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Comparison of Density of States of Fermi Gas and Bose-Einstein Condensate in Single Valence Electron Bulk Metal Elements

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Abstract- The density of states of Fermi gas and density of states of Cooper pairs that form Boson Condensate have been compared. The derived density of states of Fermi gas is elucidated using literature in a cube of size L [1]. In this line, Cooper pairs are highly coherent and responsible for super conductivity[2]. The Cooper pairs form Boson-Einstein condensate of spin quantum number s-1[3], we obtained its density of states. The ratio of Cooper pair density of states to Fermi gas density of states has been derived. To obtain the relation between the two densities of states, the condition of high coherent Cooper pairs in Bose-Einstein condensate has been taken in to account. In addition, the fact that Fermions do not take same quantum states was employed while highly coherent Cooper pairs with s-1 do take same quantum states for they are Boson particles.

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I. Introduction

Under normal condition above transition temperature (*Tc*)electrons are Fermions and they are governed by Fermi-Dirac statistics. Fermions obey Paul's exclusion principle. To this end, no two electrons occupy same quantum state. At least two electrons possess different spin quantum numbers (s) even if their principal quantum number (n) and angular momentum quantum number (l) same. Spin quantum number of electron is½. Its degeneracy is given by: *2*s + 1 = 1; 2*½ + 1 = 2 or ½,-½.* In general each electron is described by distinct quantum numbers n, l, and s in an atom[4]. For an isolated atom these quantum numbers are obtained by solving the Schrodinger equation.

In Cartesian coordinates system the time independent Schrodinger equation is given by:

$$-\frac{\hbar^2}{2m}\nabla^2\Psi(x,y,z)+V(x,y,z))\Psi(x,y,z)=E\Psi(x,y,z) \qquad (1)$$

The case is different for bulk metal elements. In such case, the electrons are free to move inside the bulk metal elements up to the surface of the metal. Under this condition interaction potential energy is constant and it can be taken as a reference (V → 0). In other words the valence electrons are not interacting with the atoms in the bulk metal and become the Fermi gas. Using this assumption the Schrodinger equation that describes the Fermi gas is reduced to;

$$-\frac{\hbar^2}{2m}\nabla^2\Psi(x,y,z)=E_k\Psi(x,y,z\,) \qquad (2)$$

If we take the electrons as a Fermi gas in side cubic metal of size L the energy of an electron of mass m is only quantized kinetic that can be described by principal quantum number *n* or wave number *k*.

The normalized wave function over dimension of the cube of solution of eq.2 is given by;

$$\Psi_{k_xk_yk_z}(x,y,z)=\left(\frac{8}{L^3}\right)^{1/2}sin(k_xx)sin(k_yy)sin(k_zz) \qquad (3)$$

Where,

$$k_x=\frac{p_x}{\hbar},\quad k_y=\frac{p_y}{\hbar}\,and k_z=\frac{p_z}{\hbar} \qquad (4)$$

Energy of the electron *En* can be put as function of wave number kas;

$$E_k=\frac{\hbar^2\boldsymbol{k}^2}{2m}=\frac{\hbar^2(k_x^2+k_y^2+k_z^2)}{2m} \qquad (5)$$

Valance energy of individual atom is discrete. However, when the number of metal atoms increases energy levels are getting overlap. In this regard, for macroscopic cubic box of size *L* the energy of electrons is continuous function of wave number *k* throughout the bulk metal.

Normally two valence electrons of distinct atoms in the metal repel each other. Under superconductive state electron and electron come together and form highly coherent pair. Attraction between electrons pair is resulting from the virtual exchange of phonons that dominates the columbic repulsive force[5]. This electrons pair is called Cooper pair after the theory of super conductivity of J. Bardeen, L.N. Cooper, and J.R. Schrieffer (BCS). The theory of BCS is regarded as one of the most land marking achievements in many body particle physics [6]. Cooper pair is highly coherent pair of delocalized electrons in superconducting state. The pairs are free to move as a unit in the superconductive material.

Mass of a Cooper pair m_c is twice mass of an electron. In bulk macroscopic cubic metal of size *L*and the motion of Cooper pair is described by (eq.2).Note that;

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$$m_c = 2m \tag{6}$$

Under this condition energy of Cooper pair is;

$$E_{kc} = \frac{\hbar^2 \boldsymbol{k}_c^2}{2m_c} = \frac{\hbar^2(k_{xc}^2 + k_{yc}^2 + k_{zc}^2)}{2m_c} \tag{7}$$

The Cooper pair momentum is vector sum of momentum of individual anti symmetric electrons that exchange momentum of phonons in the bulk metal crystal.

$$\hbar k_c = \hbar k_1 + \hbar k_2 \tag{8}$$

Below the transition temperature (*Tc*) the Cooper pairs move in the bulk crystalas a single particle and cause superconductivity.

Superconductivity is property of some materials below the transition temperature[7]. Electrons form symmetric highly coherent pairs and results in integral[8] spin s-1 particle of double mass and double charge of electron. Columbic repulsion force between the pair of electrons is balance by phonon interactions with in the crystal. Under this condition there is no more rotation of the valance electrons around the nucleus. Hence in this case rotational angular momentum becomes zero. As the result angular momentum quantum number *l = 0* and principal quantum number *n = 1*.That is all electrons reside on the ground state. Under this circumstance only the degeneracy of spin quantum number s-1 gives magnetic spin quantum numbers *1, 0, -1; where* its degeneracy number given by *2*1 + 1 = 3.*

II. Density of States of Fermi Gas in Cube Metal Elements of Size L

In wave number space representation the total numbers N_s of individual electron is calculated as follows. Volume of single state (*Vss*) in cube of size L is given by;

$$V_{ss} = \frac{\pi^3}{L^3} \tag{9}$$

And volume of sphere (V_{sp})in k- space is;

$$V_{sp} = \frac{4\pi k^3}{3} \tag{10}$$

Then, the number of filled states *NsF* of Fermi gas in a sphere of radius *k* becomes;

$$N_{sF} = \frac{1}{8}(2 * 1/2 + 1)\frac{V_{sp}}{V_{ss}} = \frac{L^3 k^3}{3\pi^2} \tag{11}$$

The number of electron states *D(k)dk* of Fermi gas between wave number *k* and *k + dk* is defined as;

$$D(k) = \frac{dN_{sF}}{dk} = \frac{L^3 k^2}{\pi^2} \tag{12}$$

Since energy of an electron in the in wave number k representation is given by;

$$E = \frac{\hbar^2 k^2}{2m} \tag{13}$$

Now, combining eq. (11) and (13) the number of field states of Fermi gas as function of energy takes the form;

$$N_{sF} = \frac{1}{8}(2 * 1/2 + 1)\frac{V_{sp}}{V_{ss}} \tag{14a}$$

$$N_{sF} = \frac{L^3}{3\pi^2}\left(\frac{2mE}{\hbar^2}\right)^{3/2} \tag{14b}$$

Then, the number of electron states of Fermi gas between energy *E* and *E + dE* obtained from eq.(14b) as follows;

$$D_F(E) = \frac{dN_{sF}}{dE} = \frac{L^3}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} E^{1/2} \tag{15}$$

III. Density of States of Bose-Einstein Condensate in Cube Metal Elements of Size L

Below *Tc* the highly coherent electrons pairs that are known as Cooper pairs form Bose-Einstein condensate. For Cooper pairs the number of filled states in the sphere of radius *k* are directly obtained using s = *1* and in eq. (13) and (14). Then, the number of Cooper pairs electrons filled states *Nsc* for Bose-Einstein condensate is given by;

$$N_{sc} = \frac{1}{8}(2 * 1 + 1)\frac{V_{sp}}{V_{ss}} \tag{16}$$

Using eq. (9) and (10) it follows;

$$N_{sc} = \frac{L^3 k^3}{2\pi^2} \tag{17}$$

In an analogous way, the number of Cooper pairs states D(k)dk of Bose-Einstein condensate between wave number k and k + dk becomes;

$$D_C(k) = \frac{dN_{sc}}{dk} = \frac{3L^3 k^2}{\pi^2} \tag{18}$$

Finally, the result of number of Cooper pairs states in eq. (16) as function of energy takes the form;

$$D_C(E) = \frac{dN_{sC}}{dE} = \frac{1}{2}\left(\frac{2m_C}{\hbar^2}\right)^{3/2}\frac{L^3 E^{1/2}}{\pi^2} \tag{19}$$

Comparison of density of states of Cooper pairs and density of states of Fermi gas states represented by eq.(15)&(19)gives the following result;

$$D_C(E) = 2^{3/2} D_F(E) \tag{20}$$

Eq. (19) shows density of Cooper pairs states are more than density of Fermi gas states in a cube of metal elements of same size. In this regard, within same volume of metal elements, we have gotten more charge carriers below *Tc* in the case of highly coherent Cooper pairs than Fermi gas. To this end, thermal contraction of the metal elements and highly coherence of Cooper pairs provides more density of states in the metal elements below *Tc*.

IV. Conclusion

The density of states of Fermi gas and density of states of highly coherent Cooper pairs in metal elements of size L have been analyzed. In this regard, the fact of s-½ and s-1 has been used for Fermi gas and Cooper pairs that form Bose-Einstein condensate, respectively.We solved the Schrödinger equation and obtained the expression of energy for both particles. At low temperature, in the absence of rotational motion of Fermi gas and Cooper pairs all particles are in the ground state and possess only translational momentum that exhibit continues energy in the metal elements. For the considered bulk metal elements density of states of both Fermi gas and highly coherent Cooper pairs are found to be continuous function of energy.

Comparison of density of states has been done. The ratio of density of states of highly coherent Cooper pairs to Boson condensate is obtained to be $2^{3/2}$. Cooper pairs are formed only below transition temperature. The derivation has been taking in to account this assumption. Therefore the dominating density of state is that of highly coherent Cooper pairs that form Boson Condensate.

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