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An Unified Study of Some Multiple Integrals

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5. K. C. Gupta and R.Jain, A unified study of some multiple integrals, Soochow Journal of Mathematics, 19(1) (1993), 73-81.

An Unified Study of Some Multiple Integrals

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Keywords: multivariable A-function, a sequence of functions, multiple integrals, multivariable I-function, class of multivariable polynomials, H-function, generalized hypergeometric function, A-function.

I. Introduction and Preliminaries

Gupta and Jain [5] have studied unified multiple integrals involving the generalized hypergeometric function, class of multivariable polynomials [9] and multivariable H-function [13,14]. The aim of this paper is to establish a general finite multiple integrals about the generalized hypergeometric function, sequence of functions, general class of multivariable polynomials, the series expansion of the A-function [4] and multivariable I-function defined by Prasad [6].

For this study, we need the following series formula for the general sequence of functions introduced by Agrawal and Chaubey [1] and was established by Salim [7].

$$R_n^{\alpha,\beta}[x;E,F,g,h;p,q;\gamma;\delta;e^{-\mathfrak{s}x^{\tau}}]=\sum_{w,v',u,t',e,k_1,k_2}\psi(w,v',u,t',e,k_1,k_2)x^R \tag{1.1}$$

where $\psi(w,v',u,t',e,k_1,k_2)=\dfrac{(-)^{t'+w+k_2}(-v')_u(-t')_e(\alpha)_t l^n}{w!v'!u!t'!e!l'_n k_1!k_2!}\dfrac{\mathfrak{s}^{w+k_1}F^{\gamma n-t'}}{(1-\alpha-t')_e}(-\alpha-\gamma n)_e(-\beta-\delta n)_{v'}$

$$g^{v+k_2}h^{\delta n-v-k_2}\,(v'-\delta n)_{k_2}E^{t'}\left(\frac{pe+\tau w+\lambda+qu}{l}\right)_n \tag{1.2}$$

and $\displaystyle\sum_{w,v',u,t',e,k_1,k_2}=\sum_{w=0}^{\infty}\sum_{v'=0}^{n}\sum_{u=0}^{v'}\sum_{t'=0}^{n}\sum_{e=0}^{t'}\sum_{k_1,k_2=0}^{\infty}$

The infinite series on the right-hand side of (1.3) is convergent and $R=ln+qv+pt'+\tau w+\tau k_1+k_2q$
We shall note $R_n^{\alpha,\beta}[x;E,F,g,h;p,q;\gamma;\delta;e^{-\mathfrak{s}x^{\tau}}]=R_n^{\alpha,\beta}(x)$ (1.3)

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The generalized multivariablepolynomials defined by Srivastava [9], is given in the following manner :

$$S_{N_1,\cdots,N_v}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}[y_1,\cdots,y_v]=\sum_{K_1=0}^{[N_1/\mathfrak{M}_1]}\cdots\sum_{K_v=0}^{[N_v/\mathfrak{M}_v]}\frac{(-N_1)_{\mathfrak{M}_1K_1}}{K_1!}\cdots\frac{(-N_v)_{\mathfrak{M}_vK_v}}{K_v!}A[N_1,K_1;\cdots;N_v,K_v]y_1^{K_1}\cdots y_v^{K_v} \quad (1.4)$$

where $\mathfrak{M}_1,\cdots,\mathfrak{M}_v$ are arbitrary positive integers and the coefficients $A[N_1,K_1;\cdots;N_v,K_v]$ are arbitrary constants Real or complex. On suitably specializing the coefficients, $A[N_1,K_1;\cdots;N_v,K_v]$, $S_{N_1,\cdots,N_v}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}[y_1,\cdots,y_v]$ yields a Number of known polynomials, the Laguerre polynomials, the Jacobi polynomials, and several other ([15], page. 158-161]. We shall note

$$a_v=\frac{(-N_1)_{\mathfrak{M}_1K_1}}{K_1!}\cdots\frac{(-N_v)_{\mathfrak{M}_vK_v}}{K_v!}A[N_1,K_1;\cdots;N_v,K_v] \quad (1.5)$$

The series representation of the multivariable A-function is given by Gautam [4] as

$$A[u_1,\cdots,u_r]=A_{A,C:(M',N');\cdots;(M^{(r)},N^{(r)})}^{0,\lambda:(\alpha',\beta');\cdots;(\alpha^{(r)},\beta^{(r)})}\left(\begin{array}{c|c} \mathrm{u}_1 & [(\mathrm{g}_j);\gamma',\cdots,\gamma^{(r)}]_{1,A}: \\ \cdot & \cdots \\ \cdot & [(\mathrm{f}_j);\xi',\cdots,\xi^{(r)}]_{1,C}: \\ \mathrm{u}_r & \end{array}\right.$$

$$\left.\begin{array}{c} (\mathrm{q}^{(1)},\eta^{(1)})_{1,M^{(1)}};\cdots;(q^{(r)},\eta^{(r)})_{1,M^{(r)}} \\ \cdots \\ \cdots \\ (\mathrm{p}^{(1)},\epsilon^{(1)})_{1,N^{(1)}};\cdots;(p^{(r)},\epsilon^{(r)})_{1,N^{(r)}} \end{array}\right)=\sum_{G_i=1}^{\alpha^{(i)}}\sum_{g_i=1}^{\infty}\phi\frac{\prod_{i=1}^r\phi_i u_i^{\eta_{G_i,g_i}}(-)^{\sum_{i=1}^r g_i}}{\prod_{i=1}^r\epsilon_{G_i}^{(i)}g_i!} \quad (1.6)$$

where

$$\phi=\frac{\prod_{j=1}^{\lambda}\Gamma\left(1-g_j+\sum_{i=1}^r\gamma_j^{(i)}\eta_{G_i,g_i}\right)}{\prod_{j=\lambda'+1}^{A}\Gamma\left(g_j-\sum_{i=1}^r\gamma_j^{(i)}U_i\right)\prod_{j=1}^{C}\Gamma\left(1-f_j+\sum_{i=1}^r\xi_j^{(i)}\eta_{G_i,g_i}\right)} \quad (1.7)$$

$$\phi_i=\frac{\prod_{j=1,j\neq m_i}^{\alpha^{(i)}}\Gamma\left(p_j^{(i)}-\epsilon_j^{(i)}\eta_{G_i,g_i}\right)\prod_{j=1}^{\beta^{(i)}}\Gamma\left(1-q_j^{(i)}+\eta_j^{(i)}\eta_{G_i,g_i}\right)}{\prod_{j=\alpha^{(i)}+1}^{N^{(i)}}\Gamma\left(1-p_j^{(i)}+\epsilon_j^{(i)}\eta_{G_i,g_i}\right)\prod_{j=\beta^{(i)}+1}^{M^{(i)}}\Gamma\left(q_j^{(i)}-\eta_j^{(i)}\eta_{G_i,g_i}\right)},i=1,\cdots,r \quad (1.8)$$

and

$$\eta_{G_i,g_i}=\frac{p_{G_i}^{(i)}+g_i}{\epsilon_{G_i}^{(i)}},i=1,\cdots,r \quad (1.9)$$

which is valid under the following conditions : $\epsilon_{m_i}^{(i)}[p_j^{(i)}+p_i']\neq\epsilon_j^{(i)}[p_{m_i}+g_i]$
and

$$u_i\neq 0,\sum_{j=1}^{A}\gamma_j^{(i)}-\sum_{j=1}^{C}\xi_j^{(i)}+\sum_{j=1}^{M^{(i)}}\eta_j^{(i)}-\sum_{j=1}^{N^{(i)}}\epsilon_j^{(i)}<0,i=1,\cdots,r \quad (1.10)$$

Here $\lambda,A,C,\alpha_i,\beta_i,m_i,n_i\in\mathbb{N}^*;i=1,\cdots,r;f_j,g_j,p_j^{(i)},q_j^{(i)},\gamma_j^{(i)},\xi_j^{(i)},\eta_j^{(i)},\epsilon_j^{(i)}\in\mathbb{C}$

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9. H. M. Srivastava, A multilinear generating function for the Konhauser set of biorthogonal polynomials suggested by Laguerre polynomial, Pacific. J. Math. 177(1985), 183-191.

Ref
6. Y. N. Prasad, Multivariable I-function, Vijnana Parishad Anusandhan Patrika 29 (1986), 231-237.

The multivariable I-function of s-variables defined by Prasad [6] generalizes the multivariable H-function defined by Srivastava and Panda [13,14]. This representation of multiple Mellin-Barnes types integral is:

$$I(z'_1,\cdots,z'_s) = I^{0,n'_2;0,n'_3;\cdots;0,n'_s:m'^{(1)},n'^{(1)};\cdots;m'^{(s)},n'^{(s)}}_{p'_2,q'_2,p'_3,q'_3;\cdots;p'_s,q'_s:p'^{(1)},q'^{(1)};\cdots;p'^{(s)},q'^{(s)}}\left(\begin{array}{c|c} \mathrm{z}'_1 & (\mathrm{a}'_{2j};\alpha'^{(1)}_{2j},\alpha'^{(2)}_{2j})_{1,p_2};\cdots; \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}'_s & (\mathrm{b}'_{2j};\beta'^{(1)}_{2j},\beta'^{(2)}_{2j})_{1,q_2};\cdots; \end{array}\right.$$

$$\left.\begin{array}{c} (\mathrm{a}'_{sj};\alpha'^{(1)}_{sj},\cdots,\alpha'_{sj}{}^{(s)})_{1,p'_s}:(a'^{(1)}_j,\alpha'^{(1)}_j)_{1,p'^{(1)}};\cdots;(a'_j{}^{(s)},\alpha'_j{}^{(s)})_{1,p'^{(s)}} \\ \\ (\mathrm{b}'_{rj};\beta'^{(1)}_{sj},\cdots,\beta'_{sj}{}^{(s)})_{1,q_s}:(b'^{(1)}_j,\beta'^{(1)}_j)_{1,q'^{(1)}};\cdots;(b'_j{}^{(s)},\beta'_j{}^{(s)})_{1,q'^{(s)}} \end{array}\right)$$

$$= \frac{1}{(2\pi\omega)^s}\int_{L'_1}\cdots\int_{L'_s}\phi(t_1,\cdots,t_s)\prod_{i=1}^{s}\phi_i(t_i)z_i'^{t_i}\mathrm{d}t_1\cdots\mathrm{d}t_s \tag{1.11}$$

where

$$\phi_i(t_i) = \frac{\prod_{j=1}^{m'^{(i)}}\Gamma(b'^{(i)}_j-\beta'^{(i)}_j t_i)\prod_{j=1}^{n'^{(i)}}\Gamma(1-a'^{(i)}_j+\alpha'^{(i)}_j t_i)}{\prod_{j=m'^{(i)}+1}^{q'^{(i)}}\Gamma(1-b'^{(i)}_j+\beta'^{(i)}_j t_i)\prod_{j=n'^{(i)}+1}^{p'^{(i)}}\Gamma(a'^{(i)}_j-\alpha'^{(i)}_j t_i)}\ ,i=1,\cdots,s \tag{1.12}$$

and

$$\phi(t_1,\cdots,t_s) = \frac{\prod_{j=1}^{n'_2}\Gamma(1-a'_{2j}+\sum_{i=1}^{2}\alpha'^{(i)}_{2j}t_i)\prod_{j=1}^{n'_3}\Gamma(1-a'_{3j}+\sum_{i=1}^{3}\alpha'^{(i)}_{3j}t_i)\cdots}{\prod_{j=n'_2+1}^{p_2}\Gamma(a'_{2j}-\sum_{i=1}^{2}\alpha'^{(i)}_{2j}t_i)\prod_{j=n'_3+1}^{p'_3}\Gamma(a'_{3j}-\sum_{i=1}^{3}\alpha'^{(i)}_{3j}t_i)\cdots}$$

$$\frac{\cdots\prod_{j=1}^{n'_s}\Gamma(1-a'_{sj}+\sum_{i=1}^{s}\alpha'^{(i)}_{sj}t_i)}{\cdots\prod_{j=n'_s+1}^{p'_s}\Gamma(a'_{sj}-\sum_{i=1}^{s}\alpha'^{(i)}_{sj}t_i)\prod_{j=1}^{q'_2}\Gamma(1-b'_{2j}-\sum_{i=1}^{2}\beta'^{(i)}_{2j}t_i)}$$

$$\times\frac{1}{\prod_{j=1}^{q'_3}\Gamma(1-b'_{3j}+\sum_{i=1}^{3}\beta'^{(i)}_{3j}t_i)\cdots\prod_{j=1}^{q'_s}\Gamma(1-b'_{sj}-\sum_{i=1}^{s}\beta'^{(i)}_{sj}t_i)} \tag{1.13}$$

About the above integrals and these existence and convergence conditions, see Prasad [4] for more details. Throughout the present document, we assume that the existence and convergence conditions of the multivariable I-function. We have:

$|argz'_i| < \frac{1}{2}\Omega'_i\pi$, where

$$\Omega'_i = \sum_{k=1}^{n'^{(i)}}\alpha'_k{}^{(i)} - \sum_{k=n'^{(i)}+1}^{p'^{(i)}}\alpha'_k{}^{(i)} + \sum_{k=1}^{m'^{(i)}}\beta'_k{}^{(i)} - \sum_{k=m'^{(i)}+1}^{q'^{(i)}}\beta'_k{}^{(i)} + \left(\sum_{k=1}^{n'_2}\alpha'_{2k}{}^{(i)} - \sum_{k=n'_2+1}^{p'_2}\alpha'_{2k}{}^{(i)}\right) +$$

$$+\cdots+\left(\sum_{k=1}^{n'_s}\alpha'^{(i)}_{sk} - \sum_{k=n'_s+1}^{p'_s}\alpha'^{(i)}_{sk}\right) - \left(\sum_{k=1}^{q'_2}\beta'^{(i)}_{2k} + \sum_{k=1}^{q'_3}\beta'^{(i)}_{3k} + \cdots + \sum_{k=1}^{q'_s}\beta'^{(i)}_{sk}\right) \tag{1.14}$$

The complex numbers z_i are not zero.Throughout this document, we assume the existence and absolute convergence conditions of the multivariable I-function.

We may establish the asymptotic expansion in the following convenient form :

$$I(z'_1,\cdots,z'_s)=0(\,|z'_1|^{\alpha'_1},\cdots,|z'_s|^{\alpha'_s}\,)\,,max(\,|z'_1|,\cdots,|z'_s|\,)\to 0$$

$$I(z'_1,\cdots,z'_s)\;=0(\,|z'_1|^{\beta'_1},\cdots,|z'_s|^{\beta'_s}\,)\,,min(\,|z'_1|,\cdots,|z'_s|\,)\to\infty$$

where: $k=1,\cdots,s:\alpha''_k=min[Re(b_j'^{(k)}/\beta_j'^{(k)})],j=1,\cdots,m'_k$ and

$$\beta''_k=max[Re((a_j'^{(k)}-1)/\alpha_j'^{(k)})],j=1,\cdots,n'_k$$

II. Main Integral

We have the following unified multiple integrals formula.

Theorem

$$\int_0^{a_1}\cdots\int_0^{a_t}\prod_{l=1}^{t}\left[x_l^{\rho_l-1}(a_l-x_l)^{\sigma_l}\left\{1+(b_lx_l)^{g_l}\right\}^{-\lambda_l}\right]R_n^{\alpha,\beta}\left[y\prod_{l=1}^{t}\left[x_l^{e_l}(a_l-x_l)^{f_l}\left\{1+(b_lx_l)^{g_l}\right\}^{-h_l}\right]\right]$$

$$S^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}_{N_1,\cdots,N_v}\left(\begin{array}{c}\mathrm{y}_1\left[\prod_{l=1}^{t}\left[x_l^{e_l''^{(1)}}(a_l-x_l)^{f_l''^{(1)}}\left\{1+(b_lx_l)^{g_l}\right\}^{-h_l''^{(1)}}\right]\right]\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{y}_v\left[\prod_{l=1}^{t}\left[x_l^{e_l''^{(v)}}(a_l-x_l)^{f_l''^{(v)}}\left\{1+(b_lx_l)^{g_l}\right\}^{-h_l''^{(v)}}\right]\right]\end{array}\right)$$

$$A\left(\begin{array}{c}\mathrm{z}_1\left[\prod_{l=1}^{t}\left[x_l^{e_l'^{(1)}}(a_l-x_l)^{f_l'^{(1)}}\left\{1+(b_lx_l)^{g_l}\right\}^{-h_l'^{(1)}}\right]\right]\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_r\left[\prod_{l=1}^{t}\left[x_l^{e_l'^{(r)}}(a_l-x_l)^{f_l'^{(r)}}\left\{1+(b_lx_l)^{g_l}\right\}^{-h_l'^{(r)}}\right]\right]\end{array}\right)$$

$$I\left(\begin{array}{c}\mathrm{z}'_1\left[\prod_{l=1}^{t}\left[x_l^{e_l^{(1)}}(a_l-x_l)^{f_l^{(1)}}\left\{1+(b_lx_l)^{g_l}\right\}^{-h_l^{(1)}}\right]\right]\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}'_s\left[\prod_{l=1}^{t}\left[x_l^{e_l^{(s)}}(a_l-x_l)^{f_l^{(s)}}\left\{1+(b_lx_l)^{g_l}\right\}^{-h_l^{(s)}}\right]\right]\end{array}\right)$$

Notes

$$ {}_PF_Q\left[(A_P);(B_Q);\sum_{l=1}^{t}B_lx_l^{\mu_l}(a_l-x_l)^{\upsilon_l}\left\{1+(b_lx_l)^{g_l}\right\}^{-\omega_l}\right]\mathrm{d}x_1\cdots\mathrm{d}x_t=\frac{\prod_{j=1}^{Q}\Gamma(B_j)}{\prod_{j=1}^{P}\Gamma(A_j)} $$

$$ \prod_{l=1}^{t}a_l^{\rho_l+\sigma_l}\sum_{w,v',u,t',e,k_1,k_2}\sum_{K_1=0}^{[N_1/\mathfrak{M}_1]}\cdots\sum_{K_v=0}^{[N_v/\mathfrak{M}_v]}\sum_{G_i=1}^{\alpha^{(i)}}\sum_{g_i=1}^{\infty}\phi_1\frac{\prod_{i=1}^{r}\phi_iz_i^{\eta_{G_i,g_i}}(-)^{\sum_{i=1}^{r}g_i}}{\prod_{i=1}^{r}\epsilon_{G_i}^{(i)}g_i!} $$

$$ a_vy_1^{K_1}\cdots y_v^{K_v}\psi(w,v',u,t',e,k_1,k_2)\,y^R\prod_{i=1}^{v}\prod_{l=1}^{t}a_l^{(e_l''^{(i)}+f_l''^{((i)})K_i}\prod_{j=1}^{r}\prod_{l=1}^{t}a^{(e_l'^{(j)}+f_l'^{((j)})\eta_{G_j,g_j}} $$

$$ I_{U:p_s'+3t+P,q_s'+2t+Q;Y}^{V;0,n_s'+3t+P;X}\left(\begin{array}{c|c} \mathrm{z}_1'\prod_{l=1}^{t}a_l^{(e_l^{(1)}+f^{(1)})} & \mathrm{A}\\ \cdots & \cdot\\ \cdots & \cdot\\ \mathrm{z}_s'\prod_{l=1}^{t}a_l^{(e_l^{(s)}+f^{(s)})} & \cdot\\ \text{-B}_1a_1^{\mu_1+\upsilon_1} & \cdot\\ \cdots & \cdot\\ \cdots & \cdot\\ \text{-B}_ta_t^{\mu_t+\upsilon_t} & \cdot\\ (\mathrm{a}_1b_1)^{g_1} & \cdot\\ \cdots & \cdot\\ \cdots & \mathrm{B}\\ (\mathrm{a}_tb_t)^{g_t} & \end{array}\right) \tag{2.1} $$

We obtain a Prasad's I-function of $(s+2t)$-variables.

Provided that

$min\{e_l''^{(i)},f_l''^{(i)},h_l''^{(i)},e_l'^{(j)},f_l'^{(j)},h_l'^{(j)},e_l^{(k)},f_l^{(k)},h_l^{(k)},e_l,f_l,h_l,\mu_l,\upsilon_l,\omega_l\}>0;(i=1,\cdots,u;j=1,\cdots,r;k=1,\cdots,s;l=1,\cdots,t)$

$Re(\lambda_l)>0,g_l>0;l=1,\cdots,t$

$$ Re\left(\rho_l+e_lR+\sum_{j=1}^{r}e_l'^{(j)}\eta_{G_j,g_j}\right)+\sum_{k=1}^{s}e_l^{(k)}\min_{1\leqslant K\leqslant m'^{(k)}}Re\left(\frac{b_K'^{(k)}}{\beta_K'^{(k)}}\right)>0\,;(l=1,\cdots,t) $$

$$ Re\left(\sigma_l+f_lR+\sum_{j=1}^{r}f_l'^{(j)}\eta_{G_j,g_j}\right)+\sum_{k=1}^{s}f_l^{(k)}\min_{1\leqslant K\leqslant m'^{(k)}}Re\left(\frac{b_K'^{(k)}}{\beta_K'^{(k)}}\right)>0\,;(l=1,\cdots,t) $$

$$ z_i\neq0,\sum_{j=1}^{A}\gamma_j^{(i)}-\sum_{j=1}^{C}\xi_j^{(i)}+\sum_{j=1}^{M^{(i)}}\eta_j^{(i)}-\sum_{j=1}^{N^{(i)}}\epsilon_j^{(i)}<0,i=1,\cdots,r $$

with $\lambda,A,C,\alpha_i,\beta_i,m_i,n_i\in\mathbb{N}^*;i=1,\cdots,r\,;f_j,g_j,p_j^{(i)},q_j^{(i)},\gamma_j^{(i)},\xi_j^{(i)},\eta_j^{(i)},\epsilon_j^{(i)}\in\mathbb{C}$

$$ \left|arg\left(z_k'\prod_{l=1}^{t}\left[(x_l^{e_l^{(k)}}(a_l-x_l)^{f_l^{(k)}}\left\{1+(b_lx_l)^{g_l}\right\}^{-h_l^{(k)}}\right)\right|<\frac{1}{2}\Omega_i''\pi\ \ (a_l\leqslant x_l\leqslant b_l;k=1,\cdots,s;l=1,\cdots,t) $$

where, $\Omega''_i = \Omega'_i - (e_l^{(i)} + f_l^{(i)} + h_l^{(i)})$, Ω'_i is defined by (1.14). $P \leqslant Q+1$, and the multiple series on a left-hand side of (2.1) converges absolutely, where

$$U = p'_2, q'_2; p'_3, q'_3; \cdots ; p'_{s-1}, q'_{s-1} \tag{2.2}$$

$$V = 0, n'_2; 0, n'_3; \cdots ; 0, n'_{s-1} \tag{2.3}$$

$$X = m'^{(1)}, n'^{(1)}; \cdots ; m'^{(s)}, n'^{(s)}; 1,0; \cdots ; 1,0; 1,0; \cdots ; 1,0 \tag{2.4}$$

$$Y = p'^{(1)}, q'^{(1)}; \cdots ; p'^{(s)}, q'^{(s)}; 0,1; \cdots ; 0,1; 0,1; \cdots ; 0,1 \tag{2.5}$$

$$A = (a'_{2k}; \alpha'^{(1)}_{2k}, \alpha''^{(2)}_{2k})_{1,p_2}; \cdots ; (a'_{(s-1)k}; \alpha'^{(1)}_{(s-1)k}, \alpha'^{(2)}_{(s-1)k}, \cdots , \alpha'^{(s-1)}_{(s-1)k})_{1,p'_{s-1}};$$

$$(1-\rho_1 - e_1 R - \sum_{i=1}^{v} e''^{(i)}_1 K_i - \sum_{j=1}^{r} e'^{(j)}_1 \eta_{G_j,g_j}; e_1^{(1)}, \cdots , e_1^{(s)}, \mu_1, \underbrace{0, \cdots , 0}_{t-1}, g_1, \underbrace{0, \cdots , 0}_{t-1}), \cdots ,$$

$$(1-\rho_t - e_t R - \sum_{i=1}^{v} e''^{(i)}_t K_i - \sum_{j=1}^{r} e'^{(j)}_t \eta_{G_j,g_j}; e_t^{(1)}, \cdots , e_t^{(s)}, \underbrace{0, \cdots , 0}_{t-1}, \mu_t, \underbrace{0, \cdots , 0}_{t-1}, g_t),$$

$$(-\sigma_1 - f_1 R - \sum_{i=1}^{v} f''^{(i)}_1 K_i - \sum_{j=1}^{r} f'^{(j)}_1 \eta_{G_j,g_j}; f_1^{(1)}, \cdots , f_1^{(s)}, \upsilon_1, \underbrace{0, \cdots , 0}_{2t-1}), \cdots ,$$

$$(-\sigma_t - f_t R - \sum_{i=1}^{v} f''^{(i)}_t K_i - \sum_{j=1}^{r} f'^{(j)}_t \eta_{G_j,g_j}; f_t^{(1)}, \cdots , f_t^{(s)}, \underbrace{0, \cdots , 0}_{t-1}, \upsilon_t, \underbrace{0, \cdots , 0}_{t}),$$

$$(1-\lambda_1 - h_1 R - \sum_{i=1}^{v} h''^{(i)}_1 K_i - \sum_{j=1}^{r} h'^{(j)}_1 \eta_{G_j,g_j}; h_1^{(1)}, \cdots , h_1^{(s)}, \omega_1, \underbrace{0, \cdots , 0}_{t-1}, 1, \underbrace{0, \cdots , 0}_{t-1}), \cdots ,$$

$$(1-\lambda_t - h_t R - \sum_{i=1}^{v} h''^{(i)}_t K_i - \sum_{j=1}^{r} h'^{(j)}_t \eta_{G_j,g_j}; h_t^{(1)}, \cdots , h_t^{(s)}, \underbrace{0, \cdots , 0}_{t-1}, \omega_t, \underbrace{0, \cdots , 0}_{t-1}, 1),$$

$$(1-E_j; \underbrace{0, \cdots , 0}_{s}, \underbrace{1, \cdots , 1}_{t}, \underbrace{0, \cdots , 0}_{t})_{1,P}, \ (a'_{sk}; \alpha'^{(1)}_{sk}, \alpha'^{(2)}_{sk}, \cdots , \alpha'^{(s)}_{sk}, \underbrace{0, \cdots , 0}_{2t})_{1,p'_s} :$$

$$(a_k'^{(1)}, \alpha_k'^{(1)})_{1,p^{(1)}}; \cdots ; (a_k'^{(s)}, \alpha_k'^{(s)})_{1,p'^{(s)}}; \ -; \cdots ; - \tag{2.7}$$

$$B = (b'_{2k}; \beta'^{(1)}_{2k}, \beta'^{(2)}_{2k})_{1,q'_2}; \cdots ; (b'_{(s-1)k}; \beta'^{(1)}_{(s-1)k}, \beta'^{(2)}_{(s-1)k}, \cdots , \beta'^{(s-1)}_{(s-1)k})_{1,q'_{s-1}}; (b'_{sk}; \beta'^{(1)}_{sk}, \beta'^{(2)}_{sk}, \cdots , \beta'^{(s)}_{sk}, \underbrace{0, \cdots , 0}_{2t})_{1,q'_s},$$

$$(\rho_1 - \sigma_1 - (e_1+f_1)R - \sum_{i=1}^{v} (e''^{(i)}_1 + f''^{(i)}_1) K_i - \sum_{j=1}^{r} (e'^{(j)}_1 + f'^{(j)}_1)\eta_{G_j,g_j}; e_1^{(1)} + f_1^{(1)}, \cdots , e_1^{(s)} + f_1^{(s)}, \mu_1 + \upsilon_1, \underbrace{0, \cdots , 0}_{t-1}, g_1, \underbrace{0, \cdots , 0}_{t-1}), \cdots ,$$

Notes

$$(\rho_t-\sigma_t-(e_t+f_t)R-\sum_{i=1}^{v}(e_t''^{(i)}+f_t''^{(i)})K_i-\sum_{j=1}^{r}(e_t'^{(j)}+f_t'^{(j)})\eta_{G_j,g_j};e_t^{(1)}+f_t^{(1)},\cdots,e_t^{(s)}+f_t^{(s)},\underbrace{0,\cdots,0}_{t-1},\mu_t+\upsilon_t,\underbrace{0,\cdots,0}_{t-1},g_t),$$

$$(1-\lambda_1-h_1R-\sum_{i=1}^{v}h_1''^{(i)}K_i-\sum_{j=1}^{r}h_1'^{(j)}\eta_{G_j,g_j};h_1^{(1)},\cdots,h_1^{(s)},\omega_1,\underbrace{0,\cdots,0}_{2t-1}),\cdots,$$

$$(1-\lambda_t-h_tR-\sum_{i=1}^{v}h_t''^{(i)}K_i-\sum_{j=1}^{r}h_t'^{(j)}\eta_{G_j,g_j};h_t^{(1)},\cdots,h_t^{(s)},\underbrace{0,\cdots,0}_{t-1},\omega_t,\underbrace{0,\cdots,0}_{t}),$$

$$(1-F_j;\underbrace{0,\cdots,0}_{s},\underbrace{1,\cdots,1}_{t},\underbrace{0,\cdots,0}_{t})_{1,Q}:(b_k'^{(1)},\beta_k'^{(1)})_{1,q'^{(1)}};\cdots;(b_k'^{(s)},\beta_k'^{(s)})_{1,q'^{(s)}};\underbrace{(0;1);\cdots;(0;1)}_{2t} \tag{2.8}$$

and

$$\sum_{G_i=1}^{\alpha^{(i)}}\sum_{g_i=0}^{\infty}=\sum_{G_1,\cdots,G_r=1}^{\alpha^{(1)},\cdots,(r)}\sum_{g_1,\cdots,g_r=0}^{\infty}$$

Proof

To evaluate the multiple integrals (2.1), we first express the class of multivariable polynomials $S_{N_1,\cdots,N_v}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}[.]$ in series the multivariable A-function $A(z_1,\cdots,z_r)$ in serie, the sequence of functions $R_n^{(\alpha,\beta)}[.]$ in series with the help of equations (1.4), (1.6) and (1.1) respectively.Then we change the order of the multiple series and the $(x_1,\cdots,x_t)$-Integrals. Nest, we express the generalized hypergeometric function ${}_PF_Q[.]$ regarding a generalized Kampé de Fériet function of t-variables with the help of the formula ([11], page.39 Eq. (30)), and express this function of an H-function Of t variables with the help of the result ([12], page. 272, Eq. (4.7)). Next, we express the H-function of t-variables and The I-function of s-variables regarding their respective Mellin-Barnes integrals contour. Now we change the order of the $(t_1,\cdots,t_s),(\eta_1,\cdots,\eta_t)$ and $(x_1,\cdots,x_t)$-integrals which are permissible under the conditions stated with (2.1). Finally, on evaluating the $(x_1,\cdots,x_t)$-integrals thus got with the help of a case of the result ([10], page. 61, Eq. (5.2.1)) and we obtain the following result (say L.H.S.) :

$$\text{L.H.S}=\sum_{w,v',u,t',e,k_1,k_2}\sum_{K_1=0}^{[N_1/\mathfrak{M}_1]}\cdots\sum_{K_v=0}^{[N_v/\mathfrak{M}_v]}\sum_{G_i=1}^{\alpha^{(i)}}\sum_{g_i=1}^{\infty}\phi\frac{\prod_{i=1}^{r}\phi_i z_i^{\eta_{G_i,g_i}}(-)^{\sum_{i=1}^{r}g_i}}{\prod_{i=1}^{r}\epsilon_{G_i}^{(i)}g_i!}a_v y_1^{K_1}\cdots y_v^{K_v}\psi(w,v',u,t',e,k_1,k_2)$$

$$\frac{1}{(2\pi\omega)^{s+t}}\int_{L_1}\cdots\int_{L_s}\int_{L_{s+1}}\cdots\int_{L_{s+t}}\phi(t_1,\cdots,t_s)\prod_{i=1}^{s}\phi_i(t_i)z_i'^{t_i}\frac{\prod_{j=1}^{P}\Gamma(E_j+\sum_{k=1}^{t}\eta_k)}{\prod_{j=1}^{Q}\Gamma(F_j+\sum_{k=1}^{t}\eta_k)}\left[\prod_{l=1}^{t}\{\Gamma(-\eta_l)(-B_l)^{\eta_l}\right.$$

$$a_k^{\rho_l+\sigma_l+(e_l+f_l)R+\sum_{i=1}^{v}(e_l''^{(i)}+f_l''^{(i)})K_i+\sum_{j=1}^{r}(e_l'^{(j)}+f_l'^{(j)})\eta_{G_j,g_j}+\sum_{k=1}^{s}(e_l^{(k)}+f_l^{(k)})t_k+(\mu_l+\upsilon_l)\eta_l}$$

$$\frac{\Gamma(1+f_lR+\sigma_l+\sum_{i=1}^{v}f_l''^{(i)}K_i+\sum_{j=1}^{r}f_l'^{(j)}\eta_{G_j,g_j}+\sum_{k=1}^{s}f_l^{(k)}t_k+\upsilon_l\eta_l)}{\Gamma(\lambda_l+h_lR+\sum_{i=1}^{v}h_l''^{(i)}K_i+\sum_{j=1}^{r}h_l'^{(j)}\eta_{G_j,g_j}+\sum_{k=1}^{s}h_l^{(k)}t_k+\omega_l\eta_l)}H_{2,2}^{1,2}\left[\left.(\mathrm{b}_l a_l)^{g_j}\ \right|\begin{matrix}\mathrm{C,D}\\ .\\ \mathrm{E}\end{matrix}\right]\Bigg]$$

$$\mathrm{d}t_1\cdots\mathrm{d}t_s\,\mathrm{d}\eta_1\cdots\mathrm{d}\eta_t$$

where

$$C=(1-\lambda_l-f_lR-\sum_{i=1}^{u}f_l''^{(i)}K_i-\sum_{j=1}^{r}f_l'^{(j)}\eta_{G_j,g_j}-\sum_{k=1}^{s}f_l^{(k)}t_k-\omega_l\eta_l;1),$$

Ref

11. H. M. Srivastava and P. W. Karlson, Multiple Gaussian hypergeometric series, John, Wiley and Sons (Ellis Horwood Ltd.), New York, 1985.

$$D=(1-\rho_l-h_lR-\sum_{i=1}^{u}g_l^{\prime\prime(i)}K_i-\sum_{j=1}^{r}g_l^{\prime(j)}\eta_{G_j,g_j}-\sum_{k=1}^{s}g_l^{(k)}t_k-\mu_l\eta_l;g_l)\text{ and}$$

$$E=(0;1;1),(-\rho_l-\sigma_l-(f_l+h_l)R-\sum_{i=1}^{u}(f_l^{\prime\prime(i)}+g_l^{\prime\prime(i)})K_i-\sum_{j=1}^{r}(f_l^{\prime(j)}+g_l^{\prime(j)})\eta_{G_j,g_j}-\sum_{k=1}^{s}(f_l^{(k)}+g_l^{(k)})t_k-(\mu_k+\upsilon_k)\eta_l;g_l)$$

Now, if we express the product of the H-functions of one variable occurring in the above expression regarding their respective Mellin-Barnes integrals contour and reinterpreting the result thus obtained regarding the Prasad's I-function Of $(s+2t)$-variables, we arrive at the desired formula after algebraic manipulations.

III. Corollaries and Special Case

If the generalized multivariable polynomials, the multivariable A-function and multivariable I-function reduce respectively to a class of polynomials of one variable [8], A-function defined by Gautam and Asgar [3] and H-function defined by Fox [2], we get the following multiple integrals :

Corollary 1.

$$\int_0^{a_1}\cdots\int_0^{a_t}\prod_{l=1}^{t}\left[x_l^{\rho_l-1}(a_l-x_l)^{\sigma_l}\left\{1+(b_lx_l)^{g_l}\right\}^{-\lambda_l}\right]R_n^{\alpha,\beta}\left[y\prod_{l=1}^{t}\left[x_l^{e_l}(a_l-x_l)^{f_l}\left\{1+(b_lx_l)^{g_l}\right\}^{-h_l}\right]\right]$$

$$S_{N_1}^{\mathfrak{M}_1}\left(\ \mathrm{y}_1\left[\prod_{l=1}^{t}\left[x_l^{e_l^{\prime\prime(1)}}(a_l-x_l)^{f_l^{\prime\prime(1)}}\left\{1+(b_lx_l)^{g_l}\right\}^{-h_l^{\prime\prime(1)}}\right]\right]\ \right)$$

$$A\left(\ \mathrm{z}_1\left[\prod_{l=1}^{t}\left[x_l^{e_l^{\prime(1)}}(a_l-x_l)^{f_l^{\prime(1)}}\left\{1+(b_lx_l)^{g_l}\right\}^{-h_l^{\prime(1)}}\right]\right]\ \right)$$

$$H\left(\ \mathrm{z}_1'\left[\prod_{l=1}^{t}\left[x_l^{e_l^{(1)}}(a_l-x_l)^{f_l^{(1)}}\left\{1+(b_lx_l)^{g_l}\right\}^{-h_l^{(1)}}\right]\right]\ \right)$$

$${}_PF_Q\left[(A_P);(B_Q);\sum_{l=1}^{t}B_lx_l^{\mu_l}(a_l-x_l)^{\upsilon_l}\left\{1+(b_lx_l)^{g_l}\right\}^{-\omega_l}\right]\mathrm{d}x_1\cdots\mathrm{d}x_t=\frac{\prod_{j=1}^{Q}\Gamma(B_j)}{\prod_{j=1}^{P}\Gamma(A_j)}$$

$$\prod_{l=1}^{t}a_l^{\rho_l+\sigma_l}\sum_{w,v',u,t',e,k_1,k_2}\sum_{K=0}^{[N_1/\mathfrak{M}_1]}\sum_{G_1=1}^{\alpha^{(1)}}\sum_{g_1=1}^{\infty}\frac{\phi_1z_1^{\eta_{G_1,g_1}}(-)^{g_1}}{\epsilon_{G_1}^{(1)}g_1!}\frac{(-\mathfrak{N})_{\mathfrak{M}K}A_{\mathfrak{N},K}}{K!}y_1^K$$

$$\psi(w,v',u,t',e,k_1,k_2)\,y^R\prod_{l=1}^{t}a_l^{(e_l^{\prime\prime(1)}+f_l^{\prime\prime((1)})K}\prod_{l=1}^{t}a^{(e_l^{\prime(1)}+f_l^{\prime((1)})\eta_{G_1,g_1}}$$

$$H_{p^{(1)}+3t+P,q^{(1)}+2t+q;Y'}^{m^{(1)},n^{(1)}+3t+P;X'}\left(\begin{array}{c|c}\mathrm{z}_1'\prod_{l=1}^{t}a_l^{(e_l^{(1)}+f^{(1)})} & \mathrm{A'}\\ \text{-B}_1a_1^{\mu_1+\upsilon_1} & \cdot\\ \cdots & \cdot\\ \cdots & \cdot\\ \text{-B}_ta_t^{\mu_t+\upsilon_t} & \cdot\\ (\mathrm{a}_1b_1)^{g_1} & \cdot\\ \cdots & \cdot\\ \cdots & \cdot\\ (\mathrm{a}_tb_t)^{g_t} & \mathrm{B'}\end{array}\right)\qquad(3.1)$$

Ref

3. B. P. Gautam, A. S. Asgar and A. N. Goyal. The A-function. Revista. Mathematica. Tucuman (1980).

We obtain an H-function of to $(1+2t)$-variables.
Provided that

$$min\{e_l''^{(1)}, f_l''^{(1)}, h_l''^{(1)}, e_l'^{(1)}, f_l'^{(1)}, h_l'^{(1)}, e_l^{(1)}, f_l^{(1)}, h_l^{(1)}, e_l, f_l, h_l, \mu_l, \upsilon_l, \omega_l\} > 0; (l = 1, \cdots, t)$$

$$Re(\lambda_l) > 0, g_l > 0; l = 1, \cdots, t$$

Notes

$$Re\left(\rho_l + e_l R + e_l'^{(1)}\eta_{G_1,g_1}\right) + e_l^{(1)} \min_{1 \leqslant K \leqslant m'^{(1)}} Re\left(\frac{b_K'^{(1)}}{\beta_K'^{(1)}}\right) > 0\,; (l = 1, \cdots, t)$$

$$Re\left(\sigma_l + f_l R + f_l'^{(1)}\eta_{G_1,g_1}\right) + f_l^{(1)} \min_{1 \leqslant K \leqslant m'^{(1)}} Re\left(\frac{b_K'^{(1)}}{\beta_K'^{(1)}}\right) > 0\,; (l = 1, \cdots, t)$$

$$z_1 \neq 0, \sum_{j=1}^{A} \gamma_j^{(1)} - \sum_{j=1}^{C} \xi_j^{(1)} + \sum_{j=1}^{M^{(1)}} \eta_j^{(i)} - \sum_{j=1}^{N^{(1)}} \epsilon_j^{(i)} < 0$$

with $\lambda, A, C, \alpha_1, \beta_1, m_1, n_1 \in \mathbb{N}^*; ; f_j, g_j, p_j^{(1)}, q_j^{(1)}, \gamma_j^{(1)}, \xi_j^{(1)}, \eta_j^{(1)}, \epsilon_j^{(1)} \in \mathbb{C}$

$$\left|arg\left(z_1' \prod_{l=1}^{t}\left[(x_l^{e_l^{(1)}}(a_l - x_l)^{f_l^{(1)}}\ \{1 + (b_l x_l)^{g_l}\}^{-h_l^{(1)}}\right)\right| < \frac{1}{2}\Omega_i''\pi \ \ (a_l \leqslant x_l \leqslant b_l; l = 1, \cdots, t)$$

where $$\Omega_1'' = = \sum_{k=1}^{n'^{(1)}} \alpha_k'^{(1)} - \sum_{k=n'^{(1)}+1}^{p'^{(1)}} \alpha_k'^{(1)} + \sum_{k=1}^{m'^{(1)}} \beta_k'^{(1)} - \sum_{k=m'^{(1)}+1}^{q'^{(i)}} \beta_k'^{(1)} - (e_l^{(1)} + f_l^{(1)} + h_l^{(1)})$$

$P \leqslant Q + 1$, and the multiple series on the left-hand side of (2.1) converges absolutely, where

$$X' = m'^{(1)}, n'^{(1)}; 1, 0; \cdots; 1, 0; 1, 0; \cdots; 1, 0 \tag{3.2}$$

$$Y' = p'^{(1)}, q'^{(1)}; 0, 1; \cdots; 0, 1; 0, 1; \cdots; 0, 1 \tag{3.3}$$

$$A' = (1 - \rho_1 - e_1 R - e_1''^{(i)} K - e_1'^{(1)}\eta_{G_1,g_1}; e_1^{(1)}, \mu_1, \underbrace{0, \cdots, 0}_{t-1}, g_1, \underbrace{0, \cdots, 0}_{t-1}), \cdots,$$

$$(1 - \rho_t - e_t R - e_t''^{(1)} K - e_t'^{(1)}\eta_{G_1,g_1}; e_t^{(1)}, \underbrace{0, \cdots, 0}_{t-1}, \mu_t, \underbrace{0, \cdots, 0}_{t-1}, g_t),$$

$$(-\sigma_1 - f_1 R - f_1''^{(1)} K - f_1'^{(1)}\eta_{G_1,g_1}; f_1^{(1)}, \upsilon_1, \underbrace{0, \cdots, 0}_{2t-1}), \cdots,$$

$$(-\sigma_t - f_t R - f_t''^{(1)} K - f_t'^{(1)}\eta_{G_1,g_1}; f_t^{(1)}, \underbrace{0, \cdots, 0}_{t-1}, \upsilon_t, \underbrace{0, \cdots, 0}_{t}),$$

$$(1 - \lambda_1 - h_1 R - h_1''^{(1)} K - h_1'^{(1)}\eta_{G_1,g_1}; h_1^{(1)}, \omega_1, \underbrace{0, \cdots, 0}_{t-1}, 1, \underbrace{0, \cdots, 0}_{t-1}), \cdots,$$

$$(1-\lambda_t-h_tR-h_t''^{(1)}K-h_t'^{(1)}\eta_{G_1,g_1};h_t^{(1)},\underbrace{0,\cdots,0}_{t-1},\omega_t,\underbrace{0,\cdots,0}_{t-1},1),$$

$$(1-E_j;\underbrace{0,\cdots,0}_{s},\underbrace{1,\cdots,1}_{t},\underbrace{0,\cdots,0}_{t})_{1,P},\ [(a_k'^{(1)},\alpha_k'^{(1)})_{1,p^{(1)}},\underbrace{0,\cdots,0}_{2t}];-;\cdots;- \tag{3.4}$$

$$B'=[(b_k'^{(1)},\beta_k'^{(1)})_{1,q'^{(1)}},\underbrace{0,\cdots,0}_{2t}],$$

$$(\rho_1-\sigma_1-(e_1+f_1)R-(e_1''^{(i)}+f_1''^{(1)})K-(e_1'^{(1)}+f_1'^{(1)})\eta_{G_1,g_1};e_1^{(1)}+f_1^{(1)},\mu_1+\upsilon_1,\underbrace{0,\cdots,0}_{t-1},g_1,\underbrace{0,\cdots,0}_{t-1})$$

$$,\cdots,(\rho_t-\sigma_t-(e_t+f_t)R-(e_t''^{(1)}+f_t''^{(1)})K-(e_t'^{(1)}+f_t'^{(1)})\eta_{G_1,g_1};e_t^{(1)}+f_t^{(1)},\underbrace{0,\cdots,0}_{t-1},\mu_t+\upsilon_t,\underbrace{0,\cdots,0}_{t-1},g_t),$$

$$(1-\lambda_1-h_1R-h_1''^{(1)}K_1-h_1'^{(1)}\eta_{G_1,g_1};h_1^{(1)},\omega_1,\underbrace{0,\cdots,0}_{2t-1}),\cdots,$$

$$(1-\lambda_t-h_tR-h_t''^{(1)}K-h_t'^{(1)}\eta_{G_1,g_1};h_t^{(1)},\underbrace{0,\cdots,0}_{t-1},\omega_t,\underbrace{0,\cdots,0}_{t}),$$

$$(1-F_j;\underbrace{0,\cdots,0}_{s},\underbrace{1,\cdots,1}_{t},\underbrace{0,\cdots,0}_{t})_{1,Q}:\underbrace{(0;1);\cdots;(0;1)}_{2t} \tag{3.5}$$

By applying our result given in (4.1) and (4.4) to the case the Laguerre polynomials ([16], page 101, eq.(15.1.6)) and ([15] ,page 159) and by setting

$$S_N^1(x)\rightarrow L_N^{\alpha'}(x)$$

In which case $\mathfrak{M}=1, A_{N,K}=\binom{N+\alpha'}{N}\frac{1}{(\alpha'+1)_K}$ we obtain the following multiple integrals.

Corollary 2.

$$\int_0^{a_1}\cdots\int_0^{a_t}\prod_{l=1}^{t}\left[x_l^{\rho_l-1}(a_l-x_l)^{\sigma_l}\left\{1+(b_lx_l)^{g_l}\right\}^{-\lambda_l}\right]R_n^{\alpha,\beta}\left[y\prod_{l=1}^{t}\left[x_l^{e_l}(a_l-x_l)^{f_l}\left\{1+(b_lx_l)^{g_l}\right\}^{-h_l}\right]\right]$$

$$L_N^{\alpha'}\Big(\ \mathrm{y}_1\left[\textstyle\prod_{l=1}^t\left[x_l^{e_l''^{(1)}}(a_l-x_l)^{f_l''^{(1)}}\left\{1+(b_lx_l)^{g_l}\right\}^{-h_l''^{(1)}}\right]\right]\ \Big)$$

$$A\Big(\ \mathrm{z}_1\left[\textstyle\prod_{l=1}^t\left[x_l^{e_l'^{(1)}}(a_l-x_l)^{f_l'^{(1)}}\left\{1+(b_lx_l)^{g_l}\right\}^{-h_l'^{(1)}}\right]\right]\ \Big)$$

$$H\Big(\ \mathrm{z}_1'\left[\textstyle\prod_{l=1}^t\left[x_l^{e_l^{(1)}}(a_l-x_l)^{f_l^{(1)}}\left\{1+(b_lx_l)^{g_l}\right\}^{-h_l^{(1)}}\right]\right]\ \Big)$$

$${}_PF_Q\left[(A_P);(B_Q);\sum_{l=1}^{t}B_lx_l^{\mu_l}(a_l-x_l)^{\upsilon_l}\left\{1+(b_lx_l)^{g_l}\right\}^{-\omega_l}\right]\mathrm{d}x_1\cdots\mathrm{d}x_t=\frac{\prod_{j=1}^{Q}\Gamma(B_j)}{\prod_{j=1}^{P}\Gamma(A_j)}$$

Ref

16. C. Szego, (1975), Orthogonal polynomials. Amer. Math. Soc. Colloq. Publ. 23 fourth edition. Amer. Math. Soc. Providence. Rhodes Island, 1975.

$$\prod_{l=1}^{t} a_l^{\rho_l+\sigma_l} \sum_{w,v',u,t',e,k_1,k_2} \sum_{K=0}^{N} \sum_{G_1=1}^{\alpha^{(1)}} \sum_{g_1=1}^{\infty} \frac{\phi_1 z_1^{\eta_{G_1,g_1}}(-)^{g_1}}{\epsilon_{G_1}^{(1)} g_1!} \psi(w,v,u,t',e,k_1,k_2)$$

$$y^R \prod_{l=1}^{t} a_l^{(e_l''^{(1)}+f_l''^{((1)})K} \prod_{l=1}^{t} a^{(e_l'^{(1)}+f_l'^{((1)})\eta_{G_1,g_1}} \frac{(-N)_K}{K!} \binom{N+\alpha'}{N} \frac{1}{(\alpha'+1)_K} y_1^K$$

$$H^{m^{(1)},n^{(1)}+3t+P;X'}_{p^{(1)}+3t+P,q^{(1)}+2t+q;Y'} \left(\begin{array}{c|c} \mathrm{z}_1' \prod_{l=1}^{t} a_l^{(e_l^{(1)}+f^{(1)})} & \mathrm{A'} \\ \text{-}\mathrm{B}_1 a_1^{\mu_1+\upsilon_1} & \cdot \\ \cdots & \cdot \\ \cdots & \cdot \\ \text{-}\mathrm{B}_t a_t^{\mu_t+\upsilon_t} & \cdot \\ (\mathrm{a}_1 b_1)^{g_1} & \cdot \\ \cdots & \cdot \\ \cdots & \cdot \\ (\mathrm{a}_t b_t)^{g_t} & \mathrm{B'} \end{array} \right) \tag{3.6}$$

under the same notations and existence conditions that (3.1).

If, $s=t=2$, the general polynomial S_N^M reduces to the Jacobi polynomials $P_n^{(\alpha,\beta)}(1-2x)$, the H-function of two variables into Appell's function F_3 and the generalized hypergeometric function ${}_PF_Q$ into the Bessel's function J_υ with the help of results ([15], page.159, Eq. (1.6)), ([10], page. 89, Eq. (6.4.6) ,page.18 Eq. (2.6.3) (2.6.5)), respectively and the A-function and a sequence of functions vanish, we arrive at the following double integrals after simplifications (see Gupta and Jain [5] for more details):

$$\int_0^{a_1}\int_0^{a_2} \prod_{l=1}^{2} \left[x_l^{\rho_l-1}(a_l-x_l)^{\sigma_l} \left\{1+(b_l x_l)^{h_l}\right\}^{-\lambda_l} \right] P_n^{(\alpha,\beta)}[1-2yx_1^{e_1}x_2^{e_2}] \left[2\sqrt{B_1x_1^{\mu_1}+B_2x_2^{\mu_2}}\right]^{-\frac{v}{2}}$$

$$F_3[k_1,k_2,h_1,h_2;L;z_1x_1^{u_1},z_2x_2^{u_2}] J_\upsilon[2\sqrt{B_1x_1^{\mu_1}+B_2x_2^{\mu_2}}]\,\mathrm{d}x_1\mathrm{d}x_2 =$$

$$\frac{\Gamma(L)\Gamma(1+\sigma_1)\Gamma(1+\sigma_2)a_1^{\rho_1+\sigma_1}a_2^{\rho_2+\sigma_2}}{\Gamma(k_1)\Gamma(k_2)\Gamma(h_1)\gamma(h_2)\Gamma(\lambda_1)\Gamma(\lambda_2)} \sum_{R=0}^{n} \frac{(-n)_R\binom{\alpha+n}{n}(\alpha+\beta+n+1)_R(ya_1^{e_1}a_2^{e_2})^R}{R!(\alpha+1)_R}$$

$$H^{0,2;1,2;1,2;1,0;1,0;1,1;1,1}_{2,4;2,1;2,1;0,1;0,1;1,1;1,1} \left(\begin{array}{c|c} \text{-}\mathrm{z}_1 a_1^{u_1} & \mathrm{A}_2 \\ \text{-}\mathrm{z}_2 a_2^{u_2} & \cdot \\ \mathrm{B}_1 a_1^{\mu_1} & \cdot \\ \mathrm{B}_2 a_2^{\mu_2} & \cdot \\ \mathrm{a}_1 b_1 & \mathrm{B}_2 \\ \mathrm{a}_2 b_2 & \end{array} \right) \tag{3.7}$$

with

$$A_2 = (1-\rho_1-e_1R;u_1,0,\mu_1,0,1,0),(1-\rho_2-e_2R;0,u_2,0,\mu_2,0,1):(1-k_1;1),(1-k_2;1);(1-h_1;1),(1-h_2;1)$$

$$-;-;(1-\lambda_1;1);(1-\lambda_2;1) \tag{3.8}$$

$$B_2 = (-v;0,0,,1,1,0,0),(-\rho_1-\sigma_1-e_1R;u_1,0,\mu_1,0,1,0),(-\rho_2-\sigma_2-e_2R;0,u_2,0,\mu_2,0,1),$$

$$(1-L;1,1,0,0,0,0,):(0,1);(0,1);(0,1);(0,1);(0,1);(0,1) \tag{3.9}$$

Ref

5. K. C. Gupta and R.Jain, A unified study of some multiple integrals, Soochow Journal of Mathematics, 19(1) (1993), 73-81.

IV. Conclusion

In this paper, we have evaluated unified multiple integrals involving the product of an expansion of the multivariable A-function, multivariable I-function defined by Prasad [6], a sequence of functions and class of multivariable polynomials defined by Srivastava [9] with general arguments. The formula established in this paper is very general nature. Thus, the results established in this research work would serve as a formula from which, upon specializing the parameters, as many as desired results involving the special functions of one and several variables, multiple integrals can be obtained.

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