Global Journal of Science Frontier Research: F
Mathematics and Decision Sciences
Volume 18 Issue 7 Version 1.0 Year 2018
Type: Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Fractional Integration of the Product of Two Multivariable Gimel-Functions and A General Class of Polynomials

By Frederic Ayant

Abstract- A significantly large number of earlier works on the subject of fractional calculus give the interesting account of the theory and applications of fractional calculus operators in many different areas of mathematical analysis (such as ordinary and partial differential equations, integral equations, special functions, the summation of series, etc.). The object of the present paper is to study and develop the Saigo-Maeda operators. First, we establish four results that give the images of the product of two multivariable Gimel-functions and a general class of multivariable polynomials in Saigo-Maeda operators. On account of the general nature of the Saigo-Maeda operators, multivariable Gimel-functions and a class multivariable polynomials a large number of new and known theorems involving Riemann-Liouville and Erdelyi-Kober fractional integral operators and several special functions.

Keywords: *general class of multivariable polynomial, saigo-maeda operator, saigo operator, multivariable gimel-function.*

GJSFR-F Classification: *FOR Code: MSC 2010: 30C10*

FRACTIONALINTEGRATIONOFTHEPRODUCTOFTWOMULTIVARIABLEGIMELFUNCTIONSANDAGENERALCLASSOFPOLYNOMIALS

Strictly as per the compliance and regulations of:

Global Journal of Science Frontier Research (F) Volume XVIII Issue VII Version I Year 2018

Fractional Integration of the Product of two Multivariable Gimel-Functions and a General Class of Polynomials

Frederic Ayant

Abstract- A significantly large number of earlier works on the subject of fractional calculus give the interesting account of the theory and applications of fractional calculus operators in many different areas of mathematical analysis (such as ordinary and partial differential equations, integral equations, special functions, the summation of series, etc.). The object of the present paper is to study and develop the Saigo-Maeda operators. First, we establish four results that give the images of the product of two multivariable Gimel-functions and a general class of multivariable polynomials in Saigo-Maeda operators. On account of the general nature of the Saigo-Maeda operators, multivariable Gimel-functions and a class multivariable polynomials a large number of new and known theorems involving Riemann-Liouville and Erdelyi-Kober fractional integral operators and several special functions.

Keywords: general class of multivariable polynomial, saigo-maeda operator, saigo operator, multivariable gimel-function.

I. Introduction And Preliminaries

The fractional integral operator involving various special functions has found significant importance and applications in Various subfields of applicable mathematical analysis. Since the last four decades, some workers like Love [17], McBride [20], Kalla [8,9], Kalla and Saxena [10,11], Saxena et al. [29], Saigo [24,25], Kilbas [12], Kilbas and Sebastian [14] and Kiryakova [16,17] have studied in depth the properties, applications and different extensions of Various hypergeometric operators of fractional integration. A detailed account of such operators along with their properties and applications can be found in the research monographs by Samko, Kilbas, and Marichev [26], Miller and Ross [22], Kilbas, Srivastava, and Trujillo [15] and Debnath and Bhatta [6]. A useful generalization of the hypergeometric fractional integrals, including the Saigo operators [23,24], has been introduced by Marichev [18], see Samko et al. [28] and also see Kilbas and Saigo [13] for more details. The generalized fractional integral operator of arbitrary order, involving Appell function F_3 in the kernel defined and studied by Saigo and Maeda [27, p. 393, Eq (4.12)] and (4.13)] in the following manner:

Let $\alpha, \alpha', \beta, \beta', \eta$ be complex numbers and, $x, Re(\eta) > 0$, we have, see Saigo and Maeda [28, p. 393, Eq (4.12)]

Definition.1

$$I_{0,x}^{\alpha,\alpha',\beta,\beta',\eta} f(x) = \frac{z^{-\alpha}}{\Gamma(\eta)} \int_0^x (x-t)^{\eta-1} t^{-\alpha'} F_3\left[\alpha,\alpha',\beta,\beta';\eta;1-\frac{t}{x},1-\frac{x}{t}\right] f(t)\mathrm{d}t \tag{1.1}$$

and

Definition 2

$$I_{x,\infty}^{\alpha,\alpha',\beta,\beta',\eta} f(x) = \frac{x^{-\alpha}}{\Gamma(\eta)} \int_0^x (t-x)^{\eta-1} t^{-\alpha'} F_3\left[\alpha,\alpha',\beta,\beta';\eta;1-\frac{x}{t},1-\frac{t}{x}\right] f(t)\mathrm{d}t \tag{1.2}$$

Author: *Teacher in High School, France. e-mail: fredericayant@gmail.com*

Ref

6. L. Debnath and D. Bhatta, Integral Transforms and Their Applications, Chapman and Hall/CRC Press, Boca Raton FL, 2006.

We have the following two results due to Saigo [25] where $Re(\eta) > 0$

Definition 3

$$I_{0^+}^{\alpha,\beta,\eta} f(z) = \frac{x^{-\alpha-\beta}}{\Gamma(\eta)} \int_0^x (x-t)^{\alpha-1} F\left[\alpha+\beta, -\eta; \alpha; 1-\frac{t}{x}\right] f(t)\mathrm{d}t \tag{1.3}$$

Definition 4

$$I_{0^-}^{\alpha,\beta,\eta} f(z) = \frac{1}{\Gamma(\eta)} \int_x^\infty t^{-\alpha-\beta}(x-t)^{\alpha-1} F\left[\alpha+\beta, -\eta; \alpha; 1-\frac{x}{t}\right] f(t)\mathrm{d}t \tag{1.4}$$

F is the Gaussian hypergeometric function. We obtain the following lemmas.

Lemma 1.

$$\left(I_{0,x}^{\alpha,\alpha',\beta,\beta',\eta} t^{\mu-1}\right) = \frac{\Gamma(u)\Gamma(\mu+\eta-\alpha-\alpha'-\beta)(\mu+\beta'-\alpha')}{\Gamma(\mu+\eta-\alpha-\alpha')\Gamma(\mu+\eta-\alpha'-\beta)\Gamma(\mu+\beta')} x^{\mu-\alpha-\alpha'+\eta-1} \tag{1.5}$$

where $\alpha, \alpha', \beta, \beta', \eta \in \mathbb{C}, Re(\mu) > max\{0, \Re(\alpha+\alpha'+\beta-\eta), Re(\alpha'-\beta')\}$

Lemma 2.

$$\left(I_{x;\infty}^{\alpha,\alpha',\beta,\beta',\eta} t^{\mu-1}\right) = \frac{\Gamma(1+\alpha+\alpha'-\eta-\mu)\Gamma(1+\alpha+\beta'-\eta-\mu)\Gamma(1-\beta-\mu)}{\Gamma(1-\mu)\Gamma(1-\mu-\eta+\alpha+\alpha'+\beta')\Gamma(1+\alpha-\beta-\mu)} x^{\mu-\alpha-\alpha'+\eta-1} \tag{1.6}$$

where $\alpha, \alpha', \beta, \beta', \eta \in \mathbb{C}, Re(\eta) > 0, Re(\mu) < min\{Re(-\beta), \Re(\alpha+\alpha'-\eta), Re(\alpha'+\beta'-\eta)\}$

Lemma 3.

$$\left(I_{0,x}^{\alpha,\beta,\eta} t^{\mu-1}\right) = \frac{\Gamma(u)\Gamma(\mu+\eta-\beta)}{\Gamma(\mu+\eta+\alpha+\eta)\Gamma(\mu-\beta)} x^{\mu-\beta-1} \tag{1.7}$$

$$\alpha, \beta, \eta \in \mathbb{C}, Re(> 0, Re(\mu) > max\{0, \Re(\beta-\eta), Re(\alpha'-\beta')\}$$

Lemma 4.

$$\left(I_{x;\infty}^{\alpha,\beta,\eta} t^{\mu-1}\right) = \frac{\Gamma(\beta-\mu+1)\Gamma(\eta-\mu+1)}{\Gamma(1-\mu)\Gamma(\alpha+\beta+\eta-\mu+1)} x^{\mu-\beta-1} \tag{1.8}$$

where $\alpha, \beta, \eta \in \mathbb{C}, Re(\alpha) > 0, Re(\mu) < 1 + min\{Re(\beta), \Re(\eta)\}$

Recently, Gupta et al. [7] have obtained the images of the product of two H-functions in Saigo operator given by (1.3) and (1.4) and thereby generalized several results obtained earlier by Kilbas, Kilbas and Sebastian [14] and Saxena et al. [29] as mentioned in this paper cited above. It has recently become a subject of interest for many researchers in the field of fractional calculus and its applications. Motivated by these avenues of applications, a number of workers have made use of the fractional calculus operators to obtain the image formulas. The aim of this paper is to obtain four results that give the theorems of the product of two multivariable Gimel functions and a general class of multivariable polynomials [30] in Saigo-Maeda operators and Saigo operators.

II. Multivariable Gimel-Function

We throughout this paper, let $\mathbb{C}, \mathbb{R}$, and $\mathbb{N}$ be set of complex numbers, real numbers and positive integers respectively. Also. $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$. We define a generalized transcendental function of several complex variables.

Ref

7. K. C. Gupta, K. Gupta and A. Gupta, Generalized fractional integration of the product of two H-functions. J. Rajasthan Acad. Phy. Sci., 9(3), 203–212(2010).

$$\gimel(z_1,\cdots,z_r)=\gimel^{0,n_2;0,n_3;\cdots;0,n_r:m^{(1)},n^{(1)};m^{(2)},n^{(2)};\cdots;m^{(r)},n^{(r)}}_{p_{i_2},q_{i_2},\tau_{i_2};R_2;p_{i_3},q_{i_3},\tau_{i_3};R_3;\cdots;p_{i_r},q_{i_r},\tau_{i_r}:R_r:p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)};\cdots;p_{i^{(r)}},q_{i^{(r)}};\tau_{i^{(r)}};R^{(r)}}\left(\begin{array}{c|}\mathrm{z}_1\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_r\end{array}\right.$$

$$[(\mathrm{a}_{2j};\alpha_{2j}^{(1)},\alpha_{2j}^{(2)};A_{2j})]_{1,n_2},[\tau_{i_2}(a_{2ji_2};\alpha_{2ji_2}^{(1)},\alpha_{2ji_2}^{(2)};A_{2ji_2})]_{n_2+1,p_{i_2}};[(a_{3j};\alpha_{3j}^{(1)},\alpha_{3j}^{(2)},\alpha_{3j}^{(3)};A_{3j})]_{1,n_3},$$
$$[\tau_{i_2}(b_{2ji_2};\beta_{2ji_2}^{(1)},\beta_{2ji_2}^{(2)};B_{2ji_2})]_{1,q_{i_2}};$$

$$[\tau_{i_3}(a_{3ji_3};\alpha_{3ji_3}^{(1)},\alpha_{3ji_3}^{(2)},\alpha_{3ji_3}^{(3)};A_{3ji_3})]_{n_3+1,p_{i_3}};\cdots;[(\mathrm{a}_{rj};\alpha_{rj}^{(1)},\cdots,\alpha_{rj}^{(r)};A_{rj})]_{1,n_r},$$
$$[\tau_{i_3}(b_{3ji_3};\beta_{3ji_3}^{(1)},\beta_{3ji_3}^{(2)},\beta_{3ji_3}^{(3)};B_{3ji_3})]_{1,q_{i_3}};\cdots;\qquad\cdots$$

$$[\tau_{i_r}(a_{rji_r};\alpha_{rji_r}^{(1)},\cdots,\alpha_{rji_r}^{(r)};A_{rji_r})]_{n_r+1,p_r}:\ [(\mathrm{c}_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{1,n^{(1)}}],[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)},\gamma_{ji^{(1)}}^{(1)};C_{ji^{(1)}}^{(1)})]_{n^{(1)}+1,p_i^{(1)}}$$
$$[\tau_{i_r}(b_{rji_r};\beta_{rji_r}^{(1)},\cdots,\beta_{rji_r}^{(r)};B_{rji_r})]_{1,q_r}:\ [(\mathrm{d}_j^{(1)}),\delta_j^{(1)};D_j^{(1)})]_{1,m^{(1)}},[\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)},\delta_{ji^{(1)}}^{(1)};D_{ji^{(1)}}^{(1)})]_{m^{(1)}+1,q_i^{(1)}}$$

$$\left.\begin{array}{l};\cdots;[(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})]_{1,m^{(r)}},[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)},\gamma_{ji^{(r)}}^{(r)};C_{ji^{(r)}}^{(r)})]_{m^{(r)}+1,p_i^{(r)}}\\ ;\cdots;[(d_j^{(r)},\delta_j^{(r)};D_j^{(r)})]_{1,n^{(r)}},[\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)},\delta_{ji^{(r)}}^{(r)};D_{ji^{(r)}}^{(r)})]_{n^{(r)}+1,q_i^{(r)}}\end{array}\right)$$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)z_k^{s_k}\,\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{2.1}$$

with $\omega=\sqrt{-1}$

$$\psi(s_1,\cdots,s_r)=\frac{\prod_{j=1}^{n_2}\Gamma^{A_{2j}}(1-a_{2j}+\sum_{k=1}^{2}\alpha_{2j}^{(k)}s_k)}{\sum_{i_2=1}^{R_2}[\tau_{i_2}\prod_{j=n_2+1}^{p_{i_2}}\Gamma^{A_{2ji_2}}(a_{2ji_2}-\sum_{k=1}^{2}\alpha_{2ji_2}^{(k)}s_k)\prod_{j=1}^{q_{i_2}}\Gamma^{B_{2ji_2}}(1-b_{2ji2}+\sum_{k=1}^{2}\beta_{2ji2}^{(k)}s_k)]}$$

$$\frac{\prod_{j=1}^{n_3}\Gamma^{A_{3j}}(1-a_{3j}+\sum_{k=1}^{3}\alpha_{3j}^{(k)}s_k)}{\sum_{i_3=1}^{R_3}[\tau_{i_3}\prod_{j=n_3+1}^{p_{i_3}}\Gamma^{A_{3ji_3}}(a_{3ji_3}-\sum_{k=1}^{3}\alpha_{3ji_3}^{(k)}s_k)\prod_{j=1}^{q_{i_3}}\Gamma^{B_{3ji_3}}(1-b_{3ji3}+\sum_{k=1}^{3}\beta_{3ji3}^{(k)}s_k)]}$$

$$\begin{array}{cccc}\cdot&\cdot&\cdot&\cdot\\ \cdot&\cdot&\cdot&\cdot\\ \cdot&\cdot&\cdot&\cdot\end{array}$$

$$\frac{\prod_{j=1}^{n_r}\Gamma^{A_{rj}}(1-a_{rj}+\sum_{k=1}^{r}\alpha_{rj}^{(k)}s_k)}{\sum_{i_r=1}^{R_r}[\tau_{i_r}\prod_{j=n_r+1}^{p_{i_r}}\Gamma^{A_{rji_r}}(a_{rji_r}-\sum_{k=1}^{r}\alpha_{rji_r}^{(k)}s_k)\prod_{j=1}^{q_{i_r}}\Gamma^{B_{rji_r}}(1-b_{rjir}+\sum_{k=1}^{r}\beta_{rjir}^{(k)}s_k)]} \tag{2.2}$$

and

$$\theta_k(s_k)=\frac{\prod_{j=1}^{m^{(k)}}\Gamma^{D_j^{(k)}}(d_j^{(k)}-\delta_j^{(k)}s_k)\prod_{j=1}^{n^{(k)}}\Gamma^{C_j^{(k)}}(1-c_j^{(k)}+\gamma_j^{(k)}s_k)}{\sum_{i^{(k)}=1}^{R^{(k)}}[\tau_{i^{(k)}}\prod_{j=m^{(k)}+1}^{q_{i^{(k)}}}\Gamma^{D_{ji^{(k)}}^{(k)}}(1-d_{ji^{(k)}}^{(k)}+\delta_{ji^{(k)}}^{(k)}s_k)\prod_{j=n^{(k)}+1}^{p_{i^{(k)}}}\Gamma^{C_{ji^{(k)}}^{(k)}}(c_{ji^{(k)}}^{(k)}-\gamma_{ji^{(k)}}^{(k)}s_k)]} \tag{2.3}$$

The contour L_k is in the $s_k(k=1,\cdots,r)$- plane and runs from $\sigma - i\infty$ to $\sigma + i\infty$ where σ if is a real number with loop, if necessary to ensure that the poles of $\Gamma^{A_{2j}}\left(1-a_{2j}+\sum_{k=1}^{2}\alpha_{2j}^{(k)}s_k\right)$ $(j=1,\cdots,n_2),\Gamma^{A_{3j}}\left(1-a_{3j}+\sum_{k=1}^{3}\alpha_{3j}^{(k)}s_k\right)$ $(j=1,\cdots,n_3),\cdots,\Gamma^{A_{rj}}\left(1-a_{rj}+\sum_{i=1}^{r}\alpha_{rj}^{(i)}\right)(j=1,\cdots,n_r)$, $\Gamma^{C_j^{(k)}}\left(1-c_j^{(k)}+\gamma_j^{(k)}s_k\right)(j=1,\cdots,n^{(k)})(k=1,\cdots,r)$ to the right of the contour L_k and the poles of $\Gamma^{D_j^{(k)}}\left(d_j^{(k)}-\delta_j^{(k)}s_k\right)(j=1,\cdots,m^{(k)})(k=1,\cdots,r)$ lie to the left of the contour L_k. The condition for absolute convergence of multiple Mellin-Barnes type contour (1.1) can be obtained of the corresponding conditions for multivariable H-function given by as

$|arg(z_k)| < \frac{1}{2}A_i^{(k)}\pi$ where

$$A_i^{(k)} = \sum_{j=1}^{m^{(k)}} D_j^{(k)}\delta_j^{(k)} + \sum_{j=1}^{n^{(k)}} C_j^{(k)}\gamma_j^{(k)} - \tau_{i^{(k)}}\left(\sum_{j=m^{(k)}+1}^{q_i^{(k)}} D_{ji^{(k)}}^{(k)}\delta_{ji^{(k)}}^{(k)} + \sum_{j=n^{(k)}+1}^{p_i^{(k)}} C_{ji^{(k)}}^{(k)}\gamma_{ji^{(k)}}^{(k)}\right) \tag{2.4}$$

$$-\tau_{i_2}\left(\sum_{j=n_2+1}^{p_{i_2}} A_{2ji_2}\alpha_{2ji_2}^{(k)} + \sum_{j=1}^{q_{i_2}} B_{2ji_2}\beta_{2ji_2}^{(k)}\right) - \cdots - \tau_{i_r}\left(\sum_{j=n_r+1}^{p_{i_r}} A_{rji_r}\alpha_{rji_r}^{(k)} + \sum_{j=1}^{q_{i_r}} B_{rji_r}\beta_{rji_r}^{(k)}\right)$$

Following the lines of Braaksma ([4] p. 278), we may establish the asymptotic expansion in the following convenient form

$$\aleph(z_1,\cdots,z_r) = 0(\,|z_1|^{\alpha_1},\cdots,|z_r|^{\alpha_r}\,)\,, max(\,|z_1|,\cdots,|z_r|\,) \to 0$$

$\aleph(z_1,\cdots,z_r) = 0(\,|z_1|^{\beta_1},\cdots,|z_r|^{\beta_r}\,)\,, min(\,|z_1|,\cdots,|z_r|\,) \to \infty$ where $i=1,\cdots,r:$

$\alpha_i = \min_{1\leqslant j\leqslant m^{(i)}} Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right]$ and $\beta_i = \max_{1\leqslant j\leqslant n^{(i)}} Re\left[C_j^{(i)}\left(\frac{c_j^{(i)}-1}{\gamma_j^{(i)}}\right)\right]$

Remark 1.

If $n_2=\cdots=n_{r-1}=p_{i_2}=q_{i_2}=\cdots=p_{i_{r-1}}=q_{i_{r-1}}=0$. $A_{2j}=A_{2ji_2}=B_{2ji_2}=\cdots=A_{rj}=A_{rji_r}=B_{rji_r}=A_{rj}=A_{rji_r}=B_{rji_r}=1$, then the multivariable Gimel-function reduces in multivariable Aleph- function defined by Ayant [3].

Remark 2.

If $n_2=\cdots=n_r=p_{i_2}=q_{i_2}=\cdots=p_{i_r}=q_{i_r}=0$. $\tau_{i_2}=\cdots=\tau_{i_r}=\tau_{i^{(1)}}=\cdots=\tau_{i^{(r)}}=R_2=\cdots=R_r=R^{(1)}=\cdots=R^{(r)}=1$, then the multivariable Gimel-function reduces in multivariable I-function defined by Prathima et al. [23].

Remark 3.

If $A_{2j}=A_{2ji_2}=B_{2ji_2}=\cdots=A_{rj}=A_{rji_r}=B_{rji_r}=1$. $\tau_{i_2}=\cdots=\tau_{i_r}=\tau_{i^{(1)}}=\cdots=\tau_{i^{(r)}}=R_2=\cdots=R_r=R^{(1)}=\cdots=R^{(r)}=1$, then the generalized multivariable Gimel-function reduces in multivariable I-function defined by Prasad [22].

Remark 4.

If the three above conditions are satisfied at the same time, then the generalized multivariable Gimel-function reduces in the H-function of several defined by Srivastava and Panda [32,33]. About the simplified notations, see Ayant ([4], page 248-255)

Now, we define the second Gimel function of s variables, the parameters are identical to the Gimel function of r variables with the prim sign and the validities conditions are equivalent.

The generalized polynomials of multivariable defined by Srivastava [30], is given in the following manner:

$$S_{N_1,\cdots,N_v}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}[y_1,\cdots,y_v] = \sum_{K_1=0}^{[N_1/\mathfrak{M}_1]}\cdots\sum_{K_v=0}^{[N_v/\mathfrak{M}_v]}\frac{(-N_1)_{\mathfrak{M}_1K_1}}{K_1!}\cdots\frac{(-N_v)_{\mathfrak{M}_vK_v}}{K_v!}A[N_1,K_1;\cdots;N_v,K_v]y_1^{K_1}\cdots y_v^{K_v} \tag{2.5}$$

Global Journal of Science Frontier Research (F) Volume XVIII Issue VII Version I Year 2018

Ref

4. F.Y. Ayant, Some transformations and identities form multivariable Gimel-function, International Journal of Matematics and Technology (IJMTT), 59(4) (2018), 248-255.

where $\mathfrak{M}_1, \cdots, \mathfrak{M}_\mathfrak{v}$ are arbitrary positive integers and the coefficients $A[N_1, K_1; \cdots; N_v, K_v]$ are arbitrary constants, real or complex.

We shall note $a_v = \dfrac{(-N_1)_{\mathfrak{M}_1 K_1}}{K_1!} \cdots \dfrac{(-N_v)_{\mathfrak{M}_\mathfrak{v} K_v}}{K_v!} A[N_1, K_1; \cdots; N_v, K_v]$

Notes

III. Main Results

We shall note

$$U = 0, n_2; 0, n_3; \cdots; 0, n_{r-1}; 0, n'_2; 0, n'_3; \cdots; 0, n'_{s-1} \tag{3.1}$$

$$V = m^{(1)}, n^{(1)}; m^{(2)}, n^{(2)}; \cdots; m^{(r)}, n^{(r)}; m'^{(1)}, n'^{(1)}; m'^{(2)}, n'^{(2)}; \cdots; m'^{(s)}, n'^{(s)} \tag{3.2}$$

$$X = p_{i_2}, q_{i_2}, \tau_{i_2}; R_2; \cdots; p_{i_{r-1}}, q_{i_{r-1}}, \tau_{i_{r-1}} : R_{r-1};\ p'_{i_2}, q'_{i'_2}, \tau'_{i_2}; R'_2; \cdots; p'_{i'_{s-1}}, q'_{i'_{s-1}}, \tau'_{i'_{s-1}} : R'_{s-1} \tag{3.3}$$

$$Y = p_{i^{(1)}}, q_{i^{(1)}}, \tau_{i^{(1)}}; R^{(1)}; \cdots; p_{i^{(r)}}, q_{i^{(r)}}; \tau_{i^{(r)}}; R^{(r)}; p'_{i'^{(1)}}, q'_{i'^{(1)}}, \tau'_{i'^{(1)}}; R'^{(1)}; \cdots; p'_{i'^{(s)}}, q'_{i^{(s)}}; \tau'_{i'^{(s)}}; R'^{(s)} \tag{3.4}$$

Theorem 1.

$$\left\{\left(I_{0,x}^{\alpha,\alpha',\beta,\beta',\eta} t^{\mu-1}(b-at)^{-\upsilon} S_{N_1,\cdots,N_v}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_\mathfrak{v}}\begin{pmatrix} \mathrm{c}_1 t^{\lambda_1}(b-at)^{-\delta_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{c}_v t^{\lambda_v}(b-at)^{-\delta_v}\end{pmatrix} \gimel \begin{pmatrix} \mathrm{z}_1 t^{\sigma_1}(b-at)^{-\omega_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r t^{\sigma_r}(b-at)^{-\omega_r}\end{pmatrix}\right.\right.$$

$$\left.\left.\gimel \begin{pmatrix} \mathrm{z}'_1 t^{\sigma'_1}(b-at)^{-\omega'_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}'_s t^{\sigma'_s}(b-at)^{-\omega'_s}\end{pmatrix}\right)\right\}(x) = b^{-\upsilon} x^{\mu-\alpha-\alpha'+\eta-1} \sum_{K_1=1}^{[N_1/M_1]} \cdots \sum_{K_v=1}^{[N_v/M_v]} a_v c_1^{K_1} \cdots c_v^{K_v}$$

$$v^{-\sum_{j=1}^{v} \delta_i K_j} x^{\sum_{j=1}^{v} \lambda_i K_j} \gimel^{U;0,n_r+n'_s+4:V,1,0}_{X;p_{i_r}+p'_{i'_s}+4,q_{i_r}+q'_{i'_s}+4,\tau_{i_r},\tau'_{i'_s}:R_r:R'_s:Y;0,1}\left(\begin{array}{c|c} \mathrm{z}_1 \frac{x^{\sigma_1}}{b^{\omega_1}} & \mathbb{A}, ,\mathrm{A}_1, \mathbf{A} : A \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{z}_r \frac{x^{\sigma_r}}{b^{\omega_r}} & \cdot \\ \mathrm{z}_1 \frac{x^{\sigma'_1}}{b^{\omega'_1}} & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{z}'_s \frac{x^{\sigma'_s}}{b^{\omega'_s}} & \mathbb{B}, \mathbf{B}, \mathrm{B}_1 : B, (0,1;1) \\ \frac{ax}{b} & \end{array}\right) \tag{3.5}$$

where

$\mathbb{A} = [(\mathrm{a}_{2j}; \alpha_{2j}^{(1)}, \alpha_{2j}^{(2)}; A_{2j})]_{1,n_2}, [\tau_{i_2}(a_{2ji_2}; \alpha_{2ji_2}^{(1)}, \alpha_{2ji_2}^{(2)}; A_{2ji_2})]_{n_2+1,p_{i_2}}, [(a_{3j}; \alpha_{3j}^{(1)}, \alpha_{3j}^{(2)}, \alpha_{3j}^{(3)}; A_{3j})]_{1,n_3},$

$[\tau_{i_3}(a_{3ji_3}; \alpha_{3ji_3}^{(1)}, \alpha_{3ji_3}^{(2)}, \alpha_{3ji_3}^{(3)}; A_{3ji_3})]_{n_3+1,p_{i_3}}; \cdots; [(\mathrm{a}_{(r-1)j}; \alpha_{(r-1)j}^{(1)}, \cdots, \alpha_{(r-1)j}^{(r-1)}; A_{(r-1)j})]_{1,n_{r-1}},$

$[\tau_{i_{r-1}}(a_{(r-1)ji_{r-1}}; \alpha_{(r-1)ji_{r-1}}^{(1)}, \cdots, \alpha_{(r-1)ji_{r-1}}^{(r-1)}; A_{(r-1)ji_{r-1}})]_{n_{r-1}+1,p_{i_{r-1}}},$

$$[(\mathrm{a'}_{2j};\alpha'^{(1)}_{2j},\alpha'^{(2)}_{2j};A'_{2j})]_{1,n'_2},[\tau'_{i'_2}(a'_{2ji_2};\alpha'^{(1)}_{2ji'_2},\alpha'^{(2)}_{2ji'_2};A'_{2ji'_2})]_{n'_2+1,p'_{i'_2}},[(a'_{3j};\alpha'^{(1)}_{3j},\alpha'^{(2)}_{3j},\alpha'^{(3)}_{3j};A'_{3j})]_{1,n'_3},$$

$$[\tau'_{i'_3}(a'_{3ji'_3};\alpha'^{(1)}_{3ji'_3},\alpha'^{(2)}_{3ji'_3},\alpha'^{(3)}_{3ji'_3};A'_{3ji'_3})]_{n'_3+1,p'_{i'_3}};\cdots;[(\mathrm{a'}_{(s-1)j};\alpha'^{(1)}_{(s-1)j},\cdots,\alpha'^{(s-1)}_{(s-1)j};A'_{(s-1)j})]_{1,n'_{s-1}},$$

$$[\tau'_{i'_{s-1}}(a'_{(s-1)ji'_{s-1}};\alpha'^{(1)}_{(s-1)ji'_{s-1}},\cdots,\alpha'^{(s-1)}_{(s-1)ji'_{s-1}};A'_{(s-1)ji'_{s-1}})]_{n'_{s-1}+1,p'_{i'_{s-1}}} \tag{3.6}$$

Notes

$$A_1=(1-v+\sum_{j=1}^{v}\lambda_jK_j;\sigma_1,\cdots,\sigma_r,\sigma'_1,\cdots,\sigma'_s,1;1),(1+\alpha+\alpha'+\beta-v-\eta-\sum_{j=1}^{K}\delta_jK_j;\omega_1,\cdots,\omega_r,\omega'_1,\cdots,\omega'_s,1;1)$$

$$(1-\mu+\sum_{j=1}^{v}\lambda_jK_j;\sigma_1,\cdots,\sigma_r,\sigma'_1,\cdots,\sigma'_s,1;1),(1+\alpha'+\beta-\mu-\eta-\sum_{j=1}^{K}\delta_jK_j;\omega_1,\cdots,\omega_r,\omega'_1,\cdots,\omega'_s,1;1) \tag{3.7}$$

$$\mathbf{A}=[(\mathrm{a}_{rj};\alpha^{(1)}_{rj},\cdots,\alpha^{(r)}_{rj},\underbrace{0,\cdots,0}_{s+1},A_{rj})]_{1,n_r},[\tau_{i_r}(a_{rji_r};\alpha^{(1)}_{rji_r},\cdots,\alpha^{(r)}_{rji_r},\underbrace{0,\cdots,0}_{s+1};A_{rji_r})]_{\mathfrak{n}+1,p_{i_r}},$$

$$[(\mathrm{a}_{sj};\underbrace{0,\cdots,0}_{r}\alpha'^{(1)}_{sj},\cdots,\alpha'^{(r)}_{sj},0,A'_{sj})]_{1,n'_s},[\tau'_{i'_s}(a'_{sji'_s};\underbrace{0,\cdots,0}_{r}\alpha'^{(1)}_{sji'_s},\cdots,\alpha^{(s)}_{sji'_s},0;A'_{sji'_s})]_{n'_s+1,p'_{i'_s}} \tag{3.8}$$

$$A=[(\mathrm{c}^{(1)}_j,\gamma^{(1)}_j;C^{(1)}_j)_{1,n^{(1)}}],[\tau_{i^{(1)}}(c^{(1)}_{ji^{(1)}},\gamma^{(1)}_{ji^{(1)}};C^{(1)}_{ji^{(1)}})]_{n^{(1)}+1,p^{(1)}_i};\cdots;$$

$$[(c^{(r)}_j,\gamma^{(r)}_j;C^{(r)}_j)_{1,m^{(r)}}],[\tau_{i^{(r)}}(c^{(r)}_{ji^{(r)}},\gamma^{(r)}_{ji^{(r)}};C^{(r)}_{ji^{(r)}})]_{m^{(r)}+1,p^{(r)}_i},$$

$$[(\mathrm{c'}^{(1)}_j,\gamma'^{(1)}_j;C'^{(1)}_j)_{1,n'^{(1)}}],[\tau'_{i^{(1)}}(c'^{(1)}_{ji'^{(1)}},\gamma^{(1)}_{ji'^{(1)}};C'^{(1)}_{ji'^{(1)}})]_{n'^{(1)}+1,p'^{(1)}_i},$$

$$\cdots,[(c'^{(s)}_j,\gamma'^{(s)}_j;C'^{(s)}_j)]_{1,m'^{(s)}},[\tau'_{i'^{(s)}}(c'^{(s)}_{ji'^{(s)}},\gamma'^{(s)}_{ji'^{(s)}};C'^{(s)}_{ji'^{(s)}})]_{m'^{(s)}+1,p'^{(r)}_{i'}};-: \tag{3.9}$$

$$\mathbb{B}=[\tau_{i_2}(b_{2ji_2};\beta^{(1)}_{2ji_2},\beta^{(2)}_{2ji_2};B_{2ji_2})]_{1,q_{i_2}},[\tau_{i_3}(b_{3ji_3};\beta^{(1)}_{3ji_3},\beta^{(2)}_{3ji_3},\beta^{(3)}_{3ji_3};B_{3ji_3})]_{1,q_{i_3}};\cdots;$$

$$[\tau_{i_{r-1}}(b_{(r-1)ji_{r-1}};\beta^{(1)}_{(r-1)ji_{r-1}},\cdots,\beta^{(r-1)}_{(r-1)ji_{r-1}};B_{(r-1)ji'_{r-1}})]_{1,q_{i_{r-1}}};$$

$$[\tau'_{i'_2}(b_{2ji'_2};\beta'^{(1)}_{2ji'_2},\beta'^{(2)}_{2ji'_2};B'_{2ji'_2})]_{1,q'_{i'_2}},[\tau'_{i'_3}(b'_{3ji'_3};\beta'^{(1)}_{3ji'_3},\beta'^{(2)}_{3ji'_3},\beta'^{(3)}_{3ji'_3};B'_{3ji'_3})]_{1,q'_{i'_3}};\cdots;$$

$$[\tau'_{i'_{s-1}}(b'_{(s-1)ji'_{s-1}};\beta'^{(1)}_{(s-1)ji'_{s-1}},\cdots,\beta'^{(s-1)}_{(s-1)ji'_{s-1}};B'_{(s-1)ji'_{s-1}})]_{1,q'_{i'_{s-1}}}$$

$$\mathbf{B}=[\tau_{i_r}(b_{rji_r};\beta^{(1)}_{rji_r},\cdots,\beta^{(r)}_{rji_r},\underbrace{0,\cdots,0}_{s+1};B_{rji_r})]_{1,q_{i_r}},\quad[\tau'_{i'_s}(b'_{sji'_s};\underbrace{0,\cdots,0}_{r};\beta'^{(1)}_{sji'_s},\cdots,\beta'^{(s)}_{sji'_s},0;B'_{sji'_s})]_{1,q'_{i'_s}} \tag{3.10}$$

$$B_1=(1-v-\sum_{j=1}^{v}\lambda_jK_j;\sigma_1,\cdots,\sigma_r,\sigma'_1,\cdots,\sigma'_s,0;1),(1+\alpha+\alpha'-v-\eta-\sum_{j=1}^{K}\delta_jK_j;\omega_1,\cdots,\omega_r,\omega'1,\cdots,\omega'_s,1;1)$$

$$(1-\mu-\beta'-\sum_{j=1}^{v}\lambda_jK_j;\sigma_1,\cdots,\sigma_r,\sigma'_1,\cdots,\sigma'_s,1;1),(1+\alpha'+\beta-\mu-\eta-\sum_{j=1}^{K}\delta_jK_j;\omega_1,\cdots,\omega_r,\omega'_1,\cdots,\omega'_s,1;1) \tag{3.11}$$

$$\mathrm{B} = [(\mathrm{d}_j^{(1)}, \delta_j^{(1)}; D_j^{(1)})]_{1,m^{(1)}}, [\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)}, \delta_{ji^{(1)}}^{(1)}; D_{ji^{(1)}}^{(1)})]_{m^{(1)}+1,q_i^{(1)}}; \cdots ;$$

$$[(\mathrm{d}_j^{(r)}, \delta_j^{(r)}; D_j^{(r)})]_{1,m^{(r)}}, [\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)}, \delta_{ji^{(r)}}^{(r)}; D_{ji^{(r)}}^{(r)})]_{m^{(r)}+1,q_i^{(r)}},$$

$$[(\mathrm{d}'^{(1)}_j, \delta'^{(1)}_j; D'^{(1)}_j)]_{1,m'^{(1)}}, [\tau'_{i'^{(1)}}(d'^{(1)}_{ji'^{(1)}}, \delta'^{(1)}_{ji'^{(1)}}; D'^{(1)}_{ji'^{(1)}})]_{m'^{(1)}+1,q'^{(1)}_i}; \cdots ;$$

$$[(\mathrm{d}'^{(s)}_j, \delta'^{(s)}_j; D'^{(s)}_j)]_{1,m'^{(s)}}, [\tau'_{i'^{(s)}}(d'^{(s)}_{ji'^{(s)}}, \delta'^{(s)}_{ji'^{(s)}}; D'^{(s)}_{ji'(s)})]_{m'^{(s)}+1,q'^{(s)}_i}; \cdot - . \tag{3.12}$$

In our investigation, we will use these simplified notations cited above.

Provided

$$a, b, \alpha, \beta, \eta, \mu, \upsilon, \delta_k, \omega_i, \omega'_j \in \mathbb{C}, k = 1, \cdots, v; i = 1 \cdots, r; j = 1, \cdots, s$$

$$\lambda_k, \sigma_i, \sigma'_j > 0; \quad k = 1, \cdots, v; i = 1 \cdots, r; j = 1, \cdots, s$$

$|arg\,(z_i)| < \frac{1}{2}\pi A_i^{(k)}$ and $A_i^{(k)}$ is defined by (2.4), $|arg\,(z'_i)| < \frac{1}{2}\pi A_i'^{(k)}; \left|\frac{a}{b}x\right| < 1$

$$Re(\mu) + \sum_{i=1}^{r} \sigma_i \min_{1\leqslant j\leqslant m^{(i)}} Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right] + \sum_{i=1}^{s} \sigma'_i \min_{1\leqslant j\leqslant m'^{(i)}} Re\left[D'^{(i)}_j\left(\frac{d'^{(i)}_j}{\delta'^{(i)}_j}\right)\right] > max[0, Re(\alpha+\alpha'+\beta-\eta), Re(\alpha'-\beta')]$$

$$Re(\upsilon) + \sum_{i=1}^{r} \omega_i \min_{1\leqslant j\leqslant m^{(i)}} Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right] + \sum_{i=1}^{s} \omega'_i \min_{1\leqslant j\leqslant m'^{(i)}} Re\left[D'^{(i)}_j\left(\frac{d'^{(i)}_j}{\delta'^{(i)}_j}\right)\right] > max[0, Re(\alpha+\alpha'+\beta-\eta), Re(\alpha'-\beta')]$$

Proof

To prove (3.1), we first express the class of multivariable polynomials $S_{N_1,\cdots,N_v}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}[y_1,\cdots,y_v]$ in series with the help of (2.13), the multivariable Gimel-functions regarding Mellin-Barnes type integrals contour with the help of (2.1). Now interchange the order of summations and two multiple Mellin-Barnes integrals contour, respectively and taking the fractional integral operator inside (which is permissible under the stated conditions) and make simplifications. Next, we express the terms $(b-ax)^{-\upsilon-\sum_{k=1}^{v} - \sum_{j=1}^{r}\omega s_i - \sum_{j=1}^{s}\omega' t_i}$ in terms of Mellin-Barnes integrals contour (Srivastava et al. [31], page 18, (2.6.3) and after algebraic manipulations, we obtain

$$\text{L.H.S} = b^{-\upsilon} \sum_{R_1,\cdots,R_u=0}^{h_1R_1+\cdots h_uR_u\leqslant L} y_1^{K_1}\cdots y_v^{K_v} c_1^{K_1}\cdots c_v^{K_v} b^{-\sum_{j=1}^{v}\delta_j K_j} \left(\frac{1}{(2\pi\omega)}\right)^{r+s+1}$$

$$\int_{L_1}\cdots\int_{L_r}\int_{L'_1}\cdots\int_{L'_s} \psi(s_1,\cdots,s_r)\psi(t'_1,\cdots,t'_s)\prod_{k=1}^{r}\theta_k(s_k)z_k^{s_k}\prod_{j=1}^{s}\theta'_k(s_k)t_k^{t_k}\, b^{-\upsilon-\sum_{j=1}^{r}\omega s_i - \sum_{j=1}^{s}\omega' t_i}$$

$$\int_L \frac{\Gamma(\upsilon+\sum_{k=1}^{v}\delta_k K_k+\sum_{i=1}^{r}\omega_i s_i+\sum_{j=1}^{s}\omega'_i st_i+u)}{\Gamma(\upsilon+\sum_{k=1}^{v}\delta_k K_k+\sum_{i=1}^{r}\omega_i s_i+\sum_{j=1}^{s}\omega'_i st_i)\Gamma(1+u)}\left(-\frac{a}{b}\right)^u$$

$$\left(I_{0,x}^{\alpha,\alpha',\beta,\beta'\eta} t^{\mu+\sum_{k=1}^{v}\lambda_k K+\sum_{i=1}^{r}\sigma_i s_i+\sum_{i=1}^{r}\sigma'_i t_i}\right)(x)\mathrm{d}u\mathrm{d}s_1\cdots\mathrm{d}s_r\mathrm{d}t_1\cdots\mathrm{d}t_s$$

Now using the lemma 1. Finally interpreting the resulting Mellin-Barnes integrals contour as a multivariable Gimel-function of $(r+s+1)$-variables, we obtain the desired result (3.1).

Let

Ref

31. H. M. Srivastava, K.C. Gupta and S. P. Goyal, The H-function of One and Two Variables with Applications, South Asian Publications, New Delhi, Madras, 1982. [32].

$$A_2 = (1-\upsilon-\sum_{j=1}^{\upsilon}\lambda_j K_j;\sigma_1,\cdots,\sigma_r,\sigma'_1,\cdots,\sigma'_s,1;1),(\eta+\mu-\alpha-\alpha'+\sum_{j=1}^{K}\delta_j K_j;\omega_1,\cdots,\omega_r,\omega'1,\cdots,\omega'_s,1;1)$$

$$(\beta+\mu+\sum_{j=1}^{\upsilon}\lambda_j K_j-\alpha-\alpha';\omega_1,\cdots,\omega_r,\omega'_1,\cdots,\omega'_s,1;1),(\mu+\eta-\alpha-\beta'+\sum_{j=1}^{K}\delta_j K_j;\omega_1,\cdots,\omega_r,\omega'_1,\cdots,\omega'_s,1;1) \quad (3.13)$$

Notes

$$B_2 = (\mu+\sum_{j=1}^{\upsilon}\lambda_j K_j;\sigma_1,\cdots,\sigma_r,\sigma'_1,\cdots,\sigma'_s,1;1),(\beta+\mu-\alpha+\sum_{j=1}^{K}\delta_j K_j;\omega_1,\cdots,\omega_r,\omega'1,\cdots,\omega'_s,1;1)$$

$$(1-\mu-\beta'-\sum_{j=1}^{\upsilon}\lambda_j K_j;\sigma_1,\cdots,\sigma_r,\sigma'_1,\cdots,\sigma'_s,1;1),(1+\alpha'+\beta-\mu-\eta-\sum_{j=1}^{K}\delta_j K_j;\omega_1,\cdots,\omega_r,\omega'_1,\cdots,\omega'_s,1;1) \quad (3.14)$$

Global Journal of Science Frontier Research (F) Volume XVIII Issue VII Version I Year 2018

We have the following resulting

Theorem 2.

$$\left\{\left(I_{x,\infty}^{\alpha,\alpha',\beta,\beta',\eta} t^{\mu-1}(b-at)^{-\upsilon} S_{N_1,\cdots,N_\upsilon}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_\upsilon}\begin{pmatrix} c_1 t^{\lambda_1}(b-at)^{-\delta_1} \\ \cdot \\ \cdot \\ \cdot \\ c_\upsilon t^{\lambda_\upsilon}(b-at)^{-\delta_\upsilon}\end{pmatrix} \gimel \begin{pmatrix} z_1 t^{\sigma_1}(b-at)^{-\omega_1} \\ \cdot \\ \cdot \\ \cdot \\ z_r t^{\sigma_r}(b-at)^{-\omega_r}\end{pmatrix}\right.\right.$$

$$\left.\left.\gimel\begin{pmatrix} z'_1 t^{\sigma'_1}(b-at)^{-\omega'_1} \\ \cdot \\ \cdot \\ \cdot \\ z'_s t^{\sigma'_s}(b-at)^{-\omega'_s}\end{pmatrix}\right)\right\}(x) = b^{-\upsilon} x^{\mu-\alpha-\alpha'+\eta-1}\sum_{K_1=1}^{[N_1/M_1]}\cdots\sum_{K_\upsilon=1}^{[N_\upsilon/M_\upsilon]} a_\upsilon c_1^{K_1}\cdots c_\upsilon^{K_\upsilon}$$

$$\upsilon^{-\sum_{j=1}^{\upsilon}\delta_i K_j} x^{\sum_{j=1}^{\upsilon}\lambda_i K_j} \gimel_{X;p_{i_r}+p'_{i'_s}+4,q_{i_r}+q'_{i'_s}+4,\tau_{i_r},\tau'_{i'_s}:R_r:R'_s:Y;0,1}^{U;0,n_r+n'_s+4:V,1,0}\left(\begin{array}{c|c} z_1\frac{x^{\sigma_1}}{b^{\omega_1}} & \mathbb{A},\ ,\mathrm{A}_2,\mathbf{A}:A \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ z_r\frac{x^{\sigma_r}}{b^{\omega_r}} & \cdot \\ z_1\frac{x^{\sigma'_1}}{b^{\omega'_1}} & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ z'_s\frac{x^{\sigma'_s}}{b^{\omega'_s}} & \mathbb{B},\mathbf{B},\ \mathrm{B}_2:B,(0,1;1) \\ \frac{ax}{b} & \end{array}\right) \quad (3.14)$$

Provided

$a,b,\alpha,\beta,\eta,\mu,\upsilon,\delta_k,\omega_i,\omega'_j \in \mathbb{C}, k=1,\cdots,\upsilon; i=1\cdots,r; j=1,\cdots,s$

$\lambda_k,\sigma_i,\sigma'_j > 0;\ \ k=1,\cdots,\upsilon; i=1\cdots,r; j=1,\cdots,s$

$|arg\,(z_i)| < \frac{1}{2}\pi A_i^{(k)}$ and $A_i^{(k)}$ is defined by (2.4), $|arg\,(z'_i)| < \frac{1}{2}\pi A_i'^{(k)}; \left|\frac{a}{b}x\right| < 1$

$$Re(\mu)-\sum_{i=1}^{r}\sigma_i\min_{1\leqslant j\leqslant m^{(i)}}Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right]-\sum_{i=1}^{s}\sigma'_i\min_{1\leqslant j\leqslant m'^{(i)}}Re\left[D_j'^{(i)}\left(\frac{d_j'^{(i)}}{\delta_j'^{(i)}}\right)\right]<1+min[Re(-\beta),Re(\alpha+\alpha'-\eta),Re(\alpha+\beta'-\eta)]$$

$$Re(v)-\sum_{i=1}^{r}\omega_i\min_{1\leqslant j\leqslant m^{(i)}}Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right]-\sum_{i=1}^{s}\omega'_i\min_{1\leqslant j\leqslant m'^{(i)}}Re\left[D_j'^{(i)}\left(\frac{d_j'^{(i)}}{\delta_j'^{(i)}}\right)\right]<1+max[Re(-\beta),Re(\alpha+\alpha'-\eta),Re(\alpha+\beta'-\eta)]$$

Notes

To prove the equation (3.14), we use the similar method that formula (3.5) by using the lemma 2.
Let

$$A_3=(1-\mu-\sum_{j=1}^{v}\lambda_jK_j;\sigma_1,\cdots,\sigma_r,\sigma'_1,\cdots,\sigma'_s,1;1),=(1-v-\sum_{j=1}^{v}\delta_jK_j;\omega_1,\cdots,\omega_r,\omega'_1,\cdots,\omega'_s,1;1),$$

$$(1-\mu-\eta+\beta-\sum_{j=1}^{v}\lambda_jK_j;\sigma_1,\cdots,\sigma_r,\sigma'_1,\cdots,\sigma'_s,1;1) \tag{3.15}$$

$$B_3=(1-v-\sum_{j=1}^{v}\delta_jK_j;\omega_1,\cdots,\omega_r,\omega'_1,\cdots,\omega'_s,0;1),(1+\beta-\mu-\sum_{j=1}^{K}\lambda_jK_j;\sigma_1,\cdots,\sigma_r,\sigma'_1,\cdots,\sigma'_s,1;1)$$

$$(1-\mu-\eta-\alpha-\sum_{j=1}^{K}\lambda_jK_j;\sigma_1,\cdots,\sigma_r,\sigma'_1,\cdots,\sigma'_s,1;1) \tag{3.16}$$

Theorem 3.

$$\left\{\left(I_{0+}^{\alpha,\beta,\eta}t^{\mu-1}(b-at)^{-v}S_{N_1,\cdots,N_v}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}\begin{pmatrix}c_1t^{\lambda_1}(b-at)^{-\delta_1}\\ \cdot\\ \cdot\\ \cdot\\ c_vt^{\lambda_v}(b-at)^{-\delta_v}\end{pmatrix}\gimel\begin{pmatrix}z_1t^{\sigma_1}(b-at)^{-\omega_1}\\ \cdot\\ \cdot\\ \cdot\\ z_rt^{\sigma_r}(b-at)^{-\omega_r}\end{pmatrix}\right.\right.$$

$$\left.\left.\gimel\begin{pmatrix}z'_1t^{\sigma'_1}(b-at)^{-\omega'_1}\\ \cdot\\ \cdot\\ \cdot\\ z'_st^{\sigma'_s}(b-at)^{-\omega'_s}\end{pmatrix}\right)\right\}(x)=b^{-v}x^{\mu-\alpha-\alpha'+\eta-1}\sum_{K_1=1}^{[N_1/M_1]}\cdots\sum_{K_v=1}^{[N_v/M_v]}a_vc_1^{K_1}\cdots c_v^{K_v}$$

$$v^{-\sum_{j=1}^{v}\delta_iK_j}x^{\sum_{j=1}^{v}\lambda_iK_j}\gimel_{X;p_{i_r}+p'_{i'_s}+3,q_{i_r}+q'_{i'_s}+3,\tau_{i_r},\tau'_{i'_s}:R_r:R'_s:Y;0,1}^{U;0,n_r+n'_s+3:V,1,0}\left(\begin{array}{c|c}z_1\frac{x^{\sigma_1}}{b^{\omega_1}} & \mathbb{A},\ ,\mathrm{A}_3,\mathbf{A}:A\\ \cdot & \cdot\\ \cdot & \cdot\\ \cdot & \cdot\\ z_r\frac{x^{\sigma_r}}{b^{\omega_r}} & \cdot\\ z_1\frac{x^{\sigma'_1}}{b^{\omega'_1}} & \cdot\\ \cdot & \cdot\\ \cdot & \cdot\\ \cdot & \cdot\\ z'_s\frac{x^{\sigma'_s}}{b^{\omega'_s}} & \mathbb{B},\mathbf{B},\ \mathrm{B}_3:B,(0,1;1)\\ \frac{ax}{b} & \end{array}\right) \tag{3.17}$$

Provided

$$a,b,\alpha,\beta,\eta,\mu,v,\delta_k,\omega_i,\omega'_j\in\mathbb{C},k=1,\cdots,v;i=1\cdots,r;j=1,\cdots,s$$

$$\lambda_k,\sigma_i,\sigma'_j>0;\ \ k=1,\cdots,v;i=1\cdots,r;j=1,\cdots,s$$

$|arg\,(z_i)| < \frac{1}{2}\pi A_i^{(k)}$ and $A_i^{(k)}$ is defined by (2.4), $|arg\,(z'_i)| < \frac{1}{2}\pi A_i'^{(k)}; \left|\frac{a}{b}x\right| < 1$

$$Re(\mu) + \sum_{i=1}^{r} \sigma_i \min_{1\leqslant j\leqslant m^{(i)}} Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right] + \sum_{i=1}^{s} \sigma'_i \min_{1\leqslant j\leqslant m'^{(i)}} Re\left[D_j'^{(i)}\left(\frac{d_j'^{(i)}}{\delta_j'^{(i)}}\right)\right] > max[0, Re(\beta-\eta)]$$

$$Re(\upsilon) + \sum_{i=1}^{r} \omega_i \min_{1\leqslant j\leqslant m^{(i)}} Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right] + \sum_{i=1}^{s} \omega'_i \min_{1\leqslant j\leqslant m'^{(i)}} Re\left[D_j'^{(i)}\left(\frac{d_j'^{(i)}}{\delta_j'^{(i)}}\right)\right] > max[0, Re(\beta-\eta)]$$

Notes

To prove the formula (3.17), we use the similar method that the theorem 1 by using the lemma 3.

Let

$$A_4 = (1-\mu-\sum_{j=1}^{v}\delta_j K_j;\omega_1,\cdots,\omega_r,\omega'_1,\cdots,\omega'_s,1;1),(-\eta+\mu+\sum_{j=1}^{v}\lambda_j K_j;\sigma_1,\cdots,\sigma_r,\sigma'_1,\cdots,\sigma'_s,1;1),$$

$$(1-\mu-\eta+\beta-\sum_{j=1}^{v}\lambda_j K_j;\sigma_1,\cdots,\sigma_r,\sigma'_1,\cdots,\sigma'_s,1;1) \tag{3.18}$$

$$B_4 = (-\upsilon-\sum_{j=1}^{v}\delta_j K_j;\eta_1,\cdots,\omega_r,\omega'_1,\cdots,\omega'_s,0;1),(\mu+\sum_{j=1}^{K}\lambda_j K_j;_1,\cdots,\sigma_r,\sigma'_1,\cdots,\sigma'_s,1;1)$$

$$(-\alpha-\beta-\eta+\mu+\sum_{j=1}^{K}\lambda_j K_j;\sigma_1,\cdots,\sigma_r,\sigma'_1,\cdots,\sigma'_s,1;1) \tag{3.19}$$

We have the formula.

Theorem 4.

$$\{(I_{x,\infty}^{\alpha,\beta,\eta}t^{\mu-1}(b-at)^{-\upsilon}S_{N_1,\cdots,N_v}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}\begin{pmatrix} c_1t^{\lambda_1}(b-at)^{-\delta_1}\\ \cdot\\ \cdot\\ \cdot\\ c_vt^{\lambda_v}(b-at)^{-\delta_v}\end{pmatrix}\gimel\begin{pmatrix} z_1t^{\sigma_1}(b-at)^{-\omega_1}\\ \cdot\\ \cdot\\ \cdot\\ z_rt^{\sigma_r}(b-at)^{-\omega_r}\end{pmatrix}$$

$$\gimel\begin{pmatrix} z'_1t^{\sigma'_1}(b-at)^{-\omega'_1}\\ \cdot\\ \cdot\\ \cdot\\ z'_st^{\sigma'_s}(b-at)^{-\omega'_s}\end{pmatrix}\Bigg)\Bigg\}(x) = b^{-\upsilon}x^{\mu-\alpha-\alpha'+\eta-1}\sum_{K_1=1}^{[N_1/M_1]}\cdots\sum_{K_v=1}^{[N_v/M_v]}a_v c_1^{K_1}\cdots c_v^{K_v}$$

$$v^{-\sum_{j=1}^{v}\delta_i K_j}x^{\sum_{j=1}^{v}\lambda_i K_j}\gimel^{U;0,n_r+n'_s+3:V,1,0}_{X;p_{i_r}+p'_{i'_s}+3,q_{i_r}+q'_{i'_s}+3,\tau_{i_r},\tau'_{i'_s}:R_r:R'_s:Y;0,1}\left(\begin{array}{c|c} z_1\frac{x^{\sigma_1}}{b^{\omega_1}} & \mathbb{A}, ,A_4,\mathbf{A}:A\\ \cdot & \cdot\\ \cdot & \cdot\\ \cdot & \cdot\\ z_r\frac{x^{\sigma_r}}{b^{\omega_r}} & \cdot\\ z_1\frac{x^{\sigma'_1}}{b^{\omega'_1}} & \cdot\\ \cdot & \cdot\\ \cdot & \cdot\\ \cdot & \cdot\\ z'_s\frac{x^{\sigma'_s}}{b^{\omega'_s}} & \\ \frac{ax}{b} & \mathbb{B},\mathbf{B}, B_4:B,(0,1;1)\end{array}\right) \tag{3.20}$$

Notes

Provided

$$a,b,\alpha,\beta,\eta,\mu,\upsilon,\delta_k,\omega_i,\omega'_j \in \mathbb{C}, k=1,\cdots,v; i=1\cdots,r; j=1,\cdots,s$$

$$\lambda_k,\sigma_i,\sigma'_j>0;\;\; k=1,\cdots,v; i=1\cdots,r; j=1,\cdots,s$$

$|arg\,(z_i)| < \frac{1}{2}\pi A_i^{(k)}$ and $A_i^{(k)}$ is defined by (2.4), $|arg\,(z'_i)| < \frac{1}{2}\pi A_i'^{(k)}; \left|\frac{a}{b}x\right| < 1$

$$Re(\mu)-\sum_{i=1}^{r}\sigma_i \min_{1\leqslant j\leqslant m^{(i)}} Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right]-\sum_{i=1}^{s}\sigma'_i \min_{1\leqslant j\leqslant m'^{(i)}} Re\left[D_j'^{(i)}\left(\frac{d_j'^{(i)}}{\delta_j'^{(i)}}\right)\right]<1+min[Re(-\beta),Re(\alpha+\alpha'-\eta),Re(\alpha+\beta'-\eta)]$$

$$Re(\upsilon)-\sum_{i=1}^{r}\omega_i \min_{1\leqslant j\leqslant m^{(i)}} Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right]-\sum_{i=1}^{s}\omega'_i \min_{1\leqslant j\leqslant m'^{(i)}} Re\left[D_j'^{(i)}\left(\frac{d_j'^{(i)}}{\delta_j'^{(i)}}\right)\right]<1+max[Re(-\beta),Re(\alpha+\alpha'-\eta),Re(\alpha+\beta'-\eta)]$$

Provided

$$a,b,\alpha,\beta,\eta,\mu,\upsilon,\delta_k,\omega_i,\omega'_j \in \mathbb{C}, k=1,\cdots,v; i=1\cdots,r; j=1,\cdots,s$$

$$\lambda_k,\sigma_i,\sigma'_j>0;\;\; k=1,\cdots,v; i=1\cdots,r; j=1,\cdots,s$$

$|arg\,(z_i)| < \frac{1}{2}\pi A_i^{(k)}$ and $A_i^{(k)}$ is defined by (2.4), $|arg\,(z'_i)| < \frac{1}{2}\pi A_i'^{(k)}; \left|\frac{a}{b}x\right| < 1$

$$Re(\mu)-\sum_{i=1}^{r}\sigma_i \min_{1\leqslant j\leqslant m^{(i)}} Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right]-\sum_{i=1}^{s}\sigma'_i \min_{1\leqslant j\leqslant m'^{(i)}} Re\left[D_j'^{(i)}\left(\frac{d_j'^{(i)}}{\delta_j'^{(i)}}\right)\right]<1+min[Re(Re(\eta),Re(\beta)]$$

$$Re(\upsilon)-\sum_{i=1}^{r}\omega_i \min_{1\leqslant j\leqslant m^{(i)}} Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right]-\sum_{i=1}^{s}\omega'_i \min_{1\leqslant j\leqslant m'^{(i)}} Re\left[D_j'^{(i)}\left(\frac{d_j'^{(i)}}{\delta_j'^{(i)}}\right)\right]<1+max[Re(\beta),Re(\eta)]$$

To prove the theorem 4, we use the similar method that the equation (3.5) by using the lemma 4.

IV. Particular Cases

In this section, we shall see four particular cases.

If we put $\beta=-\alpha$ in the theorem three, we get

Corollary 1.

$$\left\{\left(I_-^{\alpha}\, t^{\mu-1}(b-at)^{-\upsilon} S_{N_1,\cdots,N_v}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}\begin{pmatrix} c_1 t^{\lambda_1}(b-at)^{-\delta_1} \\ \cdot \\ \cdot \\ \cdot \\ c_v t^{\lambda_v}(b-at)^{-\delta_v}\end{pmatrix} \gimel \begin{pmatrix} z_1 t^{\sigma_1}(b-at)^{-\omega_1} \\ \cdot \\ \cdot \\ \cdot \\ z_r t^{\sigma_r}(b-at)^{-\omega_r}\end{pmatrix}\right.\right.$$

$$\left.\left.\gimel\begin{pmatrix} z'_1 t^{\sigma'_1}(b-at)^{-\omega'_1} \\ \cdot \\ \cdot \\ \cdot \\ z'_s t^{\sigma'_s}(b-at)^{-\omega'_s}\end{pmatrix}\right)\right\}(x) = b^{-\upsilon} x^{\mu-\beta-1}\sum_{K_1=1}^{[N_1/M_1]}\cdots\sum_{K_v=1}^{[N_v/M_v]} a_v c_1^{K_1}\cdots c_v^{K_v}$$

$$v^{-\sum_{j=1}^{v}\delta_i K_j} x^{\sum_{j=1}^{v}\lambda_i K_j} \gimel^{U;0,n_r+n'_s+2:V,1,0}_{X;p_{i_r}+p'_{i'_s}+2,q_{i_r}+q'_{i'_s}+2,\tau_{i_r},\tau'_{i'_s}:R_r:R'_s:Y;0,1}\left(\begin{array}{c|c} \mathrm{z}_1\frac{x^{\sigma_1}}{b^{\omega_1}} & \mathbb{A},\ ,\mathrm{A}_5,\mathbf{A}:A \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{z}_r\frac{x^{\sigma_r}}{b^{\omega_r}} & \cdot \\ \mathrm{z}_1\frac{x^{\sigma'_1}}{b^{\omega'_1}} & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{z}'_s\frac{x^{\sigma'_s}}{b^{\omega'_s}} & \mathbb{B},\mathbf{B},\ \mathrm{B}_5:B,(0,1;1) \\ \frac{ax}{b} & \end{array}\right) \tag{4.1}$$

where

$$A_5 = (1-v-\sum_{j=0}^{v}\delta_j K_j;\omega_1,\cdots,\omega_r,\omega'_1,\cdots,\omega'_s,1;1),(1-\mu-\sum_{j=1}^{v}\lambda_j K_j,\sigma_1,\cdots,\sigma_r,\sigma'_1,\cdots,\sigma'_s,1;1) \tag{4.2}$$

$$A_5 = (1-v-\sum_{j=0}^{v}\delta_j K_j;\omega_1,\cdots,\omega_r,\omega'_1,\cdots,\omega'_s,0;1), B_5 = (1-\mu-\alpha-\eta-\sum_{j=1}^{v}\lambda_j K_j,\sigma_1,\cdots,\sigma_r,\sigma'_1,\cdots,\sigma'_s,1;1) \tag{4.3}$$

under the same existence conditions that formula (3.17) with. $\beta=-\alpha$.

If $\beta=0$ in theorem three, we have

Corollary 2.

$$\left(I^{+}_{\eta,\alpha}\, t^{\mu-1}(b-at)^{-v} S^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}_{N_1,\cdots,N_v}\begin{pmatrix} \mathrm{c}_1 t^{\lambda_1}(b-at)^{-\delta_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{c}_v t^{\lambda_v}(b-at)^{-\delta_v}\end{pmatrix} \gimel \begin{pmatrix} \mathrm{z}_1 t^{\sigma_1}(b-at)^{-\omega_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r t^{\sigma_r}(b-at)^{-\omega_r}\end{pmatrix}\right.$$

$$\left.\gimel\begin{pmatrix} \mathrm{z}'_1 t^{\sigma'_1}(b-at)^{-\omega'_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}'_s t^{\sigma'_s}(b-at)^{-\omega'_s}\end{pmatrix}\right)\Bigg\}(x) = b^{-v}x^{\mu-\beta-1}\sum_{K_1=1}^{[N_1/M_1]}\cdots\sum_{K_v=1}^{[N_v/M_v]} a_v c_1^{K_1}\cdots c_v^{K_v}$$

$$v^{-\sum_{j=1}^{v}\delta_i K_j} x^{\sum_{j=1}^{v}\lambda_i K_j} \gimel^{U;0,n_r+n'_s+2:V,1,0}_{X;p_{i_r}+p'_{i'_s}+2,q_{i_r}+q'_{i'_s}+2,\tau_{i_r},\tau'_{i'_s}:R_r:R'_s:Y;0,1}\left(\begin{array}{c|c} \mathrm{z}_1\frac{x^{\sigma_1}}{b^{\omega_1}} & \mathbb{A},\ ,\mathrm{A}_6,\mathbf{A}:A \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{z}_r\frac{x^{\sigma_r}}{b^{\omega_r}} & \cdot \\ \mathrm{z}_1\frac{x^{\sigma'_1}}{b^{\omega'_1}} & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{z}'_s\frac{x^{\sigma'_s}}{b^{\omega'_s}} & \mathbb{B},\mathbf{B},\ \mathrm{B}_6:B,(0,1;1) \\ \frac{ax}{b} & \end{array}\right) \tag{4.4}$$

Notes

Notes

where

$$A_6 = (1 - \upsilon - \sum_{j=0}^{v} \delta_j K_j; \omega_1, \cdots, \omega_r, \omega'_1, \cdots, \omega'_s, 0; 1), (1 - \mu - \eta - \sum_{j=1}^{v} \lambda_j K_j, \sigma_1, \cdots, \sigma_r, \sigma'_1, \cdots, \sigma'_s, 1; 1) \tag{4.5}$$

$$B_6 = (1 - \upsilon - \sum_{j=0}^{v} \delta_j K_j; \omega_1, \cdots, \omega_r, \omega'_1, \cdots, \omega'_s, 0; 1), (1 - \mu - \alpha - \eta - \sum_{j=1}^{v} \lambda_j K_j, \sigma_1, \cdots, \sigma_r, \sigma'_1, \cdots, \sigma'_s, 1; 1) \tag{4.6}$$

provided that

$a, b, \alpha, \beta, \eta, \mu, \upsilon, \delta_k, \omega_i, \omega'_j \in \mathbb{C}, k = 1, \cdots, v; i = 1 \cdots, r; j = 1, \cdots, s$

$\lambda_k, \sigma_i, \sigma'_j > 0;\ \ k = 1, \cdots, v; i = 1 \cdots, r; j = 1, \cdots, s$

$|arg\,(z_i)| < \frac{1}{2}\pi A_i^{(k)}$ and $A_i^{(k)}$ is defined by (2.4), $|arg\,(z'_i)| < \frac{1}{2}\pi A_i'^{(k)}; \left|\frac{a}{b}x\right| < 1$

$$Re(\mu) + \sum_{i=1}^{r} \sigma_i \min_{1 \leqslant j \leqslant m^{(i)}} Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right] + \sum_{i=1}^{s} \sigma'_i \min_{1 \leqslant j \leqslant m'^{(i)}} Re\left[D_j'^{(i)}\left(\frac{d_j'^{(i)}}{\delta_j'^{(i)}}\right)\right] > max[0, Re(-\eta)]$$

$$Re(\upsilon) + \sum_{i=1}^{r} \omega_i \min_{1 \leqslant j \leqslant m^{(i)}} Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right] + \sum_{i=1}^{s} \omega'_i \min_{1 \leqslant j \leqslant m'^{(i)}} Re\left[D_j'^{(i)}\left(\frac{d_j'^{(i)}}{\delta_j'^{(i)}}\right)\right] > max[0, Re(-\eta)]$$

If we put $\beta = -\alpha$ in the equation (3.20), we get

Corollary 3.

$$\left\{\left(I_-^{\alpha} t^{\mu-1}(b-at)^{-\upsilon} S_{N_1,\cdots,N_v}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}\begin{pmatrix} c_1 t^{\lambda_1}(b-at)^{-\delta_1} \\ \cdot \\ \cdot \\ \cdot \\ c_v t^{\lambda_v}(b-at)^{-\delta_v} \end{pmatrix} \gimel \begin{pmatrix} z_1 t^{\sigma_1}(b-at)^{-\omega_1} \\ \cdot \\ \cdot \\ \cdot \\ z_r t^{\sigma_r}(b-at)^{-\omega_r} \end{pmatrix}\right.\right.$$

$$\left.\left.\gimel \begin{pmatrix} z'_1 t^{\sigma'_1}(b-at)^{-\omega'_1} \\ \cdot \\ \cdot \\ \cdot \\ z'_s t^{\sigma'_s}(b-at)^{-\omega'_s} \end{pmatrix}\right)\right\}(x) = b^{-\upsilon} x^{\mu+\alpha-1} \sum_{K_1=1}^{[N_1/M_1]} \cdots \sum_{K_v=1}^{[N_v/M_v]} a_v c_1^{K_1} \cdots c_v^{K_v}$$

$$\upsilon^{-\sum_{j=1}^{v}\delta_i K_j} x^{\sum_{j=1}^{v}\lambda_i K_j} \gimel_{X; p_{i_r}+p'_{i'_s}+2, q_{i_r}+q'_{i'_s}+2, \tau_{i_r}, \tau'_{i'_s}: R_r: R'_s: Y; 0,1}^{U; 0, n_r+n'_s+2: V, 1, 0} \left(\begin{array}{c|c} z_1 \frac{x^{\sigma_1}}{b^{\omega_1}} & \mathbb{A}, \ , \mathrm{A}_7, \mathbf{A} : A \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ z_r \frac{x^{\sigma_r}}{b^{\omega_r}} & \cdot \\ z_1 \frac{x^{\sigma'_1}}{b^{\omega'_1}} & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ z'_s \frac{x^{\sigma'_s}}{b^{\omega'_s}} & \mathbb{B}, \mathbf{B}, \ \mathrm{B}_7 : B, (0, 1; 1) \\ \frac{ax}{b} & \end{array} \right) \tag{4.7}$$

Global Journal of Science Frontier Research (F) Volume XVIII Issue VII Version I Year 2018

where

$$A_7 = (1-\upsilon-\sum_{j=0}^{v}\delta_j K_j;\omega_1,\cdots,\omega_r,\omega'_1,\cdots,\omega'_s,1;1),(\alpha+\mu+\sum_{j=1}^{v}\lambda_j K_j,\sigma_1,\cdots,\sigma_r,\sigma'_1,\cdots,\sigma'_s,1;1) \tag{4.8}$$

$$A_7 = (1-\upsilon-\sum_{j=0}^{v}\delta_j K_j;\omega_1,\cdots,\omega_r,\omega'_1,\cdots,\omega'_s,0;1), B_5 = (\mu+\sum_{j=1}^{v}\lambda_j K_j,\sigma_1,\cdots,\sigma_r,\sigma'_1,\cdots,\sigma'_s,1;1) \tag{4.9}$$

under the same existence conditions that formula (3.20) with. $\beta=-\alpha$

If $\beta=0$ in theorem four, we have

Corollary 4.

Let

$$A_8 = (1-\upsilon-\sum_{j=0}^{v}\delta_j K_j;\omega_1,\cdots,\omega_r,\omega'_1,\cdots,\omega'_s,1;1),(-\alpha-\eta+\mu+\sum_{j=1}^{v}\lambda_j K_j,\sigma_1,\cdots,\sigma_r,\sigma'_1,\cdots,\sigma'_s,1;1) \tag{4.10}$$

$$B_8 = (1-\upsilon-\sum_{j=0}^{v}\delta_j K_j;\omega_1,\cdots,\omega_r,\omega'_1,\cdots,\omega'_s,0;1),(\mu-\alpha-\eta+\sum_{j=1}^{v}\lambda_j K_j,\sigma_1,\cdots,\sigma_r,\sigma'_1,\cdots,\sigma'_s,1;1) \tag{4.11}$$

$$\{K^{-}_{\eta,\alpha} t^{\mu-1}(b-at)^{-\upsilon} S^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}_{N_1,\cdots,N_v}\begin{pmatrix} \mathrm{c}_1 t^{\lambda_1}(b-at)^{-\delta_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{c}_v t^{\lambda_v}(b-at)^{-\delta_v}\end{pmatrix} \gimel \begin{pmatrix} \mathrm{z}_1 t^{\sigma_1}(b-at)^{-\omega_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r t^{\sigma_r}(b-at)^{-\omega_r}\end{pmatrix}$$

$$\gimel\begin{pmatrix} \mathrm{z}'_1 t^{\sigma'_1}(b-at)^{-\omega'_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}'_s t^{\sigma'_s}(b-at)^{-\omega'_s}\end{pmatrix}\Bigg)\Bigg)\Bigg\}(x) = b^{-\upsilon}x^{\mu-1}\sum_{K_1=1}^{[N_1/M_1]}\cdots\sum_{K_v=1}^{[N_v/M_v]} a_v c_1^{K_1}\cdots c_v^{K_v}$$

$$\upsilon^{-\sum_{j=1}^{v}\delta_i K_j} x^{\sum_{j=1}^{v}\lambda_i K_j} \gimel^{U;0,n_r+n'_s+2:V,1,0}_{X;p_{i_r}+p'_{i'_s}+2,q_{i_r}+q'_{i'_s}+2,\tau_{i_r},\tau'_{i'_s}:R_r:R'_s:Y;0,1}\left(\begin{array}{c|c} \mathrm{z}_1\frac{x^{\sigma_1}}{b^{\omega_1}} & \\ \cdot & \mathbb{A},\ ,\mathrm{A}_8,\mathbf{A}:A \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{z}_r\frac{x^{\sigma_r}}{b^{\omega_r}} & \cdot \\ \mathrm{z}_1\frac{x^{\sigma'_1}}{b^{\omega'_1}} & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{z}'_s\frac{x^{\sigma'_s}}{b^{\omega'_s}} & \mathbb{B},\mathbf{B},\ \mathrm{B}_8:B,(0,1;1) \\ \frac{ax}{b} & \end{array}\right) \tag{4.12}$$

Provided

$a,b,\alpha,\beta,\eta,\mu,\upsilon,\delta_k,\omega_i,\omega'_j\in\mathbb{C}, k=1,\cdots,v; i=1\cdots,r; j=1,\cdots,s$

$\lambda_k,\sigma_i,\sigma'_j>0;\ \ k=1,\cdots,v; i=1\cdots,r; j=1,\cdots,s$

Notes

under the same existence conditions that equation (3.20) with. $\beta = 0$.

Remark: By the similar procedure, the results of this document can be extended to the product of any finite number of multivariable Gimel-functions and a class of multivariable polynomials defined by Srivastava [30].
Agarwal [1,2] has studied the fractional integration about the multivariable H-function.

V. Conclusion

In this paper, we have obtained several theorems of the generalized fractional integral operators given by Saigo-Maeda and Saigo. The images have been developed regarding the product of the two multivariable Gimel-functions and a general class of multivariable polynomials in a compact and elegant form with the help of Saigo-Maeda and Saigo operators. Most of the results obtained in this paper are useful in deriving the composition formulae involving Riemann–Liouville, Erdelyi–Kober fractional calculus operators and multivariable Gimel functions. The findings of this paper provide an extension of the results given earlier by Kilbas [12], Kilbas and Saigo [13], Kilbas and Sebastain [14], Saxena et al.[29] and Gupta et al.[7] as mentioned earlier.

Ref

30. H.M. Srivastava, A multilinear generating function for the Konhauser set of biorthogonal polynomials suggested by Laguerre polynomial, Pacific. J. Math. 177(1985), 183-191.

References Références Referencias

1. P. Agarwal, Fractional integration of the product of two H-functions and a general class of polynomials, Asian J. Appl. Sci. 5(3) (2012), pp. 144–153.
2. P. Agarwal, Fractional integration of the product of two multivariable H-functions and a general class of polynomials, advances in Applied Mathematics and Approximation Theory, Springer (2013), Chapter 23, 359-374.
3. F.Y. Ayant, A Fractional Integration of the Product of Two Multivariable Aleph-Functions and a General Class of Polynomials I, Int. Jr. of Mathematical Sciences & Applications, 7(2) (2017), 181-198.
4. F.Y. Ayant, Some transformations and idendities form multivariable Gimel-function, International Journal of Matematics and Technology (IJMTT), 59(4) (2018), 248-255.
5. B.L.J. Braaksma, Asymptotics expansions and analytic continuations for a class of Barnes-integrals, Compositio Math. 15 (1962-1964), 239-341.
6. L.Debnath and D. Bhatta, Integral Transforms and Their Applications, Chapman and Hall/CRC Press, Boca Raton FL, 2006.
7. K. C. Gupta, K. Gupta and A. Gupta, Generalized fractional integration of the product of two H-functions. J. Rajasthan Acad. Phy. Sci., 9(3), 203–212(2010).
8. S. L. Kalla, Integral operators involving Fox's H-function I, Acta Mexicana Cienc. Tecn. 3, 117–122, (1969).
9. S. L. Kalla, Integral operators involving Fox's H-function II, Acta Mexicana Cienc. Tecn. 7, 72–79, (1969).
10. S. L. Kalla and R. K. Saxena, Integral operators involving hypergeometric functions, Math. Z. 108, 231–234, (1969)
11. S. L. Kalla and R. K. Saxena, Integral operators involving hypergeometric functions II, Univ. Nac. Tucuman, Rev. Ser., A24, 31–36, (1974).
12. A. A. Kilbas, Fractional calculus of the generalized Wright function, Fract.Calc.Appl.Anal.8 (2), 113–126, (2005).
13. A. Kilbas and M. Saigo, H-transforms, theory and applications, Chapman & Hall/CRC, Press, Boca Raton, FL, 2004.
14. A. Kilbas and N. Sebastain, Generalized fractional integration of Bessel function of first kind, Integral transform and Spec. Funct. 19(12), 869–883,(2008).
15. A. Kilbas, H. M. Srivastava and J. J. Trujillo, Theory and Applications of Fractional Differential Equations, (North-Holland Mathematics), Elsevier, 540, 2006.
16. Kiryakova, Generalized Fractional Calculus and Applications, Longman Scientific & Tech., Essex, 1994.
17. V. Kiryakova, A brief story about the operators of the generalized fractional calculus, Fract. Calc. Appl. Anal. 11 (2), 203–220, (2008).

18. E. R. Love, Some integral equations involving hypergeometric functions, Proc. Edin. Math. Soc. 15 (3), 169–198, (1967).
19. O. I. Marichev, Volterra equation of Mellin convolution type with a Horn function in the kernel (In Russian). Izv. AN BSSR Ser. Fiz.-Mat. Nauk 1, 128–129, (1974).
20. A. C. McBride, Fractional powers of a class of ordinary differential operators, Proc. London, Math. Soc. (III) 45, 519–546, (1982).
21. K. S. Miller and B. Ross An Introduction to the Fractional Calculus and Differential Equations, A Wiley Interscience Publication, John Wiley and Sons Inc., New York, 1993.
22. Y.N. Prasad, Multivariable I-function , Vijnana Parisha Anusandhan Patrika 29 (1986), 231-237.
23. J. Prathima, V. Nambisan and S.K. Kurumujji, A Study of I-function of Several Complex Variables, International Journal of Engineering Mathematics Vol (2014), 1-12.
24. M. Saigo, A remark on integral operators involving the Gauss hypergeometric functions, Math. Rep. Kyushu Univ. 11, 135–143, (1978).
25. M. Saigo, A certain boundary value problem for the Euler-Darboux equation I, Math. Japonica, 24 (4) (1979), 377–385, 139.
26. M. Saigo, A certain boundary value problem for the Euler-Darboux equation II, Math. Japonica 25 (2), 211–220, (1980).
27. M. Saigo and N. Maeda, More Generalization of Fractional Calculus, Transform Methods and Special Functions, Varna, Bulgaria, 1996, pp. 386–400.
28. S. Samko, A. Kilbas and O. Marichev Fractional Integrals and Derivatives. Theory and Applications, Gordon & Breach Sci. Publ., New York, 1993.
29. R. K. Saxena, J. Ram and D. L. Suthar, Fractional calculus of generalized Mittag- Leffler functions, J. India Acad. Math.(1),165–172,(2009).
30. H.M. Srivastava, A multilinear generating function for the Konhauser set of biorthogonal polynomials suggested by Laguerre polynomial, Pacific. J. Math. 177(1985), 183-191.
31. H. M. Srivastava, K.C. Gupta and S. P. Goyal, The H-function of One and Two Variables with Applications, South Asian Publications, New Delhi, Madras, 1982. [32].
32. H.M. Srivastava and R. Panda, Some expansion theorems and generating relations for the H - function of several complex variables. Comment. Math. Univ. St. Paul. 24 (1975), 119-137.
33. H.M. Srivastava and R. Panda, Some expansion theorems and generating relations for the H-function of several complex variables II. Comment. Math. Univ. St. Paul. 25 (1976), 167-197.

Notes