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On a General Class of Multiple Eulerian Integrals with Multivariable Gimel-Function

By Frederic Ayant

Abstract- Recently, Raina and Srivastava [11] and Srivastava and Hussain [16] have provided closed-form expressions for a number of a Eulerian integral about the multivariable H-functions. Motivated by these recent works, we aim at evaluating a general multiple Eulerian integrals involving the product of multivariable I-function defined by Prathima et al. [10], a class of multivariable polynomials, a generalization of the Mittag-Leffler functions and multivariable Gimel-function.

Keywords: *multivariable gimel-function, multiple eulerian integral, multivariable polynomials, generalization of the mittag-leffler function, multivariable i-function.*

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Abstract- Recently, Raina and Srivastava [11] and Srivastava and Hussain [16] have provided closed-form expressions for a number of a Eulerian integral about the multivariable H-functions. Motivated by these recent works, we aim at evaluating a general multiple Eulerian integrals involving the product of multivariable I-function defined by Prathima et al. [10], a class of multivariable polynomials, a generalization of the Mittag-Leffler functions and multivariable Gimel-function.

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I. Introduction And Prerequisites

The well-known Eulerian Beta integral

$$\int_a^b (z-a)^{\alpha-1}(b-t)^{\beta-1}\mathrm{d}t = (b-a)^{\alpha+\beta-1}B(\alpha,\beta)(Re(\alpha)>0, Re(\beta)>0, b>a) \tag{1.1}$$

Is a basic result for evaluation of numerous other potentially useful integrals involving various special functions and polynomials. The authors Raina and Srivastava [11], Saigo and Saxena [12], Srivastava and Hussain [16], Srivastava and Garg [15] etc. have established a number of Eulerian integrals involving a various general class of polynomials, Meijer's G-function and Fox's H-function of one and more variables with general arguments. Recently, several authors study some multiple Eulerian integrals, see Bhargava et al. [4], Goyal and Mathur [6] , Ayant [1] and others. In this paper we obtain general multiple Eulerian integrals of the product of multivariable I-function defined by Prathima et al. [10], a class of multivariable polynomials, a generalization of the Mittag-Leffler functions and multivariable Gimel function.

For this study, we need the following function appointed Generalized multiple-index Mittag-Leffler function

A further generalization of the Mittag-Leffler functions is proposed recently in Paneva-Konovska [7]. These are 3mparametric Mittag-Leffler type functions generalizing the Prabhakar [8] 3-parametric function, defined as:

$$E_{(\alpha_i),(\beta_i)}^{(\gamma_i),m}(z) = \sum_{k=0}^{\infty}\frac{(\gamma_1)_k\cdots(\gamma_m)_k}{\Gamma(\alpha_1 k+\beta_1)\cdots\Gamma(\alpha_m k+\beta_m)}\frac{z^k}{k!} \tag{1.2}$$

where $\alpha_i,\beta_i,\gamma_i\in\mathbb{C}, i=1,\cdots,m, Re(\alpha_i)>0$

We shall note

$$E_k = \frac{(\gamma_1)_k\cdots(\gamma_m)_k}{\Gamma(\alpha_1 k+\beta_1)\cdots\Gamma(\alpha_m k+\beta_m)} \tag{1.3}$$

The generalized polynomials defined by Srivastava [14], is given in the following manner:

$$S_{N_1,\cdots,N_u}^{M_1,\cdots,M_u}[y_1,\cdots,y_u] \quad = \sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]}\frac{(-N_1)_{M_1K_1}}{K_1!}\cdots\frac{(-N_u)_{M_uK_u}}{K_u!}$$

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Ref

8. T. R. Prabhakar, A singular integral equation with a generalizedMittag-Leffler function in the kernel.Yokohama Math. J.19(1971), 7–15.

$$A[N_1,K_1;\cdots;N_u,K_u]y_1^{K_1}\cdots y_u^{K_u} \tag{1.4}$$

Where $M_1,\cdots,M_u$ are arbitrary positive integers and the coefficients $A[N_1,K_1;\cdots;N_u,K_u]$ are arbitrary constants, real or complex.

We shall note

$$A_u=\frac{(-N_1)_{M_1K_1}}{K_1!}\cdots\frac{(-N_u)_{M_uK_u}}{K_u!}A[N_1,K_1;\cdots;N_u,K_u] \tag{1.5}$$

The multivariable I-function defined by Prathima et al. [10] have expressed in term of multiple Mellin-Barnes types integrals :

$$\bar{I}(z_1,\cdots,z_s) = I^{0,N:M_1,N_1;\cdots;M_r,N_r}_{P,Q:P_1,Q_1;\cdots;P_r,Q_r}\left(\begin{array}{c|c} \mathrm{z}_1 & (\mathrm{a}_j;\alpha_j^{(1)},\cdots,\alpha_j^{(s)};A_j)_{1,P}: \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_s & (\mathrm{b}_j;\beta_j^{(1)},\cdots,\beta_j^{(s)};B_j)_{1,Q}: \end{array}\right.$$

$$\left.\begin{array}{c} (\mathrm{c}_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{1,N_1},(c_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{N_1+1,P_1};\cdots;(c_j^{(s)},\gamma_j^{(s)};C_j^{(r)})_{1,N_r},(c_j^{(s)},\gamma_j^{(s)};C_j^{(r)})_{N_s+1,P_s} \\ \\ (\mathrm{d}_j^{(1)},\delta_j^{(1)};1)_{1,M_1},(d_j^{(1)},\delta_j^{(1)};D_1)_{M_1+1,Q_1};\cdots;(d_j^{(s)},\delta_j^{(s)};1)_{1,M_s},(d_j^{(s)},\delta_j^{(s)};D_s)_{M_s+1,Q_s} \end{array}\right) \tag{1.6}$$

$$=\frac{1}{(2\pi\omega)^s}\int_{L_1}\cdots\int_{L_s}\phi(t_1,\cdots,t_s)\prod_{i=1}^{s}\theta_i(t_i)z_i^{t_i}\mathrm{d}t_1\cdots\mathrm{d}t_s \tag{1.7}$$

where $\phi(t_1,\cdots,t_s),\theta_i(s_i),i=1,\cdots,r$ are given by :

$$\phi(t_1,\cdots,t_s)=\frac{\prod_{j=1}^{N}\Gamma^{A_j}\left(1-aj+\sum_{i=1}^{s}\alpha_j^{(i)}t_j\right)}{\prod_{j=N+1}^{P}\Gamma^{A_j}\left(a_j-\sum_{i=1}^{s}\alpha_j^{(i)}t_j\right)\prod_{j=1}^{Q}\Gamma^{B_j}\left(1-bj+\sum_{i=1}^{s}\beta_j^{(i)}t_j\right)} \tag{1.8}$$

$$\phi_i(t_i)=\frac{\prod_{j=1}^{N_i}\Gamma^{C_j^{(i)}}\left(1-c_j^{(i)}+\gamma_j^{(i)}t_i\right)\prod_{j=1}^{M_i}\Gamma\left(d_j^{(i)}-\delta_j^{(i)}t_i\right)}{\prod_{j=N_i+1}^{P_i}\Gamma^{C_j^{(i)}}\left(c_j^{(i)}-\gamma_j^{(i)}t_i\right)\prod_{j=M_i+1}^{Q_i}\Gamma^{D_j^{(i)}}\left(1-d_j^{(i)}+\delta_j^{(i)}t_i\right)} \tag{1.9}$$

For more details, see Prathima et al. [10].

We can obtain the series representation and behavior for small values for the function $\bar{I}(z_1,\cdots,z_s)$ defined and represented by (1.16). The series representation may be given as follows :

which is valid under the following conditions:

$$\delta_i^{(h)}[d_i^{(j)}+r]\neq\delta_i^{(j)}[d_i^{(h)}+\mu] \text{ for } j\neq h, j,h=1,\cdots,M_i\,,s,\mu=0,1,2,\cdots$$

$$U_i=\sum_{j=1}^{P}A_j\alpha_j^{(i)}-\sum_{j=1}^{Q}B_j\beta_j^{(i)}+\sum_{j=1}^{P_i}C_j^{(i)}\gamma_j^{(i)}-\sum_{j=M_i+1}^{Q_i}D_j^{(i)}\delta_j^{(i)}\leqslant 0,i=1,\cdots,s \quad\text{and}\quad z_i\neq 0$$

and if all the poles of (1.7) are simple. Then the integral (1.7) can be evaluated with the help of the Residue theorem to give

$$\bar{I}(z_1,\cdots,z_s)=\sum_{h_i=1}^{M_i}\sum_{g_i=1}^{\infty}\phi_1\frac{\prod_{i=1}^{s}\phi_i z_i^{\eta_{h_i,g_i}}(-)^{\sum_{i=1}^{s}g_i}}{\prod_{i=1}^{s}\delta_{h^{(i)}}^{(i)}\prod_{i=1}^{s}g_i!} \tag{1.10}$$

Ref

10. J. Prathima, V. Nambisan and S.K. Kurumujji, A Study of I-function of Several Complex Variables, International Journal of Engineering Mathematics Vol (2014), 1-12.

where ϕ_1 and ϕ_i are defined by

$$\phi_1 = \frac{\prod_{j=1}^{N}\Gamma^{A_j}\left(1-aj+\sum_{i=1}^{s}\alpha_j^{(i)}\eta_{h_i,g_i}\right)}{\prod_{j=N+1}^{P}\Gamma^{A_j}\left(a_j-\sum_{i=1}^{s}\alpha_j^{(i)}\eta_{h_i,g_i}\right)\prod_{j=1}^{Q}\Gamma^{B_j}\left(1-bj+\sum_{i=1}^{s}\beta_j^{(i)}\eta_{h_i,g_i}\right)} \tag{1.11}$$

and

$$\phi_i = \frac{\prod_{j=1}^{N_i}\Gamma^{C_j^{(i)}}\left(1-c_j^{(i)}+\gamma_j^{(i)}\eta_{h_i,g_i}\right)\prod_{j=1}^{M_i}\Gamma\left(d_j^{(i)}-\delta_j^{(i)}\eta_{h_i,g_i}\right)}{\prod_{j=N_i+1}^{P_i}\Gamma^{C_j^{(i)}}\left(c_j^{(i)}-\gamma_j^{(i)}\eta_{h_i,g_i}\right)\prod_{j=M_i+1}^{Q_i}\Gamma^{(D_j^{(i)}}\left(1-d_j^{(i)}+\delta_j^{(i)}\eta_{h_i,g_i}\right)}, i=1,\cdots,s \tag{1.12}$$

where $\eta_{h_i,g_i} = \dfrac{d_{h^{(i)}}^{(i)}+g_i}{\delta_{h^{(i)}}^{(i)}}, i=1,\cdots,s.$

Throughout this paper, let $\mathbb{C},\mathbb{R}$ and $\mathbb{N}$ be set of complex numbers, real numbers and positive integers respectively.

Also, $\mathbb{N}_0=\mathbb{N}\cup\{0\}$. We define a generalized transcendental function of several complex variables, see Ayant [2] for more details,

$$\gimel(z_1,\cdots,z_r) = \gimel^{0,n_2;0,n_3;\cdots;0,n_r:m^{(1)},n^{(1)};m^{(2)},n^{(2)};\cdots;m^{(r)},n^{(r)}}_{p_{i_2},q_{i_2},\tau_{i_2};R_2;p_{i_3},q_{i_3},\tau_{i_3};R_3;\cdots;p_{i_r},q_{i_r},\tau_{i_r}:R_r:p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)};\cdots;p_{i^{(r)}},q_{i^{(r)}};\tau_{i^{(r)}};R^{(r)}}\left(\begin{array}{c} \mathrm{z}_1 \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r \end{array}\right|$$

$$[(\mathrm{a}_{2j};\alpha_{2j}^{(1)},\alpha_{2j}^{(2)};A_{2j})]_{1,n_2},[\tau_{i_2}(a_{2ji_2};\alpha_{2ji_2}^{(1)},\alpha_{2ji_2}^{(2)};A_{2ji_2})]_{n_2+1,p_{i_2}};[(a_{3j};\alpha_{3j}^{(1)},\alpha_{3j}^{(2)},\alpha_{3j}^{(3)};A_{3j})]_{1,n_3},$$
$$[\tau_{i_2}(b_{2ji_2};\beta_{2ji_2}^{(1)},\beta_{2ji_2}^{(2)};B_{2ji_2})]_{1,q_{i_2}};$$

$$[\tau_{i_3}(a_{3ji_3};\alpha_{3ji_3}^{(1)},\alpha_{3ji_3}^{(2)},\alpha_{3ji_3}^{(3)};A_{3ji_3})]_{n_3+1,p_{i_3}};\cdots;[(\mathrm{a}_{rj};\alpha_{rj}^{(1)},\cdots,\alpha_{rj}^{(r)};A_{rj})]_{1,n_r},$$
$$[\tau_{i_3}(b_{3ji_3};\beta_{3ji_3}^{(1)},\beta_{3ji_3}^{(2)},\beta_{3ji_3}^{(3)};B_{3ji_3})]_{1,q_{i_3}};\cdots;\qquad\cdots$$

$$[\tau_{i_r}(a_{rji_r};\alpha_{rji_r}^{(1)},\cdots,\alpha_{rji_r}^{(r)};A_{rji_r})]_{n_r+1,p_r}:[(\mathrm{c}_j^{(1)},\gamma_j^{(1)};C_j^{(1)})]_{1,n^{(1)}},[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)},\gamma_{ji^{(1)}}^{(1)};C_{ji^{(1)}}^{(1)})]_{n^{(1)}+1,p_i^{(1)}}$$
$$[\tau_{i_r}(b_{rji_r};\beta_{rji_r}^{(1)},\cdots,\beta_{rji_r}^{(r)};B_{rji_r})]_{1,q_r}:[(\mathrm{d}_j^{(1)}),\delta_j^{(1)};D_j^{(1)})]_{1,m^{(1)}},[\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)},\delta_{ji^{(1)}}^{(1)};D_{ji^{(1)}}^{(1)})]_{m^{(1)}+1,q_i^{(1)}}$$

$$\left.\begin{array}{l};\cdots;[(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})]_{1,m^{(r)}},[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)},\gamma_{ji^{(r)}}^{(r)};C_{ji^{(r)}}^{(r)})]_{m^{(r)}+1,p_i^{(r)}} \\ ;\cdots;[(d_j^{(r)},\delta_j^{(r)};D_j^{(r)})]_{1,n^{(r)}},[\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)},\delta_{ji^{(r)}}^{(r)};D_{ji^{(r)}}^{(r)}]_{n^{(r)}+1,q_i^{(r)}}\end{array}\right)$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)z_k^{s_k}\,\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.13}$$

with $\omega=\sqrt{-1}$

$$\psi(s_1,\cdots,s_r) = \frac{\prod_{j=1}^{n_2}\Gamma^{A_{2j}}(1-a_{2j}+\sum_{k=1}^{2}\alpha_{2j}^{(k)}s_k)}{\sum_{i_2=1}^{R_2}[\tau_{i_2}\prod_{j=n_2+1}^{p_{i_2}}\Gamma^{A_{2ji_2}}(a_{2ji_2}-\sum_{k=1}^{2}\alpha_{2ji_2}^{(k)}s_k)\prod_{j=1}^{q_{i_2}}\Gamma^{B_{2ji_2}}(1-b_{2ji2}+\sum_{k=1}^{2}\beta_{2ji2}^{(k)}s_k)]}$$

$$\frac{\prod_{j=1}^{n_3}\Gamma^{A_{3j}}(1-a_{3j}+\sum_{k=1}^{3}\alpha_{3j}^{(k)}s_k)}{\sum_{i_3=1}^{R_3}[\tau_{i_3}\prod_{j=n_3+1}^{p_{i_3}}\Gamma^{A_{3ji_3}}(a_{3ji_3}-\sum_{k=1}^{3}\alpha_{3ji_3}^{(k)}s_k)\prod_{j=1}^{q_{i_3}}\Gamma^{B_{3ji_3}}(1-b_{3ji3}+\sum_{k=1}^{3}\beta_{3ji3}^{(k)}s_k)]}$$

$$\cdot \qquad \cdot \qquad \cdot \qquad \cdot$$

Ref

2. F. Ayant, An expansion formula for multivariable Gimel-function involving generalized Legendre Associated function, International Journal of Mathematics Trends and Technology (IJMTT), 56(4) (2018), 223-228.

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$$\frac{\prod_{j=1}^{n_r}\Gamma^{A_{rj}}(1-a_{rj}+\sum_{k=1}^{r}\alpha_{rj}^{(k)}s_k)}{\sum_{i_r=1}^{R_r}[\tau_{i_r}\prod_{j=n_r+1}^{p_{i_r}}\Gamma^{A_{rji_r}}(a_{rji_r}-\sum_{k=1}^{r}\alpha_{rji_r}^{(k)}s_k)\prod_{j=1}^{q_{i_r}}\Gamma^{B_{rji_r}}(1-b_{rji_r}+\sum_{k=1}^{r}\beta_{rji_r}^{(k)}s_k)]} \quad (1.14)$$

and

$$\theta_k(s_k)=\frac{\prod_{j=1}^{m^{(k)}}\Gamma^{D_j^{(k)}}(d_j^{(k)}-\delta_j^{(k)}s_k)\prod_{j=1}^{n^{(k)}}\Gamma^{C_j^{(k)}}(1-c_j^{(k)}+\gamma_j^{(k)}s_k)}{\sum_{i^{(k)}=1}^{R^{(k)}}[\tau_{i^{(k)}}\prod_{j=m^{(k)}+1}^{q_{i^{(k)}}}\Gamma^{D_{ji^{(k)}}^{(k)}}(1-d_{ji^{(k)}}^{(k)}+\delta_{ji^{(k)}}^{(k)}s_k)\prod_{j=n^{(k)}+1}^{p_{i^{(k)}}}\Gamma^{C_{ji^{(k)}}^{(k)}}(c_{ji^{(k)}}^{(k)}-\gamma_{ji^{(k)}}^{(k)}s_k)]} \quad (1.15)$$

1) $[(c_j^{(1)};\gamma_j^{(1)}]_{1,n_1}$ stands for $(c_1^{(1)};\gamma_1^{(1)}),\cdots,(c_{n_1}^{(1)};\gamma_{n_1}^{(1)})$.

2) $n_2,\cdots,n_r,m^{(1)},n^{(1)},\cdots,m^{(r)},n^{(r)},p_{i_2},q_{i_2},R_2,\tau_{i_2},\cdots,p_{i_r},q_{i_r},R_r,\tau_{i_r},p_{i^{(r)}},q_{i^{(r)}},\tau_{i^{(r)}},R^{(r)}\in\mathbb{N}$ and verify :

$0\leqslant m_2,\cdots,0\leqslant m_r,0\leqslant n_2\leqslant p_{i_2},\cdots,0\leqslant n_r\leqslant p_{i_r},0\leqslant m^{(1)}\leqslant q_{i^{(1)}},\cdots,0\leqslant m^{(r)}\leqslant q_{i^{(r)}}$

$0\leqslant n^{(1)}\leqslant p_{i^{(1)}},\cdots,0\leqslant n^{(r)}\leqslant p_{i^{(r)}}$.

3) $\tau_{i_2}(i_2=1,\cdots,R_2)\in\mathbb{R}^+;\tau_{i_r}\in\mathbb{R}^+(i_r=1,\cdots,R_r);\tau_{i^{(k)}}\in\mathbb{R}^+(i=1,\cdots,R^{(k)}),(k=1,\cdots,r)$.

4) $\gamma_j^{(k)},C_j^{(k)}\in\mathbb{R}^+;(j=1,\cdots,n^{(k)});(k=1,\cdots,r);\delta_j^{(k)},D_j^{(k)}\in\mathbb{R}^+;(j=1,\cdots,m^{(k)});(k=1,\cdots,r)$.

$\mathrm{C}_{ji^{(k)}}^{(k)}\in\mathbb{R}^+,(j=m^{(k)}+1,\cdots,p^{(k)});(k=1,\cdots,r);$

$\mathrm{D}_{ji^{(k)}}^{(k)}\in\mathbb{R}^+,(j=n^{(k)}+1,\cdots,q^{(k)});(k=1,\cdots,r)$.

$\alpha_{kj}^{(l)},A_{kj}\in\mathbb{R}^+;(j=1,\cdots,n_k);(k=2,\cdots,r);(l=1,\cdots,k)$.

$\alpha_{kji_k}^{(l)},A_{kji_k}\in\mathbb{R}^+;(j=n_k+1,\cdots,p_{i_k});(k=2,\cdots,r);(l=1,\cdots,k)$.

$\beta_{kji_k}^{(l)},B_{kji_k}\in\mathbb{R}^+;(j=m_k+1,\cdots,q_{i_k});(k=2,\cdots,r);(l=1,\cdots,k)$.

$\delta_{ji^{(k)}}^{(k)}\in\mathbb{R}^+;(i=1,\cdots,R^{(k)});(j=m^{(k)}+1,\cdots,q_{i^{(k)}});(k=1,\cdots,r)$.

$\gamma_{ji^{(k)}}^{(k)}\in\mathbb{R}^+;(i=1,\cdots,R^{(k)});(j=n^{(k)}+1,\cdots,p_{i^{(k)}});(k=1,\cdots,r)$.

5) $c_j^{(k)}\in\mathbb{C};(j=1,\cdots,n^{(k)});(k=1,\cdots,r);d_j^{(k)}\in\mathbb{C};(j=1,\cdots,m^{(k)});(k=1,\cdots,r)$.

$a_{kji_k}\in\mathbb{C};(j=n_k+1,\cdots,p_{i_k});(k=2,\cdots,r)$.

$b_{kji_k}\in\mathbb{C};(j=1,\cdots,q_{i_k});(k=2,\cdots,r)$.

$d_{ji^{(k)}}^{(k)}\in\mathbb{C};(i=1,\cdots,R^{(k)});(j=m^{(k)}+1,\cdots,q_{i^{(k)}});(k=1,\cdots,r)$.

$\gamma_{ji^{(k)}}^{(k)}\in\mathbb{C};(i=1,\cdots,R^{(k)});(j=n^{(k)}+1,\cdots,p_{i^{(k)}});(k=1,\cdots,r)$.

The contour L_k is in the $s_k(k=1,\cdots,r)$- plane and runs from $\sigma-i\infty$ to $\sigma+i\infty$ where σ if is a real number with loop, if necessary to ensure that the poles of $\Gamma^{A_{2j}}\left(1-a_{2j}+\sum_{k=1}^{2}\alpha_{2j}^{(k)}s_k\right)(j=1,\cdots,n_2)$, $\Gamma^{A_{3j}}\left(1-a_{3j}+\sum_{k=1}^{3}\alpha_{3j}^{(k)}s_k\right)$

Notes

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$(j=1,\cdots,n_3),\cdots,\Gamma^{A_{rj}}\left(1-a_{rj}+\sum_{i=1}^{r}\alpha_{rj}^{(i)}\right)(j=1,\cdots,n_r),\Gamma^{C_j^{(k)}}\left(1-c_j^{(k)}+\gamma_j^{(k)}s_k\right)(j=1,\cdots,n^{(k)})(k=1,\cdots,r)$ to the right of the contour L_k and the poles of $\Gamma^{D_j^{(k)}}\left(d_j^{(k)}-\delta_j^{(k)}s_k\right)(j=1,\cdots,m^{(k)})(k=1,\cdots,r)$ lie to the left of the contour L_k. The condition for absolute convergence of multiple Mellin-Barnes type contour (1.1) can be obtained of the corresponding conditions for multivariable H-function given by: as

$|arg(z_k)|<\frac{1}{2}A_i^{(k)}\pi$ where

$$A_i^{(k)}=\sum_{j=1}^{m^{(k)}}D_j^{(k)}\delta_j^{(k)}+\sum_{j=1}^{n^{(k)}}C_j^{(k)}\gamma_j^{(k)}-\tau_{i^{(k)}}\left(\sum_{j=m^{(k)}+1}^{q_i^{(k)}}D_{ji^{(k)}}^{(k)}\delta_{ji^{(k)}}^{(k)}+\sum_{j=n^{(k)}+1}^{p_i^{(k)}}C_{ji^{(k)}}^{(k)}\gamma_{ji^{(k)}}^{(k)}\right)+$$

$$-\tau_{i_2}\left(\sum_{j=n_2+1}^{p_{i_2}}A_{2ji_2}\alpha_{2ji_2}^{(k)}+\sum_{j=1}^{q_{i_2}}B_{2ji_2}\beta_{2ji_2}^{(k)}\right)-\cdots-\tau_{i_r}\left(\sum_{j=n_r+1}^{p_{i_r}}A_{rji_r}\alpha_{rji_r}^{(k)}+\sum_{j=1}^{q_{i_r}}B_{rji_r}\beta_{rji_r}^{(k)}\right) \tag{1.16}$$

Following the lines of Braaksma ([5] p. 278), we may establish the asymptotic expansion in the following convenient form:

$$\aleph(z_1,\cdots,z_r)=0(\,|z_1|^{\alpha_1},\cdots,|z_r|^{\alpha_r}\,),max(\,|z_1|,\cdots,|z_r|\,)\rightarrow 0$$

$$\aleph(z_1,\cdots,z_r)=0(\,|z_1|^{\beta_1},\cdots,|z_r|^{\beta_r}\,),min(\,|z_1|,\cdots,|z_r|\,)\rightarrow\infty \text{ where } i=1,\cdots,r:$$

$$\alpha_i=\min_{1\leqslant j\leqslant m^{(i)}}Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right] \text{ and } \beta_i=\max_{1\leqslant j\leqslant n^{(i)}}Re\left[C_j^{(i)}\left(\frac{c_j^{(i)}-1}{\gamma_j^{(i)}}\right)\right]$$

Remark 1.

If $n_2=\cdots=n_{r-1}=p_{i_2}=q_{i_2}=\cdots=p_{i_{r-1}}=q_{i_{r-1}}=0$ and $A_{2j}=A_{2ji_2}=B_{2ji_2}=\cdots=A_{rj}=A_{rji_r}=B_{rji_r}=1$ $A_{rj}=A_{rji_r}=B_{rji_r}=1$, then the multivariable Gimel-function reduces in the multivariable Aleph- function defined by Ayant [3].

Remark 2.

If $n_2=\cdots=n_r=p_{i_2}=q_{i_2}=\cdots=p_{i_r}=q_{i_r}=0$ and $\tau_{i_2}=\cdots=\tau_{i_r}=\tau_{i^{(1)}}=\cdots=\tau_{i^{(r)}}=R_2=\cdots=R_r=R^{(1)}=\cdots=R^{(r)}=1$, then the multivariable Gimel-function reduces in a multivariable I-function defined by Prathima et al. [10].

Remark 3.

If $A_{2j}=A_{2ji_2}=B_{2ji_2}=\cdots=A_{rj}=A_{rji_r}=B_{rji_r}=1$ and $\tau_{i_2}=\cdots=\tau_{i_r}=\tau_{i^{(1)}}=\cdots=\tau_{i^{(r)}}=R_2=\cdots=R_r=R^{(1)}=\cdots=R^{(r)}=1$, then the generalized multivariable Gimel-function reduces in multivariable I-function defined by Prasad [9].

Remark 4.

If the three above conditions are satisfied at the same time, then the generalized multivariable Gimel-function reduces in the multivariable H-function defined by Srivastava and Panda [18,19].

II. Integral Representation of Generalized Hypergeometric Function

The following generalized hypergeometric function regarding multiple integrals contour is also [12,p. 39, Eq. (30)]

$$\frac{\prod_{j=1}^{P}\Gamma(A_j)}{\prod_{j=1}^{Q}\Gamma(B_j)}\,{}_PF_Q\left[(A_P);(B_Q);-(x_1+\cdots+x_r)\right]$$

Ref

3. F. Ayant, An integral associated with the Aleph-functions of several variables. International Journal of Mathematics Trends and Technology (IJMTT), 31(3) (2016), 142-154.

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\frac{\prod_{j=1}^{P}\Gamma\left(A_j+\sum_{i=1}^{r}s_i\right)}{\prod_{j=1}^{Q}\Gamma\left(B_j+\sum_{i=1}^{r}s_i\right)}\Gamma(-s_1)\cdots\Gamma(-s_r)x_1^{s_1}\cdots x_r^{s_r}\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{2.1}$$

where the contours are of Barnes type with indentations, if necessary , to ensure that the poles $\Gamma\left(A_j+\sum_{i=1}^{r}s_i\right)$ are separated from those of $\Gamma(-s_j), j=1,\cdots,r$. The above result (2.1) is easily established by an appeal to the calculus of residues by calculating the residues at the poles of $\Gamma(-s_j), j=1,\cdots,r$.

III. Main Integral

Notes

We shall use the following notations :

$$\mathbb{A} = [(\mathrm{a}_{2j};\alpha_{2j}^{(1)},\alpha_{2j}^{(2)};A_{2j})]_{1,n_2},[\tau_{i_2}(a_{2ji_2};\alpha_{2ji_2}^{(1)},\alpha_{2ji_2}^{(2)};A_{2ji_2})]_{n_2+1,p_{i_2}},[(a_{3j};\alpha_{3j}^{(1)},\alpha_{3j}^{(2)},\alpha_{3j}^{(3)};A_{3j})]_{1,n_3},$$

$$[\tau_{i_3}(a_{3ji_3};\alpha_{3ji_3}^{(1)},\alpha_{3ji_3}^{(2)},\alpha_{3ji_3}^{(3)};A_{3ji_3})]_{n_3+1,p_{i_3}};\cdots;[(\mathrm{a}_{(r-1)j};\alpha_{(r-1)j}^{(1)},\cdots,\alpha_{(r-1)j}^{(r-1)};A_{(r-1)j})]_{1,n_{r-1}},$$

$$[\tau_{i_{r-1}}(a_{(r-1)ji_{r-1}};\alpha_{(r-1)ji_{r-1}}^{(1)},\cdots,\alpha_{(r-1)ji_{r-1}}^{(r-1)};A_{(r-1)ji_{r-1}})]_{n_{r-1}+1,p_{i_{r-1}}} \tag{3.1}$$

$$\mathbf{A} = [(\mathrm{a}_{rj};\alpha_{rj}^{(1)},\cdots,\alpha_{rj}^{(r)},\underbrace{0,\cdots,0}_{l+T};A_{rj})]_{1,n_r},[\tau_{i_r}(a_{rji_r};\alpha_{rji_r}^{(1)},\cdots,\alpha_{rji_r}^{(r)},\underbrace{0,\cdots,0}_{l+T};A_{rji_r})]_{\mathfrak{n}+1,p_{i_r}} \tag{3.2}$$

$$A_1 = \left[1+\sigma_j^{(1)}-\sum_{l=1}^{u}K_l\rho_j''^{(1,l)}-\sum_{i=1}^{s}\eta_{G_i,g_i}\rho_j^{(1,i)}-\theta_j^{(1)}k;\rho_j'^{(1,1)},\cdots,\rho_j'^{(1,r)}\tau_j^{(1,1)},\cdots,\tau_j^{(1,l)},1,\underbrace{0,\cdots,0}_{T-1};1\right]_{1,s},\cdots,$$

$$\left[1+\sigma_j^{(T)}-\sum_{l=1}^{u}K_l\rho_j''^{(T,l)}-\sum_{i=1}^{s}\eta_{G_i,g_i}\rho_j^{(T,i)}-\theta_j^{(T)}k;\rho_j'^{(T,1)},\cdots,\rho_j'^{(T,r)}\tau_j^{(T,1)},\cdots,\tau_j^{(T,l)},1,\underbrace{0,\cdots,0}_{T-1};1\right]_{s1,},$$

$$[1-A_j;\underbrace{0,\cdots,0}_{r},\underbrace{1,\cdots,1}_{l},\underbrace{0,\cdots,0}_{T};1]_{1,P},$$

$$\left[1-\alpha_j-\sum_{l=1}^{u}K_l\delta_j''^{(l)}-\sum_{i=1}^{s}\eta_{G_i,g_i}\delta_j^{(i)}-\zeta_j^{(1)}k;\delta_j'^{(1)},\cdots,\delta_j'^{(r)}\mu_i^{(1)},\cdots,\mu_j^{(l)},\underbrace{1,\cdots,1}_{W},\underbrace{0,\cdots,0}_{W+1,T};1\right]_{1,s},$$

$$\left[1-\beta_j-\sum_{l=1}^{u}K_l\eta_j''^{(l)}-\sum_{i=1}^{s}\eta_{G_i,g_i}\eta_j^{(i)}-\lambda_j^{(1)}k;\eta_j'^{(1)},\cdots,\eta_j'^{(r)}\theta_i^{(1)},\cdots,\theta_j^{(l)},\underbrace{1,\cdots,1}_{W},\underbrace{0,\cdots,0}_{W+1,T};1\right]_{1,s} \tag{3.3}$$

$$A = [(\mathrm{c}_j^{(1)},\gamma_j^{(1)};C_j^{(1)}]_{1,n^{(1)}},[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)},\gamma_{ji^{(1)}}^{(1)};C_{ji^{(1)}}^{(1)})]_{n^{(1)}+1,p_i^{(1)}};\cdots;$$

$$[(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})]_{1,m^{(r)}},[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)},\gamma_{ji^{(r)}}^{(r)};C_{ji^{(r)}}^{(r)})]_{m^{(r)}+1,p_i^{(r)}}\underbrace{(1,0;1),\cdots,(1,0;1)}_{l},\underbrace{(1,0;1),\cdots,(1,0;1)}_{T} \tag{3.4}$$

$$\mathbb{B} = [\tau_{i_2}(b_{2ji_2};\beta_{2ji_2}^{(1)},\beta_{2ji_2}^{(2)};B_{2ji_2})]_{1,q_{i_2}},[\tau_{i_3}(b_{3ji_3};\beta_{3ji_3}^{(1)},\beta_{3ji_3}^{(2)},\beta_{3ji_3}^{(3)};B_{3ji_3})]_{1,q_{i_3}};\cdots;$$

$$[\tau_{i_{r-1}}(b_{(r-1)ji_{r-1}};\beta_{(r-1)ji_{r-1}}^{(1)},\cdots,\beta_{(r-1)ji_{r-1}}^{(r-1)};B_{(r-1)ji_{r-1}})]_{1,q_{i_{r-1}}} \tag{3.5}$$

$$B_1 = \left[1+\sigma_j^{(1)} - \sum_{l=1}^{u} K_l \rho_j''^{(1,l)} - \sum_{i=1}^{s} \eta_{G_i,g_i} \rho_j^{(1,i)} - \theta_j^{(1)} k; \rho_j'^{(1,1)}, \cdots, \rho_j'^{(1,r)} \tau_j^{(1,1)}, \cdots, \tau_j^{(1,l)}, \underbrace{0,\cdots,0}_{T}; 1\right]_{1,s}, \cdots,$$

$$\left[1+\sigma_j^{(T)} - \sum_{l=1}^{u} K_l \rho_j''^{(T,l)} - \sum_{i=1}^{s} \eta_{G_i,g_i} \rho_j^{(T,i)} - \theta_j^{(T)} k; \rho_j'^{(T,1)}, \cdots, \rho_j'^{(T,r)} \tau_j^{(T,1)}, \cdots, \tau_j^{(T,l)}, \underbrace{0,\cdots,0}_{T}; 1\right]_{1,s}, \cdots,$$

Notes

$$[1-B_j; \underbrace{0,\cdots,0}_{r}, \underbrace{1,\cdots,1}_{l}, \underbrace{0,\cdots,0}_{T}; 1]_{1,Q},$$

$$\left[1-\alpha_j - \sum_{l=1}^{u} K_l \delta_j''^{(l)} - \sum_{i=1}^{s} \eta_{G_i,g_i} \delta_j^{(i)} - \zeta_j^{(1)} k; \delta_j'^{(1)}, \cdots, \delta_j'^{(r)} \mu_i^{(1)}, \cdots, \mu_j^{(l)}, \underbrace{1,\cdots,1}_{W}, \underbrace{0,\cdots,0}_{W+1,T}; 1\right]_{1,s},$$

$$\left[1-\alpha_j-\beta_j - \sum_{l=1}^{u} K_l(\delta_j''^{(l)} + \eta_j''^{(l)}) - \sum_{i=1}^{s} \eta_{G_j,g_j}(\delta_j^{(k)} + \eta_j^{(k)}) - k(\zeta_j+\lambda_j),\right.$$

$$\left.(\delta_j'^{(1)} + \eta_j'^{(1)}), \cdots, (\delta_j'^{(r)} + \eta_j'^{(r)}), (\mu_j^{(1)} + \theta_j^{(1)}), \cdots, (\mu_j^{(l)} + \theta_j^{(l)}), \underbrace{1,\cdots,1}_{T}; 1\right] \tag{3.6}$$

$$\mathbf{B} = [\tau_{i_r}(b_{rji_r}; \beta_{rji_r}^{(1)}, \cdots, \beta_{rji_r}^{(r)}, \underbrace{0,\cdots,0}_{l+T}; B_{rji_r})]_{1,q_{i_r}} \tag{3.7}$$

$$\mathrm{B} = [(\mathrm{d}_j^{(1)}, \delta_j^{(1)}; D_j^{(1)})]_{1,m^{(1)}}, [\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)}, \delta_{ji^{(1)}}^{(1)}; D_{ji^{(1)}}^{(1)})]_{m^{(1)}+1,q_i^{(1)}}; \cdots;$$

$$[(\mathrm{d}_j^{(r)}, \delta_j^{(r)}; D_j^{(r)})]_{1,m^{(r)}}, [\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)}, \delta_{ji^{(r)}}^{(r)}; D_{ji^{(r)}}^{(r)})]_{m^{(r)}+1,q_i^{(r)}}; \underbrace{(0,1;1),\cdots,(0,1;1)}_{l}; \underbrace{(0,1;1),\cdots,(0,1;1)}_{T} \tag{3.8}$$

$$U = 0, n_2; 0, n_3; \cdots; 0, n_r; V = m^{(1)}, n^{(1)}; m^{(2)}, n^{(2)}; \cdots; m^{(r)}, n^{(r)}; \underbrace{(1,0),\cdots,(1,0)}_{l}; \underbrace{(1,0),\cdots,(1,0)}_{T}; \tag{3.9}$$

$$X = p_{i_2}, q_{i_2}, \tau_{i_2}; R_2; \cdots; p_{i_{r-1}}, q_{i_{r-1}}, \tau_{i_{r-1}} : R_{r-1}; Y = p_{i^{(1)}}, q_{i^{(1)}}, \tau_{i^{(1)}}; R^{(1)}; \cdots; p_{i^{(r)}}, q_{i^{(r)}}; \tau_{i^{(r)}}; R^{(r)}$$

$$\underbrace{(0,1),\cdots,(0,1)}_{l}; \underbrace{(0,1),\cdots,(0,1)}_{T} \tag{3.10}$$

Theorem

$$\int_{u_1}^{v_1} \cdots \int_{u_t}^{v_t} \prod_{i=1}^{t} \left[(x_i-u_i)^{\alpha_i-1}(v_i-x_i)^{\beta_i-1} \prod_{j=1}^{T} (U_i^{(j)} x_i + V_i^{(j)})^{\sigma_i^{(j)}}\right]$$

$$\bar{I}\begin{pmatrix} \mathrm{z}_1 \prod_{i=1}^{t} \frac{(x_i-u_i)^{\delta_i^{(1)}}(v_i-x_i)^{\eta_i^{(1)}}}{\prod_{j=1}^{T}(U_i^{(j)}x_i+V_i^{(j)})^{\rho_i^{(j,1)}}} \\ \vdots \\ \mathrm{z}_s \prod_{i=1}^{t} \frac{(x_i-u_i)^{\delta_i^{(s)}}(v_i-x_i)^{\eta_i^{(s)}}}{\prod_{j=1}^{T}(U_i^{(j)}x_i+V_i^{(j)})^{\rho_i^{(j,s)}}} \end{pmatrix} \gimel \begin{pmatrix} \mathrm{z}_1' \prod_{i=1}^{t} \frac{(x_i-u_i)^{\delta_i'^{(1)}}(v_i-x_i)^{\eta_i'^{(1)}}}{\prod_{j=1}^{T}(U_i^{(j)}x_i+V_i^{(j)})^{\rho_i'^{(j,1)}}} \\ \vdots \\ \mathrm{z}_r' \prod_{i=1}^{t} \frac{(x_i-u_i)^{\delta_i'^{(r)}}(v_i-x_i)^{\eta_r'^{(r)}}}{\prod_{j=1}^{T}(U_i^{(j)}x_i+V_i^{(j)})^{\rho_i'^{(j,r)}}} \end{pmatrix}$$

$$S^{M_1,\cdots,M_u}_{N_1,\cdots,N_u}\left(\begin{array}{c} \mathrm{z}''_1 \prod_{i=1}^{t} \frac{(x_i-u_i)^{\delta_i''^{(1)}}(v_i-x_i)^{\eta_i''^{(1)}}}{\prod_{j=1}^{T}(U_i^{(j)}x_i+V_i^{(j)})^{\rho_i''^{(j,1)}}} \\ \cdot \\ \cdot\ \cdot \\ \mathrm{z}''_u \prod_{i=1}^{t} \frac{(x_i-u_i)^{\delta_i''^{(u)}}(v_i-x_i)^{\eta_r''^{(u)}}}{\prod_{j=1}^{T}(U_i^{(j)}x_i+V_i^{(j)})^{\rho_i''^{(j,u)}}} \end{array}\right) E^{(\gamma_i),m}_{(\alpha_i),(\beta_i)}\left[z\prod_{j=1}^{j}\left[\frac{(x_i-u_i)^{\zeta_i}(v_i-x_i)^{\lambda_i}}{\prod_{j=1}^{T}(U_i^{(j)}x+V_i^{(j)})^{\theta_i^{(j)}}}\right]\right]$$

$${}_PF_Q\left[(A_P);(B_Q);-\sum_{k=1}^{l} g_k\left[\prod_{i=1}^{t}\left[(x_i-u_i)^{u_i^{(k)}}(v_i-x_i)^{\theta_i^{(k)}}\prod_{j=1}^{T}(U_i^{(j)}x_i+V_i^{(j)})^{\theta_i^{(j)}}\right]\right]\right]$$

$$=\frac{\prod_{j=1}^{Q}\Gamma(B_j)}{\prod_{j=1}^{P}\Gamma(A_j)}\prod_{j=1}^{t}\left[(v_i-u_i)^{\alpha_i+\beta_i+1}\prod_{j=1}^{W}(u_iU_i^{(j)}+v_i^{(j)})^{\sigma_i^{(j)}}\prod_{j=W+1}^{T}(u_iU_i^{(j)}+v_i^{(j)})^{\sigma_i^{(j)}}\right]$$

$$\sum_{k=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]}\sum_{h_i=1}^{M_i}\sum_{g_i=1}^{\infty}\phi_1\frac{\prod_{i=1}^{s}\phi_i z_i^{\eta_{h_i,g_i}}(-)^{\sum_{i=1}^{s}g_i}}{\prod_{i=1}^{s}\delta^{(i)}_{h^{(i)}}\prod_{i=1}^{s}g_i!}\frac{z^k}{k!}E_kA_u z_1''^{K_1}\cdots z_u''^{K_u}A_{ij}$$

$$\gimel^{U;0,n_r+sT+P+2s:V}_{X;p_{i_r}+sT+2s,q_{i_r}+sT+s,\tau_{i_r}:R_r:Y}\left(\begin{array}{c|c} \mathrm{z}'_1w_1 & \\ \cdot & \mathbb{A};\ \mathrm{A}_1,\mathbf{A}:A \\ \cdot & \cdot \\ \mathrm{z}'_rw_r & \cdot \\ \mathrm{g}_1W_1 & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{g}_lW_l & \cdot \\ \mathrm{G}_1 & \cdot \\ \cdot & \mathbb{B}\ ;\mathbf{B},\ \mathrm{B}_1:B \\ \cdot & \\ \mathrm{G}_T & \end{array}\right) \tag{3.11}$$

where

$$A_{ij}=\frac{1}{\prod_{i=1}^{t}\prod_{j=1}^{W}(u_iU_i^{j)}+V_i^{(j)})^{\sum_{k'=1}^{u}\rho_i''^{(j,k')}K_{k'}+\sum_{k''=1}^{s}\rho_i^{(j,k'')}+\theta_i^{(j)}k}}$$

$$\frac{(v_i-u_i)^{\sum_{k'=1}^{u}(\delta_i''^{(k')}+\eta_i''^{(k')})K_{k'}+\sum_{k''=1}^{s}(\delta_i^{(k'')}+\eta_i^{(k'')})\eta_{G_{k''},g_{k''}}+(\zeta_i+\lambda_i)k}}{\prod_{i=1}^{t}\prod_{j=W+1}^{T}(u_iU_i^{j)}+V_i^{(j)})^{\sum_{k'=1}^{u}\rho_i''^{(j,k')}K_{k'}+\sum_{k''=1}^{s}\rho_i^{(j,k'')}+\theta_i^{(j)}k}} \tag{3.12}$$

$$w_K=\prod_{i=1}^{t}\left[(v_i-u_i)^{\delta_i'^{(K)}+\eta_i'^{(K)}}\prod_{j=1}^{W}(u_iU_i^{(j)}+v_i^{(j)})^{-\rho_i'^{(j,K)}}\prod_{j=W+1}^{T}(u_iU_i^{(j)}+v_i^{(j)})^{-\rho_i'^{(j,K)}}\right]:K=1,\cdots,s \tag{3.13}$$

$$W_L=\prod_{i=1}^{t}\left[(v_i-u_i)^{\mu_i'^{(L)}+\theta_i'^{(L)}}\prod_{j=1}^{W}(u_iU_i^{(j)}+v_i^{(j)})^{-\tau_i^{(j,L)}}\prod_{j=W+1}^{T}(u_iU_i^{(j)}+v_i^{(j)})^{-\tau_i^{(j,L)}}\right]:L=1,\cdots,l \tag{3.14}$$

$$G_j=\prod_{i=1}^{t}\left[\frac{(v_j-u_i)U_i^{(j)}}{u_iU_i^{(j)}+V_i^{(j)}}\right];j=1,\cdots,W \tag{3.15}$$

Notes

$$G_j = -\prod_{i=1}^{t}\left[\frac{(v_j-u_i)U_i^{(j)}}{u_iU_i^{(j)}+V_i^{(j)}}\right]; j = W+1,\cdots,T \tag{3.16}$$

provided

$W \in [0,T]; u_i, v_i \in \mathbb{R}; i = 1,\cdots,t$

$min\{\delta_i^{(g)}, \eta_i^{(g)}, \delta_i'^{(h)}, \eta_i'^{(h)}, \delta_i''^{(k)}, \eta_i''^{(k)}, \zeta_i, \eta_i\} \geqslant 0; g = 1,\cdots,s; i = 1,\cdots,t; h = 1,\cdots,r; k = 1,\cdots,u$

$min\{\rho_i^{(j,g)}, \rho_i'^{(j,h)}, \rho_i'^{(j,k')}, \theta_i^{j)}, \tau_i^{(j,k)}\} \geqslant 0; j = 1,\cdots,T; i = 1,\cdots,t; g = 1,\cdots,s; h = 1,\cdots,r; k' =,\cdots,v; k = 1,\cdots,l.$

$\sigma_i^{(j)} \in \mathbb{R}, U_i^{(j)}, V_i^{(j)} \in \mathbb{C}, z_{i'}, z_{j'}z_k'', G_j \in \mathbb{C}, i = 1,\cdots,t; j = 1,\cdots,T; i' = 1,\cdots,s, j' = 1,\cdots,r; k' = 1,\cdots,v; k = 1,\cdots,l.$

$\alpha_i, \beta_i, \gamma_i \in \mathbb{C}, i = 1,\cdots,m, Re(\alpha_i) > 0$

$max\left[\left|\frac{(v_j-u_i)U_i^{(j)}}{u_iU_i^{(j)}+V_i^{(j)}}\right|\right] < 1; i = 1,\cdots,s; j = 1,\cdots,W$ and

$max\left[\left|\frac{(v_j-u_i)U_i^{(j)}}{u_iU_i^{(j)}+V_i^{(j)}}\right|\right] < 1; i = 1,\cdots,s; j = W+1,\cdots,T$

$$\left|arg\left(z_i'\prod_{j=1}^{T}(U_i^{(j)}x_i+V_i^{(j)})^{\rho_i'^{(j,k)}}\right)\right| < \frac{1}{2}\left(A_i'^{(k)} - \delta_i'^{(k)} - \eta_i'^{(k)} - \sum_{j=1}^{T}\rho_i'^{(j,k)}\right)\pi > 0$$

where $A_i^{(k)}$ is defined by (1.16).

$$Re\left(\alpha_i + \zeta_i k + \sum_{j=1}^{r}\delta_i^{(j)}\eta_{G_j,g_j}\right) + \sum_{K=1}^{r}\delta_i'^{(K)}\min_{1\leqslant j\leqslant m^{(i)}} Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right] > 0 \text{ and}$$

$$Re\left(\beta_i + \lambda_i k + \sum_{j=1}^{s}\eta_i^{(j)}\eta_{Gj,g_j}\right) + \sum_{K=1}^{r}\eta_i'^{(K)}\min_{1\leqslant j\leqslant m^{(i)}} Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right] > 0 \text{ for } i = 1,\cdots,t.$$

$P \leqslant Q+1$. The equality holds, also,

$$\text{either } P > P \text{ and } \sum_{k=0}^{l}\left|g_k\left(\prod_{j=1}^{T}(U_i^{(j)}x_i+V_i^{(j)})^{\tau_i^{(i,k)}}\right)\right|^{\frac{1}{Q-P}} < 1 \quad (u_i \leqslant x_i \leqslant v_i; i = 1,\cdots,t)$$

$$P \leqslant Q \text{ and } \max_{1\leqslant k\leqslant l}\left|g_k\left(\prod_{j=1}^{T}(U_i^{(j)}x_i+V_i^{(j)})^{\tau_i^{(i,k)}}\right)\right| < 1 \quad (u_i \leqslant x_i \leqslant v_i; i = 1,\cdots,t)$$

Proof

To establish the main theorem, we first express the class of multivariable polynomials $S_{N_1,\cdots,N_u}^{M_1,\cdots,M_u}[.]$, the multivariable Ifunction defined by Prathima et al. [10], the 3m-parametric Mittag-Leffler type functions in series with the help of (1.4), (1.10) and (1.2) respectively, Further, using the Melin-Barnes multiple integrals contour representation for the multivariable Gimel-function and use the multiple integrals contour representation with the help of (2.1) for the generalized hypergeometric function ${}_PF_Q$. Interchanging the order of integrations and summations suitably, which ispermissible under the conditions stated above. Now we write

Ref

10. J. Prathima, V. Nambisan and S.K. Kurumujji, A Study of I-function of Several Complex Variables, International Journal of Engineering Mathematics Vol (2014), 1-12.

$$\prod_{j=1}^{T}(U_i^{(j)}x_i+V_i^{(j)})^{K_i^{(j)}}=\prod_{j=1}^{W}(U_i^{(j)}x_i+V_i^{(j)})^{K_i^{(j)}}\prod_{j=W+1}^{T}(U_i^{(j)}x_i+V_i^{(j)})^{K_i^{(j)}} \tag{3.17}$$

Where

$$K_i^{(j)}=v_i^{(j)}-\theta_i^{(j)}k-\sum_{l=1}^{s}\rho_i^{(j,l)}\eta_{G_l,g_l}-\sum_{l=1}^{r}\rho_i^{\prime(j,l)}\psi_l-\sum_{l=1}^{u}\rho_i^{\prime\prime(j,l)}K_l \text{ where } i=1,\cdots t; j=1,\cdots,T$$

and express the factor occurring in right hand side of (3.11), regarding the following Mellin-Barnes integrals, we obtain,

$$\prod_{j=1}^{W}(U_i^{(j)}x_i+V_i^{(j)})^{K_i^{(j)}}=\prod_{j=1}^{W}\left[\frac{U_i^{(j)}u_i+V_i^{(j)})^{K_i^{(j)}}}{\Gamma(-K_i^{(j)})}\right]\frac{1}{(2\pi\omega)^{W}}\int_{L_1'}\cdots\int_{L_W'}\prod_{j=1}^{W}\left[\Gamma(-\xi_j')\Gamma(-K_i^{(j)}+\xi_j')\right)$$

$$\prod_{j=1}^{W}\left[\frac{U_i^{(j)}(x_i-u_i)}{u_iU_i^{(j)}+V_i^{(j)}}\right]^{\xi_i'}\mathrm{d}\xi_1'\cdots\mathrm{d}\xi_W' \tag{3.18}$$

and

$$\prod_{j=1}^{T}(U_i^{(j)}x_i+V_i^{(j)})^{K_i^{(j)}}=\prod_{j=W+1}^{T}\left[\frac{U_i^{(j)}u_i+V_i^{(j)})^{K_i^{(j)}}}{\Gamma(-K_i^{(j)})}\right]\frac{1}{(2\pi\omega)^{T-W}}\int_{L_{W+1}'}\cdots\int_{L_{L_T'}}\prod_{j=W+1}^{T}\left[\Gamma(-\xi_j')\Gamma(-K_i^{(j)}+\xi_j')\right)$$

$$\prod_{j=1}^{W}\left[\frac{U_i^{(j)}(x_i-u_i)}{u_iU_i^{(j)}+V_i^{(j)}}\right]^{\xi_i'}\mathrm{d}\xi_1'\cdots\mathrm{d}\xi_W' \tag{3.19}$$

We apply the Fubini theorem for multiple integrals. Evaluating the innermost x-integral with the help of (1.1) and finally, reinterpreting the multiple Mellin-Barnes integrals contour regarding of multivariable Gimel-function of $(r+l+T)$, we obtain the desired result.

Remarks:
We obtain the same multiple Eulerian integrals about the functions cited in section I.
We obtain the same multiple eulerian integral about the general class of polynomials introduced, and studied by Srivastava [13].

$$S_V^U(x)=\sum_{\eta=0}^{[V/U]}\frac{(-V)_{U\eta}A_{V,\eta}}{\eta!}x^{\eta} \tag{3.20}$$

Where $V=0,1,\cdots$ and U is an arbitrary positive integer. The coefficients $A_{V,\eta}(V,\eta\geqslant 0)$ are arbitrary constants, real or complex. On suitably specializing the coefficients, $A_{V,\eta}$, $S_V^U(x)$, yields some of known polynomials, these include the Jacobi polynomials, Laguerre polynomials and others polynomials ([20], p. 158-161.)

IV. Conclusion

The importance of our all the results lies in their manifold generality. Firstly, in view of the multiple Eulerian integrals involving general class of multivariable polynomials, the multivariable I-function, the multivariable Gimel-function and the 3m-parametric Mittag-Leffler type functions with general arguments utilized in this study, we can obtain a large variety of single, double and several dimenionals Eulerian integrals. Secondly by specializing the various parameters as well as variables in the generalized multivariable Gimel-function, we get several formulae involving a remarkably wide variety of

Ref

13. H.M. Srivastava, A contour integral involving Fox's H-function, Indian. J. Math. 14(1972), 1-6.

useful functions (or product of such functions) which are expressible regarding the E, F, G, H, I, Alephfunction of one and several variables and simpler special functions of one and several variables. Hence the formulae derived in this paper are most general nature and may prove to be useful in several interesting cases appearing in the literature of Pure and Applied Mathematics and Mathematical Physics.

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Notes

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