



Wind-Wave Relation during Hurricane Wilma and its Applications for Marine Science and Engineering

By S. A. Hsu

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Abstract- Analysis of datasets available from the literature indicates that, during tropical cyclones at sea, the barometric pressure is approximately negatively linearly related to the wind speed as well as to the wave height. During Hurricane Wilma in 2005, simultaneous meteorological-oceanographic (met-ocean) measurements were made by the National Data Buoy Center (NDBC) at the Data Buoy Station 42056 in the northwestern Caribbean Sea. Further analysis of these datasets showed that, when $U_{10} \geq 9$ m s⁻¹ during wind seas (when $H_s/L_p \geq 0.020$), $H_s = 0.43 U_{10} - 2$. Here, parameter H_s is the significant wave height (in meters), U_{10} is the wind speed (in m s⁻¹) at 10 m, $L_p (= 1.56 T_p^2)$ is the dominant wave length (in meters), and T_p is the peak wave period (in seconds). Applications of this proposed formula were successful during Hurricane Jose in 2017, Typhoon Russ in 1990 by NDBC Buoy 52009 near Guam, Typhoon Krosa in 2007 by a data buoy near Taiwan, and Typhoon Soudelor in 2015 by Jason -2 altimeter satellite. Also, its applications to rapid estimations of peak wave period, sea-surface currents and storm surge potentials were presented.

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Abstract- Analysis of datasets available from the literature indicates that, during tropical cyclones at sea, the barometric pressure is approximately negatively linearly related to the wind speed as well as to the wave height. During Hurricane Wilma in 2005, simultaneous meteorological-oceanographic (met-ocean) measurements were made by the National Data Buoy Center (NDBC) at the Data Buoy Station 42056 in the northwestern Caribbean Sea. Further analysis of these datasets showed that, when $U_{10} \geq 9 \text{ m s}^{-1}$ during wind seas (when $H_s/L_p \geq 0.020$), $H_s = 0.43 U_{10} - 2$. Here, parameter H_s is the significant wave height (in meters), U_{10} is the wind speed (in m s^{-1}) at 10 m, $L_p (= 1.56 T_p^2)$ is the dominant wave length (in meters), and T_p is the peak wave period (in seconds). Applications of this proposed formula were successful during Hurricane Jose in 2017, Typhoon Russ in 1990 by NDBC Buoy 52009 near Guam, Typhoon Krosa in 2007 by a data buoy near Taiwan, and Typhoon Soudelor in 2015 by Jason -2 altimeter satellite. Also, its applications to rapid estimations of peak wave period, sea-surface currents and storm surge potentials were presented.

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1. INTRODUCTION

This research article is motivated by Fig. 1 as presented in Bancroft (2016 available online at <http://www.vos.noaa.gov/MWL/201604/northpacific.shtml#contents>), who states that, in August 2015 "Super-Typhoon Soudelor was a strengthening typhoon while tracking northwest and crossing near 16N 144E at 0000 UTC August 3rd with sustained winds of 115 knots (or 59 m s^{-1}). It became a super-typhoon 12 hours later and after another six hours reached maximum intensity with sustained winds of 155 knots (80 m s^{-1}). At 1200 UTC on the 4th Soudelor was a super typhoon near 19N 137E with sustained winds 140 knots (72 m s^{-1}). Fig.1 is an infrared satellite view of Soudelor, and it is just a coincidence that we have a Jason-2 altimeter pass through the eye wall of Soudelor. Note the highest significant wave height of 90.55 feet (or 27.6 m) in the northwest eye wall. This is the highest satellite detected wave height that is known of by the author. Later on the 4th Soudelor passed west of the area with a weakening trend".

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The purpose of this investigation is to validate whether these measurements are reasonable. If they are verified, then, measurements of ocean waves routinely available by satellites may be applied to air-sea interaction studies, particularly to the relation between wind and waves, see, e.g. Csanady (2001), Drennan et al. (2005), Holthuijsen et al. (2012), and Bryant and Akbar (2016).

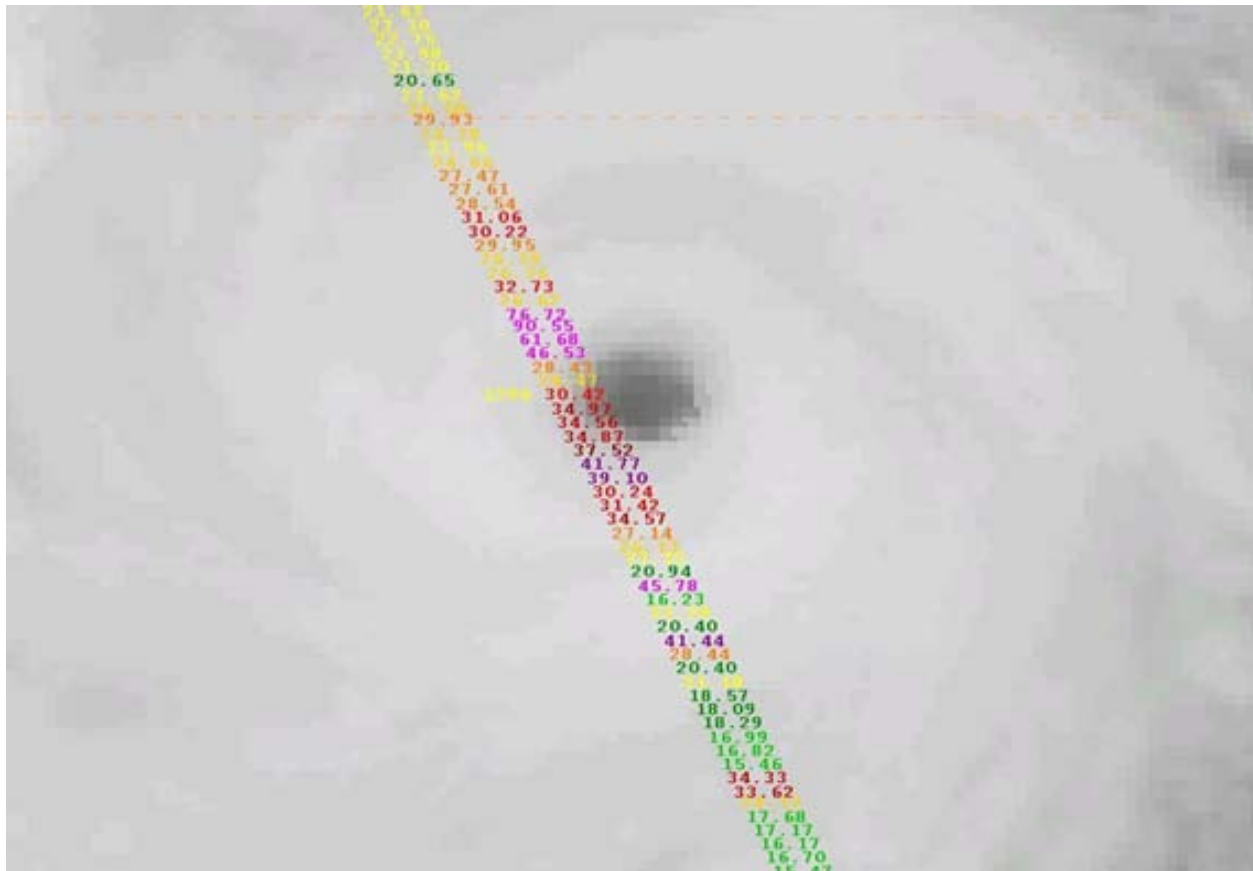


Fig. 1: A zoomed-in infrared satellite Image of Super-Typhoon Soudelor valid 1232 UTC August 4, 2015. A Jason -2 altimeter pass appears as a swath of significant wave heights given in feet to two decimal places cutting across the central core of Soudelor (for more detail, see Bancroft, 2016 at <http://www.vos.noaa.gov/MWL/201604/northpacific.shtml#contents>)

II. WIND-WAVE RELATION DURING WIND SEAS

Analytical formulas for the wind-wave relation are available in the literature. Examples are presented in Hsu et al. (2017a), who also provided following equation when the sea surface is aerodynamically rough (Hsu et al. 2017b) under the conditions of $U_{10} \geq 9 \text{ m s}^{-1}$ and $H_s/L_p \geq 0.020$, such that

$$U_{10} = 35H_s/T_p, \quad (1)$$

Here, parameter H_s is the significant wave height (in meters), U_{10} is the wind speed (in m s^{-1}) at 10 m, $L_p (= 1.56 T_p^2)$ is the dominant wave length (in meters), and T_p is the peak wave period (in seconds).

By simultaneous measurements of U_{10} , H_s and T_p at the National Data Buoy Center (NDBC) Station 42003 during Hurricane Ivan in 2004 and Katrina in 2005 and at 42056 during Wilma, Eq. (1) is further verified as shown in Fig. 2. For tracks and datasets of these three hurricanes, all located on the right side of the storm track, see www.nhc.noaa.gov and www.ndbc.noaa.gov, respectively. However, because T_p is not available from Fig. 1, one needs to correlate U_{10} and H_s directly rather

than using Eq. (1). This was accomplished in the next Section.

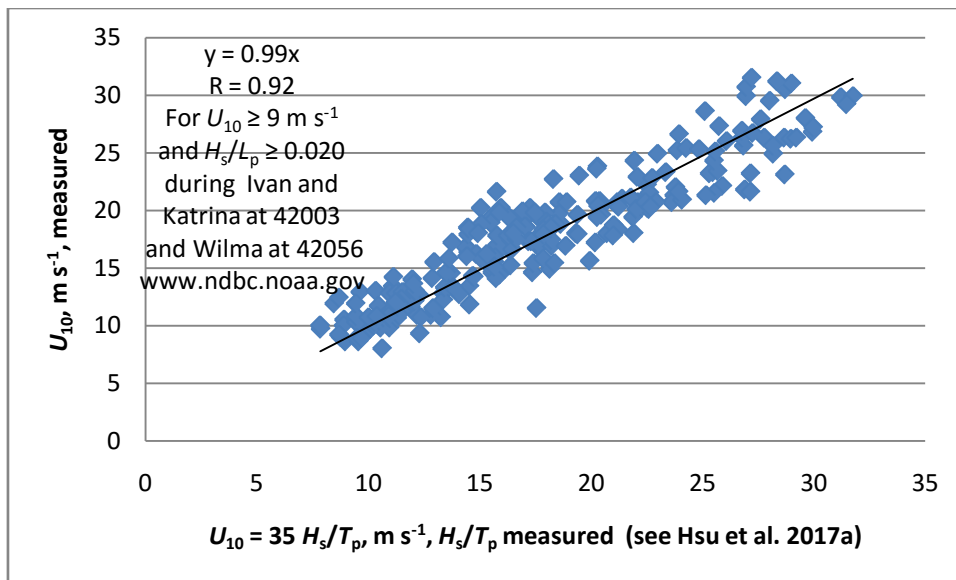


Fig. 2: Validation of Eq. (1) during Hurricanes Ivan, Katrina, and Wilma

III. RELATION BETWEEN BAROMETRIC PRESSURE AND MET-OCEAN PARAMETERS

Because the most important meteorological-oceanographic (met-ocean) parameter during a storm at sea is the barometric pressure, we first relate it to the wind speed and wave height. To validate Fig. 1, all similar circle-eye tropical cyclone datasets including barometric pressure (p_c) and the wind speed (V_{max}) available in Li et al. (2013) were incorporated in Fig. 3. It is found that they are negatively linearly related to a correlation coefficient $R = 0.89$. By Abel et al. (1989),

the wave height may also be related negatively linearly to the barometric pressure as shown in Fig. 4. Therefore, it was postulated that the wind speed and wave height are positively linearly related so that

$$H_s = aU_{10} - b, \tag{2a}$$

Here, coefficients “a” and “b” are the slope and the intercept of this proposed linear relation between H_s and U_{10} , respectively. Note that these coefficients may vary with different storms and needed to be determined from field measurements.

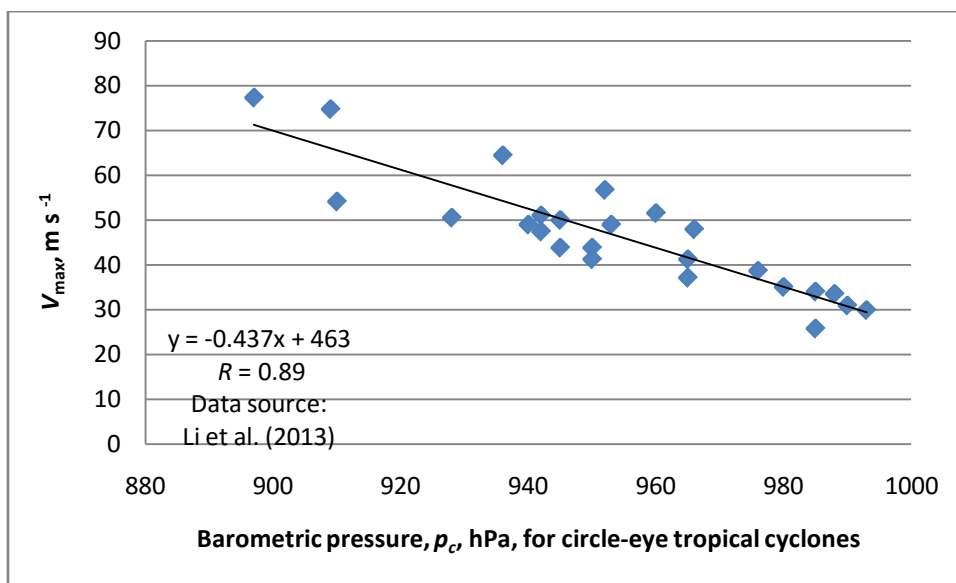


Fig. 3: Relation between barometric pressure and the maximum wind speed during circle-eye tropical cyclones

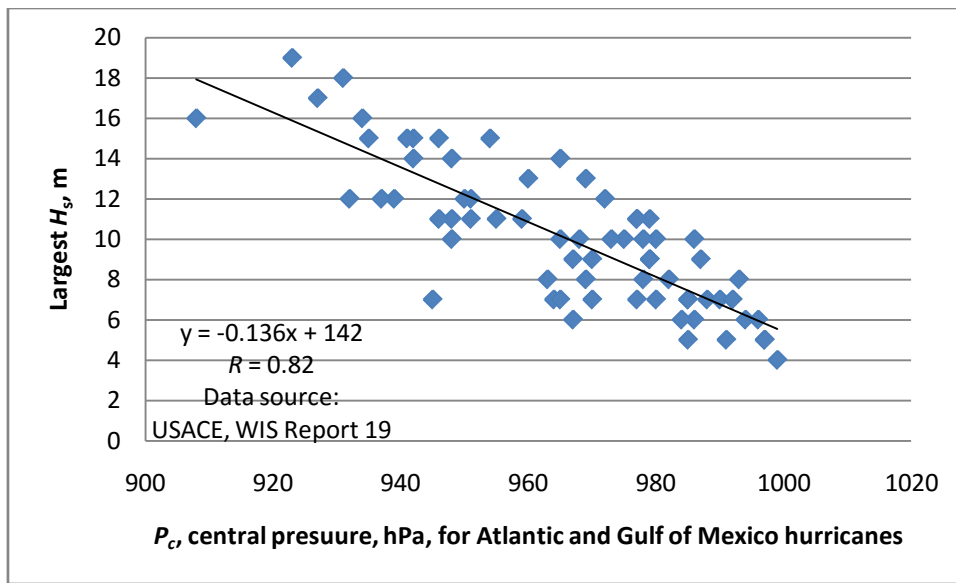


Fig. 4: Relation between barometric pressure and largest wave height during hurricanes

IV. WIND-WAVE RELATION DURING WILMA

To verify Eq. (1), pertinent data from Hurricane Wilma (see Figs. 5 thru 7) are employed. The buoy used was 42056, which was located on the right side of the storm’s track as shown in Fig. 6 in the northwestern Caribbean Sea. Similar to Figs. 3 and 4, the negatively linearly relation between U_{10} and barometric pressure as well as between H_s and barometric pressure was presented in Figs. 8 and 9, respectively. According to Hasse and Weber (1985), overwater stability categories may be estimated using a graphic approach from the measurements of wind speed and air and sea temperature difference. On the basis of Fig. 10, stability “D” prevailed during the entire period (as used in this study) from 14UTC on 19 thru 23UTC on 23, 2005 at Buoy 42056, indicating that the stability is near-neutral so that the logarithmic wind profile law is valid (see, e.g., Hsu2003; Vickery et al.2009). Our results to verify Eq. (1) is presented in Fig.10that

$$H_s = 0.43U_{10} - 2, \tag{2b}$$

Since the correlation coefficient $R = 0.91$, Eq. (2b) is useful.

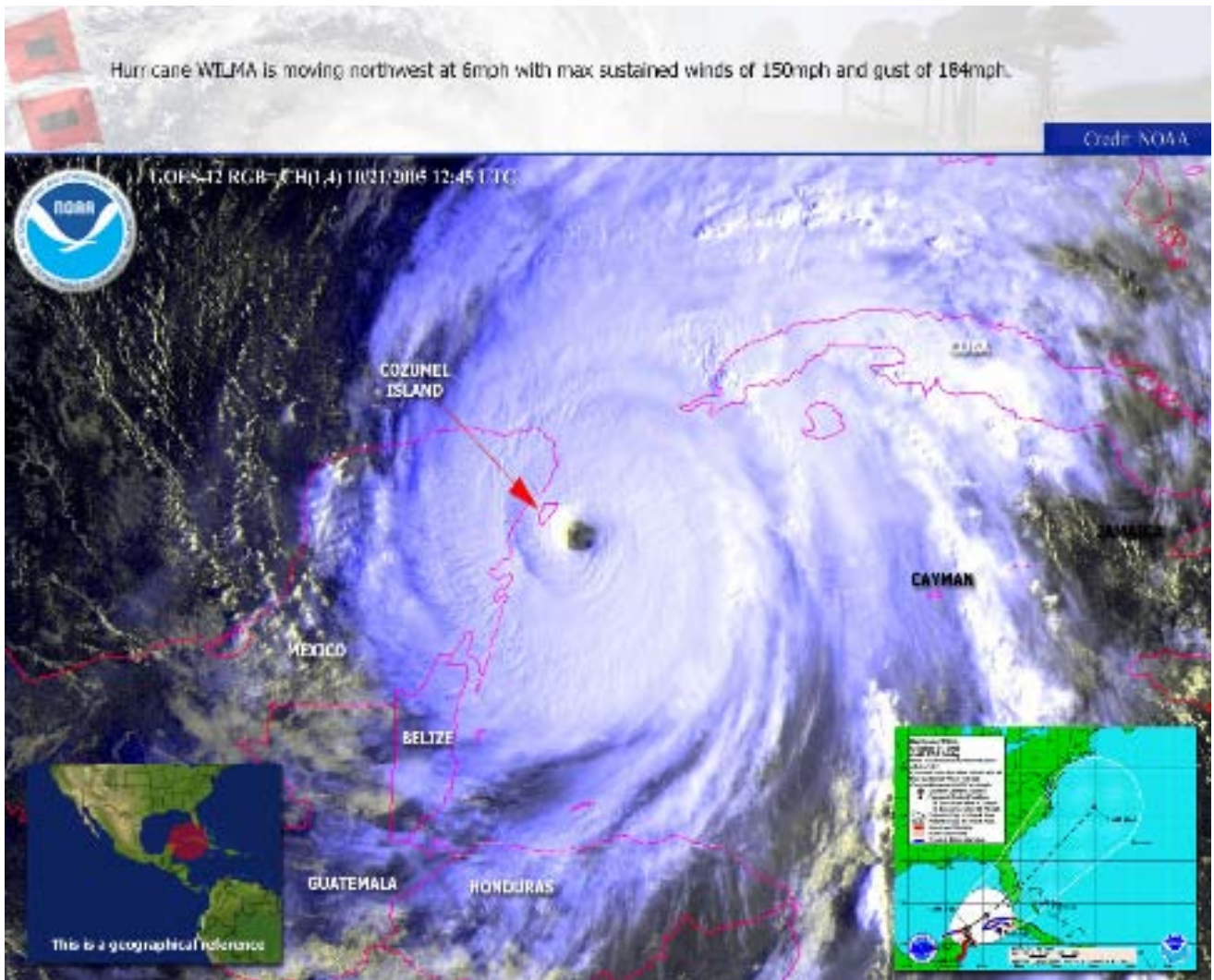


Fig. 5: Satellite view of Hurricane Wilma over the northwestern Caribbean Sea

NDBC Stations within 300 NM of Hurricane Wilma's Track 15 - 25 October 2005

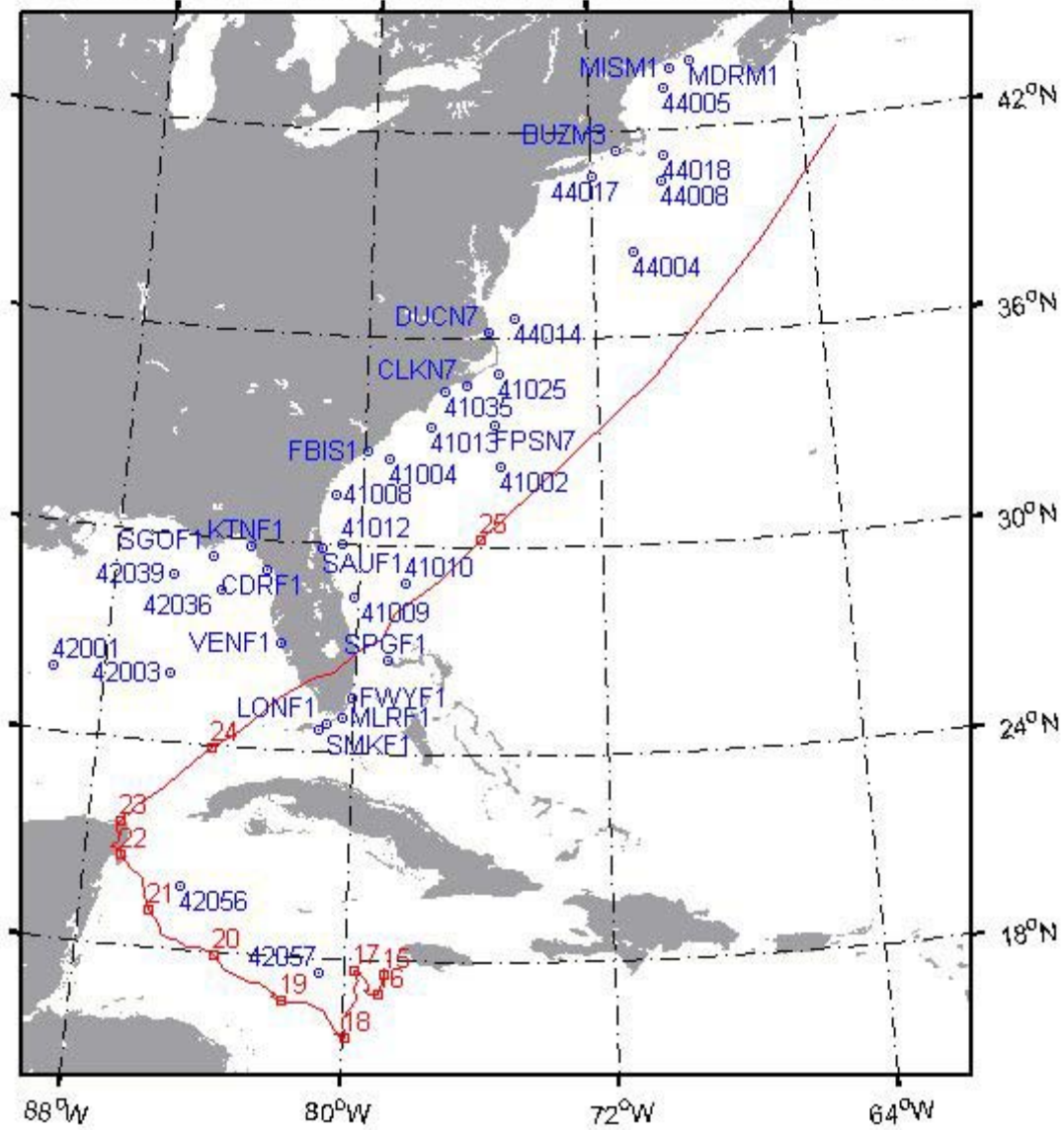


Fig. 6: Wilma's track (in red) and the location of Buoy 42056 in the northwestern Caribbean Sea (see <http://www.ndbc.noaa.gov/hurricanes/2005/wilma/>).

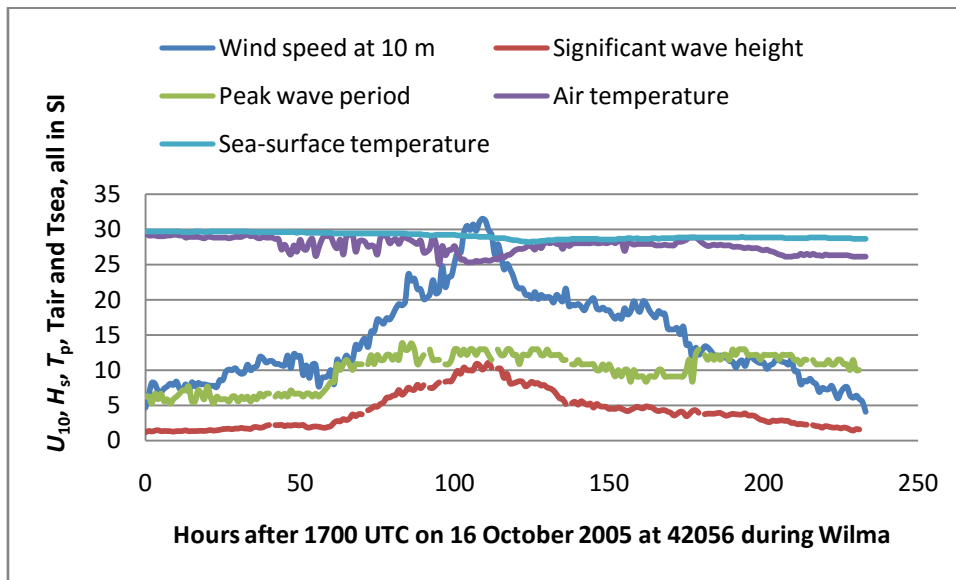


Fig. 7: Met-ocean measurements at NDBC Buoy 42056 during Hurricane Wilma

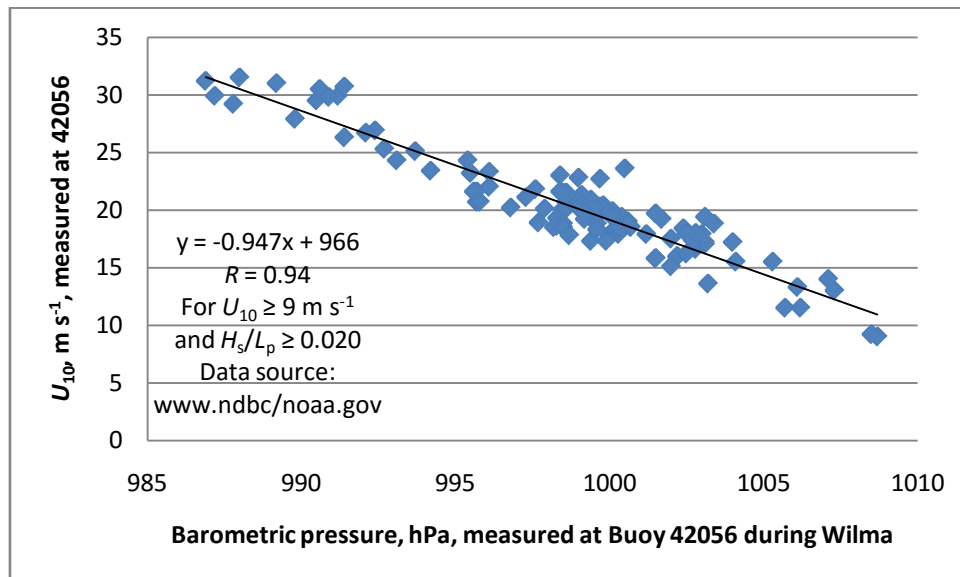


Fig. 8: Relation between wind speed and barometric pressure at 42056 during the period shown in Fig. 6

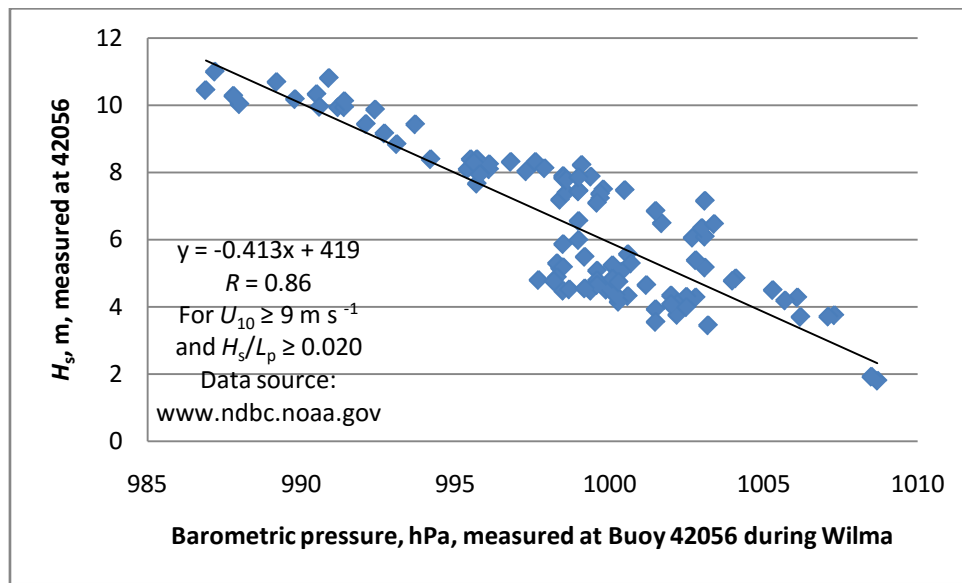


Fig. 9: Relation between H_s and barometric pressure at 42056 during the period shown in Fig. 6

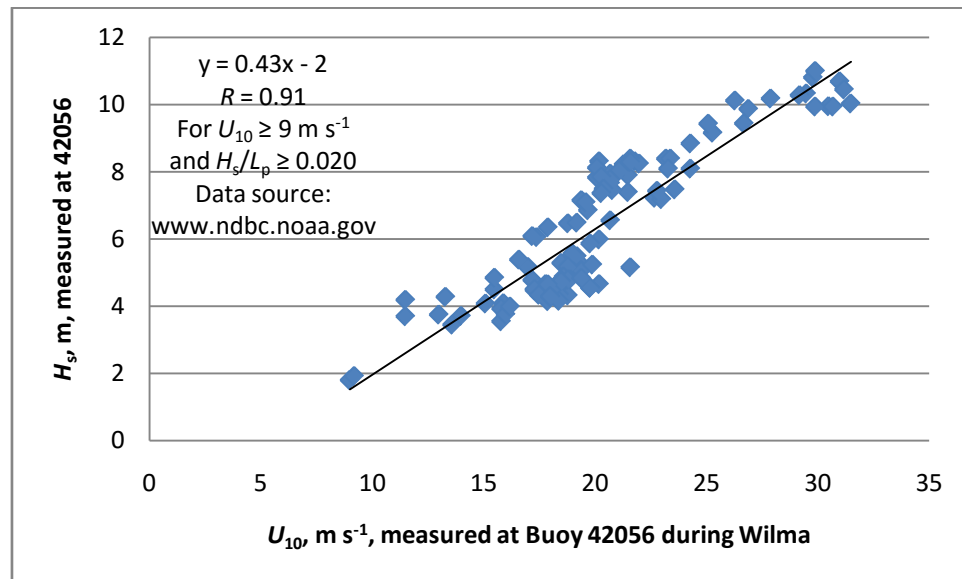


Fig. 10: Relation between H_s and U_{10} as measured at 42056 during Wilma

V. APPLICATIONS

a) Wind-wave relation during Hurricane Jose in 2017

In September 2017 Hurricane Jose, a Category 4 hurricane, passed near the data buoy 41043, located approximately 170 n.m. NNE of San Juan, Puerto Rico. Simultaneous measurements of U_4 , H_s and T_p , were available (for the track and buoy measurements, see www.nhc.noaa.gov and www.ndbc.noaa.gov, respectively). Using the method presented in Hsu et al. (2017a), Eq. (2b) is verified in Fig. 11.

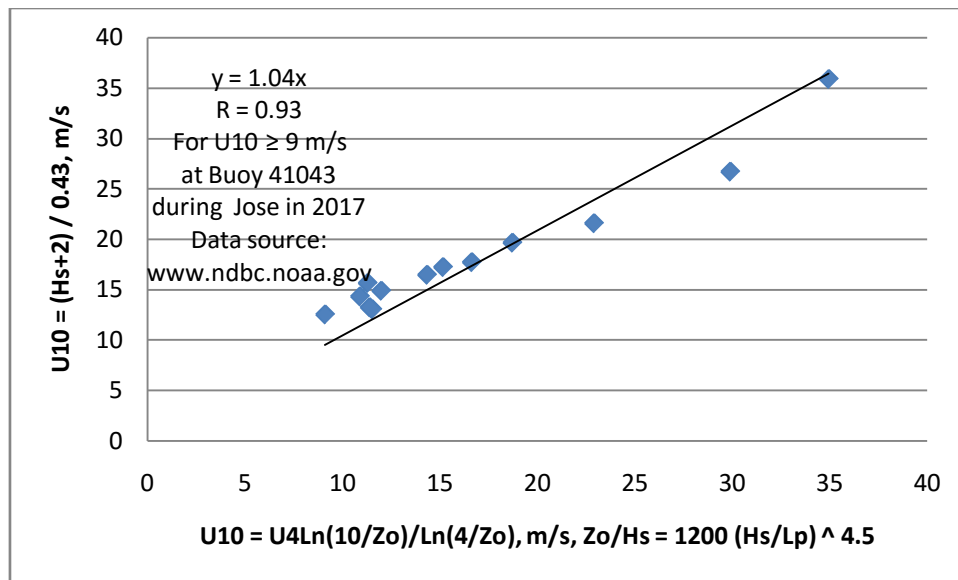


Fig. 11: Verification of Eq. (2b) at Buoy 41043 during Hurricane Jose in 2017

b) Wind-wave relation during Typhoon Russ in 1990

According to the Joint Typhoon Warning Center (<http://www.usno.navy.mil/NOOC/nmfc-ph/RSS/jtwc/atcr/1990atcr.pdf>), Typhoon Russ, the last western Pacific tropical cyclone of 1990, was the most severe to strike Guam in 14 years. Russ formed in the Marshall Islands, tracked west-northwestward and intensified to near super typhoon intensity as it approached Guam. The typhoon passed within 55 km of the southern tip of Guam and brought typhoon force winds which caused extensive damage, especially to the southern portion of the island. After leaving Guam, Russ slowly weakened, recurved and became an extratropical cyclone.

During Typhoon Russ, the National Data Buoy Center (NDBC) (see www.ndbc.noaa.gov) operated a data buoy (Station 52009) near Gaum. Before it was destroyed by Russ, met-ocean conditions were presented in Fig. 12. Using a graphic determination of overwater stability as constructed by Hasse and Weber (1985) based on the wind speed measurement at 5m, U_5 , and the difference in the air (T_{air}) and sea-surface (T_{sea}) temperatures, the stability during the period as shown was near neutral.

To reduce the effect of swell, the criterion suggested by Drennan et al. (2005) was adopted that a wind sea is defined when

$$H_s/L_p \geq 0.020, \tag{3}$$

$$L_p = (g/2\pi)T_p^2 = 1.56T_p^2, \tag{4}$$

Here, L_p is the dominant wave length, g is the gravitational acceleration, and T_p is the peak or dominant wave period. Note that the parameter H_s/L_p is called wave steepness.

According to the National Data Buoy Center (see http://www.ndbc.noaa.gov/station_page.php?

station=52009), Buoy 52009 located near Gaum was capsized during Russ. However, before its capsizing, that buoy provided some wind and wave measurements (see Fig.12) that can be employed to validate Eq. (2b). Because the wind speeds were recorded at 5m instead of 10m, one needs to adjust U_5 to U_{10} using the power-law wind profile (see, e.g., Panofsky and Dutton, 1984) and Hsu (2003) that

$$U_{10} / U_5 = (10/5)^p, \tag{5}$$

Here $p = (U_{gust}/U_5 - 1)/2$ and U_{gust} is the wind gust measured at the buoy (see, Hsu, 2003). Fig.13 shows the result that

$$U_{10} = 1.1 U_5, \tag{6}$$

Figs. 14 and 15 show that, similar to Figs.8 and 9, Eq. (1) should exist. Fig. 16 is a validation of Eq. (2b). Since the slope is unity and $R = 0.96$, Eq. (2b) can be applied to typhoon conditions.

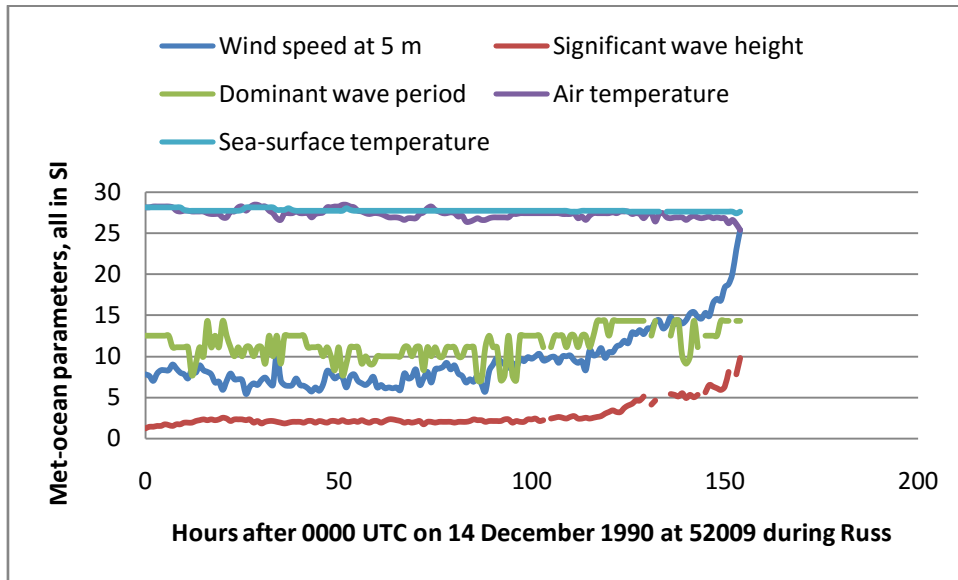


Fig. 12: Met-ocean conditions at NDBC Buoy 52009 before its capsizing by Typhoon Russ (Data source: www.ndbc.noaa.gov)

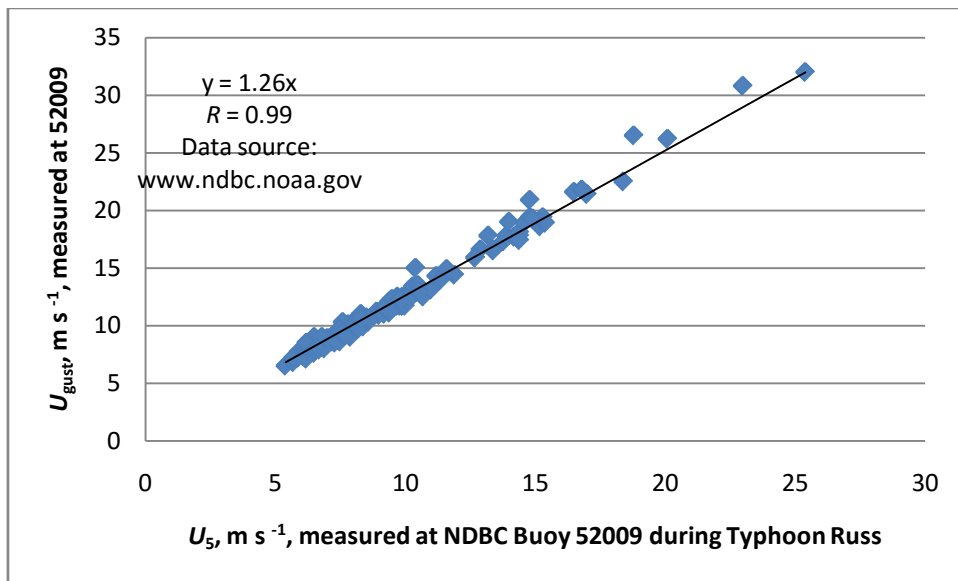


Fig. 13: The relation between the wind speed at 5 m, U_5 , and wind gust, U_{gust} , at 52009 during Russ

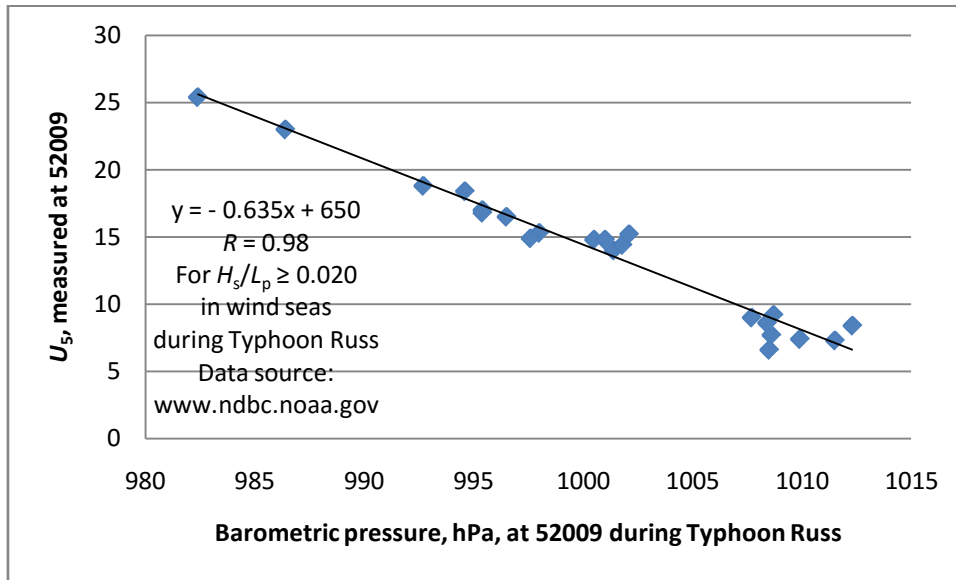


Fig. 14: Relation between U_s and barometric pressure at NDBC Buoy 52009 during Russ

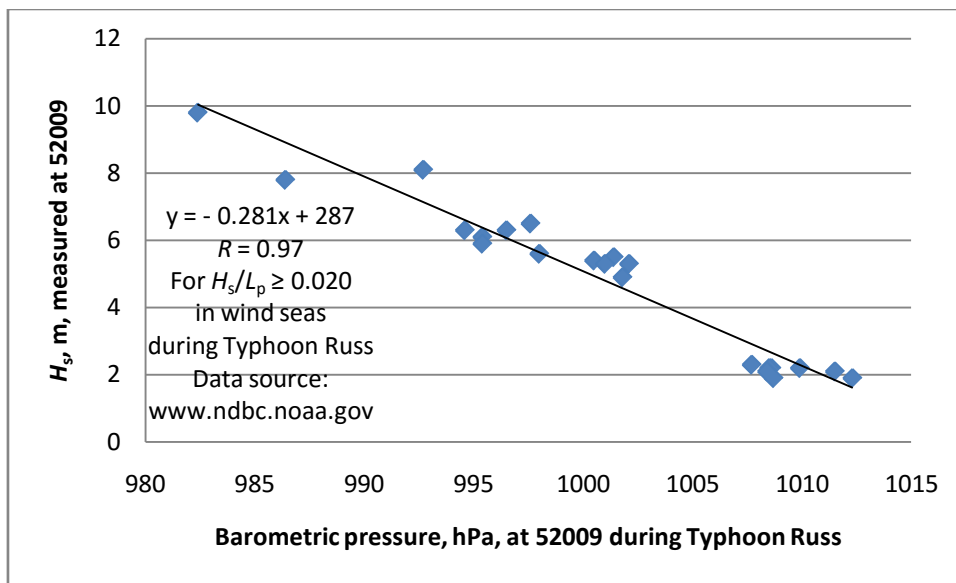


Fig. 15: The relation between H_s and barometric pressure at NDBC Buoy 52009 during Russ

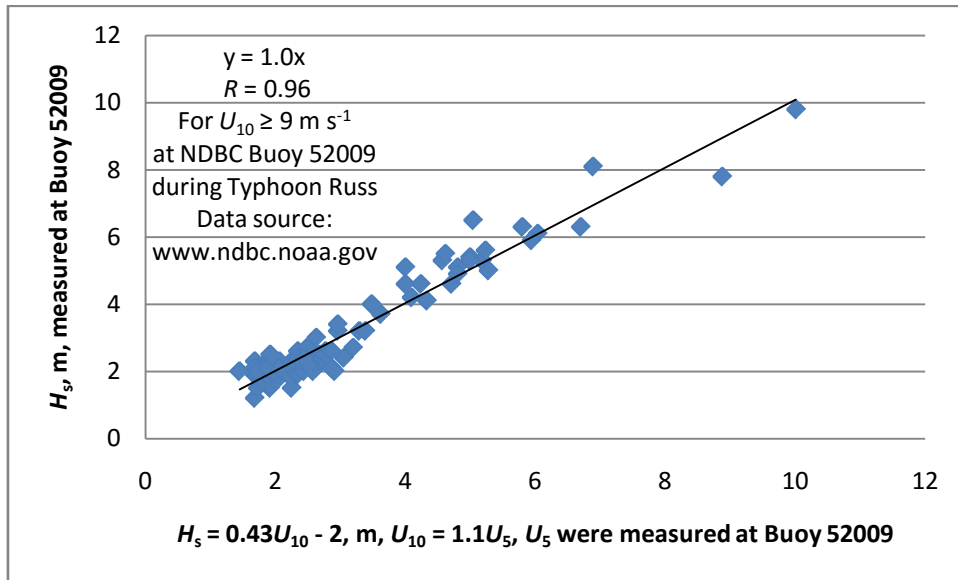


Fig.16: The relation between H_s and U_{10} at NDBC Buoy 52009 during Russ

c) Typhoon Krosa in 2007

According to Liu et al. (2008), an extreme $H_s = 23.9$ m was measured by a data buoy near Taiwan. The best track of Typhoon Krosa in 2007 is provided in Fig. 17 (see Joint Typhoon Warning Center available online at <http://www.usno.navy.mil/NOOC/nmfc-ph/RSS/jtwc/atcr/2007atcr.pdf>). Because the wind speed was 125

knots and 115 knots at 00 and 12 UTC on 6th, respectively, we use the average near-surface wind speed (a surrogate of U_{10}) of 120 knots (62 m s^{-1}). Now, substituting the U_{10} value into Eq. (6), $H_s = 23.8$ m, which is in excellent agreement with that of 23.9 m as measured.

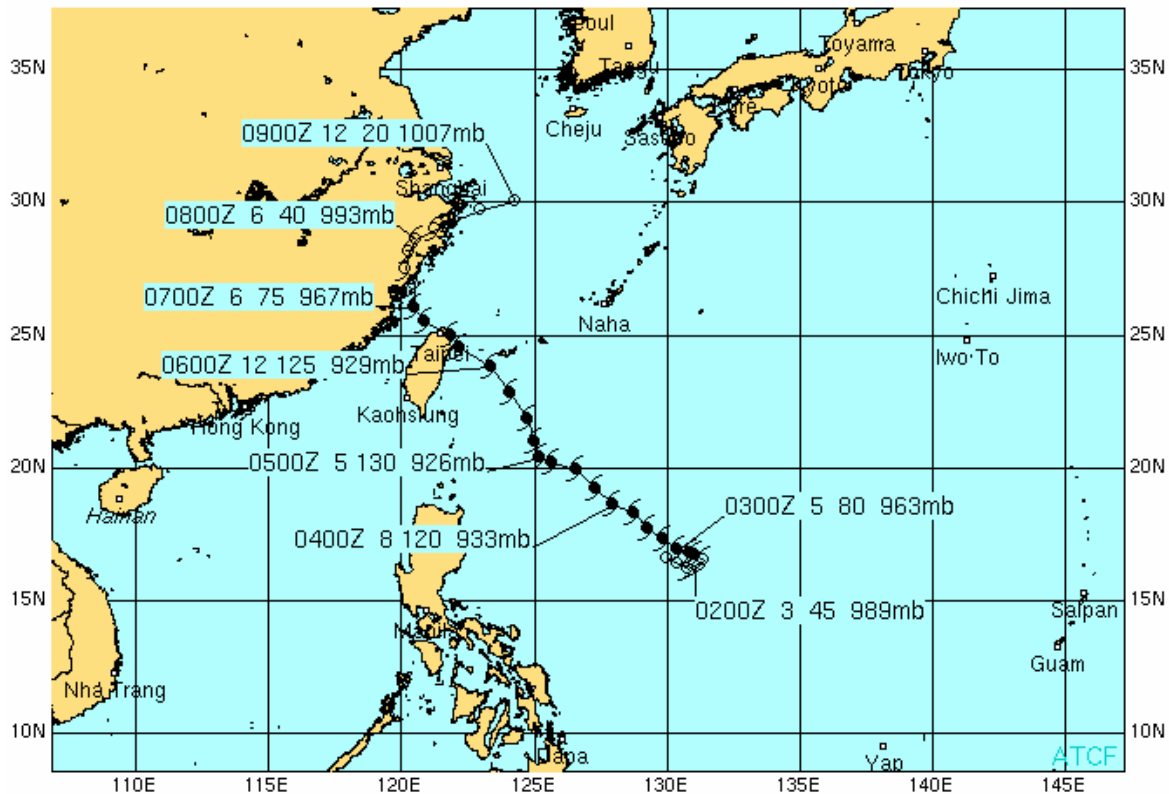


Fig. 17: Track of Typhoon Krosa in October 2007 (<http://www.usno.navy.mil/NOOC/nmfc-ph/RSS/jtwc/atcr/2007atcr.pdf>).

d) Typhoon Soudelor in 2015

From Fig. 1, $U_{10} = 72 \text{ m s}^{-1}$. Substituting this value into Eq. (2b), $H_s = 29.0 \text{ m}$, which is also in reasonable agreement with that of 27.6 m as measured by the altimeter on Jason-2 satellite as discussed in the Introduction.

e) Relation between U_* and H_s

According to Andreas et al. (2012), overwater friction velocity (U_*) is linearly related to U_{10} such that For $U_{10} \geq 9 \text{ m s}^{-1}$,

$$U_* = 0.0583 U_{10} - 0.243,$$

According to Edson et al. (2013, Eq. 22),

$$U_* = 0.062 U_{10} - 0.28,$$

And under hurricane conditions, Hsu et al. (2017b) suggest that,

$$U_* = 0.062 U_{10} - 0.29,$$

By of simultaneous measurements of U_* , U_{10} and H_s by Geernaert et al. (1987), linear relations amongst all three met-ocean parameters (see Figs. 18 thru 20) are

For $U_{10} \geq 9 \text{ m s}^{-1}$,

$$U_* = 0.062 U_{10} - 0.26, \tag{7}$$

$$U_* = 0.12H_s + 0.34, \tag{8}$$

$$H_s = 0.45U_{10} - 4, \tag{9}$$

It is interesting to note that Equations (2b) and (9) are in good agreement numerically, although the former is based on Hurricane Wilma in the Caribbean Sea whereas the latter was based on extra-tropical cyclones over the North Sea.

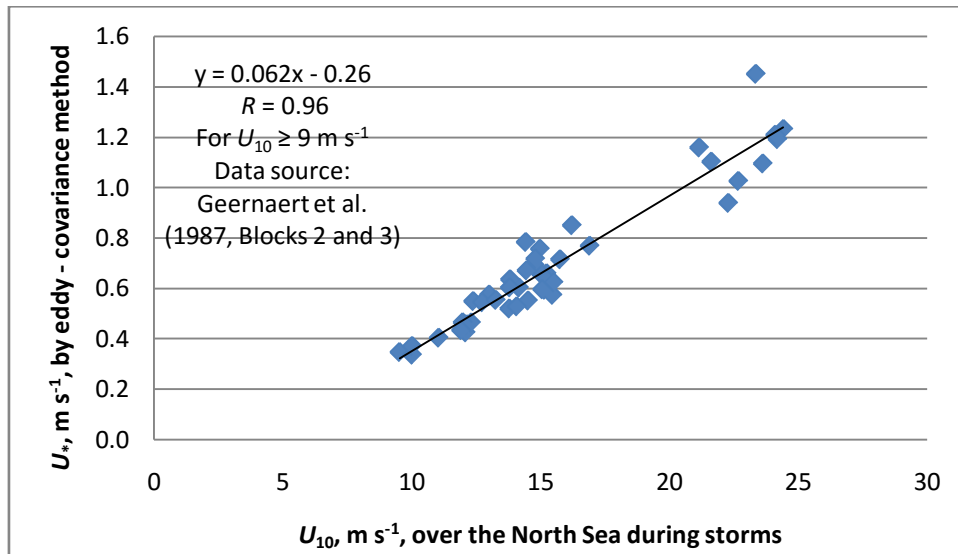


Fig. 18: Relation between U_* and U_{10} in the North Sea during storms

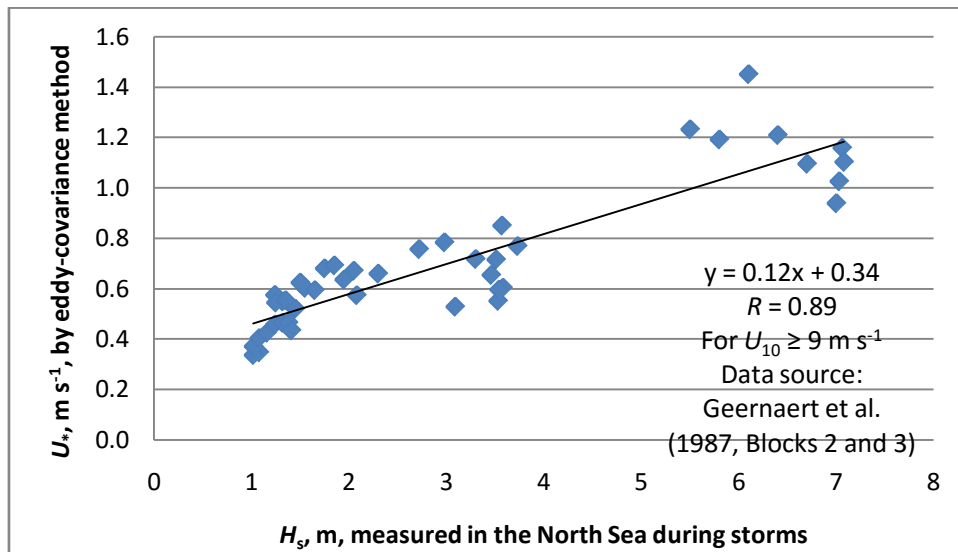


Fig. 19: Relation between U_* and H_s in the North Sea during storms

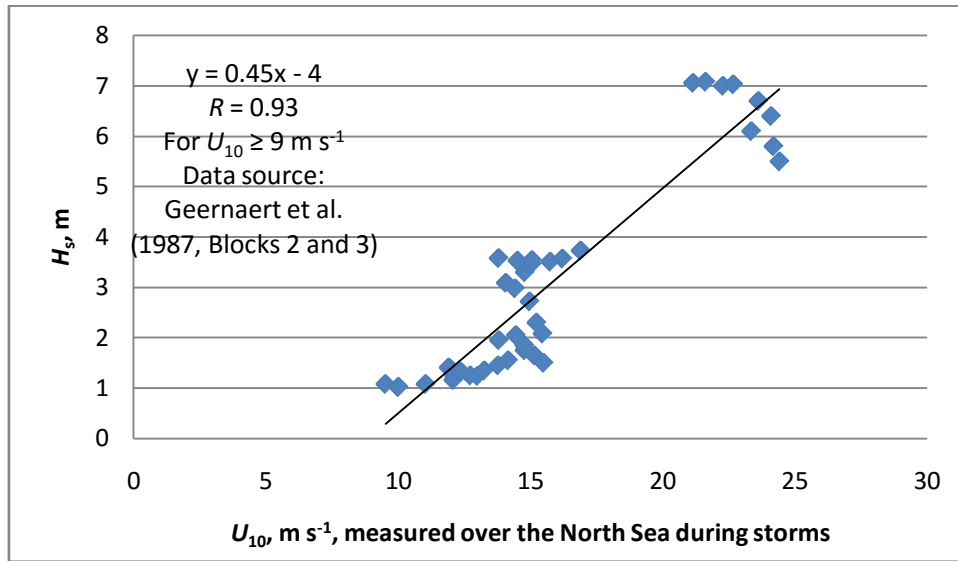


Fig. 20: Relation between H_s and U_{10} in the North Sea during storms

f) Estimating wind-driven surface currents

According to Wu (1975), the sea-surface drift velocity, U_{sea} , may be estimated from U^* that

$$U_{sea} = 0.55U_{*}, \tag{10}$$

Now, substituting Equations (7) into (10), we have

$$U_{sea} = 0.034 U_{10} - 0.14, \tag{11}$$

Similarly, substituting Equations (8) into (10), one gets

$$U_{sea} = 0.066H_s + 0.19, \tag{12}$$

An evaluation of Eq. (11) is presented in Fig.21, indicating that Eq. (11) may be useful to estimate the wind-induced surface velocity. Note that the estimates of sea-surface drift velocity using Eq. (11) are in reasonable agreement with those when a typhoon's forward moving speed is slow moving ($< 4 m s^{-1}$). For more details about drifter measurements, see Chang et al. (2014).

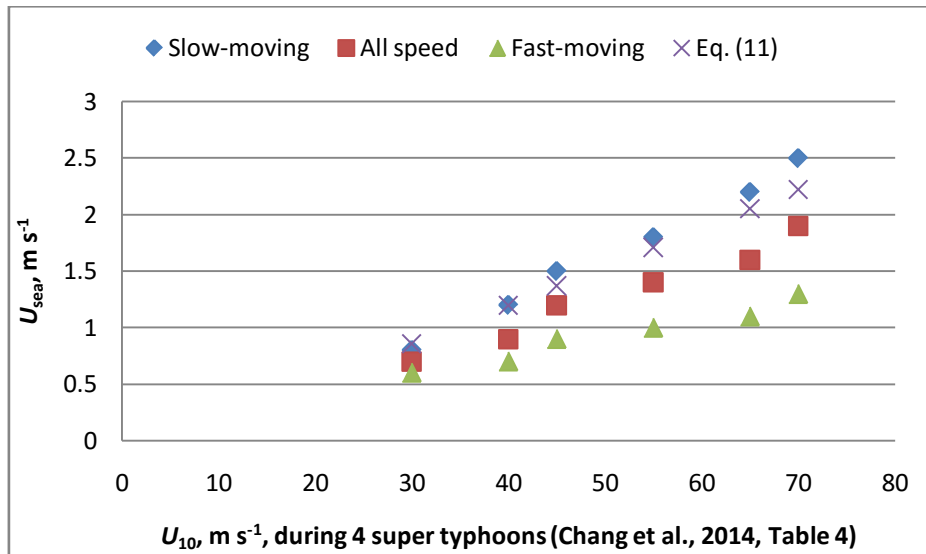


Fig. 21: An evaluation of Eq. (11) during typhoons (Data source: Chang et al., 2014)

g) Rapid estimation of storm surge potential

According to Hsu (2013), the storm surge, S , in meters, may be estimated rapidly using following formula that,

$$S = KV^2, \tag{13}$$

Here K is a proportional constant for a given location (e.g., $K = 0.0051$ for the New York region) and V is the wind speed in $m s^{-1}$.

During Hurricane Katrina in 2005 (see Knabb et al., 2005 or http://www.nhc.noaa.gov/data/tcr/AL122005_Katrina.pdf, pages 10 and 28), an extreme

storm surge value of 27.8 feet or approximate 8.5 m was observed at Pass Christian, Mississippi. In the deep-water region south of this max storm surge, an extreme significant wave height of 55 feet or approximately 17m was measured by NDBC Buoy 42040 (see <http://www.ndbc.noaa.gov/hurricanes/2005/katrina/>). Substituting the max H_s value into Eq. (2), $U_{10} = 44 \text{ ms}^{-1}$, respectively. Now, substituting both values of $S = 8.5 \text{ m}$ and $U_{10} = 44 \text{ m s}^{-1}$ into Eq. (13), we get $K = 0.0044$ so that Eq. (12) becomes

$$S = 0.0044V^2. \quad (14)$$

Eq. (14) is further verified during Hurricane Isaac in 2012 as follows: According to Berg (2013, see http://www.nhc.noaa.gov/data/tcr/AL092012_Isaac.pdf, pages 71 and 73), max S was 11 feet (3.4 m) located east of New Orleans, Louisiana and max V (surrogate of U_{10}) was 54 knots or 28 m s^{-1} , which is located south of that storm surge measurement place in the Gulf of Mexico. Substituting $U_{10} = 28 \text{ m s}^{-1}$ into Eq. (13), $S = 3.4\text{m}$, which is identical to the measurement of 3.4 m. Therefore, it is recommended that for a coastal region, the variation of coefficient K for Eq. (12) may be determined using historical datasets of simultaneous

observations of both S (from water level measurements) and V or H_s [using Eq.(2a) for the offshore regions such as best storm track data by National Hurricane Center or Joint Typhoon Warning Center].

h) Comparison between Equations (1) and (2b)

Because both Equations (1) and (2b) have been verified, it is prudent to compare these two formulas. Fig. 22 shows the result. Since the slope is nearly unity and the correlation coefficient $R = 0.98$, one can say that Eq. (2b) is very useful operationally because the significant wave height is available routinely from a satellite as illustrated in Fig.1. Note that the reason to employ this dataset is that, during Katrina, maximum H_s of 16.91 m was recorded at NDBC Buoy 42040 (<http://www.ndbc.noaa.gov/hurricanes/2005/katrina/>). Now, by substituting U_{10} from Equations (1) into (2b) and rearranging, we have

$$T_p = 15H_s / (H_s + 2). \quad (15)$$

Therefore, for rapid estimation of the peak wave period, Eq. (15) may be employed as a first approximation.

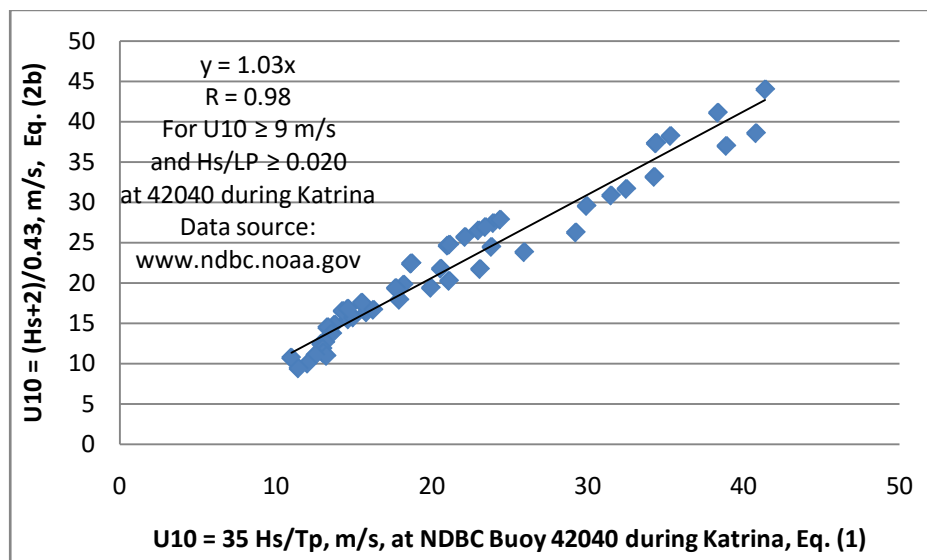


Fig. 22: Comparison between Equations (1) and (2b) at NDBC Buoy 42040 during Katrina

VI. CONCLUSIONS

By aforementioned analyses and discussions, it was concluded that, during wind seas when the wind speed at 10m exceeds 9 m s^{-1} , Eq. (2b) as deduced from the met-ocean measurements during Hurricane Wilma can explain the wind-wave relation to other tropical cyclones. Also, applications of this formula for rapid estimations of overwater friction velocity, sea-surface drift velocity, storm surges, and peak wave period were provided. These topics are needed in marine science and engineering.

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