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Deformation in a Three-Phase-Lag Model of Orthotropic Thermoviscoelastic Material

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I. Introduction

Most materials experience volumetric variations when are subjected to temperature variations and the consequent thermal stresses developed due to temperature gradient in the surface vicinity results in micro-crack and others imperfection development at the surface of anisotropic materials. Thus owing to anisotropic material's applications in aeronautics, astronautics, plasma physics, nuclear reactors and high-energy particle and in various others engineering sciences, theory of thermoelasticity has aroused intense attention in our challenge to understand the nature of the interaction between temperature and strain fields.

Thermo elasticity theory, Chadwick (1960, 1979) and Nowacki (1962, 1975), of thermal disturbances has aroused considerable interest in the last century, but systematic research started only after thermal waves – called second sound – were first measured in materials like solid helium, bismuth and sodium fluoride. Thus, the thermoelasticity theories, which admit a finite speed for thermal signals, have been receiving a lot of attention for the past thirty years. In contrast to the conventional coupled thermoelasticity theory basedon a parabolic heat equation, Biot (1956), which predicts an infinite speed for the propagation of heat, these theories involve a hyperbolic heat equation and are referred to as generalized thermoelasticity theories.

The Lord and Shulman (1967) theory introduces a single time constant todictate the relaxation of thermal propagation as well as the rate of change ofstrain rate and the rate of change of heat generation, and obtained a wave-type heat equation by postulating a new law of heat conduction to replace the classical Fourier law for isotropic bodies. Green and Lindsay (1972) developed a temperature rate dependent

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thermoelasticity that includes two thermal relaxation times and does not violate the classical Fourier law of heat conduction, when body under consideration has center of symmetry.

Dhaliwal and Sherief (1980) derived the governing field equations of generalized thermoelasticity for anisotropic media and also developed a variational principle for these equations. Dhaliwal and Rokne (1989) investigated the one dimensional thermal shock problem with two relaxation times. Simionescu (1992) studied the effect of concentrated loads in quasi-static coupled thermoelasticity.

Green and Naghdi (1993) proposed a new theory of thermoelasticity without energy dissipation and presented the derivation of a complete set of governing equations of the linearized version of the theory for homogenous and isotropic materials in terms of displacement and temperature fields and proved the uniqueness of the solutions of the corresponding initial mixed boundary value problem. An important feature of this theory, which is not present in other theories, is that this theory does not accommodate dissipation of thermal energy.

Tzou(1995) and Chandrasekharaiah(1998) developed dual-phase-lags thermoelastic model, In these models two different phase-lags, i.e.,one for the heat flux vector and other for the temperature gradient have been introduced in theFourier's law. Cimmelli(1998) studied thermodynamics of anisotropie solids near absolute zero.

Das and Lahiri (2001) employed the eigen value approach to determine the thermal stress in an orthotropic elastic slab due to prescribed surface temperatures.

Kumar and Rani(2004)investigated the disturbance due to mechanical and thermal sources in generalized orthorhombic thermoelastic material.

Kumar and Rani(2007) considered a two-diamensional problem of thermoelasticity and discussed the effects of mechanical and thermal sources in generalized orthorhombic thermoelastic material. Ieşan and Quintanilla(2009) studied inner structure and microtemperatures of thermoelastic bodies. Dolotov and Kill (2012) considered a dynamic problem for an elastic half-space with asymmetric normal loading on its boundary and obtained expressions for the components of the stress tensor in the form of series, possessing asymptotic properties, which converge for short values of the time. Liu, Lin and Li(2013) discussed convergence result for the *thermoelasticity* of type III.

Sherief and El-Latief (2014) applied the fractional order theory of thermoelasticity to a 2D problem for a half-space, solved it with Laplace and exponential Fourier transform techniques and study the effect of the fractional derivative parameter on the behavior of the solution. Marotti de Sciarra and Salerno (2014) discussed thermodynamic functions in *thermoelasticity* without energy dissipation. Yermolenko and Ivanov(2014) developed the principles of correspondence between static boundary value problems of thermoviscoelasticity and *thermoelasticity* Abbas, Kumar and Rani (2015)studied ramp-type heating in a thermally conducting cubic crystal. Zhyhailo and Bajkowski (2016) discussed axisymmetrical problem of *thermoelasticity* for halfspace with gradient coating. El-Karamany and Ezzat (2016)proposed three models of generalized thermoelasticity: a single –phase - lag Green–Naghdi theory of type III, a dual-phase–lag Green–Naghdi theory of type II and of type III for linear anisotropic inhomogeneous material. *Fernández and Masid(2017) studied mixture of thermoelastic solids with two temperatures*

The present work aims to determine the distributions of the displacement component, stresses and temperature distribution in a three-phase-lag model of

Notes

homogeneous, thermally conducting, orthotropic materialdue to thermal loading in presence and absence of the viscosity for two values of time. Expressions for the physical quantities are obtained using eigen value approach and are presented graphically. The results of the problem may be applied to a wide class of geophysical problems involving temperature change. The physical applications are encountered in the context of problems such as ground explosions and oil industries. This problem is also useful in the field of geomechanics, where the interest is in various phenomenon occurring in earthquakes and measurement of displacements, stresses and temperature field due to the presence of certain sources.

Notes

II. Basic Equations

The constitutive relations for orthotropicthermoelastic medium following Dhaliwal and Sherief (1980) and Green and Lindsay (1972) are given by

$$\mathrm{t}_{\mathrm{ij}} = c_{ijkl}e_{kl} - \beta_{\mathrm{ij}}\left(1+\tau_a \frac{\partial}{\partial t}\right)T_{,i}, \qquad \beta_{\mathrm{ij}} = c_{ijkl}\alpha_{kl} \quad (\mathrm{i, j, k, l} = 1,2,3) \tag{1}$$

Equation of motion for anorthotropic thermoelastic medium in the absence of body force is given by

$$t_{ij,j} = \rho \ddot{u}_i, \tag{2}$$

The heat conduction equation following Green and Nadhdi (1993) and Choudhuri (2007) is

$$K_{\mathrm{ij}}\left(1+\tau_T \frac{\partial}{\partial t}\right)\dot{T}_{,\mathrm{ij}} + K^*{}_{\mathrm{ij}}\left(1+\tau_v \frac{\partial}{\partial t}\right)T_{,\mathrm{ij}} = \left(1+\tau_q \frac{\partial}{\partial t} + \tau_q^2 \frac{\partial^2}{\partial t^2}\right)(\rho c_e \ddot{T} + \mathrm{T_o}\beta_{\mathrm{ij}}\ddot{u}_{i,j}) \tag{3}$$

where list of symbols has been given at the end of the paper. The comma notation is used for spatial derivatives and dot notation represents time differentiation.

c_{ijkl} *satisfies the (Green) symmetry conditions:*

$$c_{ijkl} = c_{klij} = c_{ijlk} = c_{jikl}.$$

III. Formulation And Solution Of The Problem

We consider a homogenous, orthotropicthermoelastic half-space in the undeformed state at uniform temperature T_0. The rectangular Cartesian co-ordinate system (x,y,z) having origin on the plane surface z=0 with z-axis pointing vertically into medium is introduced. The boundary of the half-space is affected by thermal loading, which depends on time t and spatial coordinate z ($-\infty < z < \infty$).

In equation (2), we have used the contracted Voigt notation for the stiffness $\mathrm{c_{ijkl}}$ to $\mathrm{c_{ij}}$ according to the scheme $11 \to 1,\quad 22 \to 2,\quad 33 \to 3,\quad 23 \to 4,\quad 13 \to 5,\quad 12 \to 6$.

In order to account for the material damping behavior the material coefficients $\mathrm{c_{ij}}$ are assumed to be function of the time operator $\mathrm{D} = \frac{\partial}{\partial \mathrm{t}}$, i.e.

$$\mathrm{c_{ij}} = \mathrm{c_{ij}^*} \tag{4}$$

where $\mathrm{c_{ij}^*} = \mathrm{c_{ij}}(D)$

Assuming that the viscoelastic nature of the material is described by the Voigt model of linear viscoelasticity (1963) , we write

$$c_{ij}(D) = c_{ij}(1+\tau_0 \frac{\partial}{\partial t}), \tag{5}$$

where τ_0 is the relaxation time assumed to be identical for each c_{ij}.

Making use of the c_{pq} from equation (2) in equation (1) then the field equations and constitutive relations for such a medium in the absence of body forces and heat sources in non-dimensional form after suppressing the primes can be rewritten as

$$u,_{xx} + c_1^* u,_{zz} + c_2^* w,_{xz} - T,_x = \ddot{u}, \tag{6}$$

$$c_1^* w,_{xx} + c_3^* w,_{zz} + c_2^* u,_{xz} - \bar{\beta} T,_z = \ddot{w}, \tag{7}$$

$$(1+\tau_T \frac{\partial}{\partial t})(\dot{T},_{xx} + \bar{K}\dot{T},_{zz}) + (1/\omega^*)(1+\tau_v \frac{\partial}{\partial t})(\bar{K}_2 T,_{xx} + \bar{K}_1 T,_{zz})$$

$$= \left(1+\tau_q \frac{\partial}{\partial t} + \frac{\tau_q^2}{2}\frac{\partial^2}{\partial t^2}\right)\{\ddot{T} + \epsilon_1 (\ddot{u},_x + \ddot{w},_z)\}, \tag{8}$$

where comma notation is used for spatial derivatives, we have defined the quantities

$$x' = \frac{\omega_1^* x}{v_1}, \quad z' = \frac{\omega_1^* z}{v_1}, \quad u' = \frac{\rho v_1 \omega_1^*}{\beta_1 T_0} u, \quad t' = \omega_1^* t, \quad w' = \frac{\rho v_1 \omega_1^*}{\beta_1 T_0} w,$$

$$T' = \frac{T}{T_0}, \quad c_1^* = \frac{c_{55}}{c_{11}}, \quad c_2^* = \frac{c_{13}+c_{55}}{c_{11}}, \quad c_3^* = \frac{c_{33}}{c_{11}}, \quad \bar{K}_1 = \frac{K_3^*}{K_1},$$

$$\omega' = \frac{\omega}{\omega_1^*}, \quad \tau_T' = \omega_1^* \tau_T, \quad \tau_v' = \omega_1^* \tau_v, \quad t_1' = \omega_1^* t_1, \quad \tau_q' = \omega_1^* \tau_q,$$

$$\bar{\beta} = \frac{\beta_3}{\beta_1}, \quad \varepsilon_1 = \frac{\beta_1^2 T_o}{\rho k_1 \omega_1^*}, \quad \bar{K}_2 = \frac{K_1^*}{K_1}, \quad a' = \frac{\omega_1^*}{c_2} a, \quad \bar{K} = \frac{K_3}{K_1}. \tag{9}$$

$$t_{zz}' = \frac{t_{zz}}{\beta_1 T_0}, \quad t_{zx}' = \frac{t_{zx}}{\beta_1 T_0}, \quad h' = \frac{h v_1}{\omega_1^*}, \tag{10}$$

and $v_1 = \left(\frac{c_{11}}{\rho}\right)^{\frac{1}{2}}$ and $\omega_1^* = \frac{c_e c_{11}}{K_1}$ are, respectively, the velocity of compressional waves in x-direction and characteristic frequency of the medium.

The initial and regularity conditions are given by

$$u(x,z,0) = 0 = \dot{u}(x,z,0),$$

$$w(x,z,0) = 0 = \dot{w}(x,z,0),$$

$$T(x,z,0) = 0 = \dot{T}(x,z,0) \quad for\ z \geq 0, \quad -\infty < x < \infty \tag{11}$$

$$and\ u(x,z,t) = w(x,z,t) = T(x,z,t) = 0 \quad for\ t > 0 \quad \text{when} \quad z \to \infty \tag{12}$$

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Applying the Laplace and Fourier transforms

$$\hat{f}(x,z,p)=\int_0^\infty f(x,z,t)e^{-pt}dt \quad \text{and}$$

on

$$\tilde{f}(\xi,\ z,\ p)=\int_{-\infty}^{\infty}\hat{f}(x,\ z,\ p)e^{i\xi x}dx \tag{13}$$

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the resulting expressions, we obtain

$$\frac{d^2\tilde{u}}{dz^2}=R_{11}\tilde{u}+R_{13}\tilde{T}+R_{15}\frac{d\tilde{w}}{dz}, \tag{14}$$

$$\frac{d^2\tilde{w}}{dz^2}=R_{22}\tilde{w}+R_{24}\frac{d\tilde{u}}{dz}+R_{26}\frac{d\tilde{T}}{dz}, \tag{15}$$

$$\frac{d^2\tilde{T}}{dz^2}=R_{31}\tilde{u}+R_{33}\tilde{T}+R_{35}\frac{d\tilde{w}}{dz}. \tag{16}$$

where

$$R_{11}=\frac{\xi^2+p^2}{c_1},\qquad R_{13}=\frac{-i\xi}{c_1},\qquad R_{15}=\frac{i\xi c_2}{c_1},$$

$$R_{22}=\frac{c_1\xi^2+p^2}{c_3},\qquad R_{26}=\frac{\overline{\beta}}{c_3},\qquad R_{24}=\frac{i\xi c_2}{c_3},$$

$$R_{31}=-\frac{i\xi\varepsilon_1 N_3 p^2}{(N_1+N_2)\overline{K}_1},\quad R_{33}=\frac{N_1\xi^2+N_2\overline{K}_2+N_3p^2}{(N_1+N_2)\overline{K}_1},\quad R_{35}=-\frac{\overline{\beta}\varepsilon_1 N_3 p^2}{(N_1+N_2)\overline{K}_1},$$

$$N_1=p+\tau_T p^2,\qquad N_2=\frac{1+\tau_v p}{\omega_1^*}\qquad N_3=1+\tau_q p+\tau_q^2 p^2.$$

The equations (14)-(16) can be written as

$$\frac{d}{dz}W(\xi,z,p)=A(\xi,p)W(\xi,z,p), \tag{17}$$

where

$$\mathrm{W}=\begin{bmatrix}\mathrm{U}\\ \mathrm{U}'\end{bmatrix},\qquad \mathrm{A}=\begin{bmatrix}\mathrm{O} & \mathrm{I}\\ \mathrm{A}_1 & \mathrm{A}_2\end{bmatrix},\qquad \mathrm{U}=\begin{bmatrix}\tilde{\mathrm{u}}\\ \tilde{\mathrm{w}}\\ \tilde{\mathrm{T}}\end{bmatrix},\qquad \mathrm{U}'=\begin{bmatrix}\tilde{\mathrm{u}}'\\ \tilde{\mathrm{w}}'\\ \tilde{\mathrm{T}}'\end{bmatrix},$$

$$\mathrm{O}=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix},\qquad \mathrm{I}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix},\qquad \mathrm{A}_1=\begin{bmatrix}0&\mathrm{R}_{15}&0\\ \mathrm{R}_{24}&0&\mathrm{R}_{26}\\ 0&\mathrm{R}_{35}&0\end{bmatrix},$$

$$\mathrm{A}_2=\begin{bmatrix}\mathrm{R}_{11}&0&\mathrm{R}_{13}\\ 0&\mathrm{R}_{22}&0\\ \mathrm{R}_{31}&0&\mathrm{R}_{33}\end{bmatrix},$$

To solve the equation (17), we take

$$W(\xi,z,p)=X(\xi,p)\,e^{qz} \tag{18}$$

so that

$A(\xi, p)W(\xi, z, p) = qW(\xi, z, p)$

which leads to an eigenvalue problem. The characterstic equation corresponding to the matrix A is given by

$$\det[A-qI]=0 \tag{19}$$

which on expansion leads to

$$q^6 - \lambda_1 q^4 + \lambda_2 q^2 - \lambda_3 = 0 \tag{20}$$

where

$$\lambda_1 = R_{15} R_{24} + R_{33} + R_{22} + R_{11} + R_{26} R_{35}$$

$$\lambda_2 = R_{15}R_{24}R_{33} - R_{13}R_{24}R_{35} + R_{22}R_{33} + R_{11}R_{26}R_{35} - R_{31}R_{15}R_{26} + R_{11}R_{33} - R_{31}R_{13} + R_{11}R_{22}$$

$$\lambda_3 = R_{22}(R_{11}R_{33} - R_{31}R_{13}),$$

The roots of equation (20) are $\pm q_\ell (\ell = 1, 2, 3)$.

The eigenvalues of the matrix A are roots of equation (20). The eigenvector X (ξ,p) corresponding to the eigenvalues q_ℓ can be determined by solving the homogeneous equation

$$[A-qI]\, X(\xi, p)=0 \tag{21}$$

The set of eigenvectors $X_\ell(\xi,p)$, $(\ell=1,2,3,4,5,6)$ may be obtained as

$$X_\ell(\xi, p) = \begin{bmatrix} X_{\ell 1}(\xi, p) \\ X_{\ell 2}(\xi, p) \end{bmatrix}$$

where

$$X_{\ell 1}(\xi,p) = \begin{bmatrix} -\xi \\ a_\ell q_\ell \\ b_\ell \end{bmatrix}, \quad X_{\ell 2}(\xi,p) = \begin{bmatrix} -\xi q_\ell \\ a_\ell q_\ell^2 \\ b_\ell q_\ell \end{bmatrix}, \quad q = q_\ell, \ell = 1,2,3.$$

$$X_{\ell_a 1}(\xi,p) = \begin{bmatrix} -\xi \\ -a_\ell q_\ell \\ b_\ell \end{bmatrix}, \quad X_{\ell_a 2}(\xi, p) = \begin{bmatrix} \xi q_\ell \\ a_\ell q_\ell^2 \\ -b_\ell q_\ell \end{bmatrix}, \quad \ell_a = \ell + 3, \quad q = -q_\ell, \ell = 1,2,3.$$

and

$$a_\ell = \frac{\{(\overline{\beta} - c_2)\xi^2 + p^2\overline{\beta} - c_1\overline{\beta}q_\ell^2\}}{\Delta_\ell},$$

$$b_\ell = \frac{\{c_1 q_\ell^2 \xi - (\xi^2 + p^2)\xi\}\{(c_1\xi^2 + p^2) - q_\ell^2(c_3 - c_2\overline{\beta})\} - q_\ell^2 c_2 \xi\ \{(\xi^2 + p^2) - c_1 q_\ell^2\}\overline{\beta} - c_2\xi^2\}}{\xi\,\Delta_\ell},$$

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$$\Delta_\ell = i\{(c_1\xi^2 + p^2) - (c_3 - c_2\overline{\beta})q_\ell^2\}, \qquad \ell = 1,2,3.$$

The solution of equation (21) is given by

$$W(\xi,z,p) = \sum_{\ell=1}^{3}[B_\ell X_\ell(\xi,p)\exp(q_\ell z) + B_{\ell+3}X_{\ell+3}(\xi,p)\exp(-q_\ell z)], \tag{22}$$

Notes

where $B_\ell\,(\ell = 1,2,3,4,5,6)$ are arbitrary constants.

Thus equation (22) represents the solution of the general problem in the plane strain case of generalized homogeneous thermoelasticity by employing the eigenvalue approach and therefore can be applied to a broad class of problems in the Laplace and Fourier transforms. Displacements and temperature distribution that satisfy the regularity conditions (12) are given by

$$\tilde{u}(\xi,z,p) = -\xi(B_4\,e^{-q_1 z} + B_5\,e^{-q_2 z} + B_6\,e^{-q_3 z}), \tag{23}$$

$$\tilde{w}(\xi,z,p) = -(a_1 q_1 B_4\,e^{-q_1 z} + a_2 q_2 B_5\,e^{-q_2 z} + a_3 q_3 B_6\,e^{-q_3 z}), \tag{24}$$

$$\tilde{T}(\xi,z,p) = (b_1 B_4\,e^{-q_1 z} + b_2 B_5\,e^{-q_2 z} + b_3 B_6\,e^{-q_3 z}), \tag{25}$$

IV. Application

a) Dynamic thermoelasticcase

i. *Thermoelastic Interactions due to Thermal Source*

The boundary conditions at the plane surface are

$$t_{zz} = 0, \quad t_{zx} = 0, \qquad \text{at } z = 0$$

$$\frac{\partial T}{\partial z}(x, z = 0) = r(x,t), \qquad \text{for the temperature gradient boundary,}$$

or

$$T(x, z = 0) = r(x,t), \qquad \text{for the temperature input boundary,.} \tag{26}$$

where $r(x,t) = \eta(x)F(t)$

Applying the Laplace and Fourier transforms defined by (12),we get

$r(\xi,p) = \tilde{\eta}(\xi)\tilde{F}(p)$

Making use of Eqs. (1), (9)-(13) in the boundary conditions given by Eq. (26) and with the help of Eqs. (23) –(25), we obtain the expressions for displacement components, stresses and temperature distribution as

$$\tilde{u} = -r(\xi,p)\frac{\xi\sum_{m=1}^{3}\Delta''_m e^{-q_m z}}{T_0\Delta_2^*},\ \tilde{w} = -r(\xi,p)\frac{\sum_{m=1}^{3}a_m q_m\Delta''_m e^{-q_m z}}{T_0\Delta_2^*},\ \tilde{t}_{zx} = r(\xi,p)\frac{\sum_{m=1}^{3}s_m\Delta''_m e^{-q_m z}}{T_0\Delta_2^*},$$

$$\tilde{t}_{zz} = r(\xi,p)\frac{\sum_{m=1}^{3}p_m\Delta''_m e^{-q_m z}}{T_0\Delta_2^*},\ \tilde{T} = -r(\xi,p)\frac{\sum_{m=1}^{3}b_m\Delta''_m e^{-q_m z}}{T_0\Delta_2^*}. \tag{27}$$

where

$$\Delta''_1 = p_2 s_3 - s_2 p_3, \quad \Delta''_2 = p_2 s_3 - s_1 p_3, \quad \Delta''_3 = p_1 s_2 - s_1 p_2,$$

$$s_m = \frac{\xi k_\ell (1 + i a_\ell)}{c_{11}}, \qquad p_\ell = \frac{i\xi c_{13} + a_\ell q_\ell^2 c_{33} - \overline{\beta} b_\ell}{c_{11}}, \quad \ell = 1,2,3.$$

On replacing Δ *by* $(\frac{\omega_1^* T_0}{V_1})\Delta_1^*$ and $T_0\Delta_2^*$ respectively, we obtain the expressions for temperature gradient boundary and temperature input boundary.

we set a triangular pulse $\eta(x) = \begin{cases} a + x, & \text{-}a \le x \le 0 \\ a - x, & 0 < x \le a \\ 0, & |x| > a \end{cases}$

in equation (26). Using equations (9)-(10) and applying the Laplace and Fourier transform defined by equation (13), we get

$$\tilde{\eta}(\xi) = \left[2\left\{ 1 - \cos(\frac{\xi c_2 a}{\omega_1^*}) \right\} \Big/ \xi^2 \right], \ \xi \neq 0.$$

which leads to an eigenvalue problem. The characterstic equation corresponding to the

Case 1: Instantaneousloading:

The plane boundary z=0 is assumed to be traction free and is subjected to an instantaneous input in temperature, i.e.

$$\mathrm{F(t)} = \mathrm{F}_0 \delta(t)$$

with

$$\tilde{F}(p) = F_0, \tag{28}$$

where F_0 is a constant representing the magnitude of constant temperature and $\delta(t)$ is the Dirac delta function.

Case 2: Continuous loading:

The plane boundary z=0 is subjected to a continuousinput in temperature, i.e. $\mathrm{F(t)} = \mathrm{F}_0 H(t)$,

with

$$\tilde{F}(p) = \frac{\mathrm{F}_0}{p} \tag{29}$$

where F_0 is a constant representing the magnitude of constant temperature and $H(t)$ is the Heaviside unit step function.

V. Special Cases

Transformed solutions of equation (27) reducesto various models of thermoelasticity as:

(1) Classical thermoelastic model - $K^*{}_{ij}$ =0.

(2) Dual phase-lag-model of thermoelasticity- K_{ij} ¿¿ $K^*{}_{ij}$, $K^*{}_{ij}$ =0.

(3) Lord shulman (L-S) model- $K^*{}_{ij}$ =0, $\tau_T = 0$, $\tau_v = 0$, $\tau_q = \tau$.

(4) Coupled thermoelasticity(CT) model- $K^*{}_{ij}$ =0, $\tau_T = \tau_q = \tau_v = 0$.

(5) Uncoupled thermoelasticity(UCT) model - $\varepsilon_1 = 0, K^*_{ij} = 0,\ \tau_T = \tau_q = \tau_v = 0.$

(6) Green-Nighdi(G-N)model(Type-1)- $K^*_{ij} = 0.$

(7) Green-Nighdi(G-N)model(Type-II)- K_{ij} ¡¡ $K^*_{ij}, \tau_T = \tau_q = \tau_v = 0.$

(8) Green-Nighdi(G-N)model(Type-III)- $\tau_T = \tau_q = \tau_v = 0.$

Notes

VI. Particular Cases

a) Transversely isotropic materials

This type of medium has only one axis of thermal and elastic symmetry. We take the z axis along the axis of symmetry. Then the non-vanishing elastic and thermal parameters are

$$c_{11} = c_{33}, \quad K_1 = K_3, \quad \alpha_1 = \alpha_3, \quad K_1^* = K_3^*.$$

b) Cubic crystal

For cubic crystals, the nonvanishing elastic and thermal parameters are

$$c_{11} = c_{33}, \quad K_1 = K_3 = K_1^* = K_3^* = K, \quad \beta_1 = \beta_3 = \beta, \quad \alpha_1 = \alpha_3 = \alpha_t$$

c) Isotropic media

For isotropic material every direction is a direction of elastic as well as thermal symmetry and the nonvanishing elastic and thermal parameters are

$$c_{11} = c_{33} = \lambda + 2\mu, \quad c_{13} = \lambda, \quad c_{55} = \mu, \quad K_1 = K_3 = K_1^* = K_3^* = K,$$

$$\alpha_1 = \alpha_3 = \alpha_t, \qquad \beta_1 = \beta_3 = \beta = (3\lambda + 2\mu)\alpha_t$$

VII. Inversion Of The Transforms

To obtain the solution of the problem in the physical domain, we must invert the transforms in equation (26) for three phase lag theory of thermoelasticity. These expressions are functions of z, the parameters of Laplace and Fourier transforms p and ξ, respectively, and hence are of the form $\tilde{f}(\xi, z, p)$. To get the function f(x,z,t) in the physical domain, first we invert the Fourier transform using

$$\hat{f}(x,z,p) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\xi x}\, \tilde{f}(\xi,z,p)\, d\xi = \frac{1}{\pi}\int_0^{\infty} (\cos(\xi x)\, f_e - i \sin(\xi x) f_0)\, d\xi, \tag{30}$$

where f_e and f_0 are, respectively, even and odd parts of the function $\tilde{f}(\xi, z, p)$.

Thus, expression (30) gives us the Laplace transform $\hat{f}(x, z, p)$ of the function f(x,z,t). Following Honig and Hirdes(1984), the Laplace transform function $\hat{f}(x, z, p)$ can be inverted to f(x,z,t).

The last step is to calculate the integral in equation (30). The method for evaluating this integral is described by Press et al. (1986), which involves the use of Romberg's integration with adaptive step size. This, also uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

VIII. Numerical Result And Discussion

Following Dhaliwal and Singh (1980), we take the case of magnesium crystal-like material for numerical calculations. The physical constants used are:

$\varepsilon = 0.0202$, $c_{11} = 5.974 \times 10^{10}$ Nm^{-2}, $c_{12} = 2.624 \times 10^{10}$ Nm^{-2},

$\rho = 1.74 \times 10^3$ kgm^{-3}, $c_{44} = 3.278 \times 10^{10}$ Nm^{-2}, $c_e = 1.04 \times 10^3$ J kg^{-1}degree^{-1}

$\omega_1^* = 3.58 x 10^{11}$ s^{-1}, $K_1 = K_3 = 1.7 \times 10^2$ Wm^{-1}degree^{-1}, $\beta_1 = \beta_3 = 2.68 \times 10^6$ Nm^{-2}degree^{-1},

$F_0 = 1$, a=1, $T_0 = 298^0$K.

The comparison of normal boundary displacement w and boundary temperature field T, and normal stress t_{zz} for instantaneous thermoviscoelastic material (ITVM) and instantaneous thermoelastic material (ITM) are depicted in Figures 1-3 and continuous thermoviscoelastic material (CTVM) and continuous thermoelastic material (CTM) are depicted in Figures 4-6.for three-phase-lag theory of thermoelasticity. The computations were carried out for two values of time t=1.0 and t=2.0, non-dimensional relaxation times $\tau_v = 0.02$, $\tau_a = 0.05$, $\tau_T = 0.04$, $\tau_q = 0.06$ at z=1.0 in the range $0 \le x \le 10$.

a) Thermoelastic Interactions due to Thermal Source (Temperature gradient boundary)

Dynamic thermoelastic case:

Figure 1. depictsthe variation of normal displacement 'w' with distance x for ITM and ITVM for different values of time in context of three phase-lag-model. At t=1.0 the viscosity effect on 'w' is prominent in the ranges $0 \le x \le 2.5$, $3.5 \le x \le 4.5$, $7 \le x \le 8.5$ and less in rest of ranges where asat t=2.0 the viscosity effect on 'w' is more in the ranges $0.5 \le x \le 3$, $7 \le x \le 9.5$ and less in rest of ranges.

Figure 2. determinethe variation of temperature distribution T with distance x. At t=1.0 the t=2.0 the ITVM and ITM show opposite oscillatory behavior in the whole range $0 \le x \le 10$.

Figure 3. displays the variation of normal stress $t_{zz}$with distance x. Near the point of application of source, the magnitude of normal stress for ITM is more ITVM and then become oscillatory in the whole range about zero for the time t=1.0 and t=2.0,respectively.

Figure 4. showsthe variation of normal displacement 'w' with distance x for CTM and CTVM. The viscosity effect is more prominent than thermal effect in the range $0 \le x \le 2.5$ for time t=1.0 and t=2.0, respectively, and less in rest of the range.

Figure 5. depictsthe variation of temperature distribution T with distance x. The values of temperature shows same oscillatory pattern in the whole range for CTVM and CTM for time t=1.0 and t=2.0,respectively.

Figure 6. displaysthe variation of normal stress $t_{zz}$with distance x.The thermal effect is more prominent than viscosity effect in the range $0 \le x \le 7$ and reverse in rest of the range $7 \le x \le 10$ for both values of time t=1.0 and t=2.0,respectively.

IX. Conclusion

The problem of investigating displacement component, temperature and stress components in anhomogeneous anisotropic thermoelastic half-space is studied in the purview of viscothermoelasticity. Eigen value technique is employed to express the

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results mathematically. Theoretically obtained field variables are also shown through a specific model to present the results in graphical form.
The results of the present work can be summarized as

1. The Laplace and Fourier techniqueis used to derive expressions for displacement components, stresses and temperature distribution for dynamic thermo-elastic case.
2. The values of all physical quantities are observed to follow oscillatory pattern about zero in the whole range with increase in distance x.
3. The viscosity effect has a significant role in the considered physical quantities.
4. The time effect has significant influence on the distribution of the considered physical quantities.

Nomenclature

$\vec{u} = (u, v, w)$ - displacement vector

T(x,y,z,t) -temperature change

c_{ijkl} - isothermal elastic parameters,

t- time

t_{ij}-stress tensor

e_{ij}-strain tensor

T_0 -uniform temperature

ρ _density

τ_T, τ_a, τ_v *and* τ_q-thermal relaxation times

α_{kl} -linear thermal expansion tensor.

$K^*_{ij} = \frac{c_e c_{11}}{4}$ -the material characteristic constant of the theory.

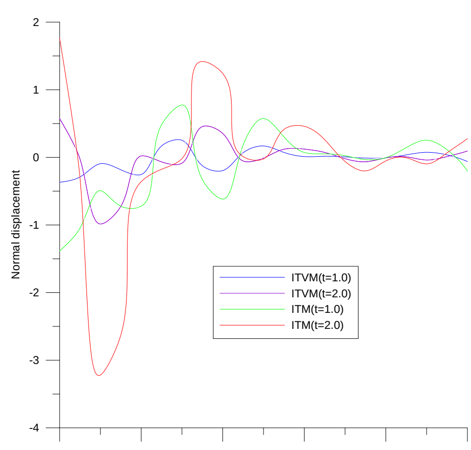

Fig. 1: Variation of Normal Displacement with Distance X.

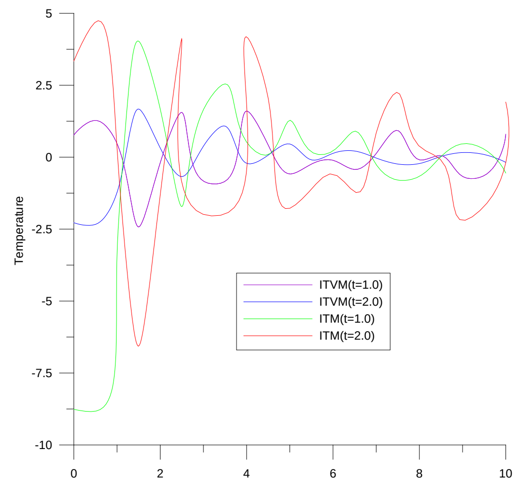

Fig. 2: Variation of Temperature with Distance X.

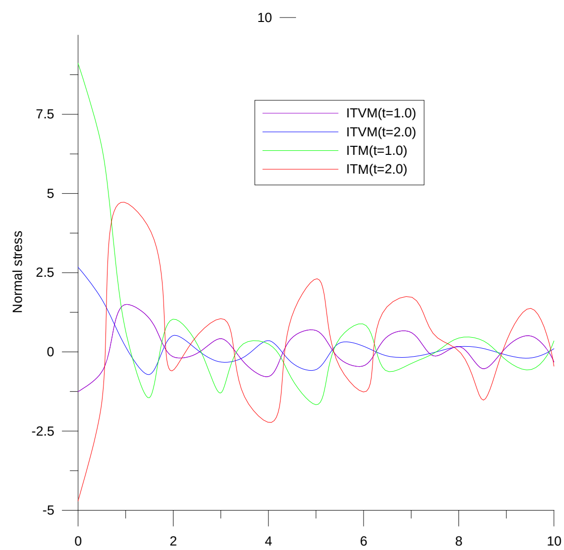

Fig. 3: Variation of Normal stress with Distance X.

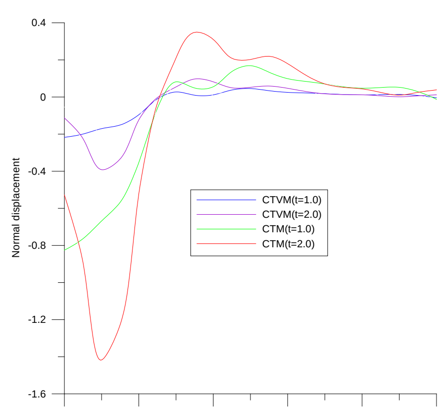

Fig. 4: Variation of Normal Displacement with Distance X.

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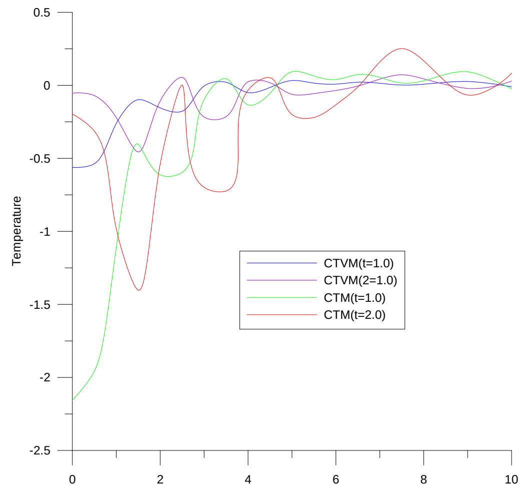

Fig. 5: Variation of Temperature with Distance X.

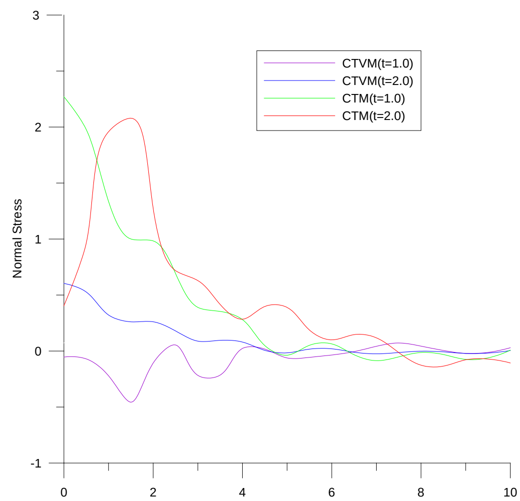

Fig. 6: Variation of Normal stress with Distance X.

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