Global Journal of Science Frontier Research: F
Mathematics and Decision Sciences
Volume 18 Issue 6 Version 1.0 Year 2018
Type: Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Certain Fractional Derivative Formulae Involving the Product of a General Class of Polynomials and the Multivariable Gimel-Function

By Frédéric Ayant

Abstract- In the present paper, we obtain three unified fractional derivative formulae. The first involves the product of a general class of polynomials and the multivariable Gimel-function. The second involves the product of a general class of polynomials and two multivariable Gimel-functions and has been obtained with the help of the generalized Leibniz rule for fractional derivatives. The last fractional derivative formulae also involves the product of a general class of polynomials and the multivariable Gimel-function but it is obtained by the application of the first fractional derivative formulae twice and, it involve two independents variables instead of one. The polynomials and the functions involved in all our fractional derivative formulae as well as their arguments which are of the type $x^{\rho}\prod_{i=1}^{s}(x^{t_j}+\alpha_j)^{\sigma_j}$ The formulae are the very general character and thus making them useful in applications. In the end, we shall give a particular case.

Keywords: multivariable gimel-function, riemann-liouville and erdélyi-kober fractional operators, general class of polynomials, fractional derivative formulae, generalized leibnitz rule.

GJSFR-F Classification: *FOR Code: 33C99, 33C60, 44A20*

CERTAINFRACTIONALDERIVATIVEFORMULAEINVOLVINGTHEPRODUCTOFAGENERALCLASSOFPOLYNOMIALSANDTHEMULTIVARIABLEGIMELFUNCTION

Strictly as per the compliance and regulations of:

Notes

Certain Fractional Derivative Formulae Involving the Product of a General Class of Polynomials and the Multivariable Gimel-Function

Frédéric Ayant

Abstract- In the present paper, we obtain three unified fractional derivative formulae. The first involves the product of a general class of polynomials and the multivariable Gimel-function. The second involves the product of a general class of polynomials and two multivariable Gimel-functions and has been obtained with the help of the generalized Leibniz rule for fractional derivatives. The last fractional derivative formulae also involves the product of a general class of polynomials and the multivariable Gimel-function but it is obtained by the application of the first fractional derivative formulae twice and, it involve two independents variables instead of one. The polynomials and the functions involved in all our fractional derivative formulae as well as their arguments which are of the type $x^{\rho}\prod_{i=1}^{s}(x^{t_j}+\alpha_j)^{\sigma_j}$. The formulae are the very general character and thus making them useful in applications. In the end, we shall give a particular case.

Keywords: *multivariable gimel-function, riemann-liouville and erdélyi-kober fractional operators, general class of polynomials, fractional derivative formulae, generalized leibniz rule.*

I. Introduction And Preliminaries

Throughout this paper, let $\mathbb{C}, \mathbb{R}$ and $\mathbb{N}$ be set of complex numbers, real numbers and positive integers respectively. Also $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$. We define a generalized transcendental function of several complex variables.

$$\gimel(z_1,\cdots,z_r) = \gimel^{0,n_2;0,n_3;\cdots;0,n_r:m^{(1)},n^{(1)};m^{(2)},n^{(2)};\cdots;m^{(r)},n^{(r)}}_{p_{i_2},q_{i_2},\tau_{i_2};R_2;p_{i_3},q_{i_3},\tau_{i_3};R_3;\cdots;p_{i_r},q_{i_r},\tau_{i_r}:R_r:p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)};\cdots;p_{i^{(r)}},q_{i^{(r)}};\tau_{i^{(r)}};R^{(r)}}\left(\begin{array}{c|c} \mathrm{z}_1 & \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_r & \end{array}\right.$$

$$\begin{array}{c}[(\mathrm{a}_{2j};\alpha^{(1)}_{2j},\alpha^{(2)}_{2j};A_{2j})]_{1,n_2},[\tau_{i_2}(a_{2ji_2};\alpha^{(1)}_{2ji_2},\alpha^{(2)}_{2ji_2};A_{2ji_2})]_{n_2+1,p_{i_2}};[(a_{3j};\alpha^{(1)}_{3j},\alpha^{(2)}_{3j},\alpha^{(3)}_{3j};A_{3j})]_{1,n_3},\\ [\tau_{i_2}(b_{2ji_2};\beta^{(1)}_{2ji_2},\beta^{(2)}_{2ji_2};B_{2ji_2})]_{1,q_{i_2}};\end{array}$$

$$\begin{array}{l}[\tau_{i_3}(a_{3ji_3};\alpha^{(1)}_{3ji_3},\alpha^{(2)}_{3ji_3},\alpha^{(3)}_{3ji_3};A_{3ji_3})]_{n_3+1,p_{i_3}};\cdots;[(\mathrm{a}_{rj};\alpha^{(1)}_{rj},\cdots,\alpha^{(r)}_{rj};A_{rj})_{1,n_r}],\\ [\tau_{i_3}(b_{3ji_3};\beta^{(1)}_{3ji_3},\beta^{(2)}_{3ji_3},\beta^{(3)}_{3ji_3};B_{3ji_3})]_{1,q_{i_3}};\cdots;\qquad\qquad \cdots\end{array}$$

$$\begin{array}{l}[\tau_{i_r}(a_{rji_r};\alpha^{(1)}_{rji_r},\cdots,\alpha^{(r)}_{rji_r};A_{rji_r})_{n_r+1,p_r}]:\ [(\mathrm{c}^{(1)}_j,\gamma^{(1)}_j;C^{(1)}_j)_{1,n^{(1)}}],[\tau_{i^{(1)}}(c^{(1)}_{ji^{(1)}},\gamma^{(1)}_{ji^{(1)}};C^{(1)}_{ji^{(1)}})_{n^{(1)}+1,p^{(1)}_i}]\\ [\tau_{i_r}(b_{rji_r};\beta^{(1)}_{rji_r},\cdots,\beta^{(r)}_{rji_r};B_{rji_r})_{1,q_r}]:\ [(\mathrm{d}^{(1)}_j),\delta^{(1)}_j;D^{(1)}_j)_{1,m^{(1)}}],[\tau_{i^{(1)}}(d^{(1)}_{ji^{(1)}},\delta^{(1)}_{ji^{(1)}};D^{(1)}_{ji^{(1)}})_{m^{(1)}+1,q^{(1)}_i}]\end{array}$$

$$\left.\begin{array}{l};\cdots;[(c^{(r)}_j,\gamma^{(r)}_j;C^{(r)}_j)_{1,m^{(r)}}],[\tau_{i^{(r)}}(c^{(r)}_{ji^{(r)}},\gamma^{(r)}_{ji^{(r)}};C^{(r)}_{ji^{(r)}})_{m^{(r)}+1,p^{(r)}_i}]\\ ;\cdots;[(d^{(r)}_j,\delta^{(r)}_j;D^{(r)}_j)_{1,n^{(r)}}],[\tau_{i^{(r)}}(d^{(r)}_{ji^{(r)}},\delta^{(r)}_{ji^{(r)}};D^{(r)}_{ji^{(r)}})_{n^{(r)}+1,q^{(r)}_i}]\end{array}\right)$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)z_k^{s_k}\,\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.1}$$

with $\omega = \sqrt{-1}$

Author: *Teacher in High School, France. e-mail: fredericayant@gmail.com*

$$\psi(s_1,\cdots,s_r)=\frac{\prod_{j=1}^{n_2}\Gamma^{A_{2j}}(1-a_{2j}+\sum_{k=1}^{2}\alpha_{2j}^{(k)}s_k)}{\sum_{i_2=1}^{R_2}[\tau_{i_2}\prod_{j=n_2+1}^{p_{i_2}}\Gamma^{A_{2ji_2}}(a_{2ji_2}-\sum_{k=1}^{2}\alpha_{2ji_2}^{(k)}s_k)\prod_{j=1}^{q_{i_2}}\Gamma^{B_{2ji_2}}(1-b_{2ji2}+\sum_{k=1}^{2}\beta_{2ji2}^{(k)}s_k)]}$$

$$\frac{\prod_{j=1}^{n_3}\Gamma^{A_{3j}}(1-a_{3j}+\sum_{k=1}^{3}\alpha_{3j}^{(k)}s_k)}{\sum_{i_3=1}^{R_3}[\tau_{i_3}\prod_{j=n_3+1}^{p_{i_3}}\Gamma^{A_{3ji_3}}(a_{3ji_3}-\sum_{k=1}^{3}\alpha_{3ji_3}^{(k)}s_k)\prod_{j=1}^{q_{i_3}}\Gamma^{B_{3ji_3}}(1-b_{3ji3}+\sum_{k=1}^{3}\beta_{3ji3}^{(k)}s_k)]}$$

. . . .

. . . .

. . . .

$$\frac{\prod_{j=1}^{n_r}\Gamma^{A_{rj}}(1-a_{rj}+\sum_{k=1}^{r}\alpha_{rj}^{(k)}s_k)}{\sum_{i_r=1}^{R_r}[\tau_{i_r}\prod_{j=n_r+1}^{p_{i_r}}\Gamma^{A_{rji_r}}(a_{rji_r}-\sum_{k=1}^{r}\alpha_{rji_r}^{(k)}s_k)\prod_{j=1}^{q_{i_r}}\Gamma^{B_{rji_r}}(1-b_{rjir}+\sum_{k=1}^{r}\beta_{rjir}^{(k)}s_k)]} \tag{1.2}$$

and

$$\theta_k(s_k)=\frac{\prod_{j=1}^{m^{(k)}}\Gamma^{D_j^{(k)}}(d_j^{(k)}-\delta_j^{(k)}s_k)\prod_{j=1}^{n^{(k)}}\Gamma^{C_j^{(k)}}(1-c_j^{(k)}+\gamma_j^{(k)}s_k)}{\sum_{i^{(k)}=1}^{R^{(k)}}[\tau_{i^{(k)}}\prod_{j=m^{(k)}+1}^{q_{i^{(k)}}}\Gamma^{D_{ji^{(k)}}^{(k)}}(1-d_{ji^{(k)}}^{(k)}+\delta_{ji^{(k)}}^{(k)}s_k)\prod_{j=n^{(k)}+1}^{p_{i^{(k)}}}\Gamma^{C_{ji^{(k)}}^{(k)}}(c_{ji^{(k)}}^{(k)}-\gamma_{ji^{(k)}}^{(k)}s_k)]} \tag{1.3}$$

For more details, see Ayant [2].

The contour L_k is in the $s_k(k=1,\cdots,r)$- plane and run from $\sigma-i\infty$ to $\sigma+i\infty$ where σ if is a real number with loop, if necessary to ensure that the poles of $\Gamma^{A_{2j}}\left(1-a_{2j}+\sum_{k=1}^{2}\alpha_{2j}^{(k)}s_k\right)(j=1,\cdots,n_2),\Gamma^{A_{3j}}\left(1-a_{3j}+\sum_{k=1}^{3}\alpha_{3j}^{(k)}s_k\right)$ $(j=1,\cdots,n_3),\cdots,\Gamma^{A_{rj}}\left(1-a_{rj}+\sum_{i=1}^{r}\alpha_{rj}^{(i)}\right)(j=1,\cdots,n_r),\quad \Gamma^{C_j^{(k)}}\left(1-c_j^{(k)}+\gamma_j^{(k)}s_k\right)(j=1,\cdots,n^{(k)})(k=1,\cdots,r)$to the right of the contour L_k and the poles of $\Gamma^{D_j^{(k)}}\left(d_j^{(k)}-\delta_j^{(k)}s_k\right)(j=1,\cdots,m^{(k)})(k=1,\cdots,r)$ lie to the left of the contour L_k. The condition for absolute convergence of multiple Mellin-Barnes type contour (1.1) can be obtained of the corresponding conditions for multivariable H-function given by as :

$|arg(z_k)|<\frac{1}{2}A_i^{(k)}\pi$ where

$$A_i^{(k)}=\sum_{j=1}^{m^{(k)}}D_j^{(k)}\delta_j^{(k)}+\sum_{j=1}^{n^{(k)}}C_j^{(k)}\gamma_j^{(k)}-\tau_{i^{(k)}}\left(\sum_{j=m^{(k)}+1}^{q_i^{(k)}}D_{ji^{(k)}}^{(k)}\delta_{ji^{(k)}}^{(k)}+\sum_{j=n^{(k)}+1}^{p_i^{(k)}}C_{ji^{(k)}}^{(k)}\gamma_{ji^{(k)}}^{(k)}\right)$$

$$-\tau_{i_2}\left(\sum_{j=n_2+1}^{p_{i_2}}A_{2ji_2}\alpha_{2ji_2}^{(k)}+\sum_{j=1}^{q_{i_2}}B_{2ji_2}\beta_{2ji_2}^{(k)}\right)-\cdots-\tau_{i_r}\left(\sum_{j=n_r+1}^{p_{i_r}}A_{rji_r}\alpha_{rji_r}^{(k)}+\sum_{j=1}^{q_{i_r}}B_{rji_r}\beta_{rji_r}^{(k)}\right) \tag{1.4}$$

Following the lines of Braaksma ([3] p. 278), we may establish the asymptotic expansion in the following convenient form :

$\aleph(z_1,\cdots,z_r)=0(|z_1|^{\alpha_1},\cdots,|z_r|^{\alpha_r}),max(|z_1|,\cdots,|z_r|)\rightarrow 0$

$\aleph(z_1,\cdots,z_r)=0(|z_1|^{\beta_1},\cdots,|z_r|^{\beta_r}),min(|z_1|,\cdots,|z_r|)\rightarrow\infty$ where $i=1,\cdots,r$:

$$\alpha_i=\min_{1\leqslant j\leqslant m^{(i)}}Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right]\text{ and }\beta_i=\max_{1\leqslant j\leqslant n^{(i)}}Re\left[C_j^{(i)}\left(\frac{c_j^{(i)}-1}{\gamma_j^{(i)}}\right)\right]$$

Ref

2. F. Ayant, An expansion formula for multivariable Gimel-function involving generalized Legendre Associated function, International Journal of Mathematics Trends and Technology (IJMTT), 56(4) (2018), 223-228.

Remark 1.
If $n_2 = \cdots = n_{r-1} = p_{i_2} = q_{i_2} = \cdots = p_{i_{r-1}} = q_{i_{r-1}} = 0$ and $A_{2j} = A_{2ji_2} = B_{2ji_2} = \cdots = A_{rj} = A_{rji_r} = B_{rji_r} = 1$ $A_{rj} = A_{rji_r} = B_{rji_r} = 1$, then the multivariable Gimel-function reduces in the multivariable Aleph- function defined by Ayant [1].

Remark 2.
If $n_2 = \cdots = n_r = p_{i_2} = q_{i_2} = \cdots = p_{i_r} = q_{i_r} = 0$ and $\tau_{i_2} = \cdots = \tau_{i_r} = \tau_{i^{(1)}} = \cdots = \tau_{i^{(r)}} = R_2 = \cdots = R_r = R^{(1)} = \cdots = R^{(r)} = 1$, then the multivariable Gimel-function reduces in a multivariable I-function defined by Prathima et al. [9].

Remark 3.
If $A_{2j} = A_{2ji_2} = B_{2ji_2} = \cdots = A_{rj} = A_{rji_r} = B_{rji_r} = 1$ and $\tau_{i_2} = \cdots = \tau_{i_r} = \tau_{i^{(1)}} = \cdots = \tau_{i^{(r)}} = R_2 = \cdots = R_r = R^{(1)} = \cdots = R^{(r)} = 1$, then the generalized multivariable Gimel-function reduces in multivariable I-function defined by Prasad [8].

Remark 4.
If the three above conditions are satisfied at the same time, then the generalized multivariable Gimel-function reduces in the multivariable H-function defined by Srivastava and Panda [15,16].

Srivastava ([14],p. 1, Eq. 1) has defined the general class of polynomials

$$S_N^M(x) = \sum_{K=0}^{[N/M]} \frac{(-N)_{MK}}{K!} A_{N,K} x^K \tag{1.5}$$

On suitably specializing the coefficients $A_{N,K}$, $S_N^M(x)$ yields some of known polynomials, these include the Jacobi polynomials, Laguerre polynomials, and others polynomials ([17],p. 158-161).

We shall define the fractional integrals and derivatives of a function $f(x)$ ([11], p. 528-529), see also [5-7] as follows :

Let α, β and γ be complex numbers. The fractional integral $(Re(\alpha) > 0)$ and derivative $(Re(\alpha) < 0)$ of a function $f(x)$ defined on $(0, \infty)$ is given by

$$I_{0,x}^{\alpha,\beta,\gamma} f(x) = \left[\begin{array}{c} \frac{x^{-\alpha-\beta}}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} F\left(\alpha+\beta, -\gamma; \alpha; 1-\frac{t}{x}\right) f(t)\mathrm{d}t (Re(\alpha) > 0). \\ . \\ \frac{d^q}{dx^q} \mathrm{I}_{0,x}^{\alpha+q,\beta-q,\gamma-q} f(x), (Re(\alpha) \leqslant 0, 0 < Re(\alpha) + q \leqslant 1, (q = 1,2,3,\cdots .), \end{array} \right. \tag{1.6}$$

where F is the Gauss hypergeometric serie.

The operator I includes both the Riemann-Liouville and the Erdélyi-Kober fractional operators as follows :

The Riemann-Liouville operator

$$R_{0,x}^{\alpha} f(x) = \left[\begin{array}{c} \mathrm{I}_{0,x}^{\alpha,-\alpha,\gamma} f(x) = \frac{1}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} f(t)\mathrm{d}t (Re(\alpha) > 0). \\ . \\ \frac{d^q}{dx^q} \mathrm{R}_{0,x}^{\alpha+q} f(x), (Re(\alpha) \leqslant 0, 0 < Re(\alpha) + q \leqslant 1, (q = 1,2,3,\cdots .), \end{array} \right. \tag{1.7}$$

The Erdélyi-Kober operators

$$E_{0,x}^{\alpha,\gamma} f(x) = I_{0,x}^{\alpha,0,\gamma} f(x) = \frac{x^{-\alpha-\gamma}}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} t^{\gamma} f(t)\mathrm{d}t, (Re(\alpha) > 0). \tag{1.8}$$

Main formulae.

Theorem 1.

$$I_{0,x}^{\alpha,\beta,\gamma} \left\{ x^{\rho} \prod_{i=1}^{s} (x^{t_i} + \alpha_i)^{\sigma_i} \prod_{j=1}^{t} S_{N_j}^{M_j} \left[e_j x^{\lambda_j} \prod_{i=1}^{s} (x^{t_i} + \alpha_i)^{\eta_i^{(j)}} \right] \gimel \left(z_1 x^{u_1} \prod_{i=1}^{s} (x^{t_i} + \alpha_i)^{-v_i'}, \cdots, z_r x^{u_r} \prod_{i=1}^{s} (x^{t_i} + \alpha_i)^{-v_i^{(r)}} \right) \right\}$$

$$= \alpha_1^{\sigma_1} \cdots \alpha_r^{\sigma_r} x^{\rho-\beta} \sum_{K_1=0}^{[N_1/M_1]} \cdots \sum_{K_t=0}^{[N_t/M_t]} \frac{(-N_1)_{M_1K_1} \cdots (-N_t)_{M_tK_t}}{K_1! \cdots K_t!} A_{N_1,K_1} \cdots A_{N_t,K_t}$$

Ref

1. F. Ayant, An integral associated with the Aleph-functions of several variables. International Journal of Mathematics Trends and Technology (IJMTT), 31(3) (2016), 142-154.

$$e_1^{K_1}\cdots e_t^{K_t}\alpha_1^{\sum_{j=1}^{t}\eta_1^{(j)}K_j}\cdots\alpha_s^{\sum_{j=1}^{t}\eta_s^{(j)}K_j}x^{\sum_{j=1}^{t}\lambda_jK_j}$$

$$\gimel^{U;0,s+2+n_r:V}_{X;s+2+p_{i_r},s+2+q_{i_r},\tau_{i_r}:R_r:Y}\left(\begin{array}{c|c} \mathrm{z}_1\alpha_1^{-v_1'}\cdots\alpha_s^{-v_s'}x^{u_1} & \mathbb{A};\mathrm{A}_1,\mathbf{A}:A \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{z}_r\alpha_1^{-v_1^{(r)}}\cdots\alpha_s^{-v_s^{(r)}}x^{u_r} & \cdot \\ \alpha_1^{-1}x^{t_1} & \cdot \\ \cdot & \cdot \\ \cdot & \\ \alpha_s^{-1}x^{t_s} & \mathbb{B};\mathbf{B},\mathrm{B}_1:B;\underbrace{(0,1;1);\cdots;(0,1;1)}_{s} \end{array}\right) \tag{2.1}$$

Notes

where

$$\mathbb{A}=[(\mathrm{a}_{2j};\alpha_{2j}^{(1)},\alpha_{2j}^{(2)};A_{2j})]_{1,n_2},[\tau_{i_2}(a_{2ji_2};\alpha_{2ji_2}^{(1)},\alpha_{2ji_2}^{(2)};A_{2ji_2})]_{n_2+1,p_{i_2}},[(a_{3j};\alpha_{3j}^{(1)},\alpha_{3j}^{(2)},\alpha_{3j}^{(3)};A_{3j})]_{1,n_3},$$

$$[\tau_{i_3}(a_{3ji_3};\alpha_{3ji_3}^{(1)},\alpha_{3ji_3}^{(2)},\alpha_{3ji_3}^{(3)};A_{3ji_3})]_{n_3+1,p_{i_3}};\cdots;[(\mathrm{a}_{(r-1)j};\alpha_{(r-1)j}^{(1)},\cdots,\alpha_{(r-1)j}^{(r-1)};A_{(r-1)j})_{1,n_{r-1}}],$$

$$[\tau_{i_{r-1}}(a_{(r-1)ji_{r-1}};\alpha_{(r-1)ji_{r-1}}^{(1)},\cdots,\alpha_{(r-1)ji_{r-1}}^{(r-1)};A_{(r-1)ji_{r-1}})_{n_{r-1}+1,p_{i_{r-1}}}] \tag{2.2}$$

$$A_1=\left(-\rho-\sum_{j=1}^{t}\lambda_jK_j;u_1,\cdots,u_r,t_1,\cdots,t_s;1\right),\left(-\beta-\gamma-\rho-\sum_{j=1}^{t}\lambda_jK_j;u_1,\cdots,u_r,t_1,\cdots,t_s;1\right),$$

$$\left(1+\sigma_1+\sum_{j=1}^{t}\eta_1^{(i)}K_j;v_1',\cdots,v_1^{(r)},1,\underbrace{0,\cdots,0}_{s-1};1\right),\cdots,\left(1+\sigma_s+\sum_{j=1}^{t}\eta_s^{(i)}K_j;v_s',\cdots,v_s^{(r)},\underbrace{0,\cdots,0}_{s-1},1;1\right)$$

$$\mathbf{A}=[(\mathrm{a}_{rj};\alpha_{rj}^{(1)},\cdots,\alpha_{rj}^{(r)},\underbrace{0,\cdots,0}_{s};A_{rj})_{1,n_r}],[\tau_{i_r}(a_{rji_r};\alpha_{rji_r}^{(1)},\cdots,\alpha_{rji_r}^{(r)},\underbrace{0,\cdots,0}_{s};A_{rji_r})_{\mathfrak{n}+1,p_{i_r}}] \tag{2.4}$$

$$A=[(\mathrm{c}_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{1,n^{(1)}}],[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)},\gamma_{ji^{(1)}}^{(1)};C_{ji^{(1)}}^{(1)})_{n^{(1)}+1,p_i^{(1)}}];\cdots;$$

$$[(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{1,m^{(r)}}],[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)},\gamma_{ji^{(r)}}^{(r)};C_{ji^{(r)}}^{(r)})_{m^{(r)}+1,p_i^{(r)}}] \tag{2.5}$$

$$\mathbb{B}=[\tau_{i_2}(b_{2ji_2};\beta_{2ji_2}^{(1)},\beta_{2ji_2}^{(2)};B_{2ji_2})]_{1,q_{i_2}},[\tau_{i_3}(b_{3ji_3};\beta_{3ji_3}^{(1)},\beta_{3ji_3}^{(2)},\beta_{3ji_3}^{(3)};B_{3ji_3})]_{1,q_{i_3}};\cdots;$$

$$[\tau_{i_{r-1}}(b_{(r-1)ji_{r-1}};\beta_{(r-1)ji_{r-1}}^{(1)},\cdots,\beta_{(r-1)ji_{r-1}}^{(r-1)};B_{(r-1)ji_{r-1}})_{1,q_{i_{r-1}}}] \tag{2.6}$$

$$\mathbf{B}=[\tau_{i_r}(b_{rji_r};\beta_{rji_r}^{(1)},\cdots,\beta_{rji_r}^{(r)},\underbrace{0,\cdots,0}_{s};B_{rji_r})_{1,q_{i_r}}] \tag{2.7}$$

$$B_1=\left(\beta-\rho-\sum_{j=1}^{t}\lambda_jK_j;u_1,\cdots,u_r,t_1,\cdots,t_s;1\right),\left(-\alpha-\gamma-\rho-\sum_{j=1}^{t}\lambda_jK_j;u_1,\cdots,u_r,t_1,\cdots,t_s;1\right),$$

$$\left(1+\sigma_1+\sum_{j=1}^{t}\eta_1^{(i)}K_j;v_1',\cdots,v_1^{(r)},\underbrace{0,\cdots,0}_{s};1\right),\cdots,\left(1+\sigma_s+\sum_{j=1}^{t}\eta_s^{(i)}K_j;v_s',\cdots,v_s^{(r)},\underbrace{0,\cdots,0}_{s};1\right) \tag{2.8}$$

Notes

$$B = [(\mathrm{d}_j^{(1)}, \delta_j^{(1)}; D_j^{(1)})_{1,m^{(1)}}], [\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)}, \delta_{ji^{(1)}}^{(1)}; D_{ji^{(1)}}^{(1)})_{m^{(1)}+1,q_i^{(1)}}]; \cdots ;$$

$$[(\mathrm{d}_j^{(r)}, \delta_j^{(r)}; D_j^{(r)})_{1,m^{(r)}}], [\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)}, \delta_{ji^{(r)}}^{(r)}; D_{ji^{(r)}}^{(r)})_{m^{(r)}+1,q_i^{(r)}}] \tag{2.9}$$

$$U = 0, n_2; 0, n_3; \cdots ; 0, n_{r-1}; V = m^{(1)}, n^{(1)}; m^{(2)}, n^{(2)}; \cdots ; m^{(r)}, n^{(r)}; \underbrace{(1,0); \cdots ; (1,0)}_{s} \tag{2.10}$$

$$X = p_{i_2}, q_{i_2}, \tau_{i_2}; R_2; \cdots ; p_{i_{r-1}}, q_{i_{r-1}}, \tau_{i_{r-1}} : R_{r-1};$$

$$Y = p_{i^{(1)}}, q_{i^{(1)}}, \tau_{i^{(1)}}; R^{(1)}; \cdots ; p_{i^{(r)}}, q_{i^{(r)}}; \tau_{i^{(r)}}; R^{(r)}; \underbrace{(0,1); \cdots ; (0,1)}_{s} \tag{2.11}$$

Provided

$$Re(\alpha) > 0, t_i, \lambda_j, \eta_i^{(j)}, u_k, v_i^{(k)} > 0; (i = 1, \cdots , s); (j = 1, \cdots , t); (k = 1, \cdots , r)$$

$$Re(\rho) + \sum_{i=1}^{r} u_i \min_{1 \leqslant j \leqslant m^{(i)}} Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right] + 1 > 0$$

$$\left| arg\left(z_k x^{u_k} \prod_{i=1}^{s} (x^{t_i} + \alpha_i)^{-v_i'}\right)\right| < \frac{1}{2}\pi A_k^{(l)}$$

Proof

To prove the fractional derivative formula, we first express the product of a general class of polynomials occurring on. Its left-hand side in the series form with the help of (1.5), next we express the Gimel-function regarding Mellin-Barnes multiple integrals contour with the help of (1.1). Interchange the order of summations and $(s_1, \cdots , s_r)$- integrals and taking the fractional derivative operator inside (which is permissible under the conditions stated above). And make simplifications. Next, we express the terms $(x^{t_1} + \alpha_1)^{\sigma_1 + \sum_{j=1}^{t} \eta_1^{(j)} k_j - \sum_{k=1}^{r} v_1^{(k)} s_k}, \cdots , (x^{t_s} + \alpha_1)^{\sigma_s + \sum_{j=1}^{t} \eta_s^{(j)} k_j - \sum_{k=1}^{r} v_s^{(k)} s_k}$ so obtained in terms of Mellin-Barnes multiple integrals contour, see ([14], p.18, Eq.(2.6.4) and p.10 Eq. (2.1.1)). Now interchanging the order of $(s_{r+1}, \cdots , s_{r+t})$ and $(s_1, \cdots , s_r)$- integrals (which is permissible under the conditions stated above). And evaluating the x-integral thus obtained by using the following formula with the help of ([10], p.16, Lemma 1)

$$I_{0x}^{\alpha,\beta,\gamma}(x^{\lambda}) = \frac{\Gamma(1+\lambda)\Gamma(1-\beta+\gamma+\lambda)}{\Gamma(1-\beta+\lambda)\Gamma(1+\alpha+\gamma+\lambda)} x^{\lambda-\beta} \tag{2.12}$$

provided $Re(\lambda) > max[0, Re(\beta-\gamma)] - 1$

and we interpret the resulting multiple integrals contour with the help of (1.1) in term of the gimel-function of $(r+s)$-variables, after algebraic manipulations, we obtain the theorem.

Theorem 2.

$$I_{0,x}^{\alpha,\beta,\gamma}\left\{x^{\rho}\prod_{i=1}^{s}(x^{t_i}+\alpha_i)^{\sigma_i}\prod_{j=1}^{t} S_{N_j}^{M_j}\left[e_j x^{\lambda_j}\prod_{i=1}^{s}(x^{t_i}+\alpha_i)^{\eta_i^{(j)}}\right]\gimel\left(z_1 x^{u_1}\prod_{i=1}^{s}(x^{t_i}+\alpha_i)^{-v_i'}, \cdots , z_r x^{u_r}\prod_{i=1}^{s}(x^{t_i}+\alpha_i)^{-v_i^{(r)}}\right)\right.$$

$$\left.\gimel\left(z_{r+1} x^{u_{r+1}}\prod_{i=1}^{s-1}(x^{t_i}+\alpha_i)^{-v_i^{(r+1)}}, \cdots , z_{r+\tau} x^{u_{r+\tau}}\prod_{i=1}^{s-1}(x^{t_i}+\alpha_i)^{-v_i^{(r+\tau)}}\right)\right\}$$

$$= \alpha_1^{\sigma_1}\cdots\alpha_s^{\sigma_s}x^{\rho-\beta}\sum_{l=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_t=0}^{[N_t/M_t]}\binom{-\beta}{l}\frac{(-N_1)_{M_1K_1}\cdots(-N_t)_{M_tK_t}}{K_1!\cdots K_s!}A_{N_1,K_1}\cdots A_{N_t,K_t}$$

$$e_1^{K_1}\cdots e_t^{K_t}\alpha_1^{\sum_{j=1}^t\eta_1^{(j)}K_j}\cdots\alpha_s^{\sum_{j=1}^t\eta_s^{(j)}K_j}x^{\sum_{j=1}^t\lambda_jK_j}e_1^{K_1}\cdots e_t^{K_t}$$

$$\gimel^{U,U';0,2s+3+n_r+n_{r+\tau},V,V'}_{X,X';p_{i_r}+p_{i_{r+\tau}}+2s+3,q_{i_r}+q_{i_{r+\tau}}+2s+3,\tau_{i_r},\tau_{i_{r+\tau}}:R_r,R_{r+\tau}:Y,Y'}\left(\begin{array}{c|c} z_1\prod_{i=1}^{s}\alpha_i^{-v_i'}x^{u_1} & \\ \cdot & \\ \cdot & \\ z_r\prod_{i=1}^{s}\alpha_i^{-v_i^{(r)}}x^{u_r} & \\ \alpha_1^{-1}x^{t_1} & \mathbb{A},\mathbb{A}';\mathrm{A}_2,\mathbf{A},\mathbf{A}':A;A' \\ \cdot & \cdot \\ \cdot & \cdot \\ \alpha_s^{-1}x^{t_s} & \cdot \\ z_{r+1}\prod_{i=1}^{s-1}\alpha_i^{-v_i^{(r+1)}}x^{u_{r+1}} & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ z_{r+\tau}\prod_{i=1}^{s-1}\alpha_i^{-v_i^{(r+\tau)}}x^{u_{r+\tau}} & \mathbb{B},\mathbb{B}';\mathbf{B},\mathbf{B}',\mathrm{B}_2:B;B' \\ \cdot & \\ \cdot & \\ \alpha_1^{-1}x^{t_1} & \\ \cdot & \\ \cdot & \\ \alpha_{s-1}^{-1}x^{t_{s-1}} & \end{array}\right) \quad (2.13)$$

where

$\mathbb{A} =[(a_{2j};\alpha_{2j}^{(1)},\alpha_{2j}^{(2)};A_{2j})]_{1,n_2},[\tau_{i_2}(a_{2ji_2};\alpha_{2ji_2}^{(1)},\alpha_{2ji_2}^{(2)};A_{2ji_2})]_{n_2+1,p_{i_2}},[(a_{3j};\alpha_{3j}^{(1)},\alpha_{3j}^{(2)},\alpha_{3j}^{(3)};A_{3j})]_{1,n_3},$

$[\tau_{i_3}(a_{3ji_3};\alpha_{3ji_3}^{(1)},\alpha_{3ji_3}^{(2)},\alpha_{3ji_3}^{(3)};A_{3ji_3})]_{n_3+1,p_{i_3}};\cdots;[(a_{(r-1)j};\alpha_{(r-1)j}^{(1)},\cdots,\alpha_{(r-1)j}^{(r-1)};A_{(r-1)j})_{1,n_{r-1}}],$

$$[\tau_{i_{r-1}}(a_{(r-1)ji_{r-1}};\alpha_{(r-1)ji_{r-1}}^{(1)},\cdots,\alpha_{(r-1)ji_{r-1}}^{(r-1)};A_{(r-1)ji_{r-1}})_{n_{r-1}+1,p_{i_{r-1}}}] \quad (2.14)$$

$\mathbb{A}' =[(a_{(r+2)j};\alpha_{(r+2)j}^{(r+1)},\alpha_{(r+2)j}^{(r+2)};A_{j(r+2)})]_{1,n_{r+2}},[\tau_{i_r+2}(a_{(r+2)ji_{r+2}};\alpha_{(r+2)ji_{r+2}}^{(1)},\alpha_{(r+2)ji_{r+2}}^{(r+2)};A_{(r+2)ji_{r+2}})]_{n_{r+2,p_{i_{r+2}}}},$

$[(a_{(r+3)j};\alpha_{(r+3)j}^{(r+1)},\alpha_{(r+3)j}^{(r+2)},\alpha_{(r+3)j}^{(r+3)};A_{(r+3)j})]_{1,n_{r+3}},$

$[\tau_{i_{r+3}}(a_{(r+3)ji_{r+3}};\alpha_{(r+3ji_{r+3}}^{(r+1)},\alpha_{(r+3)ji_{r+3}}^{(r+2)},\alpha_{(r+3)ji_{r+3}}^{(r+3)};A_{(r+3)ji_{r+3}})]_{n_{r+3}+1,p_{i_{r+3}}};\cdots;$

$[(a_{(r+\tau-1)j};\alpha_{(r+\tau-1)j}^{(r+1)},\cdots,\alpha_{(r+\tau-1)j}^{(r+\tau-1)};A_{(r+\tau-1)j})_{1,n_{r+\tau-1}}],$

$$[\tau_{i_{\tau+r-1}}(a_{(\tau+r-1)ji_{\tau+r-1}};\alpha_{(r-1)ji_{\tau+r-1}}^{(r+1)},\cdots,\alpha_{(r+\tau-1)ji_{r+\tau-1}}^{(r++\tau-1)};A_{(r+\tau-1)ji_{r+\tau-1}})_{n_{r+\tau-1}+1,p_{i_{r+\tau-1}}}] \quad (2.15)$$

$$A_2=\left(-\sum_{j=1}^{t}\lambda_jK_j;u_1,\cdots,u_r,t_1,\cdots,t_s,\underbrace{0,\cdots,0}_{\tau+s-1};1\right),\left(1-\gamma-\sum_{j=1}^{t}\lambda_jK_j;u_1,\cdots,u_r,t_1,\cdots,t_s,\underbrace{0,\cdots,0}_{\tau+s-1};1\right),$$

Notes

Global Journal of Science Frontier Research (F) Volume XVIII Issue VI Version I Year 2018

Notes

$$\left(1-\sum_{j=1}^{t}\lambda_jK_j;u_1,\cdots,u_r,t_1,\cdots,t_s,\underbrace{0,\cdots,0}_{\tau+s-1};1\right),\left(-\alpha-\gamma-\sum_{j=1}^{t}\lambda_jK_j;u_1,\cdots,u_r,t_1,\cdots,t_s,\underbrace{0,\cdots,0}_{\tau+s-1};1\right),$$

$$\left(1+\sigma_1,\underbrace{0,\cdots,0}_{r+s};v_1^{(r+1)},\cdots,v_1^{(r+\tau)},1,\underbrace{0,\cdots,0}_{s-2};1\right),\left(1+\sigma_{s-1},\underbrace{0,\cdots,0}_{r+s};v_{s-1}^{(r+s)},\cdots,v_{s-1}^{(r+\tau)},1,\underbrace{0,\cdots,0}_{s-2};1\right),$$

$$\left(1+\sum_{i=0}^{t}\eta_iK_i;v_1',\cdots,v_1^{(r)},1,\underbrace{0,\cdots,0}_{\tau+2s-2};1\right),\left(1+\sigma_s+\sum_{i=0}^{t}\eta_iK_i;v_1',\cdots,v_1^{(r)},\underbrace{0,\cdots,0}_{s-1},1,\underbrace{0,\cdots,0}_{\tau+s-1};1\right) \quad (2.16)$$

$$\mathbf{A}=[(\mathrm{a}_{rj};\alpha_{rj}^{(1)},\cdots,\alpha_{rj}^{(r)},\underbrace{0,\cdots,0}_{\tau+2s-1};A_{rj})_{1,n_r}],[\tau_{i_r}(a_{rji_r};\alpha_{rji_r}^{(1)},\cdots,\alpha_{rji_r}^{(r)},\underbrace{0,\cdots,0}_{\tau+2s-1};A_{rji_r})_{\mathfrak{n}+1,p_{i_r}}] \quad (2.17)$$

$$\mathbf{A}'=[(\mathrm{a}_{(r+\tau)j};\underbrace{0,\cdots,0}_{r+s},\alpha_{(r+\tau)j}^{(1)},\cdots,\alpha_{(r+\tau)j}^{(r+\tau)},\underbrace{0,\cdots,0}_{s-1};A_{(r+\tau)j})_{1,n_r}],$$

$$[\tau_{i_{r+\tau}}(a_{(r+\tau)ji_{r+\tau}};\underbrace{0,\cdots,0}_{r+s},\alpha_{(r+\tau)ji_{r+\tau}}^{(1)},\cdots,\alpha_{(r+\tau)ji_{r+\tau}}^{(r+\tau)},\underbrace{0,\cdots,0}_{s-1};A_{(r+\tau)ji_{r+\tau}})_{\mathfrak{n}+1,p_{i_{r+\tau}}}] \quad (2.18)$$

$$A=[(\mathrm{c}_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{1,n^{(1)}}],[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)},\gamma_{ji^{(1)}}^{(1)};C_{ji^{(1)}}^{(1)})_{n^{(1)}+1,p_i^{(1)}}];\cdots;$$

$$[(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{1,m^{(r)}}],[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)},\gamma_{ji^{(r)}}^{(r)};C_{ji^{(r)}}^{(r)})_{m^{(r)}+1,p_i^{(r)}}] \quad (2.19)$$

$$A'=[(\mathrm{c}_j^{(r+1)},\gamma_j^{(r+1)};C_j^{(r+1)})_{1,n^{(r+1)}}],[\tau_{i^{(r+1)}}(c_{ji^{(r+1)}}^{(r+1)},\gamma_{ji^{(r+1)}}^{(r+1)};C_{ji^{(r+1)}}^{(r+1)})_{n^{(r+1)}+1,p_i^{(r+1)}}];\cdots;$$

$$[(c_j^{(r+\tau)},\gamma_j^{(r+\tau)};C_j^{(r+\tau)})_{1,m^{(r+\tau)}}],[\tau_{i^{(r+\tau)}}(c_{ji^{(r+\tau)}}^{(r+\tau)},\gamma_{ji^{(r+\tau)}}^{(r+\tau)};C_{ji^{(r+\tau)}}^{(r+\tau)})_{m^{(r+\tau)}+1,p_i^{(r+\tau)}}] \quad (2.20)$$

$$\mathbb{B}=[\tau_{i_2}(b_{2ji_2};\beta_{2ji_2}^{(1)},\beta_{2ji_2}^{(2)};B_{2ji_2})]_{1,q_{i_2}},[\tau_{i_3}(b_{3ji_3};\beta_{3ji_3}^{(1)},\beta_{3ji_3}^{(2)},\beta_{3ji_3}^{(3)};B_{3ji_3})]_{1,q_{i_3}};\cdots;$$

$$[\tau_{i_{r-1}}(b_{(r-1)ji_{r-1}};\beta_{(r-1)ji_{r-1}}^{(1)},\cdots,\beta_{(r-1)ji_{r-1}}^{(r-1)};B_{(r-1)ji_{r-1}})_{1,q_{i_{r-1}}}] \quad (2.21)$$

$$\mathbb{B}'=[\tau_{i_{r+2}}(b_{(r+2)ji_{r+2}};\beta_{(r+2)ji_{r+2}}^{(r+1)},\beta_{(r+2)jir+2}^{(r+2)};B_{(r+2)ji_{r+2}})]_{1,q_{i_{r+2}}},[\tau_{i_3}(b_{3ji_3};\beta_{3ji_3}^{(1)},\beta_{3ji_3}^{(2)},\beta_{3ji_3}^{(3)};B_{3ji_3})]_{1,q_{i_3}};\cdots;$$

$$[\tau_{i_{r+3}}(b_{(r+3)ji_{r+3}};\beta_{(r+3)ji_{r+3}}^{(r+1)},\beta_{(r+3)jir+3}^{(r+2)},\beta_{(r+3)ji_{r+3}}^{(r+3)};B_{(r+3)ji_{r+3}})]_{1,q_{i_{r+3}}};\cdots;$$

$$[\tau_{i_{r+\tau-1}}(b_{(r+\tau-1)ji_{r+\tau-1}};\beta_{(r+\tau-1)ji_{r+\tau-1}}^{(r+1)},\cdots,\beta_{(r+\tau-1)ji_{r+\tau-1}}^{(r+\tau-1)};B_{(r+\tau-1)ji_{r+\tau-1}})_{1,q_{i_{r+\tau-1}}}] \quad (2.22)$$

$$\mathbf{B}=[\tau_{i_r}(b_{rji_r};\beta_{rji_r}^{(1)},\cdots,\beta_{rji_r}^{(r)},\underbrace{0,\cdots,0}_{\tau+2s-1};B_{rji_r})_{\mathfrak{n}+1,q_{i_r}}] \quad (2.23)$$

$$\mathbf{B'} = [\tau_{i_{r+\tau}}(b_{(r+\tau)ji_{r+\tau}};\underbrace{0,\cdots,0}_{r+s},\beta^{(1)}_{(r+\tau)ji_{r+\tau}},\cdots,\beta^{(r+\tau)}_{(r+\tau)ji_{r+\tau}},\underbrace{0,\cdots,0}_{s-1};B_{(r+\tau)ji_{r+\tau}})_{\mathfrak{n}+1,q_{i_{r+\tau}}}] \tag{2.24}$$

$$B_2 = \left(-\rho;\underbrace{0,\cdots,0}_{r+s},u_{r+1},\cdots,u_{r+\tau},t_1,\cdots,t_{s-1};1\right),\left(\beta-l-\rho;\underbrace{0,\cdots,0}_{r+s},u_{r+1},\cdots,u_{r+\tau},t_1,\cdots,t_{s-1};1\right),$$

$$\left(\beta-l-\gamma;\underbrace{0,\cdots,0}_{r+s},u_{r+1},\cdots,u_{r+\tau},t_1,\cdots,t_{s-1};1\right),\left(1+\sigma_s+\sum_{i=0}^{t}\eta_i K_i;v'_1,\cdots,v_1^{(r)},\underbrace{0,\cdots,0}_{\tau+2s-1};1\right)$$

$$\left(-\rho-\alpha-\gamma;\underbrace{0,\cdots,0}_{r+s},u_{r+1},\cdots,u_{r+\tau},t_1,\cdots,t_{s-1};1\right),\left(1+\sigma_{s-1};\underbrace{0,\cdots,0}_{r+s},v_{s-1}^{(r+1)},\cdots,v_{s-1}^{(r+\tau)},\underbrace{0,\cdots,0}_{s-1};1\right) \tag{2.25}$$

$$\left(1+\sigma_1;\underbrace{0,\cdots,0}_{r+s};v_1^{(r+1)},\cdots,v_1^{(r+\tau)},\underbrace{0,\cdots,0}_{s-1};1\right),\left(1+\sum_{i=0}^{t}\eta_i K_i;v'_1,\cdots,v_1^{(r)},\underbrace{0,\cdots,0}_{\tau+2s-1};1\right) \tag{2.26}$$

$$\mathrm{B} = [(\mathrm{d}_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{1,m^{(1)}}],[\tau_{i^{(1)}}(d^{(1)}_{ji^{(1)}},\delta^{(1)}_{ji^{(1)}};D^{(1)}_{ji^{(1)}})_{m^{(1)}+1,q_i^{(1)}}];\cdots;$$

$$[(\mathrm{d}_j^{(r)},\delta_j^{(r)};D_j^{(r)})_{1,m^{(r)}}],[\tau_{i^{(r)}}(d^{(r)}_{ji^{(r)}},\delta^{(r)}_{ji^{(r)}};D^{(r)}_{ji^{(r)}})_{m^{(r)}+1,q_i^{(r)}}];\underbrace{(0,1;1),\cdots,(0,1;1)}_{s} \tag{2.27}$$

$$\mathrm{B'} = [(\mathrm{d}_j^{(r+1)},\delta_j^{(r+1)};D_j^{(r+1)})_{r+1,m^{(r+1)}}],[\tau_{i^{(r+1)}}(d^{(r+1)}_{ji^{(r+1)}},\delta^{(r+1)}_{ji^{(r+1)}};D^{(r+1)}_{ji^{(r+1)}})_{m^{(r+1)}+1,q_i^{(r+1)}}];\cdots;$$

$$[(\mathrm{d}_j^{(r+\tau)},\delta_j^{(r+\tau)};D_j^{(r+\tau)})_{1,m^{(r+\tau)}}],[\tau_{i^{(r+\tau)}}(d^{(r+\tau)}_{ji^{(r+\tau)}},\delta^{(r+\tau)}_{ji^{(r+\tau)}};D^{(r+\tau)}_{ji^{(r+\tau)}})_{m^{(r+\tau)}+1,q_i^{(r+\tau)}}];\underbrace{(0,1;1),\cdots,(0,1;1)}_{s-1} \tag{2.28}$$

$$U = 0,n_2;0,n_3;\cdots;0,n_{r-1} \quad U' \text{ is similar to } U$$

$$V = m^{(1)},n^{(1)};m^{(2)},n^{(2)};\cdots;m^{(r)},n^{(r)};\underbrace{(1,0);\cdots;(1,0)}_{s} \tag{2.29}$$

$$V' = m^{(1)},n^{(r+1)};m^{(r+2)},n^{(r+2)};\cdots;m^{(r+\tau)},n^{(r+\tau)};\underbrace{(1,0);\cdots;(1,0)}_{s-1} \tag{2.30}$$

$$Y = p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)};\cdots;p_{i^{(r)}},q_{i^{(r)}};\tau_{i^{(r)}};R^{(r)};\underbrace{(0,1);\cdots;(0,1)}_{s} \tag{2.31}$$

$$Y' = p_{i^{(r+1)}},q_{i^{(r+1)}},\tau_{i^{(r+1)}};R^{(r+1)};\cdots;p_{i^{(r+\tau)}},q_{i^{(r+\tau)}};\tau_{i^{(r+\tau)}};R^{(r+\tau)};\underbrace{(0,1);\cdots;(0,1)}_{s-1} \tag{2.32}$$

Provided

$$Re(\alpha)>0,t_i,\lambda_j,\eta_i^{(j)},u_k,v_i^{(k)}>0;(i=1,\cdots,s);(j=1,\cdots,t);(k=1,\cdots,r),u_{r+l},v_i^{(r+l)}>0;(l=1,\cdots,\tau)$$

$$Re(\rho)+\sum_{i=1}^{r+\tau}u_i\min_{1\leqslant j\leqslant m^{(i)}}Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right]+1>0$$

Notes

$$\left|arg\left(z_k x^{u_k}\prod_{i=1}^{s}(x^{t_i}+\alpha_i)^{-v'_i}\right)\right|<\frac{1}{2}\pi A_k^{(l)} \quad \text{and} \quad \left|arg\left(z_{r+\tau} x^{u_{r+\tau}}\prod_{i=1}^{s-1}(x^{t_i}+\alpha_i)^{-v_i^{(r+\tau)}}\right)\right|<\frac{1}{2}\pi A_k'^{(l)}$$

Notes

Proof

We take $f(x)=x^{\rho}\prod_{i=1}^{s-1}(x^{t_i}+\alpha_i)^{\sigma_i}\gimel\left(z_{r+1}x^{u_{r+1}}\prod_{i=1}^{s-1}(x^{t_i}+\alpha_i)^{-v_i^{(r+1)}},\cdots,z_{r+\tau}x^{u_{r+\tau}}\prod_{i=1}^{s-1}(x^{t_i}+\alpha_i)^{-v_i^{(r+\tau)}}\right)$ and

$$g(x)=(x^{t_s}+\alpha_s)^{\sigma_s}\prod_{j=1}^{t}S_{N_j}^{M_j}\left[e_j x^{\lambda_j}\prod_{i=1}^{s}(x^{t_i}+\alpha_i)^{\eta_i^{(j)}}\right]\gimel\left(z_1x^{u_1}\prod_{i=1}^{s}(x^{t_i}+\alpha_i)^{-v'_i},\cdots,z_1x^{u_1}\prod_{i=1}^{s}(x^{t_i}+\alpha_i)^{-v_i^{(r)}}\right)$$

in the left-hand side of (2.13) and apply the following Leibniz rule for the fractional integrals

$$I_{0,x}^{\alpha,\beta,\gamma}\left[f(x)g(x)\right]=\sum_{l=0}^{\infty}I_{0,x}^{\alpha,\beta-l,\gamma}[f(x)]I_{0,x}^{\alpha,l,\gamma}[g(x)] \tag{2.33}$$

we obtain the fractional derivative formula (2) after simplification, using the theorem 1 and the result ({4], p. Eq. (6))

Theorem 3.

$$I_{0,x}^{\alpha,\beta,\gamma}I_{0,y}^{\alpha',\beta',\gamma'}\left\{x^{\rho}\prod_{i=1}^{s}(x^{t_i}+\alpha_i)^{\sigma_i}(y^{t'_i}+\beta_i)^{\sigma'_i}\prod_{j=1}^{t}S_{N_j}^{M_j}\left[e_jx^{\lambda_j}y^{\zeta_j}\prod_{i=1}^{s}(x^{t_i}+\alpha_i)^{\eta_i^{(j)}}(y^{t'_i}+\beta_i)^{\tau_i^{(j)}}\right.\right.$$

$$\left.\gimel\left(z_1x^{u_1}y^{u'_1}\prod_{i=1}^{s}(x^{t_i}+\alpha_i)^{-v'_i}(y^{t'_i}+\beta_i)^{-w'_i},\cdots,z_rx^{u_r}y^{u'_r}\prod_{i=1}^{s}(x^{t_i}+\alpha_i)^{-v_i'^{(r)}}(y^{t_i}+\beta_i)^{-w_i^{(r)}}\right)\right\}$$

$$=\alpha_1^{\sigma_1}\cdots\alpha_s^{\sigma_s}\beta_1^{\sigma'_1}\cdots\beta_s^{\sigma'_s}x^{\rho-\beta}y^{\rho'-\beta'}\sum_{K_1=0}^{[M_1/N_1]}\cdots\sum_{K_t=0}^{[M_t/N_t]}\frac{(-N_1)_{M_1K_1}\cdots(-N_t)_{M_tK_t}}{K_1!\cdots K_t!}A'_{N_1,K_1}\cdots A_{N_t,K_t}^{(t)}$$

$$e_1^{k_1}\cdots e_t^{k_t}\alpha_1^{\sum_{j=1}^{t}\eta_1^{(j)}K_j}\cdots\alpha_s^{\sum_{j=1}^{t}\eta_s^{(j)}K_j}\beta_1^{\sum_{j=1}^{t}\tau_1^{(j)}K_i}\cdots\beta_s^{\sum_{j=1}^{t}\tau_s^{(j)}K_i}x^{\sum_{j=1}^{t}\lambda_jK_j}y^{\sum_{j=1}^{t}\zeta_jK_j}$$

$$\gimel_{X;p_{i_r}+2s+4,q_{i_r}+s+2,\tau_{i_r}:R_r:Y}^{U;0,2s+4+n_r:V}\left(\begin{array}{c|c} z_1\prod_{i=1}^{s}\alpha_i^{-v'_i}\beta_i^{-w'_i}x^{u_1}y^{u'_1} & \\ \cdot & \mathbb{A},;\ \mathrm{A}_3,\mathbf{A}:A \\ \cdot & \cdot \\ z_r\prod_{i=1}^{s}\alpha_i^{-v_i^{(r)}}\beta_i^{-w_i^{(r)}}x^{u_r}y^{u'_r} & \cdot \\ \alpha_1^{-1}x^{t_1} & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \alpha_s^{-1}x^{t_s} & \\ \beta_1^{-1}y^{t'_1} & \mathbb{B};\ \mathbf{B},\mathrm{B}_3:B \\ \cdot & \\ \cdot & \\ \beta_s^{-1}x^{t_s} & \end{array}\right) \tag{2.34}$$

where

$$\mathbb{A}=[(\mathrm{a}_{2j};\alpha_{2j}^{(1)},\alpha_{2j}^{(2)};A_{2j})]_{1,n_2},[\tau_{i_2}(a_{2ji_2};\alpha_{2ji_2}^{(1)},\alpha_{2ji_2}^{(2)};A_{2ji_2})]_{n_2+1,p_{i_2}},[(a_{3j};\alpha_{3j}^{(1)},\alpha_{3j}^{(2)},\alpha_{3j}^{(3)};A_{3j})]_{1,n_3},$$

$$[\tau_{i_3}(a_{3ji_3};\alpha_{3ji_3}^{(1)},\alpha_{3ji_3}^{(2)},\alpha_{3ji_3}^{(3)};A_{3ji_3})]_{n_3+1,p_{i_3}};\cdots;[(\mathrm{a}_{(r-1)j};\alpha_{(r-1)j}^{(1)},\cdots,\alpha_{(r-1)j}^{(r-1)};A_{(r-1)j})_{1,n_{r-1}}],$$

$$[\tau_{i_{r-1}}(a_{(r-1)ji_{r-1}};\alpha^{(1)}_{(r-1)ji_{r-1}},\cdots,\alpha^{(r-1)}_{(r-1)ji_{r-1}};A_{(r-1)ji_{r-1}})_{n_{r-1}+1,p_{i_{r-1}}}] \tag{2.35}$$

$$A_3=\left(-\rho-\sum_{j=1}^{t}\lambda_jK_j;u_1,\cdots,u_r,\underbrace{0,\cdots,0}_{s},t_1,\cdots,t_s;1\right),\left(1+\sigma_1+\sum_{j=1}^{t}\eta_1^{(j)}K_j,v'_1,\cdots,v_1^{(r)},\underbrace{0,\cdots,0}_{s},1\underbrace{0,\cdots,0}_{s-1};1\right),$$

$$\left(1+\sigma'_s+\sum_{j=1}^{t}\tau_1^{\prime(j)}K_j,w'_1,\cdots,w_1^{(r)},1,\underbrace{0,\cdots,0}_{2s-1};1\right),\left(\beta-\gamma-\rho-\sum_{j=1}^{t}\lambda_jK_j;u_1,\cdots,u_r,\underbrace{0,\cdots,0}_{s},t_1,\cdots,t_s;1\right),$$

$$\left(\beta-\rho-\sum_{j=1}^{t}\lambda_jK_j;u_1,\cdots,u_r,\underbrace{0,\cdots,0}_{s},t_1,\cdots,t_s;1\right),\left(-\alpha-\gamma-\rho-\sum_{j=1}^{t}\lambda_jK_j;u_1,\cdots,u_r,\underbrace{0,\cdots,0}_{s},t_1,\cdots,t_s;1\right),$$

$$\left(1+\sigma'_1+\sum_{j=1}^{t}\tau_1^{\prime(j)}K_j,w'_1,\cdots,w_1^{(r)},\underbrace{0,\cdots,0}_{s},1,\underbrace{0,\cdots,0}_{s-1};1\right),$$

$$\left(1+\sigma'_1+\sum_{j=1}^{t}\tau_1^{\prime(j)}K_j,w'_1,\cdots,w_1^{(r)},1,\underbrace{0,\cdots,0}_{2s-1};1\right),\left(1+\sigma_s+\sum_{j=1}^{t}\eta_s^{(j)}K_j,v'_1,\cdots,v_1^{(r)},1,\underbrace{0,\cdots,0}_{2s-1};1\right),$$

$$\left(1+\sigma_1;\underbrace{0,\cdots,0}_{r+s};v_1^{(r+1)},\cdots,v_1^{(r+\tau)},\underbrace{0,\cdots,0}_{s-1};1\right),\left(1+\sigma'_s+\sum_{j=1}^{t}\tau_s^{\prime(j)}K_j,w'_1,\cdots,w_1^{(r)},1,\underbrace{0,\cdots,0}_{s-1};1,\underbrace{0,\cdots,0}_{s};1\right) \tag{2.36}$$

$$\mathbf{A}=[(\mathrm{a}_{rj};\alpha_{rj}^{(1)},\cdots,\alpha_{rj}^{(r)},\underbrace{0,\cdots,0}_{2s};A_{rj})_{1,n_r}],[\tau_{i_r}(a_{rji_r};\alpha_{rji_r}^{(1)},\cdots,\alpha_{rji_r}^{(r)},\underbrace{0,\cdots,0}_{2s};A_{rji_r})_{\mathfrak{n}+1,p_{i_r}}] \tag{2.37}$$

$$A=[(\mathrm{c}_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{1,n^{(1)}}],[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)},\gamma_{ji^{(1)}}^{(1)};C_{ji^{(1)}}^{(1)})_{n^{(1)}+1,p_i^{(1)}}];\cdots;$$

$$[(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{1,m^{(r)}}],[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)},\gamma_{ji^{(r)}}^{(r)};C_{ji^{(r)}}^{(r)})_{m^{(r)}+1,p_i^{(r)}}] \tag{2.39}$$

$$\mathbb{B}=[\tau_{i_2}(b_{2ji_2};\beta_{2ji_2}^{(1)},\beta_{2ji_2}^{(2)};B_{2ji_2})]_{1,q_{i_2}},[\tau_{i_3}(b_{3ji_3};\beta_{3ji_3}^{(1)},\beta_{3ji_3}^{(2)},\beta_{3ji_3}^{(3)};B_{3ji_3})]_{1,q_{i_3}};\cdots;$$

$$[\tau_{i_{r-1}}(b_{(r-1)ji_{r-1}};\beta^{(1)}_{(r-1)ji_{r-1}},\cdots,\beta^{(r-1)}_{(r-1)ji_{r-1}};B_{(r-1)ji_{r-1}})_{1,q_{i_{r-1}}}] \tag{2.40}$$

$$\mathbf{B}=[\tau_{i_r}(b_{rji_r};\beta_{rji_r}^{(1)},\cdots,\beta_{rji_r}^{(r)},\underbrace{0,\cdots,0}_{2s};B_{rji_r})_{\mathfrak{n}+1,q_{i_r}}] \tag{2.41}$$

$$B_3=\left(\beta'-\gamma'-\rho'-\sum_{j=1}^{t}\lambda_jK_j;u_1,\cdots,u_r,\underbrace{0,\cdots,0}_{s},t_1,\cdots,t_s;1\right),$$

Notes

$$\left(\beta' - \rho' - \sum_{j=1}^{t} \lambda_j K_j; u_1, \cdots, u_r, \underbrace{0, \cdots, 0}_{s}, t_1, \cdots, t_s; 1\right),$$

$$\left(-\alpha' - \gamma' - \rho' - \sum_{j=1}^{t} \zeta_j K_j; u'_1, \cdots, u'_r, \underbrace{0, \cdots, 0}_{s}, t_1, \cdots, t_s; 1\right),$$

$$\left(-\rho' - \sum_{j=1}^{t} \zeta_j K_j; u'_1, \cdots, u'_r, \underbrace{0, \cdots, 0}_{s}, t_1, \cdots, t_s; 1\right),$$

$$\left(1 + \sigma_1 + \sum_{j=1}^{t} \eta_1^{(j)} K_j, v'_1, \cdots, v_1^{(r)}, \underbrace{0, \cdots, 0}_{2s}; 1\right), \left(1 + \sigma_s + \sum_{j=1}^{t} \eta_s^{(j)} K_j, v'_1, \cdots, v_1^{(r)}, \underbrace{0, \cdots, 0}_{2s-1}; 1\right),$$

$$\left(1 + \sigma_s + \sum_{j=1}^{t} \eta_s^{(j)} K_j, v'_1, \cdots, v_1^{(r)}, \underbrace{0, \cdots, 0}_{2s}; 1\right), \left(1 + \sigma_1 + \sum_{j=1}^{t} \eta_1^{(j)} K_j, v'_1, \cdots, v_1^{(r)}, \underbrace{0, \cdots, 0}_{s}, 0, \underbrace{0, \cdots, 0}_{2s-1}\right),$$

$$\left(1 + \sigma'_1 + \sum_{j=1}^{t} \tau_1^{\prime(j)} K_j, w'_1, \cdots, w_1^{(r)}, \underbrace{0, \cdots, 0}_{2s}; 1\right), \cdots, \left(1 + \sigma'_s + \sum_{j=1}^{t} \tau_1^{\prime(j)} K_j, w'_1, \cdots, w_1^{(r)}, \underbrace{0, \cdots, 0}_{2s}; 1\right) \quad (2.42)$$

$$\mathrm{B} = [(\mathrm{d}_j^{(1)}, \delta_j^{(1)}; D_j^{(1)})_{1,m^{(1)}}], [\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)}, \delta_{ji^{(1)}}^{(1)}; D_{ji^{(1)}}^{(1)})_{m^{(1)}+1,q_i^{(1)}}]; \cdots;$$

$$[(\mathrm{d}_j^{(r)}, \delta_j^{(r)}; D_j^{(r)})_{1,m^{(r)}}], [\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)}, \delta_{ji^{(r)}}^{(r)}; D_{ji^{(r)}}^{(r)})_{m^{(r)}+1,q_i^{(r)}}]; \underbrace{(0,1;1), \cdots, (0,1;1)}_{2s} \quad (2.43)$$

$$U = 0, n_2; 0, n_3; \cdots; 0, n_{r-1} : V = m^{(1)}, n^{(1)}; m^{(2)}, n^{(2)}; \cdots; m^{(r)}, n^{(r)}; \underbrace{(1,0); \cdots; (1,0)}_{2s} \quad (2.44)$$

$$Y = p_{i^{(1)}}, q_{i^{(1)}}, \tau_{i^{(1)}}; R^{(1)}; \cdots; p_{i^{(r)}}, q_{i^{(r)}}; \tau_{i^{(r)}}; R^{(r)}; \underbrace{(0,1); \cdots; (0,1)}_{2s} \quad (2.45)$$

Provided

$$Re(\alpha), Re(\alpha') > 0, \lambda_j, \eta_i^{(j)} > 0, t_i, t'_i, \tau_i^{(j)}, u_k, u'_k, v_i^{(k)}, w_i^{(k)} > 0 (i = 1, \cdots, s); (j = 1, \cdots, k); (k = 1, \cdots, r)$$

$$Re(\rho) + \sum_{i=1}^{r} u_i \min_{1 \leqslant j \leqslant m^{(i)}} Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right] + 1 > 0, Re(\rho') + \sum_{i=1}^{r} u'_i \min_{1 \leqslant j \leqslant m^{(i)}} Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right] + 1 > 0,$$

$$\left|\bar{a}rg\left(z_i x^{u_i} y^{u_i} \prod_{i=1}^{s} (x^{t_i} + \alpha_i)^{-v_i^{(r)}} (y^{t_i} + \beta_i)^{-w_i^{(r)}}\right)\right| < \frac{1}{2}\pi A_k^{(l)}$$

Proof

To prove the theorem 3, we use the theorem 1 twice, first with respect to the variable y and then with respect to the variable x, in this situation,the variables x and y are independent variables.

II. Special Case

In this section, we shall see the particular case studied by Soni and Singh ([12],p. 561, Eq. (14))

Consider the theorem 1, if we take $t = 2$ and reduce the polynomial $S_{N_1}^{M_1}$ to the Hermite polynomial ([17], p. 158. Eq.

Notes

(1.4)), the polynomial $S_{N_2}^{M_2}$ to the Laguerre polynomial ([17], p. 159. Eq.(1.8)), the multivariable Gimel-function to the product of r different Whittaker functions ([14], p. 18, Eq. (2.6.7)), we obtain the following result

Corollary.

$$I_{0,x}^{\alpha,\beta,x}\left\{x^{+\sum_{l=1}^{r}b_l+\frac{n_1}{2}}\prod_{i=1}^{s}(x^{t_i}+\alpha_i)^{\sigma_i}H_{n_1}\left(\frac{1}{2\sqrt{x}}\right)\prod_{l=1}^{r}e^{-\frac{z_lx}{2}}W_{\mu_l v_l}(z_lx)\right\}$$

$$\frac{\prod_{l=1}^{r}z_l^{-b_l}\alpha_1^{\sigma_1}\cdots\alpha_s^{\sigma_s}x^{\rho-\beta}}{\Gamma(-\sigma_1)\cdots\Gamma(-\sigma_s)}\sum_{K_1=0}^{[N_1/2]}\sum_{K_2=0}^{N_2}\frac{(-N_1)_{2K_1}(-N_2)_{K_2}}{K_1!K_2!}(-)^{K_1}\binom{N_2+\theta}{N_2}\frac{x^{K_1+K_2}}{(\theta+1)_{K_2}}$$

$$H_{2,2:1,2,\cdots,1,1;1,2,\cdots,1,1}^{0,2:2,0;\cdots;2,0;1,1;\cdots;1,1}\left(\begin{array}{c|c} z_1x & (-\rho-K_1-K_2;1,\cdots,1,t_1,\cdots,t_s),(\beta-\gamma-\rho-K_1-K_2;1,\cdots,1,t_1,\cdots,t_s); \\ \cdot & \cdot \\ \cdot & \cdot \\ z_rx & \cdot \\ \alpha_1^{-1}x^{t_1} & \cdot \\ \cdot & \cdot \\ \cdot & \\ \alpha_s^{-1}x^{t_s} & (\beta-\rho-K_1-K_2;1,\cdots,1,t_1,\cdots,t_s),(-\alpha-\gamma-\rho-K_1-K_2;1,\cdots,1,t_1,\cdots,t_s): \end{array}\right.$$

$$\left.\begin{array}{c} (b_1-u_1+1,1);\cdots;(b_r-u_r+1,1);(1+\sigma_1,1);\cdots;(1+\sigma_s,1) \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \left(b_1\pm v_1+\frac{1}{2},1\right);\cdots;\left(b_r\pm v_r+\frac{1}{2},1\right);\underbrace{(0,1),\cdots,(0,1)}_{s} \end{array}\right) \tag{3.1}$$

The validity conditions mentioned above are verified.

Remarks :

We obtain easily the same relations with the functions defined in section 1.

Soni and Singh [12] have obtained the same relations with the multivariable H-function.

III. Conclusion

The fractional derivative formulae evaluated in this study are unified in nature and act as key formulae. Thus the general class of polynomials involved here reduce to a large variety of polynomials and so from theorems 1, 2 and 3; we can further obtain various fractional derivatives formulae involving a number of simpler polynomials. Secondly by specializing the various parameters as well as variables in the generalized multivariable Gimel-function, we get several formulae involving a remarkably wide variety of useful functions (or product of such functions) which are expressible in terms of E, F, G, H, I, Aleph-function of one and several variables and simpler special functions of one and several variables. Hence the formulae derived in this paper are most general forms and may prove to be useful in several interesting cases appearing in the literature of Pure and Applied Mathematics and Mathematical Physics.

References Références Referencias

[1] F. Ayant, An integral associated with the Aleph-functions of several variables. International Journal of Mathematics Trends and Technology (IJMTT), 31(3) (2016), 142-154.

[2] F. Ayant, An expansion formula for multivariable Gimel-function involving generalized Legendre Associated function, International Journal of Mathematics Trends and Technology (IJMTT), 56(4) (2018), 223-228.

[3] B.L.J. Braaksma, Asymptotics expansions and analytic continuations for a class of Barnes-integrals, Compositio Math. 15 (1962-1964), 239-341.

[4] K.C. Gupta and R.C. Soni, A study of H-functions of one and several variables, J. Rajasthan Acad. Phys. Sci. 1(2002), 89-94.

[5] K.S. Miller and B. Ross, An introduction, to the fractional calculus and fractional differential equation (New York: John Wiley and Sos), 1993.

[6] K.B. Oldham and J. Spanier, The fractional calculus (New York : Academic press), 1974.

Ref

12. R.C. Soni and D. Singh, certain fractional derivative formulae involving the product of a general class of polynomials and the multivariable H-function, Proc. Indian. Acad. Sci. 112(4) (2002), 551-562.

Notes

[7] M. Saigo, A remark on integral operators involving the Gauss hypergeometric functions, Math. Rep. Kyushu Univ. 11(1978), 135-143.

[8] Y.N. Prasad, Multivariable I-function , Vijnana Parisha Anusandhan Patrika 29 (1986), 231-237.

[9] J. Prathima, V. Nambisan and S.K. Kurumujji, A Study of I-function of Several Complex Variables, International Journal of Engineering Mathematics Vol (2014), 1-12.

[10] M. Saigo and R.K. Raina, Fractional calculus operators associated with a general class of polynomials, Fukuoka Univ. Sci. Report 18 (1988), 15-22.

[11] M. Saigo and R.K. Raina, Fractional calculus operators associated with the H-function of several variables: Analysis, Geometry and Groups: Hadronic Press (1993), 527-538.

[12] R.C. Soni and D. Singh, certain fractional derivative formulae involving the product of a general class of polynomials and the multivariable H-function, Proc. Indian. Acad. Sci. 112(4) (2002), 551-562.

[13] H.M. Srivastava, A contour integral involving Fox's H-function, Indian. J. Math. 14(1972), 1-6.

[14] H.M. Srivastava, KC. Gupta and S.P. Goyal, The H-function of one and two variables with applications, South Asian Publisher, New Delhi and Madras, 1982.

[15] H.M. Srivastava and R. Panda, Some expansion theorems and generating relations for the H-function of several complex variables. Comment. Math. Univ. St. Paul. 24 (1975),119-137.

[16] H.M. Srivastava and R. Panda, Some expansion theorems and generating relations for the H-function of several complex variables II. Comment. Math. Univ. St. Paul. 25 (1976), 167-197.

[17] H.M. Srivastava and N.P. Singh, The integration of certains products of the multivariable H-function with a general class of polynomials, Rend. CirC. Mat. Palermo. 32(2)(1983), 157-187.