Global Journal of Science Frontier Research: F
Mathematics and Decision Sciences
Volume 18 Issue 3 Version 1.0 Year 2018
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Unified Fractional Derivative Formulae for the Multivariable Aleph-Function

By FY. AY. Ant

Abstract- The object of this paper is to derive three unified fractional derivatives formulae for the Saigo-Maeda operators of fractional integration. The first formula deals with the product of a general class of multivariable polynomials and the multivariable Aleph-function. The second concerns the multivariable polynomials and two multivariable Aleph-functions with the help of the Leibniz rule for fractional derivatives. The last relation also implies the product of a class of multivariable polynomials and the multivariable Aleph-function but it is obtained by the application of the first formula twice and it implicates two independents variables instead of one. The polynomials and the functions have their arguments of the type $x^{\rho}\prod_{i=1}^{t}(x^{u_i}+\alpha_i)^{\sigma_i}$ are quite general nature. These formulae, besides being on very general character have been put in a compact form avoiding the occurrence of infinite series and thus making them put in applications. Our findings provide unifications and extensions of some (known and new) results. We shall give several corollaries and particular cases.

Keywords: general class of multivariable polynomial, saigo-maeda operator, multivariable aleph-function, multivariable H-function, alephfunction, fractional derivative formulae, generalized leibniz rule.

GJSFR-F Classification: FOR Code: MSC 2010: 33C60, 82C31

UNIFIEDFRACTIONALDERIVATIVEFORMULAEFORTHEMULTIVARIABLEALEPHFUNCTION

Ref

14. V. Kiryakova, A brief story about the operators of the generalized fractional calculus, Fract. Calc. Appl. Anal. 11(2), 203–220, (2008).

Unified Fractional Derivative Formulae for the Multivariable Aleph-Function

FY. AY. Ant

Abstract- The object of this paper is to derive three unified fractional derivatives formulae for the Saigo-Maeda operators of fractional integration. The first formula deals with the product of a general class of multivariable polynomials and the multivariable Aleph-function. The second concerns the multivariable polynomials and two multivariable Aleph-functions with the help of the Leibniz rule for fractional derivatives. The last relation also implies the product of a class of multivariable polynomials and the multivariable Aleph-function but it is obtained by the application of the first formula twice and it implicates two independents variables instead of one. The polynomials and the functions have their arguments of the type $x^{\rho}\prod_{i=1}^{t}(x^{u_i}+\alpha_i)^{\sigma_i}$ are quite general nature. These formulae, besides being on very general character have been put in a compact form avoiding the occurrence of infinite series and thus making them put in applications. Our findings provide unifications and extensions of some (known and new) results. We shall give several corollaries and particular cases.

Keywords: general class of multivariable polynomial, saigo-maeda operator, multivariable aleph-function, multivariable H-function, alephfunction, fractional derivative formulae, generalized leibniz rule.

I. Introduction and Preliminaries

The fractional integral operator involving various special functions has found significant importance and applications in mathematical analysis. Since last four decades, some workers like Love [14], McBride [18], Kalla [6,7], Kalla and Saxena [8,9], Saxena et al. [28], Saigo [21,22], Kilbas [10], Kilbas and Sebastian [11] have studied in depth the properties, applications and different extensions of various hypergeometric operators of Fractional integration. A detailed account of such operators along with their properties and applications can be found in the research monographs by Samko et al. [25], Miller and Ross [19], Kiryakova [13,14], Kilbas, Srivastava and Trujillo [12] and Debnath and Bhatta [3]. A useful generalization of the hypergeometric fractional integrals, including the Saigo operators [22,23], has been introduced by Marichev [15] (see details in Samko et al. [23] and also see Kilbas and Saigo [13]). The generalized fractional integral operator of arbitrary order, involving Appell function F_3 in the kernel defined and studied by Saigo and Maeda [24, p. 393, Eq (4.12) and (4.13)] in the following manner :

Let $\alpha,\alpha'\beta,\beta',\gamma$ be complex numbers. The fractional integral $(Re(\alpha)>0)$ and derivative $(Re(\alpha)<0)$ of a function $f(x)$ defined on $(0,\infty)$ is given by :

$$I_{0,x}^{\alpha,\alpha',\beta,\beta',\gamma}f(z)=\left[\begin{array}{l}\frac{x^{-\alpha}}{\Gamma(\gamma)}\int_0^x(x-t)^{\gamma-1}t^{-\alpha'}F_3\left[\alpha,\alpha',\beta,\beta';\gamma;1-\frac{t}{x};1-\frac{x}{t}\right]f(t)\mathrm{d}t, Re(\gamma)>0\\ \left(\frac{d}{dx}\right)^k\left(I_{0,x}^{\alpha,\alpha',\beta+k,\beta',\gamma+k}f\right)(x), Re(\gamma)\leqslant 0; k=[-Re(\gamma)]+1\end{array}\right. \tag{1.1}$$

and

$$I_{x,\infty}^{\alpha,\alpha',\beta,\beta',\gamma}f(z)=\left[\begin{array}{l}\frac{x^{-\alpha'}}{\Gamma(\gamma)}\int_x^{\infty}(t-x)^{\gamma-1}t^{-\alpha}F_3\left[\alpha,\alpha',\beta,\beta';\gamma;1-\frac{x}{t};1-\frac{t}{x}\right]f(t)\mathrm{d}t, Re(\gamma)>0\\ \left(-\frac{d}{dx}\right)^k\left(I_{x,\infty}^{\alpha,\alpha',\beta,\beta'+k,\gamma+k}f\right)(x), Re(\gamma)\leqslant 0; k=[-Re(\gamma)]+1\end{array}\right. \tag{1.2}$$

Author: Teacher in High School, France. e-mail: fredericayant@gmail.com

The Appell hypergeometric function of the third type denoted F_3 is defined by :

$$F_3(\alpha,\alpha',\beta,\beta';\gamma;z,t)=\sum_{m,n=0}^{\infty}\frac{(\alpha)_m(\alpha')_n(\beta)_m(\beta')_n}{(\gamma)_{m+n}}\frac{z^m t^n}{m!n!}\qquad |z|<1,|t|<1 \tag{1.3}$$

Recently, Agrawal [1], Soni and Singh [26], Ram and Suthar [20], Singh and Mandia [28] have studied several formulae about the fractional operator involving the product of a general class of polynomials of one variable defined by Srivastava [29] and multivariable H-functions introduced by Srivastava and Panda [34,35]. In this paper, we shall obtain three results that give the theorems of the product of two multivariable Aleph-functions and a general class of multivariable polynomials [30] in Saigo-Maeda operators.

The Aleph-function of several variables is an extension of the multivariable I-function defined by Sharma and Ahmad [25], itself is a generalization of G and H-functions of several variables defined by Srivastava et Panda [34,35]. The multiple Mellin-Barnes integral occurring in this paper will be referred to as the multivariable Aleph-function of $r-$ variables throughout our present study and will be defined and represented as follows (see Ayant [2]).

We have : $\aleph(z_1,\cdots,z_r)=\aleph^{0,\mathfrak{n}:m_1,n_1,\cdots,m_r,n_r}_{p_i,q_i,\tau_i;R:p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)};\cdots;p_{i^{(r)}},q_{i^{(r)}};\tau_{i^{(r)}};R^{(r)}}\left(\begin{array}{c|c} \mathrm{z}_1 & [(\mathrm{a}_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,\mathfrak{n}}], \\ \cdot & \cdot \\ \cdot & \cdots\cdots\cdots\cdots\cdots , \\ \cdot & \\ \mathrm{z}_r & \end{array}\right.$

$$[\tau_i(a_{ji};\alpha_{ji}^{(1)},\cdots,\alpha_{ji}^{(r)})_{\mathfrak{n}+1,p_i}]:\ [(\mathrm{c}_j^{(1)}),\gamma_j^{(1)})_{1,n_1}],[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)},\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_i^{(1)}}];\cdots;$$

$$[\tau_i(b_{ji};\beta_{ji}^{(1)},\cdots,\beta_{ji}^{(r)})_{m+1,q_i}]:[(\mathrm{d}_j^{(1)}),\delta_j^{(1)})_{1,m_1}],[\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)},\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_i^{(1)}}];\cdots;$$

$$\left.\begin{array}{l}[(\mathrm{c}_j^{(r)}),\gamma_j^{(r)})_{1,n_r}],[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)},\gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_i^{(r)}}] \\ \cdot \\ [(\mathrm{d}_j^{(r)}),\delta_j^{(r)})_{1,m_r}],[\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)},\delta_{ji^{(r)}}^{(r)})_{m_r+1,q_i^{(r)}}]\end{array}\right)=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)z_k^{s_k}\,\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.4}$$

with $\omega=\sqrt{-1}$

$$\psi(s_1,\cdots,s_r)=\frac{\prod_{j=1}^{\mathfrak{n}}\Gamma(1-a_j+\sum_{k=1}^{r}\alpha_j^{(k)}s_k)}{\sum_{i=1}^{R}[\tau_i\prod_{j=\mathfrak{n}+1}^{p_i}\Gamma(a_{ji}-\sum_{k=1}^{r}\alpha_{ji}^{(k)}s_k)\prod_{j=1}^{q_i}\Gamma(1-b_{ji}+\sum_{k=1}^{r}\beta_{ji}^{(k)}s_k)]} \tag{1.5}$$

and

$$\theta_k(s_k)=\frac{\prod_{j=1}^{m_k}\Gamma(d_j^{(k)}-\delta_j^{(k)}s_k)\prod_{j=1}^{n_k}\Gamma(1-c_j^{(k)}+\gamma_j^{(k)}s_k)}{\sum_{i^{(k)}=1}^{R^{(k)}}[\tau_{i^{(k)}}\prod_{j=m_k+1}^{q_{i^{(k)}}}\Gamma(1-d_{ji^{(k)}}^{(k)}+\delta_{ji^{(k)}}^{(k)}s_k)\prod_{j=n_k+1}^{p_{i^{(k)}}}\Gamma(c_{ji^{(k)}}^{(k)}-\gamma_{ji^{(k)}}^{(k)}s_k)]} \tag{1.6}$$

For more details, see Ayant [2]. The condition for absolute convergence of multiple Mellin-Barnes type contour can be obtained by extension of the corresponding Conditions for multivariable H-function given by as :

$|arg z_k|<\frac{1}{2}A_i^{(k)}\pi,$ where

$$A_i^{(k)}=\sum_{j=1}^{\mathfrak{n}}\alpha_j^{(k)}-\tau_i\sum_{j=\mathfrak{n}+1}^{p_i}\alpha_{ji}^{(k)}-\tau_i\sum_{j=1}^{q_i}\beta_{ji}^{(k)}+\sum_{j=1}^{n_k}\gamma_j^{(k)}-\tau_{i^{(k)}}\sum_{j=n_k+1}^{p_{i^{(k)}}}\gamma_{ji^{(k)}}^{(k)}+\sum_{j=1}^{m_k}\delta_j^{(k)}-\tau_{i^{(k)}}\sum_{j=m_k+1}^{q_{i^{(k)}}}\delta_{ji^{(k)}}^{(k)}>0 \tag{1.7}$$

Ref
26. C.K. Sharma and S.S. Ahmad, On the multivariable I-function. Acta ciencia Indica Math , 20(2) (1994), 113-116.

with, $k = 1, \cdots, r, i = 1, \cdots, R \;, i^{(k)} = 1, \cdots, R^{(k)}$

The complex numbers z_i are not zero.Throughout this document, we assume the existence and absolute convergence Conditions of the multivariable Aleph-function. We may establish the asymptotic expansion in the following convenient form :

Notes

$$\aleph(z_1, \cdots, z_r) = 0(\,|z_1|^{\alpha_1}, \cdots, |z_r|^{\alpha_r}\,)\,, max(\,|z_1|, \cdots, |z_r|\,) \rightarrow 0$$

$$\aleph(z_1, \cdots, z_r) = 0(\,|z_1|^{\beta_1}, \cdots, |z_r|^{\beta_r}\,)\,, min(\,|z_1|, \cdots, |z_r|\,) \rightarrow \infty$$

where: $k = 1, \cdots, r : \alpha_k = min[Re(d_j^{(k)}/\delta_j^{(k)})], j = 1, \cdots, m_k$ and

$$\beta_k = max[Re((c_j^{(k)} - 1)/\gamma_j^{(k)})], j = 1, \cdots, n_k$$

We shall note: $\aleph(z_1, \cdots, z_r) = \aleph_1(z_1, \cdots, z_r)$.

We define the Aleph-function of s-variable in the following manner :

$$\aleph(z_{r+1}, \cdots, z_{r+s}) = \aleph^{0,N:m_{r+1},n_{r+1},\cdots,m_{r+s},n_{r+s}}_{P_i,Q_i,\iota_i;R':p_{i(r+1)},q_{i(r+1)},\tau_{i(r+1)};R'^{(r+1)};\cdots;p_{i(r+s)},q_{i(r+s)};\tau_{i(r+s)};R'^{(r+s)}}\left(\begin{array}{c} \mathrm{z}_{r+1} \\ \cdot \\ \cdot \\ \mathrm{z}_{r+s} \end{array}\right|$$

$$[(\mathrm{a}'_j;\alpha_j^{(r+1)}, \cdots, \alpha_j^{(r+1)})_{1,N]}, [\iota_i(a'_{ji};\alpha_{ji}^{(r+1)}, \cdots, \alpha_{ji}^{(r+s)})_{N+1,P_i}] : [(\mathrm{c}_j^{(r+1)}), \gamma_j^{(r+1)})_{1,n_{r+1}}], [\tau_{i^{(r+1)}}(c_{ji^{(r+1)}}^{(r+1)}, \gamma_{ji^{(r+1)}}^{(r+1)})_{n_{r+1}+1,p_i^{(r+1)}}]$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\,, \quad [\iota_i(b'_{ji};\beta_{ji}^{(r+1)}, \cdots, \beta_{ji}^{(r+s)})_{M+1,Q_i}] : [(\mathrm{d}_j^{(r+1)}), \delta_j^{(r+1)})_{1,m_{r+1}}], [\tau_{i^{(r+1)}}(d_{ji^{(r+1)}}^{(r+1)}, \delta_{ji^{(r+1)}}^{(r+1)})_{m_{r+1}+1,q_i^{(r+1)}}]$$

$$;\cdots;[(c_j^{(r+s)}), \gamma_j^{(r+s)})_{1,n_{r+s}}], [\tau_{i^{(r+s)}}(c_{ji^{(r+s)}}^{(r+s)}, \gamma_{ji^{(r+s)}}^{(r+s)})_{n_{r+s}+1,p_i^{(r+s)}}]$$

$$;\cdots;[(d_j^{(r+s)}), \delta_j^{(r+s)})_{1,m_{r+s}}], [\tau_{i^{(r+s)}}(d_{ji^{(r+s)}}^{(r+s)}, \delta_{ji^{(r+s)}}^{(r+s)})_{m_{r+s+1,q_i^{(r+s)}}}]\right)$$

$$= \frac{1}{(2\pi\omega)^s}\int_{L_{r+1}}\cdots\int_{L_{r+s}}\psi(t_{r+1}, \cdots, t_{r+s})\prod_{k=r+1}^{r+s}\phi_k(t_k)z_k^{t_k}\,\mathrm{d}t_{r+1}\cdots\mathrm{d}t_{r+s} \tag{1.8}$$

$$\psi(t_{r+1}, \cdots, t_{r+s}) = \frac{\prod_{j=1}^{N}\Gamma(1 - a'_j + \sum_{k=r}^{r+s}\alpha_j^{(k)}t_k)}{\sum_{i=1}^{R'}[\iota_i\prod_{j=N+1}^{P_i}\Gamma(a'_{ji} - \sum_{k=r+1}^{r+s}\alpha_{ji}^{(k)}t_k)\prod_{j=1}^{Q_i}\Gamma(1 - b'_{ji} + \sum_{k=r+1}^{r+s}\beta_{ji}^{(k)}t_k)]} \tag{1.9}$$

and

$$\theta_k(t_k) = \frac{\prod_{j=1}^{m_k}\Gamma(d_j^{(k)} - \delta_j^{(k)}t_k)\prod_{j=1}^{n_k}\Gamma(1 - c_j^{(k)} + \gamma_j^{(k)}t_k)}{\sum_{i^{(k)}=1}^{R'^{(k)}}[\tau_{i^{(k)}}\prod_{j=m_k+1}^{q_{i^{(k)}}}\Gamma(1 - d_{ji^{(k)}}^{(k)} + \delta_{ji^{(k)}}^{(k)}t_k)\prod_{j=n_k+1}^{p_{i^{(k)}}}\Gamma(c_{ji^{(k)}}^{(k)} - \gamma_{ji^{(k)}}^{(k)}t_k)]}\,, k = r+1, \cdots, r+s \tag{1.10}$$

For more details, see Ayant [2]. $|argz_k| < \frac{1}{2}B_i^{(k)}\pi$, where

$$B_i^{(k)} = \sum_{j=1}^{N}\alpha_j^{(k)} - \iota_i\sum_{j=N+1}^{p_i}\alpha_{ji}^{(k)} - \iota_i\sum_{j=1}^{q_i}\beta_{ji}^{(k)} + \sum_{j=1}^{n_k}\gamma_j^{(k)} - \tau_{i^{(k)}}\sum_{j=n_k+1}^{p_{i^{(k)}}}\gamma_{ji^{(k)}}^{(k)} + \sum_{j=1}^{m_k}\delta_j^{(k)} - \iota_{i^{(k)}}\sum_{j=m_k+1}^{q_{i^{(k)}}}\delta_{ji^{(k)}}^{(k)} > 0 \quad (1.11)$$

with $k = r+1,\cdots,r+s, i = 1,\cdots,R'$, $i^{(k)} = 1,\cdots,R'^{(k)}$

The complex numbers z_i are not zero. Throughout this document, we assume the existence and absolute convergence Conditions of the multivariable Aleph-function. We may establish the asymptotic expansion in the following convenient form :

$$\aleph(z_{r+1},\cdots,z_{r+s}) = 0(\,|z_{r+1}|^{\alpha'_{r+1}},\cdots,|z_{r+s}|^{\alpha'_{r+s}}\,), max(\,|z_{r+1}|,\cdots,|z_{r+s}|\,) \to 0$$

$$\aleph(z_{r+1},\cdots,z_{r+s}) = 0(\,|z_{r+1}|^{\beta'_{r+1}},\cdots,|z_{r+s}|^{\beta'_{r+s}}\,), min(\,|z_{r+1}|,\cdots,|z_{r+s}|\,) \to \infty$$

where: $k = r+1,\cdots,r+s : \alpha'_k = min[Re(d_j^{(k)}/\delta_j^{(k)})], j = m_{r+1},\cdots,m_{r+s}$ and

$$\beta'_k = max[Re((c_j^{(k)}-1)/\gamma_j^{(k)})], j = n_{r+1},\cdots,n_{r+s}$$

We shall note: $\aleph(z_{r+1},\cdots,z_{r+s}) = \aleph_2(z_{r+1},\cdots,z_{r+s})$.

The generalized polynomials of multivariable defined by Srivastava [30], is given in the following manner :

$$S_{N_1,\cdots,N_v}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}[y_1,\cdots,y_v] = \sum_{K_1=0}^{[N_1/\mathfrak{M}_1]}\cdots\sum_{K_v=0}^{[N_v/\mathfrak{M}_v]}\frac{(-N_1)_{\mathfrak{M}_1K_1}}{K_1!}\cdots\frac{(-N_v)_{\mathfrak{M}_vK_v}}{K_v!}A[N_1,K_1;\cdots;N_v,K_v]y_1^{K_1}\cdots y_v^{K_v} \quad (1.12)$$

where $\mathfrak{M}_1,\cdots,\mathfrak{M}_v$ are arbitrary positive integers and the coefficients $A[N_1,K_1;\cdots;N_v,K_v]$ are arbitrary constants, real or complex.

We shall note $a_v = \frac{(-N_1)_{\mathfrak{M}_1K_1}}{K_1!}\cdots\frac{(-N_v)_{\mathfrak{M}_vK_v}}{K_v!}A[N_1,K_1;\cdots;N_v,K_v]$

II. Lemma

Lemma 1.

$$\left(I_{0,x}^{\alpha,\alpha',\beta,\beta',\gamma}t^{\mu-1}\right)(x) = \frac{\Gamma(\mu)\Gamma(\mu+\gamma-\alpha-\alpha'-\beta)\Gamma(\mu+\beta'-\alpha')}{\Gamma(\mu-\alpha-\alpha'+\gamma)\Gamma(\mu-\alpha'-\beta+\gamma)\Gamma(\mu+\beta')}x^{\mu-\alpha-\alpha'+\gamma-1} \quad (2.1)$$

where $\alpha,\alpha'\beta,\beta',\gamma \in \mathbb{C}, Re(\gamma) > 0, Re(\mu) > max\{0, Re(\alpha+\alpha'+\beta-\gamma), Re(\alpha'-\beta')\}$

Lemma 2.

$$\left(I_{x,\infty}^{\alpha,\alpha',\beta,\beta',\gamma}t^{\mu-1}\right)(x) = \frac{\Gamma(1+\alpha+\alpha'-\gamma-\mu)\Gamma(1+\alpha+\beta'-\gamma-\mu)\Gamma(1-\beta-\mu)}{\Gamma(1-\mu)\Gamma(1+\alpha+\alpha'+\beta'-\gamma-\mu)\Gamma(1+\alpha-\beta-\mu)}x^{\mu-\alpha-\alpha'+\gamma-1} \quad (2.2)$$

where $\alpha,\alpha'\beta,\beta',\gamma \in \mathbb{C}, Re(\gamma) > 0, Re(\mu) < 1 + min\{Re(-\beta), Re(\alpha+\alpha'-\gamma), Re(\alpha+\beta'-\gamma)\}$

Notes

III. Main Results

We have the following results.

a) Fractional derivative formula 1.

Theorem 1.

Notes

$$\left\{I_{0,x}^{\alpha,\alpha',\beta,\beta',\gamma}\left(x^{\rho}\prod_{i=1}^{t}(x^{u_i}+\alpha_i)^{\sigma_i}S_{N_1,\cdots,N_v}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}\begin{pmatrix}c_1x^{\lambda_1}\prod_{i=1}^{t}(x^{u_i}+\alpha_i)^{\eta_i^{(1)}}\\ \cdot\\ \cdot\\ \cdot\\ c_vx^{\lambda_v}\prod_{i=1}^{t}(x^{u_i}+\alpha_i)^{\eta_i^{(v)}}\end{pmatrix}\aleph_1\begin{pmatrix}z_1x^{\mu_1}\prod_{i=1}^{t}(x^{u_i}+\alpha_i)^{-\upsilon_i^{(1)}}\\ \cdot\\ \cdot\\ \cdot\\ z_rx^{\mu_r}\prod_{i=1}^{t}(x^{u_i}+\alpha_i)^{-\upsilon_i^{(r)}}\end{pmatrix}\right)\right\}$$

$$=\prod_{i=1}^{t}\alpha_i^{\sigma_i}x^{\rho-\alpha-\alpha'+\gamma}\sum_{K_1=0}^{[N_1/\mathfrak{M}_1]}\cdots\sum_{K_v=0}^{[N_v/\mathfrak{M}_v]}a_v\,c_1^{K_1}\cdots c_v^{K_v}x^{\sum_{j=1}^{v}\lambda_jK_j}\prod_{i=1}^{t}\alpha_i^{\sum_{j=1}^{v}K_j\eta_i^{(j)}}$$

$$\aleph_{p_i+t+3,q_i+t+3;\tau_i;R:W}^{0,\mathbf{n}+t+3;V}\left(\begin{array}{c|c}z_1x^{\mu_1}\prod_{i=1}^{t}\alpha_i^{-\upsilon_i^{(1)}} & \mathrm{A},\ \mathbf{A}\text{: C}\\ \cdot & \cdot\\ \cdot & \cdot\\ \cdot & \cdot\\ z_rx^{\mu_r}\prod_{i=1}^{t}\alpha_i^{-\upsilon_i^{(r)}} & \cdot\\ \alpha_1^{(-1)}x^{u_1} & \cdot\\ \cdot & \cdot\\ \cdot & \cdot\\ \cdot & \mathbf{B},\ \mathrm{B}\text{: D}\\ \alpha_t^{(-1)}x^{u_t} & \end{array}\right) \tag{3.1}$$

where

$$V=m_1,n_1;\cdots;m_r,n_r:1,0;\cdots;1,0 \tag{3.2}$$

$$W=p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)};\cdots;p_{i^{(r)}},q_{i^{(r)}},\tau_{i^{(r)}};R^{(r)};\underbrace{0,1;\cdots;0,1}_{t} \tag{3.3}$$

$$A=\Big(-\rho-\sum_{j=1}^{v}\lambda_jK_j;\mu_1,\cdots,\mu_r,u_1,\cdots,u_t\Big),\ \Big(\alpha'-\beta'-\rho-\sum_{j=1}^{v}\lambda_jK_j;\mu_1,\cdots,\mu_r,u_1,\cdots,u_t\Big),$$

$$\Big(-\rho-\gamma+\alpha'+\beta-\sum_{j=1}^{v}\lambda_jK_j;\mu_1,\cdots,\mu_r,u_1,\cdots,u_t\Big),\left(1+\sigma_1+\sum_{j=1}^{v}K_j\eta_1^{(j)};\upsilon_1^{(1)},\cdots,\upsilon_1^{(r)},1,\underbrace{0,\cdots,0}_{t-1}\right),\cdots,$$

$$\left(1+\sigma_t+\sum_{j=1}^{v}K_j\eta_t^{(j)};\upsilon_t^{(r)},\cdots,\upsilon_t^{(r)},\underbrace{0,\cdots,0}_{t-1},1\right) \tag{3.4}$$

$$\mathbf{A}=\{(a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)},\underbrace{0,\cdots,0}_{t})\}_{1,n},\{\tau_i(a_{ji};\alpha_{ji}^{(1)},\cdots,\alpha_{ji}^{(r)},\underbrace{0,\cdots,0}_{t})_{n+1,p_i}\}$$

$$C=\{(c_j^{(1)};\gamma_j^{(1)})_{1,n_1}\},\{\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)};\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_{i^{(1)}}}\};\cdots;\{(c_j^{(r)};\gamma_j^{(r)})_{1,n_r}\},\{\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)};\gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_{i^{(r)}}}\};-;\cdots;- \quad (3.5)$$

$$B=\Big(-\beta'-\rho-\sum_{j=1}^{v}\lambda_jK_j;\mu_1,\cdots,\mu_r,u_1,\cdots,u_t\Big),\ \Big(\alpha+\alpha'-\gamma-\rho-\sum_{j=1}^{v}\lambda_jK_j;\mu_1,\cdots,\mu_r,u_1,\cdots,u_t\Big),$$

$$\Big(-\rho-\gamma+\alpha'+\beta-\sum_{j=1}^{v}\lambda_jK_j;\mu_1,\cdots,\mu_r,u_1,\cdots,u_t\Big),\left(1+\sigma_i+\sum_{j=1}^{v}K_j\eta_i^{(j)};\upsilon_i^{(1)},\cdots,\upsilon_i^{(r)},\underbrace{0,\cdots,0}_{t}\right)_{1,t} \quad (3.6)$$

$$\mathbf{B}=\{\tau_i(b_{ji};\beta_{ji}^{(1)},\cdots,\beta_{ji}^{(r)},\underbrace{0,\cdots,0}_{t})_{m+1,q_i}\}:D=\{(d_j^{(1)};\delta_j^{(1)})_{1,m_1}\},\{\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)};\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_{i^{(1)}}}\};\cdots;$$

$$\{(d_j^{(r)};\delta_j^{(r)})_{1,m_r}\},\{\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)};\delta_{ji^{(r)}}^{(r)})_{m_r+1,q_{i^{(r)}}}\};\underbrace{(0;1),\cdots,(0;1)}_{t} \quad (3.7)$$

Provided that

$$Re(\gamma)>0;u_i,\lambda_j,\eta_i^{(j)},\mu_k,\upsilon_i^{(k)};i=1,\cdots,t;j=1,\cdots,v;k=1,\cdots,r.$$

$|argz_i|<\frac{1}{2}A_i^{(k)}\pi$, where $A_i^{(k)}$ is defined by (1.7).

$$Re(\rho)+\sum_{i=1}^{r}\mu_i\min_{1\leqslant l\leqslant m_i}Re\left(\frac{d_l^{(i)}}{\delta_l^{(i)}}\right)+1> max\{0,Re(\alpha+\alpha'+\beta-\gamma),Re(\alpha'-\beta')\}$$

Proof

To prove (3.1), we first express the general class of multivariable polynomials occurring on its left-hand side $S_{N_1,\cdots,N_v}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}[.]$ in series with the help of (1.12), replace the multivariable Aleph-function by its Mellin-Barnes integrals contour with the help of (1.4), interchange the order of summations and $(s_1,\cdots,s_r)$-integrals and taking the fractional derivative Operator inside (which is permissible under the stated conditions) and make a little simplification. Next, we express the Following terms $(x^{u_1}+\alpha_1)^{\sigma_1+\sum_{j=1}^{v}\eta_1^{(j)}K_j-\sum_{k=1}^{r}\upsilon_1^{(k)}s_k},\cdots,(x^{u_t}+\alpha_t)^{\sigma_t+\sum_{j=1}^{v}\eta_t^{(j)}K_j-\sum_{k=1}^{r}\upsilon_t^{(k)}s_k}$ so obtained regarding Mellin-Barnes integrals contour ([33], p. 18, eq.(2.6.4); p.10, eq.(2.1.1)). Now, interchanging the order of $(v_1,\cdots,v_s)$ and $(s_1,\cdots,s_r)$-integrals (which is permissible under the stated conditions), and evaluating the x-integral with the help of the lemma 1 and reinterpreting the multiple Mellin-Barnes integrals contour so obtained regarding the Aleph-function Of $(r+t)$-variables, we get the desired formula (3.1) after algebraic manipulations.

b) Fractional derivative formula 2

Theorem 2.

$$\left\{I_{0,x}^{\alpha,\alpha',\beta,\beta',\gamma}\left(x^{\rho}\prod_{i=1}^{t}(x^{u_i}+\alpha_i)^{\sigma_i}S_{N_1,\cdots,N_v}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}\begin{pmatrix}\mathrm{c}_1x^{\lambda_1}\prod_{i=1}^{t}(x^{u_i}+\alpha_i)^{\eta_i^{(1)}}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{c}_vx^{\lambda_v}\prod_{i=1}^{t}(x^{u_i}+\alpha_i)^{\eta_i^{(v)}}\end{pmatrix}\aleph_1\begin{pmatrix}\mathrm{z}_1x^{\mu_1}\prod_{i=1}^{t}(x^{u_i}+\alpha_i)^{-\upsilon_i^{(1)}}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_rx^{\mu_r}\prod_{i=1}^{t}(x^{u_i}+\alpha_i)^{-\upsilon_i^{(r)}}\end{pmatrix}\right.\right.$$

$$\left.\left.\aleph_2\begin{pmatrix}\mathrm{z}_{r+1}x^{\mu_{r+1}}\prod_{i=1}^{t-1}(x^{u_i}+\alpha_i)^{-\upsilon_i^{(r+1)}}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_{r+s}x^{\mu_{r+s}}\prod_{i=1}^{t-1}(x^{u_i}+\alpha_i)^{-\upsilon_i^{(r+s)}}\end{pmatrix}\right)\right\}=\prod_{i=1}^{t}\alpha_i^{\sigma_i}x^{\rho-\beta}\sum_{l=0}^{\infty}\sum_{K_1=0}^{[N_1/\mathfrak{M}_1]}\cdots\sum_{K_v=0}^{[N_v/\mathfrak{M}_v]}\binom{-\beta}{l}a_vc_1^{K_1}\cdots c_v^{K_v}$$

Notes

$$x^{\sum_{j=1}^{v}\lambda_j K_j}\prod_{i=1}^{t}\alpha^{\sum_{j=1}^{v}K_j\eta_i^{(j)}}\aleph^{0,\mathbf{n}+N+2t+6;V}_{p_i+P_i+2t+6,q_i+Q_i+2t+6;\tau_i;\iota_i;R;R':W}\left(\begin{array}{c|c} \mathrm{z}_1 x^{\mu_1}\prod_{i=1}^{t}\alpha_i^{-\upsilon_i^{(1)}} & \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_r x^{\mu_r}\prod_{i=1}^{t}\alpha_i^{-\upsilon_i^{(r)}} & \mathrm{A},\ \mathbf{A}\text{: C} \\ \alpha_1^{(-1)}x^{u_1} & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \alpha_t^{(-1)}x^{u_t} & \cdot \\ \mathrm{z}_{r+1}x^{\mu_{r+1}}\prod_{i=1}^{t}\alpha_i^{-\upsilon_i^{(r+1)}} & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{z}_{r+s}x^{\mu_{r+s}}\prod_{i=1}^{t}\alpha_i^{-\upsilon_i^{(r+s)}} & \cdot \\ \alpha_1^{(-1)}x^{u_1} & \mathbf{B},\ \text{B: D} \\ \cdot & \\ \cdot & \\ \cdot & \\ \alpha_{t-1}^{(-1)}x^{u_{t-1}} & \end{array}\right) \tag{3.8}$$

where

$$V=m_1,n_1;\cdots;m_r,n_r:1,0;\cdots;1,0;m_{r+1};\cdots;m_{r+s};1;0;\cdots;1,0 \tag{3.9}$$

$$W=p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)};\cdots;p_{i^{(r)}},q_{i^{(r)}},\tau_{i^{(r)}};R^{(r)};\underbrace{0,1;\cdots;0,1}_{t};p_{i^{(r+1)}},q_{i^{(r+1)}},\tau_{i^{(r+1)}};R'^{(r+1)};\cdots;$$

$$p_{i^{(r+s)}},q_{i^{(r+s)}},\tau_{i^{(r+s)}};R'^{(r+s)};\underbrace{0,1;\cdots;0,1}_{t-1} \tag{3.10}$$

$$A=(-\sum_{j=1}^{v}\lambda_jK_j;\mu_1,\cdots,\mu_r,u_1,\cdots,u_t,\underbrace{0,\cdots,0}_{t+s-1}),\ (\alpha'-\beta'-\sum_{j=1}^{v}\lambda_jK_j;\mu_1,\cdots,\mu_r,u_1,\cdots,u_t,\underbrace{0,\cdots,0}_{t+s-1}),$$

$$(\alpha+\alpha'+l-\gamma-\sum_{j=1}^{v}\lambda_jK_j;\mu_1,\cdots,\mu_r,u_1,\cdots,u_t,\underbrace{0,\cdots,0}_{t+s-1}),(-\beta'-\sum_{j=1}^{v}\lambda_jK_j;\mu_1,\cdots,\mu_r,u_1,\cdots,u_t,\underbrace{0,\cdots,0}_{t+s-1}),$$

$$(\alpha'+l-\gamma-\sum_{j=1}^{v}\lambda_jK_j;\mu_1,\cdots,\mu_r,u_1,\cdots,u_t,\underbrace{0,\cdots,0}_{t+s-1}),(\alpha+\alpha'-\gamma-\sum_{j=1}^{v}\lambda_jK_j;\mu_1,\cdots,\mu_r,u_1,\cdots,u_t,\underbrace{0,\cdots,0}_{t+s-1}),$$

$$\left(1+\sigma_1+\sum_{j=1}^{v}K_i\eta_1^{(i)};\upsilon_1^{(1)},\cdots,\upsilon_1^{(r)},1,\underbrace{0,\cdots,0}_{s+2t-2}\right),\cdots,\left(1+\sigma_t+\sum_{j=1}^{v}K_i\eta_t^{(i)};\upsilon_t^{(r)},\cdots,\upsilon_t^{(r)},\underbrace{0,\cdots,0}_{t-1},1,\underbrace{0,\cdots,0}_{s+t-1}\right),$$

$$\left(1+\sigma_i+\sum_{j=1}^{v}K_j\eta_i^{(j)};\upsilon_i^{(1)},\cdots,\upsilon_i^{(r)},\underbrace{0,\cdots,0}_{s+2t-1}\right)_{1,t} \tag{3.11}$$

Global Journal of Science Frontier Research (F) Volume XVIII Issue III Version I Year 2018

$$\mathbf{A}=\{(a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)},\underbrace{0,\cdots,0}_{s+2t-1})_{1,\mathfrak{n}}\},\{\tau_i(a_{ji};\alpha_{ji}^{(1)},\cdots,\alpha_{ji}^{(r)},\underbrace{0,\cdots,0}_{s+2t-1})_{n+1,p_i}\},$$

$$\{(a'_j;\underbrace{0,\cdots,0}_{r+t},\alpha_j^{(r+1)},\cdots,\alpha_j^{(r+s)},\underbrace{0,\cdots,0}_{t-1})_{1,N}\},\{\iota_i(a'_{ji};\underbrace{0,\cdots,0}_{r+t},\alpha_{ji}^{(r+1)},\cdots,\alpha_{ji}^{(r+s)},\underbrace{0,\cdots,0}_{t-1})_{N+1,P_i}\},$$

$$C=\{(c_j^{(1)};\gamma_j^{(1)})_{1,n_1}\},\{\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)};\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_{i^{(1)}}}\};\cdots;\{(c_j^{(r)};\gamma_j^{(r)})_{1,n_r}\},\{\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)};\gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_{i^{(r)}}}\};-;\cdots;-$$

$$;\cdots;\{(c_j^{(r+1)};\gamma_j^{(r+1)})_{1,n_{r+1}}\},\{\tau_{i^{(r+1)}}(c_{ji^{(r+1)}}^{(r+1)};\gamma_{ji^{(r+1)}}^{(r+1)})_{n_{r+1}+1,p_{i^{(r+1)}}}\}$$

$$\{(c_j^{(r+s)};\gamma_j^{(r+s)})_{1,n_{r+s}}\},\{\tau_{i^{(r+s)}}(c_{ji^{(r+s)}}^{(r+s)};\gamma_{ji^{(r+s)}}^{(r+s)})_{n_{r+s}+1,p_{i^{(r+s)}}}\};-;\cdots;- \tag{3.12}$$

$$B=(-\rho-\sum_{j=1}^{v}\lambda_jK_j;\underbrace{0,\cdots,0}_{r+t},\mu_{r+1},\cdots,\mu_{r+s},u_1,\cdots,u_{t-1}),\ (\alpha+\alpha'-\gamma-\rho-\sum_{j=1}^{v}\lambda_jK_j;\underbrace{0,\cdots,0}_{r+s},\mu_{r+1},\cdots,\mu_{r+s},u_1,\cdots,u_{t-1}),$$

$$(\alpha'-\beta'-\rho-\sum_{j=1}^{v}\lambda_jK_j;\underbrace{0,\cdots,0}_{r+t},\mu_{r+1},\cdots,\mu_{r+s},u_1,\cdots,u_{t-1}),\ (\alpha+\alpha'-\beta-l-\rho-\sum_{j=1}^{v}\lambda_jK_j;\underbrace{0,\cdots,0}_{r+s},\mu_{r+1},\cdots,\mu_{r+t},u_1,\cdots,u_{t-1}),$$

$$(\alpha'+\beta-l-\rho-\gamma-\sum_{j=1}^{v}\lambda_jK_j;\underbrace{0,\cdots,0}_{r+t},\mu_{r+1},\cdots,\mu_{r+s},u_1,\cdots,u_{t-1}),(-\beta-\sum_{j=1}^{v}\lambda_jK_j;\underbrace{0,\cdots,0}_{r+t},\mu_{r+1},\cdots,\mu_{r+s},u_1,\cdots,u_{t-1}),$$

$$\left(1+\sigma_{s-1};\underbrace{0,\cdots,0}_{r+t},\upsilon_{s-1}^{(r+1)},\cdots,\upsilon_{s-1}^{(r+s)},\underbrace{0,\cdots,0}_{t-2},1\right),\left(1+\sigma_{s-1};\underbrace{0,\cdots,0}_{r+t},\upsilon_{s-1}^{(r+1)},\cdots,\upsilon_{s-1}^{(r+s)},\underbrace{0,\cdots,0}_{t-2},1\right),\cdot,$$

$$\left(1+\sigma_i;\underbrace{0,\cdots,0}_{r+t},\upsilon_i^{(r+1)},\cdots,\upsilon_i^{(r+s)},\underbrace{0,\cdots,0}_{t-1}\right)_{1,t-1} \tag{3.13}$$

$$\mathbf{B}=\{\tau_i(b_{ji};\beta_{ji}^{(1)},\cdots,\beta_{ji}^{(r)},\underbrace{0,\cdots,0}_{s+2t-1})_{m+1,q_i}\},\{\iota_i(b'_{ji};\underbrace{0,\cdots,0}_{r+s},\beta_{ji}^{(r+1)},\cdots,\beta_{ji}^{(r+s)},\underbrace{0,\cdots,0}_{t-1})_{M+1,Q_i}\}:$$

$$D=\{(d_j^{(1)};\delta_j^{(1)})_{1,m_1}\},\{\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)};\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_{i^{(1)}}}\};\cdots;\{(d_j^{(r)};\delta_j^{(r)})_{1,m_r}\},\{\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)};\delta_{ji^{(r)}}^{(r)})_{m_r+1,q_{i^{(r)}}}\};\underbrace{(0;1);\cdots;(0;1)}_{t}$$

$$\{(d_j^{(r+s)};\delta_j^{(r+s)})_{1,m_{r+s}},\tau_{i^{(r+s)}}(\delta_{ji^{(r+s)}}^{(r+s)};\delta_{ji^{(r+s)}}^{(r+s)})_{m_{r+s}+1,q_{i^{(r+s)}}}\}\{(d_j^{(r+1)};\delta_j^{(r+1)})_{1,m_1},\{\tau_{i^{(r+1)}}(d_{ji^{(r+1)}}^{(r+1)};\beta_{ji^{(r+1)}}^{(r+1)})_{m_{r+1}+1,q_{i^{(r+1)}}}\}$$

$$;\cdots;\underbrace{(0;1);\cdots;(0;1)}_{t-1} \tag{3.14}$$

Notes

Provided that

$Re(\gamma) > 0; u_i, \lambda_j, \eta_i^{(j)}, \mu_k, \upsilon_i^{(k)}; i = 1, \cdots, t; j = 1, \cdots, v; k = 1, \cdots, r+s.$

$|argz_i| < \frac{1}{2}A_i^{(k)}\pi$, where $A_i^{(k)}$ is defined by (1.7).

$|argz_k| < \frac{1}{2}B_i^{(k)}\pi$, where $B_i^{(k)}$ is defined by (1.11).

$$Re(\rho) + \sum_{i=1}^{r+s} \mu_i \min_{1\leqslant l\leqslant m_i} Re\left(\frac{d_l^{(i)}}{\delta_l^{(i)}}\right) + 1 > \; max\{0, Re(\alpha+\alpha'+\beta-\gamma), Re(\alpha'-\beta')\}$$

and the multiple series on the left-hand side of (3.8) converges absolutely.

Proof

To prove the second theorem, we take

$$f(x) = x^{\rho}\prod_{i=1}^{t-1}(x^{u_i}+\alpha_i)^{\sigma_i}\aleph_2\begin{pmatrix} \mathrm{z}_{r+1}x^{\mu_{r+1}}\prod_{i=1}^{t-1}(x^{u_i}+\alpha_i)^{-\upsilon_i^{(r+1)}} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_{r+s}x^{\mu_{r+s}}\prod_{i=1}^{t-1}(x^{u_i}+\alpha_i)^{-\upsilon_i^{(r+s)}} \end{pmatrix}$$

and

$$g(x) = (x^{u_t}+\alpha_t)^{\sigma_t}S_{N_1,\cdots,N_v}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}\begin{pmatrix} \mathrm{c}_1x^{\lambda_1}\prod_{i=1}^{t}(x^{u_i}+\alpha_i)^{\eta_i^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{c}_vx^{\lambda_v}\prod_{i=1}^{t}(x^{u_i}+\alpha_i)^{\eta_i^{(v)}} \end{pmatrix}\aleph_1\begin{pmatrix} \mathrm{z}_1x^{\mu_1}\prod_{i=1}^{t}(x^{u_i}+\alpha_i)^{-\upsilon_i^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_rx^{\mu_r}\prod_{i=1}^{t}(x^{u_i}+\alpha_i)^{-\upsilon_i^{(r)}} \end{pmatrix}$$

in the left-hand side of the equation (3.8) and apply the following generalized Leibniz rule for the factional integrals

$$I_{0,x}^{\alpha,\alpha',\beta,\beta',\gamma}\{f(x)g(x)\} = \sum_{l=0}^{\infty}\binom{-\beta}{l}I_{0,x}^{\alpha,\alpha',\beta-l,\beta',\gamma}\{f(x)\}\,I_{0,x}^{\alpha,\alpha',l,\beta',\gamma}\{g(x)\} \tag{3.15}$$

We obtain the second relation of fractional derivative after algebraic manipulations on making use of theorem 1 and the result ([5],p. 91, eq. (6)).

c) Fractional derivative formula 1.

Theorem 3.

$$I_{0,x}^{\alpha_1,\alpha_1',\beta_1,\beta_1',\gamma_1}I_{0,x}^{\alpha_2,\alpha_2',\beta_2,\beta_2',\gamma_2}\left\{x^{\rho}y^{\rho'}\prod_{i=1}^{t}(x^{u_i}+\alpha_i)^{\sigma_i}\prod_{i=1}^{t}(y^{u_i'}+\beta_i)^{\sigma_i'}\right.$$

$$S_{N_1,\cdots,N_v}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}\begin{pmatrix} \mathrm{c}_1x^{\lambda_1}y^{\zeta_1}\prod_{i=1}^{t}(x^{u_i}+\alpha_i)^{\eta_i^{(1)}}(y^{u_i'}+\beta_i)^{\eta_i'^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{c}_vx^{\lambda_v}y^{\zeta_v}\prod_{i=1}^{t}(x^{u_i}+\alpha_i)^{\eta_i^{(v)}}(y^{u_i'}+\beta_i)^{\eta_i'^{(v)}} \end{pmatrix}$$

Ref

5. K.C. Gupta and R.C. Soni, A study of H-functions of one and several variables, J. Rajasthan. Acad. Phys. Sci. 1 (2002), 89-94.

$$\aleph_1\left(\begin{array}{c} z_1 x^{\mu_1} y^{\mu'_1} \prod_{i=1}^t (x^{u_i}+\alpha_i)^{-\upsilon_i^{(1)}} (x^{u'_i}+\alpha'_i)^{-\upsilon_i'^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ z_r x^{\mu_r} y^{\mu'_r} \prod_{i=1}^t (x^{u_i}+\alpha_i)^{-\upsilon_i^{(r)}} (x^{u'_i}+\alpha'_i)^{-\upsilon_i'^{(r)}} \end{array}\right)\Bigg\} = \prod_{i=1}^t \alpha_i^{\sigma_i} \prod_{i=1}^t \beta_i^{\sigma'_i} x^{\rho-\alpha_1-\alpha'_1+\gamma_1} x^{\rho'-\alpha_2-\alpha'_2+\gamma_2}$$

$$\sum_{K_1=0}^{[N_1/\mathfrak{M}_1]} \cdots \sum_{K_v=0}^{[N_v/\mathfrak{M}_v]} a_v\, c_1^{K_1} \cdots c_v^{K_v} x^{\sum_{j=1}^v \lambda_j K_j} y^{\sum_{j=1}^v \zeta_j K_j} \prod_{i=1}^t \alpha_i^{\sum_{j=1}^v K_j \eta_i^{(j)}} \beta_i^{\sum_{j=1}^v K_j \eta_i'^{(j)}}$$

$$\aleph^{0,\mathbf{n}+2t+6;V}_{p_i+2t+6,q_i+2t+6;\tau_i;R:W}\left(\begin{array}{c|c} z_1 x^{\mu_1} y^{\mu'_1} \prod_{i=1}^t \alpha_i^{-\upsilon_i^{(1)}} \beta_i^{-\upsilon_i'^{(1)}} & \\ \cdot & \mathrm{A}, \mathbf{A}: \mathrm{C} \\ \cdot & \cdot \\ \cdot & \cdot \\ z_r x^{\mu_r} y^{\mu'_r} \prod_{i=1}^t \alpha_i^{-\upsilon_i^{(r)}} \beta_i^{-\upsilon_i'^{(r)}} & \cdot \\ \alpha_1^{(-1)} x^{u_1} & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \alpha_t^{(-1)} x^{u_t} & \cdot \\ \beta_1^{(-1)} y^{u'_1} & \cdot \\ \cdot & \mathbf{B}, \mathrm{B}: \mathrm{D} \\ \cdot & \\ \cdot & \\ \alpha_t^{(-1)} y^{u'_t} & \end{array}\right) \tag{3.16}$$

where

$$V = m_1, n_1; \cdots; m_r, n_r : 1, 0; \cdots; 1, 0 \tag{3.17}$$

$$W = p_{i^{(1)}}, q_{i^{(1)}}, \tau_{i^{(1)}}; R^{(1)}; \cdots; p_{i^{(r)}}, q_{i^{(r)}}, \tau_{i^{(r)}}; R^{(r)}; \underbrace{0,1;\cdots;0,1}_{2t} \tag{3.18}$$

$$A = \Big(-\rho - \sum_{j=1}^v \lambda_j K_j; \mu_1, \cdots, \mu_r, \underbrace{0,\cdots,0}_{t}, u_1, \cdots, u_t\Big),\ \Big(-\alpha'_1 - \beta'_1 - \gamma_1 - \rho - \sum_{j=1}^v \lambda_j K_j; \mu_1, \cdots, \mu_r, \underbrace{0,\cdots,0}_{t}, u_1, \cdots, u_t\Big),$$

$$\Big(\alpha + \alpha'_1 + \beta_1 - \gamma_1 - \rho - \sum_{j=1}^v \lambda_j K_j; \mu_1, \cdots, \mu_r, \underbrace{0,\cdots,0}_{t}, u_1, \cdots, u_t\Big), \Big(-\beta'_1 - \rho - \sum_{j=1}^v \lambda_j K_j; \mu_1, \cdots, \mu_r, \underbrace{0,\cdots,0}_{t}, u_1, \cdots, u_t\Big),$$

$$\Big(\alpha_1 + \alpha'_1 - \gamma_1 - \rho - \sum_{j=1}^v \lambda_j K_j; \mu_1, \cdots, \mu_r, \underbrace{0,\cdots,0}_{t}, u_1, \cdots, u_t\Big), \Big(\beta_1 + \alpha'_1 - \gamma_1 - \rho - \sum_{j=1}^v \lambda_j K_j; \mu_1, \cdots, \mu_r, \underbrace{0,\cdots,0}_{t}, u_1, \cdots, u_t\Big),$$

$$\left(1 + \sigma_1 + \sum_{j=1}^v K_j \eta_1^{(j)}; \upsilon_1^{(1)}, \cdots, \upsilon_1^{(r)}, \underbrace{0,\cdots,0}_{t}, 1, \underbrace{0,\cdots,0}_{t-1}\right), \cdots, \left(1 + \sigma_t + \sum_{j=1}^v K_j \eta_t^{(j)}; \upsilon_t^{(r)}, \cdots, \upsilon_t^{(r)}, \underbrace{0,\cdots,0}_{2t-1}, 1\right)$$

Notes

Global Journal of Science Frontier Research (F) Volume XVIII Issue III Version I Year 2018

$$\left(1+\sigma_i+\sum_{j=1}^{v}K_j\eta_i^{(j)};\upsilon_i^{(1)},\cdots,\upsilon_i^{(r)},\underbrace{0,\cdots,0}_{2t}\right)_{1,t} \tag{3.19}$$

$$\mathbf{A}=\{(a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)},\underbrace{0,\cdots,0}_{2t})_{1,n}\},\{\tau_i(a_{ji};\alpha_{ji}^{(1)},\cdots,\alpha_{ji}^{(r)},\underbrace{0,\cdots,0}_{2t})_{n+1,p_i}\}$$

Notes

$$C=\{(c_j^{(1)};\gamma_j^{(1)})_{1,n_1}\},\{\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)};\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_{i^{(1)}}}\};\cdots;\{(c_j^{(r)};\gamma_j^{(r)})_{1,n_r}\},\{\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)};\gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_{i^{(r)}}}\};-;\cdots;-$$

$$B=(-\rho'-\sum_{j=1}^{v}\zeta_jK_j;\mu_1',\cdots,\mu_r',u_1',\cdots,u_t',\underbrace{0,\cdots,0}_{t}),\ (\alpha_2'-\beta_2'-\gamma_2-\rho'-\sum_{j=1}^{v}\zeta_jK_j;\mu_1',\cdots,\mu_r',u_1',\cdots,u_t',\underbrace{0,\cdots,0}_{t}),$$

$$(\alpha_2'+\alpha_2+\beta_2-\gamma_2-\rho'-\sum_{j=1}^{v}\zeta_jK_j;\mu_1',\cdots,\mu_r',u_1',\cdots,u_t',\underbrace{0,\cdots,0}_{t}),(-\rho'-\beta_2'-\sum_{j=1}^{v}\zeta_jK_j;\mu_1',\cdots,\mu_r',u_1',\cdots,u_t',\underbrace{0,\cdots,0}_{t}),$$

$$(\alpha_2'+\alpha_2-\gamma_2-\rho'-\sum_{j=1}^{v}\zeta_jK_j;\mu_1',\cdots,\mu_r',u_1',\cdots,u_t',\underbrace{0,\cdots,0}_{t}),(\alpha_2'-\beta_2-\gamma_2-\rho'-\sum_{j=1}^{v}\zeta_jK_j;\mu_1',\cdots,\mu_r',u_1',\cdots,u_t',\underbrace{0,\cdots,0}_{t}),$$

$$\left(1+\sigma_1'+\sum_{j=1}^{v}K_j\eta_1'^{(j)};\upsilon_1'^{(1)},\cdots,\upsilon_1'^{(r)},1,\underbrace{0,\cdots,0}_{2t-1}\right),\cdots,\left(1+\sigma_t'+\sum_{j=1}^{v}K_j\eta_t'^{(j)};\upsilon_t'^{(1)},\cdots,\upsilon_t'^{(r)},\underbrace{0,\cdots,0}_{t},1,\underbrace{0,\cdots,0}_{t-1}\right)$$

$$,\left(1+\sigma_i'+\sum_{j=1}^{v}K_j\eta_i'^{(j)};\upsilon_i'^{(1)},\cdots,\upsilon_i'^{(r)},\underbrace{0,\cdots,0}_{2t}\right)_{1,t} \tag{3.20}$$

$$\mathbf{B}=\{\tau_i(b_{ji};\beta_{ji}^{(1)},\cdots,\beta_{ji}^{(r)},\underbrace{0,\cdots,0}_{2t})_{m+1,q_i}\}:D=\{(d_j^{(1)};\delta_j^{(1)})_{1,m_1}\},\{\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)};\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_{i^{(1)}}}\};\cdots;$$

$$\{(d_j^{(r)};\delta_j^{(r)})_{1,m_r}\},\{\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)};\delta_{ji^{(r)}}^{(r)})_{m_r+1,q_{i^{(r)}}}\};\underbrace{(0;1),\cdots,(0;1)}_{2t} \tag{3.21}$$

Provided that

$$Re(\gamma_1)>0,Re(\gamma_2)>0;u_i,u_i',\lambda_j,\zeta_j,\eta_i^{(j)},\eta_i'^{(j)},\mu_k,\mu_k',\upsilon_i^{(k)},\upsilon_i'^{(k)};i=1,\cdots,t;j=1,\cdots,v;k=1,\cdots,r.$$

$|argz_i|<\frac{1}{2}A_i^{(k)}\pi$, where Ω_i is defined by (1.7).

$$Re(\rho)+\sum_{i=1}^{r}\mu_i\min_{1\leqslant l\leqslant m_i}Re\left(\frac{d_l^{(i)}}{\delta_l^{(i)}}\right)+1>\ max\{0,Re(\alpha_1+\alpha_1'+\beta_1-\gamma_1),Re(\alpha_1'-\beta_1')\}$$

$$Re(\rho')+\sum_{i=1}^{r}\mu_i'\min_{1\leqslant l\leqslant m_i}Re\left(\frac{d_l^{(i)}}{\delta_l^{(i)}}\right)+1>\ max\{0,Re(\alpha_2+\alpha_2'+\beta_2-\gamma_2),Re(\alpha_2'-\beta_2')\}$$

Proof of (3.16).
To prove the theorem 3; we use the fractional derivative formula one twice, first concerning the variable y and then concerning the variable x; here x and y are independent variables.

IV. Special Cases and Applications

The fractional derivative formulae 1, 2 and three established here are unified in nature and act as main formulae. Thus a general class of polynomials involved in fractional derivative form 1, 2 and three reduces to a wide spectrum of polynomials listed by Srivastava and Singh ([36], pp. 158–161), and so from expressions 1, 2 and three we can further obtain various fractional derivative expressions involving some simpler polynomials. Again, the multivariable H -function occurring in these formulae can be suitably specialized to a remarkably wide variety of useful functions (or product of several such functions) which are expressible in terms of E; F; G, H, ℵ and I -functions of one, two or more variables. For example, if $N = P = Q = 0$ (or $N = P = Q = 1$), the multivariable H -function occurring in the left-hand side of these formulae would reduce immediately to the product of r (or τ) different Fox's H-functions [4]. Thus the table listing various particular cases of the H -function ([16], pp. 145–159) can be used to derive from these fractional derivative forms some other fractional derivative formula involving any of these simpler special functions. On reducing the operator to the Riemann–Liouville operator, we arrive at three fractional derivative formulae involving these operators, but we do not record them here explicitly. Again, our theorems 1, 2 and three will also give rise in essence to some other fractional derivative relation lying scattered in the literature (see [31], pp. 563–564, Eq. (2.1)–(2.3), [32], pp. 644–645, Eq. (2.1)–(2.3)) on making suitable substitutions.

We have the following result,(see Soni and Singh [28] for more details).

$$I_{0,x}^{\alpha,\beta,\gamma}\left\{x^{\rho+\sum_{l=1}^{r}+\frac{n_1}{2}}\prod_{i=1}^{t}(x^{u_i}+\alpha_i)^{\sigma_i}H_{n_1}\left(\frac{1}{2\sqrt{x}}\right)L_{n_2}^{(\theta)}(x)\prod_{l=1}^{r}e^{-\frac{z_lx}{2}}W_{\mu_l v_l}(z_lx)\right.$$

$$=\frac{\prod_{l=1}^{r}z_l^{-b_l}\alpha_1^{\sigma_1}\cdots\alpha_t^{\sigma_t}x^{\rho-\beta}}{\Gamma(-\sigma_1)\cdots\gamma(-\sigma_t)}\sum_{k_1=0}^{[n_1/2]}\sum_{k_2=0}^{[n_2]}\frac{(-n_1)_{2k_1}(-n_2)_k}{k_1!k_2!}(-)^k\binom{n_2+\theta}{n_2}\frac{x^{k_1+k_2}}{(\theta+1)_{k_2}}$$

$$H_{2,2:1,2;\cdots;1,2;1,1;\cdots;1,1}^{0,2:2,0;\cdots;2,0;1,1;\cdots;1,1}\left(\begin{array}{c|c} \mathrm{z}_1 & \\ \cdot & \mathrm{A} \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{z}_r x & \cdot \\ \alpha_1^{(-1)}x^{u_1} & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \mathrm{B} \\ \alpha_t^{(-1)}x^{u_t} & \end{array}\right) \tag{4.1}$$

where

$$A=(-\rho-k_1-k_2;1,\cdots,1,u_1,\cdots,u_t),(\beta-\gamma-\rho-k_1-k_2;1,\cdots,1,u_1,\cdots,u_t),$$

$$(b_i-\mu_i+1;1)_{1,r};(1+\sigma_i;1)_{1,t} \tag{4.2}$$

$$B=(\beta-\rho-k_1-k_2;1,\cdots,1,u_1,\cdots,u_t),(-\alpha-\gamma-\rho-k_1-k_2;1,\cdots,1,u_1,\cdots,u_t),$$

$$\left(b_i\pm v_i+\frac{1}{2};1\right)_{1,r};\underbrace{(0;1);\cdots;(0;1)}_{t} \tag{4.3}$$

Concerning the corollaries, the class of multivariable polynomials $S_{N_1,\cdots,N_v}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}[.]$ vanishes and the multivariable Aleph-function reduces to Aleph-function of one variable defined by Sudland [3,38]. We shall use respectively the theorem 1 and theorem 2.

Ref

28. R.C. Soni and D. Singh, Certain fractional derivative formulae involving the product of a general class of polynomials and the multivariable H-function, Proc. Indian Acad. Sci. (Math. Sci.), 112(4) (2002), 551-562.

Notes

Corollary 1.

$$I_{0,x}^{\alpha,\alpha',\beta,\beta',\gamma}\left\{x^{\rho}\prod_{i=1}^{t}(x^{u_i}+\alpha_i)^{\sigma_i}\aleph\left(\ \mathrm{z}_1 x^{\mu_1}\textstyle\prod_{i=1}^{t}(x^{u_i}+\alpha_i)^{-v_i^{(1)}}\ \right)\right\}$$

$$=\prod_{i=1}^{t}\alpha_i^{\sigma_i}x^{\rho-\alpha-\alpha'+\gamma}\ \aleph_{t+3,t+3:W}^{0,t+3;V}\left(\begin{array}{c|c} \mathrm{z}_1 x^{\mu_1}\prod_{i=1}^{t}\alpha_i^{-v_i^{(1)}} & \mathrm{A},\ \mathbf{A} \\ \alpha_1^{(-1)}x^{u_1} & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \alpha_t^{(-1)}x^{u_t} & \mathrm{B},\ \mathbf{B} \end{array}\right) \tag{4.4}$$

where

$$V=m_1,n_1:1,0;\cdots;1,0 \tag{4.5}$$

$$W=p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)};\underbrace{0,1;\cdots;0,1}_{t} \tag{4.6}$$

$$A=\big(-\rho;\mu_1,u_1,\cdots,u_t\big),\ \big(\alpha'-\beta'-\rho;\mu_1,u_1,\cdots,u_t\big),\big(-\rho-\gamma+\alpha'+\beta;\mu_1,u_1,\cdots,u_t\big)$$

$$\left(1+\sigma_1;v_1^{(1)},1,\underbrace{0,\cdots,0}_{t-1}\right),\cdots,\left(1+\sigma_t;v_t^{(r)},\underbrace{0,\cdots,0}_{t-1},1\right), \tag{4.7}$$

$$\mathbf{A}=\{(c_j^{(1)};\gamma_j^{(1)})_{1,n_1}\},\{\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)};\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_{i^{(1)}}}\};-;\cdots;-$$

$$B=\big(-\beta'-\rho;\mu_1,u_1,\cdots,u_t\big),\ \big(\alpha+\alpha'-\gamma-\rho;\mu_1,u_1,\cdots,u_t\big),\big(-\rho-\gamma+\alpha'+\beta;\mu_1,u_1,\cdots,u_t\big)$$

$$\left(1+\sigma_i;v_i^{(1)},\underbrace{0,\cdots,0}_{t}\right)_{1,t}, \tag{4.8}$$

$$\mathbf{B}=\{(d_j^{(1)};\delta_j^{(1)})_{1,m_1}\},\{\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)};\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_{i^{(1)}}}\};\underbrace{(0;1),\cdots,(0;1)}_{t} \tag{4.9}$$

Provided that

$$Re(\gamma)>0;u_i,\mu_1,v_i^{(1)};i=1,\cdots,t$$

$$|arg z_1|<\frac{1}{2}\pi\left(\sum_{j=1}^{n_1}\gamma_j^{(1)}-\tau_{i^{(1)}}\sum_{j=n_1+1}^{p_{i^{(1)}}}\gamma_{ji^{(1)}}^{(1)}+\sum_{j=1}^{m_1}\delta_j^{(1)}-\tau_{i^{(1)}}\sum_{j=m_1+1}^{q_{i^{(1)}}}\delta_{ji^{(1)}}^{(1)}\right)$$

$$Re(\rho)+\mu_1\min_{1\leqslant l\leqslant m_1}Re\left(\frac{d_l^{(1)}}{\delta_l^{(1)}}\right)+1>\ max\{0,Re(\alpha+\alpha'+\beta-\gamma),Re(\alpha'-\beta')\}$$

Corollary 2.

$$I_{0,x}^{\alpha_1,\alpha_1',\beta_1,\beta_1',\gamma_1} I_{0,x}^{\alpha_2,\alpha_2',\beta_2,\beta_2',\gamma_2}\left\{x^{\rho}y^{\rho'}\prod_{i=1}^{t}(x^{u_i}+\alpha_i)^{\sigma_i}\prod_{i=1}^{t}(y^{u_i'}+\beta_i)^{\sigma_i'}\right.$$

$$\left.\aleph_1\Big(\ z_1 x^{\mu_1}y^{\mu_1'}\prod_{i=1}^{t}(x^{u_i}+\alpha_i)^{-v_i^{(1)}}(x^{u_i'}+\alpha_i')^{-v_i'^{(1)}}\ \Big)\right\} = \prod_{i=1}^{t}\alpha_i^{\sigma_i}\prod_{i=1}^{t}\beta_i^{\sigma_i'}x^{\rho-\alpha_1-\alpha_1'+\gamma_1}x^{\rho'-\alpha_2-\alpha_2'+\gamma_2}$$

$$\aleph_{2t+6,2t+6:W}^{0,2t+6;V}\left(\begin{array}{c|c} z_1 x^{\mu_1}y^{\mu_1'}\prod_{i=1}^{t}\alpha_i^{-v_i^{(1)}}\beta_i^{-v_i'^{(1)}} & \\ \alpha_1^{(-1)}x^{u_1} & \mathrm{A},\ \mathbf{A} \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \alpha_t^{(-1)}x^{u_t} & \cdot \\ \beta_1^{(-1)}y^{u_1'} & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \mathrm{B},\ \mathbf{B} \\ \alpha_t^{(-1)}y^{u_t'} & \end{array}\right) \tag{4.10}$$

where

$$V = m_1, n_1; 1,0;\cdots;1,0 \tag{4.11}$$

$$W = p_{i^{(1)}}, q_{i^{(1)}}, \tau_{i^{(1)}}; R^{(1)}; \underbrace{0,1;\cdots;0,1}_{2t} \tag{4.12}$$

$$A = (-\rho-;\mu_1,\underbrace{0,\cdots,0}_{t},u_1,\cdots,u_t),\ \ (-\alpha_1'-\beta_1'-\gamma_1-\rho;\mu_1,,\underbrace{0,\cdots,0}_{t},u_1,\cdots,u_t),$$

$$(\alpha+\alpha_1'+\beta_1-\gamma_1-\rho;\mu_1,\underbrace{0,\cdots,0}_{t},u_1,\cdots,u_t),(-\beta_1'-\rho;\mu_1,\underbrace{0,\cdots,0}_{t},u_1,\cdots,u_t),$$

$$(\beta_1+\alpha_1'-\gamma_1-\rho;\mu_1,\underbrace{0,\cdots,0}_{t},u_1,\cdots,u_t),(\alpha_1+\alpha_1'-\gamma_1-\rho;\mu_1,\underbrace{0,\cdots,0}_{t},u_1,\cdots,u_t),$$

$$\left(1+\sigma_1;v_1^{(1)},\underbrace{0,\cdots,0}_{t},1,\underbrace{0,\cdots,0}_{t-1}\right),\cdots,\left(1+\sigma_t;v_t^{(r)},\underbrace{0,\cdots,0}_{2t-1},1\right),\left(1+\sigma_i;v_i^{(1)},\underbrace{0,\cdots,0}_{2t}\right)_{1,t}$$

$$\mathbf{A} = \{(c_j^{(1)};\gamma_j^{(1)})_{1,n_1}\},\{\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)};\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_{i^{(1)}}}\};-;\cdots;- \tag{4.13}$$

$$B = (-\rho';\mu_1',u_1',\cdots,u_t',\underbrace{0,\cdots,0}_{t}),\ \ (\alpha_2'-\beta_2'-\gamma_2-\rho';\mu_1',u_1',\cdots,u_t',\underbrace{0,\cdots,0}_{t}),$$

Notes

Notes

$$(\alpha_2'+\alpha_2+\beta_2-\gamma_2-\rho';\mu_1',u_1',\cdots,u_t',\underbrace{0,\cdots,0}_{t}),(-\rho'-\beta_2'-\sum_{j=1}^{v}\zeta_jK_j;\mu_1',\cdots,\mu_r',u_1',\cdots,u_t',\underbrace{0,\cdots,0}_{t}),$$

$$(\alpha_2'+\alpha_2-\gamma_2-\rho';\mu_1',u_1',\cdots,u_t',\underbrace{0,\cdots,0}_{t}),(\alpha_2'-\beta_2-\gamma_2-\rho';\mu_1',u_1',\cdots,u_t',\underbrace{0,\cdots,0}_{t}),$$

$$\left(1+\sigma_1';\upsilon_1'^{(1)},1,\underbrace{0,\cdots,0}_{2t-1}\right),\cdots,\left(1+\sigma_t';\upsilon_t'^{(1)},\underbrace{0,\cdots,0}_{t-1},1,\underbrace{0,\cdots,0}_{t-1}\right),\left(1+\sigma_i';\upsilon_i'^{(1)},\underbrace{0,\cdots,0}_{2t}\right)_{1,t}$$

$$\left(1+\sigma_i';\upsilon_i'^{(1)},\underbrace{0,\cdots,0}_{2t}\right)_{1,t}, \tag{4.14}$$

$$\mathbf{B}=\{(d_j^{(1)};\delta_j^{(1)})_{1,m_1}\},\{\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)};\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_{i^{(1)}}}\};\ \underbrace{(0;1),\cdots,(0;1)}_{2t} \tag{4.15}$$

Provided that

$$Re(\gamma_1)>0, Re(\gamma_2)>0; u_i,u_i',\mu_1,\mu_1',\upsilon_i^{(1)},\upsilon_i'^{(1)}; i=1,\cdots,t$$

$$|argz_1| < \frac{1}{2}\pi\left(\sum_{j=1}^{n_1}\gamma_j^{(1)}-\tau_{i^{(1)}}\sum_{j=n_1+1}^{p_{i^{(1)}}}\gamma_{ji^{(1)}}^{(1)}+\sum_{j=1}^{m_1}\delta_j^{(1)}-\tau_{i^{(1)}}\sum_{j=m_1+1}^{q_{i^{(1)}}}\delta_{ji^{(1)}}^{(1)}\right)$$

$$Re(\rho)+\mu_1\min_{1\leqslant l\leqslant m_1}Re\left(\frac{d_l^{(1)}}{\delta_l^{(1)}}\right)+1> \ max\{0,Re(\alpha_1+\alpha_1'+\beta_1-\gamma_1),Re(\alpha_1'-\beta_1')\}$$

$$Re(\rho')+\mu_1'\min_{1\leqslant l\leqslant m_1}Re\left(\frac{d_l^{(1)}}{\delta_l^{(1)}}\right)+1> \ max\{0,Re(\alpha_2+\alpha_2'+\beta_2-\gamma_2),Re(\alpha_2'-\beta_2')\}$$

Remark: We can give the similar theorems by applying the operator $I_{x,\infty}^{\alpha,\alpha',\beta,\beta',\gamma}\{\}$.

V. Conclusion

In this paper, we have obtained the theorems about generalized fractional derivative operators given by Saigo-Maeda. The images have been developed regarding the product of one or two multivariable Aleph-functions and a general class of multivariable polynomials in a compact and elegant form with the help of Saigo-Maeda operators. Most of the results obtained in this paper are useful in deriving definite composition formulae involving Riemann−Liouville, Erdelyi−Kober fractional calculus operators and multivariable Aleph-functions. The findings of this paper provide an extension of the results given earlier by Kilbas, Kilbas and Saigo, Kilbas and Sebastian, Saxena et al. and Gupta et al. as mentioned before.

References Références Referencias

1. P. Agarwal, Fractional integration of the product of two multivariable H-functions and a general class of polynomials, Advances in Applied Mathematics and Approximation Theory, Springer (2013), Chapter 23, 359-374.

2. F. Y. Ayant, Generalized finite integral involving the multiple logarithm-function, a general class of polynomials, the multivariable Aleph-function, the multivariable I-function I, International Journal of Mathematics Trends of Technology (IJMTT), 48(1) (2017), 6-14.
3. L. Debnath and D. Bhatta, Integral Transforms and Their Applications, Chapman and Hall/CRC Press, Boca Raton FL, 2006.
4. C. Fox, The G and H-functions as symmetrical Fourier Kernels, Trans. Amer. Math. Soc. 98 (1961)., 395-429
5. K.C. Gupta and R.C. Soni, A study of H-functions of one and several variables, J. Rajasthan. Acad. Phys. Sci. 1 (2002), 89-94.
6. S. L. Kalla, Integral operators involving Fox' s H-function I, Acta Mexicana Cienc. Tecn. 3, 117–122, (1969).
7. S. L. Kalla, Integral operators involving Fox' s H-function II, Acta Mexicana Cienc. Tecn. 7, 72–79, (1969).
8. S. L. Kalla and R. K. Saxena, Integral operators involving hypergeometric functions, Math. Z. 108, 231–234, (1969)
9. S. L. Kalla and R. K. Saxena, Integral operators involving hypergeometric functions II, Univ. Nac. Tucuman, Rev. Ser., A24, 31–36, (1974).
10. A. A. Kilbas, Fractional calculus of the generalized Wright function, Fract.Calc.Appl.Anal.8 (2), 113–126, (2005).
11. A. A. Kilbas and N. Sebastain, Generalized fractional integration of Bessel function of first kind, Integral transform and Spec. Funct. 19(12), 869–883, (2008).
12. A. A. Kilbas, H. M. Srivastava and J. J. Trujillo, Theory and Applications of Fractional Differential Equations, 204(North-Holland Mathematics), Elsevier, 540, 2006.
13. V. Kiryakova, Generalized Fractional Calculus and Applications, Longman Scientific & Tech., Essex, 1994.
14. V. Kiryakova, A brief story about the operators of the generalized fractional calculus, Fract. Calc. Appl. Anal. 11(2), 203–220, (2008).
15. E. R. Love, Some integral equations involving hypergeometric functions, Proc. Edin. Math. Soc. 15 (3), 169–198, (1967).
16. O. I. Marichev, Volterra equation of Mellin convolution type with a Horn function in the kernel (In Russian). Izv. AN BSSR Ser. Fiz.-Mat. Nauk 1, 128–129, (1974).
17. A. M. Mathai and R. K. Saxena, The H-function with applications in statistics and other disciplines (New Delhi; Wiley Eastern Limited), (1978).
18. A. C. McBride, Fractional powers of a class of ordinary differential operators, Proc. London, Math. Soc. (III) 45, 519–546, (1982).
19. K. S. Miller and B. Ross An Introduction to the Fractional Calculus and Differential Equations, A Wiley Interscience Publication, John Wiley and Sons Inc., New York, 1993.
20. J. Ram and D.L. Suthar, Unified fractional derivative formulae for the multivariable H-function, Vijnana. Parishad. Anusandhan. Patrika. 49(2) (2006), 159-175.
21. M. Saigo, A remark on integral operators involving the Gauss hypergeometric functions, Math. Rep. Kyushu Univ. 11, 135–143, (1978).
22. M. Saigo, A certain boundary value problem for the Euler-Darboux equation I, Math. Japonica, 24 (4), 377–385, (1979).

Notes

Global Journal of Science Frontier Research (F) Volume XVIII Issue III Version I Year 2018

23. M. Saigo, A certain boundary value problem for the Euler-Darboux equation II, Math. Japonica 25 (2), 211–220, (1980).
24. M. Saigo and N. Maeda, More Generalization of Fractional Calculus, Transform Methods and Special Functions, Varna, Bulgaria, 1996, pp. 386–400.
25. S. Samko, A. Kilbas and O. Marichev Fractional Integrals and Derivatives. Theory and Applications, Gordon & Breach Sci. Publ., New York, 1993.
26. C.K. Sharma and S.S. Ahmad, On the multivariable I-function. Acta ciencia Indica Math , 20(2) (1994), 113-116.
27. Y. Singh and H. Mandia, On the fractional derivative formulae involving the product of a general class of polynomials and the multivariable A–function, IOSR Journal of Mathematics (IOSR- J.M.), 3 (1) (2012), 46-52.
28. R.C. Soni and D. Singh, Certain fractional derivative formulae involving the product of a general class of polynomials and the multivariable H-function, Proc. Indian Acad. Sci. (Math. Sci.), 112(4) (2002), 551-562.
29. H. M. Srivastava, A contour integral involving Fox' s H -function, Indian J. Math. 14 (1972), 1–6.
30. H. M. Srivastava, A multilinear generating function for the Konhauser set of biorthogonal polynomials suggested by Laguerre polynomial, Pacific. J. Math. 177(1985), 183-191.
31. H.M. Srivastava, R.C.S. Chandel P.K. Vishwakarma, Fractional derivatives of certain generalized hypergeometric functions of several variables, J.Math.Anal. Appl. 184 (1994), 560-572.
32. H. M. Srivastava and S. P. Goyal, Fractional derivatives of the H-function of several variables, J. Math. Anal. Appl. 11 (1985), 641-651.
33. H. M. Srivastava, K.C. Gupta and S. P. Goyal, The H-function of One and Two Variables with Applications, South Asian Publications, New Delhi, Madras, 1982.
34. H. M. Srivastava and R. Panda, Some expansion theorems and generating relations for the H-function of several complex variables. Comment. Math. Univ. St. Paul. 24 (1975), 119-137.
35. H. M. Srivastava and R. Panda, Some expansion theorems and generating relations for the H-function of several complex variables II. Comment. Math. Univ. St. Paul. 25 (1976), 167-197.
36. H. M. Srivastava and N. P. Singh, The integration n of certain products of the multivariable H-function with a general class of polynomials, Rend. Circ. Mat. Palermo, 32, 157–187, (1983).
37. N. Sudland, B. Baumann and T. F. Nonnenmacher, Open problem: who knows about the Aleph-functions? Fract. Calc. Appl. Anal., 1(4) (1998), 401-402.
38. N. Sudland, B. Baumann and T. F. Nannenmacher, Fractional drift-less Fokker-Planck equation with power law diffusion coefficients, in V.G. Gangha, E.W. Mayr, W.G. Vorozhtsov (Eds.), Computer Algebra in Scientific Computing (CASC Konstanz 2001), Springer, Berlin, 2001, 513–525

Notes