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# The Extended Q- Image (basic analogue) of Mittag-Leffler Function and it's Simply Properties

By Alok Jain, Farooq Ahmad, D. K. Jain & Renu Jain

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# The Extended Q- Image (basic analogue) of Mittag-Leffler Function and it's Simply Properties

Alok Jain [α], Farooq Ahmad [σ], D. K. Jain [ρ] & Renu Jain [ω]

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## I. Introduction and Mathematical Preliminaries

The purpose of this paper is to increase the accessibility of different dimensions of q-fractional calculus and the q-analogue of the extended Mittag-Leffler function to the real world problems of engineering science.

Let us start with giving the historical background of the ML-function. The function

$$E_\alpha(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k+1)}, \qquad \ldots(1)$$

was defined and studied by ML-function in the year 1903 in [1-3].

It is direct generalization of the exponential series, since for $\alpha = 1$, we have to exponential function.

The function is defined by

$$E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k+\beta)},$$

gives a generalization of equation (1) this generalization was studied by Wiman in 1905 [4,5]. Agarwal in 1953, and Humbert and Agarwal [6, 7] in 1953. Afterward, Prabhakar [8] introduced the generalized ML-function.

$$E^{\delta}{}_{\beta,\gamma}(z) = \sum_{k=0}^{\infty} \frac{(\delta)_k}{\Gamma(\beta k+\gamma)} \frac{z^k}{k!} \text{, where } \beta,\gamma,\delta \in C, \text{ with } \mathrm{Re}(\beta) > 0.$$

Recently, Mehmet Ali etal [9] extended ML-function as follows

$$E^{\gamma;c}{}_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{\beta(\gamma+k,c-\gamma)}{\beta(\gamma,c-\gamma)} \frac{(c)_k}{\Gamma(\alpha k+\beta)} \frac{z^k}{k!}\text{, where } \beta,\gamma,\delta \in C, \text{ with } \mathrm{Re}(\beta) > 0.$$

*Author σ: Assistant Professor (Mathematics), Govt. Degree College for Women Nawakadal Srinagar. e-mail: sheikhfarooq85@gmail.com*

*Riemann-Liouville fractional integrals:* As defined in [10, 11], the Riemann-Liouville fractional integral is given by:

$$(I_{a+}^{\alpha} \mathrm{f})x \ = \frac{1}{\Gamma(\alpha)} \int_a^x (x-t)^{\alpha-1} f(\mathrm{t}) \ \mathrm{dt}, \ (x > a \, , a \epsilon R).$$

Thus, in general the Riemann-Liouville fractional integrals of arbitrary order for a function f(t), is a natural consequence of the well-knownformula (Cauchy-Dirichlets?) that reduces the calculation of the n-fold primitive of a function f (t) to a single integral of convolution type.

Recently, Yadav et. al. [12] introduces a new q-extension of the Leibnitz rule for the derivatives of a product of two basic functions in terms of a finite q-series involving Weyl type q-derivatives of the functions in the following manner.

$$\underset{z,\ \infty,q}{\overset{\alpha}{D}}[U(v)V(z)] = \sum_{r=0}^{\infty} \frac{(-1)^r q^{r(r+1)/2}(q^{-\alpha};q)_r}{(q;q)_r} \underset{z\ \ \infty,q}{\overset{\alpha-r}{D}}[U(z)] \underset{z\ \ \infty,q}{\overset{\alpha}{D}}\left[V(zq^{\alpha-r})\right]$$

Where U(z) and V(z) are two regular functions and the functional q-differential operator $\underset{z,\ \infty,q}{\overset{\alpha}{D}}(.)$ of Weyl type is given by

$$\underset{z,\ \infty,q}{\overset{\alpha}{D}}[f(z)] = \frac{q^{\frac{-\alpha(\alpha+1)}{2}}}{\Gamma_q(-\alpha)} \int_z^{\infty} (t-z)_{-q-1} f(tq^{1+\alpha}) d_q(t)$$

In particular for f(z) = z$^{-p}$ the equation becomes:

$$\underset{z,\ \infty,q}{\overset{\alpha}{D}}\left[z^{-p}\right] = \frac{\Gamma_q(p+\alpha) q^{-\alpha p + \frac{\alpha(1-\alpha)}{2}} z^{-p-\alpha}(1-q)}{\Gamma_q(p)}$$

## II. Main Results

In this section of paper, we defined the q-analogue of Mehmet Ali etal [1] extended ML-function as follows.

We defined basic analogue of extended ML-function by using the fact that

$$\frac{(\gamma:q)_k}{(c:q)_k} = \frac{B_q(\gamma+k, c-\gamma)}{B_q(\gamma, c-\gamma)}$$

$$E^{\gamma;c}{}_{\alpha,\beta}(\mathrm{z:q}) \ = \ \sum_{k=0}^{\infty} \frac{B_q(\gamma+k,c-\gamma)}{B_q(\gamma,c-\gamma)} \frac{(c:q)_k}{\Gamma_q(\alpha k+\beta)} \frac{z^k}{(q;q)_k}.$$

The function $E^{\gamma;c}{}_{\alpha,\beta}(\mathrm{z:q})$ converges under convergence of basic analogue of H-function which are as follows. The integral converges if $Re[s\log(z) - \log \sin \pi s] < 0$, on the contour C,

*Theorem I:* (Integral representation) For the extended Mittag-Leffler function, we have

$$I_q^{\mu} \ \{E^{\gamma;c}{}_{\alpha,\beta}(\mathrm{z:q})\} = \ I_q^{\mu}\left[\sum_{k=0}^{\infty} \frac{B_q(\gamma+k,c-\gamma)}{B_q(\gamma,c-\gamma)} \frac{(c:q)_k}{\Gamma_q(\alpha k+\beta)} \frac{z^k}{(q;q)_k}\right]$$

Or

Ref

12. Yadav, R. K., & Purohit, S. D. (2006). On fractional q-derivatives and transformations of the generalized basic hypergeometric functions. *J. Indian Acad. Math, 28*(2), 321-326.

$$I_q^{\mu}\ \{E^{\gamma;c}{}_{\alpha,\beta}(\mathrm{z{:}\,q})\} = \left\{\frac{B_q(\gamma+k,c-\gamma)}{B_q(\gamma,c-\gamma)}\frac{(c{:}q)_k}{\Gamma_q(\alpha k+\beta)}\frac{1}{(q;q)_k}\right\} I_q^{\mu}\ (z^k)$$

Or

$$I_q^{\mu}\ \{E^{\gamma;c}{}_{\alpha,\beta}(\mathrm{z{:}\,q})\} = \frac{\Gamma_q(\gamma+k)\Gamma_q(c-\gamma)\Gamma_q(c)\Gamma_q(c+k)\Gamma_q(\mu+k)}{\Gamma_q(c+k)\ \Gamma_q(c-\gamma)\Gamma_q(\gamma)\Gamma_q(\mu+1+k)\Gamma_q(\beta+\alpha k)}\frac{z^{k+\mu}}{(q;q)_k}$$

Notes

Thus,

$$I_q^{\mu}\ \{E^{\gamma;c}{}_{\alpha,\beta}(\mathrm{z{:}\,q})\} = \frac{z^{\mu}}{\Gamma_q(\gamma)}\ {}^2\Psi_2\left[\begin{matrix}(\gamma,1)(\mu,1);\ z\\(\beta+\alpha,1)(\mu+1,1)\end{matrix}\right].$$

This is the proof of the theorem.

*Theorem II:* For the extended Mittag-Leffler function, we have

$$D_q^{\mu}\ \{E^{\gamma;c}{}_{\alpha,\beta}(\mathrm{z{:}\,q})\} = D_q^{\mu}\left[\sum_{k=0}^{\infty}\frac{B_q(\gamma+k,c-\gamma)}{B_q(\gamma,c-\gamma)}\frac{(c{:}q)_k}{\Gamma_q(\alpha k+\beta)}\frac{z^k}{(q;q)_k}\right]$$

Or

$$D_q^{\mu}\ \{E^{\gamma;c}{}_{\alpha,\beta}(\mathrm{z{:}\,q})\} = \left\{\frac{B_q(\gamma+k,c-\gamma)}{B_q(\gamma,c-\gamma)}\frac{(c{:}q)_k}{\Gamma_q(\alpha k+\beta)}\frac{1}{(q;q)_k}\right\} D_q^{\mu}\ (z^k)$$

Or

$$D_q^{\mu}\ \{E^{\gamma;c}{}_{\alpha,\beta}(\mathrm{z{:}\,q})\} = \frac{\Gamma_q(\gamma+k)\Gamma_q(c-\gamma)\Gamma_q(c)\Gamma_q(c+k)\Gamma_q(\mu+k)}{\Gamma_q(c+k)\ \Gamma_q(c-\gamma)\Gamma_q(\gamma)\Gamma_q(\mu-1+k)\Gamma_q(\beta+\alpha k)}\frac{z^{k+\mu}}{(q;q)_k}$$

Thus,

$$D_q^{\mu}\ \{E^{\gamma;c}{}_{\alpha,\beta}(\mathrm{z{:}\,q})\} = \frac{z^{\mu}}{\Gamma_q(\gamma)}\ {}^2\Psi_2\left[\begin{matrix}(\gamma,1)(\mu,1);\ z\\(\beta+\alpha,1)(\mu-1,1)\end{matrix}\right].$$

This is the proof of the theorem.

## III. Conclusion

In this paper, we have explored the possibility for derivation of some expansions of basic analogue ML-function. The results thus derived are general in character and likely to find certain applications in the theory of hyper geometric functions. Finally we conclude with the remark that the results and the operators proved in this paper appear to be new and likely to have useful applications to a wide range of problems of mathematics, statistics and physical sciences.

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Notes