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Kantowski-Sachs Bulk Viscous String Cosmological Model in $f(R,T)$ Gravity with Time Varying Deceleration Parameter

By P. P. Khade, A. P. Wasnik & S. P. Kandalkar
Vidyabharati Mahavidyalaya

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P. P. Khade [α], A. P. Wasnik [σ] & S. P. Kandalkar [ρ]

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I. Introduction

In the light of the recent discovery of the accelerated expansion of the universe [1-3]. However, final satisfactory explanation about physical mechanism and driving force of accelerated expansion of the universe is yet to achieve as human mind has not achieved perfection. It is known that a point of universe is filled with dark energy. It has been addressed by various slow rolling scalar fields. It is supposed that the dark energy is responsible for producing sufficient acceleration in the late time of evolution of the universe. Thus, it is much more essential to study the fundamental nature of the dark energy and several approaches have been made to understand it. The cosmological constant is assumed to be the simplest candidate of dark energy. It is the classical correction made to Einstein's field equation by adding cosmological constant to the field equations. The introduction of cosmological constant to Einstein's field equation is the most efficient way of generating accelerated expansion, but it faces serious problems like fine tuning and cosmic coincidence problem in cosmology [4, 5]. Quintessence [6], phantom [7], k-essence [8], tachyons [9], and Chaplygin gas [10] are the other representative of dark energy. However, there is no direct detection of such exotic fluids. Dark energy can be explored in several ways, and modifying the geometric part of the Einstein-Hilbert action [11] is treated as the most efficient possible way. Based on its modifications, several alternative theories of gravity came into existence. Modified theories of gravity are attracting more and more attention of cosmologists because of the fact that these theories may serve as the possible candidates for explaining the late time acceleration of the universe. Some of the modified theories of gravity are (T), (R), (G), and (R, T) gravity. These models are proposed to explore the dark energy and other cosmological problems. Sharif and Azeem [12] discussed the Cosmological evolution for dark energy models in (T) gravity. Jamil et al. [13] have studied the stability of the interactive models of the dark energy, matter, and radiation for a FRW model in (T) gravity. Generalized second law of thermodynamics in (T) gravity with entropy corrections has been studied by Bamba et al. [14]. Recently, Harko et al. [15] developed another modified gravity known as $f(R, T)$ gravity. In this theory, the gravitational Lagrangian is given by an arbitrary function of the Ricci scalar R and of the trace of T of the stress energy tensor. In this paper, we concentrate on (R, T) gravity, with f being in this case a function of both R and T, manifesting a coupling between matter and geometry. Before going into the details of (R, T) gravity, The field equations of $f(R, T)$ gravity obtained from the action

$$S = \frac{1}{16\pi}\int\left[f(R, T) + L_m\right]\sqrt{-g}\,d^4x, \tag{1}$$

where $f(R, T)$ is an arbitrary function of the Ricci scalar R, T is the stress energy tensor T_{ij} of matter and L_m is the matter Lagrangian density, are given by

$$R_{ij} - \frac{1}{2}g_{ij}R = 8\pi T_{ij} + 2f'(T)T_{ij} + \left[2pf'(T) + f(T)\right]g_{ij}. \tag{2}$$

Studies of cosmic strings and bulk viscosity are crucial as they play an important role in structure formation of the early stages of evolution of the universe. Cosmic strings are one dimensional topological defects, which may be formed during symmetry breaking phase transition in the early universe along with other defects like domain walls and monopoles. Bulk viscosity driven

Author α: Vidyabharati Mahavidylaya, Amravati.
e-mail: pramodmaths04@gmail.com
Author σ: Bhartiya Mahavidyalaya, Amravati.
Author ρ: Government Institute of Science, Nagpur.

inflation is primarily due to the negative effective pressure which may overcome the pressure due to the usual gravity of matter distribution in the universe and provides an impetus for rapid expansion of the universe. Hence construction of bulk viscous string cosmological models have received considerable attention of research workers in the field. Many authors have investigated the astrophysical and cosmological implications of the (R, T) gravity [16–19]. Jamil et al. [20] have reconstructed some cosmological models for some specific forms of (R, T) in this modified gravity. Shamir et al. [21] obtained exact solution of anisotropic Bianchi type I and type V cosmological models whereas Chaubey and Shukla [22] have obtained a newclass of Bianchi cosmological models using special law of variation of parameter. Using a decoupled form of (R, T), that is, $(R, T) = (R) + (T)$ for Bianchi type V universe, Ahmed and Pradhan [23] have studied the energy conditions of perfect fluid cosmological models and Yadav [24] obtained some string solutions. Pawar and Solanke [25] have studied cosmological model filled with perfect fluid source in (R, T) gravity. Pawar and Agrawal [26] have obtained the solutions of dark energy cosmological model in the framework of the (R, T) theory of gravity. Recently Pawar et al. [27] have explored two fluid cosmological models in (R, T) theory. Mishra and Sahoo [28] solved the field equations of Bianchi type-VI*h* cosmological model in presence of perfect fluid in f (R,T) gravity. Sahoo and Mishra [29] studied Kaluza–Klein dark energy model in form of wet dark fluid in this theory. Reddy et al. [30] presented Kantowski-Sachs bulk viscous string model in (R, T) theory. Recently, Naidu et al. [31], Kiran and Reddy [32], and Reddy et al. [33] discussed the Bianchi type-V, Bianchi type-III, Kaluza-Klein space time with cosmic strings, and bulk viscosity in $f(R, T)$ gravity, respectively. Caroll et al.[34], Nojiri and Odintsov [35-37] and Chiba et al.[38] are some of the authors who have investigated several aspects of *f (R)* gravity. Recently, Adhav [39] has obtained Bianchi type-I cosmological model in *f (R,T)* gravity. Reddy et al.[40, 41] have discussed Bianchi type-III and Kaluza-Klein cosmological models in *f (R,T)* gravity while Reddy and Shantikumar [42] studied some anisotropic cosmological models and Bianchi type-III dark energy model, respectively, in *f (R,T)* gravity. Subsequently Kiran and Reddy [43] established the non-existence of Bianchi type-III bulk viscous string cosmological model in *f (R,T)* gravity. Recently, Naidu et al. [44] presented Bianchi type-V bulk viscous string model in *f (R,T)* gravity while Reddy et al. [45] have obtained the same in Saez-Ballester theory. We describe some important features of the (R) gravity. The recent motivation for studying (R) gravity came from the necessity to explain the apparent late-time accelerating expansion of the universe. Detailed reviews on (R) gravity can be found in [46–49]. Thermodynamic aspects of (R) gravity have been investigated in the works of [50, 51].

Inspired by the above investigation and discussion, in this paper we investigate the role of variable deceleration parameter in Kantowski-Sachs space time with bulk viscous string and $f(R, T)$ gravity. The plan of this paper is as follows: In sec. 2, we derive the field equations of $f(R, T)$ gravity in Kantowski-Sach space-time when the matter source is bulk viscous fluid with on dimensional cosmic strings. In Section 3, we find the solution of the field equations and the model. Some physical and kinematical properties of the model are discussed in section 4. Concluding remarks are presented in section 5.

II. Metric and Field Equations

The Spatially homogeneous and anisotropic Kantowski-Sachs space-time in the form

$$ds^2 = dt^2 - A^2dr^2 - B^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right), \quad (3)$$

where A and B are the cosmic time t only.

The energy momentum tensor for a bulk viscous fluid containing one dimensional cosmic string is considered as

$$T_{ij} = \left(\rho + \bar{p}\right)u_i u_j - \bar{p} g_{ij} - \lambda x_i x_j, \quad (4)$$

$$\bar{p} = p - 3\xi H, \quad (5)$$

where ρ is the rest energy density of the system, $3\xi H$ is usually known as bulk viscous pressure, H is Hubble's parameter, p is the pressure and λ is the string tension density. Also $u^i = \delta^i_4$ is the four velocity vector, which satisfies

$$g_{ij}u^i u_j = -x^i x_j = 1, \qquad u^i x_i = 0 \quad (6)$$

Here, we also consider ρ, $\bar{p}$, λ as functions of cosmic time t only.

Now, by adopting commoving coordinates the field equations (2) of $f(R, T)$ gravity for the metric (3), with the help of eqs. (4)-(6), for the particular choice of the function

$$f(T) = \mu T \quad (7)$$

where μ is constant, can be written as

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{1}{B^2} = \bar{p}(8\pi + 3\mu) - \lambda(8\pi + 3\mu) - \mu\rho \quad (8)$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}=\bar{p}(8\pi+3\mu)-\mu\lambda-\mu\rho \quad (9)$$

$$2\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}^2}{B^2}+\frac{1}{B^2}=\mu\bar{p}-\rho(8\pi+3\mu)-\mu\lambda \quad (10)$$

where an overhead dot indicates differentiation with respect to t.

Spatial volume and the scale factor for the metric (3) are respectively, defined by

$$V=AB^2 \quad (11)$$

$$a=\left(AB^2\right)^{\frac{1}{3}} \quad (12)$$

The physical quantities which play a significant role in the discussion of cosmological models are expansion scalar θ the mean anisotropy parameter A_h and shear scalar σ^2 which are defined as

$$\theta=3H=3\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right) \quad (13)$$

$$3A_h=\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2,\ \Delta H_i=H_i-H,\ i=1,\,2,\,3 \quad (14)$$

$$\sigma^2=\frac{1}{2}\sigma^{ij}\sigma_{ij}=3A_h{}^2-H^2 \quad (15)$$

where H is the mean Hubble parameter.

III. Solutions of the Field Equations

The field equations (8)-(10) reduce to the following two independent equations:

$$\frac{\ddot{A}}{A}-\frac{\ddot{B}}{B}-\frac{\dot{B}^2}{B^2}+\frac{\dot{A}\dot{B}}{AB}-\frac{1}{B^2}=(8\pi+2\mu)\lambda \quad (16)$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}-\frac{\dot{B}^2}{B^2}-\frac{\dot{A}\dot{B}}{AB}-\frac{1}{B^2}=\bar{p}(8\pi+2\mu)+\rho(8\pi+2\mu) \quad (17)$$

Here there are two equations involving five unknowns. Since the field equations are highly nonlinear for the complete determinacy, we need extra conditions among the variables. We consider these conditions in the form case (i) and case (ii) as defined below

(i) The scalar expansion θ in the model is proportional to the shear scalar σ^2 which yields

$$A=B^m \quad (18)$$

where $m\neq 1$ is a constant, takes care of anisotropy of the space-time (Collins et al. [52])

(ii) The combined effect of the proper pressure and the bulk viscous pressure for barotropic fluid can be written as

$$\bar{p}=p-3\xi H=(\varepsilon_0-\gamma)\rho,\quad 0\leq\varepsilon_0\leq 1,\ \ p=\varepsilon_0\rho \quad (19)$$

where ε_0 and γ are constants.

Case (i)

The case (i) consists of (i), (ii) and special form of deceleration parameter [53]

$$q=-1+\frac{\beta}{1+a^{\beta}} \quad (20)$$

where $\beta>0$ is a constant and a is a scale factor of the metric.

In this case, we have discussed the solution of the field equations by considering the extra conditions as above.

The Hubble parameter H is defined as $H=\frac{\dot{a}}{a}$ and from (20) we obtained

$$H=\frac{\dot{a}}{a}=a_1\left(1+a^{-\beta}\right) \quad (21)$$

where a_1 is a constant of integration.

Integrating (21) and using the initial conditions $a=0\ at\ t=0$ we have found

$$a=\left(e^{a_1\beta t}-1\right)^{\frac{1}{\beta}} \quad (22)$$

The scale factor of metric (3) is defined as

$$a=\left(AB^2\right)^{\frac{1}{3}} \quad (23)$$

With the help of (18), (22)and (23), we have found

$$A=\left(e^{a_1\beta t}-1\right)^{\frac{3m}{\beta(2+m)}} \quad (24)$$

$$B=\left(e^{a_1\beta t}-1\right)^{\frac{3}{\beta(2+m)}} \quad (25)$$

Using (24) and (25), the metric (3) can be written as

$$ds^2 = dt^2 - \left(e^{a_1\beta t}-1\right)^{\frac{6m}{\beta(2+m)}} dr^2 - \left(e^{a_1\beta t}-1\right)^{\frac{6}{\beta(2+m)}}\left(d\theta^2 + \sin^2\theta\, d\phi^2\right) \tag{26}$$

From (16), with the help of (24)-(25) we obtain the string tension density λ as

$$\lambda = \frac{1}{(8\pi+2\mu)}\left\{\frac{(m-1)m_1 e^{a_1\beta t}}{\left(e^{a_1\beta t}-1\right)}\left[\beta\left(1+\beta_1\frac{e^{a_1\beta t}}{\left(e^{a_1\beta t}-1\right)}\right)+\frac{m_{11}e^{a_1\beta t}}{\left(e^{a_1\beta t}-1\right)}\right]-\frac{1}{\left(e^{a_1\beta t}-1\right)^{\frac{6}{\beta(2+m)}}}\right\} \tag{27}$$

where $m_1 = \frac{3a_1^2}{(2+m)}$, $\beta_1 = \frac{3}{\beta(2+m)}-1$, $m_{11} = \frac{3(m+1)}{\beta(2+m)}$

From (17), with the help of (19) and (25), we obtained the rest energy density ρ as

$$\rho = \frac{1}{(8\pi+2\mu)(\varepsilon_0-\gamma+1)}\left\{\frac{m_1 e^{a_1\beta t}}{\left(e^{a_1\beta t}-1\right)}\left[(m+1)\beta\left(1+\beta_1\frac{e^{a_1\beta t}}{\left(e^{a_1\beta t}-1\right)}\right)+\frac{3\left(m^2-2m-1\right)e^{a_1\beta t}}{(2+m)\left(e^{a_1\beta t}-1\right)}\right]-\frac{1}{\left(e^{a_1\beta t}-1\right)^{\frac{6}{\beta(2+m)}}}\right\} \tag{28}$$

From (19), with the help of (28), we have obtained the total pressure $\bar{p}$, proper pressure p and the coefficient of bulk viscosity ξ as follows:

$$\bar{p} = \frac{(\varepsilon_0-\gamma)}{(8\pi+2\mu)(\varepsilon_0-\gamma+1)}\left\{\frac{m_1 e^{a_1\beta t}}{\left(e^{a_1\beta t}-1\right)}\left[(m+1)\beta\left(1+\beta_1\frac{e^{a_1\beta t}}{\left(e^{a_1\beta t}-1\right)}\right)+\frac{3\left(m^2-2m-1\right)e^{a_1\beta t}}{(2+m)\left(e^{a_1\beta t}-1\right)}\right]-\frac{1}{\left(e^{a_1\beta t}-1\right)^{\frac{6}{\beta(2+m)}}}\right\} \tag{29}$$

$$p = \frac{\varepsilon_0}{(8\pi+2\mu)(\varepsilon_0-\gamma+1)}\left\{\frac{m_1 e^{a_1\beta t}}{\left(e^{a_1\beta t}-1\right)}\left[(m+1)\beta\left(1+\beta_1\frac{e^{a_1\beta t}}{\left(e^{a_1\beta t}-1\right)}\right)+\frac{3\left(m^2-2m-1\right)e^{a_1\beta t}}{(2+m)\left(e^{a_1\beta t}-1\right)}\right]-\frac{1}{\left(e^{a_1\beta t}-1\right)^{\frac{6}{\beta(2+m)}}}\right\} \tag{30}$$

$$\xi = \frac{\gamma\left(e^{a_1\beta t}-1\right)}{9a_1(8\pi+2\mu)(\varepsilon_0-\gamma+1)e^{a_1\beta t}}\left\{\frac{m_1 e^{a_1\beta t}}{\left(e^{a_1\beta t}-1\right)}\left[(m+1)\beta\left(1+\beta_1\frac{e^{a_1\beta t}}{\left(e^{a_1\beta t}-1\right)}\right)+\frac{3\left(m^2-2m-1\right)e^{a_1\beta t}}{(2+m)\left(e^{a_1\beta t}-1\right)}\right]-\frac{1}{\left(e^{a_1\beta t}-1\right)^{\frac{6}{\beta(2+m)}}}\right\} \tag{31}$$

a) *Some Physical Properties of the Model*

Some physical properties of the model are given below, which have crucial role in the discussion of cosmological models are the spatial volume V, scalar expansion θ, The Hubble's parameter H, shear scalar σ^2 and mean anisotropy parameter A_h, for the model (26) the above quantities are given by

$$V = \left(e^{a_1\beta t}-1\right)^{\frac{3}{\beta}} \tag{32}$$

$$\theta = \frac{9a_1 e^{a_1\beta t}}{\left(e^{a_1\beta t}-1\right)} \tag{33}$$

$$H = \frac{3a_1 e^{a_1\beta t}}{\left(e^{a_1\beta t}-1\right)} \tag{34}$$

$$3A_h = \frac{6+4m+2m^2}{(2+m)^2} \tag{35}$$

$$\sigma^2 = \frac{\left(6+4m+2m^2\right)^2}{3(2+m)^4} - \frac{9{a_1}^2 e^{2a_1\beta t}}{\left(e^{a_1\beta t}-1\right)^2} \tag{36}$$

In this model we observed that at initial epoch the values of energy density ρ, proper pressure p, total pressure $\bar{p}$, coefficient of bulk viscosity ξ, and Hubble prarmaeter H are very high and these quantities gradually decreases with the evolution of time; and as $t\to\infty,\ \rho, p, \bar{p}, \xi, H \to 0$. Spatial volume increases with the evolution of time; that is $t\to\infty,\ V\to\infty$, when $t=0$ spatial volume vanishes and the expansion scalar is infinite. Which shows that the universe starts evolving with zero volume. The scale factor vanishes at $t=0$ and hence the model has a point singularity at the initial epoch. As t increases, the scale factor and the spatial volume increases but the expansion scalar decreases. Anisotropy parameter is constant which shows that model is model remains anisotropic throughout the evolution of the universe. It is noted that bulk viscosity in the universe decreases with time so that, we obtain, inflationary models. In this model as t increases string tension density λ increases slowly.

Case (ii)

The case (ii) consists of (i), (ii) and special form of deceleration parameter [54]

$$q = -kt + n - 1 \tag{37}$$

From (37) we have Akarsu and Dereli [54]:

$$= {}_1 \exp\left[\frac{2}{\sqrt{n^2 - 2c_1 k}} \arctan h\left(\frac{kt - n}{\sqrt{n^2 - 2c_1 k}}\right)\right] \qquad for\ k > 0 \quad n \geq 0 \tag{38}$$

And $a = \begin{cases} k_2 (nt + c_2)^{\frac{1}{n}} & for\ k = 0 \quad n > 0 \\ k_3 e^{c_3 t} & for\ k = 0 \quad n = 0 \end{cases}$

where $k_1,\ k_2,\ k_3,\ c_1,\ c_2, c_3$ are constants of integration. The last two values of a give the constant deceleration parameter. So we neglect these values of a as $q = cons\tan t$ is studied by earlier researcher. Thus we focus on the first value of scale factor.

The scale factor a can also be expressed as follows for $n > 1 \quad c_1 = 0$

$$a = k_1 \exp\left[\frac{2}{n} \arctan h\left(\frac{kt - n}{n}\right)\right] \tag{39}$$

$$a = k_1 e^{\left[\frac{2}{n} \arctan h\left(\frac{kt}{n} - 1\right)\right]}$$

With the help of (18), (23) and (39) we have obtained

$$A = k_1^{\frac{3m}{(2+m)}} \exp\left[\frac{6m}{n(2+m)} \arctan h\left(\frac{kt}{n} - 1\right)\right] \tag{40}$$

$$B = k_1^{\frac{3}{(2+m)}} \exp\left[\frac{6}{n(2+m)} \arctan h\left(\frac{kt}{n} - 1\right)\right] \tag{41}$$

From (16), with the help of (41), we found the string tension density as

$$\lambda = \frac{1}{(8\pi + 2\mu)} \left\{ \frac{k_{11}}{(2n - kt)^2 t^2} [kt + k_{12} + k_{13}] - \frac{1}{k_1^{\frac{6}{(2+m)}} \exp\left[\frac{12}{n(2+m)} \arctan h\left(\frac{kt}{n} - 1\right)\right]} \right\} \tag{42}$$

where $k_{11} = \frac{12(m-1)}{(m+2)}, \quad k_{12} = \frac{3}{(m+2)} - n, \quad k_{13} = \frac{3(m+1)}{(m+2)}$

From (17), with the help of (19) and (42), we found the rest energy density ρ :

$$\rho=\frac{1}{(8\pi+2\mu)(\varepsilon_0-\gamma+1)}\left\{\frac{4k_{13}}{(2n-kt)^2t^2}[kt+k_{12}+k_{14}]-\frac{1}{k_1^{\frac{6}{(2+m)}}\exp\left[\frac{12}{n(2+m)}\operatorname{arctan}h\left(\frac{kt}{n}-1\right)\right]}\right\} \tag{43}$$

where $k_{14}=\dfrac{3(m^2-2m-1)}{(m^2+3m+2)}$.

From (19), with the help of (43), we obtained the total pressure $\overline{p}$, proper pressure p and the coefficient of bulk viscosity ξ as follows:

$$\overline{p}=\frac{(\varepsilon_0-\gamma)}{(8\pi+2\mu)(\varepsilon_0-\gamma+1)}\left\{\frac{4k_{13}}{(2n-kt)^2t^2}[kt+k_{12}+k_{14}]-\frac{1}{k_1^{\frac{6}{(2+m)}}\exp\left[\frac{12}{n(2+m)}\operatorname{arctan}h\left(\frac{kt}{n}-1\right)\right]}\right\} \tag{44}$$

$$p=\frac{\varepsilon_0}{(8\pi+2\mu)(\varepsilon_0-\gamma+1)}\left\{\frac{4k_{13}}{(2n-kt)^2t^2}[kt+k_{12}+k_{14}]-\frac{1}{k_1^{\frac{6}{(2+m)}}\exp\left[\frac{12}{n(2+m)}\operatorname{arctan}h\left(\frac{kt}{n}-1\right)\right]}\right\} \tag{45}$$

$$\xi=\frac{\gamma t(2n-kt)}{18(8\pi+2\mu)(\varepsilon_0-\gamma+1)}\left\{\frac{4k_{13}}{(2n-kt)^2t^2}[kt+k_{12}+k_{14}]-\frac{1}{k_1^{\frac{6}{(2+m)}}\exp\left[\frac{12}{n(2+m)}\operatorname{arctan}h\left(\frac{kt}{n}-1\right)\right]}\right\} \tag{46}$$

b) Some Physical Properties of the Model

Some physical properties of the model are given below, which have significant role in the discussion of cosmological models are the spatial volume V, scalar expansion θ, The Hubble's parameter H, shear scalar σ^2 and mean anisotropy parameter A_h, the above quantities are given by

$$V=k_1^{3}\exp\left[\frac{6}{n}\operatorname{arctan}h\left(\frac{kt}{n}-1\right)\right] \tag{47}$$

$$\theta=\frac{18}{t(2n-kt)} \tag{48}$$

$$H=\frac{6}{t(2n-kt)} \tag{49}$$

$$3A_h=\frac{6+4m+2m^2}{(2+m)^2} \tag{50}$$

$$\sigma^2=\frac{(6+4m+2m^2)^2}{3(2+m)^4}-\frac{9a_1^{\,2}e^{2a_1\beta t}}{(e^{a_1\beta t}-1)^2} \tag{51}$$

In this model the energy density ρ, proper pressure p, total pressure $\overline{p}$, coefficient of bulk viscosity ξ, and Hubble prarmaeter H gradually decrease with the evolution of time. Spatial volume increases with evolution of time, after that it also diverges. Anisotropy parameter is constant so the model is anisotropic model throughout the evolution of the universe. In this model cosmic string decreases as t increases.

IV. Concluding Remarks

In this paper we have studied the Kantowski-Sachs bulk viscous string cosmological model in $f(R,T)$ theory of gravity with variable deceleration parameters. According the choice of deceleration parameter (20) and (37) we have presented two

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cosmological models. The observations of both the models are as follows:

- It is observed that in the first case the model has point singularity at the initial epoch. In the second case there is no point type singularity.
- In both the model as $t \to \infty$ *then* $\rho, p, \bar{p}, \xi, H \to 0$
- In both cases the mean anisotropy parameter $A_h \neq 0$ the model do not approach isotropy and it is time independent in which gives the indication that the anisotropy in expansion rates is maintained throughout the cosmic evolution.
- In the first model The spatial volume is zero at $t = 0$ and the expansion scalar is infinite, which suggest that the universe starts evolving with zero volume at $t = 0$, i.e. we have big bang scenario.
- We observed that the type of time variations of deceleration parameter considered here affect the nonexistence of cosmic string in this model. Hence the consideration of variable deceleration parameter contribute towards the existence of cosmic strings in the theory of Kantowski-Sachs space time.

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