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The Effect of Parametric Amplifier on Entanglement and Statistical Properties of Nondegenerate Three-Level Laser Pumped by Electron Bombardment

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Tamirat Abebe [α] & Tamiru Deressa [σ]

Abstract- In this paper, the quantum properties of an electrically pumped non-degenerate three level laser with two-mode subharmonic generator coupled to a two-mode vacuum reservoirs via a single-port mirror whose open cavity contains *N* non-degenerate three-level cascade atoms is presented. The analysis is carried out by putting the noise operators associated with a vacuum reservoir in normal order. It is found that the photons aswell as atoms of the system are strongly entangled at steady state. It has been shown that the degree of photon entanglement greater than that of atom entanglement. Moreover, the presence of the subharmonic generator leads to an increase in the degree of entanglement.

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I. Introducion

It is now believed that the key ingredient of quantum information is entanglement which has been recognized as the essential resource for quantum teleportation, quantum dense coding, quantum computation, quantum error correction, and quantum cryptography [1-6]. Traditionally, quantum entangled states are considered to be non-classical correlations between individual qubits. However, it has been proved that continuous-variable (CV) entanglement has many advantages in some cases [7, 8]. Recently, much attention has been paid to the generation and detection of CV entanglement as it might be easier to manipulate than the discrete counterparts, quantum bits, in order to perform quantum information processing. On the other hand, the efficiency of quantum information processing highly depends on the degree of entanglement. Hence, it is desirable to generate strongly entangled continuous variable state.

A two-mode subharmonic generator at and above threshold has been theoretically predicted to be a source of light in an entangled state [8,9]. Recently, the experimental realization of the entanglement in two-mode subharmonic generator has been demonstrated by Zhang et al. [10]. Furthermore, Tan et al. [11] have extended the work of Xiong et al. and examined the generation and evolution of the entangled light in the Wigner representation using the sufficient and necessary in separability criteria for a two-mode Gaussian state proposed by Dual et al. [12] and Simon [13]. Tesfa [14] has considered a similar system when the atomic coherence is induced by superposition of atomic states and analyzed the entanglement at steady-state.

Recently, Eyob [15] has studied CV entanglement in a non-degenerate three-level laser with a parametric amplifier. In this model the injected atomic coherence introduced by initially preparing the atoms in a coherent superposition of the top and bottom levels. This combined system exhibits a two-mode squeezed light and produces light in an entangled state. In one model of such a laser, three-level atoms initially in the upper level are injected at a constant rate into the cavity and removed after they have decayed due to spontaneous emission. It appears to be quite difficult to prepare the atoms in a coherent superposition of the top and bottom levels before they are injected into the laser cavity. Besides, it should certainly be hard to find out that the atoms have decayed spontaneously before they are removed from the cavity.

In order to avoid the aforementioned problems, Fesseha [16] have considered that N two-level atoms available in a closed cavity are pumped to the top level by means of electron bombardment. He has shown that the light generated by this laser operating well above threshold is coherent and the light generated by the same laser operating below threshold is chaotic. Moreover, Fesseha [17] has studied the squeezing and the statistical properties of the light produced by a three-level laser with the atoms in a closed cavity and pumped by electron bombardment. He has shown that the maximum quadrature squeezing of the light generated by the laser, operating below threshold, is found to be 50% below the vacuum-state level. In addition, he has also found that the quadrature squeezing of the output light is equal to that of the cavity light. On the other hand, this study shows that the local quadrature squeezing is greater than the global quadrature squeezing. Furthermore, Fesseha [18] has studied the squeezing and the statistical properties of the light produced by a degenerate three-level laser with the atoms in a closed cavity and pumped by coherent light. He has shown that the maximum quadrature squeezing is 43% below the vacuum-state level, which is slightly less than the result found with electron bombardment.

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In this model, we seek to study CV entanglement for the light generated by electrically pumped non-degenerate three-level laser with two-mode subharmonic generator coupled to a two-mode vacuum reservoirs via a single-port mirror whose open cavity contains N non-degenerate threelevel cascade atoms.

In order to carry out our analysis, we put the noise operators associated with the vacuum reservoir in the normal order and we consider the interaction of the three-level atoms with a two mode vacuum reservoir. We then first drive the quantum Langevin equations for the cavity mode operators. We next determine the equations of evolution of the expectation values of atomic operators employing the pertinent master equation. Applying the steady-state solution of equations of evolution, we analyze the mean photon number, CV atomic and photon state entanglement, the quadrature squeezing as well as atom and photon number correlation.

II. Dynamics of Atomic and Cavity Mode Operators

We consider here the case in which non-degenerate three level laser dynamics with two-mode subharmonic generator coupled to two-mode vacuum reservoir whose cavity contains *N* three level atoms in cascade configuration as shown in Fig. 1. We denote the top, intermediate, and bottom levels of these atoms by $|2\rangle_j$, $|1\rangle_j$, and $|0\rangle_j$, respectively. In addition, we assume the cavity mode to be at resonance with the two transitions $|2\rangle_j \leftrightarrow |1\rangle_j$ and $|1\rangle_j \leftrightarrow |0\rangle_j$, with direct transition between levels $|2\rangle_j \leftrightarrow |0\rangle_j$ to be electric dipole forbidden. The pump mode emerging from the two-mode subharmonic generation does not couples to the top and bottom levels. The interaction of a three-level atom with cavity modes and the pump mode with the two-mode subharmonic generation can be described at resonance by the Hamiltonian

$$\hat{H}_S(t) = ig\sum_{n=1,2}\left[\hat{\sigma}_n^{\dagger j}(t)\hat{a}_n(t) - \hat{a}_n^{\dagger}(t)\hat{\sigma}_n^{j}(t)\right] + i\varepsilon(\hat{a}_1^{\dagger}\hat{a}_2^{\dagger} - \hat{a}_1\hat{a}_2), \tag{1}$$

where

$$\hat{\sigma}_1^j(t) = |0\rangle_{jj}\langle 1|, \quad \hat{\sigma}_2^j(t) = |1\rangle_{jj}\langle 2|, \tag{2}$$

are lowering atomic operators, $\hat{a}_1(t)$ and $\hat{a}_2(t)$ are the annihilation operators for the light modes

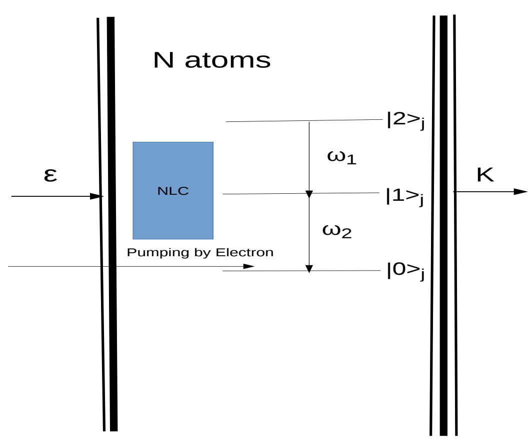

Figure 1: Schematic representation of electrically pumped non-degenerate three level laser dynamics with two-mode subharmonic generator coupled to a two-mode vacuum reservoir

a_1 and a_2, respectively, ε, assumed to be rael and constant, is proportional to the amplitude of the coherent light that drives the NLC, and *g* is the coupling constant between the atom and the cavity modes.

The master equation for a three-level atom coupled to a two-mode vacuum reservoir has the Form

$$\frac{d}{dt}\hat{\rho}(t) = -i[\hat{H}_S, \hat{\rho}] + \frac{\gamma}{2}\sum_{k=0}^{2}\left[2\hat{\sigma}_k^j\hat{\rho}\hat{\sigma}_k^{\dagger j} - \hat{\sigma}_k^{\dagger j}\hat{\sigma}_k^j\hat{\rho} - \hat{\rho}\hat{\sigma}_k^{\dagger j}\hat{\sigma}_k^j\right], \tag{3}$$

in which γ is the spontaneous emission decay constant and $\hat{\sigma}_0^j(t) = |0\rangle_{jj}\langle 2|$. Now with the aid of Eq. (1), one can put (3) in the form

$$\frac{d}{dt}\hat{\rho}(t) = g\sum_{n=1,2}\left[\hat{\sigma}_n^{\dagger j}\hat{a}_n\hat{\rho} - \hat{\rho}\hat{\sigma}_n^{\dagger j}\hat{a}_n + \hat{\rho}\hat{a}_n^{\dagger}\hat{\sigma}_n^j - \hat{a}_n^{\dagger}\hat{\sigma}_n^j\hat{\rho}\right] + \frac{\gamma}{2}\sum_{k=0}^{2}\left[2\hat{\sigma}_k^j\hat{\rho}\hat{\sigma}_k^{\dagger j} - \hat{\sigma}_k^{\dagger j}\hat{\sigma}_k^j\hat{\rho} - \hat{\rho}\hat{\sigma}_k^{\dagger j}\hat{\sigma}_k^j\right]$$

$$+\varepsilon\left[\hat{a}_1\hat{a}_2\hat{\rho} - \hat{\rho}\hat{a}_1\hat{a}_2 + \hat{\rho}\hat{a}_1^{\dagger}\hat{a}_2^{\dagger} - \hat{a}_1^{\dagger}\hat{a}_2^{\dagger}\hat{\rho}\right]. \tag{4}$$

We model that the laser cavity is coupled to a two-mode vacuum reservoir via a single-port mirror. In addition, we carry out our analysis by putting the noise operators associated with the vacuum reservoir in normal order. Thus the noise operators will not have any effect on the dynamics of the cavity mode operators. We can therefore drop the noise operators and write the quantum Langevin equation for the operators $\hat{a}_n(t)$ as

$$\frac{d}{dt}\hat{a}_n(t) = -i[\hat{a}_n(t), \hat{H}_S(t)] - \frac{1}{2}\kappa\hat{a}_n(t), \tag{5}$$

in which $n = 1, 2$, and κ is assumed to be the cavity damping constant for the light modes a_n. Then with the aid of Eqs. (1) and (5) together with the commutation relation $[\hat{a}_i, \hat{a}_j^{\dagger}] = \delta_{ij}$, we easily find

$$\frac{d}{dt}\hat{a}_n(t) = -g\hat{\sigma}_n^j - \frac{1}{2}\kappa\hat{a}_n(t) + \varepsilon\hat{a}_m^{\dagger}, \tag{6}$$

where $n, m = 1, 2$. Now employing the relation $\frac{d}{dt}\langle\hat{A}\rangle = Tr\left(\frac{d\hat{\rho}}{dt}\hat{A}\right)$, along with Eq. (4), one can readily establish that

$$\frac{d}{dt}\langle\hat{\sigma}_1^j\rangle = g[\langle\hat{\eta}_0^j\hat{a}_1\rangle - \langle\hat{\eta}_1^j\hat{a}_1\rangle - \langle\hat{a}_2^{\dagger}\hat{\sigma}_0^j\rangle] - \frac{\gamma}{2}\langle\sigma_1^j\rangle, \tag{7}$$

$$\frac{d}{dt}\langle\hat{\sigma}_2^j\rangle = g[\langle\hat{\eta}_1^j\hat{a}_2\rangle - \langle\hat{\eta}_2^j\hat{a}_2\rangle + \langle\hat{a}_1^{\dagger}\hat{\sigma}_0^j\rangle] - \gamma\langle\sigma_2^j\rangle, \tag{8}$$

$$\frac{d}{dt}\langle\hat{\sigma}_0^j\rangle = g[\langle\hat{\sigma}_1^j\hat{a}_2\rangle - \langle\hat{\sigma}_2^j\hat{a}_1\rangle] - \frac{\gamma}{2}\langle\sigma_0^j\rangle, \tag{9}$$

$$\frac{d}{dt}\langle\hat{\eta}_1^j\rangle = g[\langle\hat{\sigma}_1^{\dagger j}\hat{a}_1\rangle + \langle\hat{a}_1^{\dagger}\hat{\sigma}_1^j\rangle - \langle\hat{\sigma}_2^{\dagger j}\hat{a}_2\rangle - \langle\hat{a}_2^{\dagger}\hat{\sigma}_2^j\rangle] + \gamma(\langle\eta_2^j\rangle - \langle\eta_1^j\rangle), \tag{10}$$

$$\frac{d}{dt}\langle\hat{\eta}_2^j\rangle = g[\langle\hat{\sigma}_2^{\dagger j}\hat{a}_2\rangle + \langle\hat{a}_2^{\dagger}\hat{\sigma}_2^j\rangle] - 2\gamma\langle\eta_2^j\rangle, \tag{11}$$

$$\frac{d}{dt}\langle\hat{\eta}_0^j\rangle = -g[\langle\hat{\sigma}_1^{\dagger j}\hat{a}_1\rangle + \langle\hat{a}_1^{\dagger}\hat{\sigma}_1^j\rangle] + \gamma(\langle\eta_1^j\rangle + \langle\eta_2^j\rangle) \tag{12}$$

where $\hat{\eta}_k^j(t) = |k\rangle_{jj}\langle k|$ and $k = 0, 1, 2.$

The three-level atoms available in the cavity are pumped from the bottom level to the top level by means of electron bombardment. The pumping process must surely affect the dynamics of $\langle\hat{\eta}_2^j\rangle$ and $\langle\hat{\eta}_0^j\rangle$ If τ_a a represents the rate at which a single atom is pumped from the bottom to the top level, then $\langle\hat{\eta}_2^j\rangle$ increases at the rate of $\tau_a\langle\hat{\eta}_0^j\rangle$ and decreases at the the same rate. We see that the equations in (7)-(12) are coupled nonlinear differential equations and hence it is not possible to find exact time-dependent solutions of these equations. We intend to overcome this problem by applying the large-time approximation. Then employing this approximation scheme, we get from Eq. (6) and, the approximately valid relation

$$\hat{a}_n = \frac{2\varepsilon}{\kappa}\hat{a}_m^{\dagger} - \frac{2g}{\kappa}\hat{\sigma}_n^j(t), \tag{13}$$

where $m, n = 1, 2$. Evidently, these turn out to be exact relations at steady-state. It then follows that

$$\hat{a}_n = -\frac{4\varepsilon g}{\kappa^2 - 4\varepsilon^2}\hat{\sigma}_m^{\dagger j} - \frac{2g\kappa}{\kappa^2 - 4\varepsilon^2}\hat{\sigma}_n^j. \tag{14}$$

Now introducing Eq. (14) into Eqs. (7)-(12) and summing over the N three-level atoms, we see that

$$\frac{d}{dt}\langle\hat{\Sigma}_1\rangle = -\frac{R}{2}\langle\hat{\Sigma}_1\rangle, \tag{15}$$

$$\frac{d}{dt}\langle\hat{\Sigma}_2\rangle = -\frac{R}{2}\langle\hat{\Sigma}_2\rangle - T\langle\hat{\Sigma}_1^{\dagger}\rangle, \tag{16}$$

$$\frac{d}{dt}\langle\hat{\Sigma}_0\rangle = -\frac{R}{2}\langle\hat{\Sigma}_0\rangle + T[\langle\hat{N}_1\rangle - \langle\hat{N}_0\rangle], \tag{17}$$

$$\frac{d}{dt}\langle\hat{N}_1\rangle = -R[\langle\hat{N}_1\rangle - \langle\hat{N}_2\rangle] + T(\langle\hat{\Sigma}_0\rangle + \langle\hat{\Sigma}_0^\dagger\rangle) \tag{18}$$

$$\frac{d}{dt}\langle\hat{N}_2\rangle = -R\langle\hat{N}_2\rangle - T(\langle\hat{\Sigma}_0\rangle + \langle\hat{\Sigma}_0^\dagger\rangle) + \tau_a\langle\hat{N}_0\rangle, \tag{19}$$

$$\frac{d}{dt}\langle\hat{N}_0\rangle = -\tau_a\langle\hat{N}_0\rangle + R\langle\hat{N}_1\rangle + \gamma\langle\hat{N}_2\rangle, \tag{20}$$

Where

$$\gamma_c = \frac{4g^2}{\kappa}, \tag{21}$$

$$R = \gamma + \frac{\gamma_c\kappa^2}{\kappa^2 - 4\varepsilon^2}, \tag{22}$$

$$T = \frac{\gamma_c\varepsilon\kappa}{\kappa^2 - 4\varepsilon^2}, \tag{23}$$

$\hat{\Sigma}_1 = \sum_{j=1}^N \hat{\sigma}_1^j$, $\hat{\Sigma}_2 = \sum_{j=1}^N \hat{\sigma}_2^j$, $\hat{\Sigma}_0 = \sum_{j=1}^N \hat{\sigma}_0^j$, $\hat{N}_0 = \sum_{j=1}^N \hat{\eta}_0^j$, $\hat{N}_1 = \sum_{j=1}^N \hat{\eta}_1^j$, $\hat{N}_2 = \sum_{j=1}^N \hat{\eta}_2^j$, with the operators $\hat{N}_2$, $\hat{N}_1$, and $\hat{N}_0$ representing the number of atoms in the top, middle, and bottom levels. We prefer to call the parameter defined by Eq. (21) the stimulated emission decay constant. In addition, employing the completeness relation

$$\hat{I} = \hat{\eta}_0^j + \hat{\eta}_1^j + \hat{\eta}_2^j, \tag{24}$$

we easily arrive at

$$N = \langle\hat{N}_0\rangle + \langle\hat{N}_1\rangle + \langle\hat{N}_2\rangle. \tag{25}$$

Furthermore, applying the definition given by Eq. (2) and setting for any *j*

$$\hat{\sigma}_1^j = |0\rangle\langle 1|, \tag{26}$$

we have

$$\hat{\Sigma}_1 = N|0\rangle\langle 1|. \tag{27}$$

Following the same procedure, one can also check that $\hat{\Sigma}_2 = N|1\rangle\langle 2|$, $\hat{\Sigma}_0 = N|0\rangle\langle 2|$, $\hat{N}_0 = N|0\rangle\langle 0|$, $\hat{N}_1 = N|1\rangle\langle 1|$, and $\hat{N}_2 = N|2\rangle\langle 2|$. Moreover, using the definition

$$\hat{\Sigma} = \hat{\Sigma}_1 + \hat{\Sigma}_2 \tag{28}$$

and taking into account the above results, it can be readily established that

$$\hat{\Sigma}^\dagger\hat{\Sigma} = N(\hat{N}_1 + \hat{N}_2), \quad \hat{\Sigma}\hat{\Sigma}^\dagger = N(\hat{N}_1 + \hat{N}_0), \hat{\Sigma}^2 = N\hat{\Sigma}_0. \tag{29}$$

Upon adding the two separate equations from Eq. (6), we have

$$\frac{d}{dt}\hat{a}(t) = -g\hat{\sigma}^j - \frac{1}{2}\kappa\hat{a}(t) + \varepsilon\hat{a}^\dagger, \tag{30}$$

in which

$$\hat{a}(t) = \hat{a}_1(t) + \hat{a}_2(t), \tag{31}$$

$$\hat{\sigma}^j = \hat{\sigma}_1^j + \hat{\sigma}_2^j. \tag{32}$$

Applying large-time approximation scheme into Eq. (20), we see that

$$\hat{a} = -\frac{2g\kappa}{\kappa^2 - 4\varepsilon^2}\hat{\sigma}^j - \frac{4g\varepsilon}{\kappa^2 - 4\varepsilon^2}\hat{\sigma}^{\dagger j}(t). \tag{33}$$

Taking into account of Eq. (33) and its complex conjugate and on summing over all atoms, the commutation relation of the cavity mode operator is

$$[\hat{a}, \hat{a}^\dagger] = \frac{\gamma_c\kappa}{\kappa^2 - 4\varepsilon^2}(\hat{N}_0 - \hat{N}_2), \tag{34}$$

where $[\hat{a}, \hat{a}^\dagger] = \sum_{\neq 1}^N [\hat{a}, \hat{a}^\dagger]_j$, stands for the commutator of $\hat{a}$ and $\hat{a}^\dagger$ when the cavity mode is interacting with all the *N* three-level atoms. In the presence of N three-level atoms, we rewrite Eq. (30) as

$$\frac{d}{dt}\hat{a}(t) = -\frac{1}{2}\Big(\frac{\kappa^2-4\varepsilon^2}{\kappa}\Big)\hat{a}(t) + \lambda_1\hat{\Sigma} + \lambda_2\hat{\Sigma}^\dagger, \quad (35)$$

in which λ_1 and λ_2 is a constant whose values remains to be fixed. The steady-state solution of Eq. (35) is

$$\hat{a} = \frac{2\lambda_1\kappa}{\kappa^2-4\varepsilon^2}\hat{\Sigma} + \frac{2\lambda_2\kappa}{\kappa^2-4\varepsilon^2}\hat{\Sigma}^\dagger(t). \quad (36)$$

In view of (36) and its complex conjugate, the commutation relation for the cavity mode operator Is

$$[\hat{a},\hat{a}^\dagger] = \frac{4\kappa^2(\lambda_1^2-\lambda_2^2)}{(\kappa^2-4\varepsilon^2)^2}N(\hat{N}_0-\hat{N}_2). \quad (37)$$

Comparing Eqs. (34) and (37) to solve λ_1 and λ_2, consider the case λ_1 - $\lambda_2 \neq 0$, then we see that

$$\lambda_1 = \frac{g}{\sqrt{N}}, \quad and \quad \lambda_2 = \frac{2g\varepsilon}{\kappa\sqrt{N}}. \quad (38)$$

Then Eq. (35) can be rewritten as

$$\hat{a} = \frac{2g\kappa}{\sqrt{N}(\kappa^2-4\varepsilon^2)}\hat{\Sigma} + \frac{4g\varepsilon}{\sqrt{N}(\kappa^2-4\varepsilon^2)}\hat{\Sigma}^\dagger. \quad (39)$$

Now from Eq. (15), one can write

$$\frac{d}{dt}\langle\hat{\Sigma}_1\rangle = -\frac{1}{2}\xi\langle\hat{\Sigma}_1\rangle, \quad (40)$$

where $\xi = \frac{A}{\tau_a C}$. We notice that the steady - state solution of $\langle\hat{\Sigma}_1\rangle$ of (10) for ξ different from zero is

$$\langle\hat{\Sigma}_1\rangle = 0. \quad (41)$$

Similarly,

$$\langle\hat{\Sigma}_2\rangle = \langle\hat{\Sigma}\rangle = 0. \quad (42)$$

With the aid of (42) and the assumption that the cavity light is initially in a vacuum state, the expectation value of the solution of (35) is found to be

$$\langle\hat{a}(t)\rangle = 0. \quad (43)$$

We observe on the basis of Equations (35) and (43) that $\hat{a}$ is a Gaussian variable with zero mean. We next seek to calculate the expectation value of the operators $\hat{N}_0, \hat{N}_1, \hat{N}_2$ and the atomic operator $\hat{\Sigma}_0$. To this end, applying the large time approximation scheme to Eqs. (15)-(20) along with (25), we readily find

$$\langle\hat{N}_2\rangle = \frac{R\tau_a(R^2+\gamma\tau_a) - 4T^2(R-\tau_a)(R+\gamma)}{R^2(\tau_a+\gamma)(R+\tau_a)}N, \quad (44)$$

$$\langle\hat{N}_1\rangle = \frac{R}{R+\tau_a}N, \quad (45)$$

$$\langle\hat{N}_0\rangle = \frac{R^2+\tau_a\gamma}{(\tau_a+\gamma)(R+\tau_a)}N, \quad (46)$$

$$\langle\hat{\Sigma}_0\rangle = \frac{2T(R+\gamma)(R-\tau_a)}{R(\tau_a+\gamma)(R+\tau_a)}N. \quad (47)$$

III. The Mean Photon Number

We now proceed to obtain the mean photon number of light mode a in the entire frequency interval. The mean photon number of light mode a, represented by the operators $\hat{a}$ and $\hat{a}^\dagger$, is defined by

$$\overline{n} = \langle\hat{a}^\dagger\hat{a}\rangle. \quad (48)$$

With the aid of Eq. (39) and its complex conjugate, the mean photon number is expressible as

$$\overline{n} = \frac{\gamma_c\kappa}{(\kappa^2-4\varepsilon^2)^2}\Bigg[\kappa^2\Big(\langle\hat{N}_1\rangle + \langle\hat{N}_2\rangle\Big) + 4\varepsilon^2\Big(N - \langle\hat{N}_2\rangle\Big) + 4\kappa\varepsilon\langle\hat{\Sigma}_0\rangle\Bigg]. \quad (49)$$

For the case in which $\varepsilon = 0$, (49) turns out to be

$$\overline{n} = \frac{\gamma_c}{\kappa}\Bigg[\frac{(\gamma+\gamma_c)^2(\gamma+2\tau_a)+\gamma\tau_a^2}{(\gamma+\gamma_c)(\tau_a+\gamma)(\gamma+\gamma_c+\tau_a)}\Bigg]N. \quad (50)$$

In the absence of spontaneous emission when $(\gamma = 0)$, Eq. (50) reduces to

$$\overline{n} = \frac{2\gamma_c^2}{\kappa(\gamma_c+\tau_a)}N. \quad (51)$$

It proves to be convenient to refer to the regime of laser operation with more atoms in the top

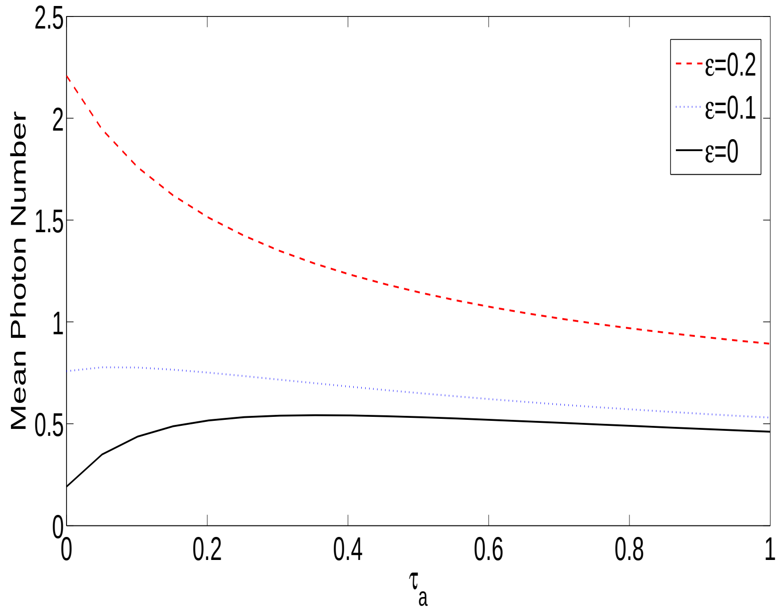

Figure 2: Plots of the mean photon number [Eq. (49)] versus τ_a for $\gamma_c = 0.4$, $\kappa = 0.8$, $N = 50$, $\gamma = 0.2$ and for different values of ε.

level than in the bottom level as above threshold, the regime of laser operation with equal number of atoms in the top and bottom levels as threshold, and the regime of laser operation with less atoms in the top level than in the bottom level as below threshold. Thus according to Eq. (51) for the laser operating above threshold $\gamma_c < \tau_a$, for the laser operating at threshold $\gamma_c = \tau_a$, and for the laser operating below threshold $\gamma_c > \tau_a$. We note that for well above threshold

$$\bar{n} = \frac{2\gamma_c}{\kappa\tau_a}N,$$

for below threshold

$$\bar{n} = \frac{2\gamma_c}{\kappa}N,$$

and at threshold

$$\bar{n} = \frac{\gamma_c}{\kappa}N.$$

IV. Photon-Number Correlation and Atom-Number Correlation

In this section we seek to analyze the degree of photon-number correlation and atom-number correlation. In the cascading transition from energy level $|2\rangle_j$ to $|0\rangle_j$ via $|1\rangle_j$, a correlation between the two emitted photons a_1 and a_2 can readily be established. Hence the photon number correlation for the cavity modes can be defined as

$$g(\hat{n}_1, \hat{n}_2) = \frac{\langle \hat{a}_1^\dagger(t)\hat{a}_1(t)\hat{a}_2^\dagger(t)\hat{a}_2(t)\rangle}{\langle \hat{a}_1^\dagger(t)\hat{a}_1(t)\rangle\langle \hat{a}_2^\dagger(t)\hat{a}_2(t)\rangle}. \quad (52)$$

On the other hand, using Eq. (6) together with (14), the equation of evolution of cavity mode operators can be rewritten as

$$\frac{d}{dt}\hat{a}_n(t) = -\frac{1}{2}\Big(\frac{\kappa^2 - 4\varepsilon^2}{\kappa}\Big)\hat{a}_n(t) - g\hat{\sigma}_n^j - \frac{2g\varepsilon}{\kappa}\hat{\sigma}_m^{\dagger j}, \quad (53)$$

where $n, m = 1, 2$. Applying the steady-state solution of Eq. (53), one can readily establish that the commutation relation of the cavity mode operators $\hat{a}_1$ and $\hat{a}_1^\dagger$ as well as $\hat{a}_2$ and $\hat{a}_2^\dagger$ with summing over all atoms. We then notice that

$$[\hat{a}_1, \hat{a}_1^\dagger] = \frac{\gamma_c\kappa}{(\kappa^2 - 4\varepsilon^2)^2}\left[4\varepsilon^2\left(\hat{N}_2 - \hat{N}_1\right) + \kappa^2\left(\hat{N}_0 - \hat{N}_1\right) + 2\kappa\varepsilon\left(\hat{\Sigma}_0 + \hat{\Sigma}_0^\dagger\right)\right] \quad (54)$$

and

$$[\hat{a}_2, \hat{a}_2^\dagger] = \frac{\gamma_c \kappa}{(\kappa^2 - 4\varepsilon^2)^2}\left[4\varepsilon^2\left(\hat{N}_1 - \hat{N}_0\right) + \kappa^2\left(\hat{N}_1 - \hat{N}_2\right) - 2\kappa\varepsilon\left(\hat{\Sigma}_0 + \hat{\Sigma}_0^\dagger\right)\right], \tag{55}$$

where

$$[\hat{a}_i, \hat{a}_k^\dagger] = \delta_{ik}\sum_{j=1}^{N}[\hat{a}_i, \hat{a}_k^\dagger]_j, \tag{56}$$

stands for the commutator of mode operators when the cavity light is interacting with all the *N* three-level atoms. In the presence of *N* three-level atoms, we rewrite Eq. (52) as

$$\frac{d}{dt}\hat{a}_n(t) = -\frac{1}{2}\Big(\frac{\kappa^2 - 4\varepsilon^2}{\kappa}\Big)\hat{a}_n(t) + \lambda_n'\hat{\Sigma}_n + \lambda_n''\hat{\Sigma}_m^\dagger, \tag{57}$$

in which λ_n', λ_n'', are constants whose values remain to be fixed. The steady-state solution of Eq. (57) is

$$\hat{a}_n = \frac{2\lambda_n'\kappa}{\kappa^2 - 4\varepsilon^2}\hat{\Sigma}_n + \frac{2\lambda_n''\kappa}{\kappa^2 - 4\varepsilon^2}\hat{\Sigma}_m^\dagger, \tag{58}$$

where $n, m = 1, 2$. In view of (58) as well as its complex conjugate, the commutation relation for the cavity mode operators is

$$[\hat{a}_1, \hat{a}_1^\dagger] = \frac{4N\kappa^2}{(\kappa^2 - 4\varepsilon^2)^2}\left[\lambda_1''^2(\hat{N}_2 - \hat{N}_1) + \lambda_1'^2(\hat{N}_0 - \hat{N}_1) + \lambda_1'\lambda_1''(\hat{\Sigma}_0 + \hat{\Sigma}_0^\dagger)\right] \tag{59}$$

and

$$[\hat{a}_2, \hat{a}_2^\dagger] = \frac{4N\kappa^2}{(\kappa^2 - 4\varepsilon^2)^2}\left[\lambda_2''^2(\hat{N}_1 - \hat{N}_0) + \lambda_2'^2(\hat{N}_1 - \hat{N}_2) - \lambda_2'\lambda_2''(\hat{\Sigma}_0 + \hat{\Sigma}_0^\dagger)\right]. \tag{60}$$

On comparing Eqs. (54) and (59) together with (55) and (60), we obtain $\lambda_1' = \lambda_2' = \lambda_1$ and $\lambda_1'' = \lambda_2'' = \lambda_2$. Hence Eq. (58) can be rewritten as

$$\hat{a}_n = \frac{2g\kappa}{\sqrt{N}(\kappa^2 - 4\varepsilon^2)}\hat{\Sigma}_n + \frac{4g\varepsilon}{\sqrt{N}(\kappa^2 - 4\varepsilon^2)}\hat{\Sigma}_m^\dagger. \tag{61}$$

Now in view of Eqs. (41) and (42) as well as the assumption that the cavity light is initially in a vacuum state, the expectation value of the solutions of Eq. (57) is found to be

$$\langle \hat{a}_n(t)\rangle = 0. \tag{62}$$

On account of Eq. (62) together with Eq. (57) that $\hat{a}_n(t)$ is a Gaussian variables with zero mean. Thus employing these results, the photon-number correlation turns out to be

$$g(\hat{n}_1, \hat{n}_2)_p = 1 + \frac{4\varepsilon\kappa\left[\left(\kappa^2 + 4\varepsilon^2\right)\langle\hat{\Sigma}_0\rangle + 2\kappa\varepsilon\left(\langle\hat{N}_2\rangle + \langle\hat{N}_0\rangle\right)\right]}{\left[\kappa^2 + 4\varepsilon^2\right]\left[\kappa^2\langle\hat{N}_2\rangle + 4\varepsilon\kappa\langle\hat{\Sigma}_0\rangle + 4\varepsilon^2\langle\hat{N}_0\rangle\right]}. \tag{63}$$

On the other hand, the atom-number correlation is defined by

$$g(\hat{n}_1, \hat{n}_2)_a = \frac{\langle\hat{\Sigma}_1^\dagger\hat{\Sigma}_1\hat{\Sigma}_2^\dagger\hat{\Sigma}_2\rangle}{\langle\hat{\Sigma}_1^\dagger\hat{\Sigma}_1\rangle\langle\hat{\Sigma}_2^\dagger\hat{\Sigma}_2\rangle}. \tag{64}$$

We recall that the atomic operators $\hat{\Sigma}_1$ and $\hat{\Sigma}_2$ are Gaussian variables with zero mean. Hence Eq. (64) can be rewritten as

$$g(\hat{n}_1, \hat{n}_2)_a = 1 + \frac{\langle\hat{\Sigma}_1^\dagger\hat{\Sigma}_2^\dagger\rangle\langle\hat{\Sigma}_1\hat{\Sigma}_2\rangle + \langle\hat{\Sigma}_1^\dagger\hat{\Sigma}_2\rangle\langle\hat{\Sigma}_1\hat{\Sigma}_2^\dagger\rangle}{\langle\hat{\Sigma}_1^\dagger\hat{\Sigma}_1\rangle\langle\hat{\Sigma}_2^\dagger\hat{\Sigma}_2\rangle}. \quad (65)$$

It then follows that

$$g(\hat{n}_1, \hat{n}_2)_a = 1. \quad (66)$$

We immediately see that the maximum degree of photon number correlation observed when more atoms in the lower energy level than on the upper level. This occurs when the three-level laser is operating below threshold. On the other hand, we note that from Eqs. (63) and (66) that unlike the photon-number correlation, the atoms in the laser cavity are not correlated. Moreover, we point out that in the absence of subharmonic generator, one can never realize correlated photons in the laser cavity.

V. Quadrature Squeezing

In this section, we wish to calculate the quadrature squeezing of the cavity light in the entire frequency interval. The squeezing properties of the cavity light are described by two quadrature operators

$$\hat{a}_+ = \hat{a}^\dagger + \hat{a}, \qquad \hat{a}_- = i(\hat{a}^\dagger - \hat{a}). \quad (67)$$

It can be readily established that

$$[\hat{a}_+, \hat{a}_-] = \left[\frac{2i\gamma_c\kappa}{\kappa^2 - 4\varepsilon^2}\right]\left[\langle\hat{N}_2\rangle - \langle\hat{N}_0\rangle\right]. \quad (68)$$

It then follows that

$$\Delta a_+ \Delta a_- \geq \left[\frac{\gamma_c\kappa}{\kappa^2 - 4\varepsilon^2}\right]\left[\langle\hat{N}_2\rangle - \langle\hat{N}_0\rangle\right]. \quad (69)$$

The variance of the quadrature operators is expressible as

$$(\Delta a_\pm)^2 = \langle\hat{a}^\dagger\hat{a}\rangle + \langle\hat{a}\hat{a}^\dagger\rangle \pm [\langle\hat{a}^2\rangle + \langle\hat{a}^{\dagger 2}\rangle] \mp [\langle\hat{a}\rangle^2 + \langle\hat{a}^\dagger\rangle^2] - 2\langle\hat{a}\rangle\langle\hat{a}^\dagger\rangle. \quad (70)$$

Since $\hat{a}$ is a Gaussian variable with zero mean, the quadrature variance turns out to be

$$(\Delta a_\pm)^2 = \langle\hat{a}^\dagger\hat{a}\rangle + \langle\hat{a}\hat{a}^\dagger\rangle \pm [\langle\hat{a}^2\rangle + \langle\hat{a}^{\dagger 2}\rangle]. \quad (71)$$

It then follows that

$$(\Delta a_\pm)^2 = \frac{\gamma_c\kappa}{(\kappa \mp 2\varepsilon)^2}[N + \langle\hat{N}_1\rangle \pm 2\langle\hat{\Sigma}_0\rangle]. \quad (72)$$

We recall that the light generated by a two-level laser operating well above threshold is coherent, the quadrature variance of which is given by [12]

$$(\Delta a_\pm)^2_c = \frac{\gamma_c}{\kappa}N. \quad (73)$$

We calculate the quadrature squeezing of the cavity light relative to the quadrature variance of the cavity coherent light. We then define the quadrature squeezing of the cavity light by

$$S = \frac{(\Delta a_\pm)^2_c - (\Delta a_-)^2}{(\Delta a_\pm)^2_c}. \quad (74)$$

Hence employing (72) and (73), one can put Eq. (74) in the form

$$S = 1 - \frac{\kappa^2[N + \langle\hat{N}_1\rangle - 2\langle\hat{\Sigma}_0\rangle]}{N(\kappa + 2\varepsilon)^2}. \quad (75)$$

We observe that, unlike the mean photon number, the quadrature squeezing does not depend on the number of atoms. This implies that the quadrature squeezing of the cavity light is independent of the number of photons.

VI. Entanglement

To this end, we prefer to analyze the entanglement of photon-states in the laser cavity. Quantum entanglement is a physical phenomenon that occurs when pairs or groups of particles cannot be described independently instead, a quantum state may be given for the system as a whole. Measurements of physical properties such as position, momentum, spin, polarization, etc. performed on entangled particles are found to be appropriately correlated. A pair of particles is taken to be entangled in quantum theory, if its states cannot be expressed as a product of the states of its individual constituents. The preparation and manipulation of these entangled states that have nonclassical and nonlocal properties lead to a better understanding of the basic quantum principles. It is in this spirit that this section is devoted to the analysis of the entanglement of the two-mode photon states. In other words, it is a well-known fact that a quantum system is said to be entangled, if it is not separable. That is, if the density operator for the combined state cannot be described as a combination of the product density operators of the constituents,

$$\hat{\rho} \neq \sum_k p_k \hat{\rho}_k^{(1)} \otimes \hat{\rho}_k^{(2)}, \tag{76}$$

in which $p_k \gg 0$ and $\sum_k p_k = 1$ to verify the normalization of the combined density states. On the other hand, a maximally entangled CV state can be expressed as a coeigenstate of a pair of EPR-type operators [19] such as $\hat{X}_2 - \hat{X}_1$ and $\hat{P}_2 - \hat{P}_1$. The total variance of these two operators reduces to zero for maximally entangled CV states. According to the criteria given by Duan et al [12], cavity photons of a system are entangled, if the sum of the variance of a pair of EPR-like operators,

$$\hat{s} = \hat{X}_2 - \hat{X}_1, \tag{77}$$

$$\hat{t} = \hat{P}_2 + \hat{P}_1, \tag{78}$$

where $\hat{X}_1 = \frac{1}{\sqrt{2}}(\hat{a}_1 + \hat{a}_1^\dagger)$, $\hat{X}_2 = \frac{1}{\sqrt{2}}(\hat{a}_2 + \hat{a}_2^\dagger)$, $\hat{P}_1 = \frac{i}{\sqrt{2}}(\hat{a}_1^\dagger - \hat{a}_1)$, and $\hat{P}_2 = \frac{i}{\sqrt{2}}(\hat{a}_2^\dagger - \hat{a}_2)$ are quadrature operators for modes a_1 and a_2, satisfy

$$(\Delta s)^2 + (\Delta t)^2 < 2N \tag{79}$$

and recalling that the cavity mode operators $\hat{a}_1$ and $\hat{a}_2$ are Gaussian variables with zero mean, we readily get

$$(\Delta s)^2 + (\Delta t)^2 = \left[\langle \hat{a}_1^\dagger \hat{a}_1\rangle + \langle \hat{a}_1 \hat{a}_1^\dagger\rangle + \langle \hat{a}_2^\dagger \hat{a}_2\rangle + \langle \hat{a}_2 \hat{a}_2^\dagger\rangle\right] - \left[\langle \hat{a}_1 \hat{a}_2\rangle + \langle \hat{a}_1^\dagger \hat{a}_2^\dagger\rangle + \langle \hat{a}_2 \hat{a}_1\rangle + \langle \hat{a}_2^\dagger \hat{a}_1^\dagger\rangle\right]. \tag{80}$$

It then follows that

$$(\Delta s)^2 + (\Delta t)^2 = 2(\Delta a_-)^2. \tag{81}$$

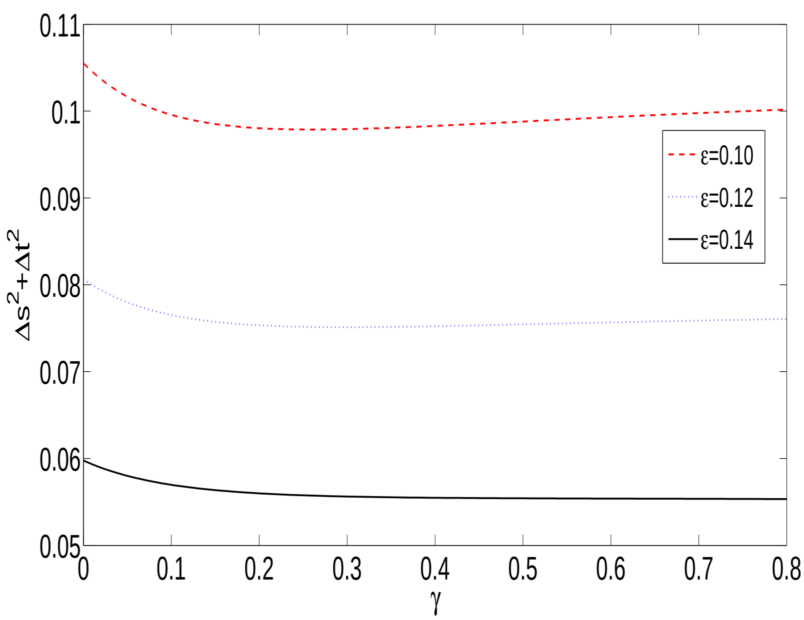

Figure 3: Plots of photon entanglement [Eq. (81)] versus γ for $\gamma_c = 0.4, \kappa = 0.8, N = 50, \tau_a = 0.2$, and for different values of ε.

In the absence of subharmonic generator, Eq. (81) takes the form

$$(\Delta s)^2 + (\Delta t)^2 = \left[\frac{\gamma_c}{\kappa} N\right]\left[\frac{2\gamma_c + 2\gamma + \tau_a}{\gamma + \gamma_c + \tau_a}\right] \tag{82}$$

We observe from Eq. (82) that as the stimulated decay constant increases, the degree of entanglement increases. On the basis of the criteria (79), we clearly see that the two states of the generated light are strongly entangled at steady-state. Moreover, the presence of the subharmonic generator leads to an increase in the degree of entanglement.

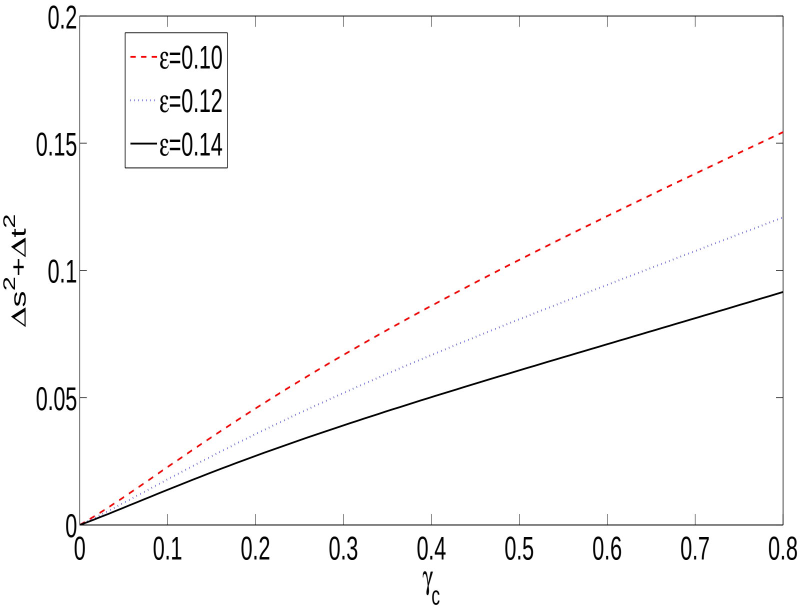

Figure 4: Plots of photon entanglement [Eq. (81)] versus γ_c for $\tau_a = 0.4,\ \kappa = 0.8,\ N = 50,\ \gamma = 0.2$, and for different values of ε.

On the other hand, cavity atoms of a system are entangled, if the sum of the variance of a pair of EPR-like operators,

$$\hat{u} = \hat{X}'_2 - \hat{X}'_1, \tag{83}$$

$$\hat{v} = \hat{P}'_2 + \hat{P}'_1, \tag{84}$$

where $\hat{X}'_1 = \frac{1}{\sqrt{2}}(\hat{\Sigma}_1 + \hat{\Sigma}_1^\dagger),\ \hat{X}'_2 = \frac{1}{\sqrt{2}}(\hat{\Sigma}_2 + \hat{\Sigma}_2^\dagger),\ \hat{P}'_1 = \frac{i}{\sqrt{2}}(\hat{\Sigma}_1^\dagger - \hat{\Sigma}_1),\ \hat{P}'_2 = \frac{i}{\sqrt{2}}(\hat{\Sigma}_2^\dagger - \hat{\Sigma}_2)$ are quadrature operators for the cavity atoms, satisfy

$$(\Delta u)^2 + (\Delta v)^2 < 2N. \tag{85}$$

Since $\hat{\Sigma}_1$ and $\hat{\Sigma}_2$ are Gaussian atomic operators with zero mean so one can easily find

$$(\Delta u)^2 + (\Delta v)^2 = \left[\langle\hat{\Sigma}_1^\dagger\hat{\Sigma}_1\rangle + \langle\hat{\Sigma}_1\hat{\Sigma}_1^\dagger\rangle + \langle\hat{\Sigma}_2^\dagger\hat{\Sigma}_2\rangle + \langle\hat{\Sigma}_2\hat{\Sigma}_2^\dagger\rangle\right]$$

$$-\left[\langle\hat{\Sigma}_1\hat{\Sigma}_2\rangle + \langle\hat{\Sigma}_2^\dagger\hat{\Sigma}_1^\dagger\rangle\right]. \tag{86}$$

It then follows that

$$(\Delta u)^2 + (\Delta v)^2 = 2N - \langle\hat{N}_1\rangle - \langle\hat{N}_2\rangle. \tag{87}$$

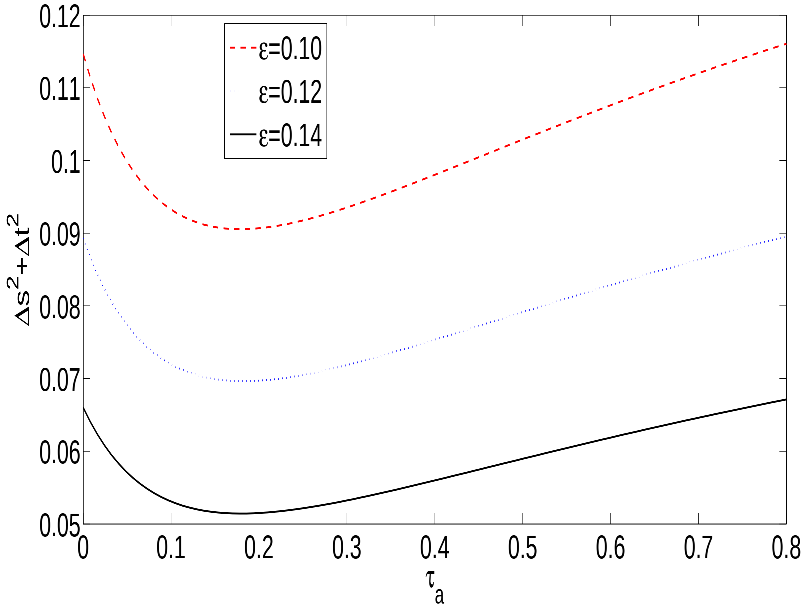

Figure 5: Plots of photon entanglement [Eq. (81)] versus τ_a for $\gamma_c = 0.4$, $\kappa = 0.8$, $N = 50$, $\gamma = 0.2$, and for different values of ε.

In the absence of subharmonic generator, Eq. (87) leads to

$$(\Delta u)^2 + (\Delta v)^2 = 2N - \left[\frac{(\gamma_c+\gamma)^2(\gamma+2\tau_a)+\gamma\tau_a^2}{(\gamma+\gamma_c)(\gamma+\tau_a)(\gamma+\gamma_c+\tau_a)}\right]N. \tag{88}$$

We immediately see note from Eqs. (82) and (88) that photon-state entanglement is greater than atom-state entanglement for the same values of κ, γ, γ_c, and τ_a.

VII. Conclusion

In this paper we present a thorough study of the squeezing as well as the statistical properties of the light produced by electrically pumped non-degenerate three-level laser, with two-mode subharmonic generator, coupled to a two-mode vacuum reservoirs via a single-port mirror whose open cavity contains N non-degenerate three-level cascade atoms. We carried out our analysis by putting the noise operators associated with a vacuum reservoir in normal order. We then first obtained the quantum Langevin equations for the cavity mode operators. We next determined the equations of evolution of the expectation values of atomic operators employing the pertinent master equation. Applying the steady-state solution of these equations, we have analyzed the mean photon number, the CV bipartite atomic and photon state entanglement as well as atom and photon number correlation. It is found that the photons and the atoms in the system are strongly entangled at steady state. Results show that the presence of parametric amplifier is to increase the squeezing and the mean photon number of the two-mode cavity light significantly. It is found that the photon- states of the system is strongly entangled at steady state where as the atomic state of the system is not entangled. In addition, we have established that the photons in the laser cavity are highly correlated and the degree of photon number correlation and entanglement increases as the amplitude of the coherent light driving the subharmonic generator increases. In addition, we have established that the presence of the subharmonic generator leads to an increase in the degree of entanglement and correlation. Moreover, we pointed out that in the absence of subharmonic generator, one can never realize correlated photons.

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