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About Structural Identifiability of Nonlinear Dynamic Systems under Uncertainty

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I. Introduction

The identification problem of dynamic systems despite the set of obtained results is one of important study directions. Fundamental results are obtained on a system parametrical identification. An approach to the identifiability estimation is based on R. Kallman ideas [1]. Further development of these ideas is given in [2, 3]. R. Li [2] gives the following identifiability definition.

Consider the system

$$\begin{aligned} X_{n+1} &= AX_n, \\ y_n &= C^T X_n, \end{aligned} \qquad (1)$$

where $X_n \in R^m$ is a state, $A \in R^{m\times m}$, $y_n \in R$ is an exit, $n = J_n = [0, N]$ is the discrete time.

Task: Determine by conditions under what the system is identified on the basis of the set

$$\mathrm{I}_o = \left\{ y_n, n = \overline{0,N}, N < \infty \right\}. \qquad (2)$$

Following sufficient and necessary conditions are obtained in [2] when $y_n \in R^m$.

Definition 1: The system described by the equation (1) is called n-identified if the matrix A is possible to determine based on measurement of the variable X.

Definition 2: The system described by the equation (1) is called 1-identified if the matrix A is possible to determine on the basis of measurement y.

The n-identifiability condition consists in that the matrix $\left[X_0 \mid AX_0 \mid A^2X_0 \mid \dots \mid A^{m-1}X_0 \right]$ was nondegenerate.

1-identifiability conditions:

1. The system (1) is n-identified.
2. The pair (A, C) is observable.

The identifiability case when the dynamic system order is less m is considered in [2].

The considered result analysis shows that the identifiability estimation of the system (1) consists in the possibility of parameters identification. Call a parametrical identifiability IP-identifiability (IPI). The publication set is devoted to the research IPI. The difference between these studies and the approach stated in [2] consists that identifiability results present in the form accepted in parametrical estimation problems. In [4] the concept of the structural identifiability is introduced.

Let two dynamic systems $S_1(U_1, Y_1, A_1)$, $S_2(U_2, Y_2, A_2)$ are considered with inputs U_1, U_2, outputs Y_1, Y_2 and parameters A_1, A_2. It corresponds to models $\mathcal{M}_1(U_1, \hat{Y}_1, \hat{A}_1)$ and $\mathcal{M}_2(U_2, \hat{Y}_2, \hat{A}_2)$.

Definition 3: [4] If the condition $\mathcal{M}_1(\hat{A}_1) \approx \mathcal{M}_2(\hat{A}_2)$ is satisfied at $U_1 = U_2$, $Y_1 = Y_2$ and $\hat{A}_1 \neq \hat{A}_2$, then models are indistinguishable on observed inputs and outputs.

Definition 4: [4] A parameter $\hat{a}_{1,i} \in \hat{A}_1$ is called structurally globally identified if the condition

$$\mathcal{M}_1(\hat{A}_1) \approx \mathcal{M}_2(\hat{A}_2) \Rightarrow \hat{a}_{1,i} = \hat{a}_{2,i}$$

is satisfied almost for any $\hat{A}_2 \in \Omega_P$ (except a zero measure subset of parametrical space Ω_P).

Definition 5 [4]: Parameter $\hat{a}_{1,i} \in \hat{A}_1$ is called structurally locally identified if such neighbourhood $O_2(\hat{A}_2)$ exists almost for any $\hat{A}_2 \in \Omega_P$ that

$$\mathcal{M}_1(\hat{A}_1) \approx \mathcal{M}_2(\hat{A}_2) \Rightarrow \hat{a}_{1,i} = \hat{a}_{2,i}$$

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follows from the condition $\hat{A}_1 \in O_2\left(\hat{A}_2\right)$.

The local identifiability is a necessary condition for the global identifiability. A parameter is called structurally globally identified if it is structurally locally identified and at the same time it is not structurally globally identified. A parameter which is not structurally locally identified is called structurally locally not identified. Different approaches and methods are applied for structural identifiability verification [5, 6].

In [7] a concept of local parametrical identifiability is introduced and given it's the theoretical justification. Consider the system described by the vector differential equation

$$\dot{X} = F(t, X, P), \quad X\left(t_0\right) = X_0, \tag{3}$$

where $X_n \in R^m$ is a system state, $P \in R^m$ is a parameter vector, $F(\cdot)$ is a nonlinear vector-function.

Definition 6: The system (3) is called locally identified in a point $P_0 \in W$ if such $\varepsilon > 0$ exist that couple $\{P_0, P_1\}$ is distinguishable for any point P_1 such that $0 < \left\| P_1 - P_0 \right\| < \varepsilon$.

Remark 1: System structure estimation methods are not considered in most papers. Therefore, the structural identifiability concept does not reflect the essence of the considered problem. As this terminology is actively applied in identifiability estimation problems, in this section we will hold to this concept. Further, we will introduce a concept which is directly related to the structural identifiability of nonlinear systems in the structural space.

Criteria are proposed in [7] for the local identifiability estimation of the linearized system (3). The case is considered when the matrix rank is equal m. The local identifiability estimation method based on the Lyapunov exponent analysis is proposed for an inhomogeneous linear system. Parametrical identifiability criteria are introduced in [8], and also generalization and development of results obtained in [7] are given. Full identifiability conditions are proposed in [9, 10] for a linear stationary system on discrete measurements of an output and of state variables.

The IPI-identifiability problem of nonlinear systems was studied by many authors (see e.g., [9-12]). The identifiability research in [10] is based on the sensitivity system analysis on the output system. This approach efficiency is illustrated in the example of the identifiability estimation of system parameter combination. This approach gives a new method the local identifiability problem solution. Local parametrical identifiability conditions are obtained in [9] for different variants of the experimental data measurement. Conditions of a joint observability and the identifiability are obtained for the linear stationary system. The critical analysis of the approaches applied to the biological model identifiability estimation is given in [11]. Models for the nonlinear system identifiability estimation are proposed on the basis of the expansion into Taylor series, tables of identifiability (contains nonzero members of Jacobian coefficients row), the algebra of differentials. Paper [12] is devoted to the practical identifiability study. Practical identifiability estimation is based on the experimental information analysis and the differential algebra application. The basis of the practical identifiability is the least-squares method and the model sensitivity analysis to the obtained parameter estimations. The proposed approach is applied to biology problems.

The identifiability of a static model is considered in [13]. The model is described by a system of the simultaneous equations

$$BY_n + \Gamma X_n = U_n, \tag{4}$$

where $B \in R^{m \times m}$ is a nonsingular matrix, $X_n \in R^k$ is an exogenous vector (external) variables, $U_n \in R^m$ is a vector of external random disturbances, $Y_n \in R^m$ is a vector of endogenous variables, $\Gamma \in R^{m \times k}$.

The a priori information is about external and internal variables, an accidental character U_t, restrictions on coefficients and the variable normalization rule.

Consider the structure $S = \left(B, \Gamma, M_n\right)$ of the model (4) where M_U is a distribution U_t. Then Y_n at specified X_n have a distribution P_n^S.

Definition 7: If $P_n^S = P_n^{\tilde{S}}$, then frameworks S, $\tilde{S}$ are observably equivalent.

Definition 8: A parameter Λ is called identified in the framework S if $\Lambda = \tilde{\Lambda}$ is true for any a framework equivalent $\tilde{S}$.

So, the parameter Λ is identified if $\Lambda = \tilde{\Lambda}$ follows from equality $P_n^S = P_n^{\tilde{S}}$ ($n = 1, 2, \ldots, N$).

Definition 9: The structure S is called identified if all its parameters are identified.

In [13] various cases of the a priori information accounting about S are considered and identifiability conditions obtained. They are restrictions on the rank of a matrix, depending on variables the system (4) which is previously subject to a normalization. The normalization is the solution of the equation (4) concerning Y_n.

Remark 2: In spite of the fact that the system (4) is static, its identifiability is interesting in the problem definition form. Here the concept structure is used also. Therefore, the existing interpretations and problem statements of the SI will be useful to compare.

So, the analysis of publications shows that the model identifiability is the estimation possibility of its parameters. The proposed methods are based on the non-degeneracy estimation of an informational matrix. Similar results are obtained in the parametrical estimation theory, and non-degeneracy condition (rank completeness) of the informational matrix is presented in easily checked the excitation constancy condition of the input and of the output system. As a rule, the model structure is specified a priori and the sense of the structural local identifiability is understandable not always. The structure concept is widely applied in identifiability estimation problems. The nonlinear system identifiability also is transformed into the parametrical identifiability problem on the basis of the different methods linearization model application on parameters. These researches extensive area does not include the structural identifiability problem of nonlinear dynamic systems in the following sense: whether have the problem decision of the structure (a form, dependence) estimation a system nonlinear part under uncertainty. The task not set was in this form.

Identifiability structural aspects of the nonlinear system are considered in such statement in this paper. This is the very complex problem as structure formalization methods of the system are not developed. The concept of the structural identifiability (h-identifiability) was introduced in [14] for nonlinear systems. The proposed approach is directed to the structure estimation problem solution of the dynamic system nonlinear part. It is based on the analysis of the framework reflecting the state of the system nonlinear part. Below we give a generalization of results obtained in [14, 15] on the h-identifiability. The IPI-identifiability problem is not considered. It's the decision can be obtained, having applied approaches considered above.

The paper has the following structure. The problem statement is given in the second section. The framework design method is stated in section three. The approach is described to the formation of a set for the construction S_{ey}-framework. Framework class properties are considered. Estimation need of the nonlinear system h-identifiability is substantiated in section four. System examples with a hysteresis are considered and the input parameters effect is analyzed on system nonlinear part properties. We will show that the input has to be constantly excited. But not every input that has excitation constancy property, gives the solution of the structural identifiability problem. Nonlinear system h-identifiability (structural identifiability) basis are stated in sections 5, 6. We introduce the concept of a system S-synchronization which the fulfillment allows solving the problem h-identifiability. The input which does not have property the S-synchronizability property gives to an "insignificant" S_{ey}-framework. Structural identifiability estimation methods are considered. We show that the h-identified framework S_{ey} have the specified dimension. Numerical modeling results are presented in section 7.

II. Problem Statement

Consider dynamic system

$$\begin{aligned} \dot{X} &= AX + B_{\varphi}\varphi(y) + B_u u, \\ y &= C^T X, \end{aligned} \tag{5}$$

where $u \in R$, $y \in R$ are the input and the output, $A \in R^{q \times q}$; $B_u \in R^q$, $B_{\varphi} \in R^q$ $C \in R^q$ are matrices of corresponding dimensions; $\varphi(y)$ is a scalar nonlinear function. A is the Hurwitz matrix.

Various assumptions are made concerning the function $\chi = \varphi(y)$ structure. They are determined by the a priori information level. Methods based on linearization procedures [16] can be applied under the a priori information. The assumption concerning the function χ is specified in the absolute stability theory in the form

$$\chi \in \mathcal{F}_{\varphi} = \left\{ \varphi(\xi)\xi \geq \xi^2,\ \xi \neq 0,\ \varphi(0) = 0 \right\}, \tag{6}$$

where $\xi \in R$ is the input of a nonlinear element.

ξ is the linear combination of vector elements X. Further generalization (6) is the sector condition

$$\begin{aligned} \chi \in \mathcal{F}_{\varphi} = \{ & \gamma_1 \xi^2 \leq \varphi(\xi)\xi \leq \gamma_2 \xi^2,\ \xi \neq 0, \\ & \varphi(0) = 0,\ \gamma_1 \geq 0, \gamma_2 < \infty \}. \end{aligned} \tag{7}$$

Often the system (5) nonlinear part is described by static (algebraic) equations. Therefore, further, we consider a case when $\varphi(y)$ is described by the algebraic equation. We believe that the function $\varphi(y)$ is smooth.

Let the informational set be known for the system (5)

$$\mathrm{I}_o = \left\{ u(t), y(t),\ t \in J = \left[t_0, t_k \right] \right\}. \tag{8}$$

Problem: Estimate the structural identifiability of the system (5) nonlinear part on the basis of the analysis and processing I_o.

Identification parametrical methods application under uncertainty does not allow obtaining the SI problem solution. Therefore, we apply to the structural identification the approach proposed in [15, 16]. It is based on the transition in a special structural space and design of the framework S_{ey} reflecting properties of the nonlinear part (5). The analysis is associated with the system structural identifiability problem solution. We use the term h-identifiability (HI) that to distinguish the proposed approach from IPI-identifiability. Describe the design method S_{ey}-framework.

III. Design Method S_{ey}-Framework

Design S_{ey}-framework demands the preliminary formation of the set $I_{N,g}$ containing an information on the function $\varphi(y)$. Describe to the obtaining $I_{N,g}$ method following [18].

a) Set for Formation S_{ey}-framework

Apply the differentiation operation to $y(t)$ and designate the obtained variable as x_1. The account x_1 expands of the system informational set: $I_{ent}=\{I_o, x_1\}$.

Remark 3: If variables u, y are measured with an error then apply to u, y the filtering procedure.

Allocate a subset $I_g \subset I_{ent}$ corresponding to the system (5) particular solution (steady state). The set does not contain data I_{tr} on the transition process in the system. Apply the mathematical model

$$\hat{x}_1^l(t) = H^T \left[1\ u(t)\ y(t)\right]^T \qquad (9)$$

for the selection of the linear component in x_1. The variable x_1 is defined on an interval $J_g = J \setminus J_{tr}$. $H \in R^3$ is the model parameter vector.

Determine by the vector H as the task solution

$$\min_H Q(e)\Big|_{e=\hat{x}_1^l - x_1} \rightarrow H_{opt}.$$

where $Q(e) = 0.5e^2$.

Find by the prediction for the variable x_1 on the basis of the model (9) and obtain the error $e(t) = \hat{x}_1^l(t) - x_1(t)$. $e(t)$ depends on the nonlinearity $\varphi(y)$ of the system (5). So, the set

$$I_{N,g} = \left\{y(t),\ e(t)\ \ t \in J_g\right\}$$

is obtained. Next, we apply the designation $y(t)$ believing that $y(t) \in I_{N,g}$.

Remark 4: The structure model (9) choice is the system (5) structural identification stage. Modeling results show that the model (9) is applicable in identification system objects with static nonlinearities. The structure model (9) choice is described in [14] for more complex nonlinearity class.

b) Frameworks S_{ey}

Application of the phase portrait S described by function $\Gamma: \{y\} \rightarrow \{y'\}$ not always the conclusion allows making about system nonlinear properties under uncertainty. Therefore, apply the set $I_{N,g}$ and go in the space $\mathcal{P}_{ye} = (y, e)$ which we will call structural.

Consider the function $\Gamma_{ey}: \{y\} \rightarrow \{e\}$ which describes the framework S_{ey} change on the plane (y,e). $I_{N,g}$ contains an information on $\varphi(y)$ therefore, S_{ey} describe the nonlinear function change in a generalized form. The system (5) input has to satisfy certain conditions for the representation obtaining of $\varphi(y)$. It is the excitation constancy property. Such input gives to the closed framework S_{ey}.

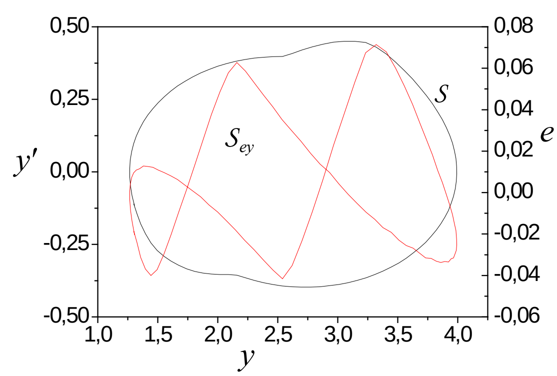
IV. About Need For Nonlinear System Identifiability Estimation

The paper review on the SI shows that the main attention is given to the IP-identifiability problem. To the urgency of h-identifiability problem understands consider arising problems on the example of the second order system (5) with parameters:

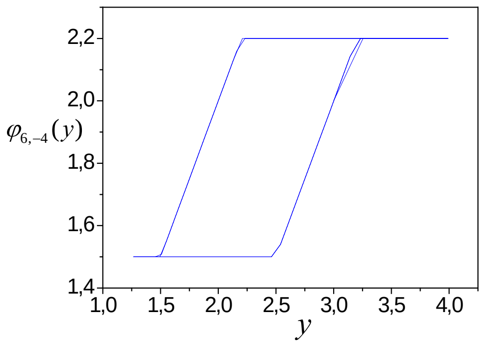
$$A = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix},\ B_u = B_\varphi = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ y(0) = 3, y'(0) = 2,$$

$$\varphi(y) = \begin{cases} 2.2 & \text{if } (y-d > 2.2) \& (y' > 0), \\ y-d & \text{if } (y-d \le 2.2) \& (y' > 0), \\ 1.5 & \text{if } (y-d \le 1.5) \& (y' > 0), \\ 2.2 & \text{if } (y > 2.2) \& (y' < 0), \\ y & \text{if } (y \le 2.2) \& (y' < 0), \\ 1.5 & \text{if } (y \le 1.5) \& (y' < 0), \end{cases} \quad d = 1.$$

Results presented below are based on the application of the approach from section 3. They show the input $u(t)$ influence on system (5) nonlinear properties. System properties are estimated on the basis of the framework S_{ey} analysis and the recovered function $\varphi(y)$ corresponding to the input $u(t)$ is presented.



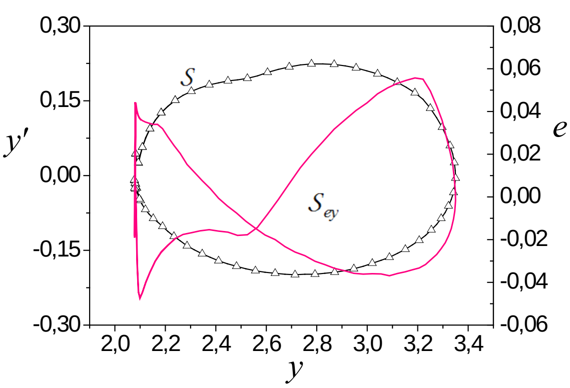
a) frameworks



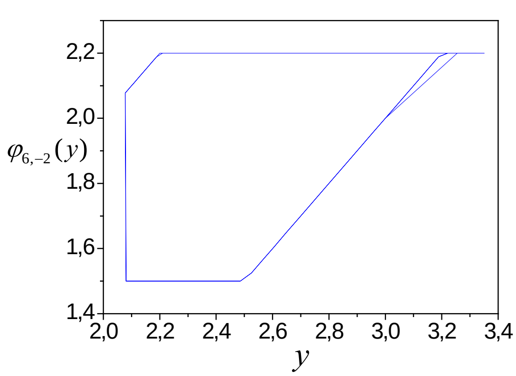
b) nonlinearity

Fig. 1: Structure estimation results for $u_{6,-4}(t)$

Fig. 1 represents the phase portrait S and the framework S_{ey} for $u_{6,-4}(t) = 6 - 4\sin(0.1\pi t)$, and also the function $\varphi(y)$ recovered from data $\{y(t), y'(t)\}$. We consider the case of the established motion. Fig. 1 shows that $u_{6,-4}(t)$ give to reference function $\varphi(y)$. The framework S_{ey} is almost symmetric and has features which are also in the framework S.



a) frameworks



b) nonlinearity

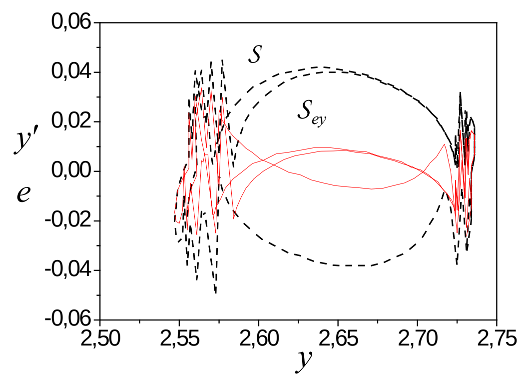
Fig. 2: Structure estimation results for $u_{6,-2}(t)$

The further decrease of the sinusoid amplitude gives to the loss of the framework S_{ey} symmetry feature. The restoration impossibility of the form function $\varphi(y)$ is the result of such input property. It shows the case presented in Fig. 2 when $u_{6,-2}(t) = 6 - 2\sin(0.1\pi t)$. We see that the sinusoid amplitude decrease gives to framework definition range compression, and the framework left part have more active changes. It gives function $\varphi_{6,-2}(y)$ saturation area reduction. This area is not recovered by the identification method application. More cardinal changes in $\varphi(y)$ gives the use

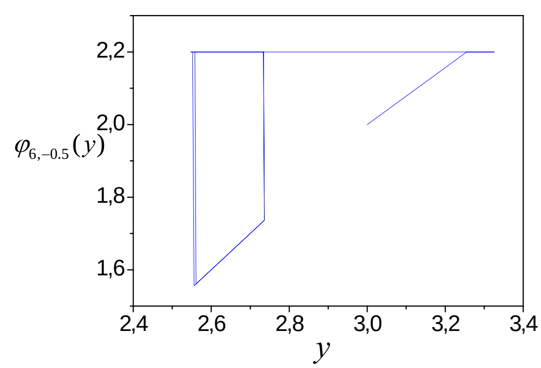
$$u_{6,-0.5}(t) = 6 - 0.5\sin(0.1\pi t).$$

Corresponding results are shown in Fig. 3.

The modeling result analysis shows that there is some parameter set of the input $u(t)$ at which the structural identifiability (structural identification) of the nonlinear system is possible. These results are presented for the system with $\omega = 0.1\pi$ in Fig. 4 where following designations are used: D_y, D_e are diameters of the variation domain y, e; a_u is the sine amplitude.



a) frameworks



b) nonlinearity

Fig. 3: Structure estimation results for $u_{6,-0.5}(t)$

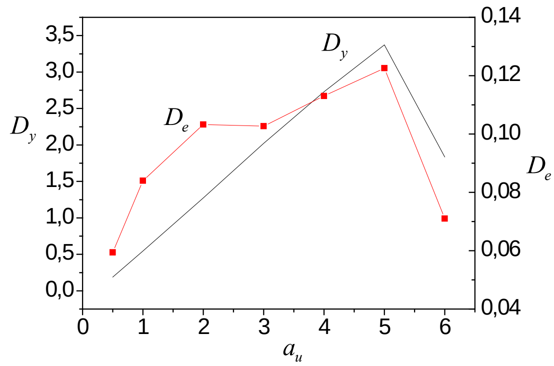
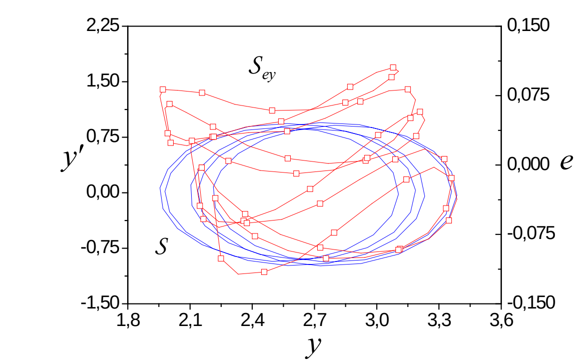


Fig. 4: Input amplitude effect on the system (5) identifiability

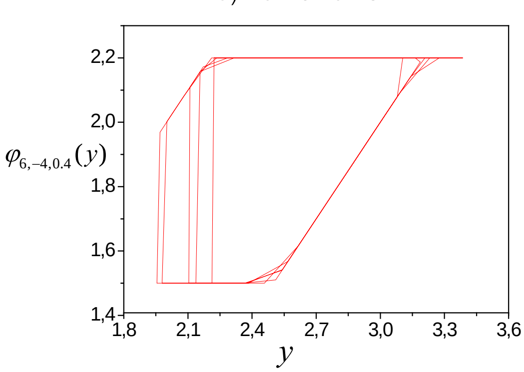
Fig. 4 shows that for the input ($a_u = 5$) which the system SI problem solves exist. The system (5) (see Fig. 1) is identified with the input having the amplitude $a_u = 4$.

We considered the input amplitude effect on system features. Similar the effect gives to the frequency influence (Fig. 5).

Remark 5: Modeling results show (Fig. 5) that ensuring the excitation constancy (EC) condition for $u(t)$ can complicate the system h-identifiability estimation. Results presented in Figures show that the requirements to the input EC have in structural and parametrical identification problems the essential distinction. It should be taking into consideration in active identification problems.



a) frameworks



b) nonlinearity

Fig. 5: Structure estimation results for $u_{6,-4,0.4}(t)$

Modeling results allow giving to h-identifiability problem statement as the following task solution: find such an input $u(t)$ for the system (5) which gives the definition range maximum for the output $y(t)$.

V. h-Identifiability

Results obtained in section 4 show that methods applied to the IP-identifiability estimation do not work in the case the h-identifiability). Further, we state to the approach to HI estimation proposed in [18].

First, consider properties of the set $\mathrm{I}_{N,g}$ allowing the problem h-identifiability to solve. The analysis $\mathrm{I}_{N,g}$ gives to the informational set I_o important properties determining the further consideration of this problem.

Let following conditions be satisfied.

B1. The set I_o gives to the parametrical identification problem solution for the model (5). It means that the input $u(t)$ is excitation constancy on an interval J.

B2. The input $u(t)$ provides obtaining the informative framework $S_{ey}\left(\mathrm{I}_{N,g}\right)$. It means that the analysis S_{ey} gives the estimates task solution of system (5) nonlinear properties.

Definition 10: The input $u(t)$ we will call representative if it satisfies conditions B1, B2.

Let the framework S_{ey} be closed and its area is not zero. Designate a height S_{ey} as $h\left(S_{ey}\right)$ where the height is the distance between two points on opposite sides of the framework S_{ey}.

Statement 1: [18]. Let: 1) the system (5) linear part is stable and the nonlinearity $\varphi(\cdot)$ satisfies the condition (7); 2) the input $u(t)$ is limited piecewise continuous and constantly excited; 3) such $\delta_S > 0$ exists that $h\left(S_{ey}\right) \geq \delta_S$. Then the framework S_{ey} is identifiable on the set $\mathrm{I}_{N,g}$.

Definition 11: Framework S_{ey} having specified properties is h-identified.

Let the framework S_{ey} be h-identified.

Concept h-identifiability features.

1. h-identifiability is a concept not parametric, but the structural identification.
2. The demand of the parametric identifiability is the h-identifiability basis.
3. h-identifiability makes more rigid demands to the system input.

Feature 3 means that "the bad" input can satisfy of the excitation constancy condition. Such input can

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give so-called an "insignificant" $\mathcal{S}_{ey}$-structure ($\mathcal{NS}_{ey}$-framework). But the $\mathcal{NS}_{ey}$-structure can be h-identified. The insignificance property gives the identification of the nonlinearity, atypical for an examined system under uncertainty.

Consider existence conditions of the $\mathcal{NS}_{ey}$-structures. Consider a class of nonlinear functions to which the homotopy operation is applicable. The homotopy [23] is the operation of obtaining one part of a geometrical figure from another part on the basis of it's the rotation and the extension about a certain point on the plane (y,e).

Consider the framework $\mathcal{S}_{ey}$. Let $\mathcal{S}_{ey} = \mathcal{F}^l_{S_{ey}} \cup \mathcal{F}^r_{S_{ey}}$, where $\mathcal{F}^l_{S_{ey}}, \mathcal{F}^r_{S_{ey}}$ are left and right fragments $\mathcal{S}_{ey}$. Determine for $\mathcal{F}^l_{S_{ey}}, \mathcal{F}^r_{S_{ey}}$ secants

$$\gamma^l_S = a^l y\,,\ \gamma^r_S = a^r y\,, \tag{10}$$

where a^l, a^r are numbers computed by means of the least-squares method (LSM).

Theorem 1 [7]. Let: i) the framework $\mathcal{S}_{ey}$ is h-identified; ii) the framework $\mathcal{S}_{ey}$ has the form $\mathcal{S}_{ey} = F^l_{S_{ey}} \cup F^r_{S_{ey}}$, where $\mathcal{F}^l_{S_{ey}}, \mathcal{F}^r_{S_{ey}}$ are left and right fragments $\mathcal{S}_{ey}$; ii) secants for $\mathcal{F}^l_{S_{ey}}, \mathcal{F}^r_{S_{ey}}$ have the form (9). Then $\mathcal{S}_{ey}$ is $\mathcal{NS}_{ey}$-structure, if

$$\left\| a^l \right| - \left| a^r \right\| > \delta_h\,, \tag{11}$$

where $\delta_h > 0$ is some specified number.

Remark 6: The theorem 1 can be proved on the basis of the homotopy of sets [24]. Estimate proximity of sets $\mathcal{F}^l_{S_{ey}}, \mathcal{F}^r_{S_{ey}}$ in this case. The approach based on the secant method is simpler in the implementation.

Remark 7: $\mathcal{NS}_{ey}$-structures are characteristic for systems with multiple-valued nonlinearities. They are the result of the inadequate application of input actions.

Consider the framework $\mathcal{S}_{ey}$. Introduce designations: $\mathcal{D}_y = \mathrm{dom}\left(\mathcal{S}_{ey}\right)$ is definition range $\mathcal{S}_{ey}$,

$$D_y = D_y\left(\mathcal{D}_y\right) = \max_t y(t) - \min_t y(t)$$

is the diameter $\mathcal{D}_y$. Let $u(t) \in \mathrm{U}$, where U is the admissible input set for the system (5).

Definition 12: If the definition range $\mathcal{D}_y$ of the framework $\mathcal{S}_{ey}$ have the maximum diameter D_y on the set $\{y(t), t \in J\}$, then the input $u(t) \in \mathrm{U}$ is S-synchronizing for the system (5).

We understand synchronization $u(t) \in \mathrm{U}$ as the choice of such input $u_h(t) \in \mathrm{U}$ which allows reflecting all features $\mathcal{S}_{ey}$ characteristic for $\varphi(y)$. It is true if $u(t)$ ensures $\max_{u_h} D_y$. Here the $u_h(t) \in \mathrm{U}$ and input property choice is directed to the possibility of obtaining the framework $\mathcal{S}_{ey} \neq \mathcal{NS}_{ey}$. The proposed concept of the synchronization by differs from oscillation theory terminology. As the choice $u_h(t) \in \mathrm{U}$ can be interpreted as the synchronization between model and system structures, then $\mathcal{d}_{h,y} = \max_{u_h} D_y$ gives the system h-identifiability.

Let the input $u_h(t)$ synchronize the set $\mathcal{D}_y$. We will write $u_h(t) \in \mathrm{S}$ if $u(t)$ is S-synchronizing. Notice that the finite set $\{u_h(t)\} \in \mathrm{S}$ exists for the system (5). The optimum choice $u_h(t)$ depends from $\mathcal{d}_{h,y}$. Ensuring this condition is one of the system (5) structural identifiability prerequisites.

Definition 13: If $\mathcal{S}_{ey}$ is h-identified and the condition $\left\| a^r \right| - \left| a^r \right\| \leq \delta_h$ is satisfied, then the framework $\mathcal{S}_{ey}$ (the system (5)) is structurally identified or h_{δ_h}-identified.

Definition 13: Shows if the system (5) is h_{δ_h}-identified, then the framework $\mathcal{S}_{ey}$ has the area $\mathcal{D}_y$ maximum diameter. Let the framework $\mathcal{S}$ contain m features. We understand function $\varphi(y)$ features as the continuity loss on some interval, and function inflection points or a function extremum. These features are signs of the examined function nonlinearity.

Definition 14: If the framework $\mathcal{S}_{ey}$ is h_{δ_h}-identifiable, then the model (9) is SM-identifying.

Theorem 2 [15]. Let: 1) the input $u(t)$ is constantly excited and guarantees the system (5) S-synchronization; 2) the system (5) phase portrait $\mathcal{S}$ have m features; 3) $\mathcal{S}_{ey}$-framework is h_{δ_h}-identifiable and have fragments corresponding to phase portrait $\mathcal{S}$ features. Then the model (9) is SM-identifying.

The theorem 2 shows if the model (9) is not SM-identifying, then the model (9) structure or an informational set have to be changed.

Consider the framework $\mathcal{S}_{ey}$. Designate the framework $\mathcal{S}_{ey}$ center on the set $J_y = \{y(t)\}$ as c_S, and the area $\mathcal{D}_y$ center as c_{D_y}.

Theorem 3. Let on the set U of system (5) representative input $u(t)$: (i) such $\varepsilon \geq 0$ exists that $\left| c_S - c_{D_y} \right| \leq \varepsilon$; (ii) the condition $\left\| a^l \right| - \left| a^r \right\| \leq \delta_h$ is satisfied. Then the system (5) is h_{δ_h}-identifiable, and the input $u_h(t) \in \mathrm{S}$.

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Proof of Theorem 3. Consider the input $u_h(t) \in \mathrm{U}$. As the condition $\left|a^l - a^r\right| \le \delta_h$ is satisfied the framework $\mathcal{S}_{ey}$ is symmetric concerning the point c_S on the plane (y,e). Therefore, fragment $\mathcal{F}^l_{\mathcal{S}_{ey}}, \mathcal{F}^r_{\mathcal{S}_{ey}}$ definition range diameters coincide within some size $\varepsilon_{\mathcal{F}} \ge 0$ on the set $\{y(t)\}$, i.e.

$$\left|D_{\mathcal{F}^l_S}\left(\mathcal{D}_{\mathcal{F}^l_S}\right) - D_{\mathcal{F}^r_S}\left(\mathcal{D}_{\mathcal{F}^r_S}\right)\right| \le \varepsilon_{\mathcal{F}}, \qquad (12)$$

where $\mathcal{D}_{\mathcal{F}^l_S}, \mathcal{D}_{\mathcal{F}^r_S}$ are ranges of definition $\mathcal{F}^l_{\mathcal{S}_{ey}}, \mathcal{F}^r_{\mathcal{S}_{ey}}$.

Then the framework $\mathcal{S}_{ey}$ center is $c_{D_y} = 0.5\left(D_{\mathcal{F}^l_S} + D_{\mathcal{F}^r_S}\right)$. As $D_{\mathcal{F}^l_S} + D_{\mathcal{F}^r_S} = D_y$, such $\varepsilon \ge 0$ exists that $\left|c_S - c_{D_y}\right| \le \varepsilon$. Fulfillment of conditions (i), (ii) guarantees $u(t) = u_h(t)$ and $\mathcal{d}_{h,y} = \max\limits_{u_h} D_y$. Therefore, the framework $\mathcal{S}_{ey}$ at $u_h(t)$ will contain all features characteristic of the function $\varphi(y)$. This implies that $u_h(t) \in \mathrm{S}$, and the system (5) is h_{δ_h}-identified. ■

Some subset $\{u_{h,i}(t)\} \subset \mathrm{U}_h \subseteq \mathrm{U}$ $(i \ge 1)$ which elements have the S-synchronizability property can exist. The framework $\mathcal{S}_{ey,i}\left(u_{h,i}\right)$ with the diameter $D_{y,i}$ of definition range $\mathcal{D}_{y,i}$ corresponds to every $u_{h,i}(t)$. As $u_{h,i}(t) \in \mathrm{S}$ the diameter $D_{y,i}$ have the $\mathcal{d}_{h,\Sigma}$-optimality property. Let the hypothetical framework $\mathcal{S}_{ey}$ of the system (5) have the diameter $\mathcal{d}_{h,\Sigma}$.

Definition 15: The framework $\mathcal{S}_{ey,i}$ have $\mathcal{d}_{h,\Sigma}$-optimality property on the set U_h, if there is such $\varepsilon_\Sigma > 0$ that $\left|\mathcal{d}_{h,\Sigma} - D_{y,i}\right| \le \varepsilon_\Sigma$ $\forall i = \overline{1, \# \mathrm{U}_h}$.

Definition 16: Let the input subset $\{u_{h,i}(t)\} = \mathrm{U}_h \subset \mathrm{U}$ $(i \ge 1)$ exist which elements $u_{h,i}(t) \in \mathrm{S}$, and frameworks $\mathcal{S}_{ey,i}\left(u_{h,i}\right)$ corresponding to them have property $\mathcal{d}_{h,\Sigma}$-optimality. Then frameworks $\mathcal{S}_{ey,i}\left(u_{h,i}\right)$ are indistinguishable on sets $\{u_{h,i}(t)\}$, $J_y\left(u(t) = u_{h,i}(t)\right) = \{y_{h,i}(t)\}$.

From definitions 15, 16 we obtain if the set U_h exists then the h_{δ_h}-identifiability estimation can be determined on any input $u(t) \subset \mathrm{U}_h$.

Remark 8: Here, the case of symmetric nonlinearities is considered. Therefore, remarks made above about the $\mathcal{NS}_{ey}$-framework existence remain are fair. If a nonlinear function does not have the symmetry property, then the research of this problem to be continued needs. Explain it with nonlinearity features. The accounting of these features is possible only under the a priori information on the system or in the further analysis of the framework $\mathcal{S}_{ey}$.

Go to estimation methods h_{δ_h}-identifiability of the system (5) now.

VI. Approach to h_{δ_h}-Identifiability Estimation

Consider the definition problem of an integral indicator which allows making the decision about the system (5) h_{δ_h}-identifiability. It is based on the analysis of framework $\mathcal{S}_{ey}$ properties.

In the nonlinear dynamics and the fractal theory, approaches based on the principle of the covering [21] are applied for the dimension estimation of a framework. Different types of the dimension are proposed. Topological dimension is one of simplest indicators. It estimates the framework geometry and is it always reflects its internal features. Attractors and fractals often are heterogeneous. The heterogeneity reflects an irregularity of point distribution on the framework. Heterogeneity estimations of frameworks obtain on the basis of parameters reflecting of system properties. It is estimated on the basis of the probability analysis of the filling with certain objects of fractal geometrically identical elements. The heterogeneity characterizes discrepancy between probabilities of the fractal filling with the specified bodies and geometrical sizes of the respective areas. Such heterogeneous fractal objects call multifractals [21]. $\mathcal{S}_{ey}$-frame works of dynamic systems with many-valued nonlinearities is the example of heterogeneous frameworks. Section 4 contains examples of heterogeneous frameworks.

Various indicators of the covering (correlation dimension, informational dimension, etc.) are approximate and labor-consuming [21]. They give an assessment of framework fragment geometrical distinction not always. Therefore, we introduce the integral characteristic of the framework which is the distribution function of the variable e on the set $\{y(t)\}$ [15]. Such approach eliminates various a priori assumptions concerning the framework covering local objects. We state to the proposed approach.

Let the framework $\mathcal{S}_{ey}$ be obtained for the system (5). Perform the fragmentation $\mathcal{S}_{ey} = \mathcal{F}^l_{\mathcal{S}_{ey}} \cup \mathcal{F}^r_{\mathcal{S}_{ey}}$ where $\mathcal{F}^l_{\mathcal{S}_{ey}}$, $\mathcal{F}^r_{\mathcal{S}_{ey}}$ are left and right parts of the framework $\mathcal{S}_{ey}$. Fragments $\mathcal{F}^l_{\mathcal{S}_{ey}}$, $\mathcal{F}^r_{\mathcal{S}_{ey}}$ are described by functions $e^l(y), e^r(y)$ where $\left\{e^l\right\} \subseteq \{e\}, \left\{e^r\right\} \subseteq \{e\}$.

Construct frequency distribution functions (histograms) $\mathcal{H}^l, \mathcal{H}^r$ for $\mathcal{F}^l_{\mathcal{S}_{ey}}, \mathcal{F}^r_{\mathcal{S}_{ey}}$. Obtain cumulative

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frequency functions $\mathcal{IH}^l, \mathcal{IH}^r$ on the basis $\mathcal{H}^l, \mathcal{H}^r$. Let $\mathrm{I}_{\mathcal{H}} = \{i\Delta e, i = \overline{1,k}\}$ is the definition range of functions. Present the value range of functions $\mathcal{H}^l, \mathcal{H}^r$ in the form of vectors

$$L(\mathcal{IH}^l) = \left[\mathcal{IH}_1^l, \mathcal{IH}_2^l, \ldots, \mathcal{IH}_k^l\right]^T,$$

$$R(\mathcal{IH}^r) = \left[\mathcal{IH}_1^r, \mathcal{IH}_2^r, \ldots, \mathcal{IH}_k^r\right]^T.$$

Here, k is the quantity of pockets set on $\mathrm{I}_{\mathcal{H}}$, Δe is the pocket size on e.
Apply the model

$$\hat{R} = a_H L(\mathcal{IH}^l) \qquad (13)$$

and determined the parameter a_H having applied the least-squares method.

The model is adequate if the parameter $a_H \in O(1)$ where $O(1)$ is the neighbourhood 1. If the condition $a_H \in O(1)$ is fair, then the system (5) is h_{δ_h}-identifiable and $\mathcal{S}_{ey} \neq \mathcal{NS}_{ey}$. Otherwise, the framework $\mathcal{S}_{ey}$ is insignificant.
So, the following statement is fair.

Statement 2: Let for the system (5): 1) the framework $\mathcal{S}_{ey} = \mathcal{F}_{S_{ey}}^l \cup \mathcal{F}_{S_{ey}}^r$ is defined on the set $\{y(t)\}$ where $\mathcal{F}_{S_{ey}}^l, \mathcal{F}_{S_{ey}}^r$ is framework $\mathcal{S}_{ey}$ fragments; 2) frequency $\mathcal{H}^l, \mathcal{H}^r$ and cumulative $\mathcal{IH}^l, \mathcal{IH}^r$ are distribution functions are known for $\mathcal{F}_{S_{ey}}^l, \mathcal{F}_{S_{ey}}^r$. Then the system (5) is h_{δ_h}-identified if $a_H \in O(1)$.

Definition 17: If the system (5) is h_{δ_h}-identifiable, then the framework $\mathcal{S}_{ey}$ have the dimension $DH_h = a_H$.

Definition 17 shows if $u(t) \in \mathrm{S}$, then dimension for the structurally identified system is approximate to 1. Such value DH_h shows that the framework $\mathcal{S}_{ey}$ does not have complex areas and fragments $\mathcal{F}_{S_{ey}}^l, \mathcal{F}_{S_{ey}}^r$ are structurally identical or homothetic. If $DH_h \notin O(1)$, then it is a sign $\mathcal{NS}_{ey}$-framework or a system with more complicated nonlinearity form. You can supplement results obtained on the basis of statement 2 with the histogram analysis for the framework $\mathcal{S}_{ey}$. Obtain $\mathcal{H}^l, \mathcal{H}^r$ and $\mathcal{IH}^l, \mathcal{IH}^r$ functions and analyze their correlations considering features $\mathcal{S}_{ey}$. Some approaches are proposed in [14, 15].

VII. Examples

Consider the system from section 4 with the input $u_{\mathcal{N}}(t) = 6 - 4\sin(0.5\pi t) + 0.4\sin(0.1\pi t)$. Frame works $\mathcal{S}$, $\mathcal{S}_{ey}$ are showed in Fig. 5 for system steady state. We see that theorem 3 conditions are not satisfied. The sector which has to belong the function $f_e = e(y)$ does not exist for $\mathcal{S}_{ey}$. Therefore, the system is not S-synchronized and $\mathcal{S}_{ey} = \mathcal{NS}_{ey}$. So, the system is not h-identified.

Let $u(t) = 6 - 2\sin(0.1\pi t)$. The system has frameworks showed in Fig. 2. Construct segments $\mathcal{F}_{S_{ey}}^l, \mathcal{F}_{S_{ey}}^r$ for the framework $\mathcal{S}_{ey}$. It can be made on the basis of Fig. 2. Secants for $\mathcal{F}_{S_{ey}}^l, \mathcal{F}_{S_{ey}}^r$ have the form

$$\gamma_S^l = -0.0359y + 0.0792, \quad \gamma_S^r = 0.0211y - 0.0649. \qquad (14)$$

The APPLICATION of theorem 1 will show that $\mathcal{S}_{ey} = \mathcal{NS}_{ey}$ i.e. the system is not h-identifiable. This conclusion is confirmed with diameters $D_{\mathcal{F}_S^l} = 0.478$, $D_{\mathcal{F}_S^r} = 0.792$. h-identifiability estimation results of the system are show in Fig. 6 on the application basis of statement 2. The model (13) has the form

$$\hat{R} = 26 + 0.656L(\mathcal{IH}^l). \qquad (15)$$

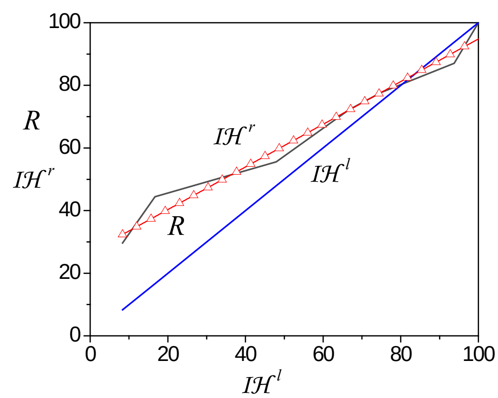

Fig. 6: Estimation h-identifiability of system on basis fragment cumulative distribution function

The model (15) adequacy is 97%. The framework $\mathcal{S}_{ey}$ dimension is 0.65. Analysis results show that the system (5) is structurally non-identifiable on input $u(t) = u_{6,-4}(t)$.

Consider the system from section 4 with

$$u(t) = u_{6,-4}(t) = 6 - 4\sin(0.1\pi t).$$

Corresponding frameworks are shown in Fig. 1. We see the framework have some asymmetry that explains with characteristics of the nonlinear function (Fig. 1b).

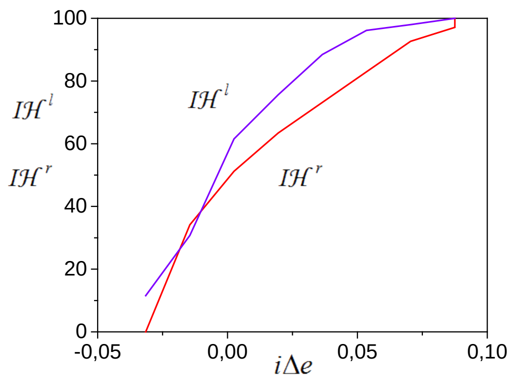

Fig. 7: Functions $\mathcal{IH}^l, \mathcal{IH}^r$

Following parameters of the secants (10) are obtained for fragments $\mathcal{F}^l_{S_{ey}}, \mathcal{F}^r_{S_{ey}}$: $a^l = -0.025$, $a^r = -0.027$. Let $\delta_h = 0.003$. The condition (11) not be satisfied and $S_{ey} \neq \mathcal{NS}_{ey}$. To confirm this inference, determine by functions $\mathcal{IH}^l, \mathcal{IH}^r$. They are shown in Fig. 7, and results of S_{ey}-framework dimension estimation are presented in Fig. 8.

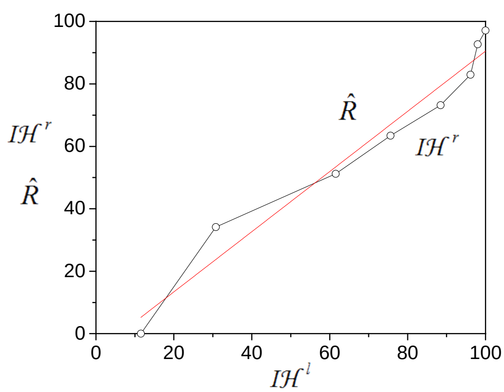

Fig. 8. h-identifiability system estimation on basis cumulative frequency function of fragment with $u_{6,-4}(t)$

The model (13) has the form

$$\hat{R} = -5.845 + 0.96L\left(\mathcal{IH}^l\right),$$

and the coefficient of determination is 96%. Statement 2 conditions are satisfied and the system is h_{δ_h}-identifiable. Framework S_{ey} dimension DH_h is 0.96. Framework fragment diameters are equal $D_{\mathcal{F}^l_S} = 1.16$, $D_{\mathcal{F}^r_S} = 1.43$. The diameter S_{ey} is equal to 2.59. This value coincides with $D_{\mathcal{F}^l_S} + D_{\mathcal{F}^r_S}$. If to choose $\varepsilon_{\mathcal{F}} = 0.4$, then the condition (12) will be satisfied. The distinction between fragment $\mathcal{F}^l_{S_{ey}}, \mathcal{F}^r_{S_{ey}}$ definition ranges depends on properties S_{ey}. The condition 2) theorem 3 is satisfied with $\varepsilon = 0$. Therefore, the system is the SI or h_{δ_h}-identifiable with $u_{6,-4}(t)$, and $u_{6,-4}(t) \in \mathrm{S}$.

VIII. Conclusion

The approach to structural identifiability analysis of nonlinear dynamic systems is proposed under uncertainty. This approach has a difference from methods applied to the structural identifiability estimation of dynamic systems in the parametrical space. We interpret the SI as structural identification possibility of the system nonlinear part. The input has to satisfy excitation the constancy condition in SI problems. This condition differs from requirements to inputs in adaptive systems. We show that the input has to have the synchronization property (S-synchronizability) for SI problem solution. The SI estimation is based on special class framework S_{ey} analysis. Therefore, the S-synchronization has to give to the maximum value of the framework definition range. Non-synchronized input gives to an insignificant framework which does not allow solving the structural identification problem. Therefore, the system is not structurally identifiable. We obtained conditions under which it is possible to estimate the system structural identifiability. An input subset is allocated has the S-synchronizability property, and frameworks S_{ey} not indistinguishable.

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