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Effect of Non-Inertial Acceleration on Thermal Convection in an Anisotropic Porous Medium with Temperature-Dependent Darcy and Brinkman Frictions

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I. Introduction

Thermal convection in a rotating porous medium is a phenomenon relevant to many fields. It has various applications in geophysics, food processing and engineering and nuclear reactors. Many authors have investigated the effect of external constraint like rotation on convection in a porous medium. Cellular convection in a rotating fluid through porous medium was studied by Rudraiah and Rohini (1975). Rudraiah and Srimani (1976) investigated thermal convection in a rotating fluid through a porous medium. Stability of finite amplitude and overstable convection of conducting fluid through fixed porous bed was studied by Rudraiah and Vortmeyer (1978). Palm and Tyvand, (1984) investigated thermal convection in a rotating porous layer. Effect of Coriolis force and non-uniform temperature gradient on the Rayleigh-Bénard convection was established by Rudraiah and Chandna (1985). Jou and Liaw (1987) studied the thermal convection in a porous medium subject to transient heating and rotation. Vadasz, 1993, 1994, 1997, 1998a, 1998b) extensively studied the flow through a porous medium with rotational effects like three dimensional free convection in a long rotating porous box, stability of free convection in a narrow porous layer subject to rotation, stability of free convection in a rotating porous layer distant from the axis of rotation, flow in rotating porous media, Coriolis effect on gravity-driven convection in a rotating porous layer heated from below and free convection in a porous media. Transition and chaos for free convection in a rotating porous layer was studied by Vadasz and Olek (1998). Straughan (2001) established a sharp nonlinear stability threshold in rotating porous convection. Govender and Vadasz (2002) made a moderate time linear study of moderate Stephan number convection in rotating mushy layers. Riahi (2003, 2006) studied stationary and oscillatory modes of flow instability in a rotating porous layer during alloy solidification and non linear convection in a rotating mushy layer. Khiri (2004) analyzed the Coriolis effect on convection for a low Prandtl number fluid. Govender (2006) studied the effect of anisotropy on stability of convection in a rotating porous layer distant from the center of rotation. Riahi (2007a, 2007b) analyzed the inertial effects on rotating flow in a porous layer and inertial and coriolis effects on oscillatory flow in a horizontal dendrite layer. Combined effect of thermal modulation and rotation on the onset of stationary convection in a porous layer was studied by Malashetty and Mahantesh Swamy (2007). Govender and Vadasz (2007) studied the effect of mechanical and thermal anisotropy on the stability of gravity driven convection in rotating porous media in the presence of thermal non-equilibrium. Effect of temperature modulation on the onset of Darcy convection in a rotating porous medium was studied by Bhadauria (2008). Linear stability of solutal convection in rotating solidifying mushy layers with permeable mush-melt interface was established by Govender (2008).

Most of the above investigators have studied convection in a low-porosity, rotating, and isotropic porous medium with constant viscosity. Temperature-dependence of viscosity gives rise to temperature-dependent Darcy and Brinkman frictions. Patil and Vaidyanathan (1983) analyzed setting up of convection currents in a rotating porous medium under the influence of variable viscosity. Richardson and Straughan (1993) studied the non-linear stability and the Brinkman effect on convection with temperature dependent viscosity (linear dependence) in a porous

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medium. Effect of radiation on non-Darcy free convection from a vertical cylinder embedded in a fluid-saturated porous medium with a temperature-dependent viscosity was studied by El-Hakiem and Rashad (2007). Vanishree and Siddheshwar (2010) investigated the effect of rotation on thermal convection in an anisotropic porous medium with temperature-dependent viscosity. Siddheshwar *et. al.* (2012) studied the heat transport in Bénard -Darcy convection with g-jitter and thermo-mechanical anisotropy in variable viscosity liquids. A good account of convection problems in porous media is given in Vafai and Hamid (2000), Ingham and Pop (2002) and Nield and Bejan (2006), Dullien (2012), Vadasz (2015).

The object of this paper is to study the effect of non-inertial acceleration on mono-diffusive convection in a high-porosity, anisotropic porous medium with temperature-dependent Darcy and Brinkman frictions.

II. Mathematical Formulation

Consider a horizontal porous layer of infinite extent occupied by a Boussinesquian fluid with temperature dependent viscosity, confined between stress free isothermal boundaries at $z = 0$ and $z = d$ at which the temperatures are T_0 and T_1 respectively, which is kept rotating at constant rate. Let Ω denote the angular velocity of rotation. The porous medium is assumed to have high porosity and hence the fluid flow is governed by Brinkman model with effect of Coriolis force and centrifugal acceleration. An appropriate single-phase heat transport equation is chosen with effective heat capacity ratio and effective thermal diffusivity. Thus the governing equations for the Rayleigh-Bénard situation in a fluid with non-Boussinesq effect occupying a rotating porous layer for temperature dependent viscosity are:

Conservation of mass

$$\nabla.\vec{q}=0, \tag{2.1}$$

Conservation of linear momentum

$$\frac{\rho_R}{\Phi}\frac{\partial\vec{q}}{\partial t}=-\nabla p+\rho\vec{g}-\mu_f\mathbf{k}.\vec{q}+\mu_e\nabla^2\vec{q} + 2\frac{\rho_R}{\Phi}\left(\vec{q}\times\vec{\Omega}\right)+\nabla.\left[\mu_p\left(\nabla\vec{q}+\nabla\vec{q}^{Tr}\right)\right], \tag{2.2}$$

Conservation of energy

$$\gamma\frac{\partial T}{\partial t}+\vec{q}.\nabla T=\chi_{Tv}\left[\eta\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\frac{\partial^2 T}{\partial z^2}\right], \tag{2.3}$$

Equation of state

$$\rho=\rho_R\left[1-\alpha\left(T-T_0\right)\right], \tag{2.4}$$

$$\mu_f(T)=\mu_1\left[1-\Gamma_1\left(T-T_0\right)-\Gamma_2\left(T-T_0\right)^2\right], \tag{2.5}$$

$$\mu_p(T)=\mu_2\left[1-\Gamma_1\left(T-T_0\right)-\Gamma_2\left(T-T_0\right)^2\right], \tag{2.6}$$

where

$$p=p^*-\frac{\rho_R}{2\Phi}\nabla\left(\left|\vec{\Omega}\times\vec{r}\right|^2\right).$$

We have neglected the inertial acceleration term in Eq. (2.2) since we are making a linear stability analysis of convection.

a) Basic state

The basic state of the liquid being quiescent is described by

$$\frac{\partial(\;)}{\partial t}=0,\ \vec{q}_b=(0,0,0),\ T=T_b(z),\quad \rho=\rho_b(z),\ \mu_f=\mu_{f_b}(z),\ \mu_p=\mu_{p_b}(z). \tag{2.7}$$

The temperature T_b, pressure p_b and density ρ_b satisfy

$$\frac{dp_b}{dz}=-\rho_b g, \tag{2.8}$$

$$\frac{d^2T_b}{dz^2}=0 \tag{2.9}$$

$$\rho_b=\rho_R\left[1-\alpha(T_b-T_0)\right], \tag{2.10}$$

$$\mu_{f_b} = \mu_1\left[1-\Gamma_1(T_b-T_0)-\Gamma_2(T_b-T_0)^2\right], \quad (2.11)$$

and

$$\mu_{p_b} = \mu_2\left[1-\Gamma_1(T_b-T_0)-\Gamma_2(T_b-T_0)^2\right]. \quad (2.12)$$

Solving Eq. (2.9) for T_b using the boundary conditions

$$T_b = T_0 \text{ at } z=0,$$

$$T_b = T_1 \text{ at } z=1,$$

we get

$$T_b - T_0 = -\beta_1 z, \quad (2.13)$$

$$\rho_b = \rho_R\left[1+\alpha\beta_1 z\right], \quad (2.14)$$

$$\mu_{f_b} = \mu_1\left[1+\Gamma_1\beta_1 z-\Gamma_2\beta_1^2 z^2\right], \quad (2.15)$$

and $$\mu_{p_b} = \mu_2\left[1+\Gamma_1\beta_1 z-\Gamma_2\beta_1^2 z^2\right]. \quad (2.16)$$

where

$$\beta_1 = \frac{T_0-T_1}{d} > 0.$$

b) Linear stability analysis

Let the basic state be disturbed by an infinitesimal thermal perturbation. We now have

$$\vec{q} = \vec{q}_b + \vec{q}', T_b = T_b(z) + T', \quad p_b = p_b(z) + p', \rho_b = \rho_b(z) + \rho', \\ \mu_f = \mu_{f_b}(z) + \mu'_f, \ \mu_p = \mu_{p_b}(z) + \mu'_p. \quad (2.17)$$

The prime indicates that the quantities are infinitesimal perturbations. Substituting Eq. (2.17) into Eqs. (2.1)-(2.3), and using the basic state solution, we get the linearized equations governing the infinitesimal perturbations in the form:

$$\nabla.\vec{q}' = 0, \quad (2.18)$$

$$\frac{\rho_R}{\Phi}\left[\frac{\partial \vec{q}'}{\partial t}\right] = -\nabla p' + \alpha\rho_R g T'\hat{k} + 2\frac{\rho_R}{k_v}\left(\vec{q}'\times\vec{\Omega}\right) \\ -\mu_{f_b}\,\mathbf{k}.\vec{q}' + \nabla\mu_{p_b}.\left(\nabla\vec{q}' + \nabla\vec{q}'^{Tr}\right) + \mu_{p_b}\nabla^2\vec{q}', \quad (2.19)$$

$$\gamma\frac{\partial T'}{\partial t} = \beta w' + \chi_{Tv}\left[\eta\left(\frac{\partial^2 T'}{\partial x^2} + \frac{\partial^2 T'}{\partial y^2}\right) + \frac{\partial^2 T'}{\partial z^2}\right]. \quad (2.20)$$

Operating curl twice on Eq. (2.19), to eliminate pressure, we get

$$-\frac{\rho_R}{\Phi}\frac{\partial}{\partial t}\left(\nabla_1^2 w'\right) = -\alpha\rho_R g\nabla_1^2 T' - \mu_{p_b}\nabla^4 w' - 2\frac{\partial\mu_{p_b}}{\partial z}\nabla^2\left(\frac{\partial w'}{\partial z}\right) \\ + \frac{\mu_{f_b}}{k_v}\nabla_1^2 w' + \frac{\mu_{f_b}}{k_v}\frac{1}{\varepsilon}\frac{\partial^2 w'}{\partial z^2} + \frac{1}{k_v}\frac{1}{\varepsilon}\frac{\partial\mu_{f_b}}{\partial z}\frac{\partial w'}{\partial z} \\ + \frac{\partial^2\mu_{p_b}}{\partial z^2}\left[\nabla_1^2 w' - \frac{\partial^2 w'}{\partial z^2}\right] + 2\frac{\rho_R}{\Phi}\Omega\frac{\partial\zeta}{\partial z}, \quad (2.21)$$

where

$\zeta = \left(\frac{\partial v'}{\partial x} - \frac{\partial u'}{\partial y}\right)$ is the z-component of vorticity, $\vec{\omega}' = \nabla \times \vec{q}'$.

Now the equation for ζ can be obtained by differentiating x and y components of Eq. (2.17) partially w.r.t. y and x respectively and then subtracting the resulting equations from one another, which is

$$\frac{\rho_R}{\Phi}\frac{\partial \zeta}{\partial t} = 2\frac{\rho_R}{\Phi}\Omega\frac{\partial w'}{\partial z} - \frac{\mu_{fb}}{k_v}\frac{1}{\varepsilon}\zeta + \frac{\partial \mu_{Pb}}{\partial z}\frac{\partial \zeta}{\partial z} + \mu_{pb}\nabla^2\zeta. \tag{2.22}$$

We now non-dimensionalize Eqs. (2.20)-(2.22) using the following definitions,

$$(x^*, y^*, z^*) = \left(\frac{x}{d}, \frac{y}{d}, \frac{z}{d}\right), w^* = \frac{w'}{\left(\chi_v/d\right)}, T^* = \frac{T'}{\beta d}, \zeta^* = \frac{\zeta}{\left(\chi_v/d^2\right)}, t^* = \frac{t}{\left(d^2/\nu\right)}. \tag{2.23}$$

Substituting Eq. (2.23) along with Eqs. (2.11) and (2.12) in Eqs (2.20)-(2.22), we get (after dropping the asterisks)

$$\begin{aligned}\frac{\partial}{\partial t}(\nabla^2 w) = {} & R\nabla_1^2 T + \Lambda(1+V_1 z - V_2 z^2)\nabla^4 w - 2\Lambda(V_1 - 2V_2 z)\nabla^2\left(\frac{\partial w}{\partial z}\right) \\ & - Da^{-1}(1+V_1 z - V_2 z^2)\left(\nabla_1^2 w + \frac{1}{\varepsilon}\frac{\partial^2 w}{\partial z^2}\right) - \frac{Da^{-1}}{\varepsilon}(1+V_1 z - V_2 z^2)\frac{\partial^2 w}{\partial z^2} \\ & - \frac{Da^{-1}}{\varepsilon}(V_1 - 2V_2 z)\frac{\partial w}{\partial z} - \sqrt{Ta}\frac{\partial \zeta}{\partial z} + 2\Lambda V_2\left(\nabla_1^2 w - \frac{\partial^2 w}{\partial z^2}\right),\end{aligned} \tag{2.24}$$

$$\begin{aligned}\frac{\partial \zeta}{\partial t} = {} & \sqrt{Ta}\frac{\partial w}{\partial z} - \frac{Da^{-1}}{\varepsilon}(1+V_1 z - V_2 z^2)\zeta - \Lambda(V_1 - 2V_2 z)\frac{\partial \zeta}{\partial z} \\ & + \Lambda(1+V_1 z - V_2 z^2)\nabla^2\zeta,\end{aligned} \tag{2.25}$$

$$Pr\frac{\partial T}{\partial t} = w + \left[\eta\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \frac{\partial^2 T}{\partial z^2}\right], \tag{2.26}$$

The dimensionless groups that appear are *R*, Thermal Rayleigh number, ε, mechanical anisotropy parameter, η, thermal anisotropy parameter, Da^{-1}, inverse Darcy number, Λ, Brinkmann number, *Pr*, Prandtl number, *Ta*, Taylor number, V_1, linear variable viscosity parameter and V_2, quadratic variable viscosity parameter. The Eqs. (2.24)-(2.26) are three equations in three unknowns *w*, ζ and *T*.

Eqs. (2.24)-(2.26) are to be solved subject to the conditions

$$w = \nabla^2 w = D\zeta = T = 0 \text{ at } z = 0,1. \tag{2.27}$$

The infinitesimal perturbations *w*, *T* and ζ are assumed to be periodic waves and hence these permit normal mode solutions in the form (see Chandrasekhar 1961)

$$\begin{bmatrix} w \\ \zeta \\ T \end{bmatrix} = e^{\sigma t}\begin{bmatrix} w(z) \\ \zeta(z) \\ T(z) \end{bmatrix} e^{i(lx+my)} \tag{2.28}$$

where the imaginary part of σ is the frequency, *w*(*z*), ζ(*z*) and *T*(*z*) are the amplitudes and l and m are the horizontal components of the wave number such that $a^2 = l^2 + m^2$. These satisfy the boundary conditions of Eqs. (2.27). Substituting Eq. (2.28) into Eqs. (2.24)-(2.26), we get

$$\Lambda(1+V_1z-V_2z^2)\left(D^2-a^2\right)^2 w-\ \sigma\left(D^2-a^2\right)w-\ Ra^2T$$
$$-2\Lambda(V_1-2V_2z)\left(D^2-a^2\right)Dw+Da^{-1}(1+V_1z-V_2z^2)a^2w$$
$$-\frac{Da^{-1}}{\varepsilon}(1+V_1z-V_2z^2)D^2w-\frac{Da^{-1}}{\varepsilon}(V_1-2V_2z)Dw \tag{2.29}$$
$$-\sqrt{Ta}\frac{d\zeta}{dz}-2\Lambda V_2\left(D^2+a^2\right)w=0,$$

$$\Lambda(1+V_1z-V_2z^2)\left(D^2-a^2\right)\zeta-\sigma\zeta+\sqrt{Ta}Dw$$
$$-\frac{Da^{-1}}{\varepsilon}(1+V_1z-V_2z^2)\zeta+\Lambda(V_1-2V_2z)D\zeta=0, \tag{2.30}$$

$$\left(D^2-\eta a^2\right)T-Pr\sigma T+w=0, \tag{2.31}$$

where σ is in general complex and $D=\frac{d}{dz}$. For marginal stability we take σ to be purely imaginary and discuss both stationary and oscillatory convection.

III. Application of Galerkin Variant of Weighted-Residuals Technique

Eqs. (2.29)-(2.31) are solved using the Galerkin variant of weighted -residuals technique (Finlayson, 1972). This method gives the general results on the eigen value of the problem using the trial functions for the lowest eigen value. We obtain an approximate solution of the differential equations with the given boundary conditions by choosing trial functions for velocity and temperature perturbations that may satisfy the boundary conditions but may not exactly satisfy the differential equations. This leads to residuals when the trial functions are substituted into the differential equations. The Galerkin variant of weighted -residuals method requires the residual to be orthogonal to each individual trial function. In the Galerkin variant of weighted -residuals procedure, we expand the velocity and temperature by

$$w(z,t)=\sum A_i(t)w_i(z),$$
$$\zeta(z,t)=\sum B_i(t)\zeta_i(z), \tag{3.1}$$
$$T(z,t)=\sum C_i(t)T_i(z),$$

where $w_i(z),\zeta_i(z)$ and $T_i(z)$ are trial functions that have to satisfy the boundary conditions. For the purpose of illustration we present below the single-term Galerkin variant of weighted -residuals technique.

Multiplying Eqs. (2.29)-(2.31) by *w*, ζ and *T* respectively and integrating the resulting equations by parts with respect to *z* between 0 and 1 and taking $w=Aw_1, T=BT_1$ and $\zeta=C\zeta_1$, in which *A*, *B* and *C* are constants and w_1, T_1 and ζ_1 are trial functions that satisfy the boundary conditions, yield the following equations for the Rayleigh number, *R* :

$$R=\frac{(G_3+Pr\sigma E_2)\left[(G_1+\sigma G_2)(G_4-\sigma F_5)+TaF_1D_9\right]}{a^2D_8^2\left(G_4-\sigma F_5\right)}, \tag{3.2}$$

where

$$G_1=\Lambda\left(D_1+a^4D_2-2a^2D_3\right)-2\Lambda\left(D_4-a^2D_5\right)+2\Lambda V_2\left(-D_6+a^2D_7\right)$$
$$+Da^{-1}a^2D_2-\frac{Da^{-1}}{\varepsilon}\left(D_3+D_5\right),$$

$$G_2 = \left(D_6 + a^2 D_7\right), G_3 = E_1 + \eta_1 a^2 E_2, G_4 = \Lambda\left(F_2 - a^2 F_3\right) + \Lambda F_4 - \frac{Da^{-1}}{\varepsilon} F_3,$$

$$D_1 = \left\langle w_1\left(1 + V_1 z - V_2 z^2\right) D^4 w_1 \right\rangle, D_2 = \left\langle w_1\left(1 + V_1 z - V_2 z^2\right) w_1 \right\rangle,$$

$$D_3 = \left\langle w_1\left(1 + V_1 z - V_2 z^2\right) D^2 w_1 \right\rangle, D_4 = \left\langle w_1 \left(V_1 - 2V_2 z\right) D^3 w_1 \right\rangle,$$

$$D_5 = \left\langle w_1 \left(V_1 - 2V_2 z\right) D w_1 \right\rangle, D_6 = \left\langle \left(D w_1\right)^2 \right\rangle, D_7 = \left\langle w_1^2 \right\rangle, D_8 = \left\langle w_1 T_1 \right\rangle,$$

$$D_9 = \left\langle w_1 D\zeta_1 \right\rangle, E_1 = \left\langle \left(D T_1\right)^2 \right\rangle, E_2 = \left\langle T_1^2 \right\rangle, F_1 = \left\langle \zeta_1 D w_1 \right\rangle,$$

$$F_2 = \left\langle \zeta_1\left(1 + V_1 z - V_2 z^2\right) D^2 \zeta_1 \right\rangle, F_3 = \left\langle \zeta_1\left(1 + V_1 z - V_2 z^2\right) \zeta_1 \right\rangle,$$

$$F_4 = \left\langle \zeta_1 \left(V_1 - 2V_2 z\right) D\zeta_1 \right\rangle, F_5 = \left\langle \zeta_1^2 \right\rangle,$$

$\langle \cdots \rangle$ denotes integration with respect to z between z = 0 and z = 1.We note here that R in Eq. (3.2) is a functional and the Euler-Lagrange equations for the extremization of R are Eqs. (2.29)-(2.31).

For stationary convection we set $\sigma = 0$ and then Eq. (3.2) becomes

$$R^s = \frac{G_3\left(G_1 G_4 + Ta\, F_1 D_9\right)}{a^2 G_4 D_8^2}. \tag{3.3}$$

For oscillatory instability we set $\sigma = i\omega$ in Eq.(3.2), which gives

$$R^o = \frac{G_3 G_4 (G_1 G_4 + Ta\, F_1 D_9) + \omega^2 \left(G_1 G_3 F_5^2 - Pr E_2 \left\{ \begin{array}{l} F_1 F_5 D_9 Ta + G_2 G_4^2 \\ + \omega^2 F_5^2 G_2 \end{array} \right\} \right) + i\omega N}{a^2 D_8^2 \left(G_4^2 + \omega^2 F_5^2\right)}, \tag{3.4}$$

where

$$N = \left(G_3 G_4 - \omega^2 Pr E_2 F_5\right)\left(G_2 G_4 - G_1 F_5\right) + \left(G_3 F_5 + Pr E_2 G_4\right)\left(G_1 G_4 + Ta F_1 D_9 + \omega^2 G_2 F_5\right). \tag{3.5}$$

Since R is a real quantity, either $\omega = 0$ (stationary) or $N = 0$ ($\omega \neq 0$, oscillatory). The latter condition, on simplification, yields the frequency of oscillations and the oscillatory Rayleigh number in the form:

$$\omega^2 = \frac{-\left[G_2 G_3 G_4^2 + G_3 F_1 F_5 D_9 Ta + Pr E_2 G_4 \left(G_1 G_4 + Ta\, F_1 D_9\right)\right]}{F_5^2 \left(Pr E_2 G_1 + G_2 G_3\right)}. \tag{3.6}$$

$$R^o = \frac{G_3 G_4 (G_1 G_4 + Ta\, F_1 D_9) + \omega^2 \left(G_1 G_3 F_5^2 - Pr E_2 \left\{ \begin{array}{l} F_1 F_5 D_9 Ta + G_2 G_4^2 \\ + \omega^2 F_5^2 G_2 \end{array} \right\} \right)}{a^2 D_8^2 \left(G_4^2 + \omega^2 F_5^2\right)}, \tag{3.7}$$

In evaluating R^s and R^o we have assumed $w_1 = T_1 = \sin(\pi z)$ and $\zeta_1 = \cos(\pi z)$. The choice of trigonometric sine and cosine as the trial function gives us the results equivalent of the higher-order Galerkin variant of weighted -residuals method.

IV. Low-Porosity Media (Vadasz, 1998) Formulation

Let us consider Eqs. (2.24)-(2.26) in a form that would facilitate arriving at the Vadasz (1998) formulation.

$$Va\frac{\partial T}{\partial \tau} = w + \left[\eta\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \frac{\partial^2 T}{\partial y^2}\right] \tag{4.1}$$

$$\begin{aligned}\frac{\partial}{\partial \tau}(\nabla^2 w) = R_D \nabla_1^2 T + Br_D(1+V_1 z - V_2 z^2)\nabla^4 w - 2Br_D(V_1 - V_2 z)\nabla^2\left(\frac{\partial w}{\partial z}\right) \\ -(1+V_1 z - V_2 z^2)\left(\nabla_1^2 w + \frac{1}{\varepsilon}\frac{\partial^2 w}{\partial z^2}\right) - \frac{1}{\varepsilon}(V_1 - V_2 z)\frac{\partial w}{\partial z} \\ + \sqrt{Va_D}\frac{\partial \varsigma}{\partial z} + 2Br_D V_2\left(\nabla_1^2 w - \frac{\partial^2 w}{\partial z^2}\right),\end{aligned} \tag{4.2}$$

$$\begin{aligned}\frac{\partial \zeta}{\partial \tau} = -\sqrt{Va_D}\frac{\partial w}{\partial z} - \frac{1}{\varepsilon}(1+V_1 z - V_2 z^2)\zeta - Br_D(V_1 - V_2 z)\frac{\partial \zeta}{\partial z} \\ + Br_D(1+V_1 z - V_2 z^2)\nabla^2 \zeta.\end{aligned} \tag{4.3}$$

In the above equations, the following dimensionless quantities are introduced as done by Vadasz (1998):

$\tau = tDa^{-1}$ (scaled dimensionless time)

$Va = PrDa$ (Vadasz number),

$\sqrt{Va_D} = \sqrt{Ta}\,Da$ (Taylor-Vadasz number),

$R_D = RDa$ (Darcy-Rayleigh number) and

$Br_D = \Lambda Da$ (Brinkman-Darcy number).

For a low-porosity media, $Br_D = 0$ and hence Eqs.(4.1)-(4.3) read as:

$$\frac{\partial}{\partial \tau}(\nabla^2 w) = R_D \nabla_1^2 T - (1+V_1 z - V_2 z^2)\left(\nabla_1^2 w + \frac{1}{\varepsilon}\frac{\partial^2 w}{\partial z^2}\right)\zeta\,\frac{1}{\varepsilon}(V_1 - V_2 z)\frac{\partial w}{\partial z} + \sqrt{Va_D}\frac{\partial}{\partial z}, \tag{4.4}$$

$$\frac{\partial \zeta}{\partial \tau} = -\sqrt{Va_D}\frac{\partial w}{\partial z}\zeta - \frac{1}{\varepsilon}(1+V_1 z - V_2 z^2)\ . \tag{4.5}$$

Eq. (4.1) remains the same for low-porosity media. Thus Eqs. (4.1), (4.4) and (4.8) are the governing equations for rotating porous-media convection with mechanical and thermal anisotropies, and also temperature-dependent viscosity. We now move on to obtain the Rayleigh number for stationary (R_D^s) and oscillatory (R_D^o) modes of convection for a constant viscosity fluid occupying an anisotropic porous medium. Assuming $V_1 = V_2 = 0$ in the Eqs. (4.4) and (4.5), we get

$$\frac{\partial}{\partial \tau}(\nabla^2 w) + \nabla_1^2 w + \frac{1}{\varepsilon}\frac{\partial^2 w \zeta}{\partial z^2} = R_D \nabla_1^2 T + \sqrt{Va_D}\frac{\partial}{\partial z}, \tag{4.6}$$

$$\frac{\partial \zeta}{\partial \tau}+\frac{1}{\varepsilon}\zeta=-\sqrt{Va_D}\,\frac{\partial w}{\partial z}. \qquad (4.7)$$

Through routine procedure, we can easily obtain R_D in the form:

$$\frac{R_D}{\pi^2}=\frac{\left(\frac{Va}{\pi^2}\sigma^*+\frac{\delta_2^2}{\pi^2}\right)\left[\left(\frac{\delta^2}{\pi^2}\sigma^*+\frac{\delta_1^2}{\pi^2}\right)\left(\sigma^*+\frac{1}{\varepsilon}\right)+Va_D\right]}{\frac{a^2}{\pi^2}\left(\sigma^*+\frac{1}{\varepsilon}\right)}. \qquad (4.8)$$

In Eq (4.8) the various symbols have the following definition:

$\delta^2=\pi^2+a^2,\ \delta_1^2=\frac{\pi^2}{\varepsilon}+a^2,\ \delta_2^2=\pi^2+\eta a^2$ and σ^* is, in general complex.

The Rayleigh numbers R_D^s and oscillatory R_D^o can be obtained by substituting $\sigma^*=0$ and $\sigma^*=i\omega$ (ω: frequency of oscillations) respectively, and doing some algebra for oscillatory mode. The expressions of R_D^s and R_D^o are:

$$R_D^s=\frac{\varepsilon\delta_2^2\left[\frac{\delta_1^2}{\varepsilon}+Va_D\pi^2\right]}{a^2}, \qquad (4.9)$$

$$R_D^o=\frac{\delta_2^2\delta_1^2\left(\omega^2+\frac{1}{\varepsilon^2}\right)-Va\,\omega^2\left(\delta^2\left(\omega^2+\frac{1}{\varepsilon^2}\right)-Va_D\right)\frac{Va_D}{\varepsilon}\delta_2^2}{a^2\left(\omega^2+\frac{1}{\varepsilon^2}\right)}. \qquad (4.10)$$

The frequency of oscillations is given by the following equation:

$$\omega^2=\frac{(\delta_2^2-\frac{Va}{\varepsilon})Va_D}{\left(\delta^2\delta_2^2+Va\delta_1^2\right)}-\frac{1}{\varepsilon^2}. \qquad (4.11)$$

One may easily check that Eq. (4.8) is, indeed, the expression obtained by Vadasz (1998) when it is assumed that $\varepsilon=\eta=1$(isotropic medium). Further, as pointed out by Vadasz (1998), the condition for oscillatory convection to manifest is

$$Va<\varepsilon\delta_2^2 \text{ and } Va_D>\frac{\delta^2\delta_2^2+Va\delta_1^2}{\varepsilon\left[\varepsilon\delta_2^2-Va\right]}.$$

We are unable to derive a condition for over stability like this for the variable viscosity case because of almost impossible tedious algebra involved.

V. Results and Discussions

In the paper a study is made of the effects of temperature-dependent Darcy and Brinkman frictions, and rigid-body rotation on convection in a fluid saturated anisotropic porous medium at the onset of convection.

The temperature dependence of the Darcy and Brinkman frictions arises due to the viscosity varying with temperature. It is important to note here that viscosity decreases with increase in temperature and this relation is exponential in many situations. However, for small temperature gradients one may take a few terms in the Taylor series expansion of the exponential term. If we take just the linear term it can be easily seen that it leads to the result that variable viscosity leads to stabilization which is quite contrary to the physics. In view of this we need to take at least terms up to the quadratic term in the Taylor series. The exponential term must, in the strictest sense, be taken as it is but with the available computational facility it was impossible to do so and hence the truncation in the Taylor series. It is important to make another observation here in regard to the $\mu-T$ relationship. One might have a doubt that some of the obtained results of the paper which seem incorrect are essentially due to the choice of T_0 (temperature of lower boundary) as reference temperature, rather than T_m (mean temperature of upper and lower boundaries). The appendix demonstrates the effect of the choice of T_m in place of T_0. Quite clearly we observe that a linear $\mu - T$ relationship (see Eqs. (A2)) again leads to the result that the variable viscosity leads to stabilization! In view of this we decided on the appropriateness of Eqs. (2.5) and (2.6) for this variable viscosity convection problem.

With the motivation of control of convection, the following effects on the classical Rayleigh- Bénard problem are considered:

i. porous medium inhibition of convection,
ii. anisotropy of the medium ,
iii. variable viscosity and
iv. Coriolis force.

These four effects are, respectively, represented by the inverse Darcy number Da^{-1}, anisotropy parameters (ε, η), variable viscosity parameters (also called thermo rheological parameters) V_1 and V_2 and the Taylor number *Ta*. The present formulation of the porous media problem for an infinite porous layer with rotation parallel to gravity is based on the Chandrasekhar (1961) formulation of the problem in a clear fluid layer. This formulation involves several assumptions (Knobloch, 1998) - the lateral boundaries are far enough not to influence rotating convection and that the Froude number is quite small. The latter assumption facilitates the restoration of the conduction state as an equilibrium solution. Experimentally, the lateral boundary effect and the centrifugal effect have been shown by Ecke *et al*. (1992) to be quite important but in a theoretical study to keep the problem manageable and focus on Bénard -like situations, it is common place to exclude these effects. The main emphasis of the present study is to consider the effect of temperature-dependent viscosity on the onset of convection via the stationary or oscillatory modes. Before embarking on a discussion of the results depicted by the figs. 1 to 10, we note that as in the case of clear fluid critical convection is always stationary as overstable motion is restricted to very low values of the Prandtl number, when Da^{-1} is quite large. For low values of Da^{-1}, the critical value is stationary or overstable.

Fig. (1a) reveals that the effect of increasing quadratic thermo rheological parameter V_2 is to destabilize the system for small rotation rates and clearly for these rotation rates the effect of varying Da^{-1} is also discernible. We further find from Fig. (1b) that, as the rotation rate is increased, its effect is the same for all values of Da^{-1}. This is due to the fact that large rotations do not allow the internal structure of the porous medium to affect convection. In addition we also observe that at large rotation rates the effect of increasing V_2 is to stabilize convection. This is one surprise result. Such surprising results also arise in the case of very small Da^{-1} with respect to the influence of the anisotropy parameters on the onset of convection. This result may also be due to the fact that the choice of terms up to the quadratic is inadequate in the viscosity-temperature relationship. As mentioned earlier computations beyond what has been done is impossible with the available computers in the department.

Fig. (2a,b) reveals that the effect of increasing Da^{-1} is to increase the cell size at the onset of convection for all rotation rates. This result can be seen from the fact that the wave length is inversely proportional to the wave number.

At small rotation rates up to a particular value of V_2 the effect of increasing ε is to decrease the value of R_c^s and then beyond that particular value the reverse effect is seen. However, at high rotation rates, we see that the effect of increasing ε is to increase R_c^s and η does not have a similar effect. The above results are seen in Fig. (3a, b, c). We have seen that for higher rotation rates the result is similar to that in Fig. (3c).

The effect of increasing V_2 and ε is to decrease the cell size at onset and the reverse effect is seen in the case of increasing η and these are clearly demonstrated in figs. (4a,b,c). The effect of high rotation rates together with the other varying parameters is as seen earlier (see Fig. 4c).

In what follows we discuss the results of oscillatory convection which is possible in the case of small Prandtl numbers for high-porosity media. It is interesting to note that this is different from the case of low-porosity media wherein oscillatory convection may exist for low values of Da^{-1} and depending on other parameters including a scaled Taylor number.

Fig. (5) shows the destabilizing effect of V_2 on R_c^o for high rotation rates. We have included the result of just two rotation rates here as a representative plot of all rotation rates in the case of high-porosity media. Fig. (6) is a result that is opposite to that of stationary convection (see Figs. 2a, b). The frequency of oscillations is, shown in Fig. (7), to increase with increase in *Ta,* and decrease with decrease in Da^{-1}. The effects of increasing ε and η are opposite to each other and the same is shown in Fig. (8).This result is similar to what we saw in the case of stationary

Global Journal of Science Frontier Research (A) Volume XVIII Issue X Version I Year 2018

convection. The surprises in results on comparing the low and high rotation ones are similar to the ones of stationary convection. Comparing Figs. (4c) and (9) we find that the results of increasing ε and η are qualitatively similar for stationary and oscillatory convection. Fig. (10) shows that the frequency of oscillations increases with increase in ε and η.

Due to the restriction of pages we now mention the result of computation on the effect of Λ on R_c^s, R_c^o, a_c^s, a_c^o and ω_c. Computation reveals a surprising result at low and high rotation rates in the case of varying Λ. We find that the effect of Λ on R_c^s and R_c^o are not the same at the extremes of low and high rotation rates. We are unable to reason this out. Clearly at low rotation rates the result of increasing Λ is the same as what we see in the case of no rotation. This aspect of the problem needs further investigation. In spite of such a result in the case of R_c^s, there is no such result in the case of a_c^s. Computation shows that the effect of Λ on R_c^o and a_c^o is similar to the case of stationary convection. In the case of low rotation rate the effect is opposite to that of the high rotation rate and this is true of many other parameters as well, as noted earlier. We find that the effect of increasing Λ, in the case of high rotation rates, is to decrease the frequency of oscillations and the opposite is seen in the case of low rotation rates. The reasoning for the results, as seen in the paper, for high rotation rates is the same as that put forward by Chandrasekhar (1961) for clear fluid and Vadasz (1998) for porous media.

We have not included a comparison of our numerical results with those of Vadasz for constant viscosity fluid occupying rotating porous media of low porosity. It is a fact that our results match quite well with those relevant ones available in the literature.

VI. Conclusions

Oscillatory convection is preferred for small values of the Prandtl number in the case of rotating convection in high-porosity media. At high rates of rotation, the internal structure of the porous medium does not affect convection, and also viscosity destabilizes the system. This is in concurrence with classical results of Chandrasekhar (1961) and Vadasz (1998). In the presence of rotation, the effect of anisotropy parameters may not be the same (in fact some times the effect is opposite) as what is observed in its absence. This result can be easily extracted if one obtains the derivative of the eigen value of the problem with respect to the parameter under question.

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Appendix A

Calculation of critical value of Rayleigh and wave numbers for the viscosity-temperature relationship as taken by Nield (1996)

Nield (1996) assumed a viscosity-temperature relationship in the form

$$\mu_f(T) = \frac{\mu_1}{1 + a(T - T_m)}, \quad \mu_p(T) = \frac{\mu_2}{1 + a(T - T_m)}. \tag{A1}$$

We have presented the $\mu - T$ relationship of Nield (1996) in a way that is in keeping with the requirements of our paper. On using Taylor series expansion in Eq. (A1) and truncating beyond the second term, we get

$$\left. \begin{aligned} \mu_f(T) &= \mu_1\left[1 - \Gamma_1\left(T - T_m\right)\right], \\ \mu_p(T) &= \mu_2\left[1 - \Gamma_1\left(T - T_m\right)\right], \end{aligned} \right\} \tag{A2}$$

where $T_m = T_0 + \frac{\Delta T}{2}$.

We assume the following equation of state:

$$\rho = \rho_R\left[1 - \alpha\left(T - T_m\right)\right] \tag{A3}$$

Adopting the procedure of the main paper with the above relationship we get the following equations in place of Eq. (2.24) and Eq. (2.25):

$$\frac{\partial}{\partial t}\nabla^2 w = R\nabla_1^2 T + \Lambda\left[1+V\left(z+\frac{1}{2}\right)\right]\nabla^4 w + 2\Lambda V\nabla^2\left(\frac{\partial w}{\partial z}\right) - Da^{-1}\left[1+V\left(z+\frac{1}{2}\right)\right]\nabla_1^2 w$$

$$-\frac{Da^{-1}}{\varepsilon}\left[1+V\left(z+\frac{1}{2}\right)\right]\frac{\partial^2 w}{\partial z^2} - \frac{Da^{-1}}{\varepsilon}V\frac{\partial w}{\partial z} - \sqrt{Ta}\frac{\partial \zeta}{\partial z}, \tag{A4}$$

$$\frac{\partial \zeta}{\partial t} = \sqrt{Ta}\frac{\partial w}{\partial z} - \frac{Da^{-1}}{\varepsilon}\left[1+V\left(z+\tfrac{1}{2}\right)\right]\zeta - \Lambda V\frac{\partial \zeta}{\partial z} + \Lambda\left[1+V\left(z+\tfrac{1}{2}\right)\right]\nabla^2\zeta, \tag{A5}$$

where $V = \Gamma_1 \Delta T,$ the variable viscosity parameter for the Nield viscosity-temperature relationship. The boundary conditions are given by Eq. (2.27). Applying the normal mode solution (2.28) into the above equations and applying one-term weighted residual Galerkin method, we get the following equation for R, for both stationary and oscillatory modes in the form:

<u>Stationary convection</u>

$$R^s = \frac{H_3\left(H_1 H_4 + Ta L_1 A_9\right)}{a^2 H_4 I_8^2}. \tag{A6}$$

<u>Oscillatory instability</u>

$$R^o = \frac{H_3 H_4 (H_1 H_4 + Ta L_1 I_9) + \omega^2\left(H_1 H_3 L_5^2 - \Pr J_2\begin{Bmatrix} L_1 L_5 I_9 Ta + H_2 H_4^2 \\ +\omega^2 L_5^2 H_2\end{Bmatrix}\right)}{a^2 I_8^2\left(H_4^2 + \omega^2 L_5^2\right)}, \tag{A7}$$

where

$$\omega^2 = \frac{-\left[H_2 H_3 H_4^2 + H_3 L_1 L_5 I_9 Ta + \Pr J_2 H_4\left(H_1 H_4 + Ta L_1 I_9\right)\right]}{L_5^2\left(\Pr J_2 H_1 + H_2 H_3\right)}, \tag{A8}$$

which is the square of the frequency of oscillations.

$$H_1 = \Lambda\left(I_1 + a^4 I_2 - 2a^2 I_3\right) - 2\Lambda\left(I_4 - a^2 I_5\right) + 2\Lambda V_2\left(-I_6 + a^2 I_7\right)$$

$$+ Da^{-1} a^2 I_2 - \frac{Da^{-1}}{\varepsilon}(I_3 + I_5),$$

$$H_2 = \left(I_6 + a^2 I_7\right), H_3 = J_1 + \eta_1 a^2 J_2, H_4 = \Lambda\left(L_2 - a^2 L_3\right) + \Lambda L_4 - \frac{Da^{-1}}{\varepsilon}L_3,$$

$$I_1 = \left\langle w_1\left[1+V\left(z+\tfrac{1}{2}\right)\right]D^4 w_1\right\rangle, I_2 = \left\langle w_1\left[1+V\left(z+\tfrac{1}{2}\right)\right]w_1\right\rangle,$$

$$I_3 = \left\langle w_1\left[1+V\left(z+\tfrac{1}{2}\right)\right]D^2 w_1\right\rangle, I_4 = \left\langle w_1 D^3 w_1\right\rangle, I_5 = \left\langle w_1 D w_1\right\rangle,$$

$$I_6 = \left\langle (D w_1)^2\right\rangle, I_7 = \left\langle w_1^2\right\rangle, I_8 = \left\langle w_1 T_1\right\rangle, I_9 = \left\langle w_1 D\zeta_1\right\rangle, J_1 = \left\langle (D T_1)^2\right\rangle, J_2 = \left\langle T_1^2\right\rangle,$$

$$L_1 = \langle \zeta_1 D w_1 \rangle, L_2 = \left\langle \zeta_1 \left[1+V\left(z+\tfrac{1}{2}\right)\right] D^2 \zeta_1 \right\rangle, L_3 = \left\langle \zeta_1 \left[1+V\left(z+\tfrac{1}{2}\right)\right] \zeta_1 \right\rangle,$$

$$L_4 = \langle \zeta_1 D \zeta_1 \rangle, L_5 = \left\langle \zeta_1^2 \right\rangle,$$

$$H_1 = \Lambda\left(I_1 + a^4 I_2 - 2a^2 I_3\right) - 2\Lambda\left(I_4 - a^2 I_5\right) + 2\Lambda V_2\left(-I_6 + a^2 I_7\right)$$
$$+ Da^{-1} a^2 I_2 - \frac{Da^{-1}}{\varepsilon}\left(I_3 + I_5\right),$$

$$H_2 = \left(I_6 + a^2 I_7\right), H_3 = J_1 + \eta_1 a^2 J_2, H_4 = \Lambda\left(L_2 - a^2 L_3\right) + \Lambda L_4 - \frac{Da^{-1}}{\varepsilon} L_3,$$

$$I_1 = \left\langle w_1 \left[1+V\left(z+\tfrac{1}{2}\right)\right] D^4 w_1 \right\rangle, I_2 = \left\langle w_1 \left[1+V\left(z+\tfrac{1}{2}\right)\right] w_1 \right\rangle,$$

$$I_3 = \left\langle w_1 \left[1+V\left(z+\tfrac{1}{2}\right)\right] D^2 w_1 \right\rangle, I_4 = \left\langle w_1 D^3 w_1 \right\rangle, I_5 = \langle w_1 D w_1 \rangle,$$

$$I_6 = \left\langle \left(D w_1\right)^2 \right\rangle, I_7 = \left\langle w_1^2 \right\rangle, I_8 = \langle w_1 T_1 \rangle, I_9 = \langle w_1 D \zeta_1 \rangle, J_1 = \left\langle \left(D T_1\right)^2 \right\rangle, J_2 = \left\langle T_1^2 \right\rangle,$$

$$L_1 = \langle \zeta_1 D w_1 \rangle, L_2 = \left\langle \zeta_1 \left[1+V\left(z+\tfrac{1}{2}\right)\right] D^2 \zeta_1 \right\rangle, L_3 = \left\langle \zeta_1 \left[1+V\left(z+\tfrac{1}{2}\right)\right] \zeta_1 \right\rangle,$$

$$L_4 = \langle \zeta_1 D \zeta_1 \rangle, L_5 = \left\langle \zeta_1^2 \right\rangle,$$

$\langle \cdots \rangle$ denotes integration with respect to z between z = 0 and z = 1.

The critical value of R and a obtained from Eqs. (A6) and (A7) are documented in tables 1 and 2 for both stationary and oscillatory modes of convection.

Table 1

Ta	Da^{-1}	ε	η	Λ	V					
					0.0		0.3		0.6	
					R_c^s	a_c^s	R_c^s	a_c^s	R_c^s	a_c^s
10^3	30	1.2	1.2	1.2	2614.2	2.8474	3143.5	2.7169	3689.2	2.6450
10^4					5956.6	4.2027	6040.1	3.7639	6255.1	3.4736
10^4	10				5983.4	4.8652	5885.6	4.3663	5889.8	4.0114
	20	1.2	1.2	1.2	5951.4	4.5069	5931.7	4.0300	6031.0	3.7026
	30				5956.6	4.2027	6040.1	3.7639	6255.1	3.4736
10^4	30	0.8			5619.5	3.9822	5824.8	3.6041	6169.8	3.3648
		1	1.2	1.2	5801.0	4.1029	5930.9	3.6883	6196.9	3.4192
		1.2			5956.6	4.2027	6040.1	3.7639	6255.1	3.4736
10^4	30	1.2	0.8		4577.9	4.4412	4737.1	4.0031	4981.8	3.7101
			1	1.2	5273.8	4.3090	5396.6	3.8708	5627.6	3.5795
			1.2		5956.6	4.2027	6040.1	3.7639	6255.1	3.4736
10^4	30	1.2	1.2	0.8	6053.6	4.6039	6068.4	4.0905	6086.3	3.7504
				1	5987.4	4.3847	6004.4	3.9126	6160.1	3.6001
				1.2	5956.6	4.2027	6040.1	3.7639	6255.1	3.4736

R_c^s **and** a_c^s for different parameters when μ - T relationship is linear.

Table 2

Ta	Da^{-1}	ε	η	Λ	V								
					0.0			0.3			0.6		
					R_c^o	a_c^o	ω_c	R_c^o	a_c^o	ω_c	R_c^o	a_c^o	ω_c
10^3	30	1.	1.2	1.2	5141.3	2.6429	----	7072.6	2.6650	----	9182.5	2.6845	----
10^4		2			5605.3	2.8548	34.229	7636.3	2.8539	----	9833.6	2.8546	----
10^5					8422.4	3.7705	155.42	11187.0	3.7208	138.62	14070.4	3.6750	118.31
10^4	10				5593.5	3.5987	176.66	7537.1	3.5573	167.31	9162.9	3.5182	157.13
	20	1.	1.2	1.2	7001.4	3.7007	165.69	9255.0	3.6545	153.00	11584	3.6113	138.49
	30	2			8422.4	3.7705	155.42	11187.0	3.7208	138.62	14070.4	3.6750	118.31
10^4	30	0.			9975.5	3.8761	141.44	13360.7	3.8279	116.82	16938.9	3.7843	82.281
		8	1.2	1.2	9039.7	3.8158	149.42	12048.2	3.7666	130.36	15203.7	3.7215	105.56
		1			8422.4	3.7705	155.42	11187.0	3.7208	138.62	14070.4	3.6750	118.31
		1.											
		2											
10^4	30	1.	0.8		7742.4	4.0924	132.86	10293.9	4.0218	109.28	12961.5	3.9575	76.662
		2	1	1.2	8088.7	3.9160	142.90	10748.9	3.8575	121.72	16425.9	3.7546	50.442
			1.2		8422.4	3.7705	155.42	11187.0	3.7208	138.62	14070.4	3.6750	118.31
10^4	30	1.	1.2	0.8	6847.6	3.9084	161.64	9270.4	3.8679	149.23	11376.4	3.8308	134.92
		2		1	7634.8	3.7882	158.73	10127.2	3.7882	144.21	12719.8	3.7462	127.10
				1.2	8422.4	3.7705	155.42	11187.0	3.7208	138.62	14070.4	3.6750	118.31

R_c^o, a_c^o **and** ω_c for different parameters when μ - T relationship is linear.

(The broken lines in certain columns indicate that oscillatory convection is not preferred for that particular parameters's combination)

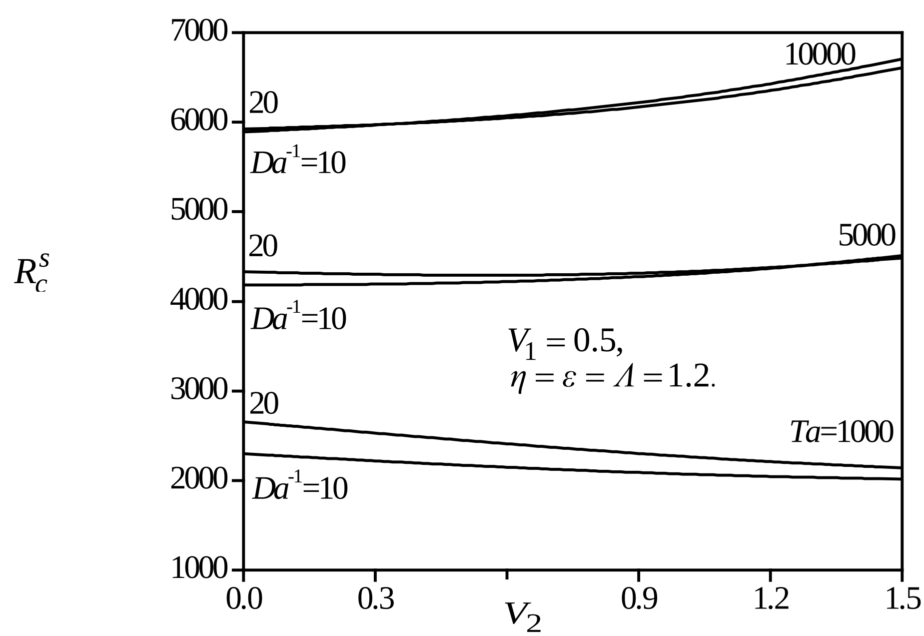

Fig. 1a: Plot of critical Rayleigh number R_c^s (stationary) Vs. quadratic variable viscosity parameter V_2 for different values of porous parameter Da^{-1} and Taylor number Ta.

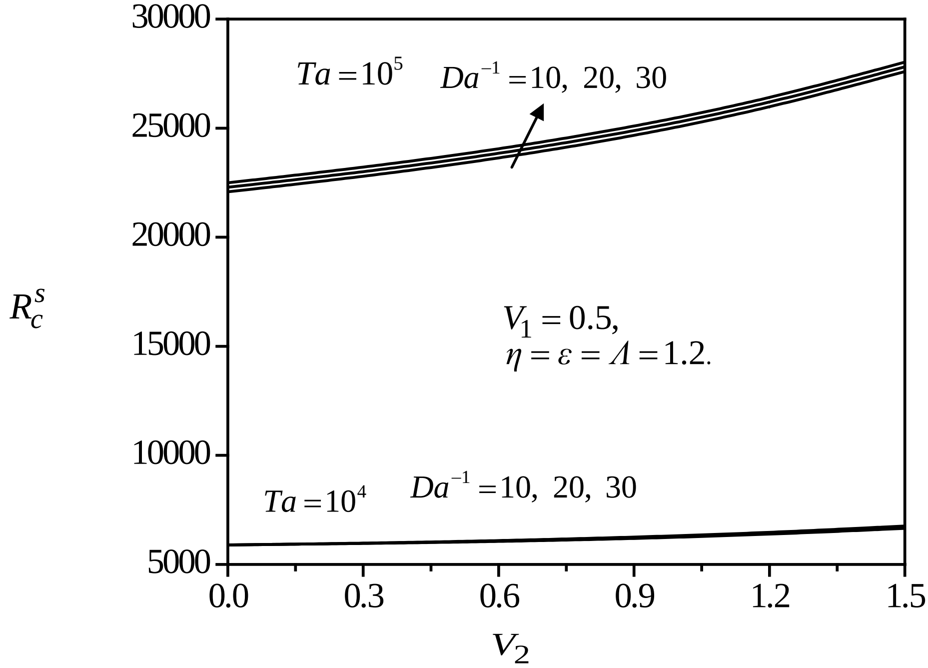

Fig. 1b: Plot of R_c^s (stationary) Vs. V_2 for different values of Da^{-1} and Ta.

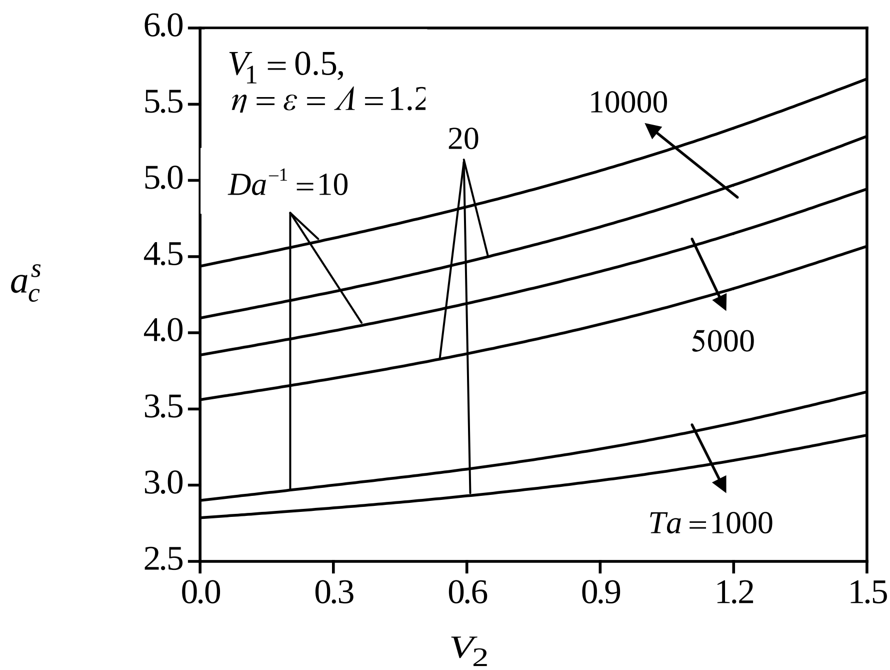

Fig. 2a: Plot of critical wave number a_c^s (stationary) Vs. V_2 for different values of Da^{-1} and Ta

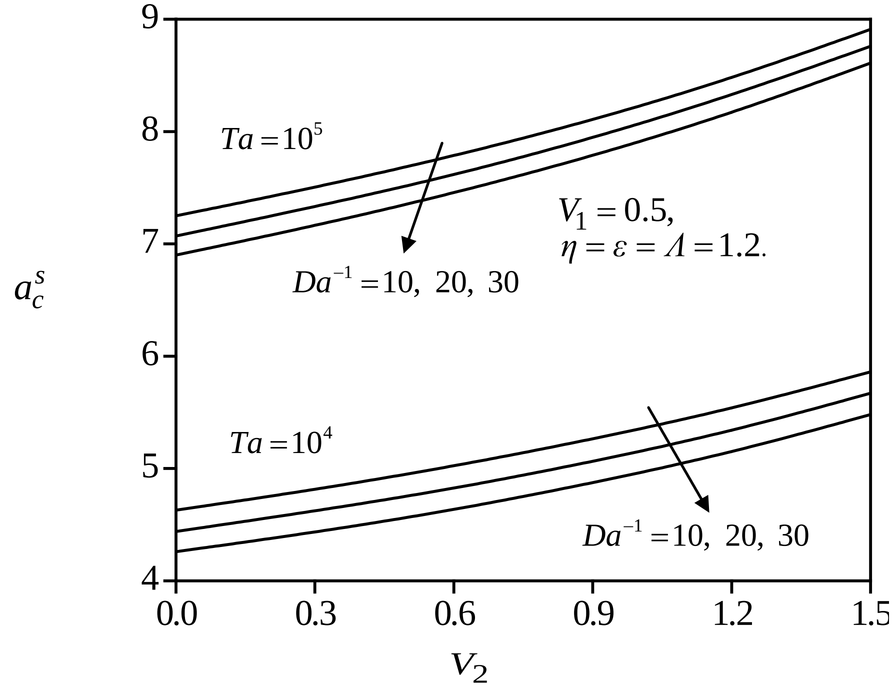

Fig. 2b: Plot of a_c^s (stationary) Vs. V_2 for different values of Da^{-1} and Ta

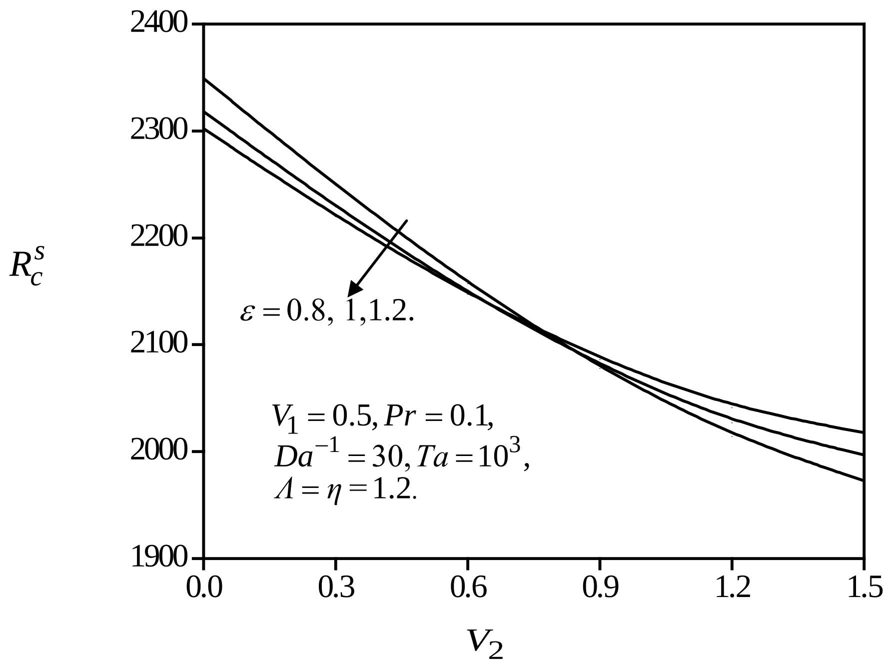

Fig. 3a: Plot of R_c^s Vs. V_2 for different values of ε.

Global Journal of Science Frontier Research (A) Volume XVIII Issue X Version I Year 2018

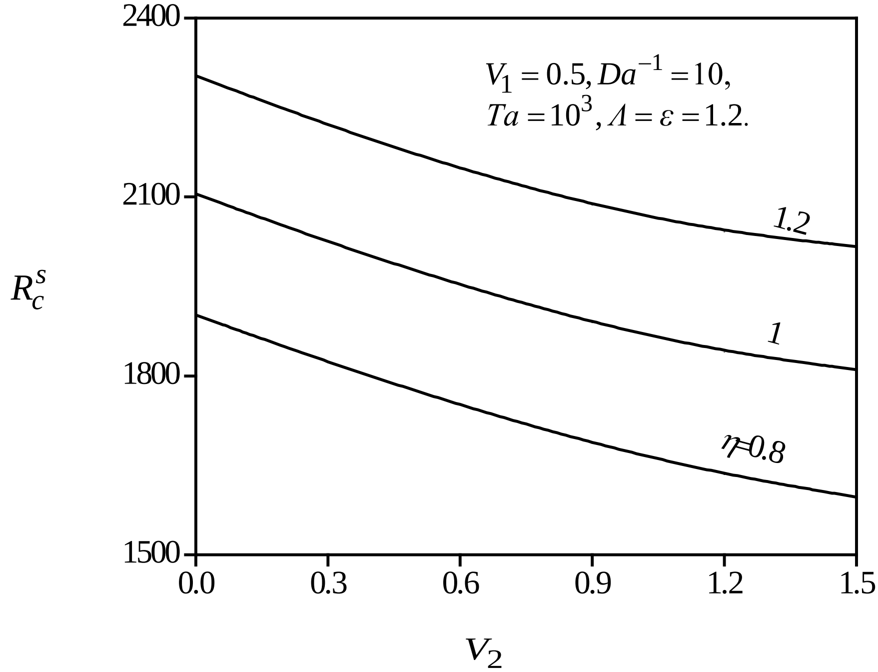

Fig. 3b: Plot of R_c^s Vs. V_2 for different values of η.

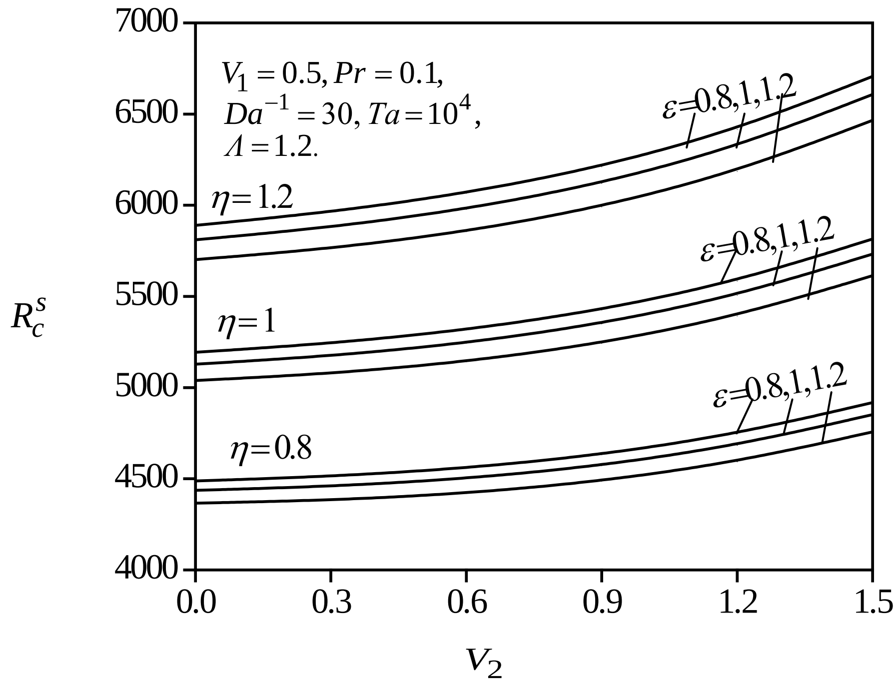

Fig. 3c: Plot of R_c^s Vs. V_2 for different values of thermal and mechanical anisotropy parameters η **and** ε.

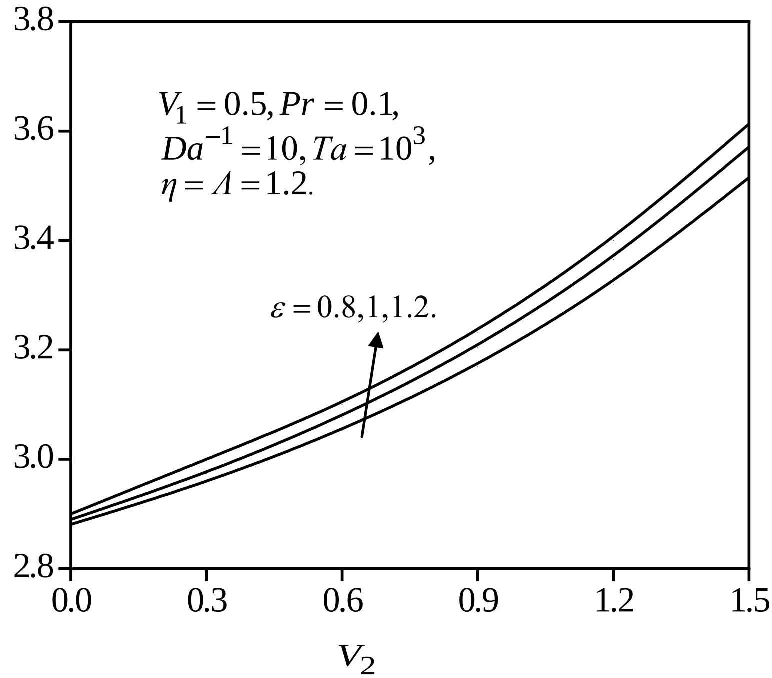

Fig. 4a: Plot of a_c^s Vs. V_2 for different values of ε.

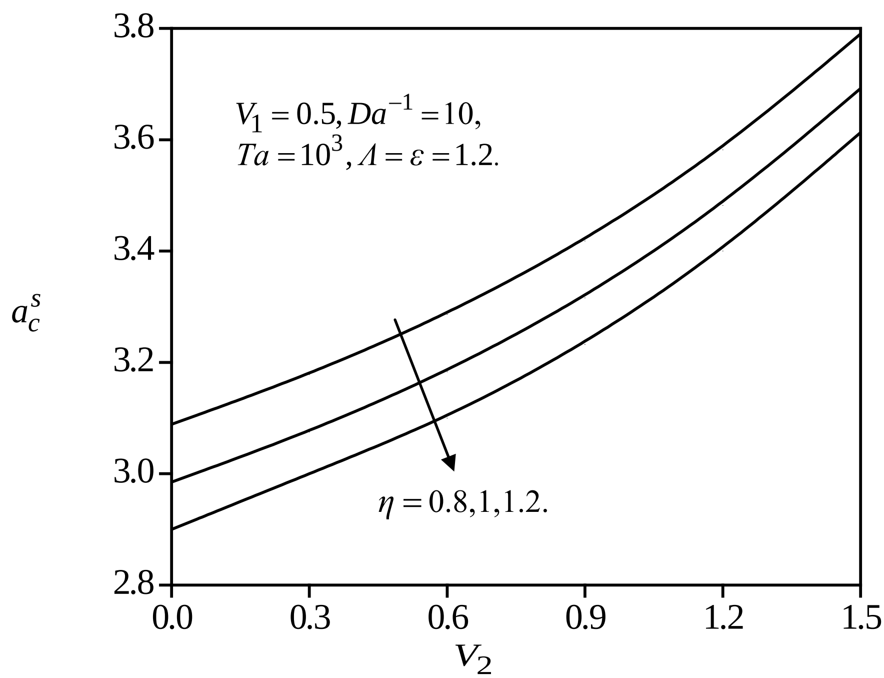

Fig. 4b: Plot of a_c^s Vs. V_2 for different values of η.

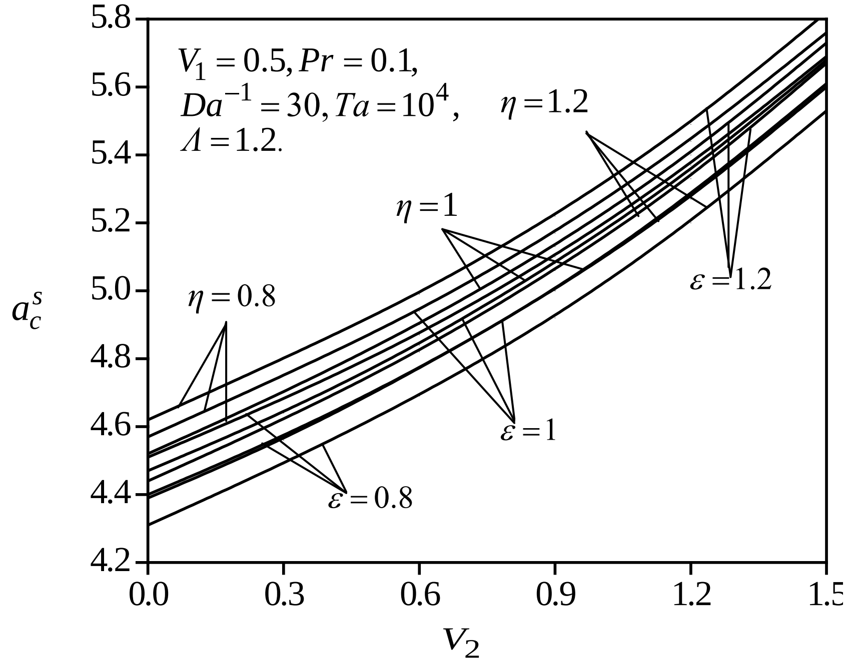

Fig. 4c: Plot of a_c^s Vs. V_2 for different values of η and ε.

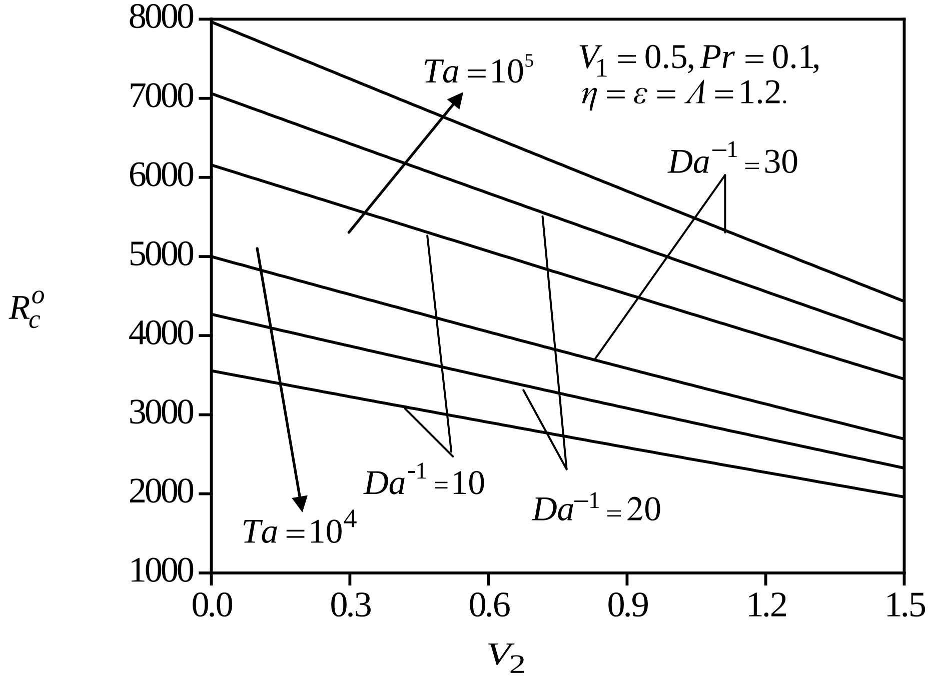

Fig. 5: Plot of R_c^o (oscillatory) Vs. V_2 for different values of Da^{-1} and Ta.

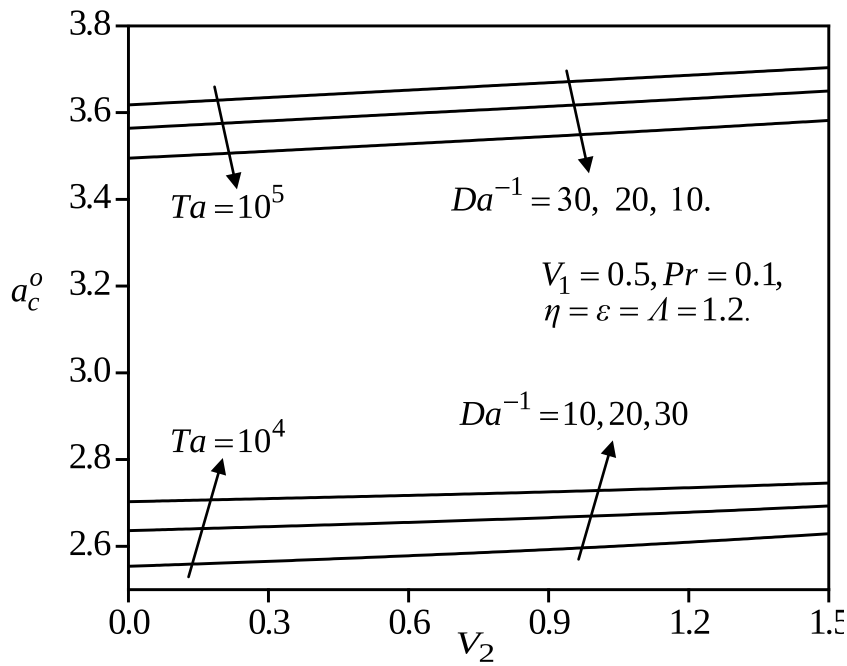

Fig. 6: Plot of a_c^o (oscillatory) Vs. V_2 for different values of Da^{-1} and Ta .

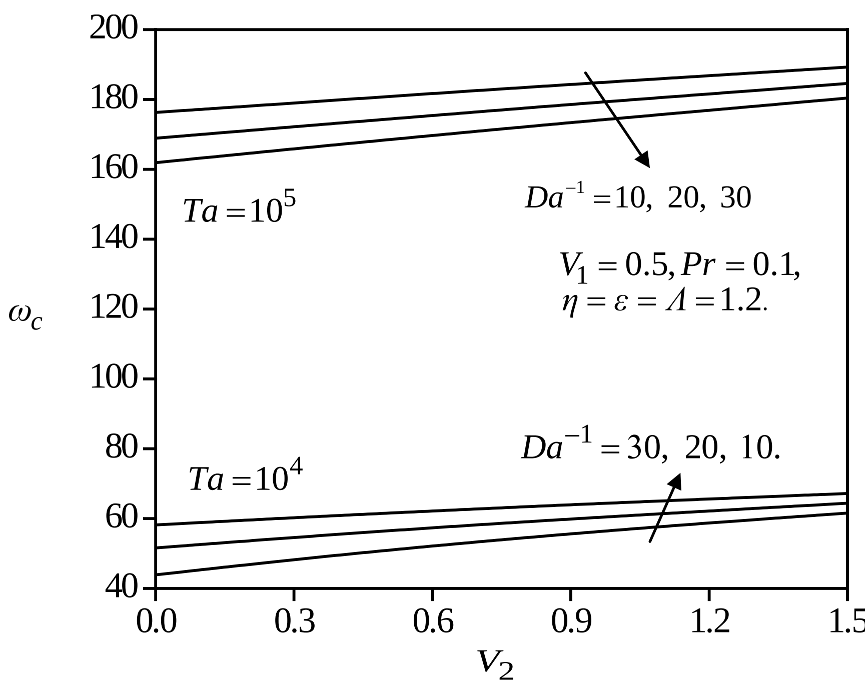

Fig. 7: Plot of ω_c Vs. V_2 for different values of Da^{-1} and Ta .

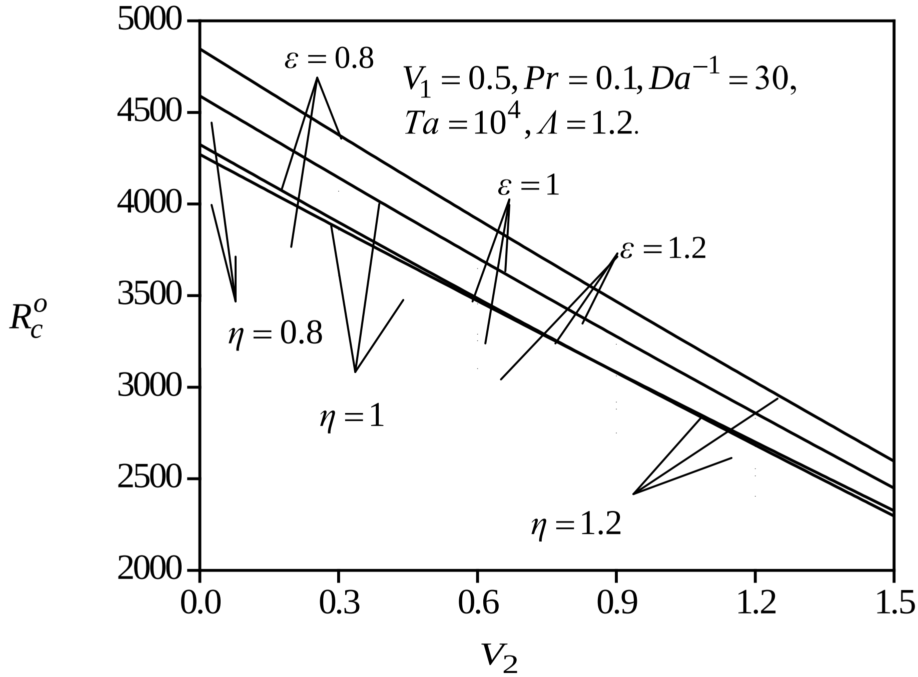

Fig. 8: Plot of R_c^o Vs. V_2 for different values of η and ε.

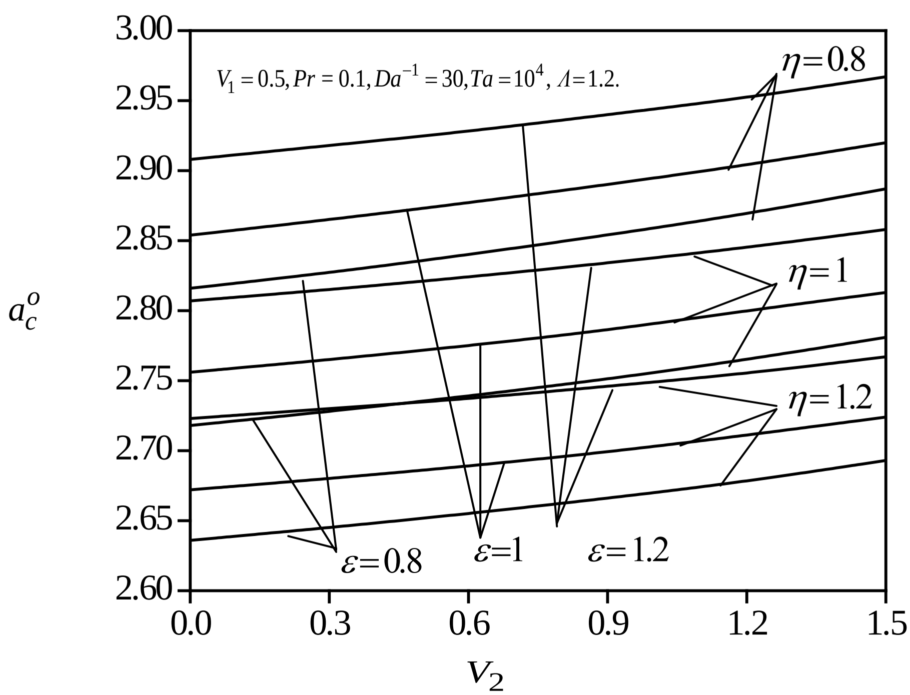

Fig. 9: Plot of a_c^o Vs. V_2 for different values of η and ε

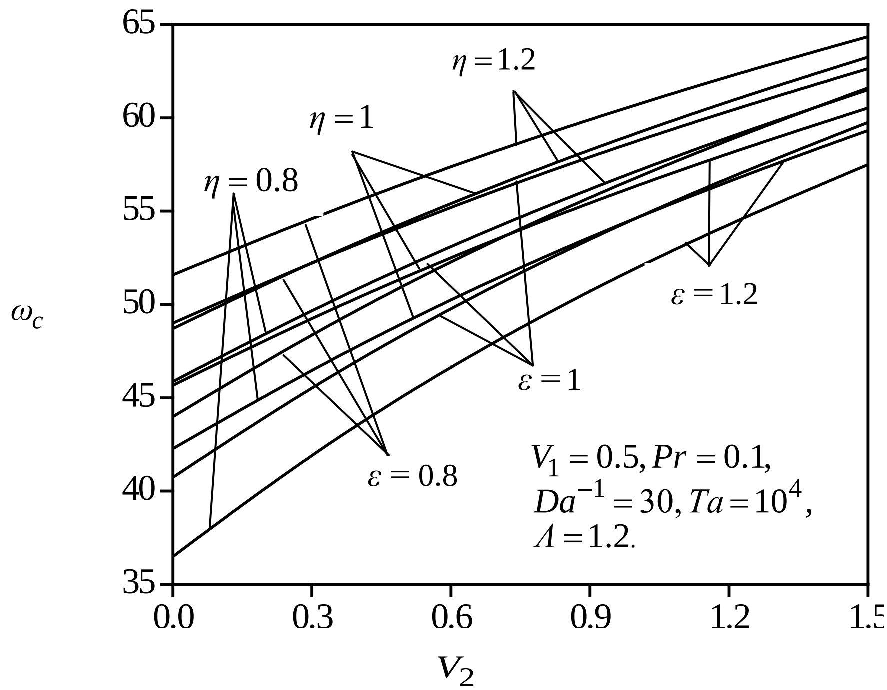

Fig. 10: Plot of ω_c Vs. V_2 for different values of η and ε.