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Realistic Polytropic Models for Neutral Stars with Vanishing Pressure Anisotropy

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Abstract- We find new exact models for the Einstein-Maxwell equations using the polytropic equation of state. The models generated satisfy uncharged star with anisotropy present. It is interesting that our anisotropic polytropic models contain isotropic case at the vanishing point of anisotropic parameters. In all models, the matter variables and gravitational potentials are well behaved. The radial and tangential pressures are compared at different values of polytropic index η.

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I. Introduction

The Einstein-Maxwell equations have been a strong tool to generate models that describe behaviors, properties and structures of relativistic stellar objects such as black holes, neutron stars, gravastars, dark energy stars and hybrid strange quark stars. Different space times geometry are considered when using the Einstein-Maxwell field equations. Using space time that is static and spherically symmetry, several findings on properties of stellar objects have been investigated. Using these field equations Sunzu et al [1, 2] found stellar masses and radii for charged quark matter consistent with several observations. Maharaj et al [3] found new exact solutions that describe Finch and Skea relativistic stars. The solutions to the field equations in static spacetimes obtained by Thirukkanesh and Maharaj [4] describe realistic compact anisotropic models. The astrophysical results obtained by Mafa Takisa and Maharaj [5] are for the anisotropic charged stars with core envelope and Matondo and Maharaj [6] obtained new models with astrophysical significance.

Stellar models with pressure anisotropy are important to be considered even in the absence of electric field. The first anisotropic model was developed by Bowers and Liang [7]. Since then there have been attention drawn by researchers to work in this direction. As indicated in the paper by Dev and Gleiser [8] that pressure anisotropy affects the structures and properties of many relativistic objects. Both the mass and redshift change with pressure anisotropy. According to Dev and Gleiser [9] stability of the stellar spheres do also depend on the pressure anisotropy. Models found by Gleiser and Dev [10] vindicate that the presence of pressure anisotropy may cause several observational effects. They proved that surface redshift for the stellar object may be large and stellar objects at large redshifts may be nearer than they really appear. This is caused by anisotropic distortions. It is shown that stars are more stable if the pressure anisotropy exists near its center. In the work by Sunzu et al [1] it was shown that anisotropic quark stars were less heavier compared to stars with isotropic pressures. Uncharged models for anisotropic bodies described by Kalam et al [11] were found to be compatible with quark strange star of the candidates SAXJ1808.4-3658, Her X-1 and 4U 1820-30. Other neutral anisotropic solutions in spacetime that is spherically symmetry are those obtained by Mak and Harko [12, 13], Harko and Mak [14], Sunzu [15], Maharaj and Chaisi [16, 17], Karmakar et al [18] and quark strange star models determined by Paul et al [19].

Several equation of states have been incorporated together with field equations in order to generate stellar models. We have many charged anisotropic stellar models obtained by considering a linear equation of state: regular compact exact models were formulated by Mafa Takisa and Maharaj [20], exact nonsingular models for quark stars are described by Sunzu et al [1, 2], Sunzu and Danford [21] and Maharaj et al [22], exact models for dark energy stars and strange stars were obtained by Thirukkanesh and Maharaj [4], isothermal anisotropic solutions were generated by Maharaj and Thirukkanesh [23] Sharma and Maharaj [24] obtained models for hybrid stars, and models for conformal invariant matter are indicated by Esculpi and Aloma [25].

Anisotropic models with linear equation of state for quark star in absence of charge are also determined by Sunzu [15]. Analytical models with a linear equation of state for isotropic charged quark stars are found by Komathiraj and Maharaj [26]. Under the same equation of state, isotropic quark star models were obtained by Bombaci [27], Sotani and Harada [28], and Sotani et al [29]. Models for strange stars with anisotropic pressures were found by Rahaman et al [30].

Models with quadratic equation of state include exact stellar models for the charged stars in presence of the pressure anisotropy found by Maharaj and Mafa Takisa [31] and Feroze and Siddiqui [32], models for compact stars obtained by Ngubelanga et al [33]. Other recent treatments in this direction are those performed

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by Sharov [34] and strange quark star model by Malaver [35]. Models generated using Van der Waals equation of state include the charged relativistic models in the paper by Malaver [36, 37], anisotropic compact models generated by Thirukkanesh and Ragel [38] and Sunzu and Mahali [39], and the treatments indicated in Lobo [40].

In the paper by Thirukkanesh and Ragel [41] models for neutral anisotropic compact objects with the polytropic equation of state are obtained. On the other hand Mafa Takisa and Maharaj [42] applied polytropic equation of state to generate anisotropic charged models. In the treatments by Shibata [43] polytropic models are performed by considering the stability of rigidly rotating objects. In the work by Lai and Xu [44] it is shown that huge quantity of gravitational energy is generated during gravitational collapse of polytropes. Most of the polytropic models are isotropic and those that are anisotropic have the anisotropy always present which is not physical. It is important to generate polytropic models with anisotropic pressures which contain isotropic pressures as a special case.

The objective of this paper is to generate uncharged anisotropic models using polytropic equation of state. The models contain isotropic pressures as a special case. In order to achieve this objective this paper is arranged as follows: In the following section, we state fundamental equations, transform the field equations and make choice for the anisotropy and one of the metric function. In Sect. (3) and Sect. (4) we formulate and find exact solutions for two polytropic models. We give discussion on the generated plots in Sect. (5) and the concluding remark is in Sect. (6).

II. Fundamental Equations

We generate models for the stellar object interior whose spacetime geometry is static and spherically symmetric and is represented by the line element

$$ds^2 = -e^{2\nu(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{1}$$

in here $\nu(r)$ and $\lambda(r)$ are functions for the gravity. The Schwarzschild exterior spacetime is defined by line element

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \left(1 - \frac{2M}{r}\right)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{2}$$

where M represents the total mass. The energy momentum tensor for neutral matter with pressure anisotropy is given by

$$T_{ij} = \mathrm{diag}\left(-\rho, p_r, p_t, p_t\right), \tag{3}$$

where ρ is the energy density, p_r is the radial pressure and p_t is the tangential pressure.

These variables are measured relative to a comoving unit timelike fluid four-velocity u^a.

The Einstein-Maxwell equations for anisotropic uncharged matter can be written in the form

$$\frac{1}{r^2}\left(1 - e^{-2\lambda}\right) + \frac{2\lambda'}{r}e^{-2\lambda} = \rho, \tag{4a}$$

$$-\frac{1}{r^2}\left(1 - e^{-2\lambda}\right) + \frac{2\nu'}{r}e^{-2\lambda} = p_r, \tag{4b}$$

$$e^{-2\lambda}\left(\nu'' + \nu'^2 - \nu'\lambda' + \frac{\nu'}{r} - \frac{\lambda'}{r}\right) = p_t, \tag{4c}$$

where primes define derivative with respect to radial distance r. In our model we are applying the units where the coupling constant $\frac{8\pi G}{c^4}$ and the speed of light c are unity.

The mass function for a neutral fluid is defined by

$$m(r) = \frac{1}{2}\int_0^r \omega^2 \rho d\omega. \tag{5}$$

We consider a polytropic equation of state which give the relationship between the radial pressure and the energy density in the form

$$p_r = \alpha\rho^{\left(1+\frac{1}{\eta}\right)}, \tag{6}$$

where η and α are real constants. It is important to note that different equation of states can be obtained by varying the value of polytropic index η.

In order to transform the field equations we use new variables given by Durgapal and Banerjee [45] as

$$x = Cr^2, \quad Z(x) = e^{-2\lambda(r)}, \quad A^2y^2(x) = e^{2\nu(r)}, \tag{7}$$

where A and C are arbitrary constants. Using this transformation the field equations become

$$\frac{1-Z}{x} - 2\dot{Z} = \frac{\rho}{C}, \tag{8a}$$

$$4Z\frac{\dot{y}}{y} + \frac{Z-1}{x} = \frac{p_r}{C}, \tag{8b}$$

$$4xZ\frac{\dot{y}}{y} + \left(4Z + 2x\dot{Z}\right)\frac{\dot{y}}{y} + \dot{Z} = \frac{p_t}{C}, \tag{8c}$$

and the line element (1) becomes

$$ds^2 = -A^2y^2dt^2 + \frac{1}{4xCZ}dx^2 + \frac{x}{C}(d\theta^2 + \sin^2\theta d\phi^2). \tag{9}$$

The transformed mass function (5) becomes

$$M(x) = \frac{1}{4C^{\frac{3}{2}}}\int_0^x \sqrt{\omega}\rho d\omega. \tag{10}$$

The field equations in the system (4) with polytropic equation of state can be written as

$$\rho = \left(\frac{1-Z}{x} - 2\dot{Z}\right)C, \tag{11a}$$

$$p_r = \alpha\rho^{\left(1+\frac{1}{\eta}\right)}, \tag{11b}$$

$$p_t = p_r + \Delta, \tag{11c}$$

$$\Delta = \left[4xZ\frac{\ddot{y}}{y} + \left(1 + 2x\frac{\dot{y}}{y}\right)\dot{Z} + \frac{1-Z}{x}\right]C. \tag{11d}$$

The variable $\Delta = p_t - p_r$ is the measure of anisotropy. This system consists of six unknown variables namely (ρ, p_r, p_t, Z, y, Δ) in four equations. The gravitational behavior of the anisotropic polytropic neutral star is governed by the system (11). Mathematically, if we specify any two of these unknown variables the system may be tractable. When $\Delta = 0$ we obtain isotropic model. From a mathematical point of view any two of the six variables can be specified in order to tract the system (11); however the choice should be made on physical grounds so that a model that is well behaved is generated.

Equation (11d) can be re-written as

$$\dot{Z} + \frac{(4x^2\ddot{y} - y)}{x(2x\dot{y} + y)}Z = \frac{\left(\frac{x\Delta}{C} - 1\right)y}{x(2x\dot{y} + y)}, \tag{12}$$

which is highly nonlinear equation in general. If y and Δ are specified then Eq. (12) is linear in the variable Z. In order to find exact solutions to this model we will specify the quantities y and Δ. We specify the metric function

$$y = (a + x^m)^n, \tag{13}$$

where a, m and n are real values. A similar choice was made by Komathiraj and Maharaj [26] and Mak and Harko [46] for a non-polytropic charged model. This choice guarantees that the metric function is regular and finite within the stellar interior. We specify the measure of anisotropy in the form

$$\Delta = A_1x + A_2x^2 + A_3x^3, \tag{14}$$

where A_1, A_2 and A_3 are real arbitrary constants. A similar choice of anisotropy was made in the paper by Maharaj et al [22] and Sunzu *et al* [1, 2] in charged models with linear equation of state. This choice is physically reasonable for it is continuous, finite and regular throughout the stellar interior. It is also possible to regain isotropic models ($\Delta = 0$) when $A_1 = A_2 = A_3 = 0$. It is important to keep this choice of the anisotropy for new polytropic model with vanishing anisotropy to be generated.

Substituting Eq. (13) and (14) in Eq. (12) we obtain the nonlinear differential equation

$$\dot{Z} + \left[\frac{-(a+x^m)^n + 4(B(x) + D(x))}{x\left[2mnx^m(a+x^m)^{n-1} + (a+x^m)^n\right]}\right]Z$$
$$= \frac{(a+x^m)^n\left(\frac{x(A_1x + A_2x^2 + A_3x^3)}{C} - 1\right)}{x\left[2mnx^m(a+x^m)^{n-1} + (a+x^m)^n\right]}, \tag{15}$$

where we have set

$$B(x) = nm^2(n-1)x^{2m}(a+x^m)^{n-2},$$
$$D(x) = mn(m-1)x^m(a+x^m)^{n-1},$$

for convenience. Once Eq. (15) is solved we can find the remaining variables ρ, p_r and p_t from the system (11). The general exact solution for Eq. (15) does not exist, however we can find its solution after specifying values for the constants m and n.

III. Polytropic Model I

We can find an exact solution to Eq. (15) when $m = 1$ and $n = 1$. For this choice the metric function (13) becomes

$$y(x) = (a + x). \tag{16}$$

The differential equation (15) becomes

$$\dot{Z}-\left(\frac{1}{x}-\frac{2}{a+3x}\right)Z=\frac{\left(\frac{A_1x+A_2x^2+A_3x^3)x}{C}-1\right)(a+x)}{x\,(a+3x)}. \tag{17}$$

Solving Eq. (17) we obtain the solution

$$\begin{aligned} Z &= \frac{1}{C}\left[\left(\frac{2}{5}a+\frac{1}{5}x\right)A_1x+\left(-\frac{3}{40}a^2+\frac{3}{20}ax+\frac{1}{8}x^2\right)A_2x\right.\\ &\left.+\left(\frac{1}{55}a^3-\frac{2}{55}a^2x+\frac{1}{11}ax^2+\frac{1}{11}x^3\right)A_3x+C+\frac{Ckx}{(a+3x)^{\frac{2}{3}}}\right], \end{aligned} \tag{18}$$

where k is a constant of integration.
Therefore the gravitational potentials and the matter variables becomes

$$e^{2\nu} = A^2(a+x)^2, \tag{19a}$$

$$\begin{aligned} e^{2\lambda} &= C\left[\left(\frac{2}{5}a+\frac{1}{5}x\right)A_1x+\left(-\frac{3}{40}a^2+\frac{3}{20}ax+\frac{1}{8}x^2\right)A_2x\right.\\ &\left.+\left(\frac{1}{55}a^3-\frac{2}{55}a^2x+\frac{1}{11}ax^2+\frac{1}{11}x^3\right)A_3x+C+\frac{Ckx}{(a+3x)^{\frac{2}{3}}}\right]^{-1}, \end{aligned} \tag{19b}$$

$$\begin{aligned} \rho &= \frac{1}{(a+3x)}\left[\left(-\frac{6}{5}a^2+\frac{23}{5}ax+3x^2\right)A_1\right.\\ &+\left(\frac{9}{40}a^3-\frac{3}{40}a^2x-\frac{25}{8}ax^2-\frac{21}{8}x^3\right)A_2\\ &-\left(\frac{3}{55}a^4-\frac{1}{55}a^3x+\frac{1}{11}a^2x^2+\frac{30}{11}ax^3+\frac{27}{11}x^4\right)A_3\\ &\left.-\frac{Ck(3a+5x)}{(a+3x)^{\frac{2}{3}}}\right], \end{aligned} \tag{19c}$$

$$\begin{aligned} p_r &= \frac{\alpha}{(a+3x)^{\left(1+\frac{1}{\eta}\right)}}\left[\left(-\frac{6}{5}a^2+\frac{23}{5}ax+3x^2\right)A_1\right.\\ &+\left(\frac{9}{40}a^3-\frac{3}{40}a^2x-\frac{25}{8}ax^2-\frac{21}{8}x^3\right)A_2\\ &-\left(\frac{3}{55}a^4-\frac{1}{55}a^3x+\frac{1}{11}a^2x^2+\frac{30}{11}ax^3+\frac{27}{11}x^4\right)A_3\\ &\left.-\frac{Ck(3a+5x)}{(a+3x)^{\frac{2}{3}}}\right]^{\left(1+\frac{1}{\eta}\right)}, \end{aligned} \tag{19d}$$

$$\begin{aligned} p_t &= A_1x+A_2x^2+A_3x^3\\ &+\frac{\alpha}{(a+3x)^{\left(1+\frac{1}{\eta}\right)}}\left[\left(-\frac{6}{5}a^2+\frac{23}{5}ax+3x^2\right)A_1\right.\\ &+\left.\frac{9}{40}a^3-\frac{3}{40}a^2x-\frac{25}{8}ax^2-\frac{21}{8}x^3\right)A_2 \end{aligned} \tag{19f}$$

$$-\left(\frac{3}{55}a^4-\frac{1}{55}a^3x+\frac{1}{11}a^2x^2+\frac{30}{11}ax^3+\frac{27}{11}x^4\right)A_3$$
$$-\left.\frac{Ck(3a+5x)}{(a+3x)^{\frac{2}{3}}}\right]^{\left(1+\frac{1}{\eta}\right)}, \tag{19e}$$
$$\Delta = A_1x+A_2x^2+A_3x^3. \tag{19f}$$

The line element corresponding to this model becomes

$$\begin{aligned} ds^2 &= -A^2(a+x)^2dt^2+ \\ &C\left[\left(\frac{2}{5}a+\frac{1}{5}x\right)A_1x+\left(-\frac{3}{40}a^2+\frac{3}{20}ax+\frac{1}{8}x^2\right)A_2x\right. \\ &\left.+\left(\frac{1}{55}a^3-\frac{2}{55}a^2x+\frac{1}{11}ax^2+\frac{1}{11}x^3\right)A_3x+C-\frac{Ckx}{(a+3x)^{\frac{2}{3}}}\right]^{-1}dr^2 \\ &+r^2(d\theta^2+\sin^2\theta d\phi^2), \end{aligned} \tag{20}$$

and mass function becomes

$$\begin{aligned} M(x) &= -\frac{x^{\frac{3}{2}}}{(a+3x)C^{\frac{3}{2}}}\left[\left(\frac{1}{5}a^2+\frac{7}{10}ax+\frac{3}{10}x^2\right)A_1\right. \\ &+\left(-\frac{3}{80}a^3-\frac{3}{80}a^2x+\frac{23}{80}ax^2+\frac{3}{16}x^3\right)A_2 \\ &+\left(\frac{1}{110}a^4+\frac{1}{110}a^3x-\frac{1}{110}a^2x^2+\frac{2}{11}ax^3+\frac{3}{22}x^4\right)A_3 \\ &\left.+\frac{1}{2}Ck(a+3x)^{\frac{1}{3}}\right]. \end{aligned} \tag{21}$$

IV. Polytropic Model II

We find second exact solution to Eq. (15) when $m = 1$ and $n = 2$. For this choice the metric function (13) becomes

$$y(x) = (a+x)^2. \tag{22}$$

Then Eq. (15) reduces to

$$\dot{Z}+\left(-\frac{1}{x}+\frac{2}{a+x}+\frac{2}{a+5x}\right)Z=\frac{\left(\frac{A_1x+A_2x^2+A_3x^3)x}{C}-1\right)(a+x)}{x(a+5x)}. \tag{23}$$

Solving Eq. (23) we obtain

$$Z=\frac{1}{(a+x)^2}\left[\left(a^2-\frac{10}{7}ax-\frac{1}{7}x^2\right)+\frac{kx}{(a+5x)^{\frac{2}{5}}}+\frac{F(x)}{C}\right], \tag{24}$$

where

$$F(x) = \left(\frac{38}{119}a^3+\frac{43}{119}a^2x+\frac{4}{17}ax^2+\frac{1}{17}x^3\right)A_1x$$

$$+\left(-\frac{109}{2618}a^4+\frac{109}{1309}a^3x+\frac{39}{187}a^2x^2+\frac{31}{187}ax^3+\frac{1}{22}x^4\right)A_2x$$

$$+\left(\frac{236}{35343}a^5-\frac{472}{35343}a^4x+\frac{236}{5049}a^3x^2+\frac{739}{5049}a^2x^3\right.$$

$$\left.+\frac{38}{297}ax^4+\frac{1}{27}x^5\right)A_3x. \tag{25}$$

Then gravitational potentials and the matter variables becomes

$$e^{2\nu} = A^2(a+x)^4, \tag{26a}$$

$$e^{2\lambda} = \frac{7(a+x)^2}{(7a^2-10ax-x^2)+\dfrac{7kx}{(a+5x)^{\frac{2}{5}}}+\frac{7F(x)}{C}}, \tag{26b}$$

$$\rho = \frac{C\left(\frac{72}{7}a^3+\frac{376}{7}a^2x+\frac{88}{7}ax^2+40x^3\right)+G(x)}{(a+x)^3(a+5x)},$$

$$+\frac{C(-3ka^2-10akx+9kx^2)}{(a+x)^3(a+5x)^{\frac{7}{5}}} \tag{26c}$$

$$p_r = \alpha\left[\frac{\left[C\left(\frac{72}{7}a^3+\frac{376}{7}a^2x+\frac{88}{7}ax^2+40x^3\right)+G(x)\right]}{(a+x)^3(a+5x)}\right.$$

$$\left.+\frac{C(-3ka^2-10akx+9kx^2)}{(a+x)^3(a+5x)^{\frac{7}{5}}}\right]^{\left(1+\frac{1}{\eta}\right)}, \tag{26d}$$

$$p_t = \alpha\left[\frac{\left[C\left(\frac{72}{7}a^3+\frac{376}{7}a^2x+\frac{88}{7}ax^2+40x^3\right)+G(x)\right]}{(a+x)^3(a+5x)}\right.$$

$$\left.+\frac{C(-3ka^2-10akx+9kx^2)}{(a+x)^3(a+5x)^{\frac{7}{5}}}\right]^{\left(1+\frac{1}{\eta}\right)}+A_1x+A_2x^2+A_3x^3, \tag{26e}$$

$$\Delta = A_1x+A_2x^2+A_3x^3, \tag{26f}$$

$$G(x) = -\left(\frac{1148}{119}a^5+\frac{747}{119}a^4x+\frac{1124}{119}a^3x^2+\frac{1342}{119}a^2x^3+\frac{110}{17}ax^4+\frac{25}{17}x^5\right)A_1$$

$$+\left(\frac{327}{2618}a^6+\frac{218}{1309}a^5x-\frac{10035}{2618}a^4x^2-\frac{12872}{1309}a^3x^3\right.$$

$$\left.-\frac{4457}{374}a^2x^4-\frac{1302}{187}ax^5-\frac{35}{22}x^6\right)A_2$$

$$-\left(\frac{236}{11781}a^7+\frac{944}{35343}a^6x-\frac{1888}{35343}a^5x^2+\frac{106973}{35343}a^4x^3\right.$$
$$\left.+\frac{47596}{5049}a^3x^4+\frac{60958}{5049}a^2x^5+\frac{2144}{297}ax^6+\frac{5}{3}x^7\right)A_3. \tag{27}$$

Note that $G(x)=0$ at the center of the stellar object and this condition is also satisfied for a model with isotropic pressures.

For this model the line element becomes

$$ds^2 = -A^2(a+x)^4dt^2+\frac{7(a+x^2)}{(7a^2-10ax-x^2)+\frac{7kx}{(a+5x)^{\frac{2}{5}}}+\frac{7F(x)}{C}}dr^2$$
$$+r^2(d\theta^2+\sin^2\theta d\phi^2), \tag{28}$$

and mass function is given by

$$M(x) = \frac{-x^{\frac{3}{2}}}{(a+x)^2(a+5x)C^{\frac{3}{2}}}\left[\left(\frac{19}{119}a^4+\frac{233}{238}a^3x+\frac{243}{238}a^2x^2+\frac{21}{34}ax^3+\frac{5}{34}x^4\right)A_1\right.$$
$$+\left(-\frac{109}{5236}a^5-\frac{327}{5236}a^4x+\frac{409}{1309}a^3x^2+\frac{113}{187}a^2x^3+\frac{327}{748}ax^4+\frac{5}{44}x^5\right)A_2$$
$$+\left(\frac{118}{35343}a^6+\frac{118}{11781}a^5x-\frac{118}{11781}a^4x^2+\frac{1919}{10098}a^3x^3\right.$$
$$\left.+\frac{1447}{3366}a^2x^4+\frac{67}{198}ax^5+\frac{5}{54}x^6\right)A_3-C\left(\frac{12}{7}a^2+\frac{64}{7}ax+\frac{20}{7}x^2\right.$$
$$\left.\left.-\frac{1}{2}k(a+5x)^{\frac{3}{5}}\right)\right]. \tag{29}$$

V. Discussion

In this section we indicate that the exact solutions for the field equations in Sect. (3) are well behaved. To do this we generate graphical plots for the gravitational potentials and matter variables. The Python programming language was used to generate graphs for the particular choices a = 5.2, A= 0.16, α = 0.33, η = 2, C = 1, k = 0, A_1 = 0.1, A_2 = 0.2, and A_3 =-0.2. The graphical plots generated are for the potential $e^{2\nu}$ (Fig. 1), potential $e^{2\lambda}$ (Fig. 2), energy density ρ (Fig. 3), radial pressure p_r (Fig. 4), tangential pressure p_t (Fig. 5), measure of anisotropy Δ (Fig. 6), and the mass M (Fig. 7). We have also generated graphs for radial pressure and tangential pressure at different values of the polytropic index η as indicated in (Fig. 8 and 9). The plots indicated in (Fig. 10-13) show comparison between radial pressure and tangential pressure at different values of η. All figures are plotted against the radial distance r. From (Fig. 1 and 2) we observe the gravitational potentials to be regular and finite which is physical. From Fig. (3 - 5) we see that the energy density, the radial pressure and the tangential pressure are decreasing functions as we approach the boundary from the center. This agrees with the physical behaviour of these variables. The anisotropy is increasing from the center to the region near the surface where it slightly decreases. We observe from Fig. (7) that the mass is monotonically increasing with the radial distance. We have different profiles for the radial and tangential pressures at different values of η. When η = 0.5 we have the model corresponding to the equation of state $p_r=\alpha\rho^3$. When η = 1.0 the equation of state becomes $p_r=\alpha\rho^2$ which is quadratic in nature. The equation of state $p_r=\alpha\rho^{\frac{3}{2}}$ when η = 2. For η = 2.5 and η = 3.0 then $p_r=\alpha\rho^{\frac{7}{5}}$ and $p_r=\alpha\rho^{\frac{4}{3}}$ respectively. Therefore the graphs presented in Fig. (8) and Fig. (9) compare the variation of these matter variables at different equation of states. However for each value of η the radial and tangential pressures are decreasing function with maximum value at the center of the stellar object. In general we observe when η is small the index $\left(1+\frac{1}{\eta}\right)$ becomes large as the result the radial pressure and tangential pressure becomes large

at the core of the stellar object. It is interesting to note that these values become equal toward the surface of the star regardless of the value of η. We also comment from Fig. (9) that the graphs for tangential pressure at $\eta = 2$ and $\eta = 3$ are merging. This may be due to the presence of polytropic index in both numerator and denominator in the equation for the tangential pressure. From Fig. (10-13) it is clear that these quantities are equal at the center and the region near the center. However at the regions away the center $p_t > p_r$. This is physical and agrees with the properties of stellar objects with astrophysical significance.

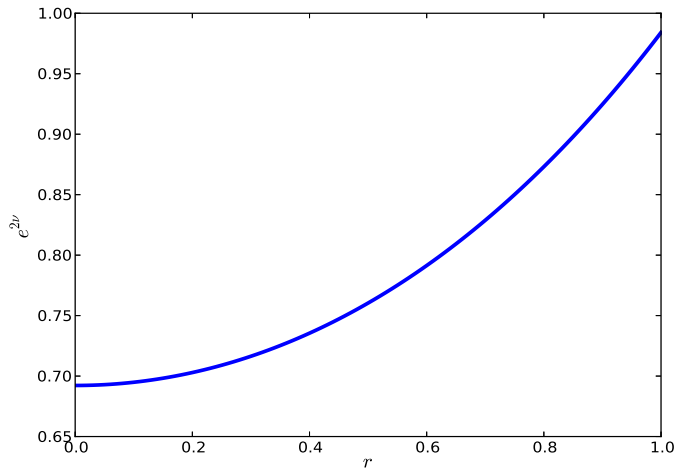

Figure 1: The potential e^{ν} against the radial distance r

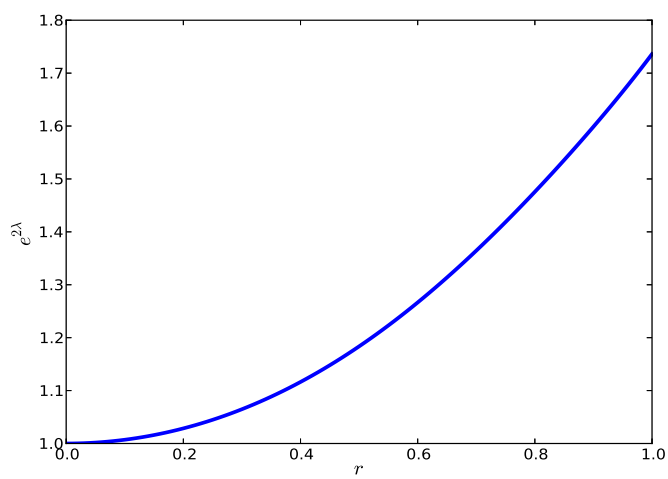

Figure 2: The potential e^{λ} against the radial distance r

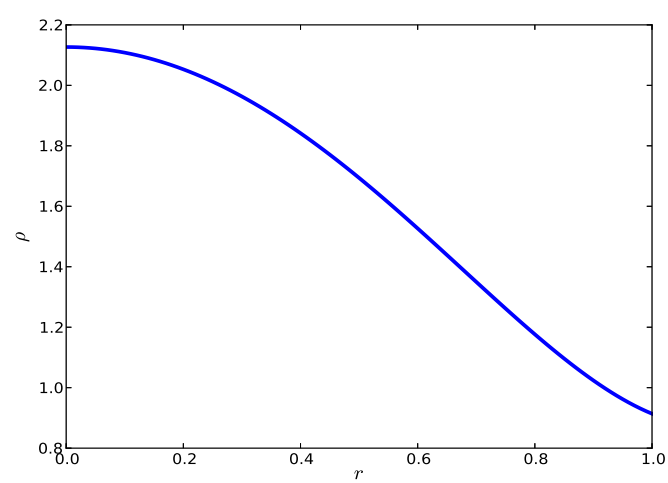

Figure 3: Energy density ρ against the radial distance r

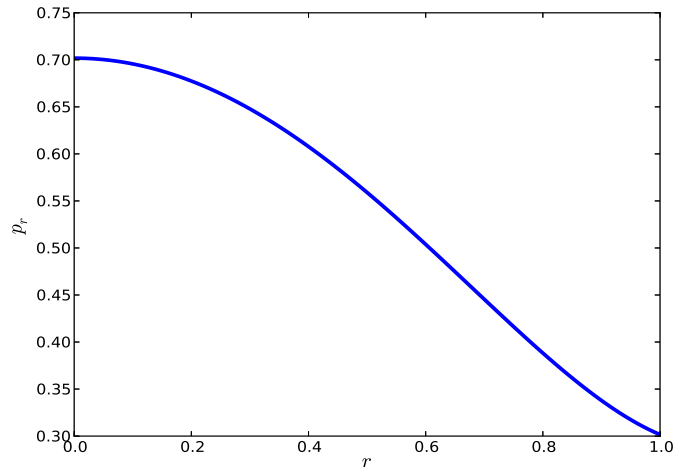

Figure 4: Radial pressure p_r against radial distance r

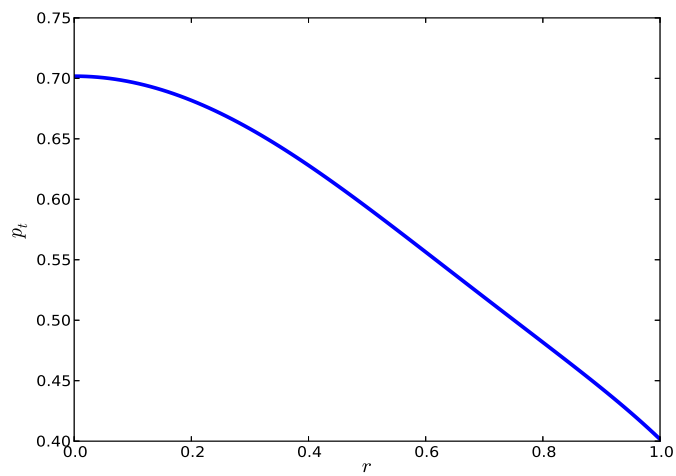

Figure 5: Tangential pressure p_t against radial distance r

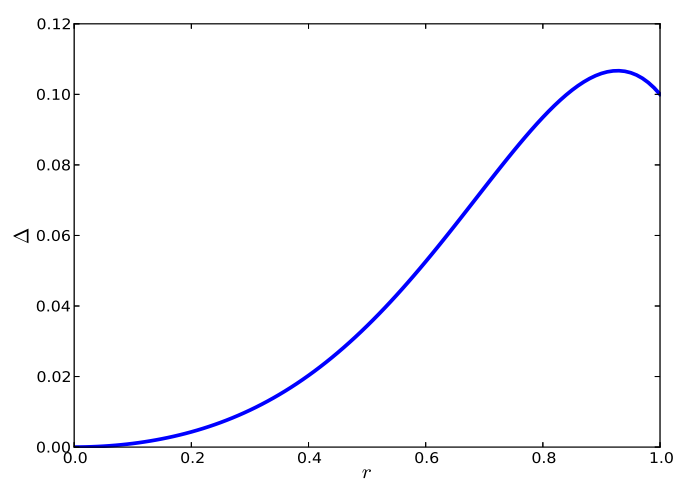

Figure 6: Measure of anisotropy Δ against radial distance r

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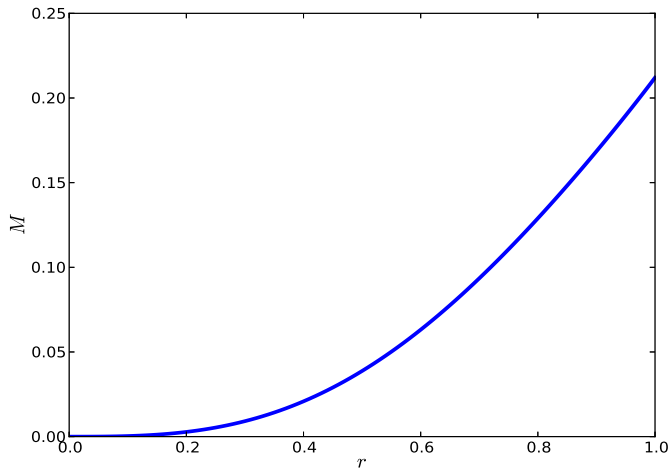

Figure 7: Mass m against radial distance r

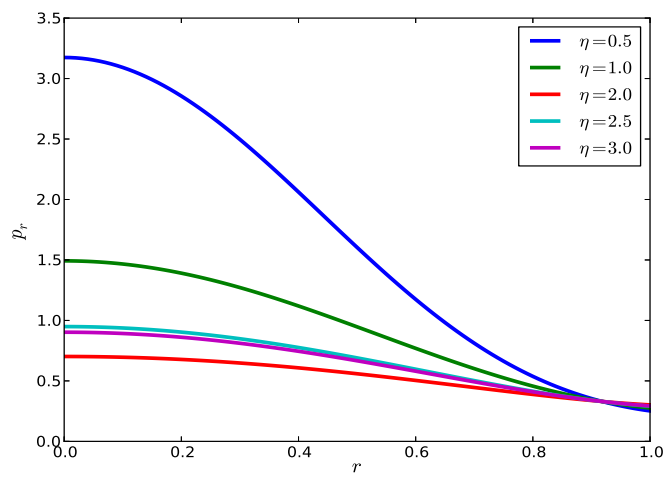

Figure 8: Radial pressures at different values of polytropic indices

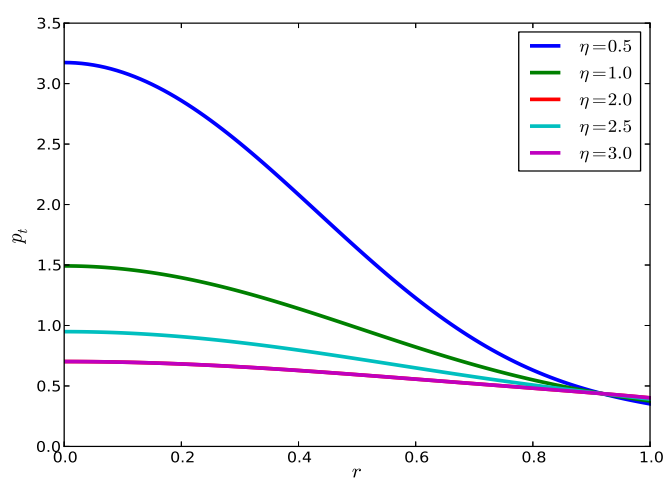

Figure 9: Tangential pressures at different values of polytropic indices

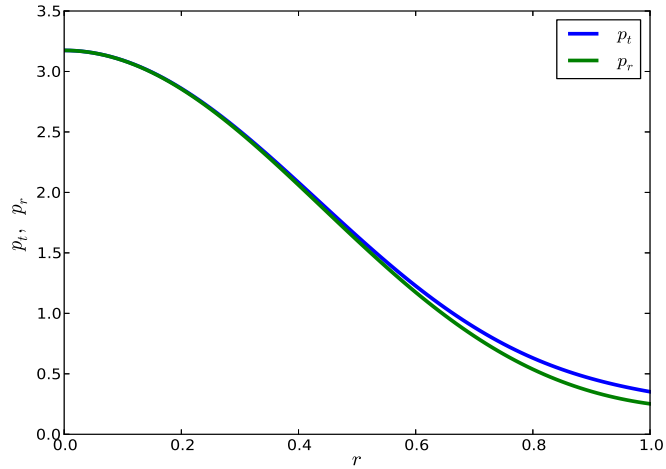

Figure 10: Comparison of the tangential and radial pressure at $\eta = 0.5$

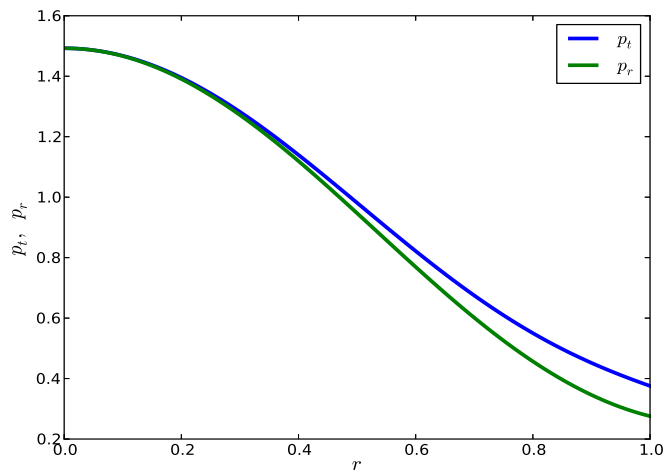

Figure 11: Comparison of the tangential and radial pressure at $\eta = 1.0$

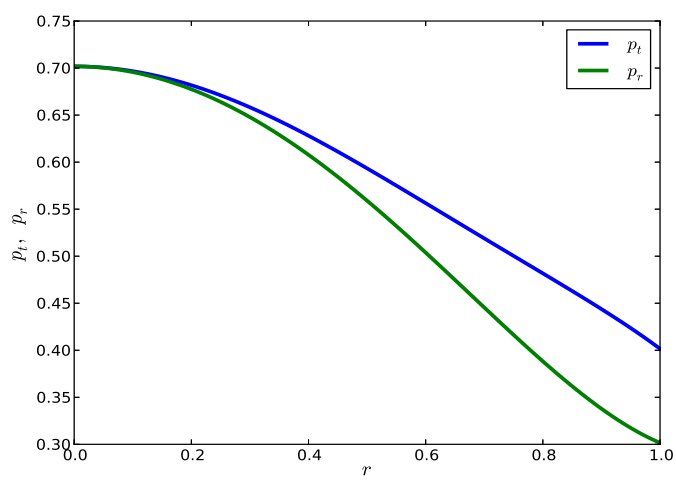

Figure 12: Comparison of the tangential and radial pressure at $\eta = 2.0$

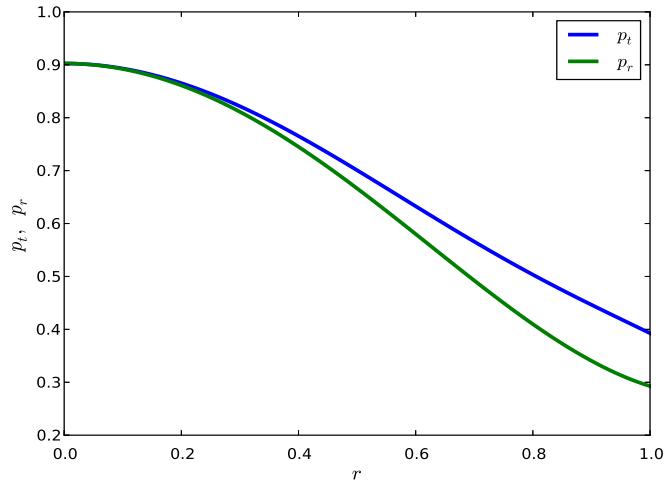

Figure 13: Comparison of the tangential and radial pressure at $\eta = 3{:}0$

VI. Conclusion

In this paper we have indicated that exact solutions for the Einstein Maxwell field equation are possible when the polytropic equations of state is incorporated. The polytropic models formulated describe relativistic stellar objects that admit no electromagnetic field distribution. We have indicated that for different values of polytropic index we obtain different variations for the radial and tangential pressures which is well behaved. We showed that the tangential and radial pressure are all equal at the center and its neighborhood, however the tangential pressure is greater than the radial pressure away the center of the stellar object. The results in this paper highlight new findings for polytropic models and are significant for the study of neutral anisotropic polytropic stellar objects. Other new results are possible when different forms for the equation of state is considered. This could be done by considering quadratic or Van der waal equation of state. Research in this direction is reserved for the future work.

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