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Vector Potential of a Strongly Inductive Impedance Media

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Vector Potential of a Strongly Inductive Impedance Media

Vasiliy K. Balkhanov [α] & Yuriy B. Bashkuev [σ]

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I. Introduction

The task of the electromagnetic wave propagation along Earth's surface more than 100 years ago first considered A. Sommerfeld. This task is expressed in the form of a Sommerfeld integrals complex, computation of which still causes great mathematical difficulty [1-6]. Sommerfeld's decision considered homogeneous media. However, there is a need to consider the diverse impedanse media, multi-layered or gradient. We consider the following to the complexity of the object - of the two media. Our consideration is that compute the vector potential in each media. Knowing him, it will be easy to find the rest of the components of the electromagnetic (EM) field.

First, we introduce the basic values needed in the course of computing, and evaluate their order of magnitude relative to each other. We assume that we have a free space where the emitter - is a vertically orientated dipole Hertz, and the receiver in the form of a vertical metal antennas. If you designate a circular frequency ω, the speed of light *c*, the square wave number k_0 in free space would be

$$k_0^{\ 2} = \frac{\omega^2}{c^2}. \qquad (1)$$

Subscript will number media. Underlying media is double layered. For definiteness, we assume that the first layer thickness *h* is a dielectric in dielectric permeability ε, for example, the pack ice of the Arctic Ocean. Square wave number k_1 in this case, you will

$$k_1^{\ 2} = k_0^{\ 2}\ \varepsilon. \qquad (2)$$

Bottom, infinite depth layer believe conducting, with electrical conductivity σ (or resistivity $\rho = 1/\sigma$). This media may be salt water in the Arctic Ocean. Square wave number k_2 in conductivity will

$$k_2^{\ 2} = k_0^{\ 2}\ \frac{i\sigma}{\varepsilon_0\ \omega}. \qquad (3)$$

In here ε_0 - permittivity of vacuum.

In general, the square of the wave number is defined as [7]

$$k^2 = \frac{\omega^2}{c^2}\left(\varepsilon + \frac{i\sigma}{\varepsilon_0\omega}\right). \qquad (4)$$

That ratio (2) and (3), a frequency range. For example, if the ice to take $\varepsilon = 40$, and salt water $\rho = 1$ Ohm·m, the equality of

$$\varepsilon_0\ \varepsilon\ \omega\ \rho = 1$$

find frequency $\omega / 2\pi = 4.5 \cdot 10^8$ Hz. The result means that formulas (2) and (3) you can use up to frequencies, smaller 450 MHz (or length of an electromagnetic wave, large ~ 1 m). In this frequency range

$$k_2 >> k_1 >> k_0. \qquad (5)$$

In this task, together with wave number appear several more units, for example,

$$\mu = \sqrt{\chi^2 - k^2}\ ,\ \chi = k_0\sqrt{1-\delta^2}. \qquad (6)$$

In here δ - powered surface impedance (furthermore, impedance). The usefulness of introducing impedance can be seen from the fact that it is directly measurable. By definition, the impedance module $|\delta|$ a small value. You can take that [8] $|\delta|$ < 0.3. Then, using the (1) to (3), we get

$$\mu_0 = i\ k_0\ \delta, \qquad (7)$$

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$$\mu_1 = i\, k_0\, \sqrt{\varepsilon}\,, \qquad (8)$$

$$\mu_2 = i\, k_0\, \frac{1-i}{\sqrt{2\varepsilon_0\, \varepsilon\, \omega\, \rho}}\,, \qquad (9)$$

And took that $\sqrt{-i} = (1-i)/\sqrt{2}$. Of inequalities (5) that

$$\mu_2 >> \mu_1 >> \mu_0\,. \qquad (10)$$

In addition to the above, during the computations appear two more values:

$$\Omega_{10} = \frac{\mu_1\, k_0^{\,2}}{\mu_0\, k_1^{\,2}}\,, \qquad \Omega_{12} = \frac{\mu_1\, k_2^{\,2}}{\mu_2\, k_1^{\,2}}\,, \qquad (11)$$

Estimates show that Ω_{10} ~ $\sqrt{\varepsilon_0 \omega\rho}$ << 1, Ω_{12} ~ $1/\sqrt{\varepsilon_0 \omega\rho}$ >> 1.

Entered values and their assessments may be needed in the course of further research.

II. Fourier Images Of Vector Potential

We assume that the point dipole of Hertz is located at distance $z = l$ from a flat border xy section "atmosphere - terrestrial two-layer media". In the interval from $z = -h$ before $z = 0$ the first layer is located. Because the boundary conditions $z = 0$ and $z = -h$ do not depend on time, frequency in all sectors will be one and the same. Therefore, temporary multiplier $\exp(-i\omega\, t)$ you can explicitly do not prescribe.

Since Hertz dipole vector, directed along the *z* axis, we have only one non-zero z-components of the vector potential $A_z = A$, which gives the following integral

$$A(x,y,z) = \frac{1}{4\pi}\int_{-\infty}^{\infty} A_\chi(z)\;H_0(\chi\, R)\;\chi\, d\chi\,. \qquad (12)$$

Here is the radial coordinate of the $R = \sqrt{x^2 + y^2}$. In the wave zone where $\chi\, R$ >> 1, function Hankel $H_0(\chi\, R)$ has the asymptotic view

$$H_0(\chi\, R) = \sqrt{\frac{2}{\pi\, i\, \chi\, R}}\exp(i\, \chi\, R)\,. \qquad (13)$$

Fourier image A_χ obeys the wave equation with source:

$$\frac{d^2 A_\chi}{d\, z^2} - \left(\chi^2 - k^2\right) A_\chi = -4\pi\; J_0\;\delta(z-l)\,. \qquad (14)$$

In here $J_0 = \mu_0\; J\; a/4\pi$, *J* –current dipole, *a* – dipole length, μ_0 - magnetic constant, $\delta(z-l)$ - the Dirac function. Integral (12) function (13) is called the Sommerfeld integral.

Integrating (14) by dz from $-\infty$ to $+\infty$, find that when $z = l$

$$A_\chi \Big|_{l-0}^{l+0} = 0\,, \qquad \frac{dA_\chi}{d\, z}\Big|_{l-0}^{l+0} = -4\pi J_0\,. \qquad (15)$$

The first equality have condition continuity vector potential. Record, for example, $A_\chi \Big|_{l-0}^{l+0} = 0$, means that $A_\chi(l+0) - A_\chi(l-0) = 0$. Condition (15) we have excluded $\delta(z-l)$ - the Dirac function, how would adding additional border in free space. For the rest of the borders $z = 0$ and $z = -h$ we have the normal boundary conditions:

$$A_\chi \Big|_{-0}^{+0} = 0\,, \qquad \frac{1}{k^2}\frac{dA_\chi}{d\, z}\Big|_{-0}^{+0} = 0\,. \qquad (16)$$

$$A_\chi \Big|_{-h-0}^{-h+0} = 0\,, \qquad \frac{1}{k^2}\frac{dA_\chi}{d\, z}\Big|_{-h-0}^{-h+0} = 0\,. \qquad (17)$$

Because inside each limited layer wave extends in both directions, the Fourier-image of the vector potential in each layer will be

$$A_\chi = W\, \exp(-\mu_0 z) \quad \text{when } z > l, \qquad (18)$$

$$A_\chi = M\, \exp(-\mu_0 z) + C\exp(\mu_0 z) \quad \text{when } l > z > 0, \qquad (19)$$

$$A_\chi = D\, \exp(\mu_1 z) + E\exp(-\mu_1 z) \quad \text{when } 0 > z > -h, \qquad (20)$$

$$A_\chi = F\, \exp(\mu_2 z) \quad \text{when } -h > z. \qquad (21)$$

In here $\mu_{012} = \sqrt{\chi^2 - k_{012}^{\;2}}$. Choice of characters in exponential is associated with the condition that the field at infinity vanishes. Substituting potentials (18)-(21) boundary conditions (15)-(17), we obtain the six equations. Simple steps allows you to write out their decision. If you enter values (11) and

$$L_- = (1+\Omega_{10})(\Omega_{12}+1) + (1-\Omega_{10})(\Omega_{12}-1)\exp(-2\mu_1 h)\,, \qquad (22)$$

$$L_+ = (1-\Omega_{10})(\Omega_{12}+1) + (1+\Omega_{10})(\Omega_{12}-1)\exp(-2\mu_1 h)\,, \qquad (23)$$

the decision will be presented in the following explicit expressions:

$$W=\left(\frac{L_-}{L_+}+\exp(2\mu_0 l)\right)\frac{2\pi\ J_0}{\mu_0}\exp(-\mu_0 l). \quad (24)$$

$$M=\frac{L_-}{L_+}\frac{2\pi\ J_0}{\mu_0}\exp(-\mu_0 l). \quad (25)$$

$$C=\frac{2\pi\ J_0}{\mu_0}\exp(-\mu_0 l); \quad (26)$$

$$D=\frac{2(\Omega_{12}+1)}{L_+}\frac{2\pi\ J_0}{\mu_0}\exp(-\mu_0 l-2\mu_1 h), \quad (27)$$

$$E=\frac{2(\Omega_{12}+1)}{L_+}\frac{2\pi\ J_0}{\mu_0}\exp(-\mu_0 l), \quad (28)$$

$$F=\frac{4\Omega_{12}}{L_+}\frac{2\pi\ J_0}{\mu_0}\exp(-\mu_0 l-\mu_1 h+\mu_2 h). \quad (29)$$

Substituting (24)-(29) in (12) get of Sommerfeld integrals, calculation who now represents the big mathematical difficulties. For example, even for a media homogeneous Sommerfeld integral method cannot pass calculated in the wave zone in only three cases, for direct, reflected and lateral EM waves.

III. Strongly Inductive Impedance Media

It is convenient to introduce the concept of effective media, when for each frequency ω media has an effective value of dielectric permittivity $\tilde{\varepsilon}$ and electrical conductivity $\tilde{\sigma}$. Then the impedance δ will have the following known species [7]:

$$\delta=\left(1+\tilde{\varepsilon}+i\tilde{\sigma}/\varepsilon_0\ \omega\right)^{-1/2}. \quad (30)$$

Square effectively wave number $\tilde{k}$ EM waves in this media

$$\tilde{k}^2=\frac{\omega^2}{c^2}\left(\tilde{\varepsilon}+\frac{i\tilde{\sigma}}{\varepsilon_0\omega}\right). \quad (31)$$

Transmitter and receiver are often above ground, in free space. As we have informed, the wave number of the EM field in free space would be $k_0=\omega/c$. Combining (30) and (31), we get:

$$\tilde{k}=k_0\sqrt{1-\delta^2}/\delta. \quad (32)$$

In this form the formula apply to any underlying media - homogeneous, layered or gradient. It should be noted that $\tilde{k}$ expressed through directly measurable value - impedance δ.

From the formula (30) see that the impedance is a complex number, which is written in two forms. Or as $\delta=|\delta|\exp(i\varphi_\delta)$, where is $|\delta|$ - module and φ_δ impedance phase. Either $\delta=\mathrm{Re}\,\delta+i\,\mathrm{Im}\,\delta$, where is $\mathrm{Re}\,\delta$ - the real and $\mathrm{Im}\,\delta$ the imaginary part of impedance. On Earth many media are strongly inductive whose impedance phase $-45.1^0 < \varphi_\delta < -89.9^0$. This year-round pack ice on the salt water of the Arctic Ocean, is also a year-round forest vegetation, the current millennium, permafrost in the northern regions and the salt lakes of Siberia in winter-spring period. In the permafrost area for one of the radiotrass on the frequency $f=255$ kHz impedance was installed [9] $\delta=0.028-i\ 0.085$, from where $|\delta|^2=0.008$, $\varphi_\delta=-71.3^0$. It is (30) easy to extend

$$\tilde{\varepsilon}=\cos(2\varphi_\delta)/|\delta|^2-1,$$

$$\tilde{\rho}=1/\tilde{\sigma}=(1/\varepsilon_0\omega)|\delta|^2/\sin(-2\varphi_\delta).$$

From where

$$\tilde{\varepsilon}=-101,\ \tilde{\rho}=950\ \text{Ом·м}.$$

Interesting, what $\tilde{\rho}$ adopted value between 0 (electrical conductor) и ∞ (air). For dielectric permittivity concept of average is not suitable. Here for air $\varepsilon=1$, for conductor $\varepsilon\approx 5$, and the average $\tilde{\varepsilon}=-101$. Because $1/\varepsilon_0\omega\rho=1.325$, then on the frequency

$$\tilde{k}^2=\frac{\omega^2}{c^2}\left(-100+i\ 1.325\right).$$

Here is the real part of the square of the effective wave number significantly more imaginary part.

In winter-spring period on one of the salt lakes of Siberia were measured surface electromagnetic wave [10]. The results of these measurements at a frequency of 10 MHz impedance has been restored $\delta=0.063-i\ 0.139$. For this case $\tilde{\varepsilon}=-29$ and $\tilde{\rho}=55$ Ohm·m. From where

$$\tilde{k}^2=\frac{\omega^2}{c^2}\left(-29+i\ 32\right).$$

We see here that the real and imaginary parts of a square wave effectively number one order. Examples of shows that the impedance

$$\delta=\mathrm{Re}\,\delta+i\,\mathrm{Im}\,\delta=\mathrm{Re}\,\delta-i|\mathrm{Im}\,\delta|,\ |\mathrm{Im}\,\delta|>\mathrm{Re}\,\delta. \quad (33)$$

Natural media with an impedance (33) are called strongly inductive impedanse media.

IV. Surface Electromagnetic Wave

A happy addition to the above three cases the calculation of the integral Sommerfeld are surface EM waves (SEW). And here there is a significant difference. To determine the trajectory of the direct, documented and side EM is possible to apply the principle of wave farm, which allows you to determine the setting for the Sommerfeld integral integration in particular point χ_0, where the integral is evaluated and pass method [11,12]. But for SEW this principle no. This wave propagates in a narrow dielectric layer, unable to escape neither the atmosphere nor bottom electrical conductor media. This means, as we shall see below, that Sommerfeld integrals and electrical conductor layer exponentially damped.

In the wave zone, away from the emitter, emitter location height *l* is almost invisible. So you can put $l = 0$. Because the dimension EM is happening in fields near the surface layer, the Sommerfeld integral in this case takes the form:

$$A_{SEW} = \frac{J_0}{\sqrt{2\pi\, i\, R}} \int_{-\infty}^{\infty} \frac{L_-}{L_+} \frac{\sqrt{\chi}}{\mu_0} \exp(-\mu_1 z + i\chi R)\, d\chi \,. \tag{34}$$

Larger values of *R* has a way of integrating into the lower half-plane χ, where near integral expression decreases rapidly. It should be reminded that great *R* meets the wave zone. At offset must bypass all zeroes of the denominator of the function L_- / L_+ and its branching point. As a result, the integral will be proportional to the $\exp f(-\mu_1(\chi_0) + \chi_0 R)$, where is χ_0 - closest to the real axis is one of the singular points. A special point of particularly easy to find in the case $h = 0$. Equating denominator zero function L_- / L_+, it is easy to find $\chi_0 = k_0 k_2 / \sqrt{k_0^{\,2} + k_2^{\,2}}$. If you put $h = \infty$, you also find $\chi_0 = k_0 k_1 / \sqrt{k_0^{\,2} + k_1^{\,2}}$. When the final *h*, according to the concept of effective media, you can put

$$\chi_0 = \frac{k_0\, \tilde{k}}{\sqrt{k_0^{\,2} + \tilde{k}^{\,2}}} \,. \tag{35}$$

Recall, $\tilde{k}$ - the effective wave number. Using its value (32), finally find a particular point Sommerfeld integral (34):

$$\chi_0 = k_0 \sqrt{1 - \delta^2} \,. \tag{36}$$

We will be interested in the spatial characteristics of the vector potential, its dependence on coordinates *R* and *z*. For this purpose it is necessary to substitute the special point (36) in the exponential (34). Next, you need to distinguish between vector potential A_{z0} in free space and vector potential A_z in impedance media. As a result, first have to free space:

$$A_{z0} = \frac{K_0}{\sqrt{R}} \exp\left(-i\, k_0 \delta\, z + i\, k_0 \sqrt{1-\delta^2}\;\, R\right). \tag{37}$$

And the vector potential in impedance media:

$$A_z = \frac{K}{\sqrt{R}} \exp\left(-i\, k_0 \frac{1-\delta^2}{\delta}\;\, z + i\, k_0 \sqrt{1-\delta^2}\;\, R\right). \tag{38}$$

Both terms all exponential phase components are expressed through impedance laying media. This is one of the manifestations of the beneficial properties of the introduction of the impedance.

Substituting the expression (33) in the formula (37) and (38), we get:

$$A_{z0}(R,z,t) = \frac{K_0}{\sqrt{R}} \exp\big[\; -i \operatorname{Re}\delta\; k_0 z - |\operatorname{Im}\delta|\;\, k_0 z \;+ \\ + \left(ik_0 R - i\omega\, t\right) - \operatorname{Re}\delta\; |\operatorname{Im}\delta|\;\, k_0 R \;\big]. \tag{39}$$

$$A_z(R,z,t) = \frac{K}{\sqrt{R}} \exp\Big[\; -i \frac{\operatorname{Re}\delta}{\operatorname{Re}^2\delta + \operatorname{Im}^2\delta}\;\, k_0 z + \frac{|\operatorname{Im}\delta|}{\operatorname{Re}^2\delta + \operatorname{Im}^2\delta}\;\, k_0 z \;+ \\ + \left(ik_0 R - i\omega\, t\right) - \operatorname{Re}\delta\; |\operatorname{Im}\delta|\;\, k_0 R \;\Big]. \tag{40}$$

Here is obviously pasted $\exp(-i\omega\, t)$, not to forget, that are considering wave motion.

Of the solutions should be that the vector potential in free space fades as

$$\exp\left(-|\operatorname{Im}\delta|\;\, k_0 z\right).$$

In a strongly inductive media vector potential fades as

$$\exp\left(+\frac{|\operatorname{Im}\delta|}{\operatorname{Re}^2\delta + \operatorname{Im}^2\delta}\;\, k_0 z\right).$$

We remind that the *z* coordinate in impedance media takes negative values. The results mean that SEW cannot "go deep" in impedanse media, and may not "escape" in free space. In general, SEW in its distribution in dielectric layer as "creeps" along the Earth's surface, not looking up from her. Therefore it is called SEW. It can be said that the existence of SEW obliged to our choice of characters in exhibitors Fourier images (18)-(21).

Now a simple differentiation of easy to install all components of EM in the field media. As we have informed, in the cylindrical coordinate system, the only non-zero component of magnetic induction

$$B_\varphi = \left(\nabla \times \vec{A}\right)_\varphi = \frac{\partial A_r}{\partial\ z} - \frac{\partial A_z}{\partial\ r}.$$

We have $B_\varphi = B_y$, and vector potential has only one non-zero component A_z. It is therefore in our legend horizontal component of magnetic induction will

$$B_y(R,z) = -\frac{\partial A_z(R,z)}{\partial\ R}. \qquad (41)$$

Because at the border $z = 0$ horizontal magnetic induction components are equal, then it follows that permanent multipliers K_0 and K in (37) and (38) equal to each other. When finding all non-zero component of the EM field to differentiate on *R* just exponential multiplier. If you differentiate before exponential multipliers, you will see small components $1/\chi\ R$, you can delete without prejudice. Clearly write the components EM will not fields, they are listed in our article [13]. Here write the absolute values of the components fields. So, for non-zero components of the magnetic field *H* in impedance media get:

$$\left|H_y(r,z)\right| = K\frac{1}{\sqrt{r}}\exp\left(-k_0\,\mathrm{Re}\,\delta\ \left|\mathrm{Im}\,\delta\right|\ r + \frac{k_0\ \left|\mathrm{Im}\,\delta\right|}{\left|\delta\right|^2}\ z\right). \qquad (42)$$

And for free space:

$$\left|H_{0y}(r,z)\right| = K\frac{1}{\sqrt{r}}\exp\left(-k_0\,\mathrm{Re}\,\delta\ \left|\mathrm{Im}\,\delta\right|\ r - k_0\ \left|\mathrm{Im}\,\delta\right|\ z\right). \qquad (43)$$

Similarly, for non-zero values component of the electric field *E* find:

$$\left|E_z(r,z)\right| = -\mu_0\ c\ \left|\delta\right|^2\ \left|H_y(r,z)\right|, \qquad (44)$$

$$\left|E_r(r,z)\right| = -\mu_0\ c\ \left|\delta\right|\ \left|H_y(r,z)\right|, \qquad (45)$$

$$\left|E_{0z}(r,z)\right| = -\mu_0\ c\ \left|H_{0y}(r,z)\right|, \qquad (46)$$

$$\left|E_{0r}(r,z)\right| = -\mu_0\ c\ \left|\delta\right|\ \left|H_{0y}(r,z)\right|. \qquad (47)$$

V. Conclusion

Installed the Fourier images of vector potential for double-layer media. Vector potentials themselves are expressed as integrated Sommerfeld integrals. It has been established that these integrals can be calculated in the wave zone for heavily inductive underlying the Earth's surface. The results are expressed in terms of real, measurable value is the surface impedance.

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