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By C. Y. Lo & Li Hua Wang

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The Measurements of Directional Dependence of Weight for Magnets

C. Y. Lo ^α & Li Hua Wang ^σ

Abstract- To explain the weight reduction of a charged capacitor and Einstein's unification, it is necessary to confirm the existence of the repulsive charge-mass interaction and the attractive current-mass interaction. Therefore, to show the existence of the current-mass interaction, one must measure the effect of a small directional dependence of weight for a magnet. To do this, one must exclude the magnetic effect from the earth. In addition, one should avoid the influence of the magnet to the electronic scale for weighing. Here, we provide a method to measure and confirm such tiny effects of weight directional dependence experimentally. However, it is not effective to measure the small current-mass interaction directly. A problem is that a current must have a maintaining source unless in the super-conducting situation. Thus, this connection to the source would affect the measurements of the small current-mass interaction. Moreover, $E = mc^2$ is not always valid and Einstein failed his unification because, unlike Maxwell, he did not recognize that some additional interactions must be added to the original theories.

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I. INTRODUCTION

The directional dependence of weight for a permanence magnet is often over-looked because there is magnetic interaction between a magnet and the earth. It was difficult to conceive the small directional dependence of weight from the directional interaction effect between the earth and the magnet. Thus, to measure such a small directional effect on the weight of a magnet, one must first find a way to cancel out the magnetic effect between the earth and the magnet.

In addition, an accurate electronic scale of weight can often be affected when magnetism is involved. Thus, to measure the weight of a magnet, a method of measurement that can essentially avoid such an effect, must be conceived. The confirmation of such a directional effect is necessary to explain the weight reduction of a charged capacitor [1] as due to that the attractive force of current-mass interaction has been changed to a static repulsive charge-mass interaction, which is crucial for Einstein's unification of electromagnetism and gravitation [1, 2]. Moreover, since a discharged capacitor would not immediately recover its previous weight until the previous temperature is recovered [1], the effect of random motion of the electrons is essentially the same as static electrons.

Thus, it is clear that an increment of temperature would lead to a reduction of gravitation. This property has been verified by experiments of heated-up metals [3 - 5]. However, if current-mass interaction can increase weight, a magnet should have directional interaction for weight because the current-mass interaction is directional.

In this paper, to measure magnets and confirm such tiny directional effects, H. W. Li provided an ingenious method. As shown in Figure 1, he simply added a sufficiently long paper tube on the weighting surface of the electronic scale such that the length of the tube would make the influence of the magnet to the scale negligible.



Figure 1: A long paper tube is put on the surface of electronic scale to make the influence of the magnet to the scale negligible.

The magnet with N-up on the left picture and the magnet with S-up is in the right picture.

It has been identified that these effects of reducing or increasing weights are due to the charge-mass and the current-mass interactions. [1] Li is known as being the first experimental physicist who showed the remarkable temperature dependence of gravitation with a torsion balance scale [4].

A problem of the charge-mass interaction and related experiments is that it cannot be adequately explained with current theories. For example, the lifter (a light capacitor) is able to lift its own weight plus a payload after being charged with a high voltage (about 40 kilovolts), but without continuous supply of electric energy [6].

A lifter could get to work by charging the wire to either a positive or a negative potential although, at the beginning, a thrust is generated by moving the electrons of the lifter with one high voltage charge [6, 7]. From a flat capacitor, it had been speculated that the weight reduction is in the direction of the electric field [8, 9]. However, the lifter shows that such a weight reduction actually has no direction.

A problem is that such effects of weight reduction cannot be explained with current theories. Perhaps, it is time to remind theorists that the current physical laws are originally obtained from and tested by observations. Thus, the lack of explanation for phenomena is a sign for the birth of new physics.

II. THE CHARGED CAPACITORS AND THE CHARGE-MASS & THE CURRENT-MASS INTERACTIONS

It is known that a charged capacitor reduces its weight [1, 2]. However, in a charged capacitor, the only change is the state of some orbital electrons that have become statically concentrated. Since such force did not appear before, it is clear that such a force was cancelled out by the force created by the motion of the electrons. In other words, the repulsive force generated by the charges of protons and the electrons was cancelled out by the force generated by the motion of the initially orbital moving charges of the electrons.

However, this repulsive force cannot be proportional to the charge density although it is acting on charged particles due to their charge. We have equal numbers of negatively charged particles (electrons) and positively charged particles (protons) and that would lead to the cancellation of the forces generated by particles of different charges. However, if such a force is proportional to the charge density square, as shown by the solution of the static Einstein equation for a charged particle [10], then these two kinds of forces from different charges would be added up, instead of cancelling out.

Moreover, since a lifter has a limited height, one should expect that this repulsive gravitational force would diminish faster than the attractive gravitational force [6, 7]. Thus, if we assume that the force is proportional to mass as usual, the static charge-mass interaction would be a repulsive force between particles with charge density D_q and another particle of mass m would have the following form,

$$F_r \leq K m D_q^2 / r^n \quad \text{where } n > 2, \quad (1)$$

r is the distance between the two particles, and K is the coupling constant. In formula (1), the coupling constant K and n the power of r can be determined by experiments. The simplest case would be $n = 3$.

Formula (1) is derived from the observations. The experimental facts are that the charged capacitors have reduced weight. This reduced weight is caused by a repulsive force that can lift a device. Thus, such an approach has to be taken because there is little in common between the charge-mass interaction and current theories. [2]

III. THE REISSNER-NORDSTROM METRIC AND THE CHARGE-MASS INTERACTION

One would immediately recognize that a term similar to Eq. (1) appears in the Reissner-Nordstrom metric [9],

$$ds^2 = \left(1 - \frac{2M}{r} + \frac{q^2}{r^2} \right) dt^2 - \left(1 - \frac{2M}{r} + \frac{q^2}{r^2} \right)^{-1} dr^2 - r^2 d\Omega^2.$$

for a particle with charge q and mass M [9], where r is the distance from the center of the particle. The static force that acts on a test particle P with mass m for the first order approximation is

$$-m \frac{M}{r^2} + m \frac{q^2}{r^3} \quad (3)$$

since $g^{rr} \cong -1$. Note that the second term is a repulsive force due to the static charge-mass interaction. According to the reaction force being equal to but in the opposite direction of the acting force, the test particle P must create a field m/r^3 that couples to q^2 . This would mean that unification between electromagnetism and gravitation is necessary [2]. In the first term of (3), M is the inertial mass of the charged particle. [9]

However, the newly discovered force was overlooked until 1997 [11] after it was recognized that the mass is not equivalent to electric energy. This overlooking was due to two misconceptions: 1) Gravity is always attractive; [4] 2) $E = mc^2$ was incorrectly considered as unconditional [2]. The non-existence of a dynamic solution for the Einstein equation [5] leads to the discovery that there must be different coupling signs for the dynamic case. This investigation on the non-

uniqueness of couplings leads to the discovery of the charge-mass interaction [12-14].

The experiments on a charged ball confirm the existence of a repulsive charge-mass interaction (3) [15], and this would confirm the unification of electromagnetism and gravitation. Einstein over-looked the coupling of charge square in the five-dimensional theory [16] because he believed that a new interaction should not be added. Since formula (3) is generated by general relativity and thus is also a crucial test for general relativity [1, 2]. However, Einstein and his peers have mistaken that the electromagnetic energy was equivalent to mass [17].

IV. THE CHARGE-MASS INTERACTION AND FIVE-DIMENSIONAL THEORY

The force on particle Q from a test particle with mass m is beyond current theoretical framework of

$$\frac{d}{ds} \left(g_{ik} \frac{dx^k}{ds} \right) = \frac{1}{2} \frac{\partial g_{kl}}{\partial x^i} \frac{dx^k}{ds} \frac{dx^l}{ds} + \left(\frac{\partial g_{5k}}{\partial x^i} - \frac{\partial g_{5i}}{\partial x^k} \right) \frac{dx^5}{ds} \frac{dx^k}{ds} - \Gamma_{i,55} \frac{dx^5}{ds} \frac{dx^5}{ds} - g_{i5} \frac{d^2 x^5}{ds^2} \text{ for } i, k, l = 0, 1, 2, 3. \tag{4a}$$

and

$$\frac{d}{ds} \left(g_{5k} \frac{dx^k}{ds} + \frac{1}{2} g_{55} \frac{dx^5}{ds} \right) = \Gamma_{k,55} \frac{dx^5}{ds} \frac{dx^k}{ds} - \frac{1}{2} g_{55} \frac{d^2 x^5}{ds^2} + \frac{1}{2} \frac{\partial g_{kl}}{\partial x^5} \frac{dx^l}{ds} \frac{dx^k}{ds}, \tag{4b}$$

The electromagnetism is included by assuming $g_{5\mu} = KA_\mu$ ($\mu = 0, 1, 2, 3$) where A_μ is the electric potential, and K is a constant. In Kaluza's theory [19], the five variables are reduced to only four variables. Einstein and Pauli [20] assumed that all the "extra" metric elements are negligible, and thus failed to include new features. In the theory of Lo, Goldstein and Napier [18], because of fewer restrictions, it can accommodate the radiation reaction force and the new charge-mass interaction.

gravitation + electromagnetism. However, although the charge square coupling is beyond general relativity, in the full five-dimensional theory [18] the geodesic equation would include the coupling of q^2 . As Kaluza [19] proposed, a five-dimensional geodesic is,

$$\frac{d^2 x^\mu}{ds^2} + \Gamma^\mu_{\alpha\beta} \frac{dx^\alpha}{ds} \frac{dx^\beta}{ds} = 0,$$

where $\Gamma^\mu_{\alpha\beta} = \frac{1}{2} (\partial_\alpha g_{\nu\beta} + \partial_\beta g_{\nu\alpha} - \partial_\nu g_{\alpha\beta}) g^{\mu\nu}$, $\tag{4}$

and $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$, and $g_{\mu\nu}$ ($\mu, \nu = 0, 1, 2, 3, 5$) are metric elements of a five-dimensional space.

After some algebraic calculation, the geodesic equation (4) can be represented as follows:

If instead of s , τ ($d\tau^2 = g_{kl} dx^k dx^l$; $k, l = 0, 1, 2, 3$) is used in (4), the Lorentz force suggests

$$\frac{q}{Mc^2} \left(\frac{\partial A_i}{\partial x^k} - \frac{\partial A_k}{\partial x^i} \right) = \left(\frac{\partial g_{i5}}{\partial x^k} - \frac{\partial g_{k5}}{\partial x^i} \right) \frac{dx^5}{d\tau}$$

Thus,

$$\frac{dx^5}{d\tau} = \frac{q}{Mc^2} \frac{1}{K}$$

$$K \left(\frac{\partial A_i}{\partial x^k} - \frac{\partial A_k}{\partial x^i} \right) = \left(\frac{\partial g_{i5}}{\partial x^k} - \frac{\partial g_{k5}}{\partial x^i} \right) \text{ and } \frac{d^2 x^5}{d\tau^2} = 0 \tag{5}$$

where K is a constant. It thus follows that

$$\frac{d}{d\tau} \left(g_{ik} \frac{dx^k}{d\tau} \right) = \frac{1}{2} \frac{\partial g_{kl}}{\partial x^i} \frac{dx^k}{d\tau} \frac{dx^l}{d\tau} + \left(\frac{\partial A_k}{\partial x^i} - \frac{\partial A_i}{\partial x^k} \right) \frac{q}{Mc^2} \frac{dx^k}{d\tau} - \Gamma_{i,55} \left(\frac{q}{Mc^2} \right)^2 \frac{1}{K^2} \tag{6a}$$

$$\frac{d}{d\tau} \left(g_{5k} \frac{dx^k}{d\tau} + \frac{1}{2} g_{55} \frac{q}{KMc^2} \right) = \Gamma_{k,55} \frac{q}{KMc^2} \frac{dx^k}{d\tau} + \frac{1}{2} \frac{\partial g_{kl}}{\partial x^5} \frac{dx^l}{d\tau} \frac{dx^k}{d\tau}. \tag{6b}$$

For a static case, it follows (6) and (4) that the forces on the charged particle Q in the ρ -direction are

$$-\frac{mM}{\rho^2} \approx \frac{M}{2} \frac{\partial g_{\mu\nu}}{\partial \rho} \frac{dct}{d\tau} \frac{dct}{d\tau} g^{\rho\rho}, \text{ and } \frac{mq^2}{\rho^3} \approx -\Gamma_{\rho,55} \frac{1}{K^2} \frac{q^2}{Mc^4} g^{\rho\rho} \tag{7a}$$

and

$$\Gamma_{k,55} \frac{q}{KMc^2} \frac{dx^k}{d\tau} = 0, \text{ where } \Gamma_{k,55} \equiv \frac{\partial g_{i5}}{\partial x^5} - \frac{1}{2} \frac{\partial g_{55}}{\partial x^k} = -\frac{1}{2} \frac{\partial g_{55}}{\partial x^k} \quad (7b)$$

in the $(-r)$ -direction. Here particle P is at the origin of spatial coordinate system $(\rho, \theta', \varphi')$. It is interesting that the same force would come from a different type of

metric element depending on the test particle used. Thus [21],

$$g_{tt} = 1 - \frac{2m}{\rho}, \text{ and } g_{55} = \frac{mMc^4}{\rho^2} K^2 + \text{constant. or } \left(\frac{1}{MK^2c^4} g_{55} = \frac{m}{\rho^2} + \text{const.} \right) \quad (8)$$

Thus, g_{55} is a repulsive potential plus a constant. and g_{55}/M is also a function of the a distant source mass m ,

Therefore, this force, though acting on a charged particle, would penetrate electromagnetic screening. Otherwise, there is no repulsive force from a charged capacitor. From (8), it is possible that a charge-mass repulsive potential would exist for a metric based on the mass M of the charged particle Q . However, since P is neutral, there is no charge-mass repulsion force (from $\Gamma_{k,55}$) on P .

To address the issue of charge-mass interaction one must have a solid theoretical ground, and thus the details of formula (3) should be completely tested with experiments.

V. THE CURRENT-MASS INTERACTION

If the electric energy leads to a repulsive force toward a mass according to general relativity, the magnetic energy would lead to an attractive force from a current toward a mass [21, 22]. Due to the fact that a charged capacitor has reduced weight, it is necessary to have the current-mass interaction to be cancelled out by the effect of the charge-mass interaction. In other words, the existence of the current-mass attractive force would solve a puzzle, i.e., why a charged capacitor exhibits the charge-mass repulsive force since a charged capacitor has no additional electric charges. Normally, the charge-mass repulsive force would be cancelled by the current-mass force as Galileo, Newton and Einstein implicitly assumed.

The existence of such a current-mass attractive force has been discovered by Martin Tajmar and Clovis de Matos [23] from the European Space Agency. Martin et al found that a spinning ring of superconducting material increases its weight more than expected. Thus, they believed that general relativity was wrong. However, according to quantum theory, spinning superconductors should produce a weak magnetic field. Thus, they also measured the current-mass interaction to the earth! From their findings, the current-mass interaction would generate a force which is perpendicular to the current.

This characteristic would explain why an alternative current on the capacitor would also make a capacitor reduce its weight as in the case of charged

capacitors. The alternative current would create an attractive force parallel to the surface of a flat capacitor. However, such current-mass interaction would not cancel the repulsive force that is perpendicular to the surface. It follows that, just as the case of a charged capacitor, there are repulsive forces in the perpendicular direction of the surface. (This weight reduction is directional.) Note that our explanation on the weight reduction of alternative current is self-consistent and very different from current theories [8].

One may ask the formula for what the current-mass force is. Unlike the charge-mass repulsive force, which can be derived from general relativity; this general force would be beyond general relativity since a current-mass interaction would involve the acceleration of a charge, this force would be time-dependent and generates electromagnetic radiation. Moreover, when the radiation is involved, the radiation reaction force and the variable of the fifth dimension must be considered [18]. Thus, we are not ready to derive the current-mass interaction yet.

Nevertheless, we may assume that, for a charged capacitor, the resulting force is the interaction of net macroscopic charges with the mass [24]. The irradiated ball has the extra electrons, and thus reduced its weight [15]. A spinning ring of superconducting material has the electric currents that are attractive to the earth. This also explains a predicted phenomenon, which is also reported by Liu [25] that it takes time for a capacitor to recover its weight after being discharged [3]. This was observed by Liu because his rolled-up capacitors keep heat better. A discharged capacitor needs time to dissipate the heat such that the motion of its charges would recover to normal.

VI. THE WEIGHTS OF A PERMANENT MAGNET AND CURRENT-MASS INTERACTION

A permanent magnet, from the view of Maxwell's electromagnetism, is essentially a group of circularly moving currents around parallel axis in the same direction. Note also that before the weighting, we must make sure that the paper tube is long enough that the magnet from such a distance would not affect the reading of the scale

Due to the magnetic effects of the earth, a magnet would have the maximum weight reading from the electronic scale for the perpendicular positions when the north end is up and a smaller weight when the south end is up. The weight average for these two-positions is the weight of the magnet in the perpendicular position without the magnetic effect from the earth.

On the other hand, the magnet would have weight readings when the magnet is in horizontal situations, which are rotated in 180 degree around the

horizontal axis. Then we would get two very close weight readings for the magnet. The average weight reading of these two positions is a weight of the magnet in the horizontal position.

We measure three kinds of magnets. As expected, the perpendicular weight of a magnet is always larger than the horizontal weight of the same magnet. Our measurements confirm our conclusions as follows:

Table 1: Some directional weight measurements of thee magnets (A, B, C)

Weight	N-up	S-up	V-average	Horizontal-1	Horizontal-2	H-average
A mag. bar	50.9735g	50.9595g	50.9665g	50.9625g	50.9565g	50.9595g
B mag. ring	75.9944g	75.9754g	75.9849g	75.9877g	75.9814g	75.9845g
C Small mag. bar	15.5118g	15.5079g	15.5099g	15.5093g	15.5077g	15.5085g

Note that the directional dependence of weight is due to the current-mass interaction, whose existence is necessary for the weight reduction for a charged capacitor. These measurements support the theory.

VII. EXPERIMENTAL VERIFICATION OF THE MASS-CHARGE REPULSIVE FORCE

The repulsive force in metric (2) can be detected with a mass. To see the repulsive effect, one must have

$$\frac{1}{2} \frac{\partial}{\partial r} \left(1 - \frac{2M}{r} + \frac{q^2}{r^2} \right) = \frac{M}{r^2} - \frac{q^2}{r^3} < 0 \quad (9)$$

Thus, repulsive gravity would be observed at $q^2/M > r$. For the electron the repulsive gravity would exist only inside the classical electron radius $r_0 (= 2.817 \times 10^{-13} \text{cm})$. Thus, it would be difficult to test a single charged particle.

$$N > \frac{R}{r_0}, \text{ (or } 0 > \left[\frac{Nm_e}{R^2} - \frac{N^2e^2}{R^3} \right] m_p) \quad \text{where } r_0 = \frac{e^2}{m_e c^2} = 2.817 \times 10^{-13} \text{cm.} \quad (10)$$

For example, if $R = 10 \text{ cm}$, then it requires $N > 3.550 \times 10^{13}$. Thus $Q = 5.683 \times 10^{-7} \text{Coulomb}$. Then, one would see the reduction of attractive gravitation when condition (10) is satisfied. For this case, the repulsive force is

$$\frac{Q^2 m_p}{R^3} \quad (11)$$

where m_p is the mass of the testing particle P, and the total force is

$$\left(\frac{M_0 + Nm_e}{R^2} - \frac{N^2e^2}{R^3} \right) m_p. \quad (12)$$

When condition (9) is satisfied for a certain R, the repulsive effect will be observed.

However, for a charged metal ball with mass M and charge Q, the formula can be $0 > M/R^2 - Q^2/R^3$, where R is the distance from the center of the ball [26]. The increased attractive effect in gravity is proportional to mass related to the number of electrons, but the repulsive effect in gravity is proportional to square of the number of electrons. Thus, when the electrons are accumulated numerous enough in a metal ball, the effect of repulsive gravity will be shown in a macroscopic distance.

Consider that Q and M consist of N electrons, i.e., $Q = Ne$, $M = Nm_e + M_0$, where M_0 is the mass of the metal ball, m_e and e are the mass and charge of an electron. To have sufficient electrons, the necessary condition is

The verification of this formula also disproves the equivalence between mass and electric energy. However, the majority of theorists failed gravity by following Einstein's error. Moreover, before the repulsive effect is detected for the reduction of attractive gravity to be seen requires only $N > R/r_0$. The relation (10) is a much easier condition to be satisfied.

This is why Tsipenyuk & Andreev [15] observed a reduction of weight after a metal ball is sufficiently charged. However, since the repulsive force is normally very small, the interference of electricity would be comparatively large. Thus, it would be desirable to screen the electromagnetic effects out. The modern capacitor is such a piece of simple equipment. When a capacitor is charged, it separates the electron from the atomic nucleus or polarizes the atom, but there is no change of mass due to increase of charged particles.

In case when such a capacitor is charged with high electric potential, it can even lift up [6-8]. This would confirm the mass-charge repulsive force, and thus the unification of electromagnetism and gravitation [1].

One may ask whether the lighter weight of a capacitor after charged could be due to a decrease of mass. Such a speculation is ruled out. Inside a capacitor the increased energy due to being charged would not be pure electromagnetic energy such that, for the total internal energy, Einstein's formula is valid.

In the case of a charged capacitor, the repulsive force would be proportional to the potential square, V^2 where V is the electric potential difference of the capacitor ($Q = CV$, C is the capacitance). This has been verified by the experiments of Musha [4-8]. Thus, the factor of charge density square in heuristic Eq. (1) is correct. It remains to verify the space dependence. However, even the $1/r^3$ factor in Eq. (1) is verified, the calculation would still depend on the detailed modeling because the repulsive force from a metal ball does not simply depend on the distance from the ball (see Appendix A). Also, although the initial thrust is directionally decided by the electric field, the weight reduction effect for charged capacitors is not directional and it stays if the potential does not change. This is also verified by Liu [25], who measured the effect of weight reduction with the roll-up capacitors.

VIII. CONCLUSIONS AND DISCUSSIONS

Here, the most important conclusion is the confirmation of the repulsive charge-mass interaction and attractive current-mass interaction in general relativity. These two interactions are crucial to verify the existence the repulsive gravitation. This repulsive gravitation had been over-looked because $E = mc^2$ has been misinterpreted by Einstein [27] as unconditionally valid.

Nevertheless, the existence of lifters [6, 7] unequivocally announces the existence of repulsive gravitation. It is also clear that the repulsive force of gravity depends on the square of charge density [1] although derivations to specifics are incomplete because of the lack of detailed modeling (see Appendix A). The improvements for such shortcomings need the input of some accurate experimental data as well as better theoretical understanding. The existence of a small directional dependence of weight for a magnet confirms the current-mass interaction.

Thus, the theoretical progress is often blocked by Einstein's own errors. Moreover, invalid derivations following Einstein's errors [27], were not discovered because of general inadequacy in mathematics. Two major errors of Einstein are that he failed to see that there is no bounded dynamic solution for the 1915 Einstein equation [12, 28] as Gullstrand suspected [29]

and $E = mc^2$ is not valid for the electromagnetic energy [11]. These two errors are intimately related. Thus, the conjecture for the existence of the black holes must be justified anew because the assumptions [28], that validity of the Einstein equation for the dynamic case and gravitation being always attractive are incorrect.

Moreover, these two major errors have subsequently developed into various invalid theories [28]. Fortunately, the new charge-mass and the current-mass interaction provide simple ways to show the errors.

The important conclusions from the weight reductions of charged capacitors are: 1) $E = mc^2$ may not be valid. 2) Einstein's conjecture of unification is established; 3) Einstein's general relativity is valid only for the static and stable cases, but is invalid for the dynamic case [11, 28]. It remains to be rectified and completed in at least two aspects: a) The exact form of the gravitational energy-stress tensor is not known; and b) The gravitational radiation reaction force is also not known [28].

Note that, due to the gravitational radiation reaction force is not included, considering general relativity as a theory of geometry is invalid [30]. Moreover, since the photons include gravitational energy [31], the unification of gravitation and electromagnetism is necessary. To this end, it is possible for a five-dimensional theory.

Moreover, because of inadequate understanding in physics, the space-time singularity theorems of Hawking and Penrose were accepted although they are based on an invalid assumption, the non-existence of the antigravity coupling [28]. Thus, their contribution to physics is essentially zero if not negative. The positive theorem of Yau and Schoen is also misleading [32] because essentially the same invalid assumption was used. The Field's Medals were awarded to Yau in 1982 and Witten in 1990 because those mathematicians do not understand physics [33]. These errors follow Einstein's mistake of believing in the existence of dynamic solutions for his equation.

According to the position on this issue, journals can be divided into three categories, namely: 1) Agreeing with Gullstrand [29] Chairman of the Nobel Committee for Physics (1922-1929) that there is no dynamic solution for the Einstein equation; 2) Disagreeing with Gullstrand that the Einstein equation has dynamic solutions, but failed to provide even one example; 3) Unable to decide whether the Einstein equation has any dynamic solution. Currently the Physical Review, Proceeding of the Royal Society A, Classical and Quantum Gravity, Gravitation and General Relativity, etc. all are against Gullstrand [29]. On the other hand, journals such as the Astrophysics Journal, Physics Essays, etc have committed to my position by publishing my paper [12-14].

However, some journals such as *Annalen der Physik* prefer not to fix their position on this. Moreover, currently some still ignore experimental facts such as the existence of lifters [6,7].

Einstein has shown that the photons are equivalent to mass [17] and this is supported by the experimental fact that a π_0 meson would decay into two light rays ($\pi_0 \rightarrow \gamma + \gamma$). Thus, Einstein had misinterpreted that the electromagnetic energy is equivalent to mass. This is impossible because the electromagnetic energy-stress tensor is traceless, and the addition of electromagnetic energies is still an electromagnetic energy. The fact is however, that the photons include not only electromagnetic energy, but also gravitational energy [31, 34]. Thus, Einstein's proposal of photons is inadequate (see Appendix B).

Einstein [27] claimed that a piece of heated-up metal would have increased weight. Experimentally, however, six kinds of heated-up metals have reduced weight [3]. Because a charged capacitor has reduced weight, it will fall slower than other neutral objects [35]. Thus, Einstein's unification would affect many areas of physics. The charge-mass interaction appears in many areas of physics, but its effects have not been considered. Thus, not only gravitational theories, but also other related physics must be reviewed carefully. Moreover, the charge-mass interaction is also has useful applications because it would lead to a new way of observation [1]. The fact that gravitation would reduced for a heat-up metal would be useful for an accurate super speed missiles trajectory.

Among many potential applications in astrophysics, it is the charge-mass interaction that provides an explanation of NASA's discovery of the Space-Probe Pioneer Anomaly [36], and a new technology of detecting matters under ground and/or under the water [1]. Thus, the discovery of the charge-mass interaction is a big step of progress in modern physics. However, some physicists have irrationally denied such repulsive gravitation.

Einstein's formula $E = mc^2$ is only conditionally valid and its misinterpretation is the source of many errors in general relativity. Obviously, the current-mass interaction is important in Einstein's unification. However, it has not been adequately addressed, and thus some issues cannot be answered. It is hoped that such work can be completed in the future. It is interesting that since Einstein's unification ia valid, Einstein is still the number one theorist in physics after the rectification of his theory although his general relativity has many problems of being misleading in physics.

$$l^2 = s^2 + r^2 - 2sr \cos \theta, \quad \text{and} \quad dm = \rho(r d\theta) dr (r \sin \theta) d\varphi = \rho r^2 dr d(-\cos \theta) d\varphi, \tag{A2}$$

where ρ is the mass density. We calculate the force potential, which is locally proportional to l^2 , and have

$$\int_0^\pi \frac{d(-\cos \theta)}{s^2 + r^2 - 2sr \cos \theta} = \frac{1}{2sr} \int_0^\pi \frac{d(-2sr \cos \theta)}{s^2 + r^2 - 2sr \cos \theta} = \frac{1}{2sr} \ln(s^2 + r^2 - 2sr \cos \theta) \Big|_0^\pi = \frac{1}{sr} \ln\left(\frac{s+r}{s-r}\right) \tag{A3}$$

Another lesson is that all human institutes are not perfect and can make scientific errors [30, 32, 37, 38]. We respect Nobel laureates for their achievements. However, they could also be incorrect in physics [38-40]. Thus experiments together with careful analysis are still the most reliable source of information. *A reason that Einstein's errors were not confirmed earlier because solid experimental proof was not found* [41]. Who would have conceived that $E = mc^2$ considered as Einstein's major achievement is actually his major error in physics. This shows that we must be more careful in physics.

Currently, a blind faith toward the opinions of Nobel Committee or the Nobel Laureates is a rather common disease.[42] Many seem to forget what Galileo taught us is that Physics must be supported by experiments. Further more, the fact is, however, only the experiment supported part is valid. For instance, Einstein's photo-electric effect shows only that a photon is a quantum of energy, but it did not show the energy is only electromagnetic energy. In addition, the formula $E = mc^2$ is inconsistent with the Einstein equation, which shows that the electromagnetic energy cannot change the curvature.

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APPENDIX A: Some Remarks on the Charge-Mass Repulsive Force

The charge-mass repulsive force between a point charge q and a point mass m is,

$$F = \frac{q^2 m}{r^3} \tag{A1}$$

would behave very differently from a Newtonian force, which is inverse proportional to the square of the distance r .

To illustrate this, let us calculate the force between a charge q and a mass M whose density ρ is uniformly distributed in a sphere of radius R . In a coordinate (r, θ, φ) , let s be the distance between the center of the sphere, located at $(0, 0, 0)$ and the charge q is at $(s, 0, 0)$. Let l be the distance between the charge q and a mass dm at a point (r, θ, φ) . Then we have

Then the total potential $V(s, R)$ would be

$$\begin{aligned}
 2V(s, R) &= \rho \int_0^R 2\pi r^2 dr \frac{1}{sr} \ln\left(\frac{s+r}{s-r}\right) = \rho \frac{2\pi}{s} \int_0^R r dr \ln\left(\frac{s+r}{s-r}\right) = \rho \frac{2\pi}{s} \int_0^R r dr \left[2\left(\frac{r}{s} + \frac{1}{3}\left(\frac{r}{s}\right)^3 + \frac{1}{5}\left(\frac{r}{s}\right)^5 + \dots + \frac{1}{2n-1}\left(\frac{r}{s}\right)^{2n-1} + \dots \right) \right] \\
 &= \rho \frac{4\pi}{s^2} \int_0^R dr r^2 \left[1 + \frac{1}{3}\left(\frac{r}{s}\right)^2 + \frac{1}{5}\left(\frac{r}{s}\right)^4 + \dots + \frac{1}{2n-1}\left(\frac{r}{s}\right)^{2n-2} + \dots \right] \\
 &= \rho \left[\frac{4\pi R^3}{3} \right] \frac{1}{s^2} \left[1 + \frac{1}{5}\left(\frac{R}{s}\right)^2 + \frac{3}{5 \cdot 7}\left(\frac{R}{s}\right)^4 + \dots + \frac{3}{(2n-1)(2n+1)}\left(\frac{R}{s}\right)^{2n-2} + \dots \right] \tag{A4}
 \end{aligned}$$

Note that $1 + \frac{1}{5} + \frac{3}{5 \cdot 7} + \dots + \frac{3}{(2n-1)(2n+1)} + \dots = \frac{3}{2} \sum_1^\infty \left[\frac{1}{2n-1} - \frac{1}{2n+1} \right] = \frac{3}{2}$, and $M = \rho \left[\frac{4\pi R^3}{3} \right]$.

Thus, the repulsive force between the charge q and the mass M is

$$\begin{aligned}
 2F(s, R) &= 2q^2V_s(s, R) = q^2M \frac{d}{ds} \left\{ \frac{1}{s^2} \left[1 + \frac{1}{5}\left(\frac{R}{s}\right)^2 + \frac{3}{5 \cdot 7}\left(\frac{R}{s}\right)^4 + \dots + \frac{3}{(2n-1)(2n+1)}\left(\frac{R}{s}\right)^{2n-2} + \dots \right] \right\} \\
 &= q^2M \frac{-1}{s^3} \left[2 + \frac{4}{5}\left(\frac{R}{s}\right)^2 + \frac{3 \cdot 6}{5 \cdot 7}\left(\frac{R}{s}\right)^4 + \dots + \frac{3(2n)}{(2n-1)(2n+1)}\left(\frac{R}{s}\right)^{2n-2} + \dots \right] \tag{A5}
 \end{aligned}$$

Note that $\frac{-3}{2s^3} \left[1 + \sum_{n=1}^\infty \frac{1}{2n+1} \right]$ does not converge.

The close forms of the repulsive potential and the repulsive force are,

$$V(s, R) = \rho \frac{\pi}{s} \int_0^R r dr \ln\left(\frac{s+r}{s-r}\right) = \rho \frac{\pi}{s} \left[\frac{R^2 - s^2}{2} \ln\left(\frac{s+R}{s-R}\right) + sR \right] \tag{A6}$$

$$F(s, R) = q^2V_s(s, R) = -q^2s\rho\pi \left(\left[1 - \left(\frac{R}{s}\right)^2 \right] \frac{1}{2} \ln\left(\frac{s+R}{s-R}\right) - \frac{R}{s} \right) \tag{A7}$$

Now, it is clear that the repulsive force on the charge q is sensitive to the surrounding of the charge.

For a charged object, the repulsive force is sensitive to the arrangements of the charges. Note that the dependence of r^{-3} in (A1) is derived from general relativity. Thus, this force provides another test of general relativity. However, it would be difficult to calculate the effects of this force without the necessary detailed information.

APPENDIX B: The Electromagnetic Wave, Photons and Anti-gravity Coupling

$$G_{ab} = -K[T(E)_{ab} - T(p)_{ab}], \text{ and } T_{ab} = -T(g)_{ab} = T(E)_{ab} - T(P)_{ab}, \tag{B1}$$

where $T(E)_{ab}$ and $T(P)_{ab}$ are the energy-stress tensors for the electromagnetic wave and the related photons. Thus, Einstein [43] was wrong, and Einstein's understanding on general relativity needs improvement. Note that the energy-stress tensor of photons has an anti-gravity coupling. The presence of the photonic energy-stress is necessary; otherwise there is no bounded gravitational wave solution for equation (B1) [31, 34].

In Einstein's assumption, the photons consist of only electromagnetic energy because general relativity has not been conceived. Since all the charged particles are massive, it is natural that a gravitational wave should

be included. If the photons consist of only electromagnetic energy, there is a conflict since the photonic energy could be equivalent to mass but the electromagnetic energy-stress tensor is traceless. Now, this conflict is resolved since the photonic energy is the sum of electro-magnetic energy and gravitational energy.

Both quantum theory and relativity are based on the phenomena of light. The gravity of photons shows that there is a link between them. It is gravity that makes photons compatible with electromagnetic waves.

It should be noted also that the anti-gravity coupling would appear in where the gravitational wave is

present. For instance, it is necessary to appear in the equation for the calculation on the gravitational waves generated by massive sources [12-14].⁹ For massive sources, the 1995 update of the Einstein equation is as follows:

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -K [T(m)_{\mu\nu} - t(g)_{\mu\nu}], \quad (B2)$$

where $t(g)_{\mu\nu}$ is the energy-stress tensors for gravity. From (B2), the equation in vacuum is,

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = K t(g)_{\mu\nu}. \quad (B3')$$

Note that $t(g)_{\mu\nu}$ is equivalent to $G^{(2)}_{\mu\nu}$ in terms of Einstein's radiation formula. When gravitational wave is present, the gravitational energy-stress tensor $t(g)_{\mu\nu}$ is non-zero.

Thus, a radiation of gravitation does carry energy-momentum as physics requires. This explains also that the absence of an anti-gravity coupling is the reason that the Einstein equation is incompatible with radiation [12-14].

Note that eq. (B2) was proposed by Lorentz [44] in 1916 and later by Levi-Civita [45] in the following form,

$$\kappa t(g)_{ab} = G_{ab} + \kappa T_{ab} \quad (B4)$$

where T_{ab} is the sum of other massive energy-stress tensors. This is the Lorentz-Levy-Einstein equation. However, Einstein [46] rejected their proposal because he did not realize that his equation does not have any bounded dynamic solution [12-14].

Nevertheless, Christodoulou¹⁰ and Klainerman [47] had claimed that they have constructed bounded dynamic solutions for the Einstein equation. However, upon close examination, it is found [48] that they actually have not completed the proof for the existence of bounded dynamic solutions, in addition to other errors in mathematics [49]. For these, one must know that the thesis advisor of Christodoulou is J. A. Wheeler, whose mathematics is so inadequate that he made crucial errors in their book [10] even at the undergraduate level [37].

Endnotes

- For a thorough discussion on the mass and the total energy of a particle, one can read the 1989 paper of L. B. Okun [50]. However, Okun [51] did not understand that the electromagnetic energy is not equivalent to mass [10]. Moreover, Einstein's proof for the equivalence of mass and the photonic energy is incomplete [41].
- Currently a major problem in general relativity is not only that Einstein's errors are over-looked, but also that some theorists [9, 47, 52-55] additionally make their own errors and ignore experimental facts.
- Due to the popular opinion that gravity is attractive, Herrera, Santos, & Skea [56] argued that M in (2) involves the electric energy. They follow the error of Whittaker [57] and Tolman [58], and interpreted that M in the Riessner-Nordstrom metric includes the electromagnetic energy. Thus, according to their interpretation, an increase of the charge would lead to the increment of weight. Thus, the charge-mass interaction was over-looked. However, their interpretation is rejected by that a charged ball does reduce its weight [14]. Nevertheless, 't Hooft claimed in his Nobel Lecture [39] that the electric energy is part of the physical mass m_{phys} of an electron. Moreover, he claimed this "physical mass" obeys Newton's second law $F = m_{phys} a$. Note that such a claim violates special relativity because part of the electric energy is far from the electron and thus cannot react immediately as an inertial mass does. Another Nobel Laureate Wilczek [40] also incorrectly applied $m = E/c^2$ without providing a justification. His problem is that he cannot be distinct about the issue that a type of energy is equivalent to mass is different from that the mass and energy are equivalent.
- In Newtonian theory, gravitation is always attractive to mass, but in general relativity the situation is different.
- Misner et al. [10], Wald [52], Christodoulou & Klainerman [47], 't Hooft [59], and etc. claimed to have explicit dynamic solutions, due to various errors in mathematics [28, 37, 59]. Note that journals such as the Physical Review, Proceeding of the Royal Society, Classical and Quantum Gravity, Gravitation and General Relativity, etc. all are against Gullstrand [29] and believed incorrectly that there are bounded dynamic solutions for the Einstein equation [11-13, 60]
- Recently, it has been found that Einstein's proof [16] of $L = mc^2$ and $E = mc^2$ is also invalid [41].
- A well-known misleading result is the positive mass theorem of Schoen & Yau [54] (and the positive energy theorem of Witten [55]). Their failure in understanding general relativity properly could be a reason that the lack of progress of the string theory in physics. They were awarded Fields Medals in 1982 and in 1990 because leading mathematicians such as M. F. Atiyah and L. D. Faddeev do not understand the related physics and relativity [32]. This was confirmed by Prof. [Peter C. Sarnak](#), the Chairman of the 2011 Shaw Prize Committee when we met at a conference in Toronto in 2016.
- I have reported these problems in $E = mc^2$ to MIT President Hockfield and the subsequent President Reif. They have promised to up-grade the related education in gravitation.
- Hod [61] invalidly claimed to have "[A simplified two-body problem in general relativity](#)" because he did

not know that the linearized equation is actually a valid linearization of only the Lorentz-Levy-Einstein equation [12],

10. The 2011 Shaw Prize has mistakenly awarded a half prize in mathematics to Christodoulou for his erroneous work against the honorable Gullstrand [29] Chairman (1922-1929) of the Nobel Committee for Physics. Christodoulou has misled many including the 1993 Nobel Committee [38]. However, his errors are now well-established since they have been illustrated with mathematics at the undergraduate level [30]. Christodoulou claimed in his autobiography that his work is based on two sources: 1) The claims of Christodoulou and Klainerman on general relativity, shown in their book *The Global Nonlinear Stability of the Minkowski Space* [47]; 2) Roger Penrose had introduced, in 1965, the concept of a trapped surface and had proved that a space-time containing such a surface cannot be complete [30]. However, this work of Penrose, which uses an implicit assumption of unique sign for all coupling constants, actually depends on the errors of Christodoulou and Klainerman.

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