2), we find a polynomial in Q^{i-j}, $(i,j=0,1,2,\cdots)$. In this polynomial equating all terms of same power and equating them

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to zero, we get a system of algebraic equations which can be solved by the Maple or Mathematica to get the unknown parameters $a_i\,(i=0,1,2,\ldots,N)$ and $b_j\,(j=0,1,2,\ldots,M)$, ω. Consequently, we obtain the exact solutions of Eq. (1).

III. Applications

a) The (2+1)-dimensional Breaking Soliton (BS) equation

In this subsection, we will implement the generalized Kudryashov method look for the exact solutions of the BS equation.

Let us consider the (2+1)-dimensional BS equation

$$u_{xxxy}-2u_y u_{xx}-4u_x u_{xy}+u_{xt}=0. \tag{6}$$

We apply the traveling wave transformation of the form

$$u(\eta)=u(x,y,t), \qquad \eta=x+y-ct, \tag{7}$$

The wave transformation (7) reduces Eq. (6) into the following ordinary differential equation

$$u^{iv}-6u'u''-cu''=0, \tag{8}$$

Integrating Eq. (8) with respect to η and neglecting the constant of integration, we obtain

$$u'''-3(u')^2-cu'=0, \tag{9}$$

Balancing homogeneously between the highest order nonlinear term $(u')^2$ and the derivative term u''' in Eq. (9), we attain

$$N=M+1$$

If we choose $M=1$ then $N=2$

Hence for $M=1$ and $N=2$ Eq. (3) reduces to

$$u(\eta)=\frac{a_0+a_1Q+a_2Q^2}{b_0+b_1Q}, \tag{10}$$

Where a_0, a_1, a_2, b_0 and b_1 are constants to be determined.

Now substituting Eq. (10) into Eq. (9), we get a polynomial in $Q(\eta)$, equating the coefficient of same power of $Q(\eta)$, we attain the following system of algebraic equations:

$$-6a_2b_1^3+3a_2^2b_1^2=0,$$

$$-6a_2^2b_1^2-24a_0b_0b_1^2+12a_2b_1^3+12a_2^2b_0b_1=0,$$

$$3a_2^2b_1^2-6a_0a_2b_1^2+48a_2b_0b_1^2+6a_1a_2b_0b_1-7a_2b_1^3-36a_2b_0^2b_1+12a_2^2b_0^2$$
$$+ca_2b_1^3-24a_2^2b_0b_1=0,$$

$$12a_2^2b_0b_1+a_2b_1^3-ca_2b_1^3+12a_1a_2b_0^2-12a_1a_2b_0b_1+12a_0a_2b_1^2-12a_0a_2b_0b_1-24a_2b_0^3$$
$$+72a_2b_0^2b_1+4ca_2b_0b_1^2-24a_2^2b_0^2-28a_2b_0b_1^2=0,$$

$$6a_0b_0^2b_1-6a_1b_0^2b_1+54a_2b_0^3-6a_0a_1b_0b_1+24a_0a_2b_0b_1+6a_1a_2b_0b_1+12a_2^2b_0^2+6a_0b_0b_1^2$$
$$+4a_2b_0b_1^2-41a_2b_0^2b_1-4ca_2b_0b_1^2+a_0b_1^3-ca_0b_1^3-24a_1a_2b_0^2-6a_0a_2b_1^2$$
$$+3a_0^2b_1^2-a_1b_0b_1^2+3a_1^2b_0^2+ca_1b_0b_1^2+5ca_2b_0^2b_1-6a_1b_0^3=0,$$

$$10a_1b_0^2b_1-6a_1^2b_0^2+12a_0a_1b_0b_1+2ca_1b_0^2b_1-ca_1b_0b_1^2-12a_0a_2b_0b_1+a_1b_0b_1^2-2ca_0b_0b_1^2$$
$$-38a_2b_0^3+12a_1a_2b_0^2-a_0b_1^3+5a_2b_0^2b_1-6a_0^2b_1^2-10a_0b_0b_1^2-5ca_2b_0^2b_1-12a_0b_0^2b_1$$
$$+ca_0b_1^3+12a_1b_0^3+2ca_2b_0^3=0,$$

$$-2ca_2b_0^3+ca_1b_0^3+3a_0^2b_1^2-7a_1b_0^3-2ca_1b_0^2b_1-6a_0a_1b_0b_1+8a_2b_0^3+3a_1^2b_0^2+7a_0b_0^2b_1$$
$$+4a_0b_0b_1^2-4a_1b_0^2b_1-ca_0b_0^2b_1+2ca_0b_0b_1^2=0,$$

$$-a_0b_0^2b_1-ca_1b_0^3+a_1b_0^3+ca_0b_0^2b_1=0.$$

Solving the above system of equations for a_0, a_1, a_2, b_0, b_1 and c, we attain the following values:

Set 1: $c=1, a_1=2b_0, a_2=0, b_1=0.$

Set 2: $c=1, a_0=\dfrac{b_0(-2b_1-2b_0+a_1)}{b_1}, a_2=0.$

Set 3: $c=1, a_0=\dfrac{b_0(a_1-2b_0)}{b_1}, a_2=2b_1.$

Set 4: $c=4, a_0=-0.50a_1, a_2=-4b_0, b_1=-2b_0.$

Set 1 Corresponds the following solutions for Breaking Soliton (BK) equation

$$u_1(\mu)=\frac{a_0+a_0A\exp(\eta)+2b_0}{(1+A\exp(\eta))b_0},$$

where $\eta=x+y-t$.

Set 2 Corresponds the following solutions for Breaking Soliton (BK) equation

$$u_2(\eta)=\frac{-2b_0b_1-2b_0b_1A\exp(\eta)-2b_0^2-2b_0^2A\exp(\eta)+a_1b_0+a_1b_0A\exp(\eta)+a_1b_1}{(b_0+b_0A\exp(\eta)+b_1)b_1}$$

where $\eta=x+y-t$.

Set 3 Corresponds the following solutions for Breaking Soliton (BK) equation

$$u_3(\eta)=\frac{a_1A\exp(\eta)-2b_0A\exp(\eta)-2b_0+a_1+2b_1}{(1+A\exp(\eta))b_1},$$

where $\eta=x+y-t$.

Set 4 Corresponds the following solutions for Breaking Soliton (BK) equation

$$u_4(\eta)=\frac{1}{2}\frac{a_1-a_1A^2\exp(2\eta)-8b_0}{(A^2\exp(2\eta)-1)b_0},$$

Notes

where $\eta = x + y - 4t$.

Remark: All of these solutions have been verified with Maple by substituting them into the original solutions.

b) The (2+1)-dimensional Burgers equation

In this subsection, we will construct the generalized Kudryashov method to find the exact traveling wave solutions of the Burgers equation. Let us consider the (2+1)-dimensional Burgers equation [25]

$$u_t - uu_x - u_{xx} - u_{yy} = 0, \tag{11}$$

Burgers equation arises in various areas of applied mathematics, such as modeling of gas dynamics and various vehicle densities in high way traffic [26].The wave transformation (7) reduces Eq. (11) into the following ordinary differential equations

$$cu' + uu' + 2u'' = 0, \tag{12}$$

Integrating Eq. (12) with respect to ξ and neglecting the constant of integration, we obtain

$$cu + \frac{u^2}{2} + 2u' = 0, \tag{13}$$

Considering the homogeneous balance between the highest order nonlinear term u^2 and the derivative term u' in Eq. (13), we attain

$$N = M + 1.$$

If we choose $M = 1$ then $N = 2$
Hence for $M = 1$ and $N = 2$ Eq. (3) reduces to

$$u(\eta) = \frac{a_0 + a_1 Q + a_2 Q^2}{b_0 + b_1 Q}, \tag{14}$$

Where a_0, a_1, a_2, b_0 and b_1 are constants to be determined.

Now substituting Eq. (14) into Eq. (13), we get a polynomial in $Q(\eta)$, equating the coefficient of same power of $Q(\eta)$, we attain the following system of algebraic equations:

$$4a_2 b_1 + a_2^2 = 0,$$

$$2ca_2 b_1 + 2a_1 a_2 + 8a_2 b_0 - 4a_2 b_1 = 0,$$

$$-4a_0 b_1 + 4a_1 b_0 + a_1^2 + 2ca_1 b_1 + 2ca_2 b_0 + 2a_0 a_2 - 8a_2 b_0 = 0,$$

$$2ca_0 b_1 + 4a_0 b_1 - 4a_1 b_0 + 2ca_1 b_0 + 2a_0 a_1 = 0,$$

$$a_0^2 + 2ca_0 b_0 = 0.$$

Ref

26. Polyanin, A. D. and Zaitsev, V. F. (2004) Handbook of Nonlinear Partial Differential Equations, Chapman & Hall/CRC, Boca Raton.

Solving the above system of equations for a_0, a_1, a_2, b_0, b_1 and c, we attain the following values:

Set 1: $c = 4, a_0 = 0, a_1 = 0, a_2 = -4b_1, b_0 = -0.50b_1$.

Set 2: $c = 2, a_0 = 0, a_1 = -4b_0 - 4b_1, a_2 = 0$.

Set 3: $c = 2, a_0 = 0, a_1 = -4b_0, a_2 = -4b_1$.

Set 4: $c = -2, a_0 = 4b_0, a_1 = -4b_0, a_2 = 0$.

Set 5: $c = -2, a_0 = -a_1 + 4b_1, a_1 = -4b_1, b_0 = b_1 - 0.25a_1$.

Set 6: $c = -4, a_0 = -4b_1, a_1 = 8b_1, a_2 = -4b_1, b_0 = -0.50b_1$.

Set 1 Corresponds the following solutions for Burgers equations

$$u_1(\eta) = \frac{8}{A^2 \exp(2\eta) - 1},$$

Where $\eta = x + y - 4t$.

Set 2 Corresponds the following solutions for Burgers equations

$$u_2(\eta) = -\frac{4(b_0 + b_1)}{b_0 + b_0 A \exp(\eta) + b_1},$$

Where $\xi = x + y - 2t$.

Set 3 Corresponds the following solutions for Burgers equations

$$u_3(\eta) = -\frac{4}{1 + A \exp(\eta)},$$

Where $\eta = x + y - 2t$.

Set 4 Corresponds the following solutions for Burgers equations

$$u_4(\eta) = \frac{4b_0 A \exp(\eta)}{b_0 + b_0 A \exp(\eta) + b_1},$$

Where $\mu = x + y + 2t$.

Set 5 Corresponds the following solutions for Burgers equations

$$u_5(\eta) = \frac{4A \exp(\eta)}{1 + A \exp(\eta)},$$

Where $\eta = x + y + 2t$.

Set 6 Corresponds the following solutions for Burgers equations

$$u_6(\eta) = \frac{8A^2 \exp(2\eta)}{A^2 \exp(2\eta) - 1},$$

Where $\eta = x + y + 4t$.

Notes

Remark: All of these solutions have been verified with Maple by substituting them into the original solutions.

c) The (2+1)-dimensional Boussinesq equation

In this subsection, we will use the generalized Kudryashov method to find the exact traveling wave solutions of the Boussinesq equation. Let us consider the (2+1)-dimensional Boussinesq equation [27] is in the form

$$u_{tt} - u_{xx} - u_{yy} - \left(u^2\right)_{xx} - u_{xxxx} = 0\,, \tag{15}$$

which describes the propagation of gravity waves on the surface of water. The wave transformation (7) reduces Eq. (15) into the following ordinary differential equations

$$\left(c^2 - 2\right)u'' - \left(u^2\right)'' - u^{iv} = 0\,, \tag{16}$$

Integrating Eq. (16) with respect to η and neglecting the constant of integration, we obtain

$$\left(c^2 - 2\right)u - u^2 - u'' = 0\,, \tag{17}$$

Considering the homogeneous balance between the highest order nonlinear term u^2 and the derivative term u'' in Eq. (13), we attain $N = M + 2$.

If we choose $M = 1$ then $N = 3$

Hence for $M = 1$ and $N = 3$ Eq. (3) reduces to

$$u(\eta) = \frac{a_0 + a_1 Q + a_2 Q^2 + a_3 Q^3}{b_0 + b_1 Q}, \tag{18}$$

where a_0, a_1, a_2, a_3, b_0 and b_1 are constants to be determined.

Now substituting Eq. (18) into Eq. (17), we get a polynomial in $Q(\eta)$, equating the coefficient of same power of $Q(\eta)$, we attain the following system of algebraic equations:

$$a_3^2 b_1 + 6a_3 b_1^2 = 0,$$

$$a_3^2 b_0 + 2a_2 a_3 b_1 + 16a_3 b_0 b_1 + 2a_2 b_1^2 - 10a_3 b_1^2 = 0,$$

$$12a_3 b_0^2 - 3a_2 b_1^2 + 6a_3 b_1^2 + 2a_2 a_3 b_0 + a_2^2 b_1 - 27a_3 b_0 b_1 + 6a_2 b_0 b_1 + 2a_1 a_3 b_1 - c^2 a_3 b_1^2 = 0,$$

$$a_2^2 b_0 + 3a_2 b_1^2 + 15a_3 b_0 b_1 - 2c^2 a_3 b_0 b_1 + 2a_0 a_3 b_1 + 6a_2 b_0^2 + 2a_1 a_3 b_0 - 21a_3 b_0^2 - c^2 a_2 b_1^2$$
$$- 9a_2 b_0 b_1 + 2a_1 a_2 b_1 = 0,$$

$$7a_2 b_0 b_1 + 11a_3 b_0^2 - a_0 b_1^2 - c^2 a_3 b_0^2 - c^2 a_1 b_1^2 + a_1^2 b_1 + 2a_1 a_2 b_0 + 2a_1 b_0^2 - 2c^2 a_2 b_0 b_1 + 2a_1 b_1^2$$
$$+ 2a_0 a_3 b_0 - 10a_2 b_0^2 - 2a_0 b_0 b_1 + 2a_0 a_2 b_1 + a_1 b_0 b_1 = 0,$$

$$2a_0 a_1 b_1 + 2a_0 a_2 b_0 + 3a_0 b_0 b_1 - c^2 a_2 b_0^2 - 2c^2 a_1 b_0 b_1 - c^2 a_0 b_1^2 + a_1^2 b_0 + 3a_0 b_1^2 - 3a_1 b_0^2$$
$$+ 3a_1 b_0 b_1 + 6a_2 b_0^2 = 0,$$

$$3a_0 b_0 b_1 + a_0^2 b_1 + 3a_1 b_0^2 - 2c^2 a_0 b_0 b_1 - c^2 a_1 b_0^2 + 2a_0 a_1 b_0 = 0,$$

$$2a_0 b_0^2 + a_0^2 b_0 - c^2 a_0 b_0^2 = 0.$$

Ref

27. Moleleki, L. D. and Khalique, C. M. (2013) Solutions and conservation laws of a (2+1) dimensional Boussinesq equation, Abstract and Applied Analysis, 2013, 8.

Solving the above system of equations for $a_0, a_1, a_2\, a_3, b_0, b_1$ and c, we attain the following values:

Set 1: $c = \pm\sqrt{3}, a_0 = 0, a_1 = 6b_0, a_2 = -6b_0 + 6b_1$.

Set 2: $c = \pm 1, a_0 = -b_0, a_1 = -b_1 + 6b_0, a_2 = -6b_0 + 6b_1$.

Set 1 Corresponds the following solutions for Boussinesq equation

$$u_1(\eta) = \frac{6A\exp(\eta)}{(1 + A\exp(\eta))^2},$$

where $\eta = x + y + \sqrt{3}t$.

Set 2 Corresponds the following solutions for Boussinesq equation

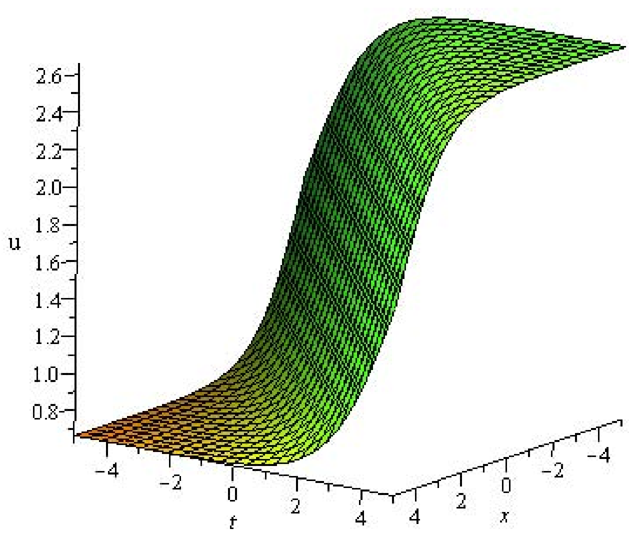
$$u_2(\eta) = -\frac{1 - 4A\exp(\eta) + A^2\exp(2\eta)}{(1 + A\exp(\eta))^2}.$$

where $\eta = x + y + t$.

Remark: All of these solutions have been verified with Maple by substituting them into the original solutions.

IV. Graphical Representation of Some Obtained Solutions

The graphical presentations of some obtained solutions are depicted in Figures 1–7 with the aid ofcommercial software Maple 13.



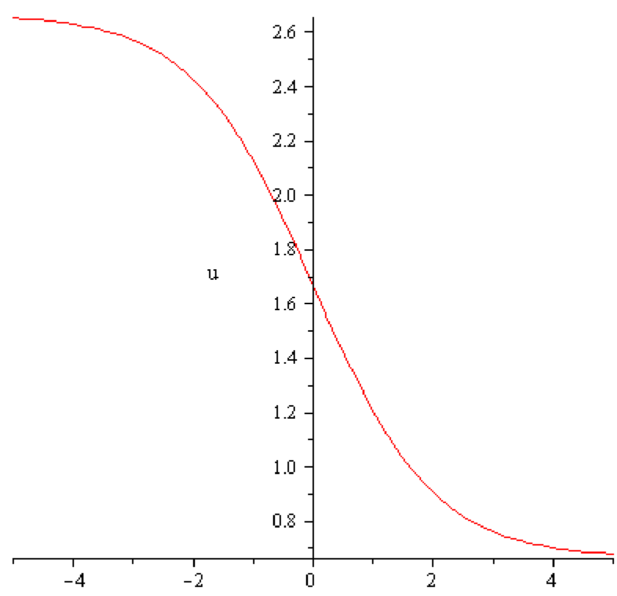


Fig. 1: Kink shaped soliton of BS equation for $A = 1, a_0 = 2, b_0 = 2, y = 0$ within the interval $-5 \le x, t \le 5$. (Only shows the shape of $u_1(\eta)$), the left figure shows the 3D plot and the right figure shows the 2D plot for $t = 0$

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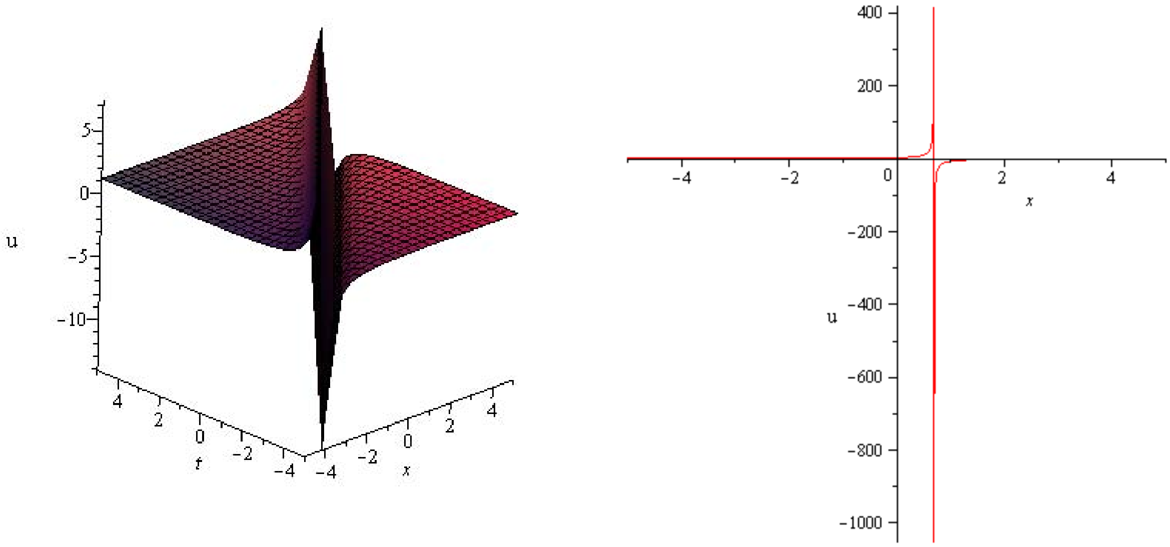

Fig. 2: Singular kink soliton of BK equation for $A=-0.50, a_1=2, b_0=3, b_1=5, y=0$ within the interval $-5\le x,t\le 5$. (Only shows the shape of $u_3(\eta)$), the left figure shows the 3D plot and the right figure shows the 2D plot for $t=0$

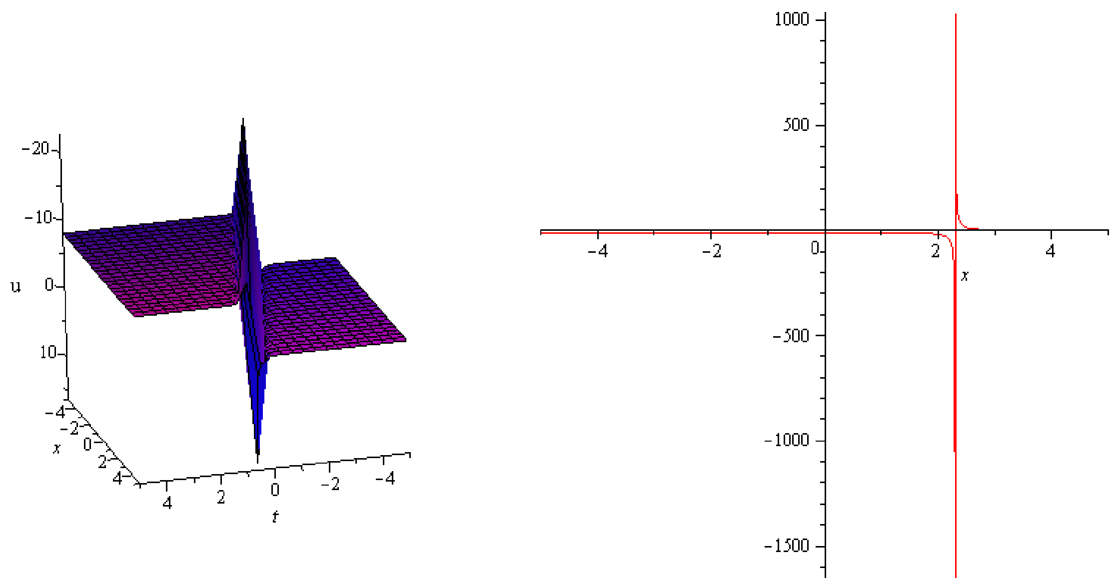

Fig. 3: Singular kink soliton of Burgers equation for $A=-0.10$, $y=0$ within the interval $-5\le x,t\le 5$. (Only shows the shape of $u_1(\eta)$), the left figure shows the 3D plot and the right figure shows the 2D plot for $t=0$

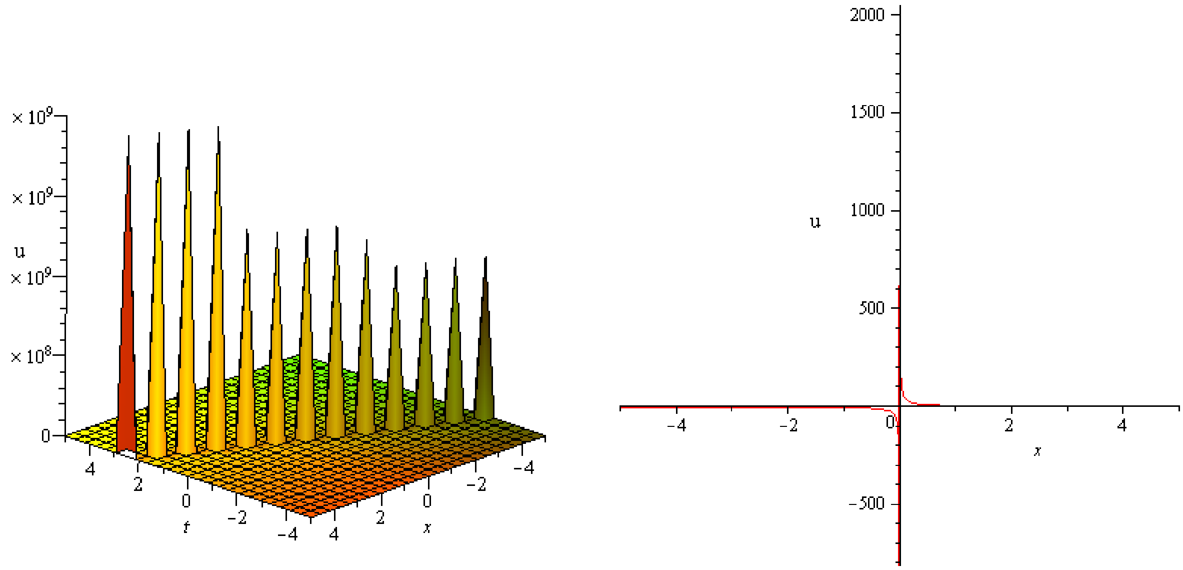

Fig. 4: Singular soliton of Burgers equation for $A=-1,\ y=0$ within the interval $-5\le x,t\le 5$. (Only shows the shape of $u_3(\eta)$), the left figure shows the 3D plot and the right figure shows the 2D plot for $t=0$

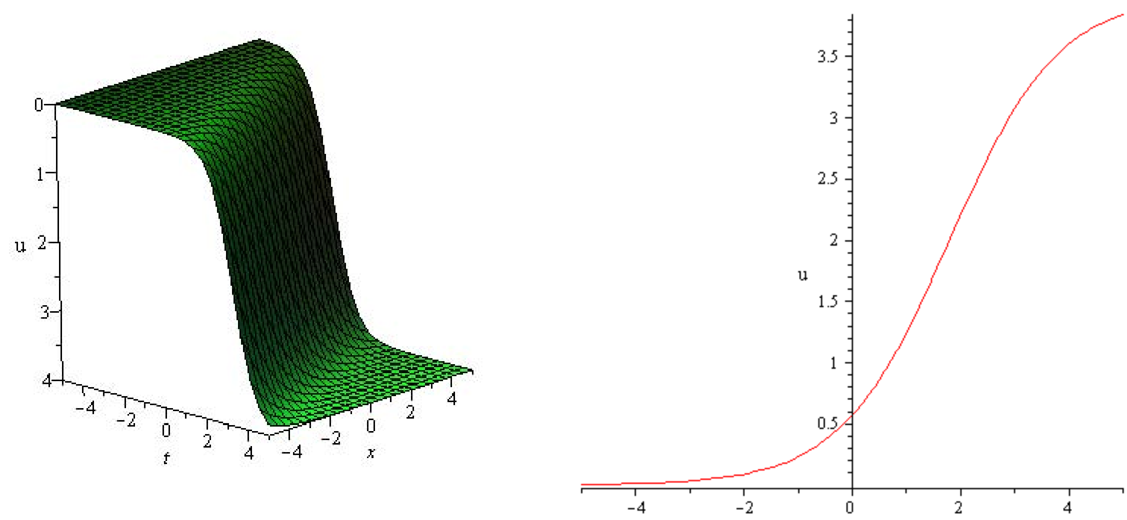

Fig. 5: Kink shaped soliton of Burgers equation for $A=0.50,\ b_0=1, b_1=2,\ y=0$ within the interval $-5\le x,t\le 5$. (Only shows the shape of $u_4(\eta)$), the left figure shows the 3D plot and the right figure shows the 2D plot for $t=0$

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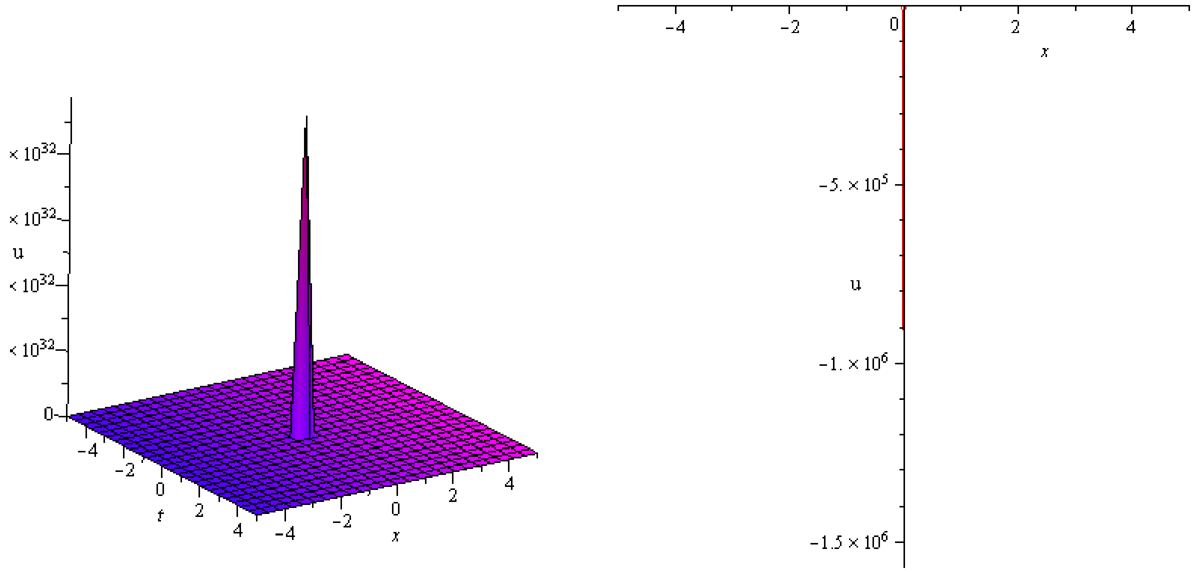

Fig. 6: Single soliton of Boussines q equation for $A=-1$, $y=0$ within the interval $-5\le x,t\le 5$. (Only shows the shape of $u_1(\eta)$), the left figure shows the 3D plot and the right figure shows the 2D plot for $t=0$

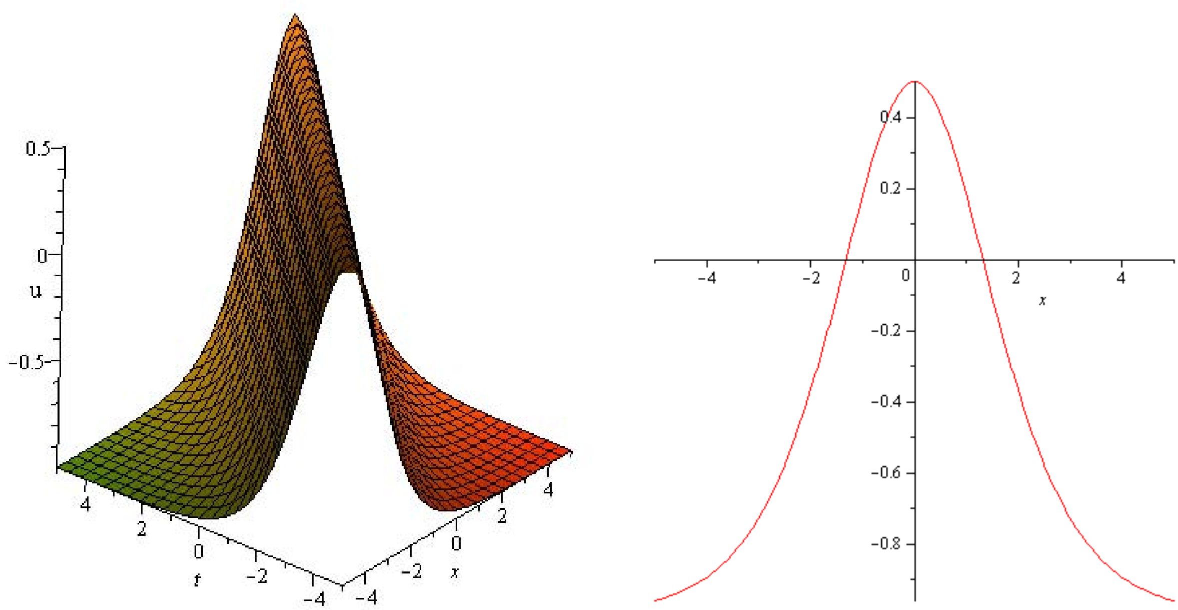

Fig. 7: Bell shaped solitonof Boussinesq equation for $A=1$, $y=0$ within the interval $-5\le x,t\le 5$. (Only shows the shape of $u_2(\eta)$), the left figure shows the 3D plot and the right figure shows the 2D plot for $t=0$

V. Conclusions

In this article, using the MAPLE 13 software the generalized Kudryashov method is executed to investigate the nonlinear evolution equations, namely (2+1)-dimensional Breaking soliton (BS) equation, (2+1)-dimensional Burgers equation, (2+1)-dimensional Boussinesq equation. All the attained solutions in this study verified these three NLEEs; we checked this using the MAPLE 13 software. Moreover, the obtained results in this work clearly demonstrate the reliability of the generalized Kudryashov method. This method can be more successfully applied to study nonlinear evolution equations, which frequently arise in nonlinear sciences.

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Notes

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Twin Prime Number Theorem

By Yin Yue Sha
Zhejiang University

Abstract- Let Pt(N) be the number of twin primes less than or equal to N, Pi (3≤Pi≤Pm) be taken over the odd primes less than or equal to √N, then exists the formula as follows:

Pt(N) ≥ INT { N×(1−1/2)×∏ (1−2/Pi) } − 2
≥ INT { Ct×2N/(Ln (N)) ^ 2 } − 2
Pt(N) ≥ INT { 0.660×2N/(Ln (N)) ^ 2 } − 2 ≥ 0.660×2N/(Ln (N)) ^ 2 − 3
∏ (Pi(Pi−2)/(Pi−1) ^ 2) ≥ Ct = 0.6601618158…

Where the INT { } expresses the taking integer operation of formula spread out type in { }.

Keywords: *twin prime, bilateral sieve method.*

GJSFR-F Classification: *MSC 2010: 70A05, 70E55, 70G10*

TWINPRIMENUMBERTHEOREM

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Notes

Twin Prime Number Theorem

Yin Yue Sha

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Pt(N) ⩾ INT { N×(1−1/2)×Π (1−2/Pi) } − 2
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Keywords: twin prime, bilateral sieve method.

I. The Twin Prime Number

There exists a prime P for which the Twin number Q=2+P is also prime. The Twin Primes shall be denoted by the representation 2=Q−P=(2+P)−P, where P and Q are primes and prime P{P<Q} is a Twin prime of even integer 2. Looking at the Twin partition a different way, we can look at the number of distinct representations (or Twin primes) that exist for 2.

For example, as noted at the beginning of this discussion:

$$2 = 05 - 03 = (2+03) - 03;\ 2 = 07 - 05 = (2+05) - 05;$$
$$2 = 13 - 11 = (2+11) - 11;\ 2 = 19 - 17 = (2+17) - 17;$$

where 3, 5, 11, and 17 are Twin primes of even integer 2.

II. The Sieve Method about the Twin Primes

The 2 is an even integer, Ti is a positive integer less than or equal to N, then exists the formula as follows:

$$2 = (2+\mathrm{Ti}) - \mathrm{Ti} \qquad (1)$$

where Ti and 2+Ti are two positive integers less than or equal to N+2.

If Ti and 2+Ti any one can be divided by the prime anyone more not large than √(N+2), then sieves out the positive integer Ti; If both Pt and 2+Pt can not be

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divided by all primes more not large than √(N+2), then both the Pt and 2+Pt are primes at the same time, where the prime Pt is a Twin prime of even integer 2.

III. The Total of Representations

The 2 is an even integer, then exists the formula as follows:

$$2 = (2+Ti)-Ti$$

where Ti is a positive integer less than or equal to N.

In terms of the above formula we can obtain the array as follows:

$$(2+1,\ 1),\ (2+2,\ 2),\ (2+3,\ 3),\ (2+4,\ 4),\ (2+5,\ 5),\cdots,\ (2+N,\ N).$$

From the above arrangement we can obtain the formula about the total of Twin numbers of even integer 2 as follows:

$$Ti(N) = N = \text{Total of integers Ti more not large than N} \tag{2}$$

IV. The Bilateral Sieve Method of Even Prime 2

It is known that the number 2 is an even prime, and above arrangement from (2+1, 1) to (2+N, N) can be arranged to the form as follows:

$$(2+1,\ 1),\ (2+3,\ 3),\ (2+5,\ 5),\ \ldots,\ (2+N-X{:}X< 2,\ N-X{:}X< 2).$$

$$(2+2,\ 2),\ (2+4,\ 4),\ (2+6,\ 6),\ \ldots,\ (2+N-X{:}X< 2,\ N-X{:}X< 2),$$

From the above arrangement we can known that: Because the even integer 2 can be divided by the even prime 2, therefore, both Ti and 2+Ti can be or can not be divided by the even prime 2 at the same time.

The number of integers Ti that Ti and 2+Ti anyone can be divided by the even prime 2 is:

$$\mathrm{INT}\ (\ N\times(1/2)\ ).$$

The number of integers Ti that both Ti and 2+Ti can not be divided by the even prime 2 is:

$$N-\mathrm{INT}(N\times(1/2))=\mathrm{INT}\{\ N-N\times(1/2)\}=\mathrm{INT}\{\ N\times(1-1/2)\} \tag{3}$$

Where the INT { } expresses the taking integer operation of formula spread out type in { }.

V. The Bilateral Sieve Method of Odd Prime 3

It is known that the number 3 is an odd prime, and above arrangement from (2+1, 1) to (2+N, N) can be arranged to the form as follows:

$$(2+1,\ 1),\ (2+4,\ 4),\ (2+7,\ 7),\ \ldots,\ (2+N-X{:}X< 3,\ N-X{:}X< 3),$$

$$(2+2,\ 2),\ (2+5,\ 5),\ (2+8,\ 8),\ \ldots,\ (2+N-X{:}X< 3,\ N-X{:}X< 3),$$

$$(2+3,\ 3),\ (2+6,\ 6),\ (2+9,\ 9),\ \ldots,\ (2+N-X{:}X< 3,\ N-X{:}X< 3).$$

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From the above arrangement we can known that:

The even integer 2 can not be divided by the odd prime 3, then both Ti and 2+Ti can not be divided by the odd prime 3 at the same time, that is the Ti and 2+Ti only one can be divided or both the Ti and 2+Ti can not be divided by the odd prime 3.

The number of integers Ti that the Ti and 2+Ti anyone can be divided by the odd prime 3 is: INT(N×(2/3)).

The number of integers Ti that both the Ti and 2+Ti can not be divided by the odd prime 3 is:

$$N-INT(N\times(2/3))=INT\{N-N\times(2/3)\}=INT\{N\times(1-2/3)\} \tag{4}$$

Where the INT { } expresses the taking integer operation of formula spread out type in { }.

VI. The Sieve Function of Bilateral Sieve Method

The 2 is an even integer, then exists the formula as follows:

$$2 = (2 + Ti) - Ti$$

where Ti is the natural integer less than or equal to N.
In terms of the above formula we can obtain the array as follows:

$$(2+1,\ 1),\ (2+2,\ 2),\ (2+3,\ 3),\ (2+4,\ 4),\ (2+5,\ 5),\ \ldots,\ (2+N,\ N).$$

Let Pi be an odd prime less than or equal to √(N+2), then the above arrangement can be arranged to the form as follows:

$$\begin{array}{c}
(2+1,\ 1),\ (2+Pi+1,\ Pi+1),\ \ldots,\ (2+N-X{:}X< Pi,\ N-X{:}X< Pi),\\
(2+2,\ 2),\ (2+Pi+2,\ Pi+2),\ \ldots,\ (2+N-X{:}X< Pi,\ N-X{:}X< Pi),\\
(2+3,\ 3),\ (2+Pi+3,\ Pi+3),\ \ldots,\ (2+N-X{:}X< Pi,\ N-X{:}X< Pi),\\
\ldots\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots\\
(2+Pi,\ Pi),\ (2+2Pi,\ 2Pi),\ \ldots,\ (2+N-X{:}X< Pi,\ N-X{:}X< Pi).
\end{array}$$

The even integer 2 can not be divided by the odd prime Pi, then both the Ti and 2+Ti can not be divided by the odd prime Pi at the same time, that is the Ti and 2+Ti only one can be divided or both the Ti and 2+Ti can not be divided by the odd prime Pi.

The number of integers Ti that the Ti and 2+Ti anyone can be divided by the odd prime Pi is INT(N×(2/Pi)).

The number of integers Ti that both the Ti and 2+Ti can not be divided by the odd prime Pi is

$$N-INT\ (N\times(2/Pi)\ )\ =\ INT\ \{\ N-N\times(2/Pi)\ \}\ =\ INT\{\ N\times(1-2/Pi)\ \} \tag{5}$$

Let Pt(N) be the number of Twin Primes less than or equal to (N+2), Pi be taken over the odd primes less than or equal to √(N+2), then exists the formulas as follows:

$$Pt(N) \geqslant INT\{ N\times(1-1/2)\times\prod(1-2/Pi) \} - 2 \quad (6)$$

Where the INT { } expresses the taking integer operation of formula spread out type in { }.

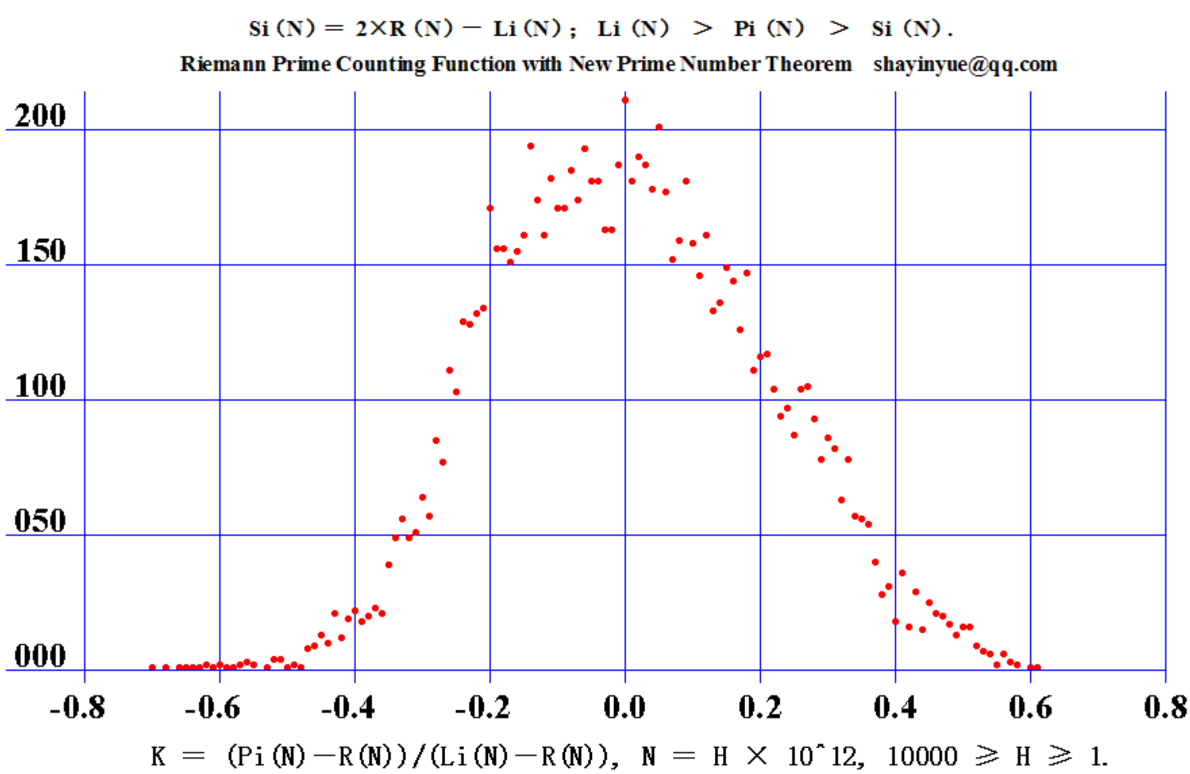
VII. New Prime Number Theorem

Let Pi(N) be the number of primes less than or equal to N, Pi $(3\leqslant Pi\leqslant Pm)$ be taken over the odd primes less than or equal to √N, then exists the formulas as follows:

$$Pi(N \mid N\geqslant 10\hat{}4) = INT\{ N\times(1-1/2)\times\prod(1-1/Pi) + m + 1 \} - 1 \quad (7)$$

$$\geqslant INT\{N\times(1-1/2)\times\prod(1-1/Pi)\} \geqslant INT\{N/Ln(N)\} \quad (8)$$

$$Pi(N) = R(N) + K\times(Li(N) - R(N)),\ 1 > K > -1. \quad (9)$$



Figure

VIII. The Twin Prime Theorem

From above we can obtain that:

Let Pt(N) be the number of Twin Primes less than or equal to (N+2), $Pi(3\leqslant Pi\leqslant Pm)$ be taken over the odd primes less than or equal to √(N+2), then exists the formulas as follows:

Notes

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$$Pt(N) \geqslant INT\ \{\ N\times(1-1/2)\times\prod (1-2/Pi)\ \} - 2 \tag{10}$$

Apply the Prime Number Theorem, from above formula we can obtain the formula as follows:

$$Pt(N \mid N\geqslant 10\hat{}4) \geq INT\ \{\ N\times(1-1/2)\times\prod (1-2/Pi)\ \} - 2$$

$$\geqslant INT\ \{Ct\times 2N/(Ln\ (N))\hat{}2\} - 2 \tag{11}$$

$$\geqslant Ct\times 2N/(Ln(N))\hat{}2 - 3 \tag{12}$$

$$\prod (Pi(Pi-2)/(Pi-1)\hat{}2) \geqslant Ct = 0.6601618158\ldots \tag{13}$$

When the number $N \to \infty$, we can obtain the formula as follows:

$$Pt(N \mid N\to\infty)\geqslant 0.660\times 2N/(Ln\ (N))\hat{}2 - 3 \to \infty \tag{14}$$

IX. Conclusion

There are infinitely many pairs of Twin primes which difference by 2.

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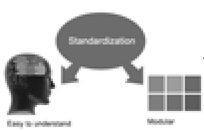

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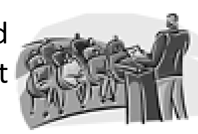

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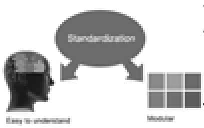

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- Do not present the similar data more than once.
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- Never confuse figures with tables - there is a difference.

Approach

- As forever, use past tense when you submit to your results, and put the whole thing in a reasonable order.
- Put figures and tables, appropriately numbered, in order at the end of the report
- If you desire, you may place your figures and tables properly within the text of your results part.

Figures and tables

- If you put figures and tables at the end of the details, make certain that they are visibly distinguished from any attach appendix materials, such as raw facts
- Despite of position, each figure must be numbered one after the other and complete with subtitle
- In spite of position, each table must be titled, numbered one after the other and complete with heading
- All figure and table must be adequately complete that it could situate on its own, divide from text

Discussion:

The Discussion is expected the trickiest segment to write and describe. A lot of papers submitted for journal are discarded based on problems with the Discussion. There is no head of state for how long a argument should be. Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implication of the study. The purpose here is to offer an understanding of your results and hold up for all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of result should be visibly described. Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved with prospect, and let it drop at that.

- Make a decision if each premise is supported, discarded, or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."
- Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work
- You may propose future guidelines, such as how the experiment might be personalized to accomplish a new idea.
- Give details all of your remarks as much as possible, focus on mechanisms.
- Make a decision if the tentative design sufficiently addressed the theory, and whether or not it was correctly restricted.
- Try to present substitute explanations if sensible alternatives be present.
- One research will not counter an overall question, so maintain the large picture in mind, where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

- When you refer to information, differentiate data generated by your own studies from available information
- Submit to work done by specific persons (including you) in past tense.
- Submit to generally acknowledged facts and main beliefs in present tense.

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	A-B	**C-D**	**E-F**
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Introduction	Containing all background details with clear goal and appropriate details, flow specification, no grammar and spelling mistake, well organized sentence and paragraph, reference cited	Unclear and confusing data, appropriate format, grammar and spelling errors with unorganized matter	Out of place depth and content, hazy format
Methods and Procedures	Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads	Difficult to comprehend with embarrassed text, too much explanation but completed	Incorrect and unorganized structure with hazy meaning
Result	Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake	Complete and embarrassed text, difficult to comprehend	Irregular format with wrong facts and figures
Discussion	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

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