Global Journal of Science Frontier Research: F
Mathematics and Decision Sciences
Volume 18 Issue 2 Version 1.0 Year 2018
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

On the Efficiency of Some Selected Designs: A Case of Randomized Block Designs

By Asogwa, O. C., & Eze, C. M.
Federal University Ndufu Alike Ikwo

Abstract- This work examines the Relative Efficiency (RE) of some selected Randomized Complete Block Designs $(RCBD)$.The efficiency of the selected designs showed that design B was the most preferable having Mean Square Error of 6.14, followed by design C with Mean Square Error of 9.11 and design A with Mean Square Error of 18.08. The results from the pair-wise relative efficiency of the selected designs show that $RE(B, A) = 0.34$ with the smallest relative efficiency value and $RE(A, B) = 2.94$ with the largest relative efficiency value. We recommended design B as the best design for this particular problem since its mean square error remains the smallest.

Keywords: *RCBD, efficiency, missing value and estimation.*

GJSFR-F Classification: *MSC 2010: 13P25*

ONTHEEFFICIENCYOFSOMESELECTEDDESIGNSACASEOFRANDOMIZEDBLOCKDESIGNS

Strictly as per the compliance and regulations of:

Notes

On the Efficiency of Some Selected Designs: A Case of Randomized Block Designs

Asogwa, O. C. [α] & Eze, C. M. [σ]

Abstract- This work examines the Relative Efficiency (RE) of some selected Randomized Complete Block Designs ($RCBD$). The efficiency of the selected designs showed that design B was the most preferable having Mean Square Error of 6.14, followed by design C with Mean Square Error of 9.11 and design A with Mean Square Error of 18.08. The results from the pair-wise relative efficiency of the selected designs show that $RE(B,A) = 0.34$ with the smallest relative efficiency value and $RE(A,B) = 2.94$ with the largest relative efficiency value. We recommended design B as the best design for this particular problem since its mean square error remains the smallest.

Keywords: RCBD, efficiency, missing value and estimation.

I. Introduction

The basic concepts of the statistical design of experiments and data analysis were discovered as early as 20th century as a cost effective research design tool to improve researches, for instance in agriculture and every other fields of study where experimentation is possible. Moreover, an experiments performed by an investigators or a researchers virtually in all the fields of inquiry, are usually, to discover something about a particular process or system, in relative to cost effectiveness.

However, the missingness of observation is common in scientific experiments. In statistical planning, it is never possible to anticipate beforehand which of the observations are going to be missing after the experiment. With this regards, the experimenter cannot redo the experiment with a different design because it costs money, time and effort, etc. One of such experimental designs in which missing observation can occur is a randomized block design. A randomized block design is a set together with a family of subsets whose members are chosen to satisfy some set of properties that are deemed useful for a particular application.

Complete block design may encounter missing observation at the process of experimentation because of some known causes which ranges from the carelessness of the experimenter, lack of response, questionable response, mixed up of values from different experimental plots, or the death of an experimental units, etc. This missing observation could inadvertently occur in various kinds of experiments, like in Agriculture, Ecology, Biology, Animal trials, etc.

This research basically aims at comparing the efficiency, as well as the relative efficiency of the selected designs, $(A, B, \& C)$. Design A analyzed the data as an incomplete design. Design B and design C computed the data when the missing

Author α: Department of Mathematics/Computer Science/Statistics/Informatics, Federal University Ndufu Alike Ikwo, Ebonyi State, Nigeria. e-mail: qackasoo@yahoo.com
Author σ: Department of Statistics, University of Nigeria, Nsukka. e-mail: chinonso.eze@unn.edu.ng

observation has been estimated by the Correct Least Square Method and the Inversion Method, respectively. Test of significances of the individual designs were equally evaluated at some levels of significance.

[5] compared the relative efficiency of two statistical experimental designs based on mean square error. The result showed that lattice design is better off than randomized complete block design. [1] studied the effect of a range of uniform plant populations on yield and yield components of canola. The result indicated a significance difference in the seed yield. More literatures can be seen in, [4], [10], [2]. Etc.

II. The Correct Least Square Approach (CLSA)

This approach stipulates that the block(s) in which the missing value(s) occur(s) is /are removed from the data and then analyzed using the available observations. This removal will normally reduce the number of available blocks to the number of blocks in which missing observation occurred. The estimates of the experimental effects realized here are always unbiased. The major drawback encounters in this approach is that the analysis is more complicated than the case when no missing observation(s) occur.

III. The Inversion Method (IM)

In this case the missing observation(s) is/are first estimated from the remaining data and the experiment is then analyzed as in the complete data case. However, there is a loss of a unit(s) in the degree of freedom due to the error, depending on the number of missing observations present. This method equally has a setback when the numbers of missing observations are many.

IV. Estimation of Missing Observations

The general procedure is to replace the missing observation by its estimate under the model. This can only be achieved if the following steps are adhered to;

- Write down the linear model for the design under consideration.
- Write down the parametric equation involving the missing observation and this identify the parameter whose estimates are required to estimate the missing observation.
- Apply the least square method to obtain the estimates of the unknown parameter.

In the case where $m > 1$, where m signifies the number of missing observation in different blocks, set up estimates for each of them using the general expression for one missing observation. In setting up the estimate each observation is treated as if it is the only one that is missing and the rest are assigned values $x_1, x_2, \dots, x_{n-1}$. This approach leads to a system of equation in n unknown whose solution gives estimate of the missing observations.

V. Randomized Compete Block Design (RCBD)

Randomized complete block design which is one of the most widely used experimental designs [8], which has its primary interest as to reduce or minimize the error or variability arising from the known nuisance sources, has been widely used in agricultural and industrial researches for many decades. It makes the experimental error or variability as small as possible by the help of its unique nature, which is blocking the variables according to their homogeneity. The blocks restrict the randomization here in the sense that randomization of the treatments is within the blocks. Usually they are more powerful, have higher external validity, are less subject to bias, and produce more

Ref

5. Irum, Raza and Masood, M. A. (2009). *Pakistan Journal of Agriculture Research.* **20**, 101.

reproducible results than the completely randomized designs typically used in research involving laboratory animals, [7].

VI. Efficiency

In mathematical or scientific terms, efficiency is a measure of the extent to which input is well used for an intended output. It measures the goodness of a design, [6]. However, In the comparison of various statistical procedures, efficiency is a measure of quality of an estimator, of an experimental design, [3] or of a hypothesis testing procedure, [9]. Essentially, a more efficient estimator, experiment, or test needs fewer observations than a less efficient one to achieve a given performance. In fact, efficiencies are often defined using the variance or mean square error (minimal) as the measure of desirability, [3].

VII. Relative Efficiency (RE)

Relative efficiency which is often used to indicate how much saving in cost can be envisaged from a design, can be symbolized as RE, which stands for relative efficiency. The relative efficiency of two procedures is the ratio of their efficiencies. If statistic X has a smaller variance than statistic Y, then statistic X is more efficient than statistic Y. The relative efficiency of two designs X and Y are expressed as $RE(A,B)$ and if this realization is greater than 1, this implies that design A is a better design than design B and this is computed generally as:

$$(a)\quad RE(A,B) = \frac{\sigma^2_{\varepsilon(B)}}{\sigma^2_{\varepsilon(A)}} \ \ldots (*) \qquad (b)\quad RE(A,B) = \frac{SS_B/d_{f_B}}{SS_A/d_{f_A}}, \text{ if } d_{f_A} \geq 20 \ \ldots (**)$$

$$(c)\quad RE(A,B) = \frac{\{(d_{f_A}+1)(d_{f_B}+3)\}MS_B}{\{(d_{f_B}+1)(d_{f_A}+3)\}MS_A}, \text{ if } d_{f_A} < 20 \ \ldots (***)$$

Where: MS_A is the mean square error of design A, d_{f_A} is the degree of freedom for the error term of design A, MS_B is the mean square error of design B, d_{f_B} is the degree of freedom for the error term of design B. SS_A and SS_B are the sum of squares for design A and B respectively.

VIII. Materials and Methodology

The data used in this study are secondary and were collected from the National Root Crops Research Institute (NRCRI) Umudike, Abia State, Nigeria. The data are on yield of cassava with different rations (kg) of Nitrogen, Phosphorous and Potassium (NPK) application. The experimenter was interested in the yield of four varieties of cassava when four different rates of NPK fertilizer were administered on them. It was administered in such manner that the fertilizer rations (kg) were blocked by the varieties of cassava. The data were arranged by the experimenter in a Randomized Complete Block Design (RCBD) layout. This arrangement was made, because Randomized Complete Block Design was deemed appropriate for the study.

a) *The Statistical model of the design*

The statistical model is given as:

$$Y_{ij} = I + \alpha_i + \beta_j + \varepsilon_{ij}\ ; \quad i = 1,2,3; \ j = 1,2,3.$$

Ref

3. Everitt, B. S. (2002). *The Cambridge Dictionary of Statistics*. Cambridge University Press. *ISBN 0-521-81099-X*.

$$\varepsilon_{ij} \sim N(0,1); \sum_{i}^{a} \alpha_i = 0; \sum_{j}^{b} \beta_j = 0;$$

Where Y_{ij} is the observed response of the ith level of the NPK on the jth yield. I is the universal constant. α_i is the effect of the ith NPK. β_j is the jth effect of the yield. ε_{ij} is the random error. The model was based on the assumptions of normality, constant variance and independence.

The data layout of the Randomized Complete Block Design is presented in table 1 below;

b) The table of the observed values

Table 1: Table of extracted data for this study (Source: NRCRI, Umudike, Abia State, Nigeria)

Fertilizers (NPK)	Cassava (Yield)			
05kg	7	8	6	Y_{14}
10kg	10	9	12	14
15kg	20	15	25	26

The missing observation in table 1above was estimated and replaced in the table 2 below;

c) Data layout with the missing observations replaced

Table 2: Table with the estimated value of of Y_{2A}

Fertilizers (NPK)	Cassava (Yield)			
05kg	7	8	6	Y_{14}= (12)
10kg	10	9	12	14
15kg	20	15	25	26

The descriptive ANOVA table of Randomized Complete Block Design is represented in the table 3 below;

d) The ANOVA Table

Table 3: ANOVA Table for the design (Randomized Complete Block Design)

Source of Variation	Degree of freedom	Sum of squares	Mean square	F-ratio
NPK$\{\alpha_i\}$	$\{p-1\}$	SS_α	MS_α	$F_\alpha = MS_\alpha/MS_e$
Yield$\{\beta_j\}$	$\{n-1\}$	SS_β	MS_β	$F_\beta = MS_\beta/MS_e$
Error $\{e\}$	$\{p-1\}\{n-1\}$	SS_e	MS_e	-
Total	$N-1$	SS_T	-	-

IX. Presentation of Tables of the Results

The result gotten when design A was analyzed is being presented in the table 4 below;

a) Table of result for design A

Table 4: ANOVA table for design A (the data were analyzed as an incomplete design)

Source of Variation	Degree of Freedom	Sum of Squares	Mean Squares	F-ratio	F-tabulated	
					$\alpha = 0.01$	$\alpha = 0.05$
Fertilizer(kg) $\{\alpha_i\}$	2	540.20	270.10	$F = 14.94$	$F_v = 10.90$	$F_v = 5.14$
Yields(tons) $\{\beta_j\}$	3	22.03	7.34	$F = 0.46$	$F_d = 9.78$	$F_d = 4.76$
Error $\{\varepsilon_{ij}\}$	6	108.47	18.08			
Total	11	670.70				

Below in table 5, the result of the analyses of design B was presented.

Notes

Notes

b) Table of result of design B

Table 5: ANOVA table for design B (the missing value in the datum was estimated & analyzed)

Source of Variation	Degree of Freedom	Sum of Squares	Mean Squares	F-ratio	F-tabulated	
					$\alpha = 0.01$	$\alpha = 0.05$
Fertilizer(kg) $\{\alpha_i\}$	2	386.20	193.10	$F = 30.12$	$F_j = 10.90$	$F_j = 5.14$
Yields(tons) $\{\beta_j\}$	3	74.03	24.68	$F = 3.85$	$F_q = 9.78$	$F_q = 4.76$
Error $\{\varepsilon_{ij}\}$	6	38.47	6.41			
Total	11	498.70				

The result of the analyses of design C was equally displayed in table 6 below;

c) Table of result of design C

Table 6: ANOVA table for design C (the block with the missing value is deleted from the design & analyzed

Source of Variation	Degree of Freedom	Sum of Squares	Mean Squares	F-ratio	F-tabulated	
					$\alpha = 0.01$	$\alpha = 0.05$
Fertilizer(kg) $\{\alpha_i\}$	2	273.55	136.78	$F = 15.01$	$F_u = 18.00$	$F_u = 6.94$
Yields(tons) $\{\beta_j\}$	2	20.22	10.11	$F = 1.11$	$F_i = 18.00$	$F_i = 6.94$
Error $\{\varepsilon_{ij}\}$	4	36.45	9.11			
Total	8	330.22				

The result of the efficiency comparisons of designs A, B and C was presented in the table 7 below;

d) Table of result comparisons of the three designs

Table 7: Tabulation of the result of the hypotheses carried out in this study

Designs	Eff. (MS_ϵ)	Sig α_i, 0.01	Sig β_j, 0.01	Sig α_i, 0.05	Sig β_j, 0.05
Design A	18.08	Sig	Not sig	Sig	Not sig
Design B	6.14	Sig	Not sig	Sig	Not sig
Design C	9.11	Not sig	Not sig	Sig	Not sig

It can be observed that in table 8, the result of the relative efficiency of the pair wise designs were presented;

e) The pair wise comparisons of the relative efficiency

Table 8: Table that compared the relative efficiency of all the possible pairs of the designs

Relative Efficiency	Values	Comparisons of the Designs
$R(A, B)$	2.94	Design A is better than Design B
$R(A, C)$	2.16	Design A is better than Design C
$R(B, C)$	0.73	Design C is better than Design B
$R(B, A)$	0.34	Design A is better than Design B
$R(C, A)$	0.46	Design A is better than Design C
$R(C, B)$	1.36	Design C is better than Design B

X. Summary and Discussion

The randomized complete block design is undoubtedly one of the most fundamental and useful tool in the analysis of variance models. The major advantage of using a randomized complete block design is that it makes reduction in error variance its primary target. The widely accepted relative precision measure is purported to evaluate the relative efficiency in terms of the ratio of error variances of both designs.

However, this relative precision measure does not take account of the loss in error degrees of freedom in a randomized complete block design with complete observations as compared with that in a randomized complete block design with a missing observation.

Unlike other researches that examine parameter values; this one focuses on the estimates of the relative efficiency measure that possess immediate applicability and practical importance. In this research, the selected randomized complete block designs presented different values of efficiency and relative efficiency, depending on the pair-wise combination of the designs considered, as can be seen in table 7, which enveloped the results of table 4, 5 and 6. This efficiency value was evaluated in terms of (1) comparing the precisions, (2) comparing the observed significance levels, while the relative efficiency was evaluated by taking the ration of the efficiencies of all the possible pair-wise combination of the selected designs with replacement.

It can be observed in table 7 that at different levels of significance considered, the treatments which is the level of the administered NPK fertilizer to the cassava (yield) is significant for all the designs, except design C at 0.01, while blocking were all not significant. The mean square error (MSE) for the selected designs showed that design B has the least (smallest) mean square error of 6.14, followed by design C and design A, with 9.11 and 18.08 values respectively and this recommend design B as the best of all. Finally, table 8, which evaluated a pair-wise relative efficiency presented $R(B, A) = 0.34,$ which is the smallest (< 1) and best recommended and $R(A, B) = 2.94,$ which is the largest (> 1) and least recommended. Both $R(B, A)$ and $R(A, B)$ agree with an existing literature like [10], which suggest design B as the best design for our problem.

References Références Referencias

1. Angadi, S.V., Cutforth, H.W., McConkey, B. G. and Gan, Y. (2003). Yield adjustment by canola grown at different plant populations under semiarid conditions. *Journals of Crop Science*. 43:1358-1366.
2. Casler, M. D. (2015). Spatial variation affects precision of perennial cool-season forage grass trials. *Agronomy Journal*. 91:75-81.
3. Everitt, B. S. (2002). *The Cambridge Dictionary of Statistics.* Cambridge University Press. *ISBN 0-521-81099-X.*
4. Gwowen, Shieh (2004). The effectiveness of randomized complete block design. *Statistica Neerlandica.* Block well publishing, UK. Vol. 58. No. 1. Pp.111-124.
5. Irum, Raza and Masood, M. A. (2009). *Pakistan Journal of Agriculture Research.* 20, 101.
6. Kuhfeld, W. F., Tobias, R. D., and Garratt, M. (1994). Efficient experimental design with marketing research applications. *Journal of Marketing Research*. 31:545-55.
7. Michael, F. W. Festing (2014). Randomized block experiment design can increase the power and producibility of laboratory Animal experiments. *Institute for laboratory Animal Research Journal.* Vol. 55. No.3. Leicester, UK: doi: 10./093/ilar/ilu045.
8. Montgomery, D. C. (1976). *Design and analysis of experiments*. 5th ed. John Wiley and Sons, Inc. Pub.
9. Nikulin, M. S. (2001), Efficiency of a statistical procedure in Hazewinkel, Michiel, *Encyclopedia of Mathematics*, *Springer*, *ISBN 978-1-55608-010-4.*

Ref

10. Nishu, Lohmor., Mujahid, Khan., Kiran, Kapoor., and Ramesh, Kumar Tripathi. (2017). Studies on the relative efficiency of different experimental designs for sunflower (Helianthus annus L). *Annual Research and review in biology* 12(14): 1-7.

10. Nishu, Lohmor., Mujahid, Khan., Kiran, Kapoor., and Ramesh, Kumar Tripathi. (2017). Studies on the relative efficiency of different experimental designs for sunflower (Helianthus annuus L). *Annual Research and review in biology* 12(14): 1-7.

Notes

This page is intentionally left blank