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Regular Exact Models with Vanishing Anisotropy Generated using Van Der Waals Equation of State

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Jefta M. Sunzu [α] & Kasongo A. Mahali [σ]

Abstract- In this paper, new exact models for Einstein field equations are generated using a Van der Waals equation of state. We consider anisotropic stellar objects with no electromagnetic field distribution. Our models contain previous results as a special case. Models generalized in our performance include a familiar uncharged Einstein model with no pressure anisotropy. It is interesting that our models indicate that when matter variables vanish, gravitational potentials remain constant. This condition agrees with Minkowski spacetime. The physical features of our models show that the gravitational potentials and matter variables are well behaved. We also compute relativistic stellar masses and radii consistent with the stars PSR J1614-2230, Vela X-1, 4U 1538-52, LMC X-4, SMC X-4, Cen X-3, Her X-1, SAX J1808.4-3658 and EXO 1785-248.

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I. Introduction

Models for gravitating spheres in general relativity are generated by utilizing the Einstein-Maxwell system of equations. In doing so some conditions may be imposed for physical acceptability. The Einstein Maxwell field equations are equations generated by equating the energy momentum tensor and the Einstein tensor involving gravitating stellar bodies with or without electric field distribution. In relativistic models, matter distribution can either be isotropic or anisotropic as Chaisi and Maharaj [1] in their work assumed that the matter distribution is isotropic so that the radial pressure is the same as the transverse pressure. A strong case can be made to study matter distributions which are anisotropic in which the radial component of the pressure is not the same as the transverse pressure.

The electric field is one of the ingredient that can be included in some of the relativistic models for charged stellar spheres. Charged models include the performance by Chattopadhyay et al [2], Maharaj and Thirukkanesh [3], Ivanov [4], Mehta et al [5], Murad and Fatema [6], Pant and Negi [7], Malaver [8], Thirukkanesh and Maharaj [9] and Maharaj and Komathiraj [10]. Mafa Takisa and Maharaj [11] obtained charged compact objects with anisotropic pressures in a core envelope setting.

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Bijalwan [12] indicated that the mass of a stellar star with electric field present is maximized with all degree of suitability. It was investigated the maximum mass of charged star to be $1{:}512 M_{\odot}$ with linear dimension 14:964 km. Maurya and Gupta [13] generated exact solutions for the Einstein's field equations for fluid spheres with pressure anisotropy. On the other hand, Mak and Harko [14] showed that strong magnetic fields could result into pressure anisotropy within stellar objects. Gupta and Maurya [15] found that the presence of electric field have effect on the gravitational collapse due to Colombian repulsive force and the pressure gradient. Neutral stellar models include results generated by Maharaj and Komathiraj [10], Sunzu [16] and Pant *et al* [17].

Relativistic models with linear equation of state have been found in the past. These include models performed by Esculpi and Aloma [18], Sharma and Maharaj [19], and Zdunik [20]. Aktas and Yilmaz [21] found linear models for Einstein field equations for spherical symmetric space-time via conformal motions. Sharma and Maharaj [22] found new exact models with linear equation of state by assuming a particular mass function. Maharaj and Chaisi [23] generated new models with linear barotropic equation of state. Kalam et al [24] proposed a relativistic model for strange quark stars within the framework of MIT bag model. Mak and Harko [14] presented exact anisotropic models consistent to stellar objects with a quark matter. Thirukkanesh and Maharaj [25] on physical grounds imposed a barotropic equation of state for the existence of strange matter. Exact anisotropic models for a charged relativistic spheres with linear equation of state were found by Maharaj and Mafa Takisa [26], Kileba Matondo and Maharaj [27], Maharaj et al [28] and Sunzu et al [29,30] and Sunzu and Danford [31]. Yilmaz and Baysal [32] investigated that quark stars are being formed during the collapsing of the core of a massive star after supernova explosion.

There are several anisotropic models generated using a quadratic equations of state for charged stellar spheres. These include the work by Maharaj and Mafa Takisa [33], Feroze and Siddiqui [34], Thirukkanesh and Maharaj [25] and Malaver [35]. Relativistic stellar models with polytropic equation of state were performed by Herrera and Barreto [36] and Dev and Gleiser [37]. It is often indicated that polytropes describe low or high

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pressure regimes especially for white dwarfs and neutron stars. Shibata [38] determined secular stability against a quasi-radial oscillation for rigidly rotating stellar objects. Lai and Xu [39] indicated that a theoretical polytropic quark star model could be tested by observations. Thirukkanesh and Ragel [40] indicated that a polytropic model is more stiffer than the conventional bag model. This is regarded more essential for modeling stars with realistic matter such as ideal gas, photon gas, degenerated Fermi gas and in particular quark matter. Other papers with polytropic equation of state include the work performed by Nilsson and Uggla [41], Spaans and Silk [42] and Mafa Takisa and Maharaj [43].

Relativistic stellar models with Van der Waals equations of state include models by Lobo [44], and Malaver [45, 46]. Thirukkanesh and Ragel [47] used Van der waals equation to generated compact anisotropic stellar models. Most of the anisotropic models with Van der waals equation have anisotropy always present and can not regain isotropic models. This is not physical. On the other hand, many of charged treatments in this direction have the electric field always present and can not regain neutral models. This is also not realistic. Uncharged anisotropic models with vanishing anisotropy using Van der waals equation of state are necessary.

The objective of this paper is to find new uncharged anisotropic models with vanishing anisotropy using Van der waals equation of state. In order to achieve this objective we arrange this paper in the following manner: In Sec. 2, we give the Einstein-Maxwell field equations for a neutral matter with anisotropic pressures. In Sect.3, we transform the field equations according to Durgapal and Bannerji [48]. In Sect. 4, we formulate the general differential equation governing the model. In Sect. 5, we generate solutions for nonsingular model I with Van derWaals equation of state. The model in this section generalizes earlier Einstein neutral model and obeys the phenomenon of Minkowski space-times. In Sect. 6, we find solutions for nonsingular model II with Van der Waals equation of state. In Sect. 7, we perform the physical analysis to indicate that the gravitational potentials and matter variables in our models are well behaved. We also generate relativistic stellar masses consistent with observations in this section. In Sect. 8 we give the conclusion.

II. The Anisotropic Model

We generate neutral anisotropic star models in a spacetime that is static and spherically symmetry. The line element in standard form is given by

$$ds^2 = -e^{2\nu(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2\left(d\theta^2 + sin^2\theta d\phi^2\right), \tag{1}$$

where $\nu(r)$ and $\lambda(r)$ are functions for gravitational potentials. The Schwarzschild [49] line element describing the exterior space time is given as

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \left(1 - \frac{2M}{r}\right)^{-1}dr^2 + r^2\left(d\theta^2 + sin^2\theta d\phi^2\right), \tag{2}$$

where *M* represents the total mass. The energy momentum tensor for neutral anisotropic matter is given by

$$\tau_{ij} = diag(-\rho, p_r, p_t, p_t), \tag{3}$$

where the energy density ρ, the radial pressure p_r and the tangential pressure p_t, are variables measured relative to a vector u. The vector u^a is comoving, unit and timelike.

According to Krasinski [50], the Einstein-Maxwell field equations for a neutral matter with anisotropic pressures can be written in the form

$$\frac{1}{r^2}\left(1 - e^{-2\lambda}\right) + \frac{2\lambda'}{r}e^{-2\lambda} = \rho, \tag{4a}$$

$$-\frac{1}{r^2}\left(1 - e^{-2\lambda}\right) + \frac{2\nu'}{r}e^{-2\lambda} = p_r, \tag{4b}$$

$$e^{-2\lambda}\left(\nu'' + \nu'^2 - \nu'\lambda' + \frac{\lambda'}{r} - \frac{\lambda'}{r}\right) = p_t. \tag{4c}$$

Where primes in the system (4) stand for differentiation with respect to the radial coordinate *r*. The mass contained within the neutral sphere is given by

$$m(r) = \frac{1}{2}\int_0^r \omega^2 \rho d\omega. \tag{5}$$

We use the Van der Waals equation of state relating the radial pressure and the energy density for the stellar object which is given as

$$p_r = \alpha\rho^2 + \frac{\beta\rho}{1+\gamma\rho}, \tag{6}$$

It is important to note that Eq. (6) becomes quadratic when $\gamma = 0$. It is in linear form when $\alpha = \gamma = 0$.

III. Transformation of the Field Equations

In order to obtain the exact solutions for the Einstein field equations, we transform the system (4) to an equivalent form by introducing independent variable x and new metric functions y and Z. These are defined as

$$x = Cr^2, Z(x) = e^{-2\lambda(r)}, A^2y^2(x) = e^{2\nu(r)}, \tag{7}$$

In the above *A* and *C* are arbitrary real constants. From Eq. (7) the line element (1) becomes

$$ds^2 = -A^2y^2(x)dt^2 + \frac{1}{Z(x)}\frac{1}{4xC}dx^2 + \frac{x}{C}\left(d\theta^2 + \sin^2\theta d\phi^2\right). \tag{8}$$

Then the mass function (5) becomes

$$M(x) = \frac{1}{4C^{\frac{3}{2}}}\int_0^x \sqrt{\omega}\rho d\omega. \tag{9}$$

The transformed Einstein-Maxwell field equations (4) with Van der Waals equation of state can be written as

$$\rho = \left(\frac{1-Z}{x} - 2\dot{Z}\right)C, \tag{10a}$$

$$p_r = \alpha\rho^2 + \frac{\beta\rho}{1+\gamma\rho}, \tag{10b}$$

$$p_t = p_r + \Delta, \tag{10c}$$

$$\Delta = \left[4xZ\frac{\ddot{y}}{y} + \left(1 + 2x\frac{\dot{y}}{y}\right)\dot{Z} + \frac{1-Z}{x}\right]C. \tag{10d}$$

In the above, $\Delta = p_t - p_r$. The system (10) above consists of six variables $(\rho, p_r, p_t, Z, y, \Delta)$ in four equations. The system can be solved if we specify any two variables. We have specified the following variables:

$$\Delta = A_0x + A_1x^2 + A_2x^3, \tag{11}$$

$$y = \frac{1-ax^m}{1+bx^n}, \tag{12}$$

where a, b, A_0, A_1, A_2, m and n are arbitrary real constants.

The metric function (12) is convenient to be used in modeling the stellar objects due to the fact that it is continuous, regular and finite. Similar choice of measure of anisotropy and metric function was made by Sunzu et al [29] in a model with electric field and a linear equation of state. However our model contain no electric field and we are using Van der waals equation of state. This choice of metric function and measure of anisotropy allows us to regain stellar models generated by Sunzu [16]. When the variable $\Delta = 0$, we generate the isotropic model. The condition of isotropic pressure is satisfied when $A_0 = A_1 = A_2 = 0$.

IV. The General Differential Equation for the Model

In this section we formulate the master differential equation governing our models. The differential equation is generated by using the measure of anisotropy and the metric function in Eq. (11) and Eq. (12) respectively. Substituting these equations into Eq. (10d) we have

$$\dot{Z} + \left[\frac{xN(x) - xP(x) - (1-ax^m)(1+bx^n)^2}{x(1+bx^n)(R(x)+Q(x))}\right]Z = \frac{\left(\frac{\Delta}{C} - \frac{1}{x}\right)R(x)}{R(x)+Q(x)}, \tag{13}$$

where for convenience we have set

$$\begin{aligned} N(x) &= (1+bx^n)\left[4ab\left(n^2 - m^2 + m - n\right)x^{m+n-1} \right. \\ &\quad \left. - 4a\left(m^2 - m\right)x^{m-1} - 4b\left(n^2 - n\right)x^{n-1}\right], \\ P(x) &= \left[8ab^2\left(n^2 - nm\right)x^{m+2n-1} - 8abnmx^{m+n-1} - 8b^2n^2x^{2n-1}\right], \\ Q(x) &= 2ab(n-m)x^{m+n} - 2amx^m - 2bnx^n, \\ R(x) &= (1-ax^m)(1+bx^n). \end{aligned}$$

We observe that Eq. (13) is in general a nonlinear differential equation in the potential *Z* which when integrated we obtain the function *Z*. We can therefore expressions for the matter variables. The general exact solution for Eq. (13) does not exist. However, we can find the exact solution for the nonlinear differential equation (13) by specifying the values for the constants. By doing so the nonlinear differential equation (13) can be linear and hence tractable. The choice for the constants should be made carefully so that the resulting model is physically well behaved and possible to regain other previous exact models.

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V. Solution for Nonsingular Model I with Van Der Waals Equation of State

In this section we use Van der waals equations of state to generate a nonsingular model for specific values of constants. Setting $m = 1$, $n = 1$, $a = 0$ and $b = 0$ we generate the first class of exact solutions for the differential equation (13) with metric function $y = 1$. Doing so Eq. (13) becomes

$$\dot{Z} - \frac{Z}{x} = \frac{A_0x + A_1x^2 + A_2x^3}{C} - \frac{1}{x}. \tag{14}$$

Solving Eq. (14) we obtain

$$Z = \frac{6A_0x^2 + 3A_1x^3 + 2A_2x^4 + 6xCk + 6C}{6C}, \tag{15}$$

where k is a constant of integration. Therefore the gravitational potentials and matter variables in system (10) become

$$e^{2\nu} = A^2, \tag{16a}$$

$$e^{2\lambda} = \frac{6C}{x\left(6A_0x + 3A_1x^2 + 2A_2x^3 + 6Ck\right) + 6C}, \tag{16b}$$

$$\rho = -5A_0x - \frac{7}{2}A_1x^2 - 3A_2x^3 - 3Ck, \tag{16c}$$

$$p_r = \alpha\left(5A_0x + \frac{7}{2}A_1x^2 + 3A_2x^3 + 3Ck\right)^2 - \frac{\beta\left(5A_0x + \frac{7}{2}A_1x^2 + 3A_2x^3 + 3Ck\right)}{1 - \gamma\left(5A_0x + \frac{7}{2}A_1x^2 + 3A_2x^3 + 3Ck\right)}, \tag{16d}$$

$$p_t = \alpha\left(5A_0x + \frac{7}{2}A_1x^2 + 3A_2x^3 + 3Ck\right)^2 - \frac{\beta\left(5A_0x + \frac{7}{2}A_1x^2 + 3A_2x^3 + 3Ck\right)}{1 - \gamma\left(5A_0x + \frac{7}{2}A_1x^2 + 3A_2x^3 + 3Ck\right)} + A_0x + A_1x^2 + A_2x^3, \tag{16e}$$

$$\Delta = A_0x + A_1x^2 + A_2x^3. \tag{16f}$$

Then the mass function (9) becomes

$$M(x) = -\frac{x^{\frac{3}{2}}}{4C^{\frac{3}{2}}}\left(2A_0x + A_1x^2 + \frac{2}{3}A_2x^3 + 2Ck\right). \tag{17}$$

The line element for the exact model in the system (16) becomes

$$ds^2 = -A^2dt^2 + \frac{6dx^2}{4x\left[x\left(6A_0x + 3A_1x^2 + 2A_2x^3 + 6Ck\right) + 6C\right]} + \frac{x}{C}\left(d\theta^2 + sin^2\theta d\phi^2\right). \tag{18}$$

For isotropic pressure ($\Delta = 0$), we have $A_0 = A_1 = A_2 = 0$. The gravitational potentials and matter variables in the system (16) becomes

$$e^{2\nu} = A^2, \tag{19a}$$

$$e^{2\lambda} = \frac{1}{kx + 1}, \tag{19b}$$

$$\begin{aligned} \rho &= -3Ck, \quad (19c)\\ p_r = p_t &= \alpha\left(3Ck\right)^2 - \frac{3\beta Ck}{1-3\gamma Ck}. \quad (19d)\end{aligned}$$

The mass function (17) becomes

$$M(x) = -\frac{x^{\frac{3}{2}}k}{2C^{\frac{1}{2}}}, \quad (20)$$

with the line element

$$ds^2 = -A^2dt^2 + \frac{dx^2}{4xC\left(kx+1\right)} + \frac{x}{C}\left(d\theta^2 + sin^2\theta d\phi^2\right). \quad (21)$$

The line element (21) can be presented as

$$ds^2 = -A^2dt^2 + \left(1-\frac{r^2}{\Gamma^2}\right)^{-1}dr^2 + r^2\left(d\theta^2 + sin^2\theta d\phi^2\right), \quad (22)$$

where $\Gamma^2 = -\frac{1}{Ck}$ and $k<0$. The line element (21) becomes a well known neutral isotropic Einstein model. This shows that our model contains other previous models as a special case. Taking $k=0$ and $\Delta=0$ we have

$$e^{2\nu} = A^2, e^{2\lambda} = 1, \rho = 0, p_r = p_t = 0, M = 0. \quad (23)$$

We see that the matter variables vanish and the gravitational potentials are constant. This agrees with Minkowski space-times.

VI. Solution for Nonsingular Model II with Van Der Waals Equation of State

We consider different values for the parameters *m, n, a* and *b* given in Eq. (13). We find other exact solutions when the metric function y is not constant. We choose $m = 1$, $n = 1$, $a \neq 0$ and $b = 0$. The metric function (12) becomes

$$y = 1 - ax, \quad (24)$$

and the differential equation (13) becomes

$$\dot{Z} - \left(\frac{1-ax}{x\left(1-3ax\right)}\right)Z = \frac{\left[x\left(A_0x + A_1x^2 + A_2x^3\right) - C\right]\left(1-ax\right)}{Cx\left(1-3ax\right)}. \quad (25)$$

Solving Eq. (25) we obtain

$$\begin{aligned} Z = &-\frac{1}{Ca^3}\left[\left(\frac{2}{5}-\frac{1}{5}ax\right)A_0a^2x\right.\\ &-\left(-\frac{3}{40}-\frac{3}{20}ax+\frac{1}{8}a^2x^2\right)A_1ax\\ &+\left(\frac{1}{55}+\frac{2}{55}ax+\frac{1}{11}a^2x^2-\frac{1}{11}a^3x^3\right)A_2x - a^3C\\ &\left.-\frac{Cka^3x}{\left(1-3ax\right)^{\frac{2}{3}}}\right]. \quad (26)\end{aligned}$$

Hence the gravitational potentials and matter variables become

$$e^{2\nu} = A^2\left(1-ax\right)^2, \quad (27a)$$

$$\begin{aligned} e^{2\lambda} &= -Ca^3\left[\left(\frac{2}{5}-\frac{1}{5}ax\right)A_0a^2x \right.\\ &\quad -\left(-\frac{3}{40}-\frac{3}{20}ax+\frac{1}{8}a^2x^2\right)A_1ax \\ &\quad +\left(\frac{1}{55}+\frac{2}{55}ax+\frac{1}{11}a^2x^2-\frac{1}{11}a^3x^3\right)A_2x-a^3C \\ &\quad \left.-\frac{Cka^3x}{(1-3ax)^{\frac{2}{3}}}\right]^{-1}, \end{aligned} \tag{27b}$$

$$\begin{aligned} \rho &= -\frac{1}{a^3(1-3ax)}\left[\left(-\frac{6}{5}-\frac{23}{5}ax+3a^2x^2\right)A_0a^2 \right.\\ &\quad -\left(\frac{9}{40}+\frac{3}{40}ax-\frac{25}{8}a^2x^2+\frac{21}{8}a^3x^3\right)A_1a \\ &\quad -\left(\frac{3}{55}+\frac{1}{55}ax+\frac{1}{11}a^2x^2-\frac{30}{11}a^3x^3+\frac{27}{11}a^4x^4\right)A_2 \\ &\quad \left.+\frac{Ca^3k(3-5ax)}{(1-3ax)^{\frac{2}{3}}}\right], \end{aligned} \tag{27c}$$

$$\begin{aligned} p_r &= \alpha\left(\frac{1}{a^3(1-3ax)}\left[\left(-\frac{6}{5}-\frac{23}{5}ax+3a^2x^2\right)A_0a^2 \right.\right.\\ &\quad -\left(\frac{9}{40}+\frac{3}{40}ax-\frac{25}{8}a^2x^2+\frac{21}{8}a^3x^3\right)A_1a \\ &\quad -\left(\frac{3}{55}+\frac{1}{55}ax+\frac{1}{11}a^2x^2-\frac{30}{11}a^3x^3+\frac{27}{11}a^4x^4\right)A_2 \\ &\quad \left.\left.+\frac{Ca^3k(3-5ax)}{(1-3ax)^{\frac{2}{3}}}\right]\right)^2-\frac{\beta\rho}{1-\gamma\rho}, \end{aligned} \tag{27d}$$

$$\begin{aligned} p_t &= \alpha\left(\frac{1}{a^3(1-3ax)}\left[\left(-\frac{6}{5}-\frac{23}{5}ax+3a^2x^2\right)A_0a^2 \right.\right.\\ &\quad -\left(\frac{9}{40}+\frac{3}{40}ax-\frac{25}{8}a^2x^2+\frac{21}{8}a^3x^3\right)A_1a \\ &\quad -\left(\frac{3}{55}+\frac{1}{55}ax+\frac{1}{11}a^2x^2-\frac{30}{11}a^3x^3+\frac{27}{11}a^4x^4\right)A_2 \\ &\quad \left.\left.+\frac{Ca^3k(3-5ax)}{(1-3ax)^{\frac{2}{3}}}\right]\right)^2-\frac{\beta\rho}{1-\gamma\rho} \\ &\quad +A_0x+A_1x^2+A_2x^3, \end{aligned} \tag{27e}$$

$$\Delta = A_0x+A_1x^2+A_2x^3. \tag{27f}$$

The mass function (9) becomes

$$M(x) = \frac{x^{\frac{3}{2}}}{a^3(1-3ax)C^{\frac{3}{2}}}\left[\left(\frac{1}{5}-\frac{7}{10}ax+\frac{3}{10}a^2x^2\right)A_0a^2 - \left(-\frac{3}{80}+\frac{3}{80}ax+\frac{23}{80}a^2x^2-\frac{3}{16}a^3x^3\right)A_1a + \left(\frac{1}{110}-\frac{1}{110}ax-\frac{1}{110}a^2x^2-\frac{2}{11}a^3x^3+\frac{3}{22}a^4x^4\right)A_2 - \frac{1}{2}a^3Ck(1-3ax)^{\frac{1}{3}}\right]. \quad (28)$$

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The line element for the model in the system (27) becomes

$$ds^2 = -A^2(1-ax)^2 - Ca^3\left[\left(\frac{2}{5}-\frac{1}{5}ax\right)A_0a^2x - \left(-\frac{3}{40}-\frac{3}{20}ax+\frac{1}{8}a^2x^2\right)A_1ax + \left(\frac{1}{55}+\frac{2}{55}ax+\frac{1}{11}a^2x^2-\frac{1}{11}a^3x^3\right)A_2x - a^3C - \frac{Cka^3x}{(1-3ax)^{\frac{2}{3}}}\right]^{-1}\frac{dx^2}{4xC}+\frac{x}{C}\left(d\theta^2+sin^2\theta d\phi^2\right). \quad (29)$$

Setting $a<0$, $\alpha=0$ and $\gamma=0$ in the system (27) we regain the exact model given by Sunzu [16].

For isotropic pressure ($\Delta=0$) we have $A_0 = A_1 = A_2 = 0$, and the gravitational potentials and matter variables in (27) become

$$e^{2\nu} = A^2(1-ax)^2, \quad (30a)$$

$$e^{2\lambda} = \frac{(1-3ax)^{\frac{2}{3}}}{kx+(1-3ax)^{\frac{2}{3}}}, \quad (30b)$$

$$\rho = -\frac{Ck(3-5ax)}{(1-3ax)^{\frac{5}{3}}}, \quad (30c)$$

$$p_r = p_t = \alpha\left(\frac{Ck(3-5ax)}{(1-3ax)^{\frac{5}{3}}}\right)^2 - \frac{\beta Ck(3-5ax)}{(1-3ax)^{\frac{5}{3}}-\gamma Ck(3-5ax)}. \quad (30d)$$

The mass function (28) becomes

$$M(x) = -\frac{x^{\frac{3}{2}}k}{2C^{\frac{1}{2}}(1-3ax)^{\frac{2}{3}}}. \quad (31)$$

with the line element

$$ds^2 = -A^2(1-ax)^2dt^2 + \frac{(1-3ax)^{\frac{2}{3}}dx^2}{4xC\left[kx+(1-3ax)^{\frac{2}{3}}\right]} + \frac{x}{C}\left(d\theta^2+sin^2\theta d\phi^2\right). \quad (32)$$

Taking $k=0$ and $\Delta=0$ we have

$$e^{2\nu}=A^2(1-ax)^2, e^{2\lambda}=1, \rho=0, p_r=p_t=0, M=0. \quad (33)$$

We see that the matter variables vanish and the gravitational potentials are constant at the centre. This condition agrees with Minkowski space-times.

VII. Discussion

In this section, we indicate that the exact solutions for the field equations given in the system (27) of Sect. (6) are well behaved. The gravitating potentials and the matter variables obtained are finite, regular and continuous. We see that isotropic results are contained in our nonsingular models as a special case. This is possible when the measure of anisotropy $\Delta=0$, the case when parameters A_0, A_1 and A_2 are set to zero. Of interest is to indicate that the physical analysis is possible. We do this by generating graphical plots for the gravitational potentials, matter variables using the model in the system (27) and mass function (28) in Sect. (6). Python programming language was used to generate these plots for the particular choices a = -3:3, A = 1:0, β = 0:5, α = 0:18, γ = 0:1, C = 1:0, k = 0:3, A_0 = -1.5, A_1 = -0.6 and A_2 = 1:0. The graphical plots generated are for the potential $e^{2\nu}$ (Fig.1), potential $e^{2\lambda}$ (Fig. 2), energy density ρ (Fig. 3), radial pressure p_r (Fig. 4), tangential pressure p_t (Fig. 5), measure of anisotropy Δ (Fig. 6) and the mass M (Fig. 7). All figures are plotted against the radial coordinate r. These quantities are regular and well behaved in the stellar interior.

The potentials in Fig.(1) and Fig. (2) are increasing functions with radial distance. They are finite, regular and continuous similar to those in Komathiraj and Maharaj [52], Sunzu [16] and Sunzu et al [29]. The energy density ρ in Fig. (3), the radial pressure p_r in Fig. (4) and the tangential pressure p_t in Fig. (5) are decreasing functions with the radial coordinate. These profiles are similar to to the one in Kalam et al [24], Sunzu [16] and Thirukkanesh and Maharaj [25]. We observe in Fig. (6) that the measure of anisotropy Δ is a decreasing function from the centre to the region near the surface. This is similar to the findings in Sunzu et al [29] and Kalam et al [24]. The mass M in Fig. (7) increase with radial distance similar to that in Sunzu [16], Sunzu et al [29] and Malaver [45, 46].

We also generate relativistic stellar masses using the transformations $\tilde{A}_0 = A_0R^2$, $\tilde{A}_1 = A_1R^2$, $\tilde{A}_2 = A_2R^2$, $\tilde{C} = CR^2$ and $\tilde{a} = aR^2$. We are using the mass function (28) of Sect. (6) to generate masses consistent with observations. We generated stellar masses consistent with the one observed by Demorest *et al* [53] for a star PSR J1614-2230, Rawls et al [54] for stars Vela X-1, 4U 1538-52, LMC X-4, SMC X-4 and Cen X-3, Abubekerov et al [55] for a star Her X-1, Elebert *et al* [56] for a star SAX J1808.4-3658 and Ozel *et al* [57] for a star EXO 1785-248. Computation is done by choosing different values for the constants $\tilde{a}, \tilde{A}_0$, $\tilde{A}_1$ and $\tilde{A}_2$. Conveniently, for computation purposes we have set R = 55.00. Therefore our exact model produce finite masses consistent with astronomical objects. The parameters producing these relativistic masses are indicated in Table (1).

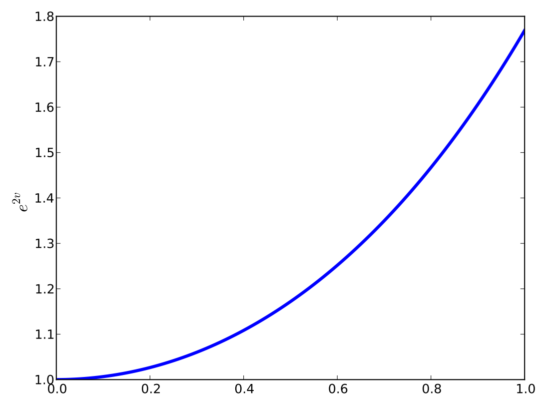

Figure 1: The gravitational potential $e^{2\nu}$ against the radial distance

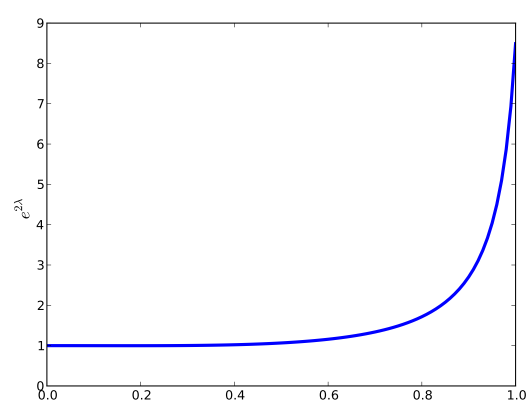

Figure 2: The gravitational potential $e^{2\lambda}$ against the radial distance

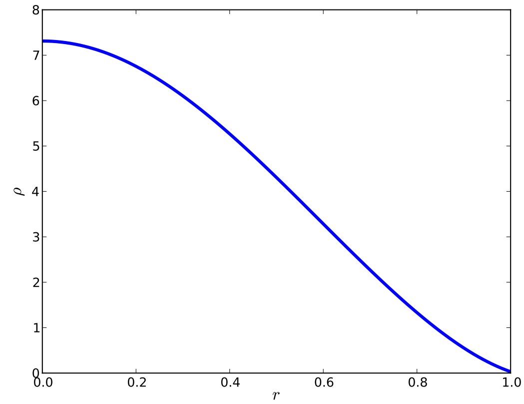

Figure 3: Energy density ρ against the radial distance

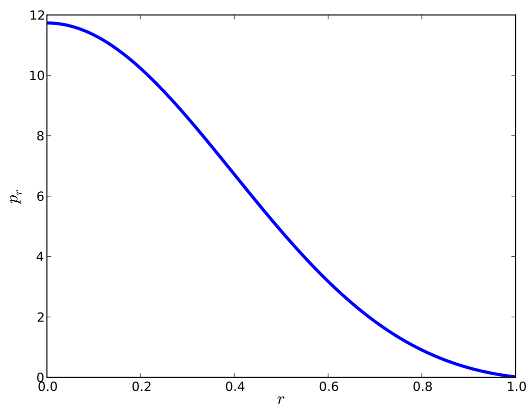

Figure 4: Radial pressure p_r against the radial distance

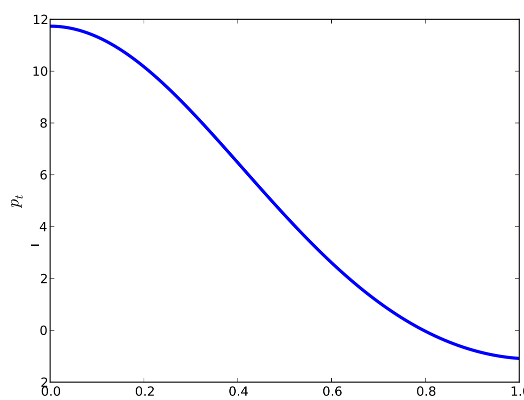

Figure 5: Tangential pressure p_t against the radial distance

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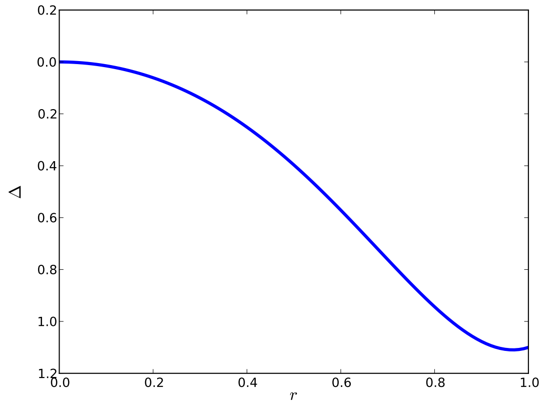

Figure 6: Measure of anisotropy Δ against the radial distance

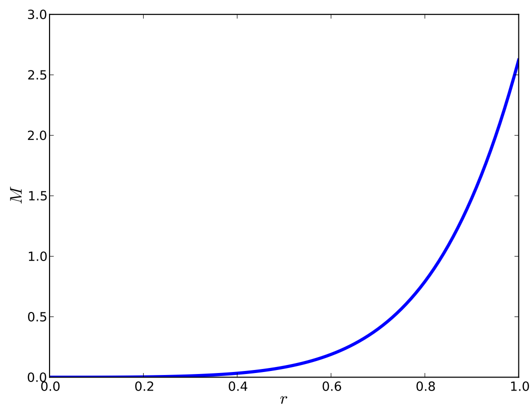

Figure 7: Mass *M* against the radial distance

Table 1: Particular stellar star masses obtained for various parameter values

$\tilde{a}$	$\tilde{C}$	$\tilde{A}_0$	$\tilde{A}_1$	$\tilde{A}_2$	$\tilde{k}$	$R(Km)$	$M(M_\odot)$	$Star$	$References$
9.5	2.0	2.0	3.0	1.9	1.0	9.69	1.97	$PSR\,J1614-2230$	Demorest *et al* [53]
9.5	2.0	2.0	1.3	1.8	1.0	9.56	1.77	$Vela\,X-1$	Rawls *et al* [54]
9.9	2.0	2.0	1.3	1.8	1.0	7.866	0.87	$4U\,1538-52$	Rawls *et al* [54]
9.9	2.0	1.1	3.5	1.8	1.0	8.301	1.04	$LMC\,X-4$	Rawls *et al* [54]
9.8	2.0	1.1	3.5	1.8	1.0	8.831	1.29	$SMC\,X-4$	Rawls *et al* [54]
9.7	2.0	1.1	3.0	1.8	1.0	9.178	1.49	$cen\,X-3$	Rawls *et al* [54]
9.7	2.0	1.1	2.0	1.5	1.0	8.1	0.85	$Her\,X-1$	Abubekerov *et al* [55]
9.7	2.0	1.1	2.0	1.7	1.0	7.951	0.9	$SAX\,J1808.4-3658$	Elebert *et al* [56]
9.6	2.0	1.1	2.5	1.7	1.0	8.849	1.3	$EXO\,1785-248$	Ozel *et al* [57]

VIII. Conclusion

We have generated new exact relativistic models for neutral anisotropic stars using Einstein-Maxwell field equations. We have used a Van der Waals equation of state relating the energy density and the radial pressure. In our new models, the energy density ρ, the radial pressure p_r and the tangential pressure p_t are finite decreasing functions. The mass M and the gravitational potentials $e^{2\nu}$, $e^{2\lambda}$ are increasing functions, continuous and finite inside the stellar interior. The measure of anisotropy Δ is a decreasing negative function showing that $p_t < p_r$. We have indicated that for specific conditions our models agree with earlier Einstein isotropic neutral model and Minkowski spacetime. In this paper neutral relativistic models described are physically reasonable and have astrophysical significance. We have generated masses consistent with observations. The masses generated are those consistent with the star PSR J1614-2230, Vela X-1, 4U 1538-52, LMC X-4, SMC X-4, Cen X-3, Her X-1, SAX J1808.4-3658 and the star EXO 1785-248. Our models are significant for studies of relativistic compact neutral stellar objects in astrophysics. For further research, these models could be used to study the interior structures of the stellar objects by considering new choice of measure of anisotropy, metric functions and equation of states.

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